Introduction to Quarkyonic Matter

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Outline

1- Introduction: Preliminary concepts

2- The quarkyonic matter

3- Motivation

4- Possible approaches

5- Summary

A brief review on Large N_c expansion concepts

ELSEVIER
Nuclear Physics B
Volume 72, Issue 3, 18 April 1974, Pages 461-473

A planar diagram theory for strong interactions

G.'t Hooft

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Abstract

A gauge theory with colour gauge group U(N) and quarks having a colour index running from one to N is considered in the limit $N \to \infty$, g^2N fixed. It is shown that only planar diagrams with the quarks at the edges dominate; the topological structure of the perturbation series in $1/N$ is identical to that of the dual models, such that the number $1/N$ corresponds to the dual coupling constant. For hadrons N is probably equal to three. A mathematical framework is proposed to link these concepts of planar diagrams with the functional integrals of Gervais, Sakita and Mandelstam for the dual string.

ELSEVIER
Nuclear Physics B
Volume 160, Issue 1, 26 November 1979, Pages 57-115

Baryons in the 1N expansion ☆

Edward Witten

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Abstract

In this paper the existing results concerning mesons and glue states in the large-N limit of QCD are reviewed, and it is shown how to fit baryons into this picture.

- 3 colors in SU(3) gauge group $\longrightarrow$ SU(N) with $N \longrightarrow \infty$ "Not-so-obvious" candidate for the QCD expansion parameter.
- Simplifies the theory: systematic expansion in powers of $1/N$.
- Good qualitative results, although it does not provide a basis for quantitative results.
- $N \longrightarrow \infty$: Many possible intermediate states in Feynman diagrams $\longrightarrow$ Sum over intermediate states generates large combinatoric factors, responsible for the nature of the large N limit.

$A^i_{\mu j}$: gluon field, $N \times N$ matrix with N^2 components
q^i, $\bar{q}_i$: quark and antiquark fields, N with components

A brief review on Large N_c expansion concepts

If g is the gauge coupling: Take $N_c \longrightarrow \infty$ holding $g^2 N_c$ fixed. 't Hooft limit!

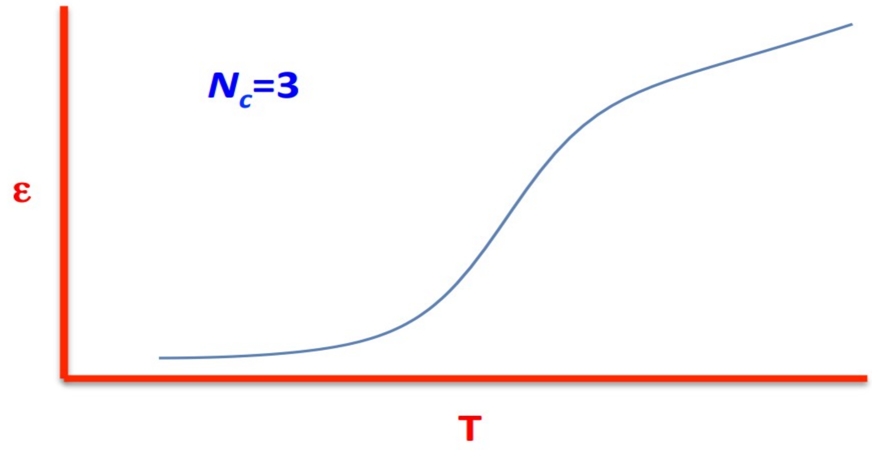
Finite Temperature: Deconfining transition when the N_c^2 term turns on in the pressure

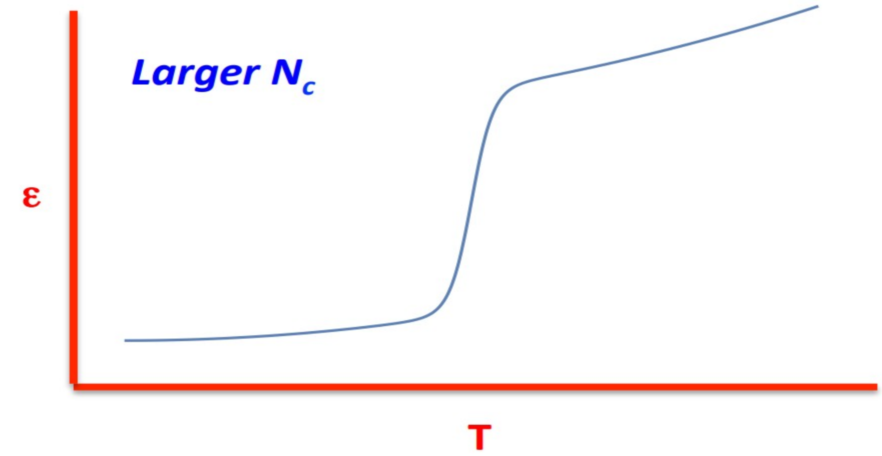
- At low T: Confined states are color singlet (mesons and glueballs) with pressure ~ N_c^0.
- At high T: Gluons deconfine and contribute with N_c^2 to the pressure.

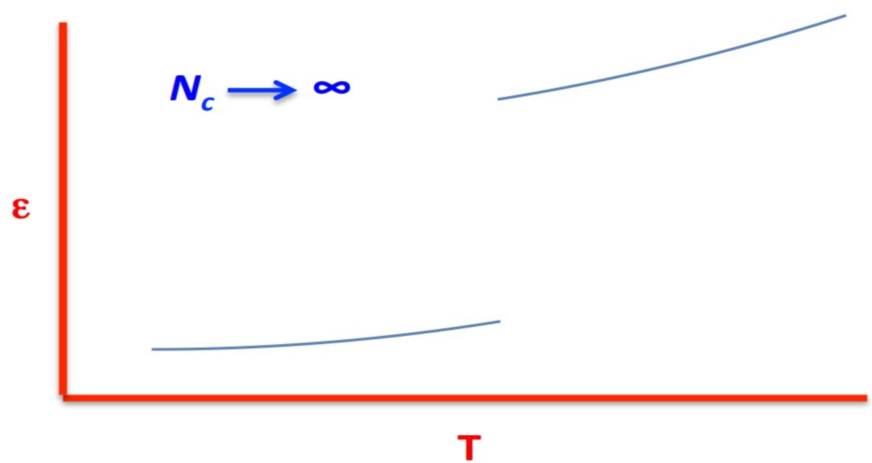
Deconfinement temperature T_d is expected to be of order 1 at large N_c, on the order of a typical QCD scale ($\Lambda_{\rm QCD}$ ~ 200 MeV).

The energy density of N_f flavors of deconfined quarks is ~ $N_c N_f$ in the large N_c limit:

Deconfinement probably drives the chiral symmetry restoration at T_d.







A crossover for N_c=3 can become increasing sharp as N_c increases and as it goes to ∞, the qualitative behavior can change from being a crossover to a first order transition—a qualitatively different behavior.

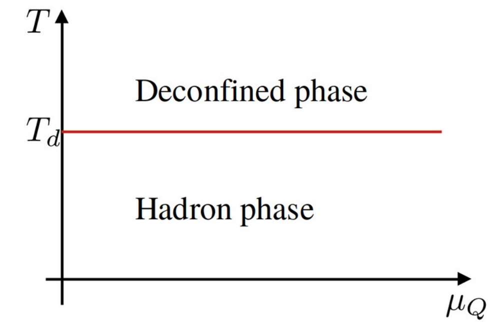
This is precisely what we believe happens for QCD.

Despite the qualitative differences there may be useful insights by considering the large N_c limit.

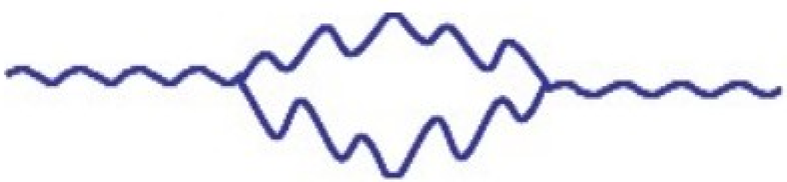
From Thomas Cohen presentation @EQCD 2020

A brief review on Large N_c expansion concepts

If each quark has the energy of ~ Λ_{QCD}: $\begin{cases} \text{Baryons: } M_B \sim N_c \Lambda_{QCD} \\ \text{Quarks: } M_Q \sim M_B / N_c \end{cases}$



Finite chemical potential: T_d does not depends on μ at μ ~ 1.



Gluon loop

Quark loop

- $g^2 N_c T^2 \sim T^2$;
- Dynamics not affected by quarks;
- Debye screening at large distances.

$$\Pi^{\mu\mu}(0) = g^2 \left[\left(N_c + \frac{N_f}{2} \right) \frac{T^2}{3} + \frac{N_f \mu^2}{2\pi^2} \right]$$

- $\sim \mu_Q^{\ 2} g^2 \Rightarrow$ Supressed by $1/N_c$ at large N_c.
- High density limit: $\mu_Q \gg \Lambda_{QCD}$, so quarks are important when $\mu_Q \sim N_c^{1/2} \Lambda_{QCD}$.
- Debye screen mass $m_D \simeq g\, \mu_Q$

Quarkyonic Matter

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Nuclear Physics A 796 (2007) 83–100

NUCLEAR PHYSICS A

Phases of dense quarks at large N_c

Larry McLerran [a,b], Robert D. Pisarski [a,*]

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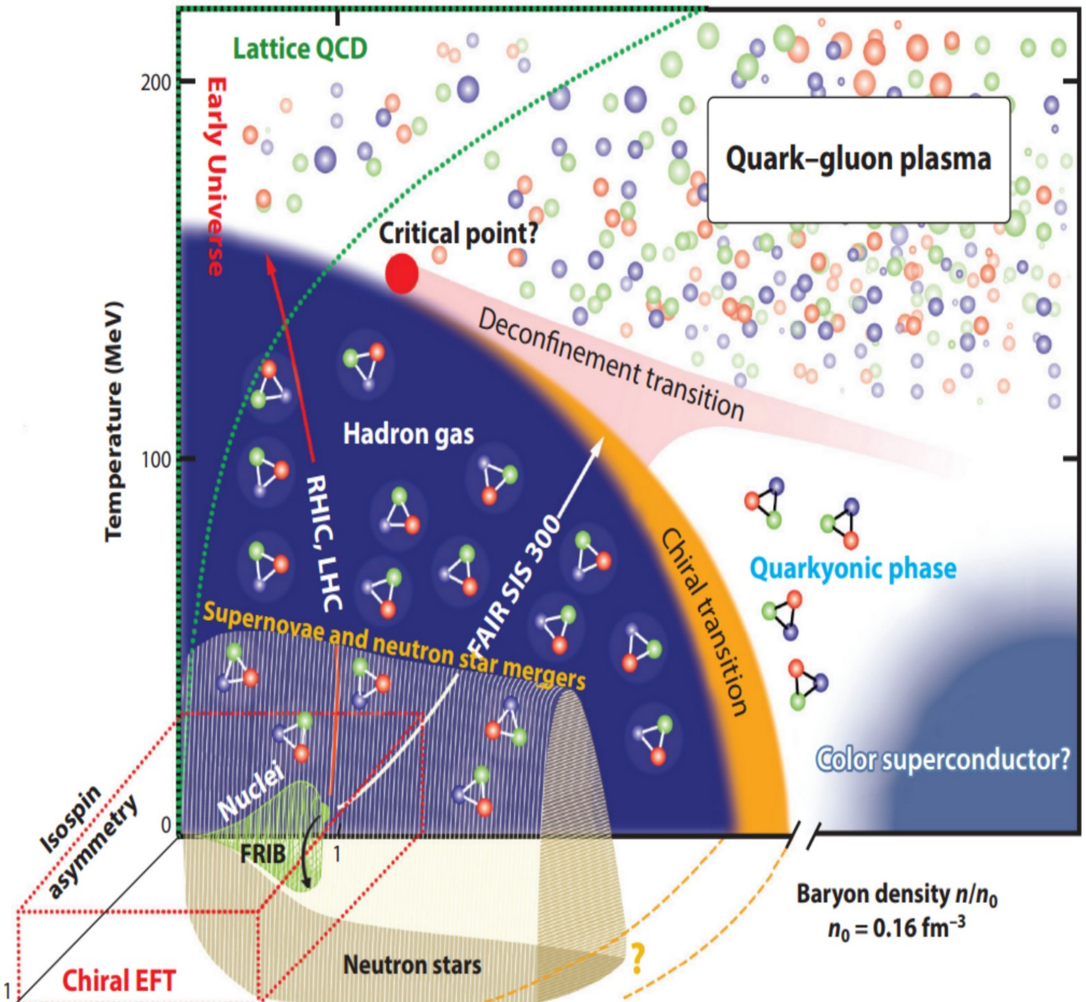
Available online 14 September 2007

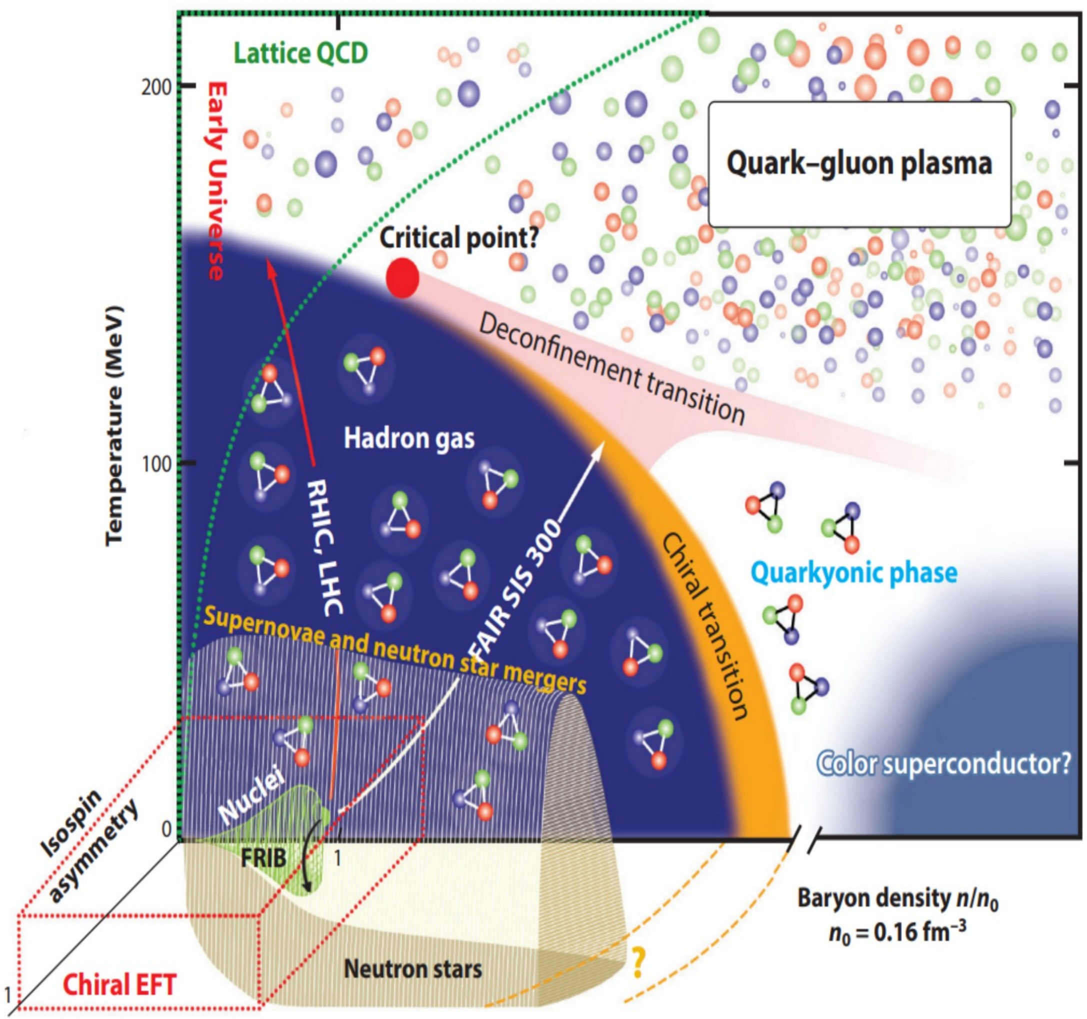
"In the limit of a large number of colors, gauge theories may exhibit several distinct phases at nonzero temperature and quark density, in addition to the familiar phase of confined hadrons and deconfined quarks and gluons."

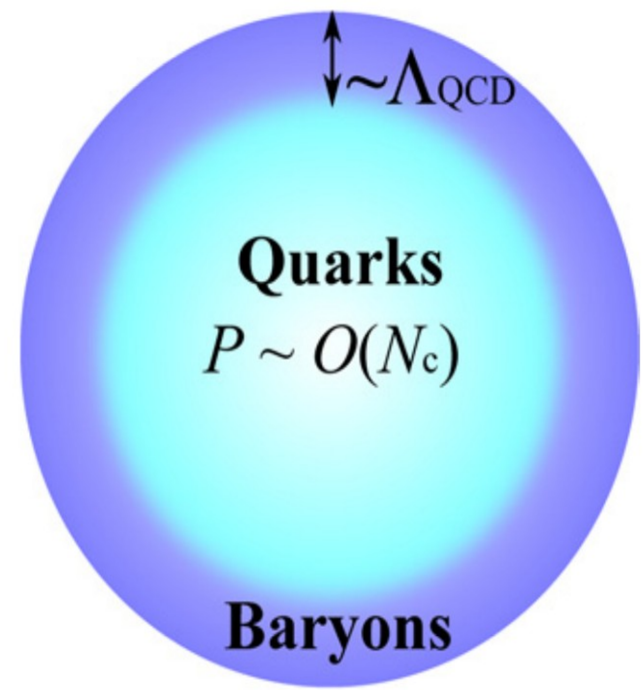
Quarkyonic Matter



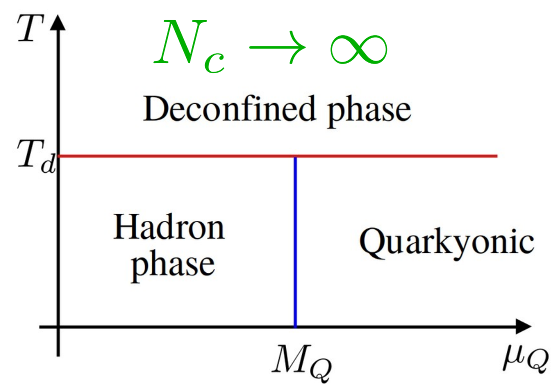
- Different phase in confined world, appear when $\mu_q > M_q$ and n_B becomes nonzero.
- Pressure changes suddenly from $O(N_c^{\,0}) \rightarrow O(N_c)$.
- Weakly interacting quark system or baryonic system?

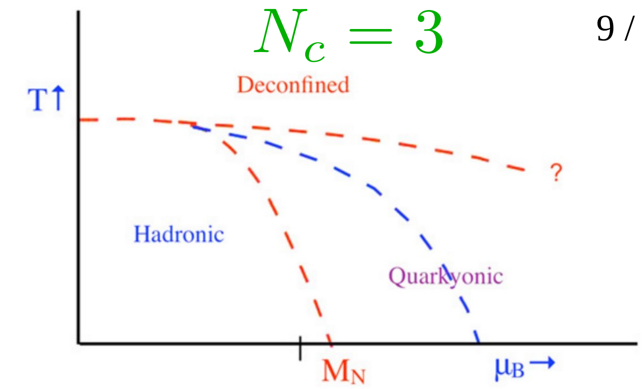
Quarkyonic Matter





Quarkyonic Matter





Nuclear ➡ Quarkyonic (at few times ρ_0)

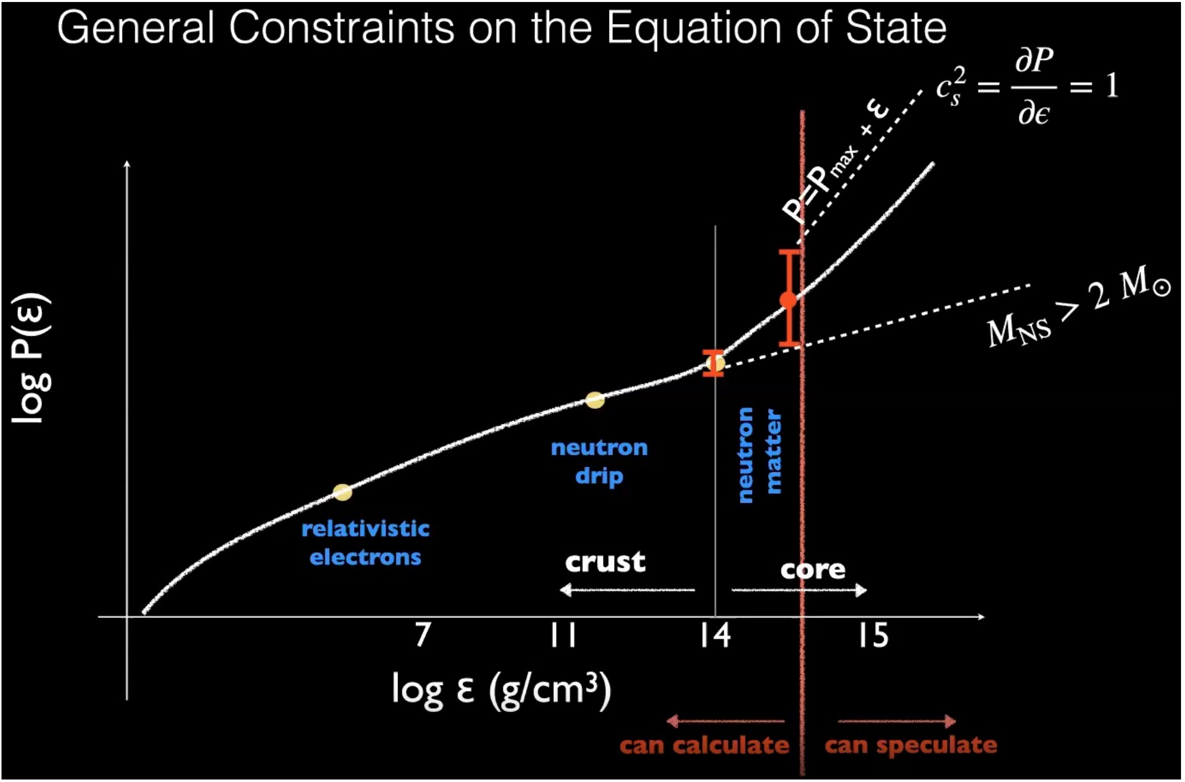
- For $k_F^B < \Lambda_{\rm QCD}$: Quarks confined in nucleons.
- For $\Lambda_{\rm QCD} \leq k_F^B \leq N_c\Lambda_{\rm QCD}$: Quarks starts to take low phase space, and a shell-like structure is formed.
- For $k_F^B \simeq N_c^{3/2}\Lambda_{\rm QCD}$: Confinement disappears.

- Total baryon density has smooth behavior and chemical potential for confined states enhance suddenly, then pressure suddenly increases.

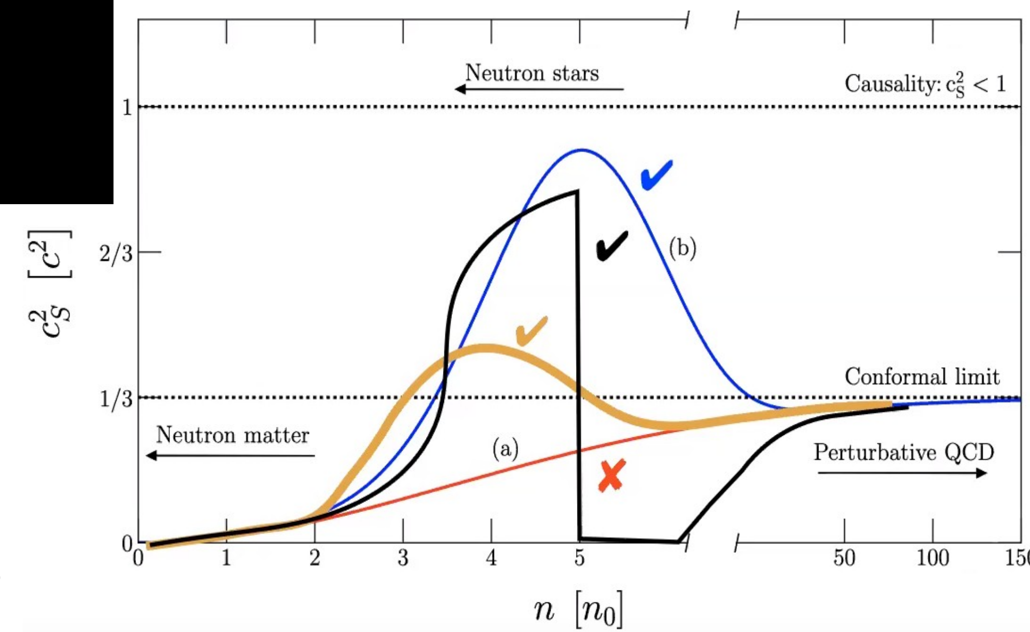
This is not an usual phase transition!

Break:

Why is this interesting?



GW provides constraints to the EoS of dense matter



GW170817

- $R_{M=1.4M_\odot} \leq 13.5$ km and $M_{max} > 2M_\odot$

GW190425

- $R_{M=1.4M_\odot} \leq 15$ km and M_{max} $(2-3)M_\odot$

Breakdown of the models!

Figs. from S. Reddy presentation @BNL (2021)

Many different approaches and possibilities!

- Extension of low-energy nuclear physics to higher densities: RMF and many body calculations with dependence of couplings and masses with n_B [Oertel et al., Rev.Mod.Phys. 89, 015007 (2017), Kaiser et al., Nucl. Phys. A. 697, 255 (2002), Drischler et al., Phys. Rev. Lett.122, 042501 (2019), Lonardoni et al., Phys. Rev. Res. 2, 022033 (2020)];

- Other interactions and degrees of freedom [Glendenning, Astrophys. J. 293, 470 (1985), Knorren, et al., Phys. Rev. C 52, 3470 (1995), Cai et Al., Phys. Rev. C 92, 015802 (2015)];

- Phase transition from nuclear to quark matter. [Annala et al. Nat. Phys. 16, 907 (2020), Nature Communications 14, 8451 (2023)].

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- ➤ Phase transition from nuclear to quark matter. [Annala et al. Nat. Phys. 16, 907 (2020), Nature Communications 14, 8451 (2023)].

Resultant model/theory must:

$\surd$ Satisfy the well-known nuclear matter properties at saturation density; $n_B \sim (1-2)\rho_0$

$\surd$ Evolve to a phase of deconfined quarks at high densities; $n_B \gtrsim (2-5)\rho_0$???

$\surd$ Recover the pQCD results at asymptotically high densities and temperatures. $n_B \gtrsim 40\rho_0$

Quarkyonic Matter: Motivation

- QCD with 3 colors: Small typical energy scales (~$1/N_c$), even for nuclear interactions of O(N_c)
  ⟹ matter behaves like a dilute gas.

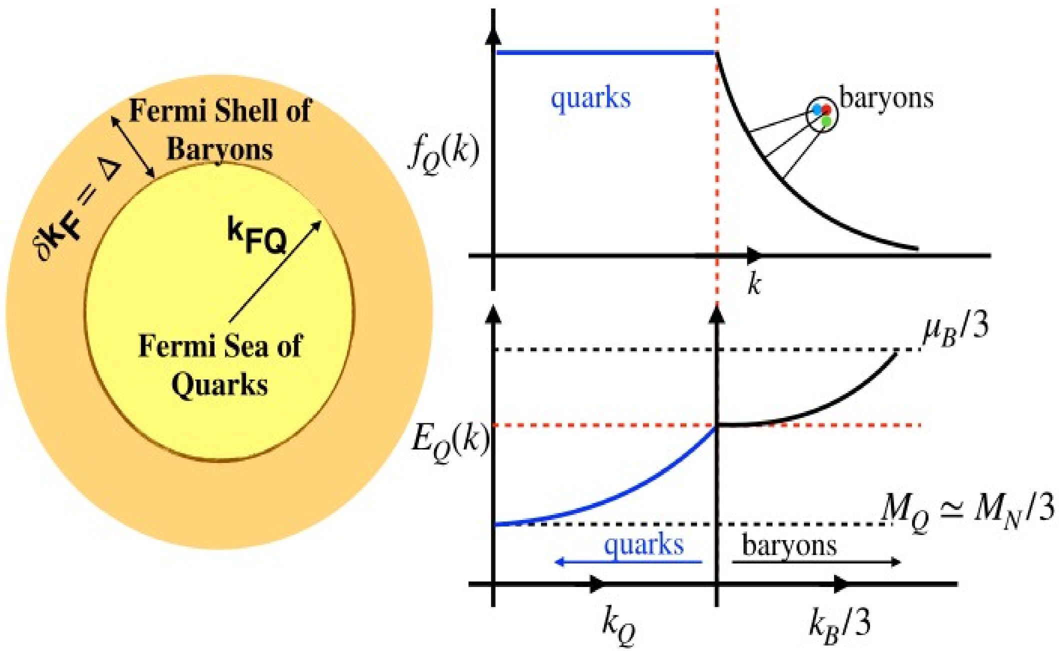
- When density increase: Typical energies become of the same order of nuclear interactions
  ⟹ hadronic matter changes properties rapidly.

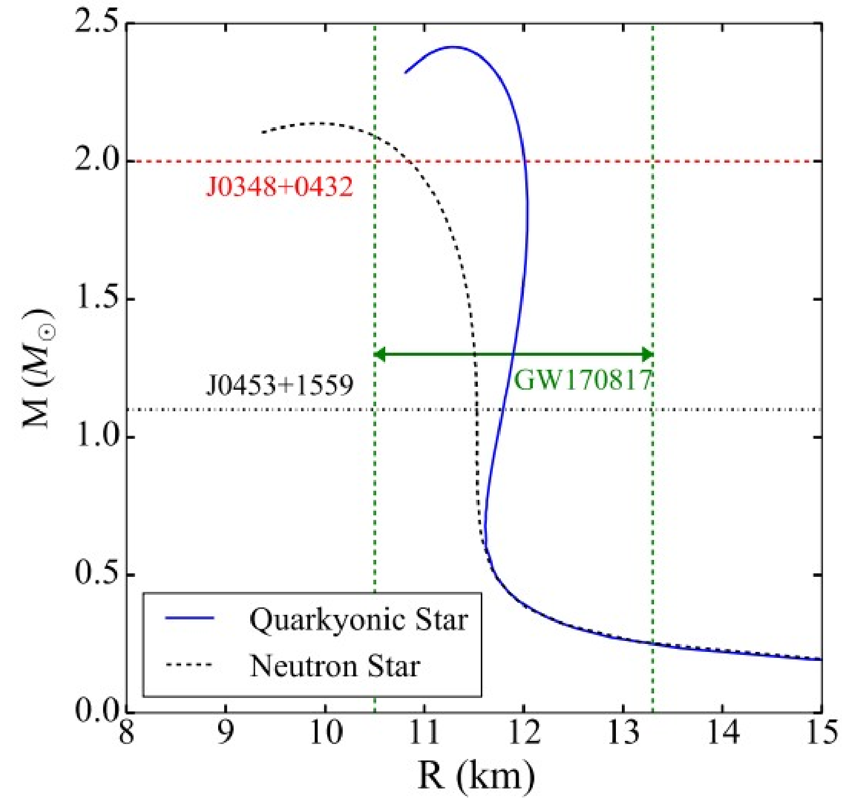
- Sound velocity increase very rapidly, exceed 1/3 at densities around (3-4) ρ_0.
  ⟹Since it needs to approach 1/3 from below we expect that sound velocity will show a minimum at some point.

$$n_B \sim (k_F^B)^3 \sim (k_F^Q)^3 \quad \text{and} \quad k_F^B \simeq N_c k_F^Q \qquad c_s^2 = \frac{n_B}{\mu_B}\frac{d\mu_B}{dn_B}$$

If nuclear matter is composed by only nucleons (quasiparticles) their phase space increase with k_F^3, and the rapid increase in phase space available without corresponding increase in n_B suggests that nucleons are only partially filling their available phase space.

Quarkyonic Matter can satisfy constraints to the EoS of NS





Model for the dependence of Δ with the baryon density:

$$\Delta = \frac{\Lambda^3}{k_{FB}^2} + \kappa \frac{\Lambda}{N_c^2}$$

McLerran, Reddy, PRL 122,122701 (2019)

Dynamically generated shell of nucleons: Excluded Volume Model

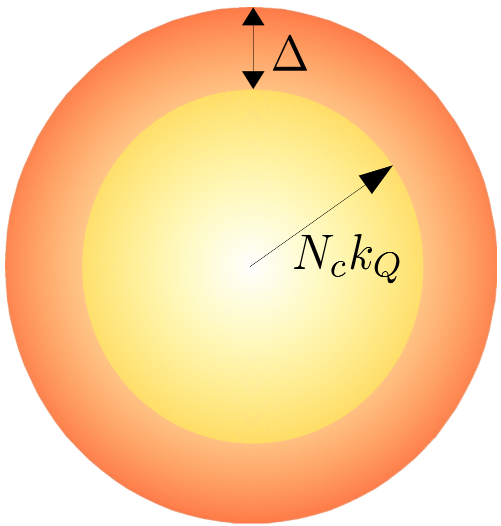
$$n_{ex} = \frac{n_N}{1 - n_N/n_0} = \frac{N_f}{\pi^2}\int_{k_F}^{k_F+\Delta} dk\, k^2$$

$$\varepsilon = \frac{N_f}{\pi^2}\left(1 - \frac{n_N}{n_0}\right)\int_{k_F}^{k_F+\Delta} dk\, k^2\sqrt{k^2+M^2} + \varepsilon_Q$$

Free gas of quarks contribution

$$\begin{cases} n_Q = \dfrac{N_f}{\pi^2}\displaystyle\int_0^{k_Q} dk\, k^2 = \dfrac{N_f}{3\pi^2}k_Q^3 \\ \varepsilon_Q = \dfrac{N_c N_f}{\pi^2}\displaystyle\int_0^{k_Q} dk\, k^2\sqrt{k^2+m^2} \end{cases}$$

$$k_Q = k_F/N_c \qquad m = M/N_c$$



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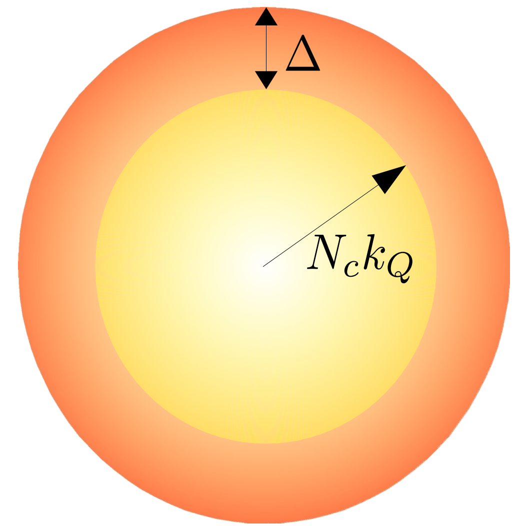

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$$\varepsilon_Q = \frac{N_c N_f}{\pi^2} \int_0^{k_Q} dk\, k^2 \sqrt{k^2 + m^2}$$

At small k_Q quark density increase very fast to generate acceptable sound velocity ➡ Modification in the low density Fermi distribution in a way that does not affect its behavior for large Fermi momenta:

$$1 \to \frac{\sqrt{k_Q^2 + \Lambda^2}}{k_Q}$$

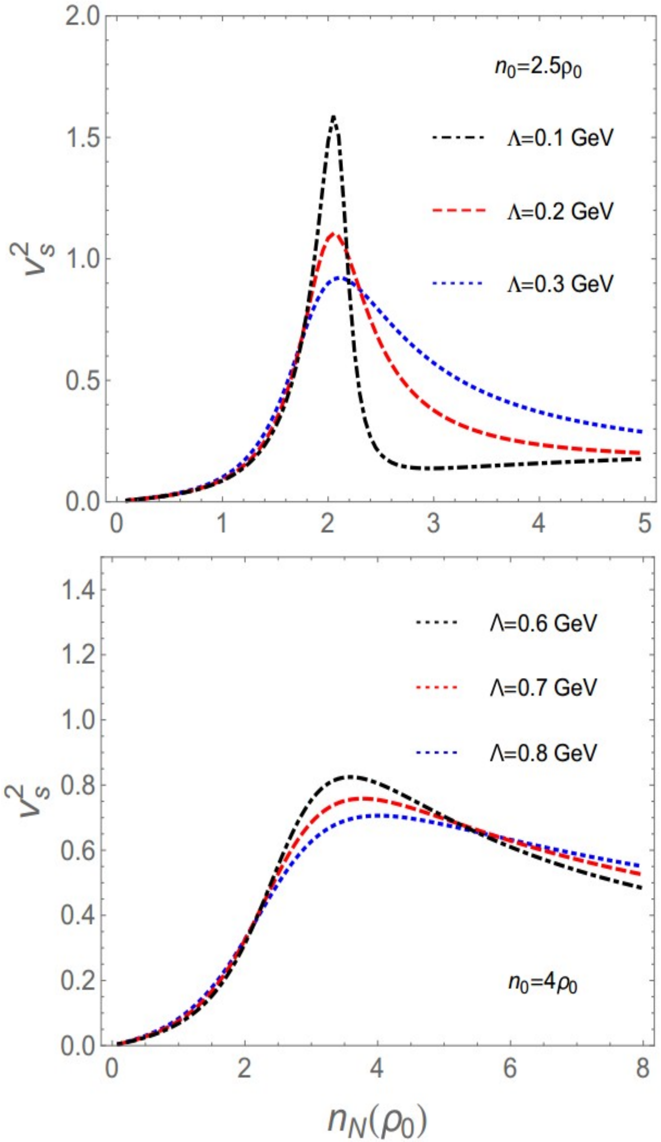
$$n_Q = \frac{N_f}{3\pi^2}\left[\left(k_Q^2 + \Lambda^2\right)^{3/2} - \Lambda^3\right]$$

At the minimum of the energy density: $\mu_N = N_c \mu_Q$

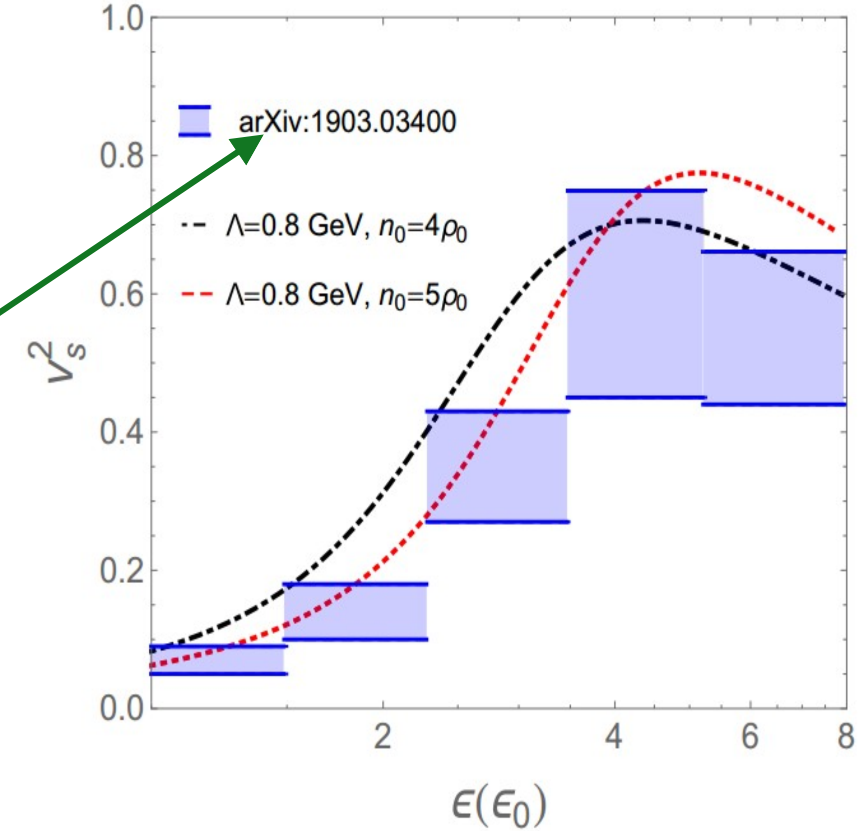
$$\varepsilon_Q = \frac{N_c N_f}{\pi^2} \int_0^{k_Q} dk\, k \sqrt{k^2 + \Lambda^2} \sqrt{k^2 + m^2}$$

K.S. Jeong, L. McLerran, S. Sen, Phys. Rev. C 101 035201 (2020)

Quarkyonic Matter can satisfy constraints to the EoS of NS



Fujimoto, Fukushima, Murase, PRD101,054016(2020)



Good agreement with sound velocity obtained from an equation of state extracted from neutron stars properties using deep neural network.

Jeong, McLerran, Sen, PRC 101,035201(2020)

3 Flavor Excluded Volume Model: Baryons

- Hard core repulsion: Scale can be measured by the effective size of the baryon.
- Protons + Neutrons + Hyperons in an excluded volume $v_0 = 1/n_0$:

$$n_N = n_p + n_n + n_\Lambda; \qquad n_{\tilde{N}} = n_p + n_n + (1+\alpha) n_\Lambda$$

$$n^{ex}_{N_i} = \frac{n_{N_i}}{1 - n_{\tilde{N}}/n_0} = 2 \int_0^{K_F^i} \frac{dk k^2}{2\pi^2}$$

$$K_F^i = \left(3\pi^2 \frac{n_i}{1 - n_{\tilde{N}}/n_0} \right)^{1/3}$$

$$\varepsilon_N = \left(1 - \frac{n_{\tilde{N}}}{n_0} \right) \sum_{i=p,n,\Lambda} \int_0^{K_F^i} \frac{dk k^2}{\pi^2} \sqrt{k^2 + m_i^2}$$

DD, K.S. Jeong, S. Hernandez-Ortiz, PRC 102, 025203(2020)

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- For neutron stars phenomenology: β-equilibrium and charge neutrality must be imposed.

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DD, K.S. Jeong, S. Hernandez-Ortiz, PRC 102, 025203(2020)

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- For neutron stars phenomenology: β-equilibrium and charge neutrality must be imposed.

Strength of Λ repulsion

$$n^{ex}_{N_i} = \frac{n_{N_i}}{1 - n_{\tilde{N}}/n_0} = 2\int_0^{K_F^i} \frac{dk k^2}{2\pi^2}$$

$$K_F^i = \left(3\pi^2 \frac{n_i}{1 - n_{\tilde{N}}/n_0}\right)^{1/3}$$

$$\varepsilon_N = \left(1 - \frac{n_{\tilde{N}}}{n_0}\right) \sum_{i=p,n,\Lambda} \int_0^{K_F^i} \frac{dk k^2}{\pi^2} \sqrt{k^2 + m_i^2} + \varepsilon_e$$

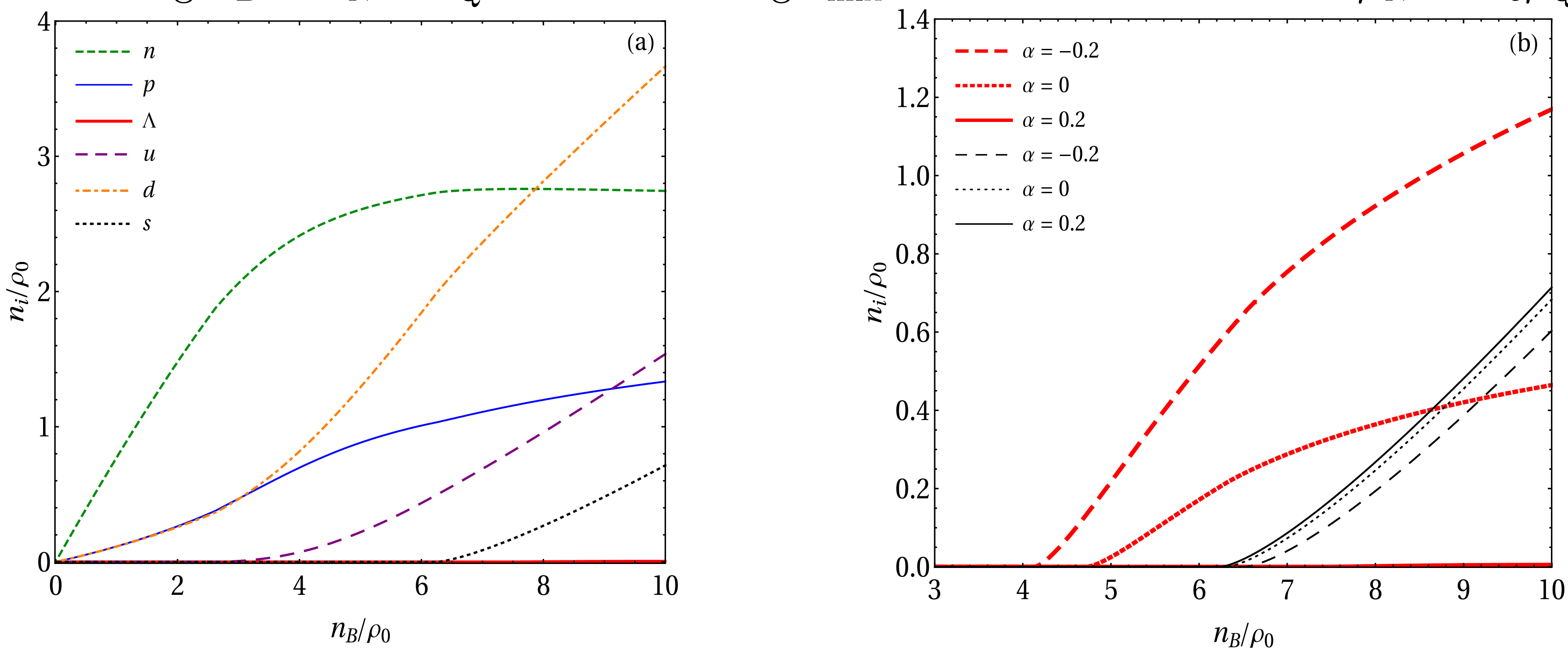
DD, K.S. Jeong, S. Hernandez-Ortiz, PRC 102, 025203(2020)

3 Flavor Excluded Volume Model: Baryons + Quarks

➞ Protons + Neutrons + Hyperons in an excluded volume + *u, d, s* quarks.

$$\varepsilon_{\rm mix} = \left(1 - \frac{n_{\tilde{N}}}{n_0}\right) \sum_{i=p,n,\Lambda} \int_0^{K_F^i} \frac{dk k^2}{\pi^2} \sqrt{k^2 + M_i^2} + \varepsilon_e + \sum_{j=u,d,s} \int_0^{k_F^{Q_j}} \frac{dk k^2}{\pi^2} \sqrt{k^2 + m_j^2}$$

Fixing $n_B = n_N + n_Q$ and minimizing $\varepsilon_{\rm mix}$ we obtain the condition $\mu_N = N_c \mu_Q$



DD, K.S. Jeong, S. Hernandez-Ortiz, PRC 102, 025203(2020)

- **For the minimum of energy density:**

$$dn_B = dn_n + dn_Q = 0$$

which results in

$$\mu_n = N_c \mu_{\tilde{d}} - \mu_e$$

- **Electromagnetic charge neutrality**

$$n_e = n_p + 2n_{\tilde{u}} - n_{\tilde{d}} - n_{\tilde{s}}$$

- **Beta equilibrium conditions**

$$\mu_n = \mu_p + \mu_e$$

$$\mu_{\tilde{d}} = \mu_{\tilde{u}} + 3\mu_e$$

- **Existence of Λ hyperon**

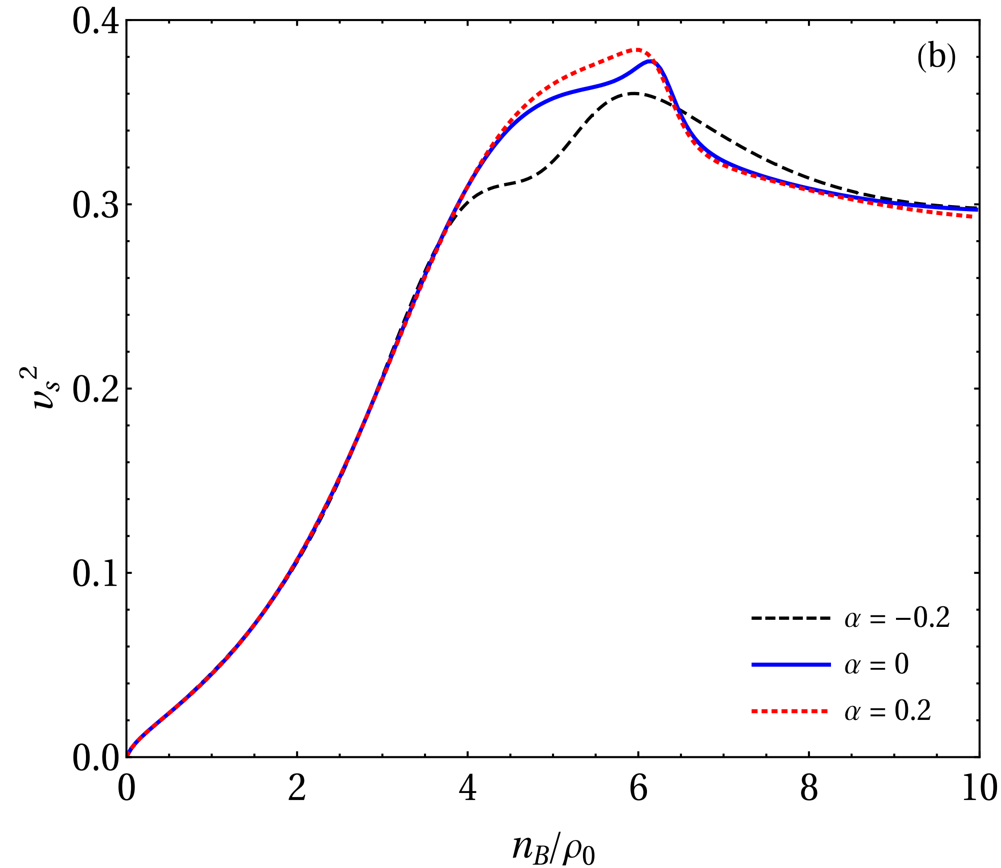
$$\mu_\Lambda = \mu_n$$

- **Existence of s quark**

$$\mu_{\tilde{s}} = \mu_{\tilde{d}}$$

$$c_s^2 = \frac{\partial P}{\partial \varepsilon_{\rm mix}} = \frac{n_B}{\mu_B}\left(\frac{\partial n_B}{\partial \mu_B}\right)^{-1}$$

$$P = -\varepsilon + \mu_B n_B$$

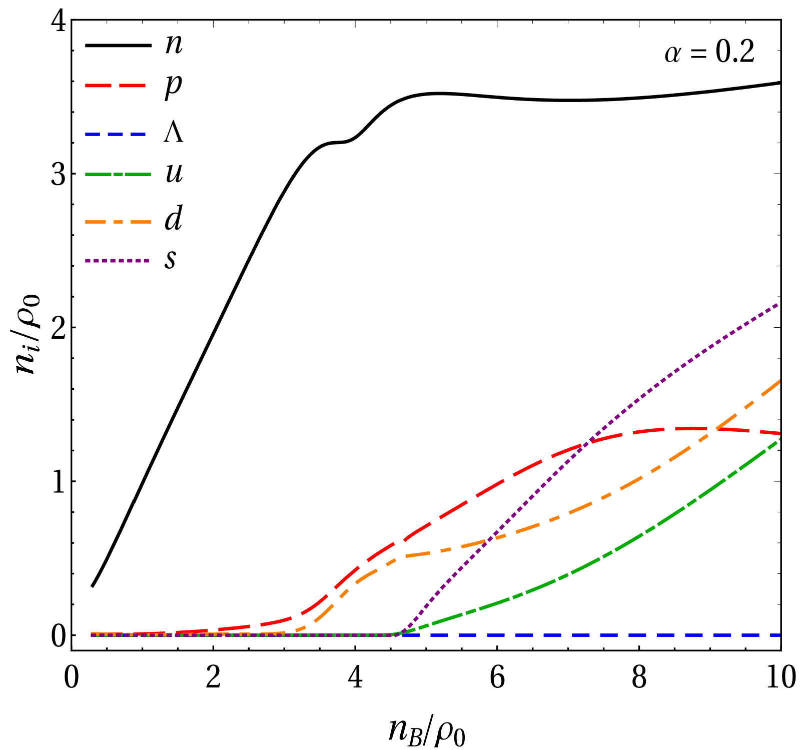


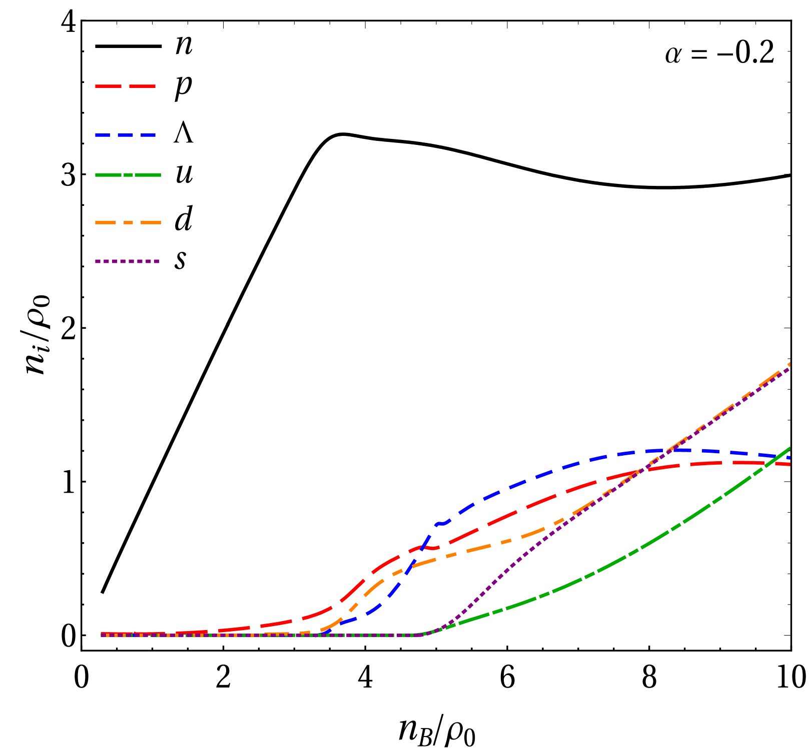
DD, K.S. Jeong, S. Hernandez-Ortiz, PRC 102, 025203(2020)

3 Flavor Excluded Volume Model: Shell-like distribution of Baryons

- Nucleons in a shell of width Δ, around a Fermi sea of quarks in β-equilibrium and charge neutral:

$$\varepsilon_{\text{qy.}} = 2\left(1-\frac{\tilde{n}_b}{n_0}\right)\sum_i^{\{n,p,\Lambda\}}\int_{k_F^{b_i}}^{[k_F+\Delta]_{b_i}}\frac{d^3k}{(2\pi)^3}\sqrt{k^2+m_{b_i}^2}+\frac{N_c}{\pi^2}\sum_j^{\{u,d,s\}}\int_0^{k_F^{Q_j}}dk\mathcal{M}_j(k^2)\sqrt{k^2+m_{Q_j}^2}+\frac{(3\pi^2)^{\frac{4}{3}}}{4\pi^2}n_e^{\frac{4}{3}}$$





DD, K.S. Jeong, S. Hernandez-Ortiz, PRC 102 065202 (2020)

3 Flavor Excluded Volume Model: Shell-like distribution of Baryons

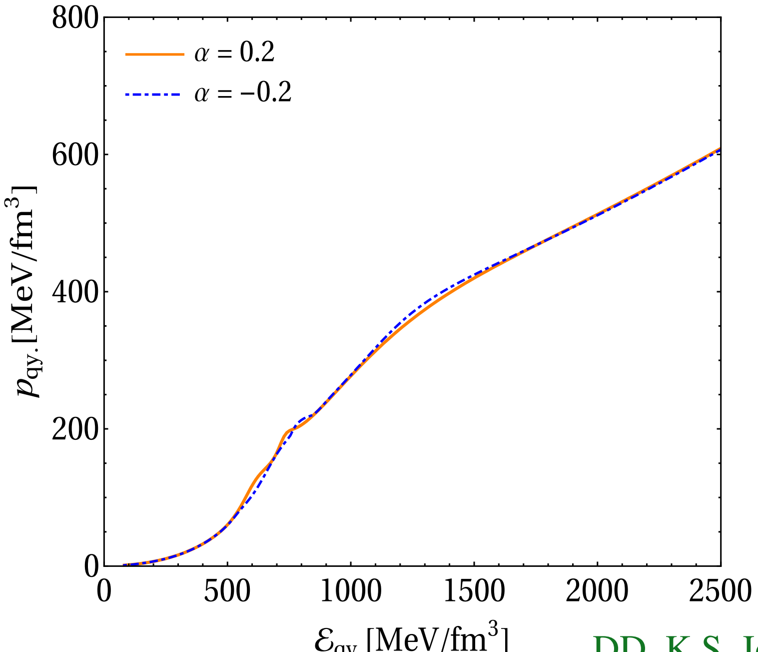
$$n_{\tilde{Q}_i} \equiv \frac{1}{\pi^2}\int_0^{k_F^{Q_i}} dk\mathcal{M}_i(k^2) = \frac{1}{\pi^2}\int_0^{k_F^{Q_i}} dk(k^2+\Lambda_{Q_i}^2)$$

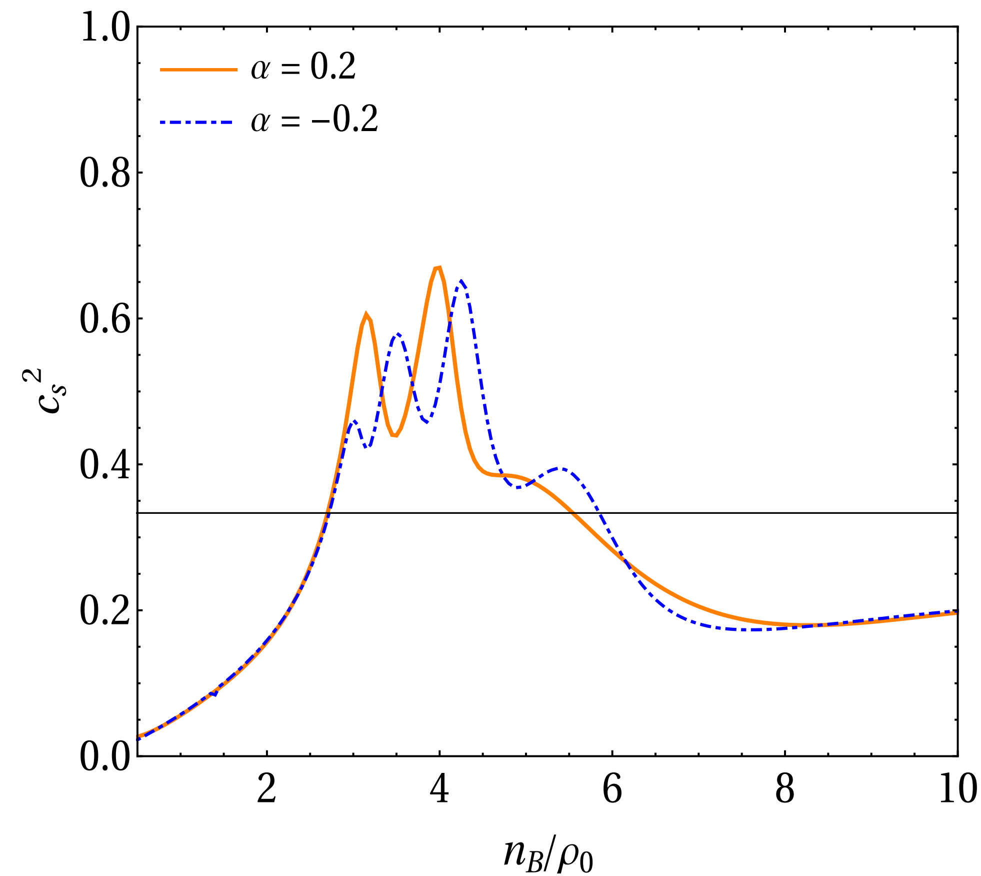
For the minimum of energy density:

$$dn_B = dn_n + dn_Q = 0$$

which results in

$$\mu_n = N_c\mu_{\tilde{d}} - \mu_e$$





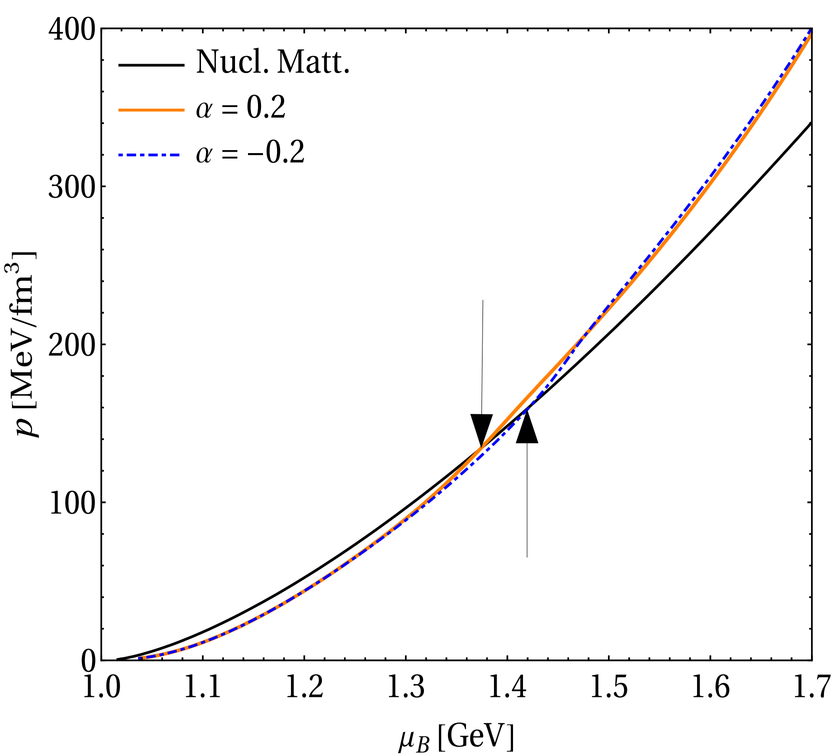
DD, K.S. Jeong, S. Hernandez-Ortiz, PRC 102 065202 (2020)

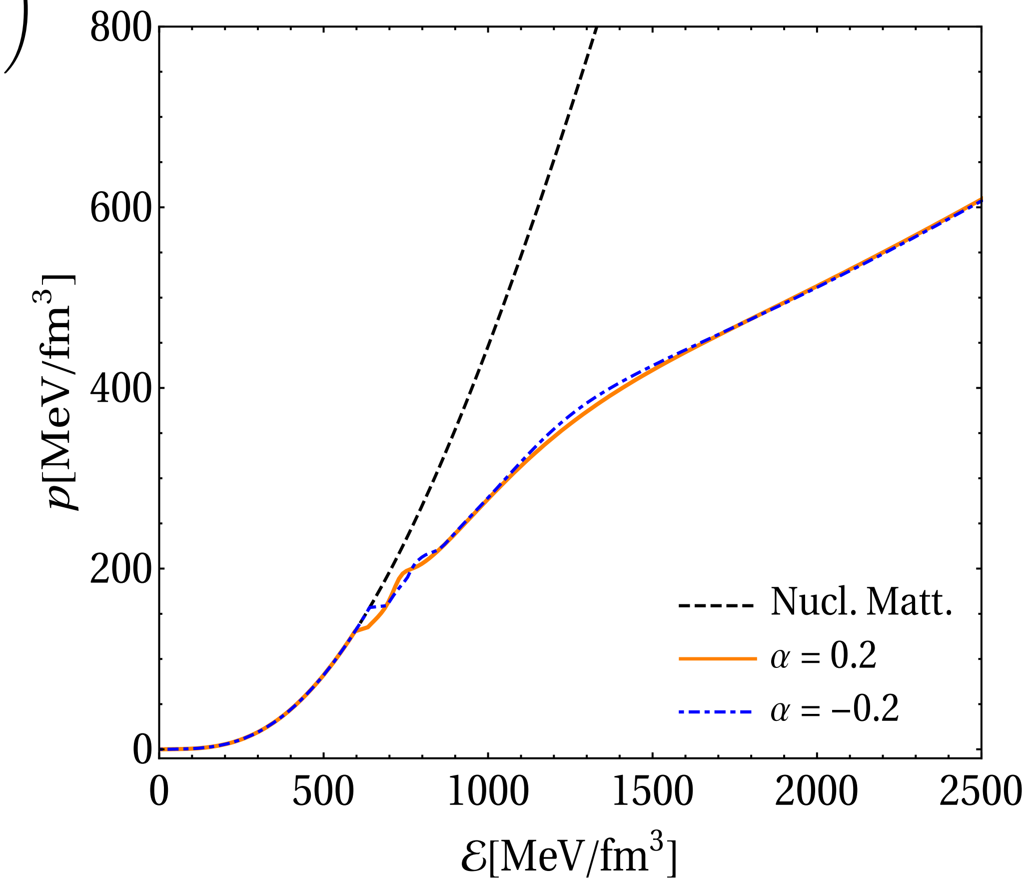
➞ Correction of low density regime EoS: use of a rich neutron matter EoS in the range $n_B < n_M$.

$$E/A = \sqrt{(p_F^n)^2 + M_n^2} - M_n + \tilde{a}\left(\frac{n_n}{\rho_0}\right) + \tilde{b}\left(\frac{n_n}{\rho_0}\right)^2$$

$\tilde{a} = -28.3$ MeV and $\tilde{b} = 10.7$ MeV

S. Gandolfi, et. al Eur. Phys. J. A 50, 10 (2014).





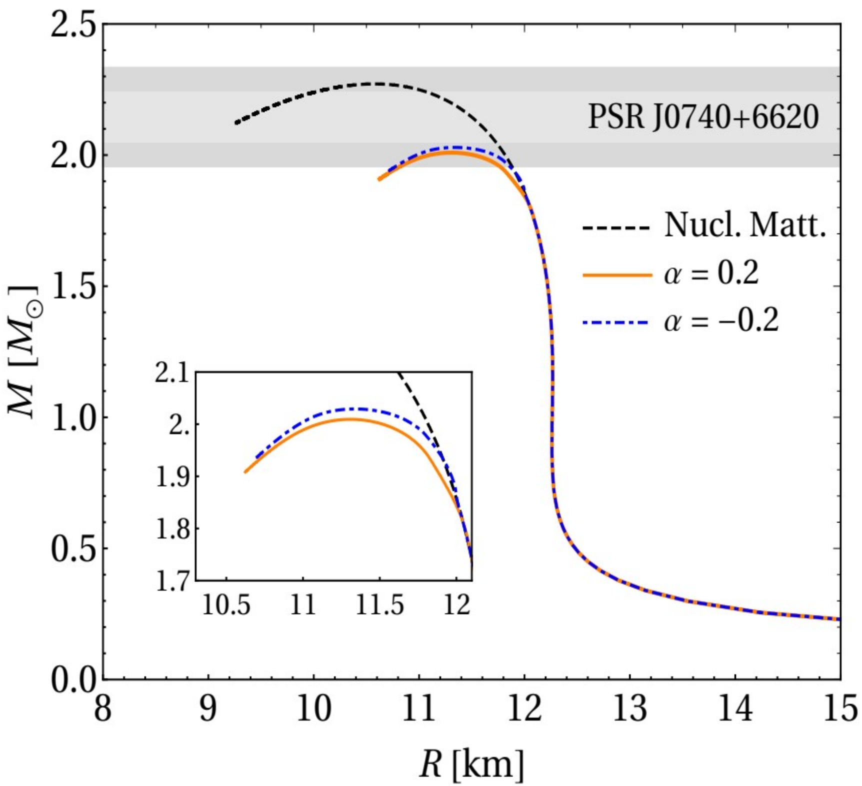
DD, K.S. Jeong, S. Hernandez-Ortiz, PRC 102 065202 (2020)

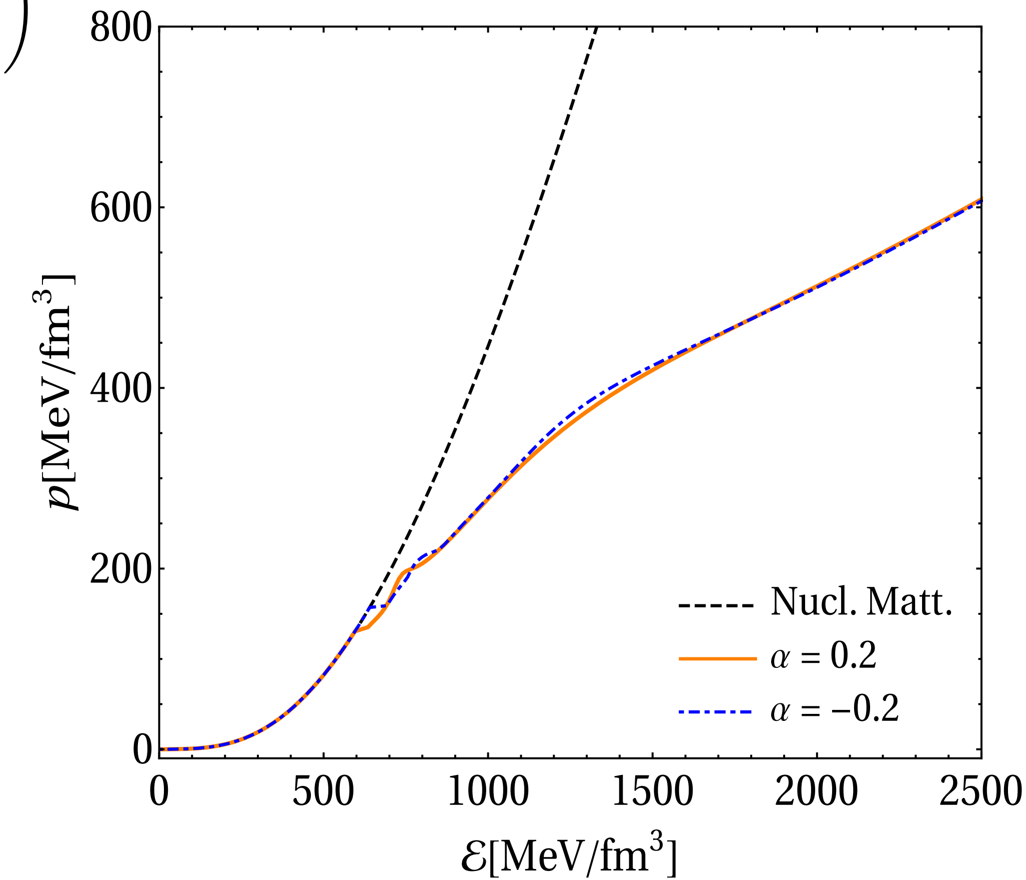
➡ Correction of low density regime EoS: use of a rich neutron matter EoS in the range $n_B < n_M$.

$$E/A = \sqrt{(p_F^n)^2 + M_n^2} - M_n + \tilde{a}\left(\frac{n_n}{\rho_0}\right) + \tilde{b}\left(\frac{n_n}{\rho_0}\right)^2$$

$\tilde{a} = -28.3$ MeV and $\tilde{b} = 10.7$ MeV

S. Gandolfi, et. al Eur. Phys. J. A 50, 10 (2014).





DD, K.S. Jeong, S. Hernandez-Ortiz, PRC 102 065202 (2020)

Summary

- We discussed some important concepts to define quarkyonic matter.
- The analysis of GW data have been providing very important insights about the properties of dense QCD matter.
- EV model improved: modifications in the low density regime to correctly describe the properties of nuclear matter at $n_B = \rho_0$: $\Lambda \to \Lambda(n_B)$ [Sen and Sivertsen, ApJ 915 109 (2021)] prevents the early appearance of quarks. The inclusion of a nuclear potential corrects the low density regime of EoS when considering the neutron quarkyonic matter.
- Extension to finite temperature: possible approach in the next lecture.

Thanks for your attention!

If we have nucleons with a hard core radius r_0, and a hard core volume $v_0 = \frac{4}{3}\pi r_0^3$, then we can define the hard core density as

$$n_0 = 1/v_0. \tag{17}$$

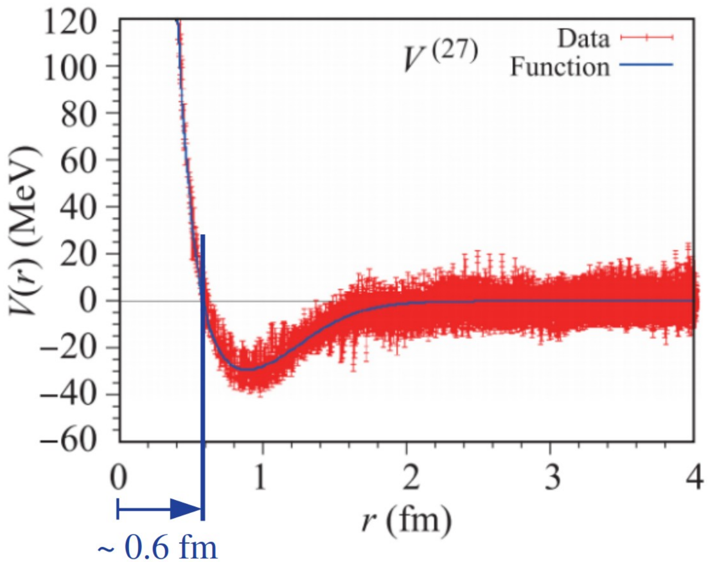
For a system with baryon density n and volume V, the excluded volume not occupied by baryon cores is

$$V_{ex} = V\ (1 - n/n_0). \tag{18}$$

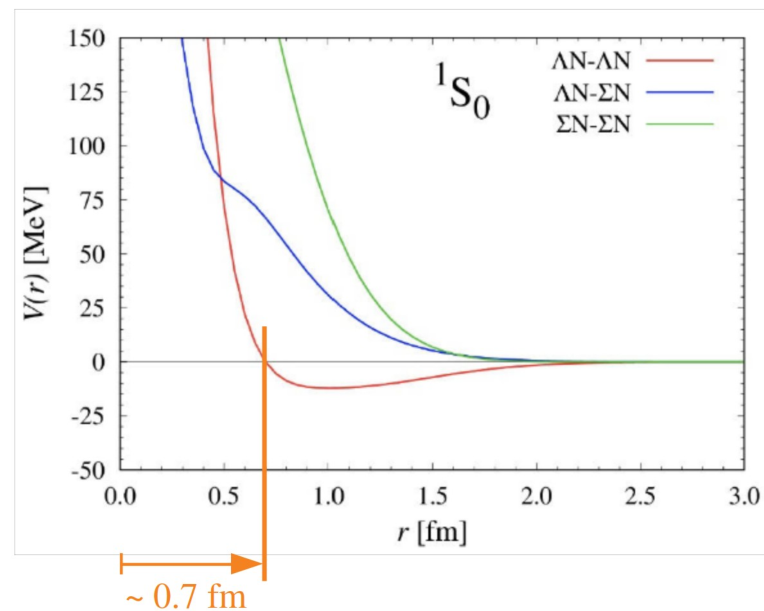
We assume the nucleons are free particles within the excluded volume so that

$$n_{ex} = \frac{n}{1 - n/n_0} = \frac{2}{(2\pi)^3} \int^{k_F} d^3p. \tag{19}$$

T. Hatsuda, Front. Phys. 13(6), 132105 (2018)



T. Hatsuda, private communication



$$n_{\tilde{B}} = n_p + n_n + (1+\alpha)n_\Lambda$$