

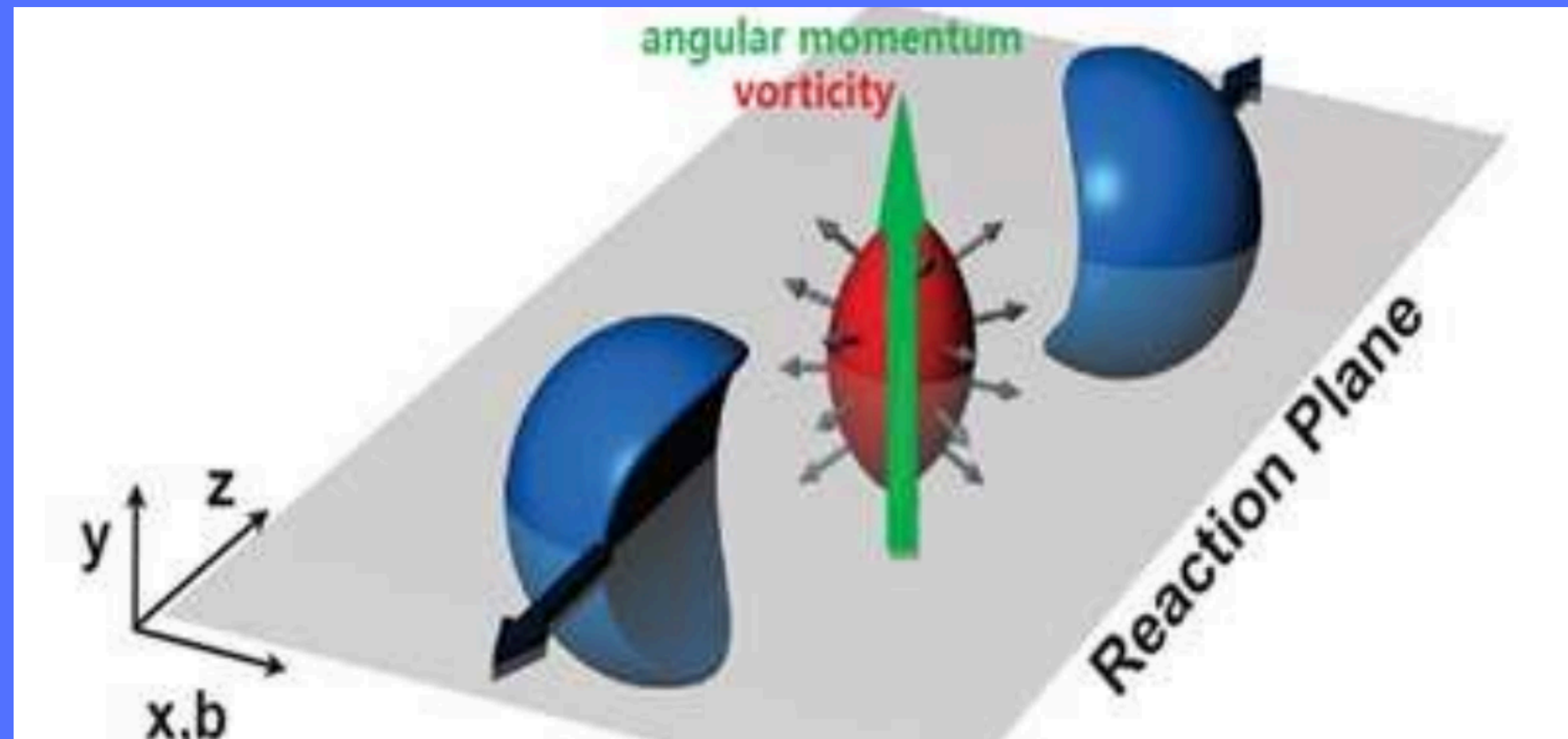
# ANOTHER PHASE DIAGRAM FOR THE STRONGLY INTERACTING MATTER

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First Latin American Workshop on Electromagnetic Effects in QCD  
july 22th, 2024

# CONTENT

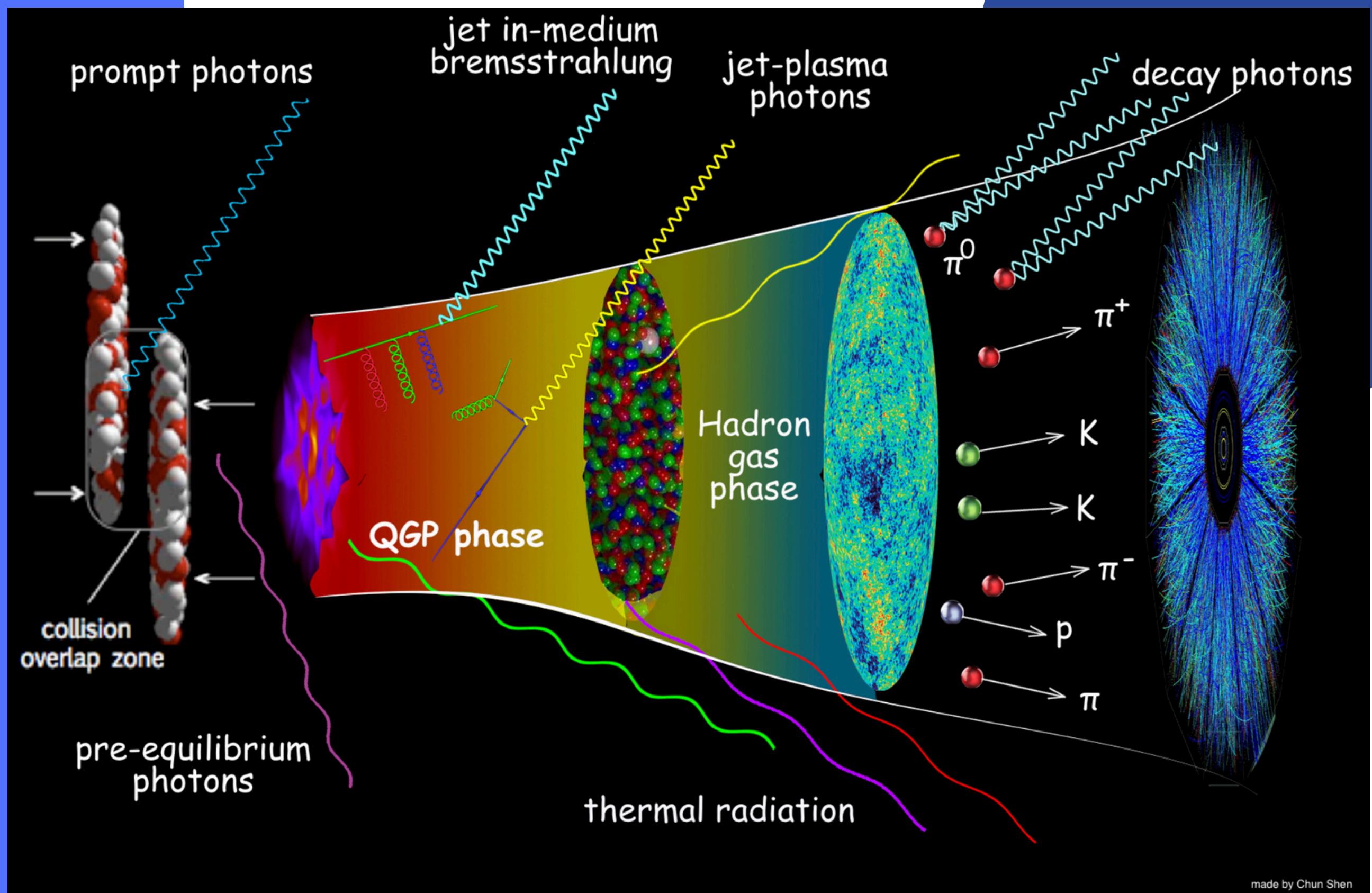


- PHYSICS  
MOTIVATION

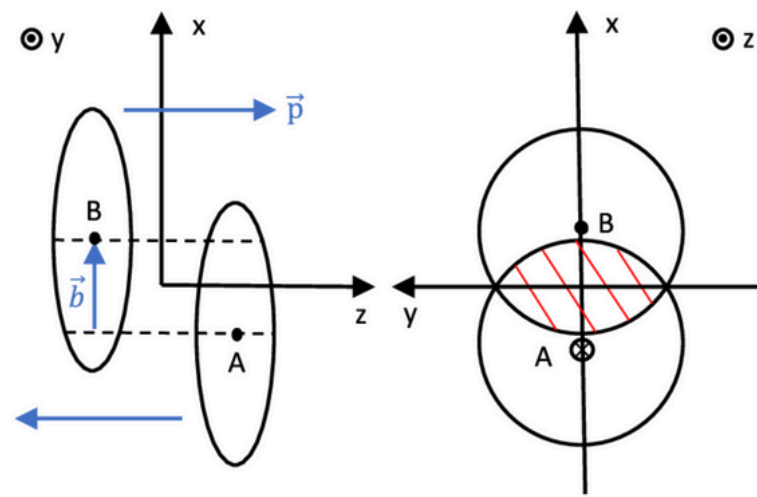
- LSMQ AND THE  
EFFECTIVE  
POTENTIAL

- PHASE DIAGRAMA  
AND FINAL  
REMARKS

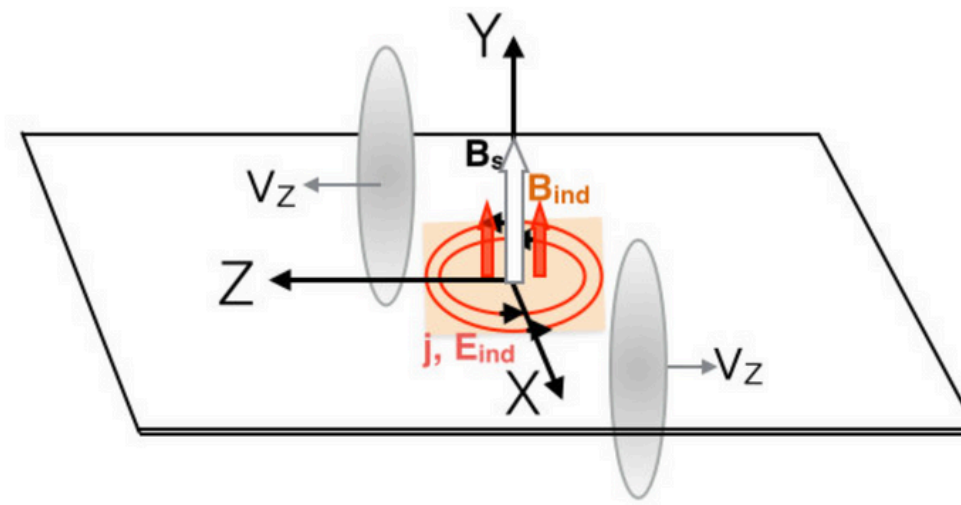
# HIC



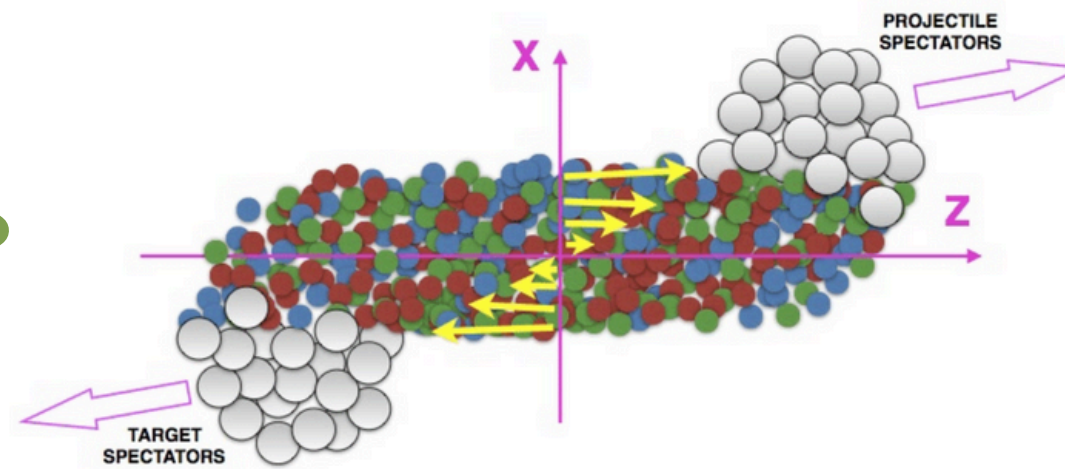
# NON CENTRAL COLISION



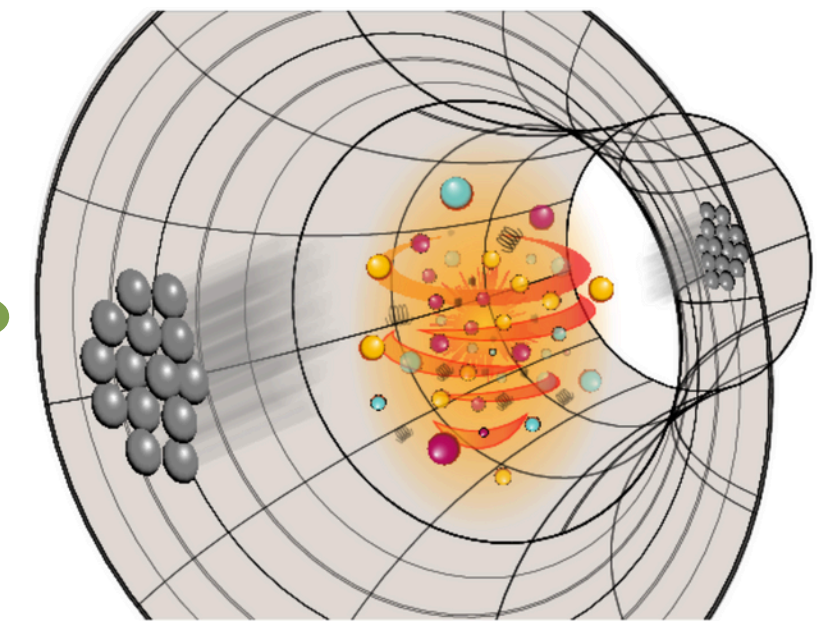
Eur.Phys.J.C 83 (2023) 1, 96



Phys.Rev.C 96 (2017) 5, 054909

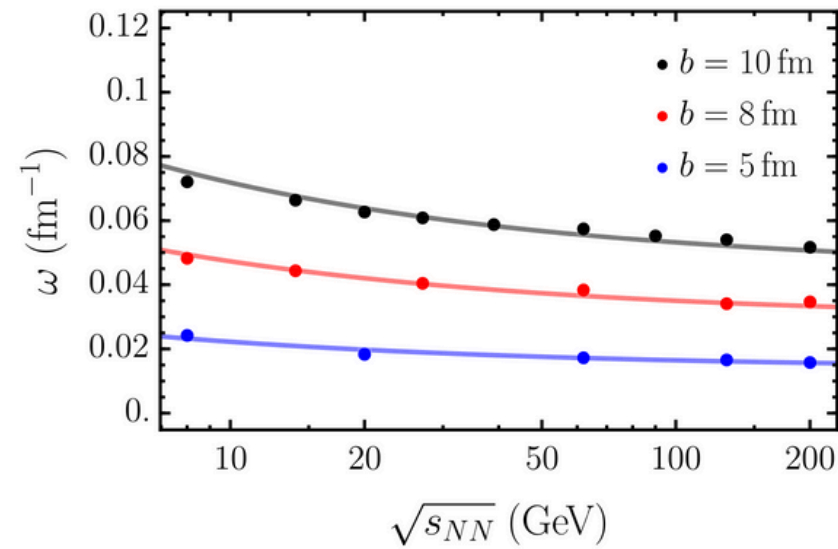


EPJ Web Conf. 171 (2018) 07002

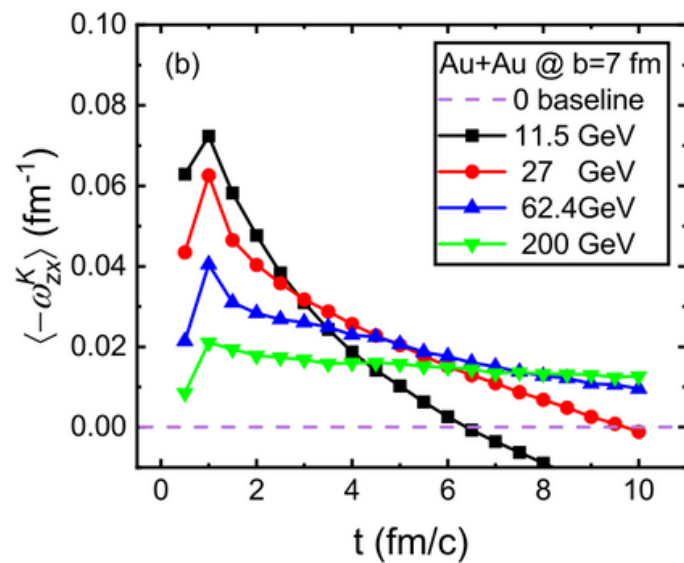


Nucl.Sci.Tech. 34 (2023) 1, 15

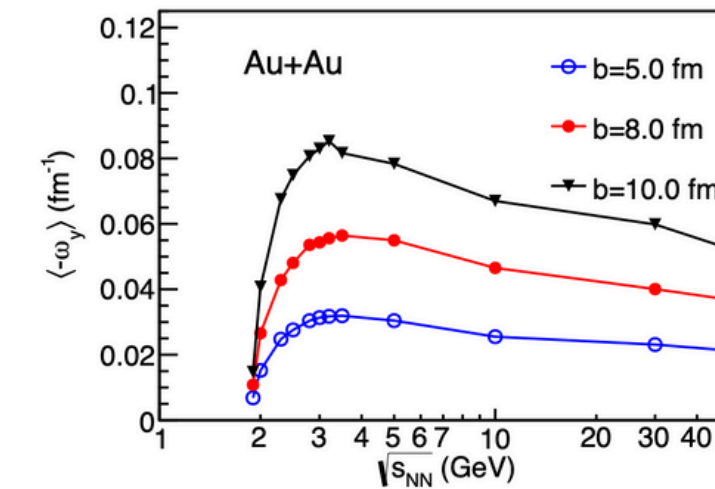
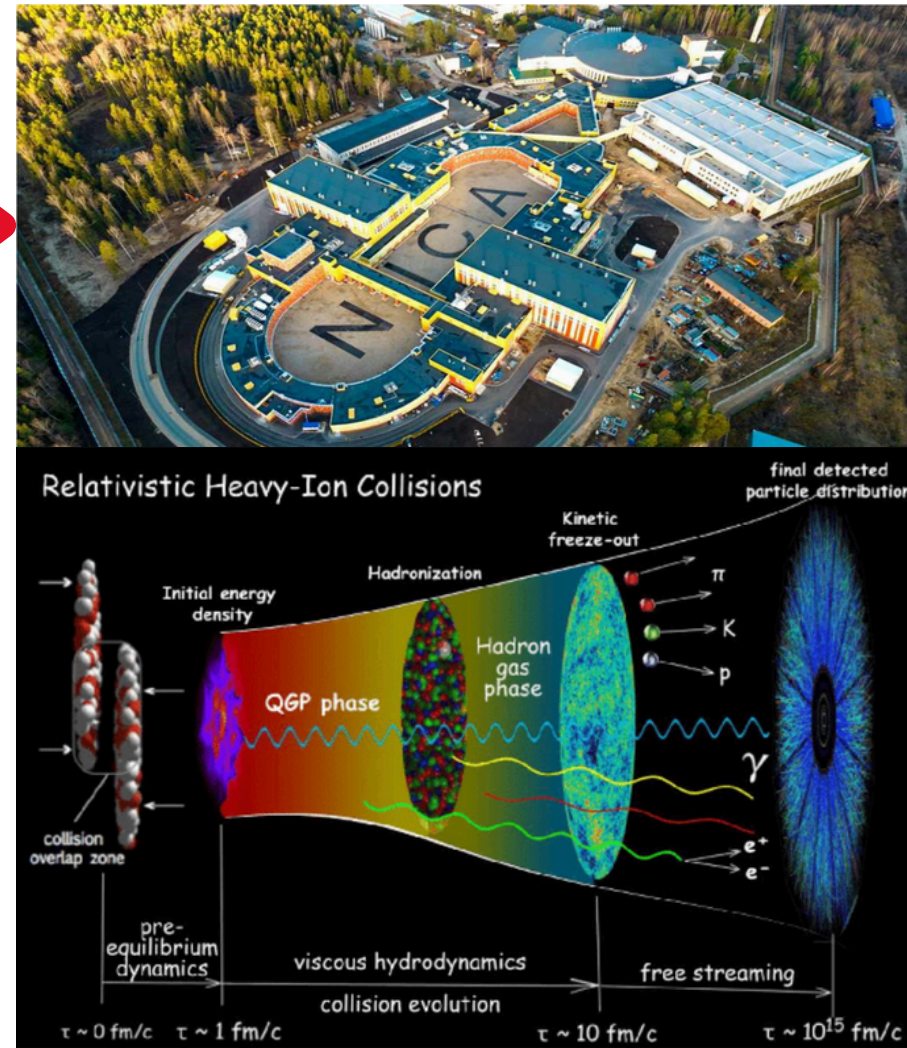
# ANGULAR VELOCITY



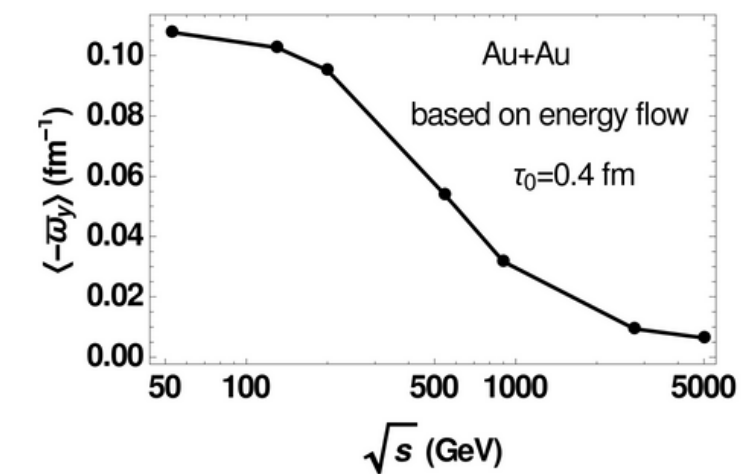
Initial angular velocity  $\omega$  for Au + Au collisions at impact parameters  $b = 5, 8, 10$  fm as functions of collision energy (UrQMD). Phys. Rev. D **102** (2020), 056019



Time evolution of angular velocity at  $b = 7$  fm and four different energies (PACIAE). Phys. Rev. C **104** (2021) 5, 054903



Initial angular velocity at mid rapidity as a function of the collision energy for impact parameters  $b = 5, 8,$  and  $10$  fm (UrQMD). Phys. Rev. C **101** (2020) 6, 064908



Angular velocity at fixed  $\tau = 0.4$  fm and  $\eta = 0$  as function of collision energy (HIJING). Phys. Rev. C **93** (2016), 064907

# HICs

1

## Phase transition

*Quark-gluon plasma  $\rightarrow$  Chiral symmetry*

2

## Non central collisions

*Finite impact parameter  $b$ .*

3

## Angular velocity

*Maximum value around  $0.1 \text{ 1/fm}$  aprox.  $20 \text{ MeV}$ .*

4

## Collision energy

*Angular velocity is more important at low collision energies.*

5

## Baryon chemical potential

*Region of maximum baryon density works MPD-NICA.*

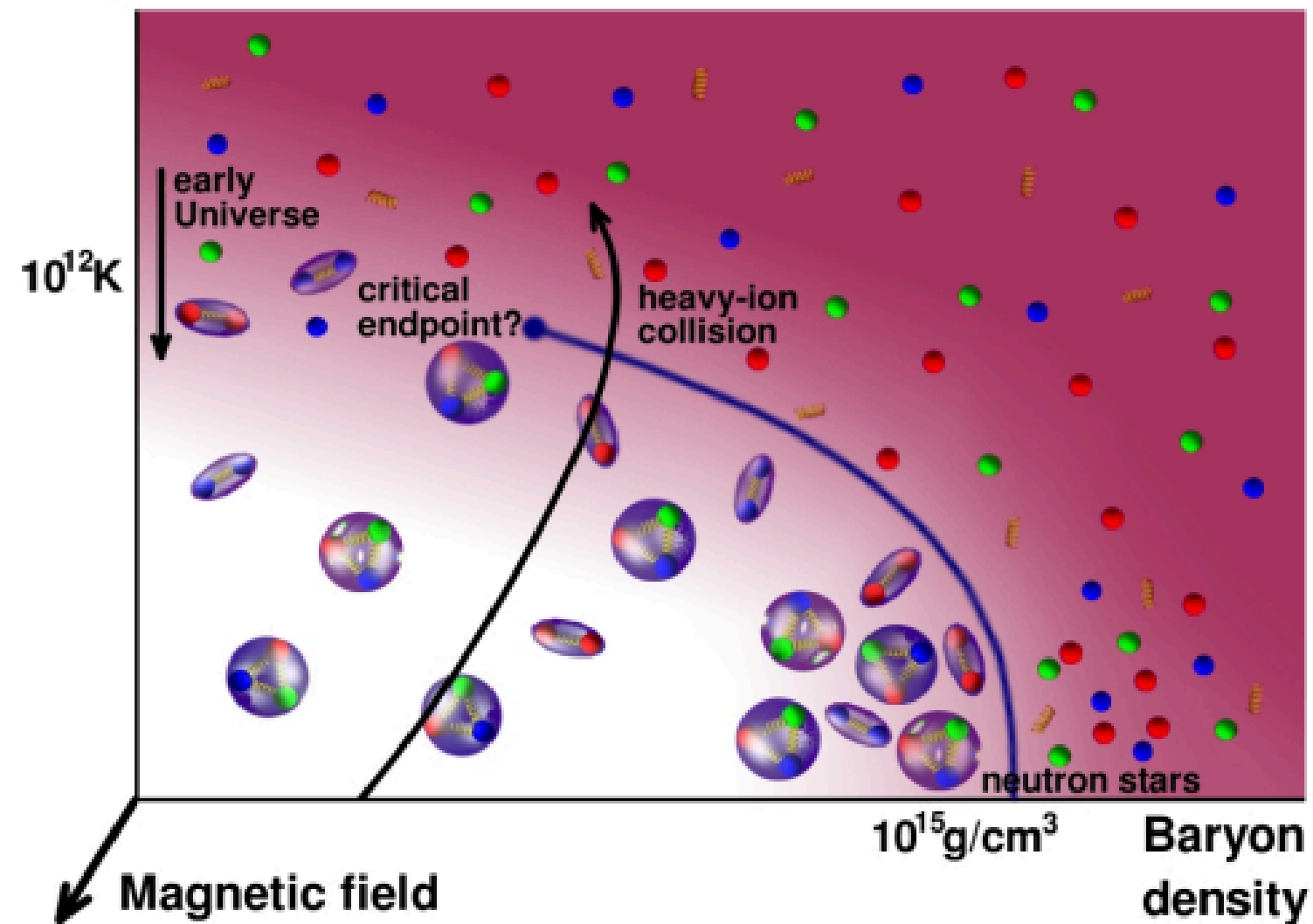
6

## Effective models

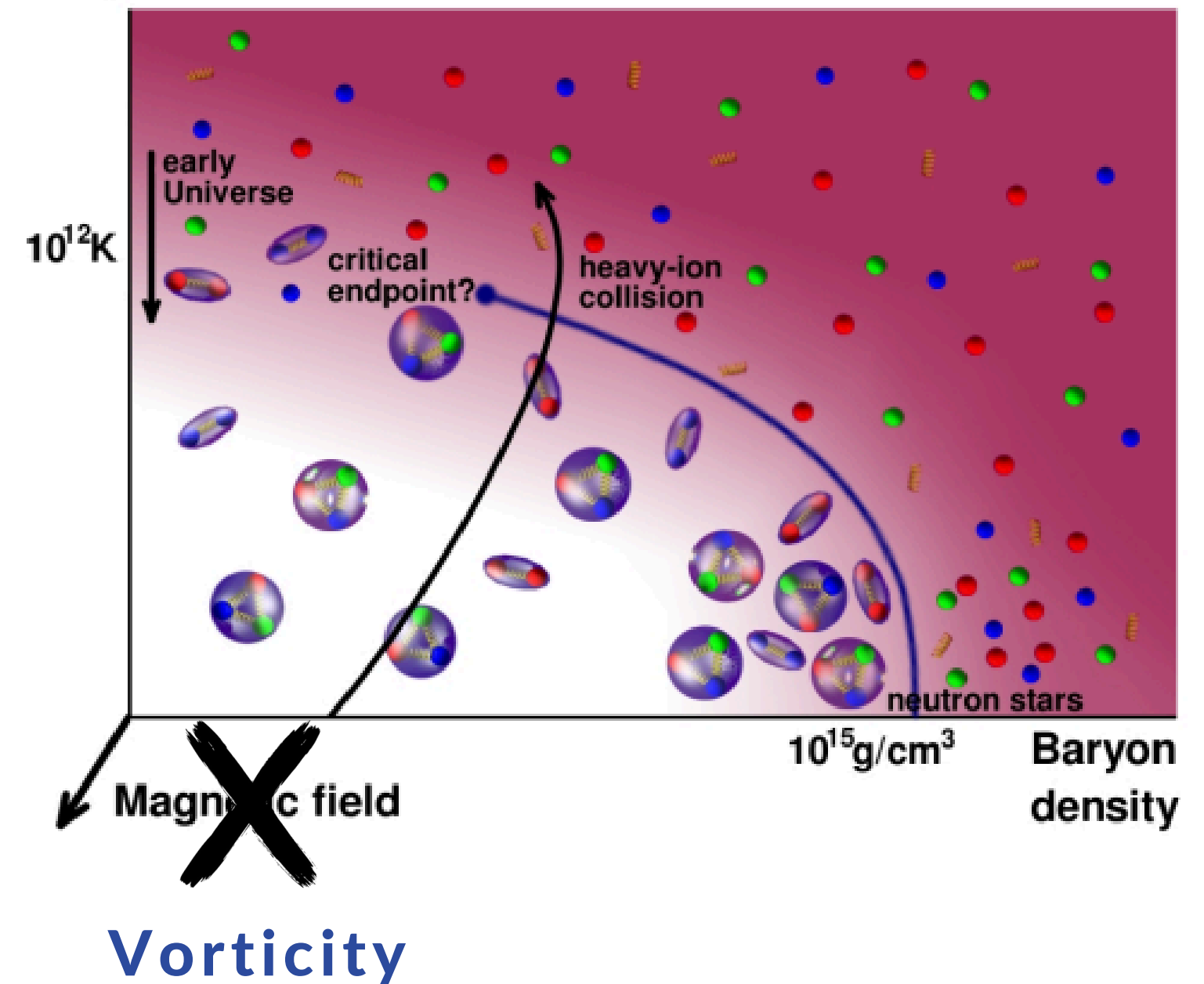
*Low energies of QCD.*

# QCD phase diagram

Temperature



Temperature



J.Phys.Conf.Ser. 503 (2014) 012009

# Linear Sigma model coupled to quarks

Effective theory which is useful to emulate the low energy regime of Quantum Chromodynamics. It exhibits a symmetry spontaneously broken.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu\partial_\mu\psi - ig\bar{\psi}\gamma^5\vec{\tau}\cdot\vec{\pi}\psi - g\bar{\psi}\psi\sigma$$

Letting the sigma-field to develop a vacuum expectation value  $v$ , we have

$$V^{tree} = -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4$$

$$m_\sigma^2 = 3\lambda v^2 - a^2, \quad m_\pi^2 = \lambda v^2 - a^2, \quad m_f = gv$$



# Effective potential

We compute the 1-loop corrections of bosons and fermions

$$V_b^1 = -\frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \ln(D_b^{-1}(k)) \quad , \quad V_f^1 = iN_c \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\ln(S_f^{-1}(k))]$$

where the propagators are

$$S(\tilde{\omega}_n, \Omega, \vec{k}) = -\frac{(i\tilde{\omega}_n + \frac{\Omega}{2} - k_z + ik_\perp)(\gamma_0 + \gamma_3) + m_f(1 + \gamma_5)}{(\tilde{\omega}_n - i\frac{\Omega}{2})^2 + \vec{k}^2 + m_f^2} \mathcal{O}_+ - \frac{(i\tilde{\omega}_n - \frac{\Omega}{2} + k_z - ik_\perp)(\gamma_0 - \gamma_3) + m_f(1 + \gamma_5)}{(\tilde{\omega}_n + i\frac{\Omega}{2})^2 + \vec{k}^2 + m_f^2} \mathcal{O}_-$$

- Phys.Rev.D **103** (2021) 7, 076021

$$D(p) = \frac{1}{(p_0 + \Omega)^2 - p_\perp^2 - p_z^2 - m^2 + i\epsilon}$$

- Phys.Rev.D **108** (2023) 9, 094020

# 1-loop boson contribution

Working within the Matsubara formalism

$$V_b^1 = T \sum_{n=-\infty}^{\infty} \int \frac{d^3 k}{(2\pi)^3} \ln D(\omega_n, \Omega, \vec{k})^{1/2},$$

We perform the sum over the Matsubara frequencies

$$V_b^1 = \frac{1}{4} \int \frac{d^3 k}{(2\pi)^3} \int dm_\phi^2 \frac{1}{E} \left[ 1 + \frac{1}{e^{\beta(E-\Omega)} - 1} + \frac{1}{e^{\beta(E+\Omega)} - 1} \right],$$

$$V_{b,\text{vac}}^1 = \frac{1}{4} \int \frac{d^3 k}{(2\pi)^3} \int dm_\phi^2 \frac{1}{\sqrt{k^2 + m_\phi^2}},$$

$$V_{b,\text{mat}}^1 = \frac{1}{4} \int \frac{d^3 k}{(2\pi)^3} \int dm_\phi^2 \frac{1}{\sqrt{k^2 + m_\phi^2}} \times \left[ \frac{1}{e^{\beta(\sqrt{k^2 + m_\phi^2} - \Omega)} - 1} + \frac{1}{e^{\beta(\sqrt{k^2 + m_\phi^2} + \Omega)} - 1} \right].$$

# 1-loop boson contribution

Vacuum term. We use the  $\overline{\text{MS}}$  scheme and then we obtain

$$V_{\text{b,vac}}^1 = -\frac{m_\phi^4}{64\pi^2} \left[ \ln\left(\frac{\mu^2}{m_\phi^2}\right) + \frac{3}{2} \right].$$



Dynamic mass of a boson

$$m_\sigma^2 = 3\lambda v^2 - a^2, \quad m_0^2 = \lambda v^2 - a^2$$

Matter term. We have

$$V_{\text{b,mat}}^1 = \frac{T^4}{2\pi^2} \int_0^\infty x^2 dx \left\{ \ln[1 - e^{-\sqrt{x^2+y^2+zy}}] + \ln[1 - e^{-\sqrt{x^2+y^2-zy}}] \right\}$$

$$V_{\text{b,mat}} = \frac{2T^{n+1}}{(4\pi)^{n/2}\Gamma(n/2)} \int_0^\infty x^{n-1} dx \times \left\{ \ln[1 - e^{-\sqrt{x^2+y^2+zy}}] + \ln[1 - e^{-\sqrt{x^2+y^2-zy}}] \right\},$$

J.Math.Phys. 23 (1982) 1852

High T approximation

$$V_{\text{b,mat}}^1 = -\frac{\pi^2 T^4}{90} + \frac{T^2}{24} (m_\phi^2 - 2\Omega^2)$$

$$- \frac{T}{12\pi} (m_\phi^2 - \Omega^2)^{3/2} - \frac{\Omega^2}{48\pi^2} (3m_\phi^2 - \Omega^2)$$

$$- \frac{m_\phi^4}{64\pi^2} \left[ \ln\left(\frac{m_\phi^2}{16\pi^2 T^2}\right) + 2\gamma_E - \frac{3}{2} \right].$$

# 1-loop fermion contribution

Working within the Matsubara formalism

$$V_f^1 = -T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \text{Tr} [\ln S(\tilde{\omega}_n, \mu_q, \Omega, \vec{k})^{-1}],$$

$$V_f^1 = -\frac{T}{2} \sum_{n=-\infty}^{\infty} \int dm_f^2 \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1}{(\omega_n - i\mu_1)^2 + \vec{k}^2 + m_f^2} + \frac{1}{(\omega_n - i\mu_2)^2 + \vec{k}^2 + m_f^2} \right]$$

with

$$\mu_1 = \mu_q + \frac{\Omega}{2}, \quad \mu_2 = \mu_q - \frac{\Omega}{2}$$

$$V_{f,\text{vac}}^1 = \frac{m_f^4}{16\pi^2} \left[ \ln \left[ \frac{\mu^2}{m_f^2} \right] + 2\gamma_E + \frac{3}{2} \right]$$

$$\begin{aligned} V_{f,\text{mat}}^1 = & \frac{m_f^4}{16\pi^2} \left[ \ln \left[ \frac{m_f^2}{\pi^2 T^2} \right] + 2\gamma_E - \frac{3}{2} \right] - \frac{7T^4\pi^2}{360} \\ & - \frac{T^2}{24} \left( \left( \mu_q + \frac{\Omega}{2} \right)^2 + \left( \mu_q - \frac{\Omega}{2} \right)^2 \right) \\ & + \frac{T^2 m_f^2}{4\pi^2} \left( \text{Li}_2 \left( -e^{-\frac{\mu+\Omega}{T}} \right) + \text{Li}_2 \left( -e^{-\frac{\mu-\Omega}{T}} \right) \right. \\ & \left. + \text{Li}_2 \left( -e^{-\frac{\mu+\Omega}{T}} \right) + \text{Li}_2 \left( -e^{-\frac{\mu-\Omega}{T}} \right) \right) \\ & + \frac{(\mu + \frac{\Omega}{2})^4 + (\mu - \frac{\Omega}{2})^4}{48\pi^2} \end{aligned}$$

# Ring diagrams

$$V^{\text{ring}} = \frac{T}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \ln (1 + \Pi D(\omega_n, \Omega, \vec{k})),$$

We work with the high T approximation

$$V^{\text{ring}} = \frac{T}{2} \int \frac{d^3k}{(2\pi)^3} \ln (1 + \Pi D(\omega_n, \Omega, \vec{k}))$$

We obtain

$$V^{\text{ring}} = -\frac{T}{12\pi} (m_\phi^2 - \Omega^2 + \Pi)^{3/2} + \frac{T}{12\pi} (m_\phi^2 - \Omega^2)^{3/2}.$$

# Effective Potential

$$\begin{aligned}
 V^{\text{eff}} = & -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 + \sum_{b=\sigma,\bar{\pi}} \left\{ -\frac{m_b^4}{64\pi^2} \left[ \ln \left( \frac{\mu^2}{16\pi^2 T^2} \right) + 2\gamma_E \right] \right. \\
 & - \frac{\pi^2 T^4}{90} + \frac{T^2}{24} (m_b^2 - 2\Omega^2) - \frac{T (\Pi + m_b^2 - \Omega^2)^{3/2}}{12\pi} \\
 & \left. - \frac{\Omega^2}{48\pi^2} (3m_b^2 - \Omega^2) \right\} + N_f N_c \left\{ \frac{m_f^4}{16\pi^2} \left[ \ln \left( \frac{\mu^2}{\pi^2 T^2} \right) + 2\gamma_E \right] \right. \\
 & - \frac{7T^4 \pi^2}{360} - \frac{T^2}{24} \left( \left( \mu_q + \frac{\Omega}{2} \right)^2 + \left( \mu_q - \frac{\Omega}{2} \right)^2 \right) \\
 & + \frac{T^2 m_f^2}{4\pi^2} \left( \text{Li}_2 \left( -e^{-\frac{\mu + \frac{\Omega}{2}}{T}} \right) + \text{Li}_2 \left( -e^{-\frac{\mu - \frac{\Omega}{2}}{T}} \right) \right) \\
 & + \text{Li}_2 \left( -e^{-\frac{\mu + \frac{\Omega}{2}}{T}} \right) + \text{Li}_2 \left( -e^{-\frac{\mu - \frac{\Omega}{2}}{T}} \right) \\
 & \left. + \frac{(\mu + \frac{\Omega}{2})^4 + (\mu - \frac{\Omega}{2})^4}{48\pi^2} \right\}
 \end{aligned}$$

where the self-energy is

$$\Pi_\sigma = \frac{\lambda}{4} [12l(m_\sigma) + 4l(m_0) + 8l(m_b)] + N_f N_c \Pi_f$$

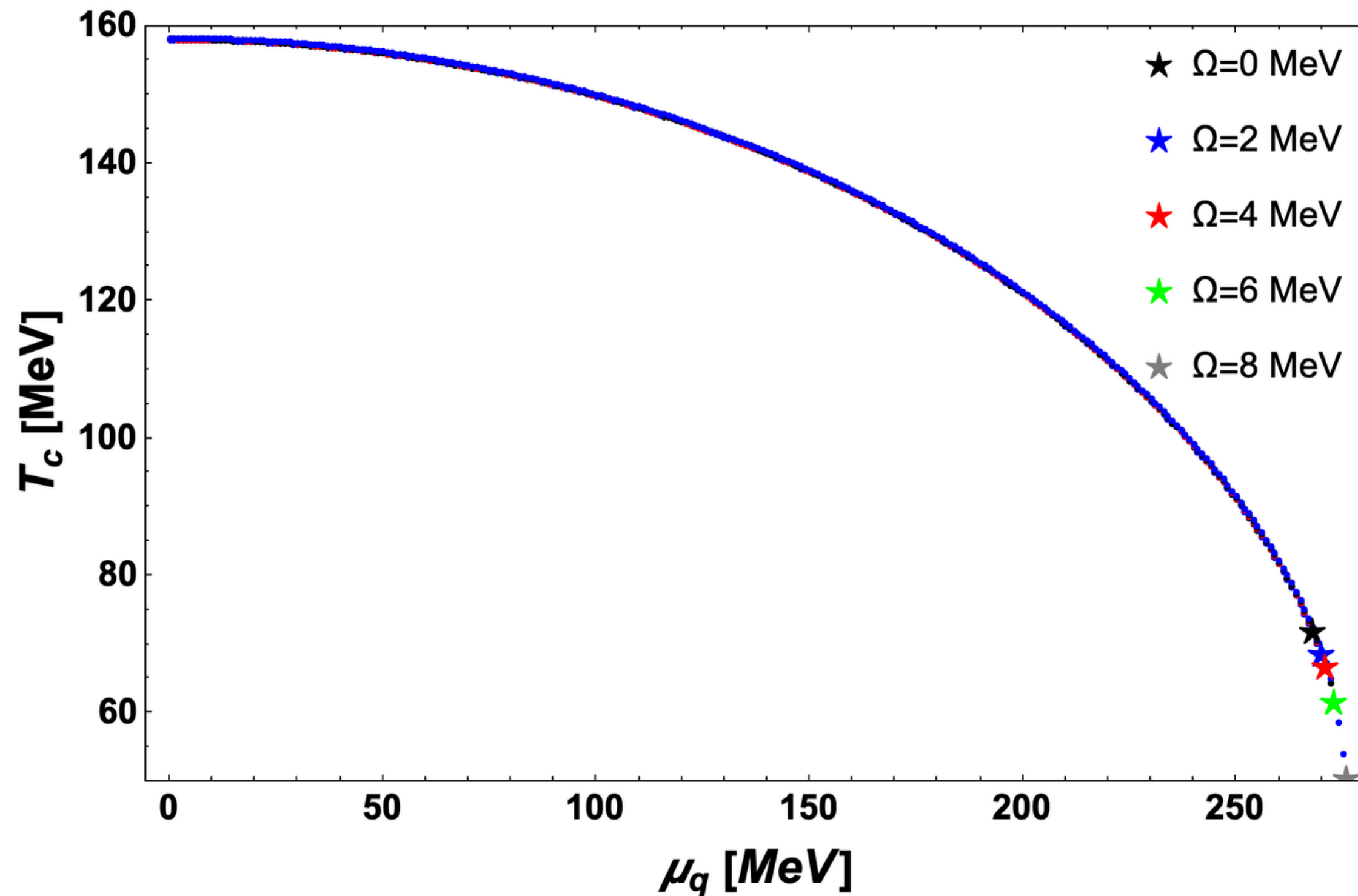
$$\Pi_0 = \frac{\lambda}{4} [4l(m_\sigma) + 12l(m_0) + 8l(m_b)] + N_f N_c \Pi_f$$

with

$$l(m_b) = 2 \frac{dV_b^1}{dm_b^2}$$

$$\Pi_f = 2g^2 \frac{dV_f^1}{dm_f^2}$$

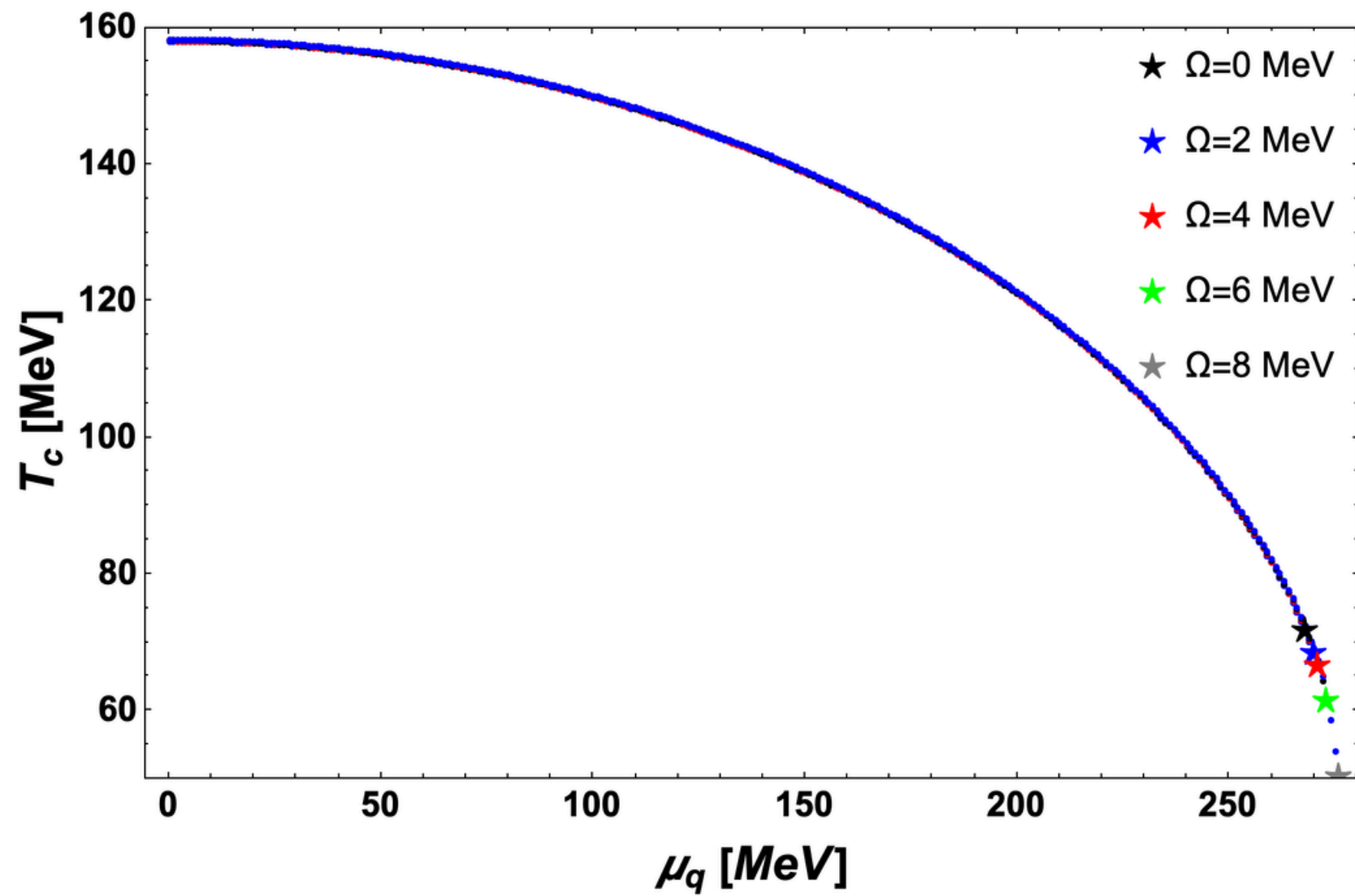
# Phase diagram



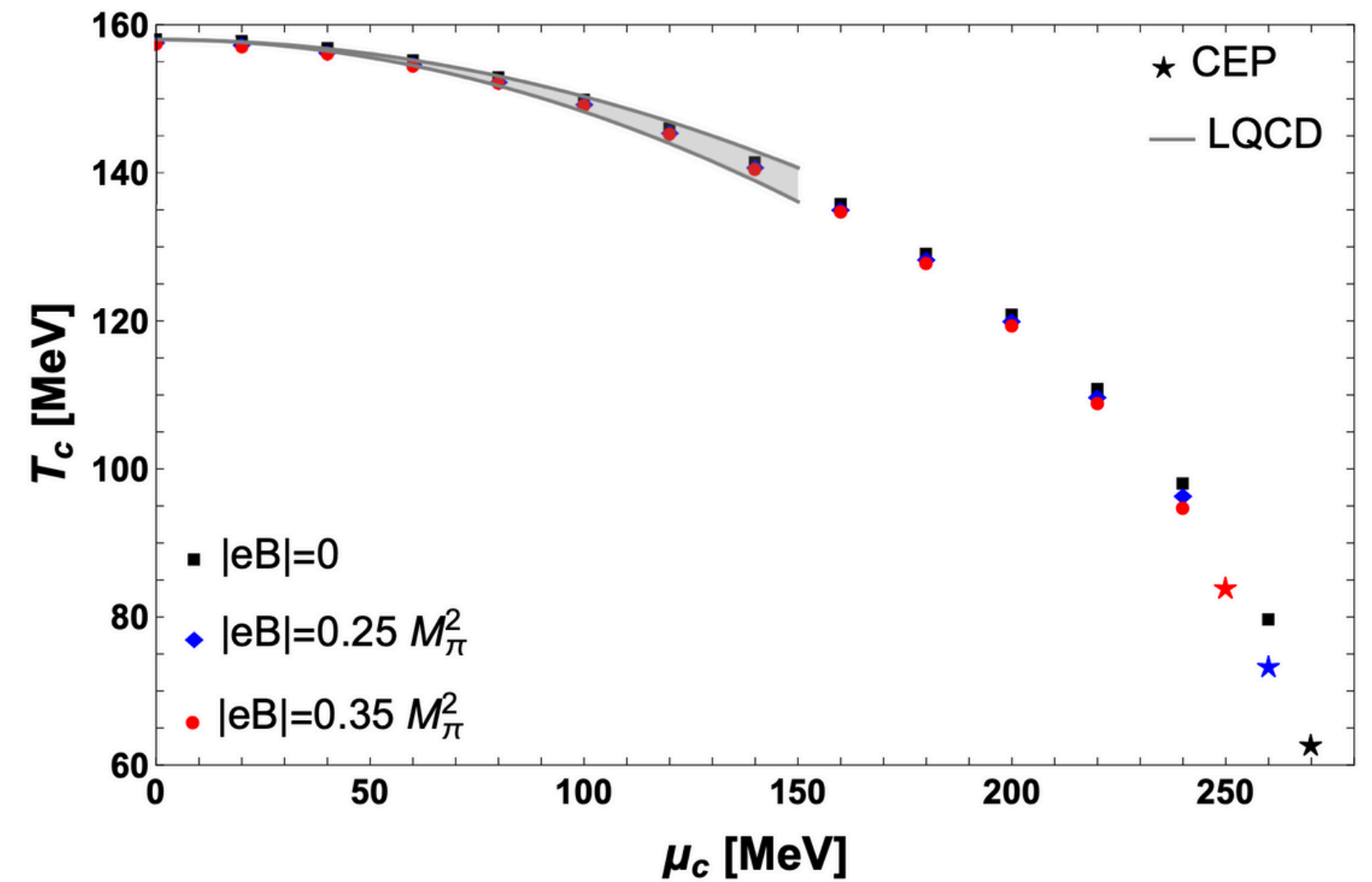
- $T_c$  decreases as angular velocity increases (Inverse Vortical Catalysis).
- The CEP is moving to the right and downwards

# Phase diagram

Eur.Phys.J.A 57 (2021) 7, 234



Vorticity



Magnetic field



## Conclusions and

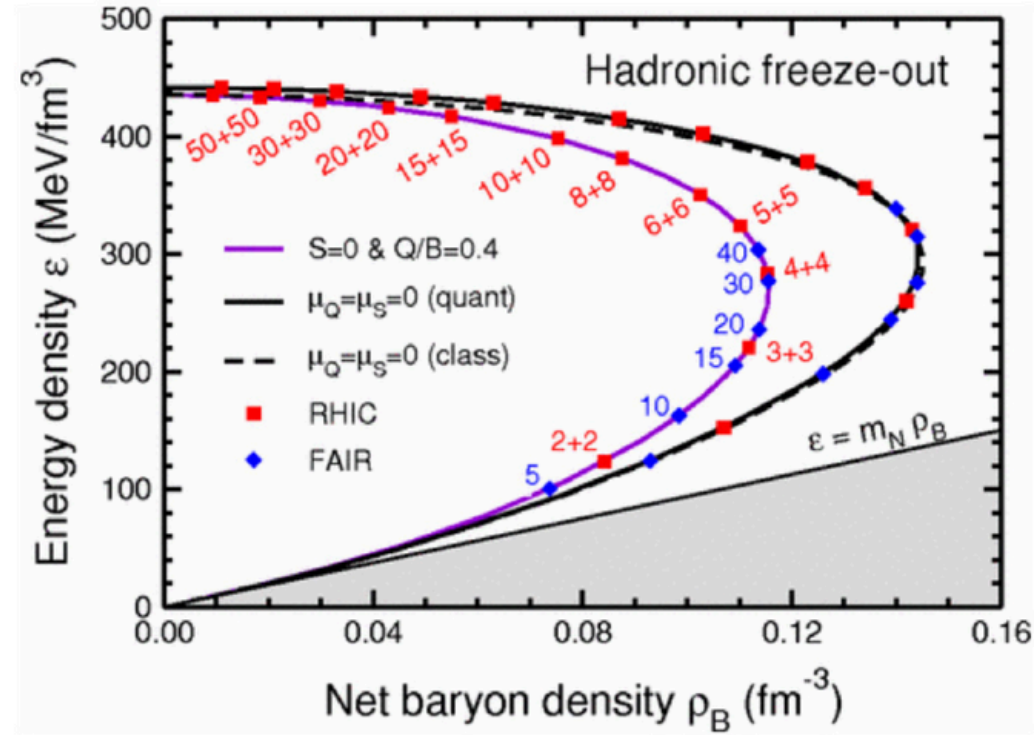
- 1 The angular velocity promotes the restoration of chiral symmetry
- 2 The CEP changes its position as a function of angular velocity
- 3 **Computation of the low T approximation**
- 4 **Enough equations to fix the free parameters.**
- 5 **Translate the information found to conditions of the collision.**



**¡Muchas gracias  
por su atención!**

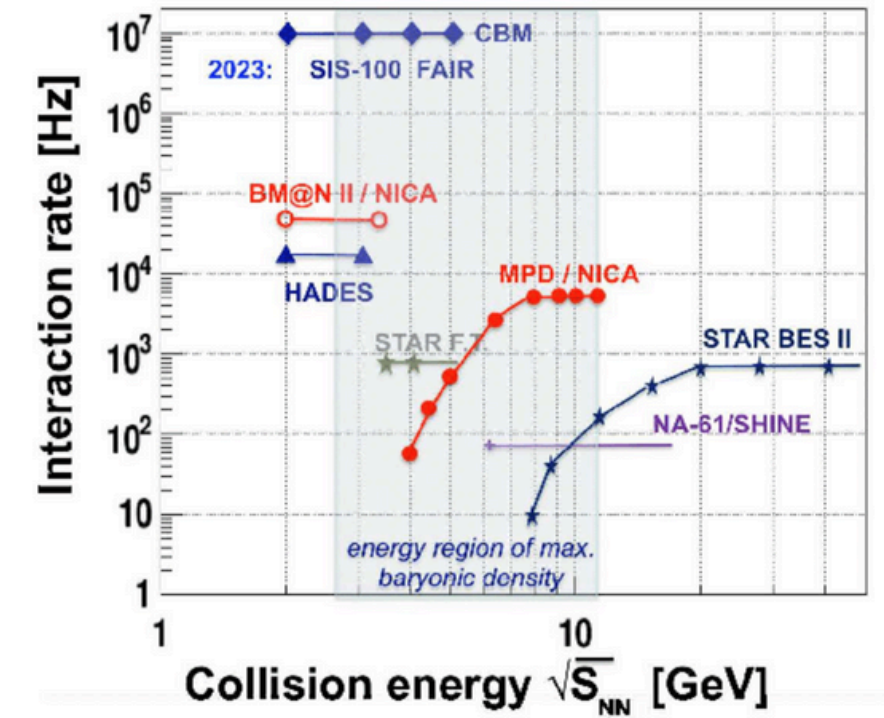
# Signals of criticality

Phys.Rev.C 74 (2006) 047901

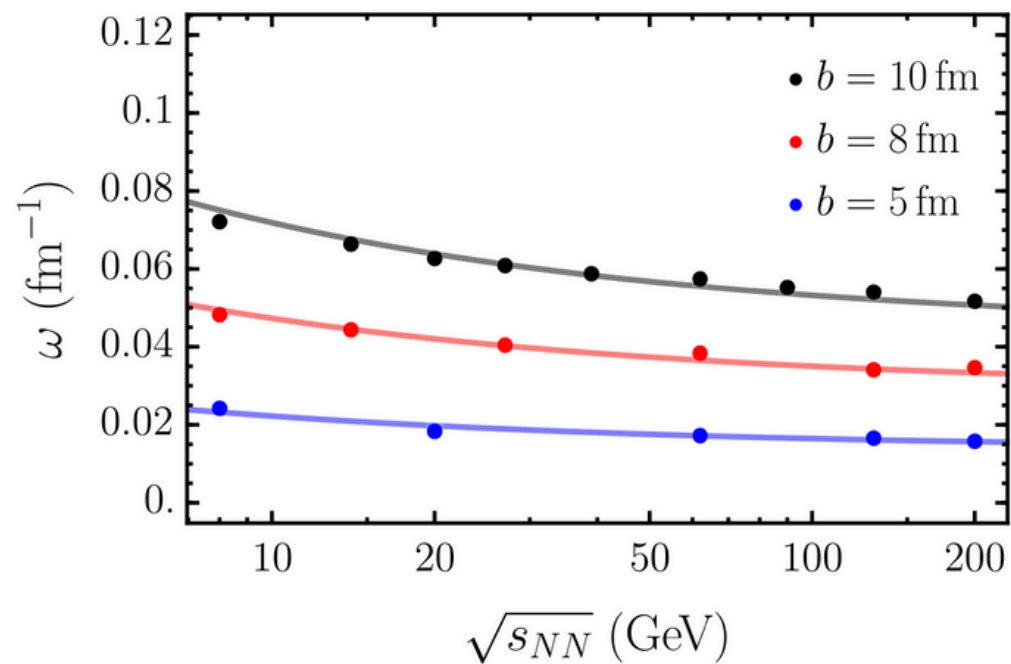


$$\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}},$$

PoS ICHEP2018 (2019) 493

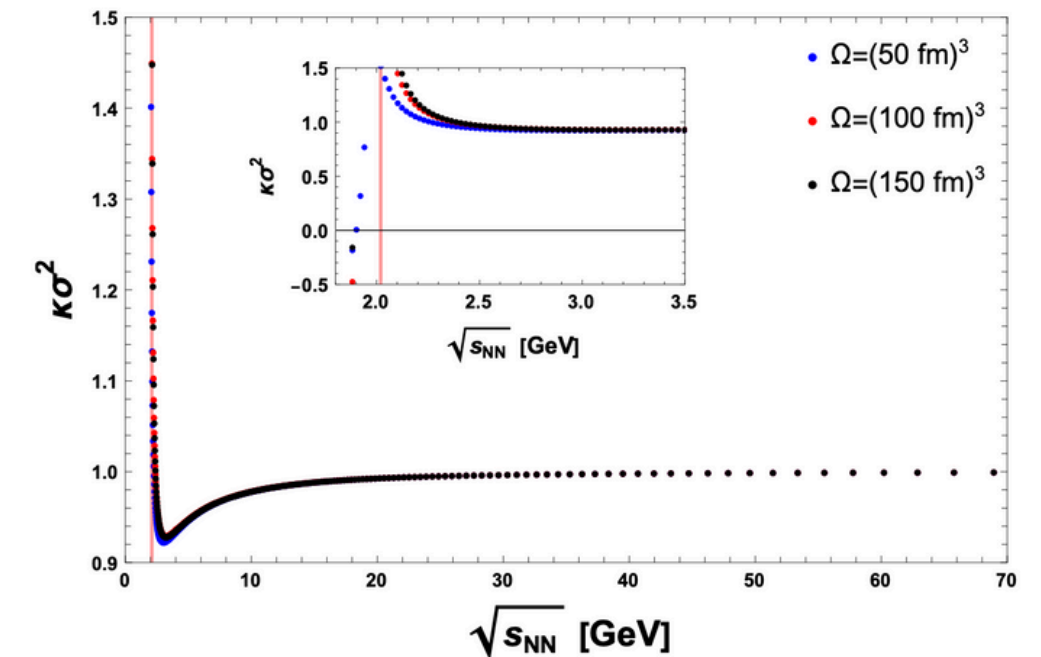


Phys. Rev. D 102 (2020), 056019

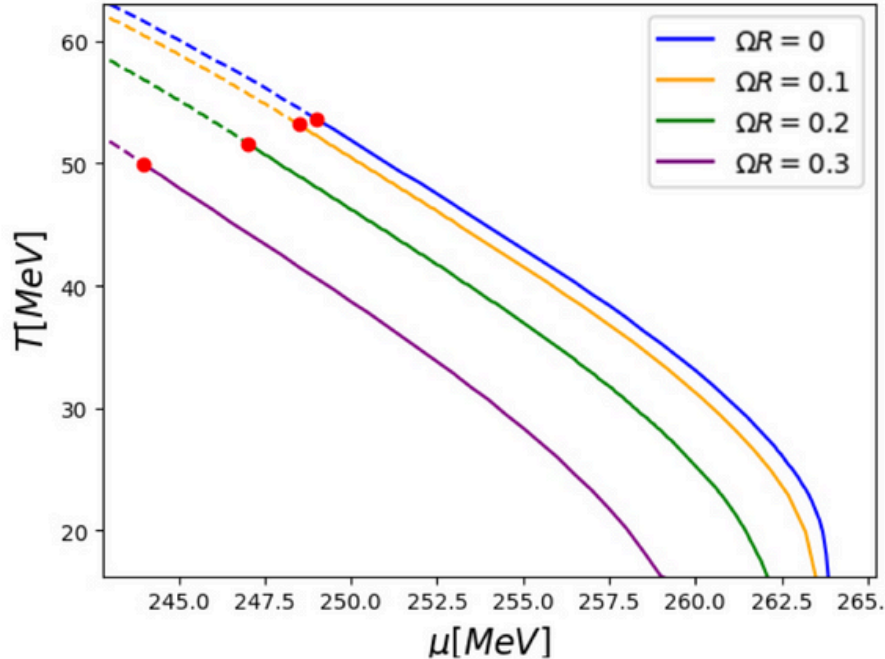
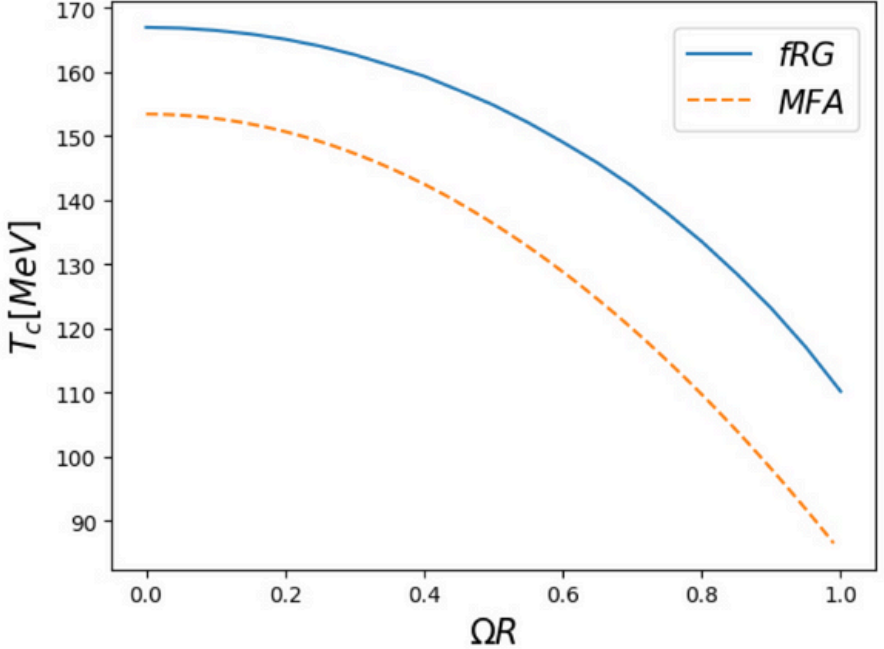


$$\omega = \frac{\omega_0 b^2}{2 V_N} \left[ 1 + 2 \left( \frac{m_N}{\sqrt{s_{NN}}} \right)^{1/2} \right],$$

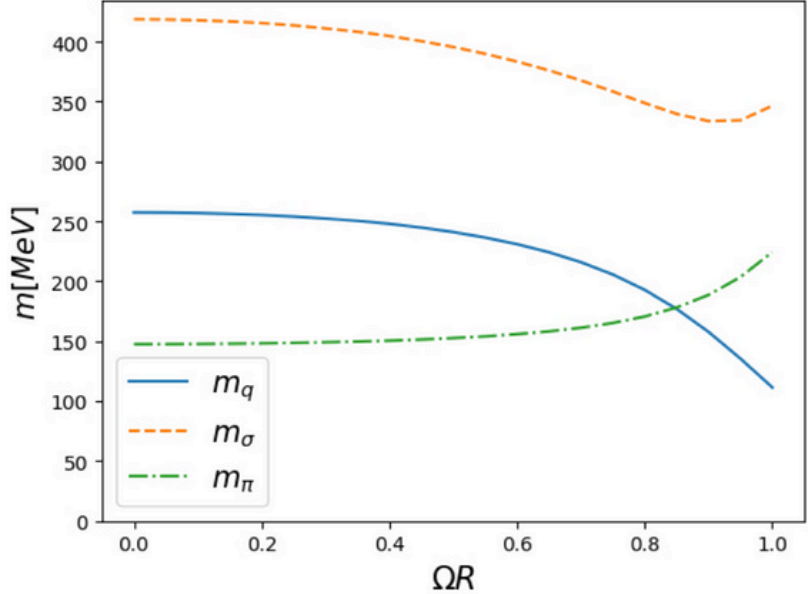
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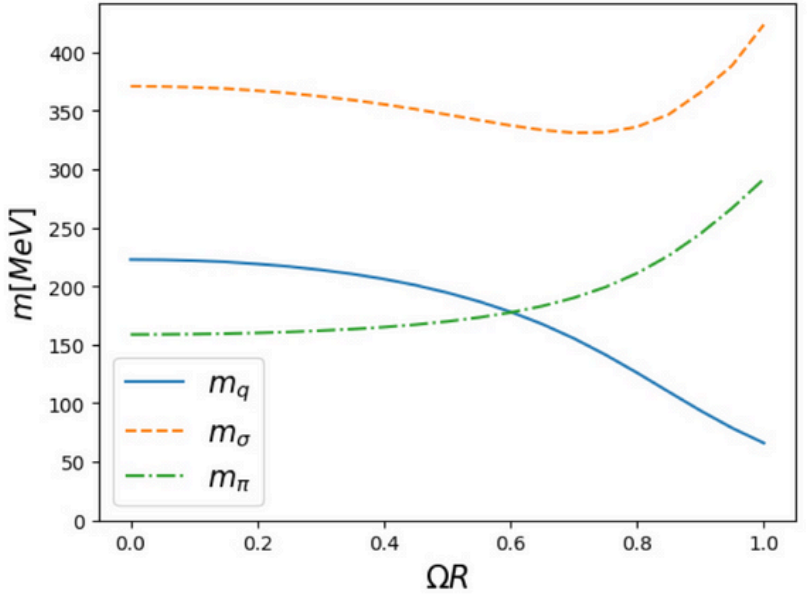
# Result in the literature with Quark-Meson model



fRG-> functional renormalization group  
 MFA->mean-field approximation



T=120 MeV



T=140 MeV