The Replica Trick and Temperature

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Scope

- Replica trick.
- Example: Sherrington-Kirkpatrick model.
- Example: Fermions in a magnetic Field.
- Finite temperature.
- Deconfinement transition.

Replica trick I

- The replica method was introduced through the spin glass model:
$$H = -\sum_{i,k} J_{ik}\sigma_i\sigma_k,$$
where the J_{ik} are uncorrelated Gaussian random variables with zero mean and variance $\overline{J_{ik}^2} = K_{ik}$.
- It is necessary to obtain the proper averaged thermodynamic potential
$$F = -kT\langle \ln Z\rangle_{\text{ave}}\ .$$

Replica trick II

- In order to average over macroscopic samples wherein a vast number of different configurations of the J_{ik} are operative, they introduced the so-called "replica trick":[1]

$$\ln Z = \lim_{n \to 0} \frac{Z^n - 1}{n}$$

- The average is computed before taking the limit $n \to 0$. It was introduced by Parisi as a method to average the free energy, defined via the logarithm of the partition function $\ln Z$, of a system over quenched (or frozen) disorder.

[1] Mézard M, Parisi G, Virasoro M. 1987. Spin glass theory and beyond: An **An Introduction to the Replica Method and Its Applications**. World Scientific, Singapore. 476pg.

Replica trick III

It is used in

- Spin glasses,
- Polymer networks,
- Z_n field theory,
- intermittency of turbulence,
- Euclidean random matrices,
- granular matter
- AdS/CFT
- etc.

Example: Sherrington-Kirkpatrick model I

Given the Hamiltonian

$$\mathcal{H}[\sigma] = -\sum_{\langle i,j \rangle}^{N} J_{ij}\sigma_i\sigma_j - h\sum_{i}^{N}\sigma_i$$

where $\sigma_i \in \{-1, 1\}$, h is the uniform magnetic field and the couplings are random variables extracted from the distribution

$$P(J) = \sqrt{\frac{N}{2\pi}}\exp\left(-\frac{NJ^2}{2}\right).$$

Example: Sherrington-Kirkpatrick model II

Therefore the solution of the statics of such a system in presence of quenched disorder requires the computation of the average of the logarithm of the (sample dependent) partition function

$$Z_J = \sum_{\{\sigma\}} \exp(-\beta \mathcal{H}[\sigma]) = \sum_{\{\sigma\}} \exp\left(+\beta \sum_{\langle i,j \rangle} J_{ij}\sigma_i\sigma_j + \beta h \sum_i \sigma_i \right),$$

Since we need to compute the average of the free energy, we use the trick

$$\lim_{n\to 0} \frac{\overline{Z^n} - 1}{n} = \overline{\log Z} \text{ or } \lim_{n\to 0} \frac{\log\left(\overline{Z^n}\right)}{n} = \overline{\log Z}.$$

Example: Sherrington-Kirkpatrick model III

Computing the average of the n-th power of the partition function is a much easier task than computing the average of its logarithm:

$$\overline{Z_J^n} = \int \left(\prod_{i<j} dJ_{ij} P\left(J_{ij}\right) \right)$$

$$\times \sum_{\{\underline{\sigma}\}} \exp \left(\beta \sum_a \sum_{\langle i,j \rangle} J_{ij} \sigma_i^a \sigma_j^a + \beta h \sum_a \sum_i \sigma_i^a \right).$$

Example: Sherrington-Kirkpatrick model IV

Since we know that

$$\int d^M x \exp\left(-\sum_{i,k=1,M} A_{i,k} x_i x_k + \sum_{i,M} B_i x_i\right)$$
$$= \sqrt{\frac{\pi}{\det(A)}} \exp\left(\frac{1}{4} \sum_{i,k=1,M} \left(\hat{A}^{-1}\right)_{i,k} B_i B_k\right),$$

Then, we end with an effective lagrangian

$$\overline{Z_J^n} = \sum_{\{\varsigma\}} \exp\left(\beta h \sum_i \sum_a \sigma_i^a + \frac{\beta^2}{2N} \sum_{i<j} \sum_{a,b} \sigma_i^a \sigma_i^b \sigma_j^a \sigma_j^b\right).$$

Example: Sherrington-Kirkpatrick model V

The interaction term can be rewritten in the following way:

$$\sum_{i<j}\sum_{a,b}\sigma_i^a\sigma_i^b\sigma_j^a\sigma_j^b = N^2\sum_{a<b}\left(\frac{1}{N}\sum_i \sigma_i^a\sigma_i^b\right)^2 + \frac{N^2n - Nn^2}{2}.$$

The replicated partition function reads

$$\overline{Z_J^n} = \sum_{\{\underline{\sigma}\}}\exp\left(\beta h\sum_i\sum_a \sigma_i^a\right)\exp\left(\frac{\beta^2}{4}\left(Nn - n^2\right)\right)$$
$$\times\prod_{a<b}\exp\left(\frac{\beta^2}{2N}\left(\sum_i \sigma_i^a\sigma_i^b\right)^2\right).$$

Example: Sherrington-Kirkpatrick model VI

$$\overline{Z_J^n} = \sum_{\{\underline{\sigma}\}} \exp\left(\beta h \sum_i \sum_a \sigma_i^a + \frac{\beta^2}{4}\left(Nn - n^2\right)\right)\left(\frac{2\pi\beta^2}{N}\right)^{\frac{n(n-1)}{2}}$$

$$\prod_{a<b}\int dQ_{ab}\exp\left(-\frac{N}{2}\beta^2 Q_{ab}^2 + \beta^2\sum_i \sigma_i^a\sigma_i^b Q_{ab}\right),$$

We are now interested in computing the free energy

$$f(\beta,h) = \lim_{n\to 0}\lim_{N\to\infty}\left(-\frac{1}{\beta N n}\ln\overline{Z_J^n}\right).$$

Example: Sherrington-Kirkpatrick model VII

In principle, the two limits should have been taken in the opposite order, nevertheless the calculation would be impossible with the right order of the limits, we can express the replicated partition function as follows

$$\overline{Z_J^n} \propto \int dQ \exp(-N\mathcal{S}[Q,h])$$

with

$$\mathcal{S}[Q,h] = -\frac{\beta^2 n}{4} + \frac{\beta^2}{2}\sum_{a<b} Q_{ab}^2 - \mathcal{W}[Q]$$

and

$$\mathcal{W}[Q] = \ln \sum_{\{\underline{\sigma}\}} \exp\left(\beta h \sum_a \sigma^a + \beta^2 \sum_{a<b} \sigma^a \sigma^b Q_{ab}\right)$$

Example: Sherrington-Kirkpatrick model I

Taking the saddle point w.r.t. the order parameter Q we obtain that the free energy can be written as

$$f(\beta, h) = \lim_{n \to 0} \frac{1}{\beta n} \operatorname{extr}_Q \mathcal{S}[Q, h].$$

Fermions in a magnetic field I[2]

We consider a physical scenario where a classical and static magnetic field background, **possessing random spatial fluctuations**, modifies the quantum dynamics of a system of fermions general, we distinguish three main c. Inontributions to the gauge field:

$$A^{\mu}(x) \rightarrow A^{\mu}(x) + A^{\mu}_{\mathrm{BG}}(x) + \delta A^{\mu}_{\mathrm{BG}}(\mathbf{x}),$$

Where $A^{\mu}(x)$ denotes the dynamical quantum field, BG stands for "background", referring to the classical magnetic field imposed by experimental conditions.

[2] Castaño-Yepes, J.D.,Loewe, M.,Muñoz, E.,Rojas, J.C. and Zamora, R. **QED fermions in a noisy magnetic field background**, Phys.Rev.D 107 (2023) 9, 096014

Fermions in a magnetic field II

We consider the effect of static (quenched) white noise spatial fluctuations $\delta A^{\mu}_{\mathrm{BG}}(\mathbf{x})$ with respect to the mean value $A^{\mu}_{\mathrm{DC}}(x)$, satisfying the statistical properties

$$\left\langle \delta A^{j}_{\mathrm{BG}}(\mathbf{x}) \delta A^{k}_{\mathrm{BG}}\left(\mathbf{x}'\right)\right\rangle = \Delta_B \delta_{j,k} \delta^3\left(\mathbf{x} - \mathbf{x}'\right),$$
$$\left\langle \delta A^{\mu}_{\mathrm{BG}}(\mathbf{x})\right\rangle = 0.$$

These statistical properties are represented by a Gaussian functional distribution of the form

$$dP\left[\delta A^{\mu}_{\mathrm{BG}}\right] = \mathcal{N} e^{-\int d^3x \frac{\left[\delta A^{\mu}_{\mathrm{BG}}(\mathbf{x})\right]^2}{2\Delta_B}} \mathcal{D}\left[\delta A^{\mu}_{\mathrm{BG}}(\mathbf{x})\right].$$

Fermions in a magnetic field III

We split the lagrangian in to terms:

$$\mathcal{L} = \mathcal{L}_{\text{FBG}} + \mathcal{L}_{\text{NBG}},$$

where

$$\mathcal{L}_{\text{FBG}} = \bar{\psi}\left(\mathrm{i}\ \partial\!\!\!/ - e\mathcal{A}_{\text{BG}} - e\mathcal{A} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

represents a Lagrangian which describes a fermion interaction with quantized photons and a background magnetic field. The other term is

$$\mathcal{L}_{\text{NBG}} = \bar{\psi}\left(-e\delta\mathcal{A}_{\text{BG}}\right)\psi$$

representing the interaction with ther spatial fluctuation.

Fermions in a magnetic field IV

The generating functional is given by

$$Z[A] = \int \mathcal{D}[\bar{\psi}, \psi] e^{\mathrm{i} \int d^4x [\mathcal{L}_{\mathrm{FBG}} + \mathcal{L}_{\mathrm{NBG}}]},$$

in order to compute the statistical average over the magnetic background noise, we apply the replica trick:

$$\overline{\ln Z[A]} = \lim_{n \to 0} \frac{\overline{Z^n[A]} - 1}{n}.$$

Fermions in a magnetic field VI

After the computating the Gaussian integral, we end with an effective Lagrangian

$$\bar{S}\left[\bar{\psi}^a, \psi^a; A\right] = \int d^4x \left(\sum_a \bar{\psi}^a \left(\mathrm{i}\,\not{\partial} - e\mathbb{A}_{\mathrm{BG}} - e\,\not{A} - m\right)\psi^a - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right)$$

$$+\,\mathrm{i}\frac{e^2\Delta_B}{2}\int d^4x \int d^4y \sum_{a,b}\sum_{j=1}^{3} \bar{\psi}^a(x)\gamma^j\psi^a(x)\bar{\psi}^b(y)\gamma_j\psi^b(y)\delta^3(\mathbf{x}-\mathbf{y}).$$

To be continued ... (EM or ML).

Finite temperature I

We assume a non-equilibrium scenario, where temperature is not defined uniformly through the whole system, but smaller regions may still be pictured as nearly-thermalized subsystems. Therefore, we model this situation by an ensemble of subsystems whose individual temperatures $T = T_0 + \delta T$ are subjected to stochastic fluctuations with zero mean $\overline{\delta T} = 0$, but finite variance $\overline{\delta T^2} = \Delta$. In terms of the inverse temperature

$$\beta = (T_0 + \delta T)^{-1} = T_0^{-1} - \frac{\delta T}{T_0^2} = \beta_0 + \delta\beta.$$

Finite temperature II

So

$$\delta\beta = -\frac{\delta T}{T_0^2},$$
$$\overline{\delta\beta} = -T_0^{-2}\overline{\delta T} = 0,$$
$$\overline{\delta\beta^2} = T_0^{-4}\overline{\delta T^2} = \beta_0^4\Delta = \Delta_\beta.$$

Assuming gaussian distribution

$$dP[\delta\beta] = \frac{d(\delta\beta)}{\sqrt{2\pi\Delta_\beta}}e^{-\frac{\delta\beta^2}{2\Delta_\beta}}.$$

Finite temperature III

Begining with the Grand-Canonical ensemble

$$Z(\mu, \mathcal{V}, T) = \mathrm{Tr}\left[e^{-\beta(\hat{H}-\mu\hat{N})}\right]$$

where, in particular, we shall focus on a system of QED fermions, described by the Hamiltonian operator including the chemical potential

$$\begin{aligned}\hat{H}-\mu\hat{N} &= \int d^3x\hat{\psi}^{\dagger}(\mathbf{x})\gamma^0\left[\gamma\cdot(-\mathrm{i}\nabla)+m-\gamma^0\mu\right]\hat{\psi}(\mathbf{x})\\ &\equiv \hat{K}.\end{aligned}$$

Finite temperature IV

As we know, we must compute the average

$$\overline{Z^n} = \mathrm{Tr}\left[\exp\left\{-(\beta_0+\delta\beta)\sum_{a=1}^{n}\hat{K}^{(a)}\right\}\right] = \int dP[\delta\beta]\,\mathrm{Tr}\left[e^{-\beta_0\sum_{a=1}^{n}\hat{K}^{(a)}}\left(1+\sum_{j=1}^{\infty}\frac{(-1)^j(\delta\beta)^j}{j!}\left(\sum_{a=1}^{n}\hat{K}^{(a)}\right)^j\right)\right],$$

$$= \mathrm{Tr}\left[e^{-\beta_0\sum_{a=1}^{n}\hat{K}^{(a)}}\left(1+\sum_{j=1}^{\infty}\frac{\Delta_\beta^j}{(2j)!}(2j-1)!!\left(\sum_{a=1}^{n}\hat{K}^{(a)}\right)^{2j}\right)\right],$$

$$= \left(1+\sum_{j=1}^{\infty}\frac{\Delta_\beta^j}{(2j)!}(2j-1)!!\frac{\partial^{2j}}{\partial\beta_0^{2j}}\right)\mathrm{Tr}\left[e^{-\beta_0\sum_{a=1}^{n}\hat{K}^{(a)}}\right].$$

So, we have

$$\overline{Z^n} = \left(1+\sum_{j=1}^{\infty}\frac{\Delta_\beta^j}{(2j)!}(2j-1)!!\frac{\partial^{2j}}{\partial\beta_0^{2j}}\right)Z_0^n.$$

Finite temperature V

In order to compute the

$$Z_0^n = \prod_{a=1}^{n} \int \mathcal{D}\left[\psi_a^\dagger, \psi_a\right]$$

$$\times \exp\left[-\int_0^{\beta_0} d\tau \sum_{a=1}^{n} \psi_a^\dagger(\mathbf{x},\tau)\gamma^0\left(\gamma^0\left(\partial_\tau - \mu\right) + \gamma\cdot\mathbf{p} + m\right)\psi_a(\mathbf{x},\tau)\right]$$

$$= \det\left[\partial_\tau - \mu + \gamma^0\gamma\cdot\mathbf{p} + m\gamma^0\right]^n$$

$$= \exp\left\{n\,\mathrm{Tr}\ln\left[\partial_\tau - \mu + \gamma^0\gamma\cdot\mathbf{p} + m\gamma^0\right]\right\}$$

$$= \exp\left(n\ln Z_0\right)$$

Here, we also defined the partition function for the fermion gas

$$\ln Z_0 = \mathrm{Tr}\ln\left[\partial_\tau - \mu + \gamma^0\gamma\cdot\mathbf{p} + m\gamma^0\right]$$

$$= \mathcal{V}\int \frac{d^3p}{(2\pi)^3} \sum_{k\in\mathbb{Z}} \mathrm{tr}\ln\left[\mathrm{i}\omega_k - \mu + \gamma^0\gamma\cdot\mathbf{p} + m\gamma^0\right]$$

Finite temperature VI

From the eigenvalues of the matrix in the argument of the logarithm: $\mathrm{i}\omega_k - \mu \pm E_{\mathbf{p}}$, with $E_{\mathbf{p}} = \sqrt{p^2 + m^2}$, it is obtained

$$
\begin{aligned}
\ln Z_0 =& 2\mathcal{V} \int \frac{d^3p}{(2\pi)^3} \sum_{k\in\mathbb{Z}} \left\{ \ln\left[\mathrm{i}\omega_k - \mu + E_{\mathbf{p}}\right] \right. \\
& \left. + \ln\left[\mathrm{i}\omega_k - \mu - E_{\mathbf{p}}\right] \right\}, \\
=& 2\mathcal{V} \int \frac{d^3p}{(2\pi)^3} \left\{ \ln\left(1 + e^{\beta_0(\mu - E_{\mathbf{p}})}\right) \right. \\
& \left. + \ln\left(1 + e^{\beta_0(\mu + E_{\mathbf{p}})}\right) \right\}.
\end{aligned}
$$

Finite temperature VII

The average of the partition function is then:

$$\overline{\ln Z} = \lim_{n\to 0}\frac{\overline{Z^n}-1}{n} = \left(1+\sum_{j=1}^{\infty}\frac{\Delta_\beta^j}{(2j)!}(2j-1)!!\frac{\partial^{2j}}{\partial\beta_0^{2j}}\right)\lim_{n\to 0}\frac{e^{n\ln Z_0}-1}{n},$$

$$= \left(1+\sum_{j=1}^{\infty}\frac{(\Delta_\beta/2)^j}{j!}\frac{\partial^{2j}}{\partial\beta_0^{2j}}\right)\ln Z_0,$$

$$= \exp\left[\frac{\Delta_\beta}{2}\frac{\partial^2}{\partial\beta_0^2}\right]\ln Z_0,$$

Finite temperature VIII

In the weak limit, $m^2\Delta_\beta \ll 1$. The expansion is truncated up to first order in the fluctuation Δ, to obtain

$$\begin{aligned}\overline{\ln Z/Z_0} &= \frac{\Delta_\beta}{2}\frac{\partial^2}{\partial\beta_0^2}\ln Z_0 + O\left(\Delta_\beta^2\right)\\ &= \beta_0\left(\mathcal{P}\mathcal{V} - (\mathcal{P}\mathcal{V})_{\text{ig}}\right)\end{aligned}$$

where $\beta_0(\mathcal{P}\mathcal{V})_{ig} = \ln Z_0$ is the equation of state for the ideal Fermi gas. Therefore, up to $O\left(\Delta^2\right)$, the excess pressure $\delta\mathcal{P} \equiv \mathcal{P} - \mathcal{P}_{\text{ig}}$ of the Fermi gas due to the average effect of the temperature fluctuation is

$$\delta\mathcal{P} \equiv \mathcal{P} - \mathcal{P}_{\text{ig}} = \frac{\Delta_\beta}{2\mathcal{V}\beta_0}\frac{\partial^2}{\partial\beta_0^2}\ln Z_0 + O\left(\Delta^2\right)$$

Finite temperature IX

From the differential form of the Grand Potential for the ideal reference system, we have

$$d\Omega_0 = -\mathcal{P}d\mathcal{V} - SdT_0 - Nd\mu,$$

it is concluded that

$$S = -\left.\frac{\partial\Omega_0}{\partial T_0}\right|_{\mu,\mathcal{V}}.$$

Finite temperature X

On the other hand, using $T_0 = \beta_0^{-1}$ and the definition $\Omega_0 = -T_0 \ln Z_0$, it is possible to show the identity

$$
\begin{aligned}
\frac{\Delta_\beta}{2\beta_0 \mathcal{V}} \frac{\partial^2}{\partial \beta_0^2} \ln Z_0 &= \frac{\Delta_\beta T_0^2}{2\mathcal{V}} T_0 \frac{\partial S}{\partial T_0}\bigg|_{\mu,\mathcal{V}}, \\
&= \frac{\Delta_\beta T_0^2}{2\mathcal{V}} \left[C_v + \frac{\left(T_0 \frac{\partial N}{\partial T_0}\Big|_{\mu,\mathcal{V}} \right)^2}{\left\langle (\delta \hat{N})^2 \right\rangle} \right], \\
&\geq 0.
\end{aligned}
$$

Finite temperature XI

We expect for the excess pressure due to random temperature fluctuations in the ensemble to be positive $\delta\mathcal{P} \geq 0$ at first order in Δ. Indeed, a direct calculation of the second derivative of the Grand Partition function, leading to the explicit formula

$$\delta\mathcal{P} = \frac{\Delta_\beta}{\beta_0} \sum_{s=\pm 1} \int \frac{d^3p}{(2\pi)^3} \left(E_{\mathbf{p}} + s\mu\right)^2 n_{\mathrm{F}} \left(\frac{E_{\mathbf{p}} + s\mu}{T_0}\right) \times \left[1 - n_{\mathrm{F}} \left(\frac{E_{\mathbf{p}} + s\mu}{T_0}\right)\right],$$

where $n_F(x) = \left(e^x + 1\right)^{-1}$ is the Fermi distribution.

Finite temperature XII

Defining the dimensionless variables $x \equiv E/m, y \equiv T_0/m, z = \mu/m$, and $\widetilde{\Delta} \equiv \Delta/m^2$, such that we have

$$\delta P = \frac{m^4\widetilde{\Delta}}{2\pi^2 y^3} \sum_{s=\pm 1} \int_1^\infty dx x(x+sz)^2\sqrt{x^2-1} \\ \times n_{\mathrm{F}}\left(\frac{x+sz}{y}\right) n_{\mathrm{F}}\left(-\frac{x+sz}{y}\right),$$

where we used the property $n_{\mathrm{F}}(-x) = 1 - n_{\mathrm{F}}(x)$.

Finite temperature XIII

We can repeat the analysis for an ideal gas of massless Bosons (with chemical potential $\mu_B = 0$), whose Grand-potential is

$$\Omega_0^B = -T_0 \ln Z_0^B = -\frac{\mathcal{V} T_0^4}{6\pi^2} \int_0^\infty dx \frac{x^3}{e^x - 1}$$
$$= -\nu_B \mathcal{V} \frac{\pi^2 T_0^4}{90}$$

where ν_B represents the total number of discrete degrees of freedom. The ideal gas pressure for Bosons in equilibrium at temperature T_0 is thus

$$\mathcal{P}_{\mathrm{ig}}^B = \nu_B \frac{\pi^2 T_0^4}{90}$$

Finite temperature XIV

while the corresponding excess pressure due to nonequilibrium thermal fluctuations will be,

$$
\begin{aligned}
\delta\mathcal{P}^B = \mathcal{P} - \mathcal{P}^B_{\mathrm{ig}} &= \frac{\Delta_\beta}{2\beta_0\mathcal{V}}\frac{\partial^2}{\partial\beta_0^2}\ln Z_0^B \\
&= \nu_B\frac{\pi^2}{15}\Delta_\beta\beta_0^{-6} = \nu_B\frac{\pi^2}{15}\Delta T_0^2 > 0
\end{aligned}
$$

Finite temperature XV

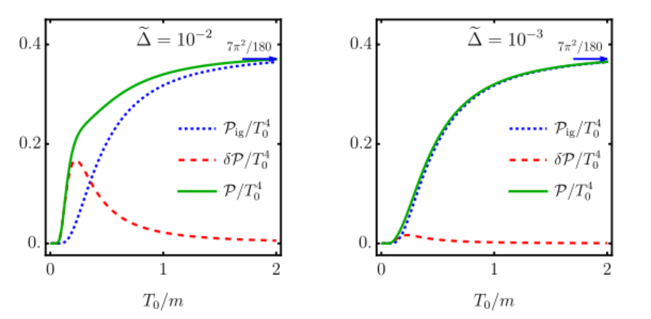

Figure: Excess pressure of the Fermi gas, computed up to order $O(\Delta)$, as a function of the average temperature T_0, and the chemical potential μ.

Finite temperature XVI

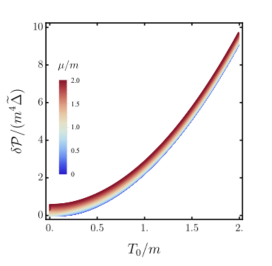

Figure: Pressure normalized to T_0^4 when $\mu = 0$ for the ideal fermion gas (dotted line), the excess pressure (dashed line), and the total pressure (continuous line). The arrow indicates the asymptotic ideal gas limit at high temperatures.

Deconfinement transition I

Using bag model considerations. and assuming that the Hadronic phase is mainly constituted by pions (with $\mu = 0$ and $\nu_B = 3$ for charged states $0, \pm$), applying our expression, we have that its pressure, including the excess pressure effect due to temperature fluctuations, would be

$$\mathcal{P}_{\text{Had}} = 3\frac{\pi^2 T_0^4}{90} + \delta\mathcal{P}^{\text{Had}}.$$

For the QG plasma phase we have $\nu_F = 2 \cdot 3 \cdot 2 = 12$ for quarks, and $\nu_B = 2 \cdot (3^2 - 1) = 16$ for gluons, such that

$$\begin{aligned} \mathcal{P}_{\textsf{Plasma}} &= \left(\nu_B + \frac{7}{4}\nu_F\right)\frac{\pi^2 T_0^4}{90} + \delta\mathcal{P}^{\textsf{Plasma}} - B \\ &= \frac{37\pi^2}{90} T_0^4 + \delta\mathcal{P}^{\textsf{Plasma}} - B \end{aligned}$$

Deconfinement transition II

Where $B \sim 200\mathrm{MeV}$. The critical temperature T_c is obtained by imposing the condition of equal pressures at both phases at the phase transition, i.e.

$$3\frac{\pi^2 T_c^4}{90} = \frac{37\pi^2}{90} T_c^4 + \delta\mathcal{P}^{\mathrm{Net}} - B,$$

where we defined the net excess pressure as

$$\begin{aligned}\delta\mathcal{P}^{\textsf{Net}} &= \delta\mathcal{P}^{\textsf{Plasma}} - \delta\mathcal{P}^{\mathrm{Had}} = \delta\mathcal{P}^{G} - \delta\mathcal{P}^{\mathrm{Had}} + \delta\mathcal{P}^{Q}, \\ &= 13\frac{\pi^2}{15}\Delta T_0^2 + \delta\mathcal{P}^{Q} > 0.\end{aligned}$$

Deconfinement transition III

Finally, solving for T_c, we obtain

$$T_c = T_c^0 \left(1 - \frac{\delta \mathcal{P}^{\mathrm{Net}}}{\left(T_c^0\right)^4}\right)^{1/4} \leq T_c^0,$$

with $T_c^0 = (45B/17\pi^2)^{1/4} \sim 144\mathrm{MeV}$.

Summary I

- Replica trick is used in order to obtain an effective action due to classical random fluctuations.
- We have considered an ensemble of subsystems at different temperatures $T = T_0 + \delta T$, with average T_0 and standard deviation $\overline{\delta T^2} = \Delta$. These statistical properties imply that the inverse temperature $\beta = \beta_0 + \delta\beta$ can be modeled by a Gaussian distributed fluctuation $\delta\beta$, with zero mean and standard deviation $\overline{\delta\beta^2} = \Delta_\beta = \beta_0^4\Delta$.

Summary II

- By first considering a non-interacting system of QED fermions, we applied the replica trick to obtain the statistical average of the Grand Potential as a series expansion at all orders in the parameter Δ.
- From our expression, we obtained the excess pressure with respect to the ideal Fermi gas due to the thermal fluctuations. The same analysis can be carried out for an ideal gas of Bosons, for which we also obtained explicit results for the corresponding excess pressure.

Thanx!