Neutral pion screening mass in a magnetized medium

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ICN, UNAM, México

23/07/2024

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[1]A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Overview

- Why is strong-field physics important?
- Interplay between strong magnetic fields and QCD.
- Linear sigma model with quarks (LSMq)
- Analysis of the neutral pion screening mass
- Results
- Summary and perspectives

Strong-(Electromagnetic)Field Physics

- High energy physics (heavy ion collisions)
- Astrophysics (neutron stars)

Strong-Field Physics

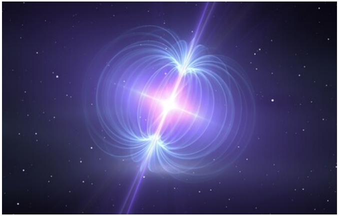


Figure: magnetar ($10^{13} - 10^{15}$ G.)

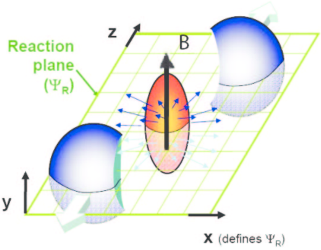


Figure: Heavy ion collisions ($10^{18} - 10^{19}$ G.)

Comparison of different magnetic fields

- Earth's magnetic field: $0.6\ G$
- Common comercial magnet: $100\ G$
- Strongest magnetic field produced in labs: $4.5 \times 10^5\ G$
- Magnetars: $(10^{13} - 10^{15}) G$
- Heavy ion collisions: $(10^{18} - 10^{19})\ G$

Interplay between strong magnetic fields and QCD

- Magnetic catalysis at zero temperature.
- Inverse magnetic catalysis around T_C.
- Chiral magnetic effect.
- Electromagnetic fields provide a powerful probe to explore the properties of the QCD vacuum.

Why screening masses of neutral pions?

Since the dynamics of chiral symmetry breaking is dominated by pions, the lightest of all quark-antiquark bound states, it then becomes important to explore how the pion mass is affected by the presence of magnetic fields.

In this work we will study the effect of a constant magnetic field B in the screening mass of neutral pions.

Screening Mass

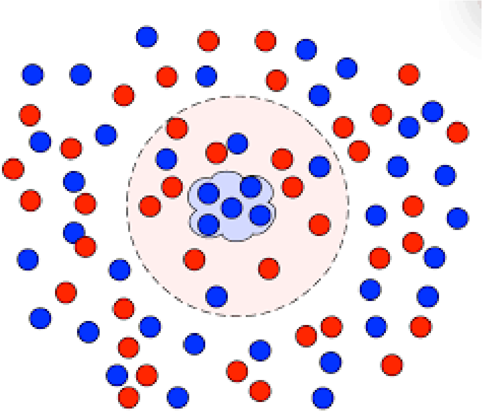
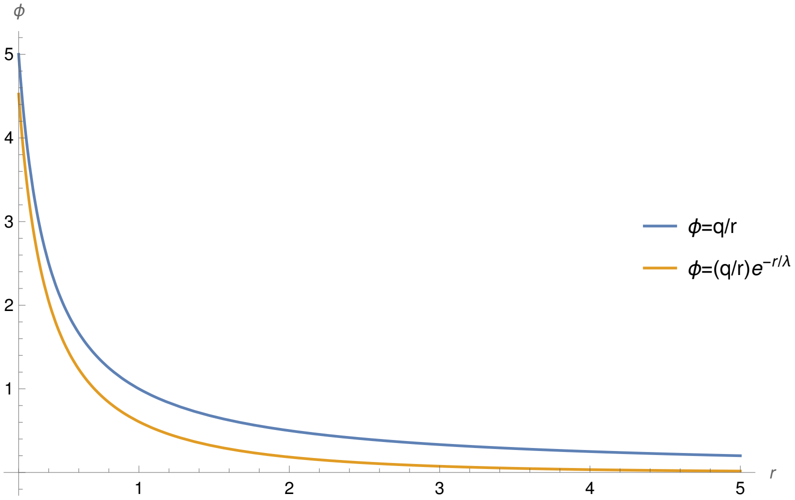


Figure: Debye mass (inverse of Debye length)

Screening Mass



Pole and screening masses ($B \neq 0$, $T = 0$)

- Pole mass

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_3^2=p_\perp^2=0} = 0$$

- Screening mass B breaks Lorentz invariance and defines $\parallel$ and $\perp$
  - Longitudinal

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2=p_\perp^2=0} = 0$$

  - Transverse

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2=p_3^2=0} = 0$$

Pole and screening masses (definitions)

$$E^2 = u_\perp^2 \mathbf{q}_\perp^2 + u_\parallel^2 q_3^2 + m_{\pi^0,pole}^2$$

$$m_{\pi^0,scr.\perp} = \frac{m_{\pi^0,pole}}{u_\perp}$$

$$m_{\pi^0,scr.\parallel} = \frac{m_{\pi^0,pole}}{u_\parallel}$$

$$u_\perp \equiv u_\perp(B,T) \ , \ \ u_\parallel \equiv u_\parallel(T)$$

Pole and screening masses (special cases)

- **(i)T=0, B=0**

$$u_\perp = u_\parallel = 1$$

$$m_{\pi^0,pole} = m_{\pi^0,scr,\parallel} = m_{\pi^0,scr,\perp}$$

- **(ii)T≠0, B=0**

$$u_\perp = u_\parallel = u \neq 1$$

$$m_{\pi^0,pole} \neq m_{\pi^0,scr,\perp} = m_{\pi^0,scr,\parallel}$$

$u < 1$ in order to satisfy causality

Pole and screening masses (special cases)

- **(iii)B≠0, T=0**

$$u_\perp \neq u_\parallel \;\; but \;\; u_\parallel = 1$$

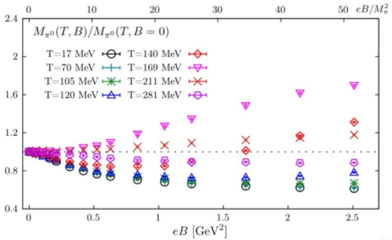
$$m_{\pi^0,pole} = m_{\pi^0,scr,\parallel} < m_{\pi^0,scr,\perp}$$

- **(iv)B≠0, T≠0**

$$u_\perp < u_\parallel < 1$$

$$m_{\pi^0,pole} < m_{\pi^0,scr,\parallel} < m_{\pi^0,scr,\perp}$$

Comparison with LQCD and the NJL model



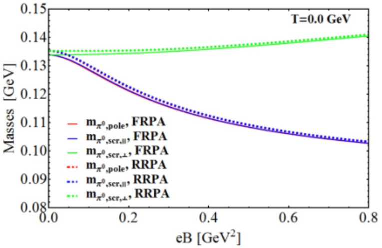


Figure: H. T. Ding, S. T. Li, J. H. Liu, and X. D. Wang, Phys. Rev D105, 034514 (2022), and B. Sheng, Y. Wang, X. Wang, and L. Yu, Phys. Rev. D103 (2021) 9, 094001.

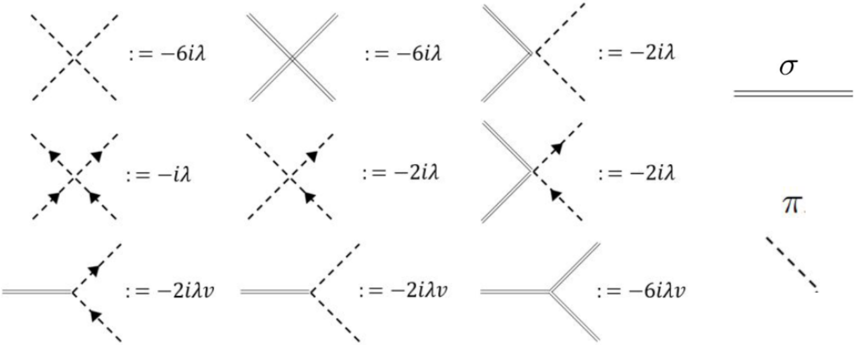
LSMq Lagrangian and features

$$
\begin{aligned}
\mathcal{L} =& \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu \partial_\mu \psi \\
&- ig\bar{\psi}\gamma^5 \bar{\psi}\vec{\tau} \cdot \vec{\pi}\psi - g\bar{\psi}\psi\sigma \,.
\end{aligned}
$$

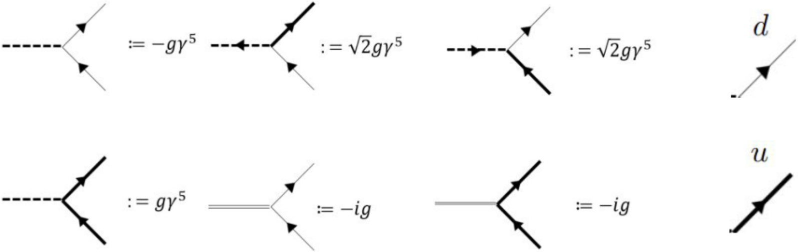
- it implements the SSB of: $SU(2)_L \times SU(2)_R \to SU(2)_V$.
- $m_\pi(v) = \sqrt{\lambda v^2 - a^2} = 0$ at VEV.
- $m_f(v) = gv$.
- $m_\sigma(v) = \sqrt{3\lambda v^2 - a^2}$

$\mathcal{L} \to \mathcal{L} + h(\sigma + v)$ in order to give the correct vacuum pion mass.

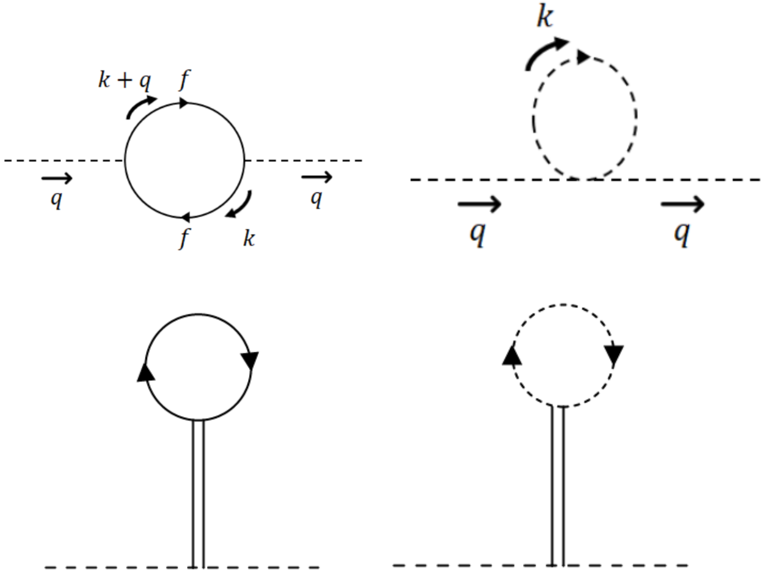
Feynman rules for boson-boson interactions



Feynman rules for boson-fermion interactions



Relevant Feynman diagrams



Feynman rules for the fermionic contribution to $\pi_{f\bar{f}}$ (vertices and propagator)

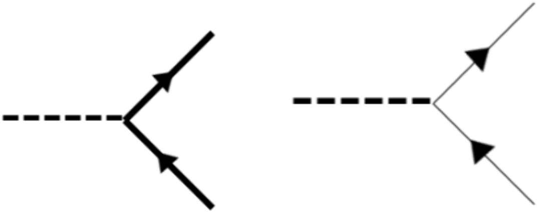


Figure: π_0-u quark vertex=gγ^5; π_0-d quark vertex=-gγ^5

$$iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp\left[is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon \right) \right]$$
$$\times \left\{ (m_f + \not{p}_\parallel)\Big(\cos(qBs) + \gamma^1\gamma^2 \sin(qBs) \Big) - \frac{\not{p}_\perp}{\cos(qBs)} \right\}$$

$$iS(p) \to i\frac{(m_f + \not{p})}{p^2 - m_f^2 + i\epsilon}$$

Neutral pion self-energy (fermion contribution)

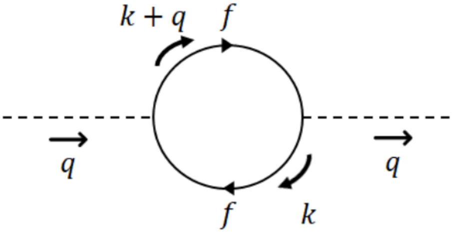


Figure: Neutral pion self-energy

$$-i\pi_{f\bar{f}} = -g^2 \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma^5 iS(k)\gamma^5 iS(k+q)\right] + c.c..$$

$$Tr[\gamma^5 m_f^2 \cos(|q_f B|s)\gamma^5 \cos(|q_f B|s')] = 4m_f^2 \cos(|q_f B|s)\cos(|q_f B|s'),$$

$$\mathrm{Tr}[\gamma^5 \not{k}_{\|} \cos(|q_f B|s)\gamma^5(\not{k}_{\|} + \not{q}_{\|})\cos(|q_f B|s')] =$$

$$-4\cos(|q_f B|s)\cos(|q_f B|s')k_{\|}\cdot(k_{\|} + q_{\|}),$$

$$\mathrm{Tr}[\gamma^5 m_f(\gamma^1\gamma^2)\sin(|q_f B|s)\gamma^5 m_f(\gamma^1\gamma^2)\sin(|q_f B|s')] =$$

$$-4m_f^2 \sin(|q_f B|s)\sin(|q_f B|s'),$$

$$\mathrm{Tr}[\gamma^5 \not{k}_{\|}(\gamma^1\gamma^2)\sin(|q_f B|s)\gamma^5(\not{k}_{\|} + \not{q}_{\|})(\gamma^1\gamma^2)\sin(|q_f B|s')] =$$

$$4\sin(|q_f B|s)\sin(|q_f B|s')k_{\|}\cdot(k_{\|} + q_{\|}),$$

$$\mathrm{Tr}\left[-\frac{\gamma^5 \not{k}_{\perp}}{\cos(|q_f B|s)}(-\gamma^5)\frac{(\not{k}_{\perp} + \not{q}_{\perp})}{\cos(|q_f B|s')}\right] = \frac{4\mathbf{k}_{\perp}\cdot(\mathbf{k}_{\perp} + \mathbf{q}_{\perp})}{\cos(|q_f B|s)\cos(|q_f B|s')}.$$

Neutral pion self-energy (fermion contribution)

$$
\begin{aligned}
& -i\pi_{f\bar{f}} = -4g^2 \int_0^\infty \int_0^\infty \frac{ds ds'}{\cos(qBs)\cos(qBs')} \\
& \times \int \frac{d^4k}{(2\pi)^4} e^{is\left(k_\parallel^2 - k_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon\right)} e^{is'\left((k+p)_\parallel^2 - (k+p)_\perp^2 \frac{\tan(qBs')}{qBs'} - m_f^2 + i\epsilon\right)} \\
& \left\{ \cos[qB(s+s')][m_f^2 - k_\parallel \cdot (k+p)_\parallel] + \frac{k_\perp \cdot (k_\perp + p_\perp)}{\cos(qBs)\cos qBs'} \right\}
\end{aligned}
$$

Important integrals and convenient change of variables

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$u = s + s'$$

$$s = u(1 - v)$$

$$s' = uv$$

$$\frac{\partial(s, s')}{\partial(u, v)} = u$$

Neutral pion self-energy (fermion contribution)

$$
\begin{aligned}
\pi_{f\bar{f}}(q) &= -4g^2 \frac{|q_f B|}{(4\pi)^2} \int_0^1 dv \int_0^\infty du \\
&\times \exp[-i \frac{q_\perp^2}{|q_f B|} \frac{\sin(|q_f B|u(1-v)) \sin(|q_f B|uv)}{\sin(|q_f B|u)}] \\
&\times e^{-iq_3^2 uv(1-v)} e^{iq_0^2 uv(1-v)} e^{-ium_f^2} e^{-u\epsilon} \\
&\times \left\{ \frac{m_f^2}{\tan(|q_f B|u)} + \frac{|q_f B|}{\sin^2(|q_f B|u)} \right. \\
&\times \left(\frac{-q_\perp^2}{|q_f B|} \frac{\sin(|q_f B|u(1-v)) \sin(|q_f B|uv)}{\sin(|q_f B|u)} - i \right) \\
&+ \left. \frac{1}{u \tan(|q_f B|u)} \left(\frac{1}{i} - uv(1-v)(q_3^2 - q_0^2) \right) \right\}.
\end{aligned} \tag{1}
$$

Analysis 1

$$f(p_0, p_\perp, p_\parallel, B) = \pi_{f\bar{f}} - \lim_{B\to 0} \pi_{f\bar{f}}$$

We start with the simplest case ($p_\perp^2 = p_0^2 = 0$). This is the 'longitudinal' screening mass which is found by solving the equation:

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_\perp^2 = 0} = 0$$

Analysis 2

$$f(0,q_3^2,0,|q_fB|,m_f) = -4g^2\frac{|q_fB|}{(4\pi)^2}\int_0^1 dv\int_0^\infty due^{-u\epsilon}$$
$$\times\left\{e^{-iX}\left[\frac{m_f^2-v(1-v)q_3^2}{\tan(|q_fB|u)}\right.\right.$$
$$\left.-i\left(\frac{|q_fB|}{\sin^2(|q_fB|u)}+\frac{1}{u\tan(|q_fB|u)}\right)\right]$$
$$\left.-e^{-iX_0}\left[\frac{m_f^2-v(1-v)q_3^2}{|q_fB|u}-\frac{2i}{|q_fB|u^2}\right]\right\}.$$

$$f(0,q_3^2,0,|q_fB|,m_f)\equiv-\frac{4g^2}{(4\pi)^2}F(q_3^2,|q_fB|,m_f)\,, \qquad (2)$$

Analysis 3

$$F(q_3^2, |q_f B|, m_f) \equiv |q_f B| \int_0^1 dv \int_0^\infty du G(u, v, q_3^2, |q_f B|, m_f),$$

$$\begin{aligned} G(u, v, q_3^2, |q_f B|, m_f) &\equiv [m_f^2 - v(1-v) q_3^2] e^{-u\epsilon} e^{-iau} \\ &\times \left[\cot(|q_f B| u) - \frac{1}{|q_f B| u} \right] \\ &- i e^{-u\epsilon} e^{-iau} \left[|q_f B| \csc^2(|q_f B| u) \right. \\ &\left. + \frac{\cot(|q_f B| u)}{u} - \frac{2}{|q_f B| u^2} \right] \\ &\equiv G_1 - i G_2 \,, \end{aligned}$$

Analysis 4

$$I_{G_1} \equiv -\frac{4g^2}{(4\pi)^2}|q_f B| \int_0^1 dv \int_0^\infty du e^{-u\epsilon}(m_f^2 - v(1-v)q_3^2) \\ \times e^{-iau}\left(\cot(|q_f B|u) - \frac{1}{|q_f B|u}\right) .$$

$$I_{G_2} \equiv \frac{ig^2}{\pi^2}|q_f B| \int_0^1 dv \int_0^\infty du e^{-u\epsilon} e^{-iau} \\ \times \left[|q_f B| \csc^2(|q_f B|u) + \frac{\cot(|q_f B|u)}{u} - \frac{2}{|q_f B|u^2}\right].$$

Contour to perform the u integral

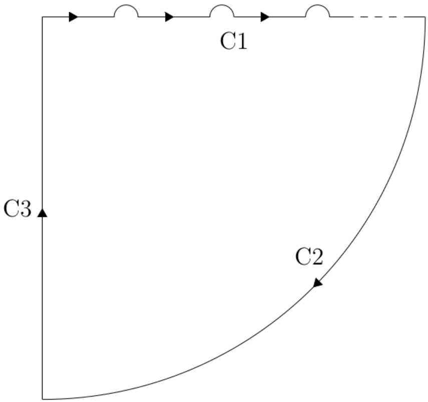


Figure: Contour to perform the u integral

Final expression

$$\Re f = lim_{\epsilon \to 0}\Bigg(-\frac{4g^2}{(4\pi)^2}\int_0^1 dv \Bigg\{ \Big(-2v(1-v)p_3^2 \Big)$$

$$\times \left[\frac{\pi}{2}\frac{\sin(\frac{a\pi}{qB})}{\cosh(\frac{\epsilon\pi}{qB}) - \cos(\frac{a\pi}{qB})} - \tan^{-1}\left(\frac{\sin(\frac{a\pi}{qB})e^{\left(\frac{-\epsilon\pi}{qB}\right)}}{1-e^{\left(\frac{-\epsilon\pi}{qB}\right)}\cos\left(\frac{a\pi}{qB}\right)} \right) \right]$$

$$+ qB\left[\frac{\epsilon\pi}{2qB} - \ln\sqrt{2\cosh\left(\frac{\epsilon\pi}{qB}\right) - 2\cos\left(\frac{a\pi}{qB}\right)} \right]$$

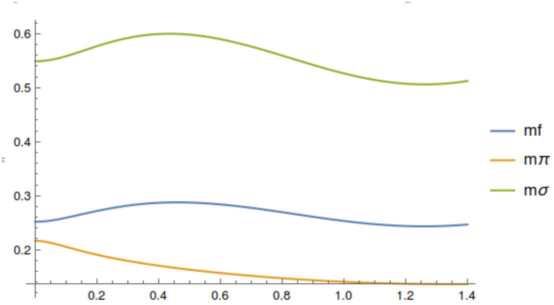
$$- \frac{qB}{\pi}\left[\Re\Big(Li_2\Big[e^{-(ia+\epsilon)\frac{\pi}{qB}} \Big] \Big) \right] \Bigg\} \Bigg)$$

Parameters

Parameters at zero magnetic field

$$g = \frac{m_f}{v} = 2.75; \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2} = 15$$

Magnetic field dependence of masses



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[2] S. S. Avancini, R. Farias, M. B. Pinto, W. R. Tavares, Phys. Letters B 767 (2017) 247-252.

Fermionic contribution for different g values

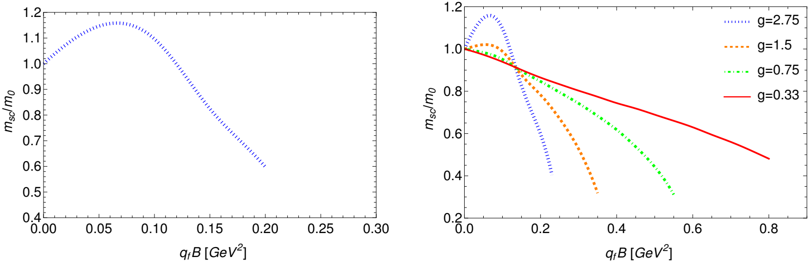

Figure: 'Longitudinal' SC mass as function of B for $g = 2.75$ (left). 'Longitudinal' SC mass as function of B for different values of g.

[3]

[3] A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Fermionic contribution + Tadpoles

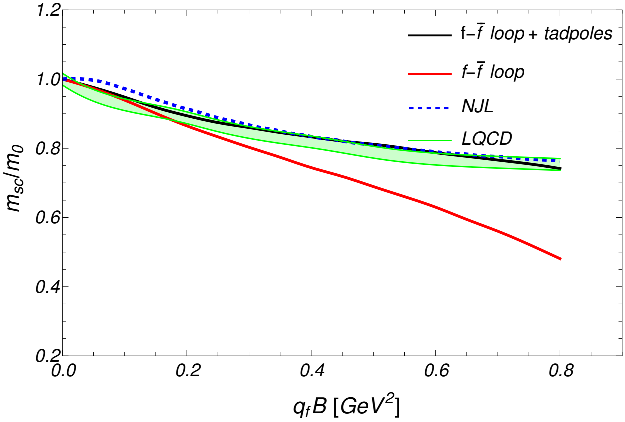

Figure: 'Longitudinal' Screening mass as a function of B, $g = 0.33, \lambda = 2.5$.

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[4] A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Summary and perspectives

Summary:

- We have calculated the neutral pion self-energy in the LSMq.
- We have obtained the 'longitudinal' screening mass as a function of B.
- We have compared our results with LQCD and NJL, and we have found a nice agreement only when we have a magnetic field dependence on the couplings and masses.

Perspectives:

- We are studying the 'transverse' screening mass as a function of B.
- We will study the case where $T \neq 0$.

Thank You

Important integrals and convenient change of variables

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$u = s + s'$$

$$s = u(1 - v)$$

$$s' = uv$$

$$\frac{\partial(s, s')}{\partial(u, v)} = u$$

Neutral pion self-energy (fermion contribution)

$$\pi_{\bar{f}f} = \frac{-4g^2 qB}{(4\pi)^2} \int_0^1 dv \int_0^\infty du \exp(-ix) \exp(-u\epsilon) \Bigg[\frac{m_f^2}{\tan(qBu)}$$
$$- \frac{qB}{\sin^2(qBu)} \left(\frac{p_\perp^2}{|qB|} \frac{\sin(qBu(1-v)) \sin(qBuv)}{\sin(qBu)} \right) - \frac{v(1-v)(p_3^2 - p_0^2)}{\tan(qBu)}$$
$$- \frac{iqB}{\sin(qBu)} - \frac{i}{u \tan(qBu)} \Bigg]$$

where x is given by:

$$x = \frac{p_\perp^2}{qB} \frac{\sin(qBu(1-v)) \sin(qBuv)}{\sin(qBu)} + p_3^2 uv(1-v) - p_0^2 uv(1-v) + m_f^2 u$$

$B \to 0$ limit of $\pi_{\bar{f}f}$

$$\lim_{B\to 0} \pi_{\bar{f}f} = -\frac{4g^2}{(4\pi)^2}\int_0^1 dv \int_0^\infty \frac{du}{u} e^{-ix_0} e^{-u\epsilon} \times \left\{ m_f^2 - v(1-v)(p_\perp^2 - p_3^2) + v(1-v)p_0^2 - \frac{2i}{u} \right\}$$

where

$$x_0 = uv(1-v)(p_\perp^2 + p_3^2) - p_0^2 uv(1-v) + m_f^2$$

Fermionic contribution with g_{eff} as a function of B

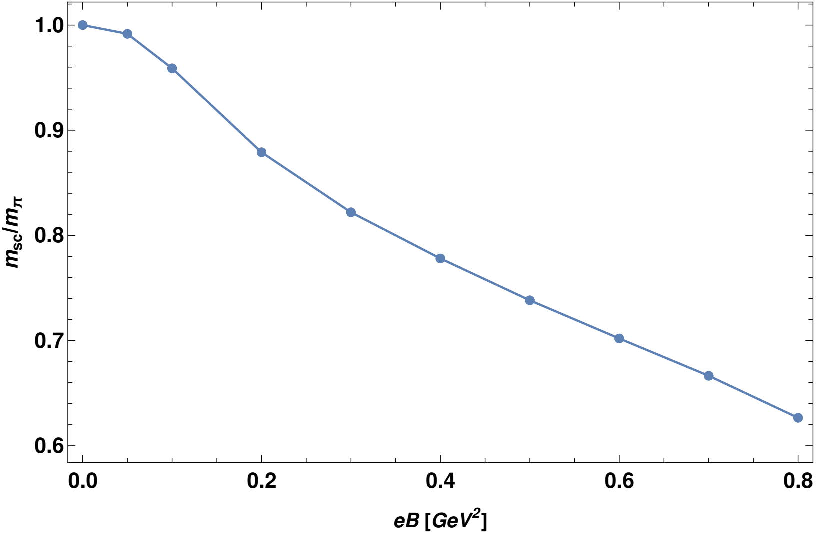

Figure: 'Longitudinal' Screening mass as a function of B, $g_{eff}(B) = 0.3 + 1.2\exp[-(7B)^2]$.

Fermionic contribution + Tadpoles

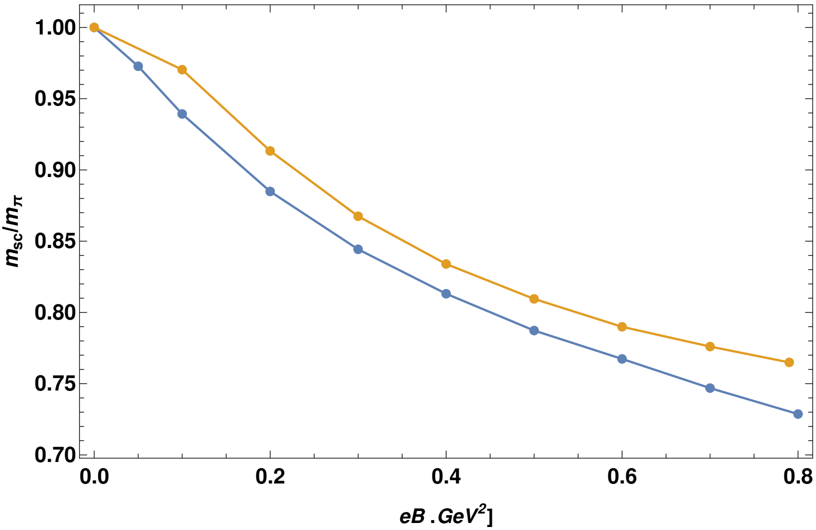

Figure: 'Longitudinal' Screening mass as a function of B vs NJL $g_{eff}(B) = 0.3, \lambda = 1.$

Preliminary results

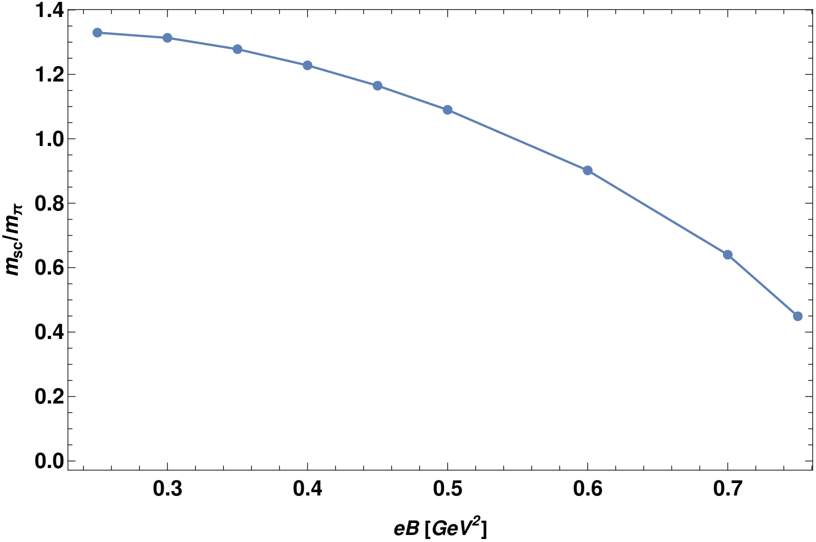

Figure: 'Longitudinal' Screening mass as a function of B.