A tale on anomalous transport

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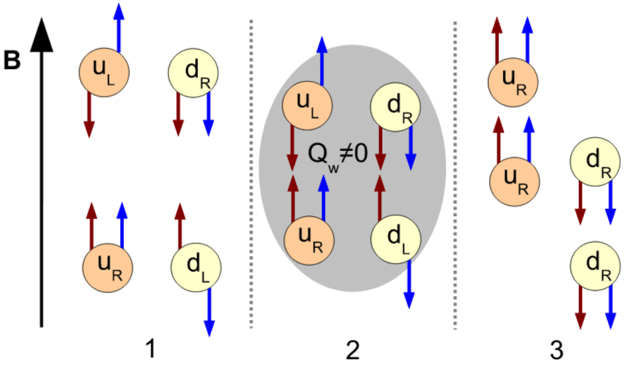
The chiral magnetic effect



[Kharzeev, McLerran and Warringa, NPA 803, 227 (2008).] Blue arrows denote spin and red arrows denote momentum.

Quantum Anomalies

Noether theorem

$$\boxed{\text{Symmetry}} \rightarrow \boxed{\text{Conserved current}} \rightarrow \boxed{\partial_\mu J^\mu = 0}$$

- Quantum anomalies: the failure of a symmetry of a theory's classical action to be a symmetry of any regularization of the full quantum theory.
- Symmetry violated by the integration measure in the path integral.

Overview

The chiral anomaly

Maxwell-Chern-Simons

The chiral magnetic effect in Weyl materials

The chiral anomaly (1+1)D

$$S = \int d^2 x i \bar{\psi} \not{\partial} \psi$$

$$S = \int d^2 x i \psi^\dagger \gamma^0 (\gamma^0 \partial_t + \gamma^1 \partial_x) \psi = \int d^2 x i \psi^\dagger (\partial_t - \gamma^5 \partial_x) \psi$$

$$\psi_\pm = \frac{1}{2} \left(1 \pm \gamma^5\right) \psi \qquad \psi_+ = \begin{pmatrix} \chi_+ \\ 0 \end{pmatrix} \quad \psi_- = \begin{pmatrix} 0 \\ \chi_- \end{pmatrix}$$

$$S = \int d^2 x \left(i \chi_+^\dagger \partial_- \chi_+ + i \chi_-^\dagger \partial_- \chi_- \right)$$

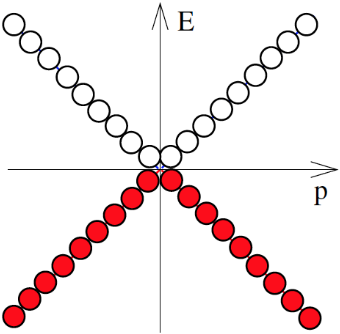
$$\partial_\pm = \partial_t \pm \partial_x$$

Equations of motion:

$\partial_- \chi_+ = 0$ with solution $\chi_+ = \chi_+(\mathrm{t} + \mathrm{x})$ (left moving fermions)

$\partial_+ \chi_- = 0$ with solution $\chi_- = \chi_-(\mathrm{t} - \mathrm{x})$ (right moving fermions)

Vacuum



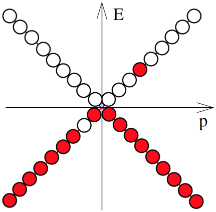
Dispersion relation $E = |p|$

The chiral anomaly (1+1)D

Symmetries of the action rotate the individual phases of χ_+ and χ_-: $\psi \to e^{i\alpha}\psi$ and $\psi \to e^{i\alpha\gamma^5}\psi$

Chiral symmetry: n_+ (number of right moving particles) and n_- (number of left moving particles) separately conserved.

What if we deform the theory? We can generate a particle-antiparticle pair. But n_+ and n_- remain the same.

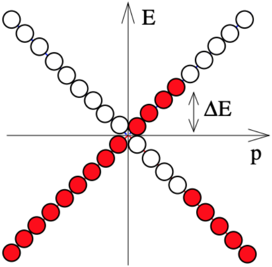


The chiral anomaly

We turn on an electric field

$$S = \int d^2 x i \bar{\psi} \not{D} \psi$$

The value of the momenta is changed by $\Delta p = e\mathcal{E}t$. Result from the sea with infinite momentum.



"Anomalies, as a manifestation of the high momentum collective motion in the vacuum", Gribov.

The chiral anomaly (1+1)D

Charge density $\rho_+ = \frac{e\mathcal{E}}{2\pi} t$ and $\rho_- = -\frac{e\mathcal{E}}{2\pi} t$.

The total number of particles is conserved: $\psi \to e^{i\alpha}\psi$

$\dot{\rho} = 0$ where $\rho = \rho_+ + \rho_-$.

The difference between fermion number is not conserved: $\psi \to e^{i\alpha\gamma^5}\psi$

$\dot{\rho}_A = \frac{e\mathcal{E}}{\pi}$ where $\rho_A = \rho_+ - \rho_-$.

Axial anomaly or chiral anomaly!

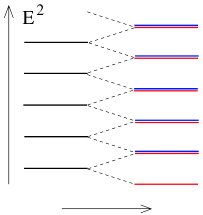
The chiral anomaly in (3+1)D

In (3+1)D:

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$S = \int d^4 x i \bar{\psi} \not{D} \psi = \int d^4 x i \psi_+^\dagger \bar{\sigma}^\mu D_\mu \psi_+ + i \psi^\dagger \sigma^\mu D_\mu \psi_-$$

$$E^2 = eB(2n+1) + p_z^2 - 2eBS_z$$



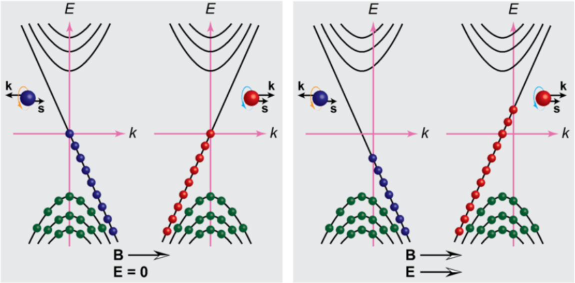
The lowest Landau level is polarized

$$\psi_+ = \begin{pmatrix} \chi_+ \\ 0 \end{pmatrix}$$

For very intense field there is a dimensional reduction: $\int d^4 x \to \int d^2 x_\parallel$

The chiral anomaly in (3+1)D

$S \sim \int dz\, dt\, i\bar{\chi}_+(\partial_t - \partial_z)\chi_+ \rightarrow$ The ψ_+ particles move to the right.



[Kharzeev,Li Nuclear Physics A, 956, 107 (2014)]

- Magnetic field: in the lowest Landau level the particles are polarized. Privileged direction.
- Electric field: imbalance in the number of particles moving to the right and to the left.

The chiral anomaly

The action of massless Dirac fermion in $d = 3+1$ dimensions, coupled to an electromagnetic gauge field is:

$$S = \int d^4 x i\bar{\psi}\not{D}\psi$$

Invariant under axial rotation:

$$\psi \to e^{i\alpha\gamma^5}\psi \qquad j^{\mu}_{\alpha} = \bar{\psi}\gamma^{\mu}\gamma^5\psi$$

Non-conservation of axial current:

$$\partial_\mu j^5_\mu = \frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma};$$

$$\partial_\mu j^5_\mu = \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B}$$

Overview

The Maxwell-Chern-Simons theory works as an effective theory. Let us consider a space-time dependence for θ

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu - \frac{c}{4}\theta\tilde{F}^{\mu\nu}F_{\mu\nu}$$

$$\theta\tilde{F}^{\mu\nu}F_{\mu\nu} = -\partial_\mu\theta K^\mu$$

$$P_\mu = \partial_\mu\theta = (M, \vec{P})$$

Maxwell equations are modified by the Chern-Simons term

$$
\begin{aligned}
\nabla \cdot E &= \rho + c\mathbf{P} \cdot \mathbf{B} \\
\nabla \cdot B &= 0 \\
\nabla \times E &= -\frac{\partial B}{\partial t} \\
\nabla \times B &= j - \frac{\partial E}{\partial t} + c\left(M\mathbf{B} - \mathbf{P} \times \mathbf{E}\right)
\end{aligned}
$$

For an external magnetic field with $\nabla \times B = 0$, $E = 0$ and $|\mathbf{P}| = 0$:

$$j = -cM\mathbf{B}$$

The conductivity is non-dissipative

Time reversal symmetry

$$t \to -t \qquad \vec{J}:\text{T-odd};\ \ \vec{B}:\text{T-odd};\ \ \vec{E}:\text{T-even}$$

$$\vec{J} = \sigma \vec{E} \ \to \ \vec{J} = -\sigma \vec{E}; \qquad \sigma = \frac{ne^2\tau}{m^*}$$

Ohmic conductivity (dissipative)

$$\frac{\partial \vec{J}}{\partial t} = \mu \vec{E} \ \to \ \frac{\partial \vec{J}}{\partial t} = \mu \vec{E}; \qquad \mu = \left(\frac{n_s e^2}{m^*}\right)$$

Superconductivity (non-dissipative)

$$\vec{J} = \sigma_{\mathrm{CME}} \vec{B} \ \to \ \vec{J} = \sigma_{\mathrm{CME}} \vec{B}$$

Chiral magnetic effect (non-dissipative)

[Q. Li (2018)]

Instantons in Yang-Mills

Action of a non-Abelian pure gauge system: $S = \frac{1}{g^2} \int d^4x \; Tr \; F^{\mu\nu} F_{\mu\nu}$

The axial current is given by: $\partial_\mu j^\mu = \frac{g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$

Topological invariant: $\nu = \frac{1}{16\pi^2} \int d^4x Tr F^{\mu\nu} \tilde{F}_{\mu\nu}$

Schwartz inequality: $tr \int \left(F_{\mu\nu} \pm \tilde{F}_{\mu\nu}\right)^2 d^4x \geq 0$

$$tr \int \left(F^{\mu\nu} F_{\mu\nu}\right) d^4x \geq |tr \int \left(F^{\mu\nu} \tilde{F}_{\mu\nu}\right) d^4x| = 16\pi^2 \nu$$

$S \geq \frac{8\pi^2}{g^2} \nu$

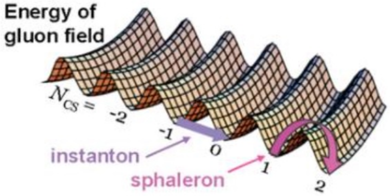
Minimum for $F = \pm \tilde{F}$ (instantons!)

Theta vacuum

Multiple vacua

$$|\theta\rangle = \sum e^{-i\nu\theta|\nu\rangle} \quad (1)$$

Effective Lagrangian: $L_e ff = L + \frac{\theta}{16\pi^2} tr\left(F^{\mu\nu} F_\mu \nu\right)$



Interacting with fermions

The Dirac operator has real eigenvalues

$$i\not{D}\phi_n = \lambda_n\phi_n$$

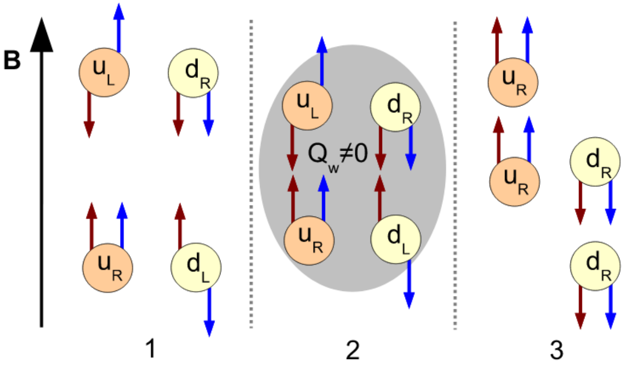
$$i\not{D}(\gamma^5\phi_n) = \lambda_n\gamma^5\phi_n$$

For the zero mode it is possible to diagonalize simultaneously $i\not{D}$ and γ^5.
Total number of zero modes: $n_+ + n_-$. The index of the operator is defined as $n_+ - n_-$.
Using the anomalous Ward identity it is possible to show that

$$\sum_n \bar{\phi}_n\gamma^5\phi_n = \frac{e^2}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

Atiyah-Singer theorem: $\nu = n_+ - n_-$.

The chiral magnetic effect



[Kharzeev, McLerran and Warringa, NPA 803, 227 (2008).] Blue arrows denote spin and red arrows denote momentum.

Overview

Graphene: the first material where relativistic-like quasi-particles were observed.

Vol 438|10 November 2005|**doi:10.1038/nature04233**

nature

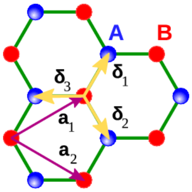
LETTERS

Two-dimensional gas of massless Dirac fermions in graphene

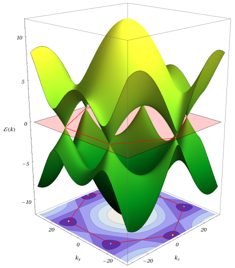
K. S. Novoselov[1], A. K. Geim[1], S. V. Morozov[2], D. Jiang[1], M. I. Katsnelson[3], I. V. Grigorieva[1], S. V. Dubonos[2] & A. A. Firsov[2]

Quantum electrodynamics (resulting from the merger of quantum mechanics and relativity theory) has provided a clear understanding of phenomena ranging from particle physics to cosmology and from astrophysics to quantum chemistry[1–3]. The ideas underlying quantum electrodynamics also influence the theory of condensed matter[4,5], but quantum relativistic effects are usually minute in the known experimental systems that can be described accurately

behaviour shows that substantial concentrations of electrons (holes) are induced by positive (negative) gate voltages. Away from the transition region $V_g \approx 0$, Hall coefficient $R_H = 1/ne$ varies as $1/V_g$, where n is the concentration of electrons or holes and e is the electron charge. The linear dependence $1/R_H \propto V_g$ yields $n = \alpha V_g$ with $\alpha \approx 7.3 \times 10^{10}\,\mathrm{cm^{-2}V^{-1}}$, in agreement with the theoretical estimate $n/V_g \approx 7.2 \times 10^{10}\,\mathrm{cm^{-2}V^{-1}}$ for the surface charge density



- Represented in terms of two triangular sublattices.
- Tight-binding approach: nearest neighboors.
- Hopping only between sublattices.



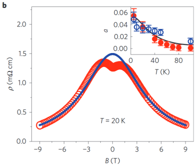
- Linear dispersion relation: $\mathcal{H} = \bar{\psi}\, \hbar v_F \gamma \cdot \boldsymbol{k}\, \psi$.
- Dirac points: valence and conduction band touch generating no gap.

Why not look for the chiral anomaly in graphene? No chiral anomaly in odd dimensions.

Chiral anomaly in 3D Weyl semi-metal $ZrTe_5$

$$J_{CME} = \frac{e^2}{2\pi^2}\mu_5 B, \qquad \mu_5 \sim E \cdot B$$

$$J_{CME} \equiv \sigma^{ik}_{CME} E^k, \sigma^{zz}_{CME} \sim B^2$$



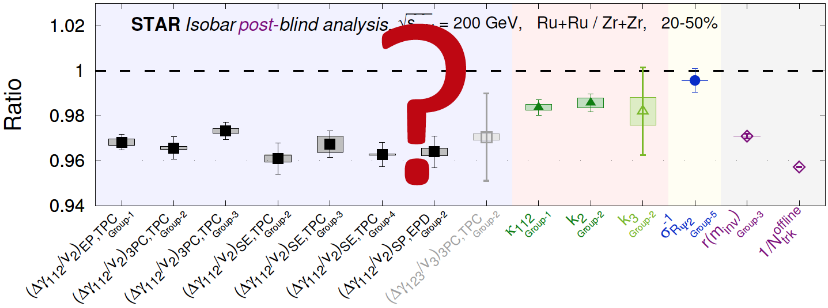
[Li et al, Nature Phys. 12, 550 (2016).]

- The magnetoresistence in $ZrTe_5$ when a magnetic field is applied parallel to an electric field is in accordance with the predictions for the CME.

After the first observation, the CME was detected in several other 3D Dirac materials.

Could it replace superconductors in certain devices? Could it perform at a higher temperature? Patent for a chiral qubit.

Isobar collisions



Isobar collisions: program dedicated to detect the CME did not find the signal. New analysis considering multiplicity difference between the isobars indicate a small signal.

Summary

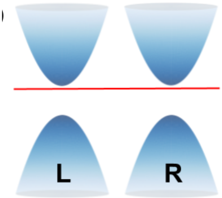
- The chiral anomaly may emerge in Abelian and non-Abelian systems
- In Abelian systems it is driven by parallel electric and magnetic field
- In non-Abelian systems it is driven by topological objects: instantons and sphalerons
- Weyl materials are relativistic-like systems that allows for the manifestation of the chiral anomaly
- The CME has not been observed in the quark-gluon plasma: can table top experiments shed light on this?

Back up slides

QED in (2+1)D

- Gaps between the conduction and valence band appear as a mass term $\hat{M}$ in the Lagrangian - interactions, deformations, substrates, doping, etc.
- $\hat{\mu}$ is a generalized chemical potential including spin interaction (Zeeman term), $\mu_\sigma = \mu - \frac{\sigma g}{2\mu_B} B$.
- QED3 fermion sector ($\hbar$=c=1):

$$\mathcal{L} = \bar{\psi}[\gamma_0(i\partial_0 + \hat{\mu}) - i(\gamma_1 D_x + \gamma_2 D_y) - \hat{M}]\psi.$$



Pseudo QED

[Marino, Nucl. Phys. B 408, 551 (1993), Gorbar, Guysinin, Miranski, PRD 64, 105028 (2001).]

- The gauge sector is not constrained to the plane.
- Coulomb rather than logarithmic interaction.
- Reduced QED: general (3+1)D theory dimensionally reduced to a non-local effective (2+1)D theory.

$$S = \int d^D X \left(\frac{1}{4e^2} F_{ab}^2 + A_a J^a - \frac{1}{2e^2\xi} \left(\partial_a A^a\right)^2 \right)$$

- $D = 4 \rightarrow$ Integrating over the gauge field and the third spatial dimension.
- Keeping $J^3 = 0$.
- Adding the fermion fields in (2+1)D.

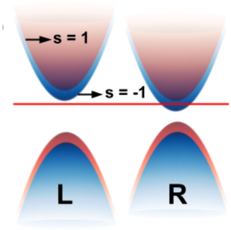
$$S = \int d^3 x \left[\bar{\psi} \left(i\not{D} + m \right) \psi + \frac{1}{2} F_{\mu\nu} \frac{1}{\sqrt{-\partial^2}} F^{\mu\nu} + \frac{1}{e^2\xi} \partial_\mu A^\mu \frac{1}{\sqrt{-\partial^2}} \partial_\nu A^\nu \right].$$

Anomalous Quantum Hall Effect

[**AJM**, A. Raya, C. Villavicencio S. Hernandez, Eur.Phy.J.C 78, 912 (2018),

[**AJM**, C. Villavicencio, D. Dudal, A. R. Rocha, F. Matusalém, Sci.Rep. 12, 5439 (2022)]

We assume different gaps for each pair of cones.



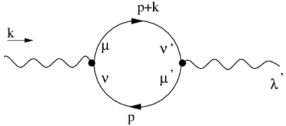
Linear response formalism: reaction of the system to external influences.

$$\delta S = \int d^4 x J_\mu(x) a_\mu(x)$$

The conductivity is given by

$$\sigma_\chi = -\lim_{\omega\to 0}\frac{1}{\hbar\omega}\tilde{\Pi}_R^{xy}$$

The polarization tensor is given by the diagram



We have shown that only 1-loop contributions are non-vanishing: Coleman-Hill theorem valid for RQED

[D. Dudal, AJM and P. Pais, PRD (2018)].

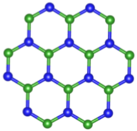
The limit can only be taken if we consider a configuration of the magnetic field that implies an electric field when the limit $\omega \to 0$ is taken.

Considering a chemical potential, we obtain for the net current

$$\sigma_\chi = \sum_s \frac{e^2}{4\pi}\left[\frac{m_{s,k}}{|m_{s,k}|}\theta(m_{s,k}^2-\mu^2) - \frac{m_{s,k'}}{|m_{s,k'}|}\theta(m_{s,k'}^2-\mu^2)\right]$$

Quantum Hall Effect, with fractional Chern-number. **TOPOLOGICALLY PROTECTED!**

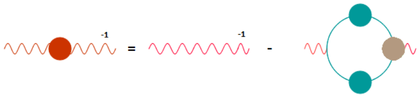
- $m_R = M_+ + M_-$, $m_L = M_+ - M_-$
- Center symmetry breaking "mass": $M_+ = m_3\gamma_3$
- This can be obtained if **sublattice symmetry is broken**.



- Broken T symmetry: complex next to nearest neighbors term. In the Lagrangian: $M_- = m_3\gamma_3\gamma_5$. **Spin orbit?**
- Both mechanisms occurring simultaneously lead to different gaps in each Dirac cone.

Schwinger-Dyson equations

SDE for the inverse fermion propagator

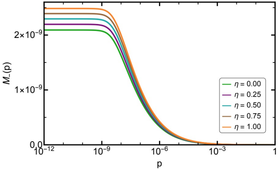


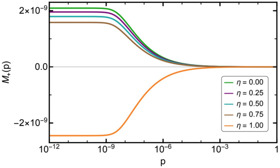
SDE for the inverse gauge boson propagator

Schwinger-Dyson equations

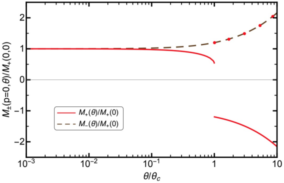
$$
\begin{aligned}
S^{-1}(p) &= S_0^{-1}(p) - \Xi(p), \\
\Delta_{\mu\nu}^{-1}(p) &= \Delta_{0\mu\nu}^{-1}(p) - \Pi_{\mu\nu}(p)
\end{aligned}
$$

Schwinger-Dyson: coupled equations





- For one chirality, the height of the mass function increases with increasing θ. For the other chirality the mass function flips sign at a critical value of θ.
- In the infrared Haldane mass is generated above a critical value of θ



- More educated truncations of SDE lower the value of the critical coupling

L. Albino, A. Bashir, **AJM**, A. Raya, PRD 106 (2022) 9, 096007