Free energy at high temperature in the presence of magnetic fields from LSMq

Gabriela Fernández

Dr. Luis Alberto Hernández Rosas & Dr. Ana Julia Mizher

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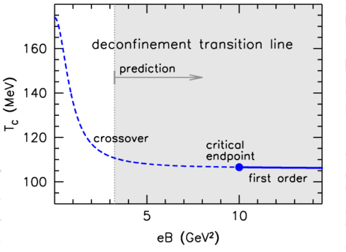
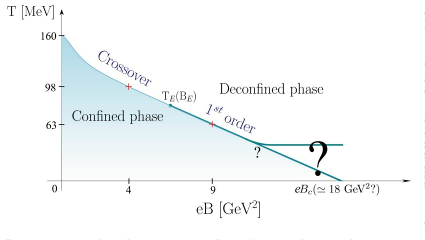
General objectives

Is it true that the CEP appears at such large eB?



G. Endrodi,JHEP **07** (2015), 173 .



M. D'Elia, L. Maio, F. Sanfilippo and A. Stanzione, Phys. Rev. D **105** (2022) no.3, 034511

Obtain the phase diagram of the strongly interacting matter, in the plane of temperature *vs* intensity of the magnetic field using the effective model, LSMq.

Calculate the free energy of the system, to study the behavior of the order parameter of the theory, associated with the breaking/restoration of chiral symmetry.

Specific objectives

Calculate the quantum corrections to the free energy, that is, calculate the effective potential in the high temperature approximation and in the presence of ultra-intense magnetic fields.

- The 1-loop correction of the potential for mesons, both neutral and charged, and for fermions.
- Ring diagrams for the case of neutral mesons.
- The self-energies of the mesons.

LSMq

The Lagrangian of the Linear Sigma Model with quarks is given by

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu \partial_\mu \psi - ig\bar{\psi}\gamma^5 \vec{\tau}.\vec{\pi}\psi - g\bar{\psi}\psi\sigma \quad (1)$$

Once spontaneous breaking of chiral symmetry is allowed and working with the physical pions, we have

$$\begin{aligned}\mathscr{L} =& \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \pi_0 \partial^\mu \pi_0 + \partial \pi_- \partial^\mu \pi_+ - \frac{1}{2}m_\sigma^2(v)\sigma^2 - \frac{1}{2}m_0^2(v)\pi_0^2 - m_0^2(v)\pi_-\pi_+ \\ &+ i\bar{\psi}\partial\!\!\!/\psi - m_f(v)\bar{\psi}\psi + \mathscr{L}_{int} - V^{tree}\end{aligned} \quad (2)$$

where

$$\begin{aligned}\mathscr{L}_{int} =& -\frac{\lambda}{4}\sigma^4 - \lambda v \sigma^3 - \lambda v^3 \sigma - \lambda \sigma^2 \pi_- \pi_+ - 2\lambda v \sigma \pi_- \pi_+ - \frac{\lambda}{2}\sigma^2 \pi_0^2 - \lambda(\pi_-\pi_+)^2 - \frac{\lambda}{4}\pi_0^4 + a^2 v \sigma \\ &- \lambda \pi_- \pi_+ \pi_0^2 - ig\sqrt{2}\left(\bar{u}\gamma^5 d \pi_+ + \bar{d}\gamma^5 u \pi_-\right) - ig\bar{u}\gamma^5 u \pi_0 + ig\bar{d}\gamma^5 d \pi_0 - g\bar{u}u\sigma - g\bar{d}d\sigma\end{aligned} \quad (3)$$

$$V^{tree} = -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 \quad (4)$$

LSMq with magnetic fields

We rewrite the covariant derivative with the minimal coupling

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - iqA_\mu \tag{5}$$

Thus, the Lagrangian is

$$\begin{aligned}\mathscr{L} =& \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi_0\partial^\mu\pi_0 + D_\mu\pi_- D^\mu\pi_+ - \frac{1}{2}m_\sigma^2(v)\sigma^2 - \frac{1}{2}m_0^2(v)\pi_0^2 \\ &- m_0^2(v)\pi_-\pi_+ + i\bar{\psi}\not{D}\psi - m_f(v)\bar{\psi}\psi + \mathscr{L}_{int} - V^{tree}\end{aligned} \tag{6}$$

Due to spontaneous symmetry breaking, the fields acquire masses given by

$$m_\sigma^2 = 3\lambda v^2 - a^2 \quad , \quad m_0^2 = \lambda v^2 - a^2 \quad , \quad m_f = gv \tag{7}$$

Considerations

For the LSMq in the presence of an external magnetic field, the 1-loop effective potential contains both bosonic and fermionic contributions.

$$V_b^1 = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4} \ln(D_b^{-1}(k)) \quad , \quad V_f^1 = iN_c\Omega^{-1}\int \frac{d^4k}{(2\pi)^4} Tr[\ln(S_f^{-1}(k))] \tag{8}$$

The propagators for each of the possible fields are as follows,

$$iG(k) = \frac{i}{k^2 - m^2} \tag{9}$$

$$iG^{LLL}(k) = 2i\frac{e^{-\frac{k_\perp^2}{|qB|}}}{k_\parallel^2 - |qB| - m_b^2} \quad , \quad iS^{LLL}(k) = 2ie^{-\frac{k_\perp^2}{|qB|}}\frac{\not{k}_\parallel + m_f}{k_\parallel^2 - m_f^2}O^+ \tag{10}$$

Neutral mesons

For neutral fields, (π_0, σ), the 1-loop potential in terms of Matsubara frequencies is as

$$V_0^1 = \frac{T}{2} \sum_n \int \frac{d^3k}{(2\pi)^3} dm^2 \frac{1}{\omega_n^2 + \vec{k}^2 + m^2}. \quad (11)$$

By summing over the Matsubara frequencies

$$V_0^1 = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} dm^2 \frac{1}{2\omega_k} \left(1 + \frac{2}{e^{\omega_k/T} - 1}\right) \quad (12)$$

with

$$\omega_k = \sqrt{\vec{k}^2 + m^2}$$

.

We solve for each term separately .

Vacuum term

The vacuum term is divergent so we resort to the dimensional regularization method. By integrating over m^2

$$V^1_{0,vac} = \frac{\mu^{3-d}}{2} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(\vec{k}^2 + m^2)^{-1/2}} \tag{13}$$

By integrating

$$V^1_{0,vac} = \frac{m^4}{64\pi^2} \left(-\frac{1}{\epsilon} + (-\frac{3}{2} + \gamma_E) - \ln\left(\frac{4\pi\mu^2}{m^2}\right) \right) \tag{14}$$

using $\overline{MS}$ to renormalize the mass

$$V^1_{0,vac} = -\frac{m^4}{64\pi^2} \left(\frac{3}{2} + \ln\left(\frac{\mu^2}{m^2}\right) \right) \tag{15}$$

Matter term

To calculate the matter term $V^1_{0,\beta} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} dm^2 \frac{1}{\sqrt{\vec{k}^2+m^2}} \frac{1}{e^{\sqrt{\vec{k}^2+m^2}/T}-1}$ (16)

In this case, we use the high temperature approximation to obtain

$$V^1_{0,\beta} = -\frac{T^4}{8\pi^2}\left(\frac{4\pi^4}{45} - \frac{\pi^2 m^2}{3T^2} + \frac{2\pi m^3}{3T^3} + \left(2\gamma_E - \frac{3}{2}\right)\frac{m^4}{8T^4} + \ln\left(\frac{m^2}{(4\pi T)^2}\right)\frac{m^4}{8T^4}\right) \quad (17)$$

finally we have

$$V^1_0 = -\frac{T^4\pi^2}{90} + \frac{m^2T^2}{24} - \frac{m^3T}{12\pi} - \frac{m^4}{64\pi^2}\left(2\gamma_E + \ln\left(\frac{\mu^2}{(4\pi T)^2}\right)\right) \quad (18)$$

Charged mesons

In terms of the Matsubara frequencies, for charged bosons

$$V_b^1 = T \sum_n \int \frac{d^3k}{(2\pi)^3} dm_b^2 \frac{e^{-\frac{k_\perp^2}{|qB|}}}{\omega_n^2 + k_3^2 + m_b^2 + |qB|} \tag{19}$$

When summing and integrating with respect to the perpendicular components of the momentum

$$V_b^1 = \frac{|qB|}{8\pi} \int \frac{dk_3}{2\pi} dm_b^2 \frac{1}{\sqrt{k_3^2 + m_b^2 + |qB|}} \left(1 + \frac{2}{e^{\sqrt{k_3^2+m_b^2+|qB|}/T} - 1} \right) \tag{20}$$

From here we can take two paths.
Integrating over the mass leads us to the expression

$$V_b^1 = \frac{|qB|}{4\pi} \int \frac{dk_3}{2\pi} \left(\sqrt{k_3^2 + m_b^2 + |qB|} + 2T \ln(1 - e^{-\sqrt{k_3^2+m_b^2+|qB|}/T}) \right) \tag{21}$$

Vacuum term

Following the same procedure as for the neutral case, that is, doing dimensional regularization

$$V^1_{b,vac} = \frac{|qB|}{4\pi} \int \frac{d^d k_3}{(2\pi)^d} \frac{\mu^{1-d}}{(k_3^2 + m_b^2 + |qB|)^{-1/2}} \tag{22}$$

and using $\overline{MS}$, we have

$$V^1_{b,vac} = \frac{|qB|}{(4\pi)^2}(m_b^2 + |qB|)\left(1 + \ln\left(\frac{\mu^2}{m_b^2 + |qB|}\right)\right) \tag{23}$$

This result is the same for any of the two integration paths chosen.

Matter term

With

$$V_{b,\beta}^{1} = \frac{T|qB|}{2\pi}\int \frac{dk_3}{2\pi}\ln\left(1 - e^{-\sqrt{k_3^2+m_b^2+|qB|}/T}\right) \tag{24}$$

To rewrite the expression we use the fact that magnetic fields are ultra-intense, i.e. $|qB| >> T$, because of this we can use that

$$\ln(1+x) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}x^n$$

so

$$V_{b,\beta}^{1} = -\frac{T|qB|}{2\pi}\int \frac{dk_3}{2\pi}\sum_{n=1}^{\infty}\frac{e^{-\sqrt{k_3^2+m_b^2+|qB|}n/T}}{n} \tag{25}$$

by integrating

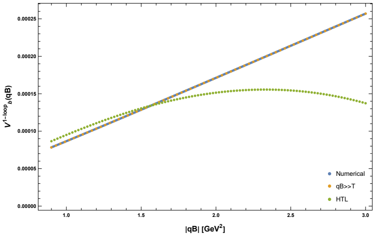
$$V_{b,\beta}^{1} - \frac{T|qB|}{2\pi^2}\sqrt{m_b^2+|qB|}\sum_{n=1}^{\infty}\frac{K_1\left(n\sqrt{m_b^2+|qB|}/T\right)}{n} \tag{26}$$

We obtain the 1-loop potential for charged bosons, with the first method, as

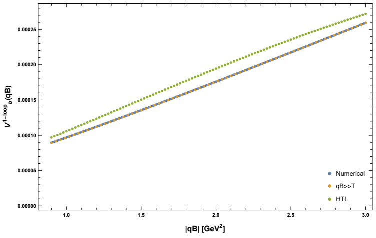
$$V_b^1 = \frac{|qB|}{(4\pi)^2}(m_b^2 + |qB|)\left(1 + \ln\left(\frac{\mu^2}{m_b^2 + |qB|}\right)\right) - \frac{T|qB|}{2\pi^2}\sqrt{m_b^2 + |qB|}K_1\left(\frac{\sqrt{m_b^2 + |qB|}}{T}\right) \quad (27)$$

Using the high-temperature approximation, the result is

$$V_b^1 = -\frac{qBT}{4\pi}\sqrt{|qB| + m_b^2} - \frac{|qB|}{(4\pi)^2}(m_b^2 + |qB|)\left(2\gamma_E + \ln\left(\frac{\mu^2}{(4\pi T)^2}\right)\right) \quad (28)$$



$V_b^1(qB)$with n=10, T=0.2, $m_b = 0{,}14$



$V_b^1(qB)$ with n=10, T=0.3, $m_b = 0{,}14$

Quarks

The 1-loop potential for fermions is

$$V_f^1 = -4N_c \sum_n T \int \frac{d^3k}{(2\pi)^3} dm_f^2 \frac{e^{-\frac{k_\perp^2}{|qB|}}}{\tilde{\omega}_n^2 + k_3^2 + m_f^2} \tag{29}$$

Before integrate into the mass but later of the integration in de perpendicular part

$$V_f^1 = -\frac{N_c|qB|}{2\pi} \int \frac{dk_3}{2\pi} dm_f^2 \frac{1}{\sqrt{k_3^2 + m_f^2}} \left(1 - \frac{2}{e^{\sqrt{k_3^2+m_f^2}/T} + 1}\right) \tag{30}$$

We can use the high temperature approximation.

Vacuum term

By doing dimensional regularization, the vacuum term is as follows

$$V^1_{f,vac} = -\frac{N_c|qB|}{2\pi}\int \frac{d^dk_3}{(2\pi)^2} dm_f^2 \frac{\mu^{1-d}}{(k_3^2 + m_f^2)^{1/2}} \tag{31}$$

with $\overline{MS}$

$$V^1_{f,vac} = -\frac{N_c|qB|}{4\pi^2}\int dm_f^2 \ln\left(\frac{\mu^2}{m_f^2}\right) \tag{32}$$

integrating over mass

$$V^1_{f,vac} = -\frac{N_c|qB|}{4\pi^2} m_f^2 \left(1 + \ln\left(\frac{\mu^2}{m_f^2}\right)\right) \tag{33}$$

Matter term

with

$$V^1_{f,\beta} = \frac{N_c|qB|}{2\pi^2}\int dk_3 dm_f^2 \frac{1}{\sqrt{k_3^2+m_f^2}}\frac{1}{e^{\sqrt{k_3^2+m_f^2}/T}+1} \tag{34}$$

in this case, we can use the high temperature approximation in this integral

$$V^1_{f,\beta} = -\frac{N_c|qB|}{\pi^2}\int dm_f^2\left(\frac{\gamma_E}{2}+\frac{1}{2}\ln\left(\frac{m_f}{\pi T}\right)\right) \tag{35}$$

thus

$$V^1_{f,\beta} = -\frac{N_c|qB|}{4\pi^2}m_f^2\left(-1+2\gamma_E+\ln\left(\frac{m_f^2}{\pi^2T^2}\right)\right) \tag{36}$$

we have

$$V^1_{f} = -\frac{N_c|qB|}{4\pi^2}m_f^2\left(2\gamma_E+\ln\left(\frac{\mu^2}{\pi^2T^2}\right)\right) \tag{37}$$

Ring diagrams

With the masses of the neutral bosons, these could be negative, so, the terms as m^3 could be imaginary and this is a problem, to resolve this is necessary add the corrections corresponding to the ring diagrams, which are like

$$V^{ring} = \frac{T}{2} \sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left(1 + \Pi_b D\right) \tag{38}$$

due to we are working with a high tempeture we can take the more dominat term wich is the Matsubara´s zero mode, it is taking $n = 0$, so

$$V^{ring} = \frac{T}{2} \int \frac{d^3k}{(2\pi)^3} \left(\ln\left(k^2 + m^2 + \Pi_b\right) - \left(k^2 + m^2\right)\right) \tag{39}$$

with solution

$$V^{ring} = -\frac{T}{12\pi}(m^2 + \Pi_b)^{3/2} + \frac{Tm^3}{12\pi} \tag{40}$$

adding this terms the 1-loop contribution result as

$$\begin{aligned} V^1 + V^{ring} &= -\frac{T^4\pi^2}{90} + \frac{m_0^2 T^2}{24} - \frac{T}{12\pi}(m_0^2 + \Pi_b)^{3/2} - \frac{m_0^4}{64\pi^2}\left(2\gamma_E + \ln\left(\frac{\mu^2}{(4\pi T)^2}\right)\right) \\ &- \frac{T^4\pi^2}{90} + \frac{m_\sigma^2 T^2}{24} - \frac{T}{12\pi}(m_\sigma^2 + \Pi_b)^{3/2} - \frac{m_\sigma^4}{64\pi^2}\left(2\gamma_E + \ln\left(\frac{\mu^2}{(4\pi T)^2}\right)\right) \\ &+ \frac{|qB|}{2\pi^2}(m_b^2 + |qB|)\left(1 + \ln\left(\frac{\mu^2}{m_b^2 + |qB|}\right)\right) - \frac{T|qB|}{\pi^2}\sqrt{m_b^2 + |qB|}K_1\left(\frac{\sqrt{m_b^2 + |qB|}}{T}\right) \\ &- \sum_f \frac{N_c|qB|}{4\pi^2} m_f^2\left(2\gamma_E + \ln\left(\frac{\mu^2}{\pi^2 T^2}\right)\right) \end{aligned} \tag{41}$$

Self-energy

Now, we have to compute the self-energies, to this we start knowing the next expresions

$$\Pi_\sigma = \frac{\lambda}{4}\left[12I(m_\sigma) + 4I(m_0) + 8I(m_b)\right] + N_f N_c \Pi_f \quad (42)$$

$$\Pi_0 = \frac{\lambda}{4}\left[4I(m_\sigma) + 12I(m_0) + 8I(m_b)\right] + N_f N_c \Pi_f \quad (43)$$

where the function $I(m_b)$ is

$$I(m_b) = 2\frac{dV_b^1}{dm_b^2} \quad (44)$$

and the self-energy for fermions is

$$\Pi_f = 2g^2\frac{dV_f^1}{dm_f^2} \quad (45)$$

In this way we can obtain by deriving, for each self-energy

$$\Pi_\sigma = -\frac{\lambda T\sqrt{\Pi + m_0^2}}{4\pi} - \frac{3\lambda T\sqrt{\Pi + m_\sigma^2}}{4\pi} - \frac{3|qB|g^2 N_c \ln\left(\frac{\mu^2}{\pi^2 T^2}\right)}{\pi^2} - \frac{6\gamma_E |qB| g^2 N_c}{\pi^2} + \frac{2|qB|\lambda \ln\left(\frac{\mu^2}{|qB|+m_b^2}\right)}{\pi^2}$$
$$+ \frac{2|qB|\lambda K_0\left(\frac{\sqrt{m_b^2+|qB|}}{T}\right)}{\pi^2} - \frac{\gamma_E \lambda m_0^2}{8\pi^2} - \frac{\lambda m_0^2 \ln\left(\frac{\mu^2}{16\pi^2 T^2}\right)}{16\pi^2} - \frac{3\gamma_E \lambda m_\sigma^2}{8\pi^2} - \frac{3\lambda m_\sigma^2 \ln\left(\frac{\mu^2}{16\pi^2 T^2}\right)}{16\pi^2} + \frac{\lambda T^2}{3} \quad (46)$$

$$\Pi_0 = -\frac{3\lambda T\sqrt{\Pi + m_0^2}}{4\pi} - \frac{\lambda T\sqrt{\Pi + m_\sigma^2}}{4\pi} - \frac{3|qB|g^2 N_c \ln\left(\frac{\mu^2}{\pi^2 T^2}\right)}{\pi^2} - \frac{6\gamma_E |qB| g^2 N_c}{\pi^2} + \frac{2|qB|\lambda \ln\left(\frac{\mu^2}{|qB|+m_b^2}\right)}{\pi^2}$$
$$+ \frac{2|qB|\lambda K_0\left(\frac{\sqrt{m_b^2+|qB|}}{T}\right)}{\pi^2} - \frac{3\gamma_E \lambda m_0^2}{8\pi^2} - \frac{3\lambda m_0^2 \ln\left(\frac{\mu^2}{16\pi^2 T^2}\right)}{16\pi^2} - \frac{\gamma_E \lambda m_\sigma^2}{8\pi^2} - \frac{\lambda m_\sigma^2 \ln\left(\frac{\mu^2}{16\pi^2 T^2}\right)}{16\pi^2} + \frac{\lambda T^2}{3} \quad (47)$$

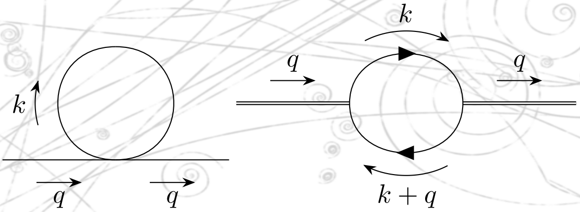
Feynman diagrams contribution

In a diferent way we calculate using for the bosons

$$-i\Pi_b = \int \frac{d^4k}{(2\pi)^4}(-2i\lambda)iD(k) \tag{48}$$

for fermions

$$-i\Pi_f = -\int \frac{d^4k}{(2\pi)^4} Tr\left[\left(\pm g\gamma^5\right) iS^{LLL}(k)\left(\pm g\gamma^5\right) iS^{LLL}(k+q)\right] + C.C \tag{49}$$



neutral bosons

In terms of Matsubara frecuencies

$$-i\Pi_0 = -2i\lambda T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + \vec{k}^2 + m^2} \tag{50}$$

by summing

$$-i\Pi_0 = -2i\lambda \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{\vec{k}^2 + m^2}} \left(\frac{1}{2} + \frac{1}{e^{\sqrt{\vec{k}^2+m^2}/T} - 1} \right) \tag{51}$$

using the high temperature approximation we obtain

$$-i\Pi_0 = \frac{i\lambda}{2\pi} mT + \frac{i\lambda}{8\pi^2} m^2 \left(2\gamma_E + \ln\left(\frac{\mu^2}{(4\pi T)^2} \right) \right) - \frac{i\lambda T^2}{6} \tag{52}$$

Charged bosons

With

$$-i\Pi_{\pm} = -4i\lambda T \sum_{n} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-k_{\perp}^2/|qB|}}{\omega_n^2 + k_3^2 + |qB| + m_b^2} \quad (53)$$

Sum over n and integrate perpendicular components

$$-i\Pi_{\pm} = -i\frac{\lambda|qB|}{2\pi} \int \frac{dk_3}{2\pi} \frac{1}{\sqrt{k_3^2 + |qB| + m_b^2}} \left(1 + \frac{2}{e^{\sqrt{k_3^2+|qB|+m_b^2}/T} - 1}\right) \quad (54)$$

rewrite as a function of a derivative of mass

$$-i\Pi_{\pm} - i\frac{\lambda|qB|}{\pi} \frac{d}{dm_b^2} \left[\int \frac{dk_3}{2\pi} \left\{\sqrt{k_3^2 + |qB| + m_b^2} + 2T \ln\left(1 - e^{-\sqrt{k_3^2+|qB|+m_b^2}/T}\right)\right\}\right] \quad (55)$$

$$-i\Pi_{\pm} - i\frac{\lambda}{4}\frac{d}{dm_b^2}\left[\frac{|qB|}{(4\pi)^2}(m_b^2 + |qB|)\left(1 + \ln\left(\frac{\mu^2}{m_b^2 + |qB|}\right)\right) - \frac{|qB|T}{2\pi^2}\sqrt{m_b^2 + |qB|}K_1\left(\frac{\sqrt{m_b^2 + |qB|}}{T}\right)\right] \quad (56)$$

Quarks

In the case of fermion self-energy, we will have two possible cases, for each of the allowed interactions, i.e., with neutral pions and with sigma mesons.

$$-i\Pi_f = -\int \frac{d^4k}{(2\pi)^4} Tr\left[\left(\pm g\gamma^5\right) iS^{LLL}(k)\left(\pm g\gamma^5\right) iS^{LLL}(k+q)\right] + C.C \tag{57}$$

$$-i\Pi_f = -\int \frac{d^4k}{(2\pi)^4} Tr\left[\left(-ig\right) iS^{LLL}(k)\left(-ig\right) iS^{LLL}(k+q)\right] + C.C \tag{58}$$

However, we prove that both results, in the high temperature approximation are equal, as

$$-i\Pi_f = i\frac{g^2|qB|}{2\pi^2}\left(2\gamma_E + \ln\left(\frac{\mu^2}{(T\pi)^2}\right)\right) \tag{59}$$

Results

We have

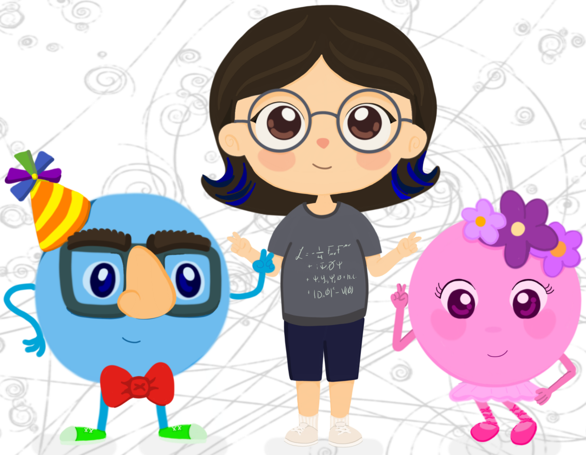
$$
\begin{aligned}
V^{eff} = & -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 + \frac{(\lambda v^2 - a^2)T^2}{24} - \frac{T}{12\pi}((\lambda v^2 - a^2) + \Pi_0)^{3/2} - \frac{(\lambda v^2 - a^2)^2}{64\pi^2}\left(2\gamma_E + \ln\left(\frac{\mu^2}{(4\pi T)^2}\right)\right) \\
& - \frac{T^4\pi^2}{45} + \frac{(3\lambda v^2 - a^2)T^2}{24} - \frac{T}{12\pi}((3\lambda v^2 - a^2) + \Pi_\sigma)^{3/2} - \frac{(3\lambda v^2 - a^2)^2}{64\pi^2}\left(2\gamma_E + \ln\left(\frac{\mu^2}{(4\pi T)^2}\right)\right) \\
& + \frac{|q_b B|}{2\pi^2}((\lambda v^2 - a^2) + |q_b B|)\left(1 + \ln\left(\frac{\mu^2}{(\lambda v^2 - a^2) + |q_b B|}\right)\right) \\
& - \frac{T|q_b B|}{\pi^2}\sqrt{(\lambda v^2 - a^2) + |q_b B|}\sum_n K_1\left(\frac{n\sqrt{(\lambda v^2 - a^2) + |q_b B|}}{T}\right) \\
& - \sum_f \frac{N_c|q_f B|}{4\pi^2}g^2 v^2\left(2\gamma_E + \ln\left(\frac{\mu^2}{\pi^2 T^2}\right)\right)
\end{aligned}
\tag{60}
$$

Conclusion

We obtained an expression for the free energy associated with LSMq, with quantum corrections due to high temperature and ultra-intense magnetic fields.

This energy is a function of the order parameter v, the temperature T, the magnetic field qB, and the coupling constants λ and g.

It will allow us to find a relation between temperature and magnetic field at which the chiral symmetry breaking/restoration occurs, in order to construct a phase diagram and study the phase transition, the kind of transition and where values we find the critical point.



THANK YOU!