CONAHCYT
CONSEJO NACIONAL DE HUMANIDADES CIENCIAS Y TECNOLOGÍAS

First Latinamerican Worshop on Electromagnetic Effects on QCD

Two-gluon one-photon vertex in a magnetic field and its explicit one-loop approximation in the intermediate field strength regime

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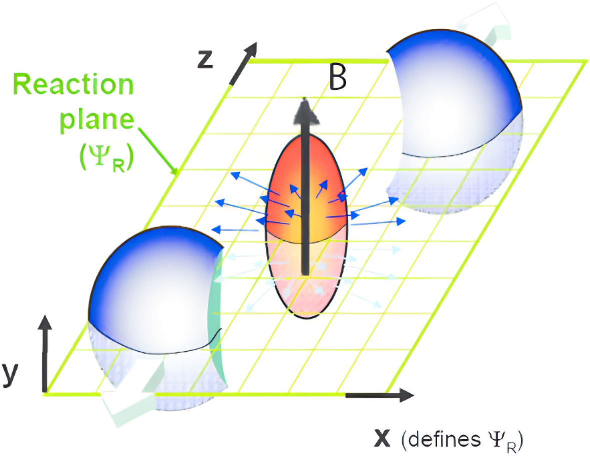
[1]



[1] Exploring dense and cold QCD in magnetic fields. The European Physical Journal A, 52, 1-7.

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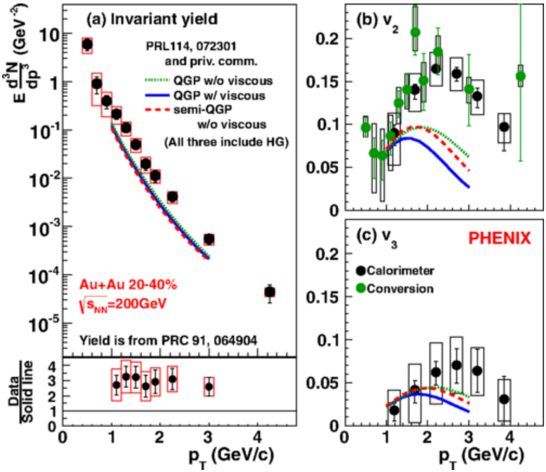

[2] Phys. Rev. C 94, 064901 (2016)

General structure of one-photon two-gluon vertex in a constant magnetic field

We will denote the one-photon two-gluon vertex as

$$\Gamma^{\mu\nu\alpha}_{ab}(p_1, p_2, q). \tag{1}$$

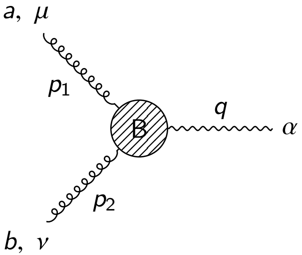

Figure: General representation of the two-gluon one-photon vertex. The shaded blob represents the effect of a magnetic field.

There are three properties that we have to analyze for the structure on the vertex. These are:

- The tensor vertex is transverse with respect the momentum.
- The tensor is symmetric in the gluon indexes.
- The tensor is invariant under CP transformation

Symmetry properties

Transverse property

$$\begin{aligned} p_{1\mu}\Gamma^{\mu\nu\alpha}_{ab}(p_1,p_2,q) &= 0, \\ p_{2\nu}\Gamma^{\mu\nu\alpha}_{ab}(p_1,p_2,q) &= 0, \\ q_{\alpha}\Gamma^{\mu\nu\alpha}_{ab}(p_1,p_2,q) &= 0. \end{aligned} \tag{2}$$

Symmetry under gluon exchange

$$\Gamma^{\mu\nu\alpha}_{ab}(p_1,p_2,q) = \Gamma^{\nu\mu\alpha}_{ba}(p_2,p_1,q). \tag{3}$$

CP invariance

$$\begin{aligned} \hat{C}\ \Gamma^{\mu\nu\alpha}_{ab}\hat{C}^{-1} &= (-1)^3\Gamma^{\mu\nu\alpha}_{ab}, \\ \hat{P}\ \Gamma^{\mu\nu\alpha}_{ab}\hat{P}^{-1} &= (-1)^3\Gamma^{\mu\nu\alpha}_{ab}. \end{aligned} \tag{4}$$

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Ritus Base

We need to translate the previuos properties to a tensor basis. For that propurse, we choose the Ritus base[3] [4]

$$
\begin{aligned}
& q^\mu \\
l_q^\mu &\equiv \hat{F}^{\mu\beta} q_\beta \\
l_q^{*\mu} &\equiv \hat{F}^{*\mu\beta} q_\beta \\
k_q^\mu &\equiv \frac{q^2}{l_q^2} \hat{F}^{\mu\beta} \hat{F}_{\beta\sigma} q^\sigma + q^\mu,
\end{aligned} \tag{5}
$$

where

$$
\hat{F}^{\mu\beta} \equiv F^{\mu\beta}/|B|, \tag{6}
$$

with $F^{\mu\beta}$ the electromagnetic field strength tensor, $F^{*\mu\beta}$ its dual and $|B|$ the strength of the magnetic field.

[3] V. O. Papanyan and V. I. Ritus, Zh. Eksp. Teor. Fiz. 61, 2231 (1971).

[4] V. O. Papanyan and V. I. Ritus, Zh. Eksp. Teor. Fiz. 65, 1756 (1973).

Expansion of the vertex in the Ritus base

We now proceed to express the vertex in terms of the external product of the polarization vectors corresponding to each of the vector particles. We can repeat the above process for each gauge particle an obtain

$$\begin{aligned} \text{photon } \alpha \quad &\longrightarrow \quad q^{\alpha},\ l_q^{\alpha},\ l_q^{*\alpha},\ k_q^{\alpha} \\ \text{gluon } \mu,\ a \quad &\longrightarrow \quad p_{1a}^{\mu},\ l_{p_1 a}^{\mu},\ l_{p_1 a}^{*\mu},\ k_{p_1 a}^{\mu} \\ \text{gluon } \nu,\ b \quad &\longrightarrow \quad p_{2b}^{\nu},\ l_{p_2 b}^{\nu},\ l_{p_2 b}^{*\nu},\ k_{p_2 b}^{\nu}. \end{aligned} \tag{7}$$

Therefore, in general the i-th basis element

$$\Gamma_{ab\ i}^{\mu\nu\alpha}(p_1, p_2, q),$$

corresponds to one of the products of three polarization vectors

$$\begin{aligned} \Gamma_{ab\ i}^{\mu\nu\alpha}(p_1, p_2, q) \quad \in \quad &\left\{ \left[\hat{l}_{p_1 a}^{\mu},\ \hat{l}_{p_1 a}^{*\mu},\ \hat{k}_{p_1 a}^{\mu} \right] \otimes \left[\hat{l}_{p_2 b}^{\nu},\ \hat{l}_{p_2 b}^{*\nu},\ \hat{k}_{p_2 b}^{\nu} \right] \right. \\ \otimes \quad &\left. \left[\hat{l}_q^{\alpha},\ \hat{l}_q^{*\alpha},\ \hat{k}_q^{\alpha} \right] \right\}. \end{aligned} \tag{8}$$

The imposing of the symmetry in gluon indexes, the CP invariane, on-shell conditions and conservation of energy-momentum allow us to write

$$\begin{aligned}\Gamma^{\mu\nu\alpha}_{ab}(p_1,p_2,q)_{\text{on-shell}} &= a_1^{++}\hat{l}^{\mu}_{p_1 a}\hat{l}^{\nu}_{p_2 b}\hat{l}^{\alpha}_{q} + a_2^{++}\hat{l}^{*\mu}_{p_1 a}\hat{l}^{*\nu}_{p_2 b}\hat{l}^{\alpha}_{q} \\ &+ \frac{a_{10}^{++}}{\sqrt{2}}\left(\hat{l}^{\mu}_{p_1 a}\hat{l}^{*\nu}_{p_2 b} + \hat{l}^{*\mu}_{p_1 a}\hat{l}^{\nu}_{p_2 b}\right)\hat{l}^{*\alpha}_{q}. \qquad (9)\end{aligned}$$

One-loop approximation for the effective one-photon two-gluon vertex

At leading order in the strong α_s and electromagnetic α_{em} couplings, the scattering process involving two gluons and a photon, either gluon fusion or splitting, is depicted in Figure 2

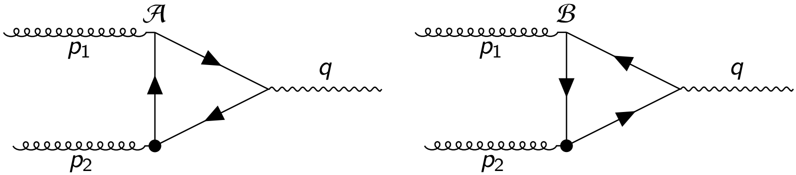

Figure: One-loop diagrams contributing to the two-gluon one-photon vertex. Diagram $\mathcal{B}$ represents the charge conjugate of diagram $\mathcal{A}$. The four-momentum vectors are chosen such that $q = p_1 + p_2$.

Schwinger phase

Each internal line corresponds to a fermion propagator in the presence of a magnetic field, which can be written as

$$S(x,x') = \Phi(x,x') \int \frac{d^4 p}{(2\pi)^4} e^{-ip\cdot(x-x')} S(p), \tag{10}$$

where $\Phi(x,x')$ is the Schwinger's phase factor given by

$$\Phi(x,x') = \exp\left\{ iq_f \int_{x'}^{x} d\xi^\mu \left[A^\mu + \frac{1}{2} F_{\mu\nu}(\xi - x')^\nu \right] \right\}, \tag{11}$$

where q_f is the charge of the quark with flavor f.

The translationally invariant part of the propagator can be written, using Schwinger's proper time representation, as

$$\begin{aligned} S(p) &= \int_0^\infty \frac{ds}{\cos(q_f Bs)} e^{is\left(p_\parallel^2 + p_\perp^2 \frac{\tan(q_f Bs)}{q_f Bs} - m_f^2 + i\epsilon\right)} \\ &\times \left[e^{iq_f Bs\Sigma_3}\left(m_f + \not{p}_\parallel\right) + \frac{\not{p}_\perp}{\cos(q_f Bs)}\right], \end{aligned} \tag{12}$$

where m_f is the mass of the quark with flavor f and $\Sigma_3 = i\gamma_1\gamma_2$.

General expression

The explicit expression for the sum of the diagrams in Fig. 2 is written as

$$\begin{aligned}
\Gamma^{\mu\nu\alpha}_{ab} &= -ig^2 q_f \int d^4x d^4y d^4z \int \frac{d^4r}{(2\pi)^4}\frac{d^4s}{(2\pi)^4}\frac{d^4t}{(2\pi)^4} \\
&\times e^{-it\cdot(y-x)} e^{-is\cdot(x-z)} e^{-ir\cdot(z-y)} e^{-ip_1\cdot z} e^{-ip_2\cdot y} e^{iq\cdot x} \\
&\times \Big\{ \mathrm{Tr}\left[\gamma_\alpha S(s)\gamma_\mu t_a S(r)\gamma_\nu t_b S(t)\right] \\
&+ \mathrm{Tr}\left[\gamma_\alpha S(t)\gamma_\nu t_b S(r)\gamma_\mu t_a S(s)\right] \Big\} \\
&\times \Phi(x,y)\Phi(y,z)\Phi(z,x), \qquad (13)
\end{aligned}$$

where g is the quark-gluon coupling, $t^a = \lambda^a/2$, $t^b = \lambda^b/2$ with λ^a and λ^b being Gell-Mann matrices.

After a lengthy but straightforward calculation we get

$$
\begin{aligned}
\Gamma^{\mu\nu\alpha}_{ab} &= -i\frac{g^2 q_f^2 B}{(2\pi^2)}\mathrm{Tr}[t_a t_b]\delta^4(p_1+p_2-q)\int_0^\infty \frac{ds_1 ds_2 ds_3}{c_1^2 c_2^2 c_3^2}\left(\frac{e^{-ism_f^2}}{s}\right) \\
&\times \left(\frac{1}{t_1 t_2 t_3 - t_1 - t_2 - t_3}\right) e^{-\frac{i}{s}\left(s_1 s_3 \omega_{p_1}^2 + s_2 s_3 \omega_{p_2}^2 + s_1 s_2 \omega_q^2\right)\frac{q_\perp^2}{\omega_q^2}} \\
&\times e^{-\frac{i}{\omega_q^2}\frac{q_\perp^2}{q_f B}\left(\frac{1}{t_1 t_2 t_3 - t_1 - t_2 - t_3}\right)\left(t_1 t_3 \omega_{p_1}^2 + t_2 t_3 \omega_{p_2}^2 + t_1 t_2 \omega_q^2\right)} \\
&\times \sum_{j=1}^{19}\left(T^{\mu\nu\alpha}_{\mathcal{A}j} + T^{\mu\nu\alpha}_{\mathcal{B}j}\right), \qquad (14)
\end{aligned}
$$

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where

$$\begin{aligned} c_j &\equiv \cos(q_f B s_j), \\ t_j &\equiv \tan(q_f B s_j), \\ e_j &\equiv c_j e^{i\text{sign}(q_f B)|q_f B| s_j \Sigma_3}, \end{aligned} \tag{15}$$

and $s = s_1 + s_2 + s_3$. Eq.(14) represents the exact one-loop result for the two-gluons one-photon vertex in the presence of a constant magnetic field of arbitrary strength. Its large field approximation has been already explored[5] [6]. Now, we are interested in the region $m_f^2 < |q_f B| < q_\perp^2$.

[5] Phys. Rev. D 96, 119901 (2017)

[6] Phys. Rev. C 106, 064905

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We can perform a change of variable $s_i \rightarrow sv_i$, where $v1 + v2 + v3 = 1$. Then, we can write the integral as

$$\sum_{j=1}^{19} \int ds\, dv_1 dv_2\, K\, e^{i\text{Arg}} \left(T^{\mu\nu\alpha}_{\mathcal{A}j} + T^{\mu\nu\alpha}_{\mathcal{B}j} \right), \tag{16}$$

$$K = -i\frac{g^2 q_f^2 B}{(2\pi^2)} \text{Tr}[t_a t_b] \frac{s}{c_1^2 c_2^2 c_3^2} \left(\frac{1}{t_1 t_2 t_3 - t_1 - t_2 - t_3} \right) \tag{17}$$

$$\begin{aligned} \text{Arg} = -s(m_f^2) - s \Big(& v_1(1 - v_1 - v_2)\omega_{p_1}^2 \\ & + v_2(1 - v_1 - v_2)\omega_{p_2}^2 + v_1 v_2 \omega_q^2 \Big) \frac{q_\perp^2}{\omega_q^2} \\ & - \frac{1}{\omega_q^2} \frac{q_\perp^2}{|q_f B|} \left(\frac{1}{t_1 t_2 t_3 - t_1 - t_2 - t_3} \right) \\ & \times \left(t_1 t_3 \omega_{p_1}^2 + t_2 t_3 \omega_{p_2}^2 + t_1 t_2 \omega_q^2 \right). \end{aligned} \tag{18}$$

Aproximation

Now, let us analyze the Arg term,

$$\mathrm{Arg} = -sm^2 - s\left(v_1 v_3 \omega_{p_1}^2 + v_2 v_3 \omega_{p_2}^2 + v_1 v_2 \omega_q^2\right)\frac{q_\perp^2}{\omega_q^2} - \frac{1}{\omega_q^2}\frac{q_\perp^2}{qB}\left(\frac{1}{t_1 t_2 t_3 - t_1 - t_2 - t_3}\right)\left(t_1 t_3 \omega_{p_1}^2 + t_2 t_3 \omega_{p_2}^2 + t_1 t_2 \omega_q^2\right). \tag{19}$$

We are going to expand to leading order the second part of the left side,

$$\mathrm{Arg} = -m^2 s + q_\perp^2 qB^2 s^3 F(v_1, v_2), \tag{20}$$

where

$$F(v_1, v_2) = \frac{1}{3\omega_q^2}\left(v_1^4 \omega_{p_1}^2 + 2v_1^3\left((v_2-1)\omega_{p_1}^2 + v_2(\omega_2^2 - \omega_q^2)\right) + v_1^2\left((3v_2^2 - 3v_2 + 1)\omega_{p_1}^2 + 3(v_2-1)v_2(\omega_2^2 - \omega_q^2)\right) + v_1 v_2\left(2v_2^2 - 3v_2 + 1\right)\left(\omega_{p_1}^2 + \omega_2^2 - \omega_q^2\right) + (v_2-1)^2 v_2^2 \omega_2^2\right). \tag{21}$$

Now, it is convenient to take $s \longrightarrow \frac{x}{qB}$

$$\text{Arg} = -\frac{m^2}{qB}x + i\frac{q_\perp^2}{qB}x^3 F(v_1, v_2) \tag{22}$$

We can further simplify the expressions recalling that in the intermediate field regime, terms proportional to m_f^2 can be neglected.

$$\begin{aligned} K &= i\frac{g^2 q_f}{16\pi^2}\,\text{Tr}[t_a t_b]\,x\csc(x) \\ &\quad \times \sec(v_1 x)\sec(v_2 x)\sec(x(1 - v_1 - v_2)), \end{aligned} \tag{23}$$

$$\text{Arg} = \frac{q_\perp^2}{|q_f B|}x^3 F(v_1, v_2). \tag{24}$$

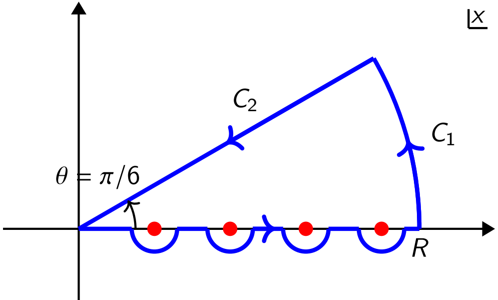
Choose of integration path

Contracting Eq. (14) with the elements of the Ritus base, we obtain three types of integrals

$$J_1 \equiv \int dv_1 dv_2 dx \ \csc^2(x) e^{i\mathrm{Arg}} G_1(x, v_1, v_2), \tag{25}$$

$$J_2 \equiv \int dv_1 dv_2 dx \ \csc^3(x) e^{i\mathrm{Arg}} G_2(x, v_1, v_2), \tag{26}$$

$$J_3 \equiv \int dv_1 dv_2 dx \ \csc^4(x) e^{i\mathrm{Arg}} G_3(x, v_1, v_2). \tag{27}$$



In the c1 trajectory, $x \longrightarrow Me^{i\theta}$, then

$$\mathrm{Arg} = i\frac{q_\perp^2}{qB}M^3 e^{i3\theta}F(v_1, v_2) \tag{28}$$

In the region where $0 < \theta < \frac{\pi}{6}$, the real part of Arg is always negative. So, in the limit where M is big, e $e^{\mathrm{Arg}} \longrightarrow 0$.
In the other hand, in c2 trajectory, $x \longrightarrow^{i\frac{\pi}{6}}$, so

$$\mathrm{Arg} = i\frac{q_\perp^2}{qB}\tau^3 e^{i\pi/2}F(v_1, v_2) = -\frac{q_\perp^2}{qB}\tau^3 F(v_1, v_2) \tag{29}$$

Therefore, the integral over x on the real axis can be written as

$$\begin{aligned}
&\int_0^\infty dx \csc^{j+1}(x) e^{i\mathrm{Arg}} G_j(x, v_1, v_2) \\
&= \sum \mathrm{Res}\left(\csc^{j+1}(x) e^{i\mathrm{Arg}} G_j(x, v_1, v_2), n\pi\right) \\
&- \int_0^\infty d\tau \csc^{j+1}(\tau e^{i\pi/6}) \\
&\times e^{-\frac{q_\perp^2}{q_f B}\tau^3 F(v_1, v_2)} G_j(\tau e^{i\pi/6}, v_1, v_2).
\end{aligned} \tag{30}$$

Stationary phase approximation

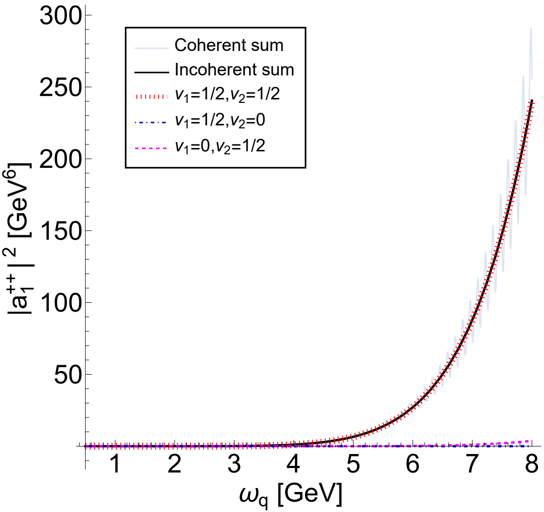
The strategy to estimate the integral over v_1, v_2 follows as this: Let us consider $\vec{v} = (v_1, v_2)$, then we have an integral in the form

$$\int dv_1 dv_2 \, \mathcal{F}(v_1, v_2) e^{i\chi\psi(v_1, v_2)}, \tag{31}$$

where the parameter $\chi = \frac{q_\perp^2}{q_f B}$ and the phase ψ has a set of critical points $\Upsilon = (v_1^0, v_2^0)$ where $\nabla\psi(v_1^0, v_2^0) = 0$. Therefore, we can make use of the stationary phase approximation, which gives the asymptotic behaviour of this integral for $\chi \gg 1$, as

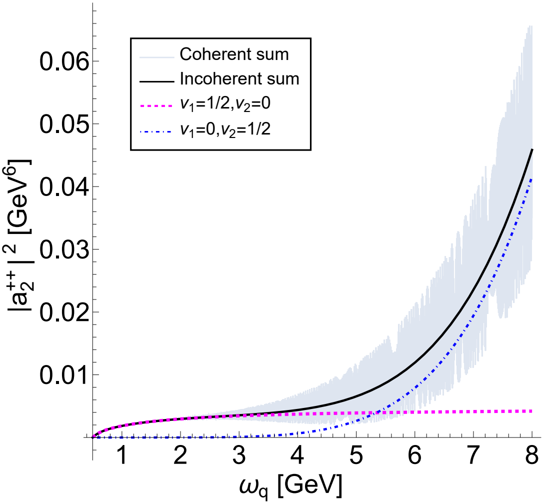
$$\begin{aligned} \int d^2v \, \mathcal{F}(\mathbf{v}) e^{i\chi\psi(\mathbf{v})} \approx & \sum_{\vec{v}^0 \in \Upsilon} e^{i\chi\psi(\vec{v}_0)} \left|\det\left(\text{Hess}(\psi(\vec{v}^0))\right)\right|^{-1/2} \\ & \times \; e^{i\frac{\pi}{4}\text{sign}(\text{Hess}(\psi(\vec{v}_0)))} \left(\frac{2\pi}{\chi}\right)^{2/2} \mathcal{F}(\vec{v}_0) \\ & + \mathcal{O}(\chi^{-1/2}), \end{aligned} \tag{32}$$

Coefficient $|a_1^{++}|^2$ as function of the photon energy



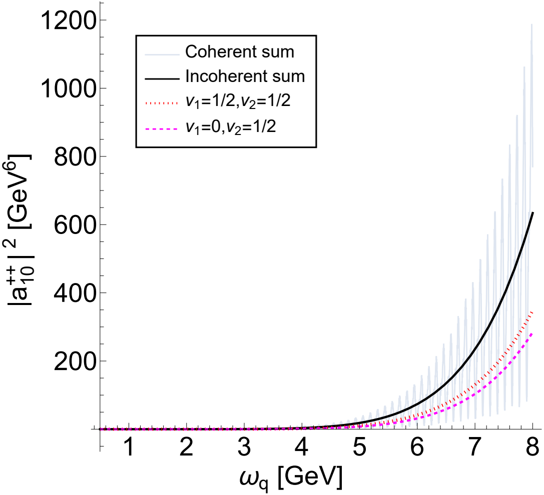
- $\theta = \pi/2$.
- $\omega_{p_1} = 0.5$ GeV.
- $|eB| = m_\pi^2$.

Coefficient $|a_2^{++}|^2$ as function of the photon energy



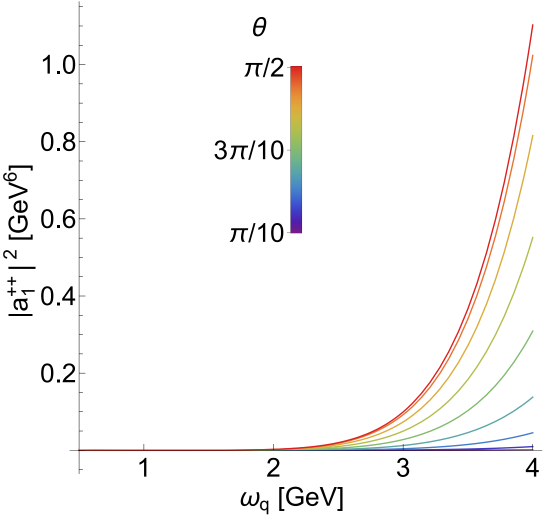
- $\theta = \pi/2$.
- $\omega_{p_1} = 0.5$ GeV.
- $|eB| = m_\pi^2$.

Coefficient $|a_{10}^{++}|^2$ as function of the photon energy



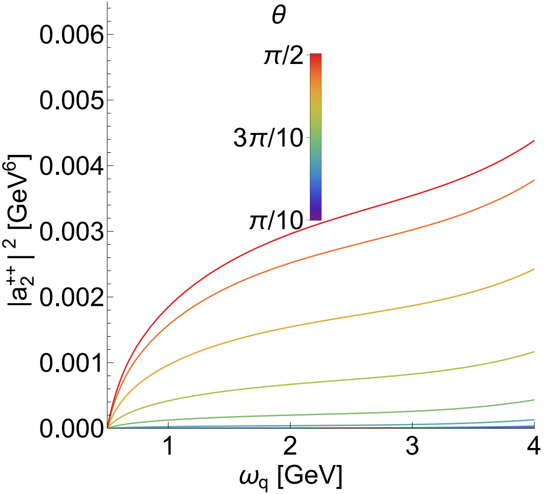
- $\theta = \pi/2$.
- $\omega_{p_1} = 0.5$ GeV.
- $|eB| = m_\pi^2$.
- The contribution from the three light quark flavors u, d, s, is accounted for.

Coefficient $|a_1^{++}|^2$ as function of the photon energy



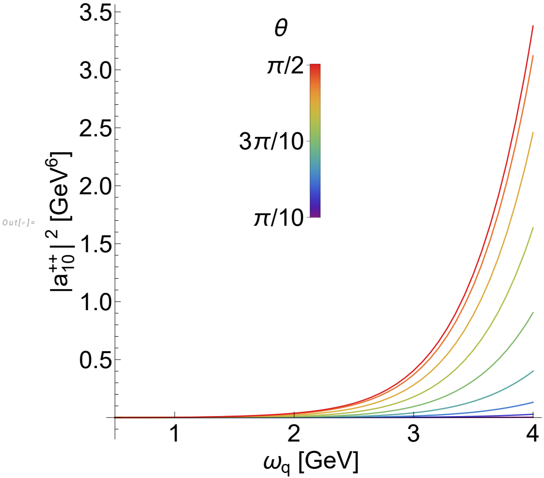
- $\omega_{p_1} = 0.5.$
- $|eB| = m_\pi^2.$

Coefficient $|a_2^{++}|^2$ as function of the photon energy



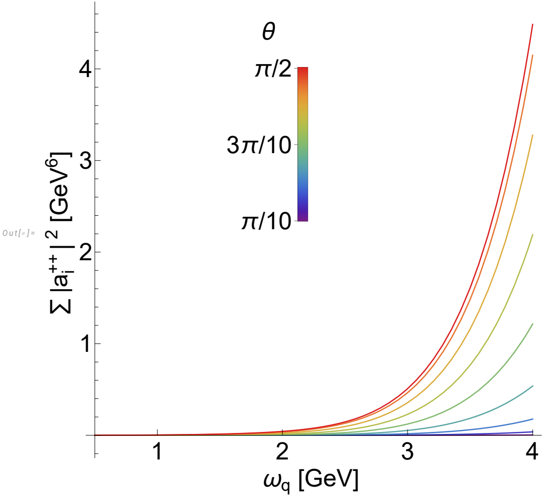
- $\omega_{p_1} = 0.5.$
- $|eB| = m_\pi^2.$

Coefficient $|a_{10}^{++}|^2$ as function of the photon energy



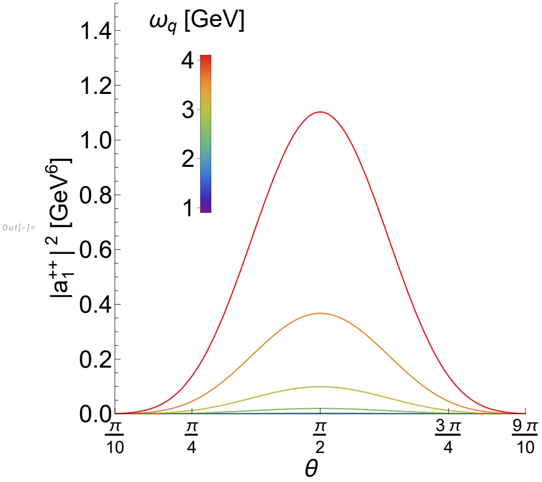
- $\omega_{p_1} = 0.5.$
- $|eB| = m_\pi^2.$

$\sum |a_i^{++}|^2$ as function of the photon energy



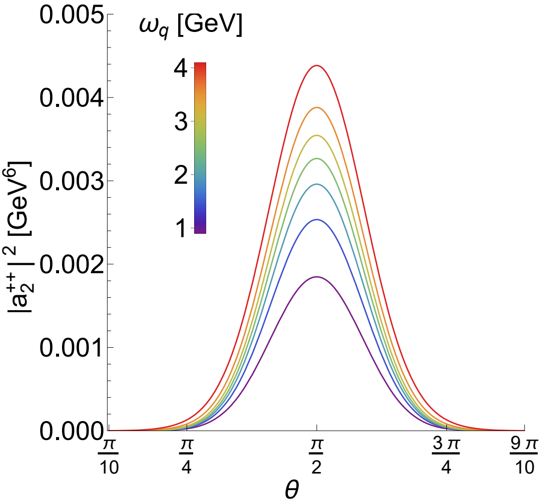
- $\omega_{p_1} = 0.5.$
- $|eB| = m_\pi^2.$

Coefficient $|a_1^{++}|^2$ as function of the angle



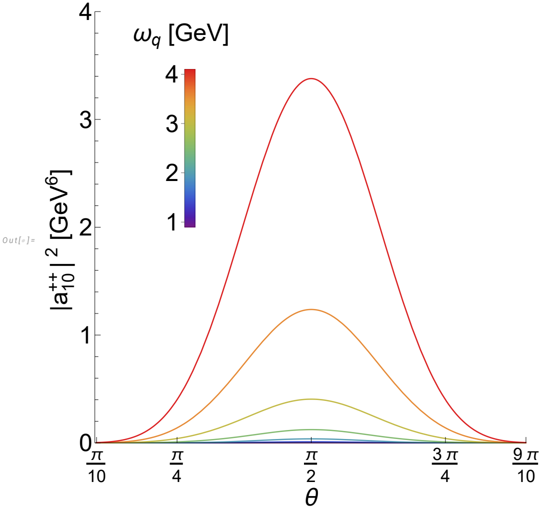
- $\omega_{p_1} = 0.5$ GeV.
- $|eB| = m_\pi^2$.

Coefficient $|a_2^{++}|^2$ as function of the angle



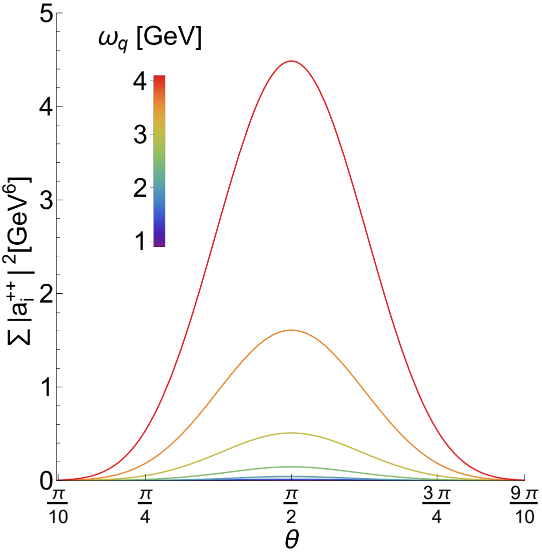
- $\omega_{p_1} = 0.5$ GeV.
- $|eB| = m_\pi^2$.

Coefficient $|a_{10}^{++}|^2$ as function of the angle



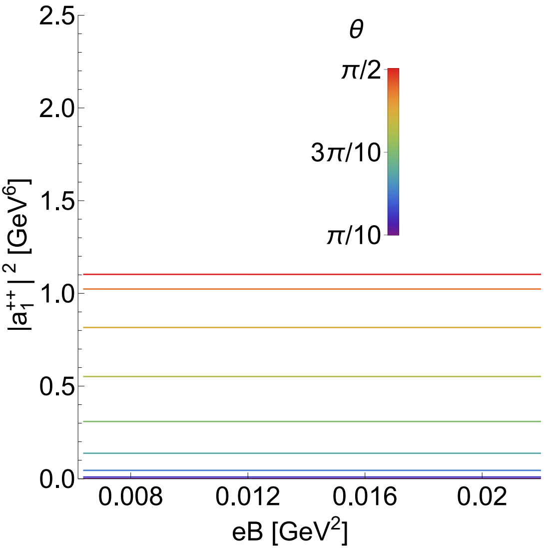
- $\omega_{p_1} = 0.5$ GeV.
- $|eB| = m_\pi^2$.

$\sum |a_i^{++}|^2$ as function of the angle



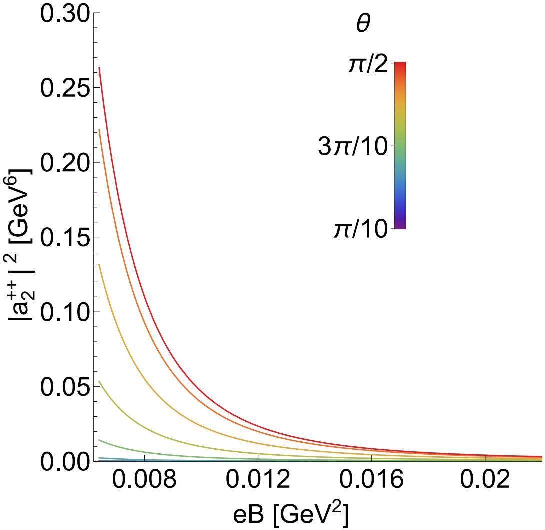
- $\omega_{p_1} = 0.5$ GeV.
- $|eB| = m_\pi^2$.

Coefficient $|a_1^{++}|^2$ as function of $|eB|$



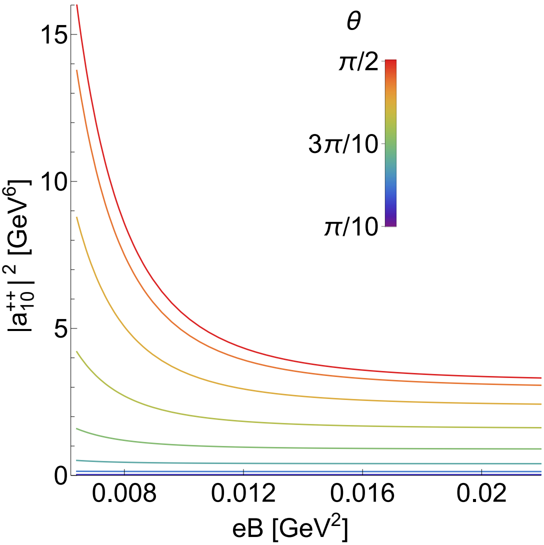
- $\omega_q = 4$ GeV.
- $\omega_{p_1} = 0.5$ GeV.

Coefficient $|a_2^{++}|^2$ as function of $|eB|$



- $\omega_q = 4$ GeV.
- $\omega_{p_1} = 0.5$ GeV.

Coefficient $|a_{10}^{++}|^2$ as function of $|eB|$



- $\omega_q = 4$ GeV.
- $\omega_{p_1} = 0.5$ GeV.

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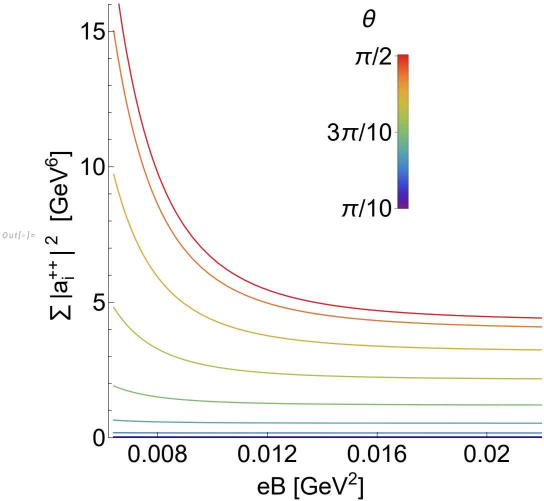
$\sum |a_i^{++}|^2$ as function of $|eB|$



- $\omega_q = 4$ GeV.
- $\omega_{p_1} = 0.5$ GeV.

Summary

- The vertex tensor was expanded in terms of the Ritus base.
- The symmetries of the vertex were used to reduce from 27 to 3 the basis coefficients.
- The approximation to intermediate field was performed to each coefficient.
- The behavior of the coefficient as function of different was analysed.

Appendix A: Explicit form of traces

$$
\begin{aligned}
T^{\mu\nu\alpha}_{\mathcal{A}1} + T^{\mu\nu\alpha}_{\mathcal{B}1} &= \mathrm{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha \mathcal{A}_b \gamma^\nu \mathcal{A}_c] + \mathrm{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu \mathcal{B}_b \gamma^\alpha \mathcal{B}_a], \\
T^{\mu\nu\alpha}_{\mathcal{A}2} + T^{\mu\nu\alpha}_{\mathcal{B}2} &= m_f^2 \left\{ \mathrm{Tr}[\gamma^\mu e_1 \gamma^\alpha e_2 \gamma^\nu \mathcal{A}_c] + \mathrm{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu e_2 \gamma^\alpha e_1] \right\}, \\
T^{\mu\nu\alpha}_{\mathcal{A}3} + T^{\mu\nu\alpha}_{\mathcal{B}3} &= m_f^2 \left\{ \mathrm{Tr}[\gamma^\mu e_1 \gamma^\alpha \mathcal{A}_b \gamma^\nu e_3] + \mathrm{Tr}[\gamma^\mu e_3 \gamma^\nu \mathcal{B}_b \gamma^\alpha e_1] \right\}, \\
T^{\mu\nu\alpha}_{\mathcal{A}4} + T^{\mu\nu\alpha}_{\mathcal{B}4} &= m_f^2 \left\{ \mathrm{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha e_2 \gamma^\nu e_3] + \mathrm{Tr}[\gamma^\mu e_3 \gamma^\nu e_2 \gamma^\alpha \mathcal{B}_a] \right\}, \\
T^{\mu\nu\alpha}_{\mathcal{A}5} + T^{\mu\nu\alpha}_{\mathcal{B}5} &= \frac{i}{s} \left\{ \mathrm{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha e_2 \gamma^\nu_\parallel e_3] + \mathrm{Tr}[\gamma^\mu e_3 \gamma^\nu_\parallel e_2 \gamma^\alpha \mathcal{B}_a] \right\}, \\
T^{\mu\nu\alpha}_{\mathcal{A}6} + T^{\mu\nu\alpha}_{\mathcal{B}6} &= \frac{i}{s} \left\{ \mathrm{Tr}[\gamma^\mu_\parallel e_1 \gamma^\alpha \mathcal{A}_b \gamma^\nu e_3] + \mathrm{Tr}[\gamma^\mu_\parallel e_3 \gamma^\nu \mathcal{B}_b \gamma^\alpha e_1] \right\}, \\
T^{\mu\nu\alpha}_{\mathcal{A}7} + T^{\mu\nu\alpha}_{\mathcal{B}7} &= \frac{i}{s} \left\{ \mathrm{Tr}[\gamma^\mu e_1 \gamma^\alpha_\parallel e_2 \gamma^\nu \mathcal{A}_c] + \mathrm{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu e_2 \gamma^\alpha_\parallel e_1] \right\}, \\
T^{\mu\nu\alpha}_{\mathcal{A}8} + T^{\mu\nu\alpha}_{\mathcal{B}8} &= -\frac{i}{s} \left\{ \mathrm{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha e_2 \gamma^\nu e_3] + \mathrm{Tr}[\gamma^\mu e_3 \gamma^\nu e_2 \gamma^\alpha \mathcal{B}_a] \right\}, \\
T^{\mu\nu\alpha}_{\mathcal{A}9} + T^{\mu\nu\alpha}_{\mathcal{B}9} &= -\frac{i}{s} \left\{ \mathrm{Tr}[\gamma^\mu e_1 \gamma^\alpha \mathcal{A}_b \gamma^\nu e_3] + \mathrm{Tr}[\gamma^\mu e_3 \gamma^\nu \mathcal{B}_b \gamma^\alpha e_1] \right\},
\end{aligned}
$$

$$T^{\mu\nu\alpha}_{\mathcal{A}10} + T^{\mu\nu\alpha}_{\mathcal{B}10} = -\frac{i}{s}\left\{\mathrm{Tr}[\gamma^\mu e_1 \gamma^\alpha e_2 \gamma^\nu \mathcal{A}_c] + \mathrm{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu e_2 \gamma^\alpha e_1]\right\},$$

$$\mathrm{T}^{\mu\nu\alpha}_{\mathcal{A}11} + T^{\mu\nu\alpha}_{\mathcal{B}11} = \frac{iq_f B}{t}\left\{\mathrm{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha \gamma^\nu] + \mathrm{Tr}[\gamma^\mu \gamma^\nu \gamma^\alpha \mathcal{B}_a]\right\},$$

$$\mathrm{T}^{\mu\nu\alpha}_{\mathcal{A}12} + T^{\mu\nu\alpha}_{\mathcal{B}12} = \frac{iq_f B}{t}\left\{\mathrm{Tr}[\gamma^\mu \gamma^\alpha \mathcal{A}_b \gamma^\nu] + \mathrm{Tr}[\gamma^\mu \gamma^\nu \mathcal{B}_b \gamma^\alpha]\right\},$$

$$\mathrm{T}^{\mu\nu\alpha}_{\mathcal{A}13} + T^{\mu\nu\alpha}_{\mathcal{B}13} = \frac{iq_f B}{t}\left\{\mathrm{Tr}[\gamma^\mu \gamma^\alpha \gamma^\nu \mathcal{A}_c] + \mathrm{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu \gamma^\alpha]\right\},$$

$$\mathrm{T}^{\mu\nu\alpha}_{\mathcal{A}14} + T^{\mu\nu\alpha}_{\mathcal{B}14} = -\frac{iq_f B}{t}\left\{\mathrm{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha \gamma^\nu_\perp] + \mathrm{Tr}[\gamma^\mu \gamma^\nu_\perp \gamma^\alpha \mathcal{B}_a]\right\},$$

$$\mathrm{T}^{\mu\nu\alpha}_{\mathcal{A}15} + T^{\mu\nu\alpha}_{\mathcal{B}15} = -\frac{iq_f B}{t}\left\{\mathrm{Tr}[\gamma^\mu_\perp \gamma^\alpha \mathcal{A}_b \gamma^\nu] + \mathrm{Tr}[\gamma^\mu_\perp \gamma^\nu \mathcal{B}_b \gamma^\alpha]\right\},$$

$$\mathrm{T}^{\mu\nu\alpha}_{\mathcal{A}16} + T^{\mu\nu\alpha}_{\mathcal{B}16} = -\frac{iq_f B}{t}\left\{\mathrm{Tr}[\gamma^\mu \gamma^\alpha_\perp \gamma^\nu \mathcal{A}_c] + \mathrm{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu \gamma^\alpha_\perp]\right\},$$

$$
\begin{aligned}
T^{\mu\nu\alpha}_{\mathcal{A}17} + T^{\mu\nu\alpha}_{\mathcal{B}17} &= \frac{iq_f B\ t_1}{2t}\{\mathrm{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha \gamma^\beta_\perp \gamma^\nu \gamma^\sigma_\perp]\hat{F}_{\beta\sigma} \\
&+ \mathrm{Tr}[\gamma^\mu \gamma^\sigma_\perp \gamma^\nu \gamma^\beta_\perp \gamma^\alpha \mathcal{B}_a]\hat{F}_{\sigma\beta}\}, \\
T^{\mu\nu\alpha}_{\mathcal{A}18} + T^{\mu\nu\alpha}_{\mathcal{B}18} &= -\frac{iq_f B\ t_2}{2t}\{\mathrm{Tr}[\gamma^\mu \gamma^\beta_\perp \gamma^\alpha \mathcal{A}_b \gamma^\nu \gamma^\sigma_\perp]\hat{F}_{\beta\sigma} \\
&+ \mathrm{Tr}[\gamma^\mu \gamma^\sigma_\perp \gamma^\nu \mathcal{B}_b \gamma^\alpha \gamma^\beta_\perp]\hat{F}_{\sigma\beta}\}, \\
T^{\mu\nu\alpha}_{\mathcal{A}19} + T^{\mu\nu\alpha}_{\mathcal{B}19} &= \frac{iq_f B\ t_3}{2t}\{\mathrm{Tr}[\gamma^\mu \gamma^\beta_\perp \gamma^\alpha \gamma^\sigma_\perp \gamma^\nu \mathcal{A}_c]\hat{F}_{\beta\sigma} \\
&+ \mathrm{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu \gamma^\sigma_\perp \gamma^\alpha \gamma^\beta_\perp]\hat{F}_{\sigma\beta}\}. \qquad (33)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{A}_a &= -\left(\frac{s_3\omega_{p_1}+s_2\omega_q}{s\omega_q}\right)\not{q}_{\parallel} e_1 + \frac{(t_3\omega_{p_1}+t_2\omega_q)\not{q}_{\perp} - t_2 t_3\omega_{p_2}\gamma^{\sigma}\hat{F}_{\sigma\beta}q_{\perp}^{\beta}}{t\omega_q}\\
\mathcal{A}_b &= \left(\frac{s_1\omega_q+s_3\omega_{p_2}}{s\omega_q}\right)\not{q}_{\parallel} e_2 - \frac{(t_3\omega_{p_2}+t_1\omega_q)\not{q}_{\perp} + t_1 t_3\omega_{p_1}\gamma^{\sigma}\hat{F}_{\sigma\beta}q_{\perp}^{\beta}}{t\omega_q}\\
\mathcal{A}_c &= \left(\frac{s_1\omega_{p_1}-s_2\omega_{p_2}}{s\omega_q}\right)\not{q}_{\parallel} e_3 + \frac{(-t_1\omega_{p_1}+t_2\omega_{p_2})\not{q}_{\perp} + t_1 t_3\omega_q\gamma^{\sigma}\hat{F}_{\sigma\beta}q_{\perp}^{\beta}}{t\omega_q}\\
\mathcal{B}_a &= \left(\frac{s_3\omega_{p_1}+s_2\omega_q}{s\omega_q}\right)\not{q}_{\parallel} e_1 - \frac{(t_3\omega_{p_1}+t_2\omega_q)\not{q}_{\perp} + t_2 t_3\omega_{p_2}\gamma^{\sigma}\hat{F}_{\sigma\beta}q_{\perp}^{\beta}}{t\omega_q}\\
\mathcal{B}_b &= -\left(\frac{s_1\omega_q+s_3\omega_{p_2}}{s\omega_q}\right)\not{q}_{\parallel} e_2 + \frac{(t_3\omega_{p_2}+t_1\omega_q)\not{q}_{\perp} - t_1 t_3\omega_{p_1}\gamma^{\sigma}\hat{F}_{\sigma\beta}q_{\perp}^{\beta}}{t\omega_q}\\
\mathcal{B}_c &= -\left(\frac{s_1\omega_{p_1}-s_2\omega_{p_2}}{s\omega_q}\right)\not{q}_{\parallel} e_3 + \frac{(t_1\omega_{p_1}-t_2\omega_{p_2})\not{q}_{\perp} + t_1 t_3\omega_q\gamma^{\sigma}\hat{F}_{\sigma\beta}q_{\perp}^{\beta}}{t\omega_q}.\\
t &= t_1 t_2 t_3 - t_1 - t_2 - t_3
\end{aligned}
$$