Finite energy sum rules under extreme conditions: A practical tool

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Motivation

Extreme temperature

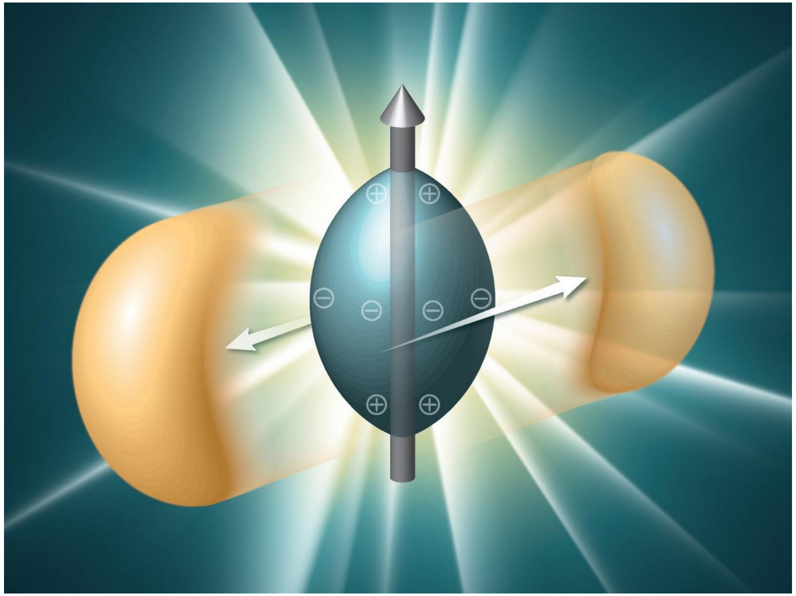
- Relativistic heavy-Ion collision experiments
- Early universe

Extreme density

- Compact stars
- Relativistic heavy-Ion collision experiments

Extreme magnetic fields

- Peripheral heavy ion collision experiments
- Magnetars



Outline

- Introduction to finite energy sum rules (FESR)
- Medium effects
- The QCD sector
- Condensates
- Hadronic sector
- FESR under external electroamgnetic effects

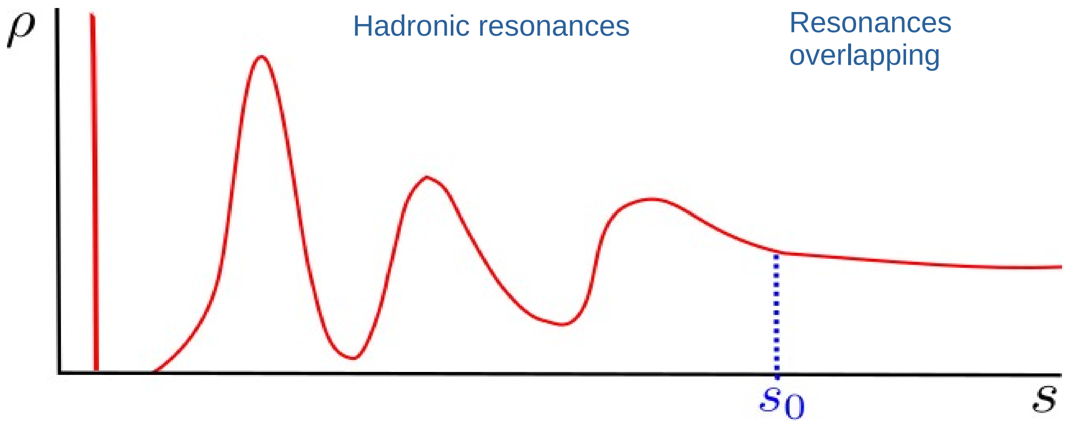
Introduction to FESR

The spectral function

Two current correlator $\Pi_{\mu\nu}(x-y) = i\langle 0|T J_\mu(x) J_\nu^\dagger(y)|0\rangle$

Fourier transformation $\Pi_{\mu\nu}(q) = q_\mu q_\nu \Pi_L(q^2) + (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_T(q^2)$

Spectral function $\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s+i\epsilon)$ $\Pi(p) = \int_0^\infty ds\, \frac{\rho(s)}{s-p^2},$



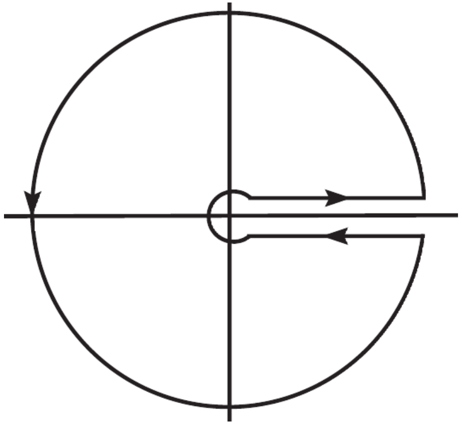
$s_0 \rightarrow$ Hadronic continuum threshold

FESR Quark-hadron duality $\Pi^{\rm Had} \leftrightarrow \Pi^{\rm QCD}$

Cauchy's theorem

$$\frac{1}{\pi}\int_0^{s_0} ds\, s^{N-1}\, {\rm Im}\Pi^{\rm had}(s+i\epsilon) \quad = \quad \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, s^{N-1}\, \Pi^{\rm QCD}(s)$$

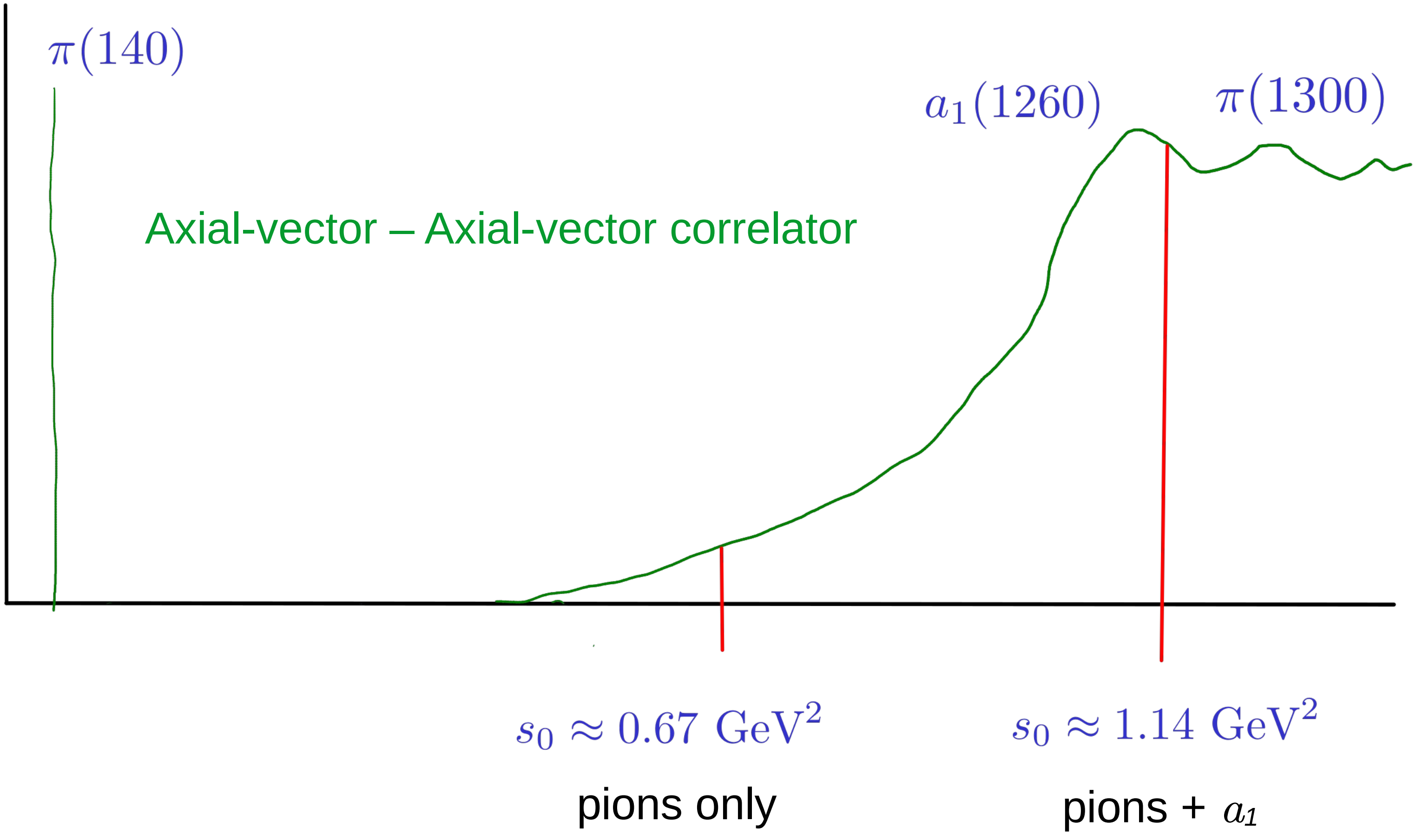
"Pac-man" contour



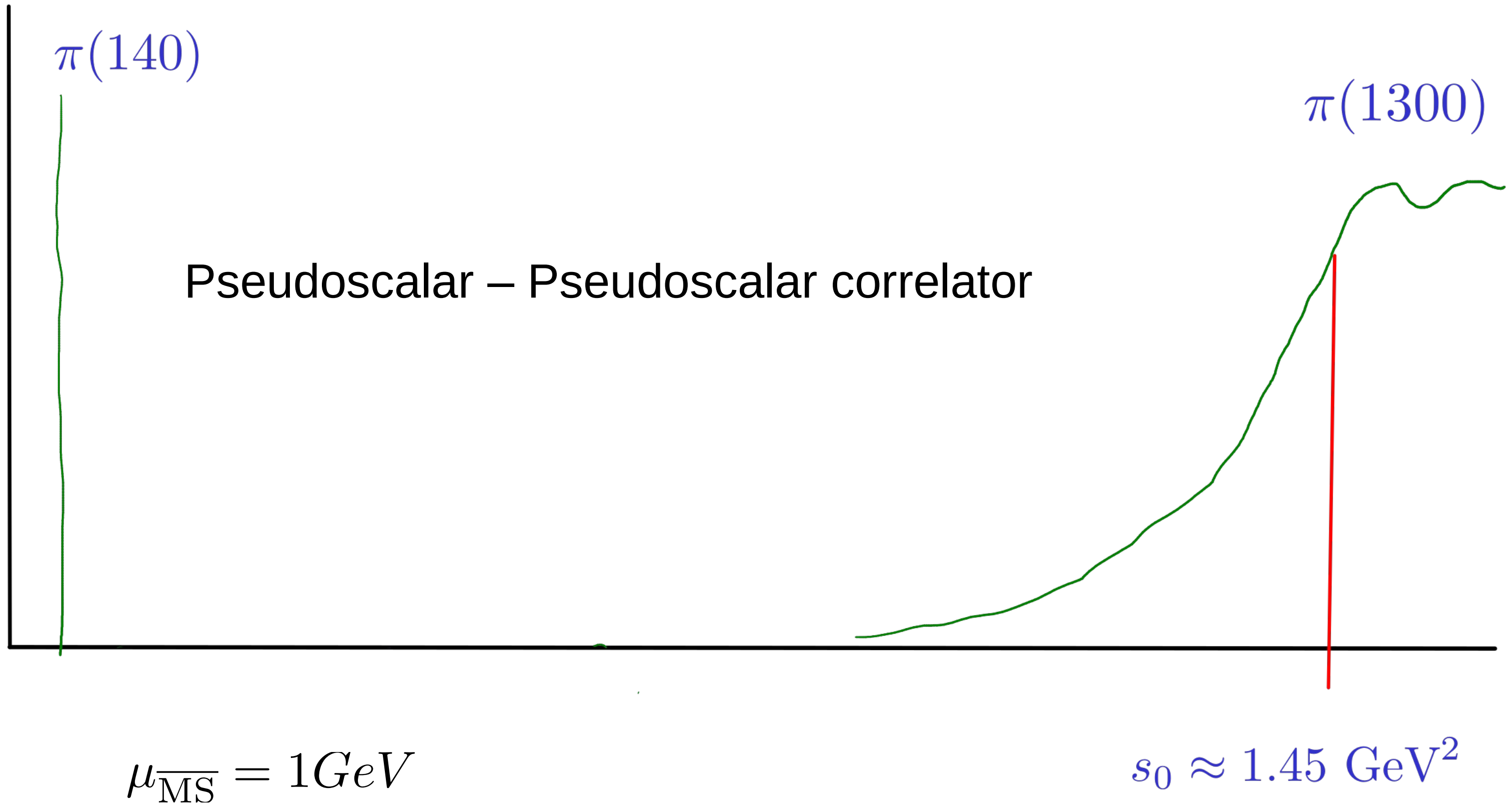
$$\oint_{s_0} \to {\rm QCD}$$

$$\int_0^{s_0} \to {\rm hadron}$$

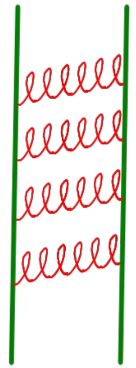
Continuum hadronic threshold



Continuum hadronic threshold



Operator Product Expansion (OPE)



$$\Pi^{\mathrm{QCD}}(x,y) = \Pi^{\mathrm{pQCD}}(x-y) + \sum_{n>0} C_{2n}(x-y)\ \langle \mathcal{O}_{2n} \rangle (x+y)$$

Short distance – high energy   large distance – low energy

C_{2n} Wilson coefficients $\langle \mathcal{O}_{2n} \rangle$ condensates

In momentum space, short distance expansion, high *s* expansion

$$\Pi^{\mathrm{QCD}}(s) = \Pi^{\mathrm{pQCD}}(s) + \sum_{n>0} \tilde{C}_{2n}(\Lambda) \frac{\langle \mathcal{O}_{2n} \rangle (\Lambda)}{s^n}$$

Operator mixing

$$\Pi(s) = \Pi_{\text{pQCD}}(s,\Lambda) + \sum_{n>0} C_n(s) \langle : \mathcal{O}_n : \rangle$$

normal ordered condensates

Problems with the chiral limit

- $\ln(-s/m_q^2)$ terms
- limits $m_u \to 0,\ m_d \to 0 \neq m_d \to 0,\ m_u \to 0 \neq m_u = m_d \to 0$

$$T q(x)\bar{q}(x) =: q(x)\bar{q}(x) : + S(x,x;\mu_{\overline{\rm MS}})$$

$$S(x,y) = \langle x | \frac{i}{i\partial\!\!\!/ - gG\!\!\!/ - m} | y \rangle$$

$$\langle \Omega | : \bar{q}q : | \Omega \rangle = \langle \Omega | T\bar{q}q | \Omega \rangle + S(x,x;\mu_{\overline{\rm MS}})$$

Solution to the chiral-limit problem

$$\langle \Omega | : \bar{q}q : | \Omega \rangle = \langle \bar{q}q \rangle(\mu_{\overline{\rm MS}}) + S(x,x;\mu_{\overline{\rm MS}})$$ Non-normal ordered condensate

→ operator mixing
→ condensates become scale dependent and renormalizable

$$\Pi(s) = \tilde{\Pi}_{\rm pQCD}(s, \mu_{\overline{\rm MS}}) + \sum_{n>0} \tilde{C}_n(s, \mu_{\overline{\rm MS}}) \langle \mathcal{O}_n \rangle (\mu_{\overline{\rm MS}})$$

Example

$$\langle : \bar{q}\, \gamma_\mu i D_\nu\, q : \rangle = \langle \bar{q}\, \gamma_\mu i D_\nu\, q \rangle + \mathrm{tr} \int \frac{d^d k}{(2\pi)^d}\, \gamma_\mu i \tilde{D}_\nu\, S_G(k; \mu_{\overline{\rm MS}})$$

In vacuum

$$\langle : \bar{q}\gamma_\mu i D_\nu q : \rangle = \langle \bar{q}\gamma_\mu i D_\nu q \rangle + \frac{N_c m^4}{8\pi^2} \left[\frac{1}{d-4} + \ln\left(\frac{m}{\mu_{\overline{\rm MS}}} \right) - \frac{3}{4} \right] g_{\mu\nu} + \frac{1}{48} \langle G^2 \rangle g_{\mu\nu}$$

then $\ln(-s/m^2) \to \ln(-s/\mu^2_{\overline{\rm MS}})$

FESR features

- cuts the OPE series (vacuum with no radiative corrections)

$$\oint_{s_0} \frac{ds}{2\pi i} s^{N-1} \sum_{n>0} C_{2n} \frac{\langle \mathcal{O}_{2n} \rangle}{s^n} = C_{2N} \langle \mathcal{O}_{2N} \rangle$$

- No need to calculate the full form factor (can integrate contour before loop momentum integration or Feynman parameters integration)

$$\oint_{s_0} ds \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p+k)^2 - m^2} \frac{1}{k^2 - m^2} \rightarrow \int \frac{d^4k}{(2\pi)^4} \oint_{s_0} ds \frac{1}{(p+k)^2 - m^2} \frac{1}{k^2 - m^2}$$

Medium effects

Medium effects:

Temperature

Chemical potential

External electromagnetic fields

$$X = T, \mu, B, E, \ldots$$

$$\Pi^{\rm QCD}(p, X) = \Pi^{\rm pQCD}(p, \mu_{\overline{\rm MS}}, X) + \sum_{n>0} C_n(p, \mu_{\overline{\rm MS}}) \, \langle \mathcal{O}_n \rangle (\mu_{\overline{\rm MS}}, X)$$

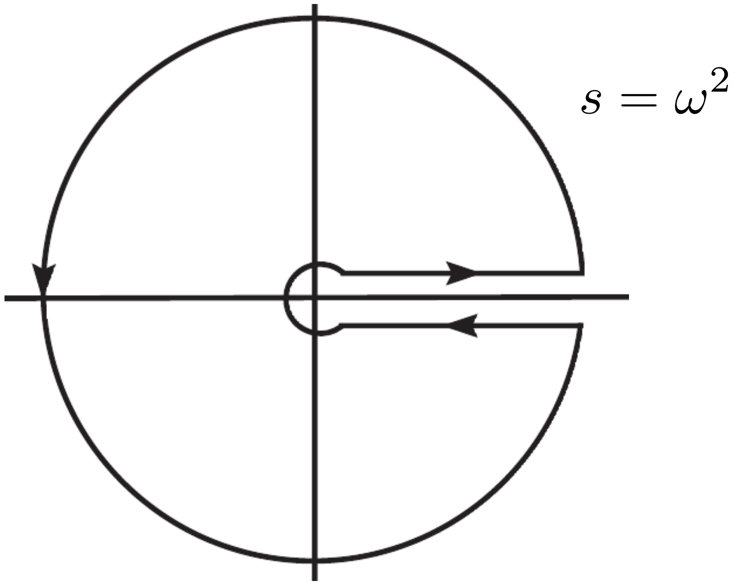
- New condensates due to symmetry breaking
- More FESR equations
- Infrared Wilson coefficients
  → is assumed all medium effects in the condensate

Finite temperature and density

$$\Pi(\omega, \boldsymbol{p}^2) = \Pi^{\text{even}}(\omega^2, \boldsymbol{p}^2) + \omega \Pi^{\text{odd}}(\omega^2, \boldsymbol{p}^2)$$

$$\int_0^{s_0} \frac{ds}{\pi} s^N \, \mathrm{Im}\Pi^{\text{had}}(s + i\epsilon, 0) = -\oint_{|s|=s_0} \frac{ds}{2\pi i} s^N \, \Pi^{\text{QCD}}(s, 0) + \operatorname*{Res}_{s\to 0} \left[s^N \Pi^{\text{QCD}}(s, 0) \right]$$

Scattering with the thermal bath term

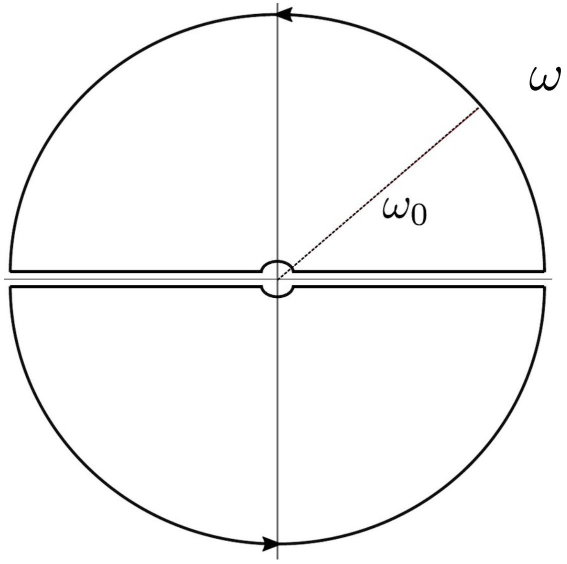


Finite temperature and density

$$\Pi(\omega, \boldsymbol{p}^2)$$

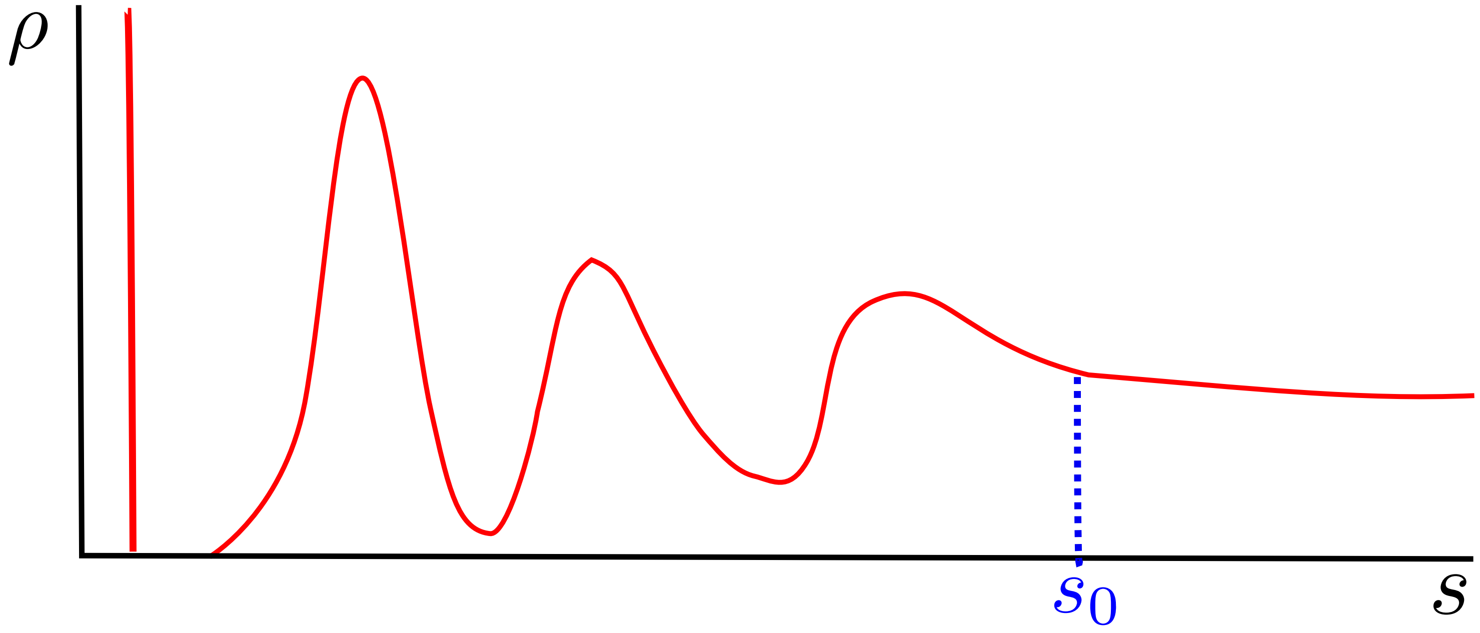
$$\int_{-\omega_0}^{\omega_0} \frac{d\omega}{\pi} \omega^{n+1} \mathrm{Im}\, \Pi^{\mathrm{had}}(\omega + i\epsilon, 0) = -\oint_{\omega_0} \frac{d\omega}{2\pi i} \omega^{n+1} \Pi^{\mathrm{QCD}}(\omega, 0) + \underset{\omega=0}{\mathrm{Res}} \left[\omega^{n+1} \Pi^{\mathrm{QCD}}(\omega, 0)\right]$$

Scattering with the thermal bath term

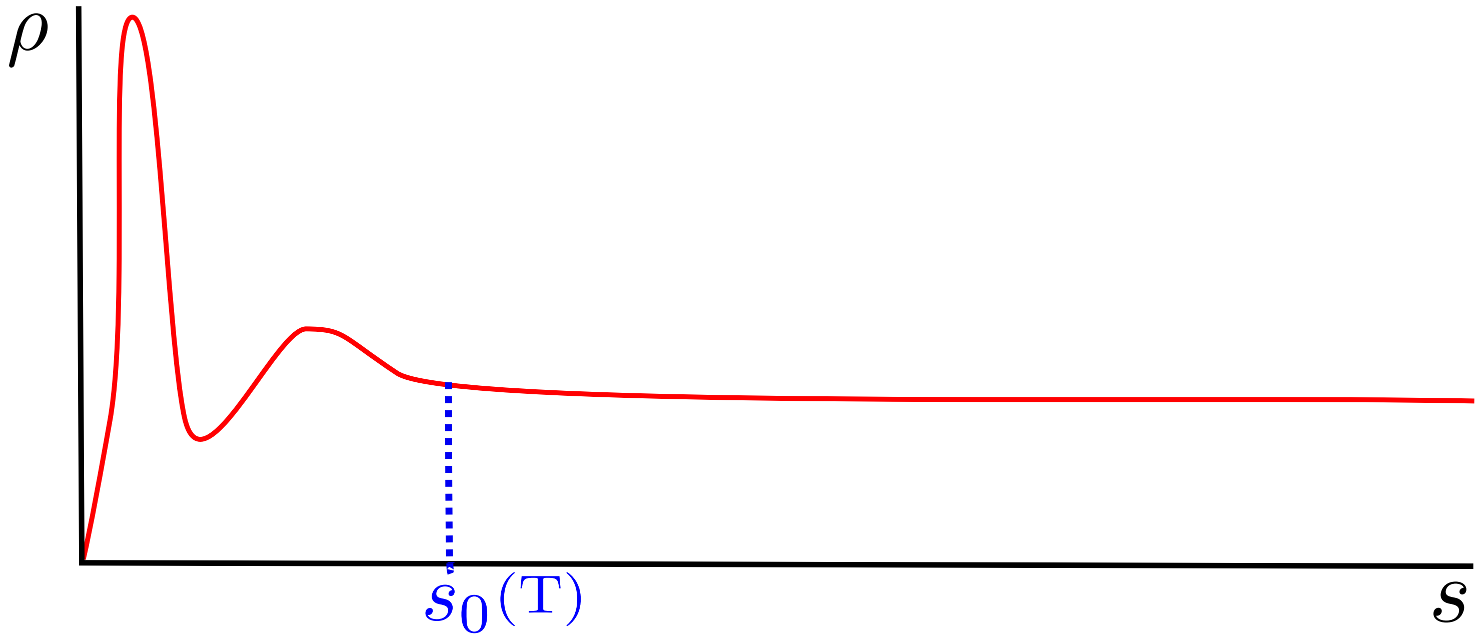


$$\Pi_i^{\mathrm{had}}(\omega, \boldsymbol{p}) = \int_0^\infty dp_0^2 \, \frac{\rho_i(p_0, \boldsymbol{p})}{p_0^2 - \omega^2}$$

Vacuum

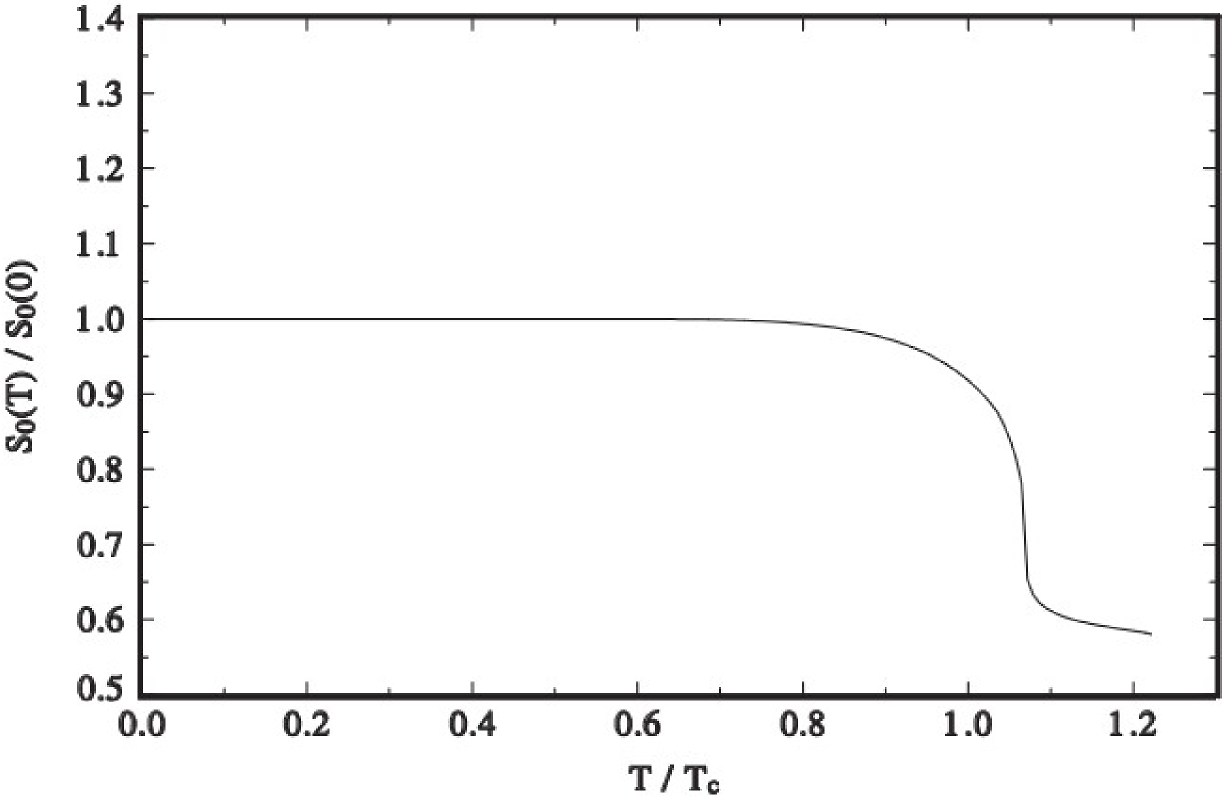


Finite temperature → Resonances “melt”

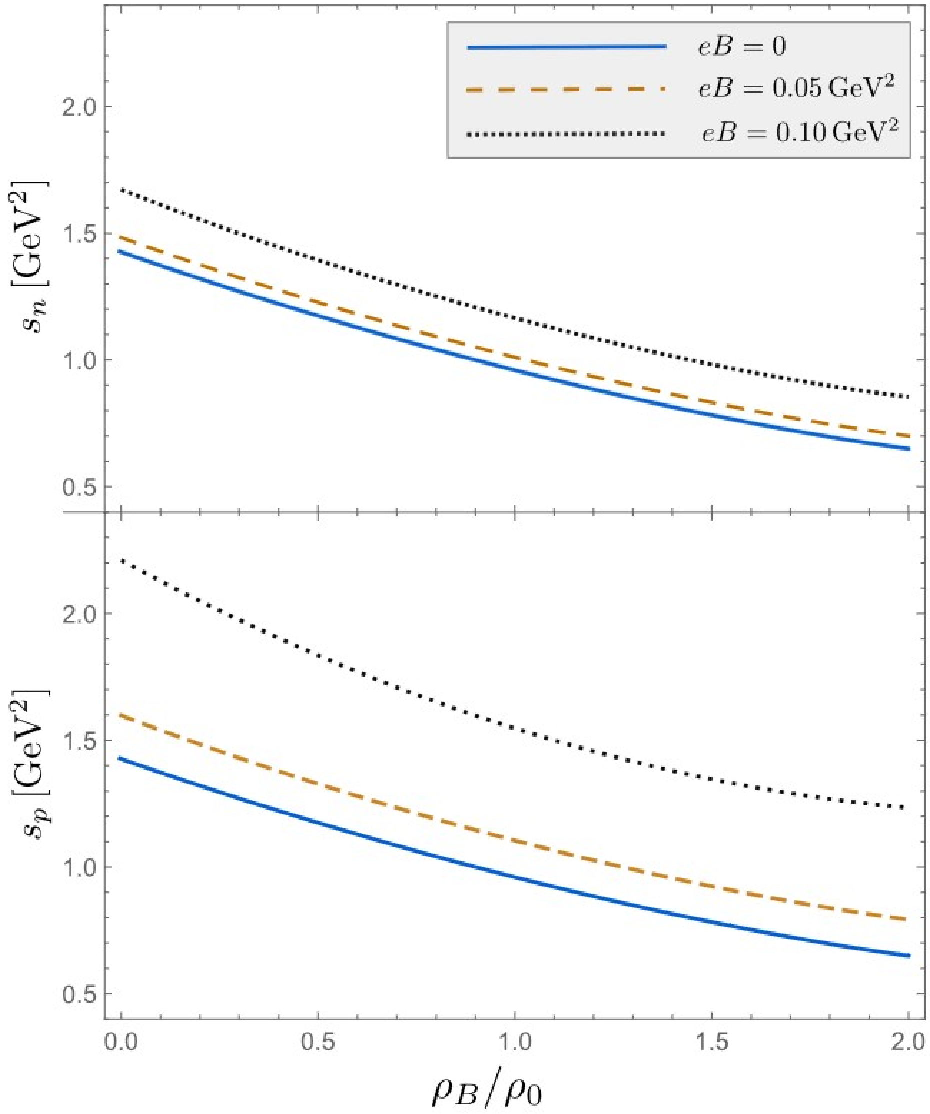


s_0 acts as an order parameter

$s_0 \rightarrow$ deconfinement



Dominguez, Loewe, Rojas, Zhang
PRD **81**, 014007 (2010)



Nucleon threshold at finite baryon density

Dominguez, Loewe, Villavicencio, Zamora
PRD **108**, 074024 (2023)

The QCD sector

Vacuum

Operators → quark and gluon condensates in combination with quark mass

$$\Pi^{\mathrm{pQCD}}(s) \sim \log(-s)$$

$$\langle O_3 \rangle \quad \sim \quad \langle \bar{q} q \rangle$$

$$\langle O_4 \rangle \quad \sim \quad \langle \alpha_s G^a_{\mu\nu} G^{a\mu\nu} \rangle, \quad m_q \langle \bar{q} q \rangle$$

$$\langle O_5 \rangle \quad \sim \quad \langle \bar{q}\, g_s G^a_{\mu\nu} t^a \sigma^{\mu\nu} q \rangle$$

Example: charged pions

Axial current, hadronic sector

$$A_\mu(x) = -f_\pi \partial_\mu \pi^+(x)$$

Axial current, QCD sector

$$A_\mu(x) = \bar{d}(x) \gamma_\mu \gamma_5 u(x)$$

Axial – Axial correlator and derivatives

$$\Pi^A_{\mu\nu}(q^2) = i\int d^4x\, e^{iqx}\, \langle 0|T[A_\mu(x)A^\dagger_\nu(0)]|0\rangle \;=(q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi_T(q^2) + g_{\mu\nu}\,\Pi_d(q^2)$$

$$\Pi_{5\nu}(q^2) = i\int d^4x\, e^{iqx}\, \langle 0|T\,[i\partial\cdot A(x)\, A^\dagger_\nu(0)]|0\rangle = \; q_\nu\,\Pi_5(q^2)$$

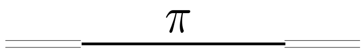
$$\psi_5(q^2) = i\int d^4x\, e^{iqx} \langle 0|T\,[\partial\cdot A(x)\, \partial\cdot A^\dagger_\nu(0)]|0\rangle$$

Ward identities ➡

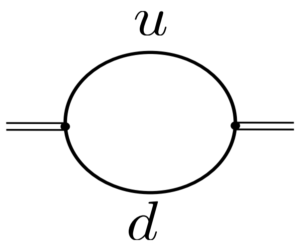
$$q^\mu \Pi^A_{\mu\nu}(q^2) = \Pi_{5\nu}(q^2) + \langle 0|[\bar{u}\gamma_\nu u - \bar{d}\gamma_\nu d]|0\rangle$$

$$q^\nu \Pi_{5\nu}(q^2) = \Psi_5(q^2) + \langle 0|[m_u \bar{u}u + m_d \bar{d}d]|0\rangle$$

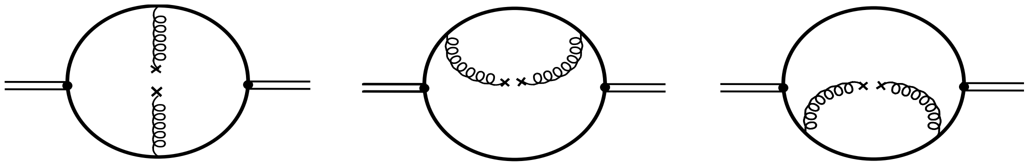
Diagramatically



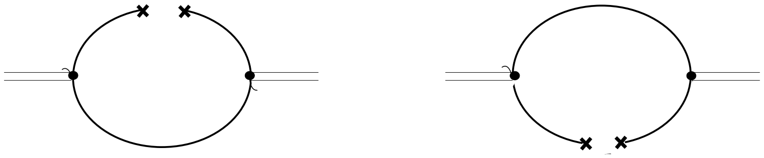
$$\sim \frac{f_\pi^2}{s - m_\pi^2}$$



$$\sim \log(-s)$$



$$\sim \frac{\langle \alpha G^2 \rangle}{s^2}$$



$$\sim \frac{m_q \langle \bar{q} q \rangle}{s^2}$$

FESR for charged pions

$$\int ds\, \Pi_T \rightarrow \qquad 2\, f_\pi^2 = \frac{s_0}{4\,\pi^2}$$

$$\int ds\, s\, \Pi_T \rightarrow \qquad 2\, f_\pi^2\, m_\pi^2 = \frac{s_0^2}{8\,\pi^2} - 2m_q\, \langle \bar{q} q \rangle - \frac{1}{12\pi}\, \langle \alpha_s\, G^2 \rangle$$

$$\int ds\, \Pi_5 \rightarrow \qquad 2\, f_\pi^2\, m_\pi^2 = -\, 4m_q \langle \bar{q} q \rangle + \frac{3}{2\pi^2} {m_q}^2 s_0$$

$$\int ds\, \Psi_5 \rightarrow \qquad 2 f_\pi^2\, m_\pi^4 = \frac{3m_q^2\, s_0^2}{4\,\pi^2} - 4m_q^3\, \langle \bar{q} q \rangle + \frac{m_q^2}{2\pi}\, \langle \alpha_s\, G^2 \rangle$$

$$+\mathcal{O}(m_q/s_0)^n$$

Nucleon – nucleon correlator

hadronic sector

$$\eta_N(x) = \lambda_N \psi(x)$$

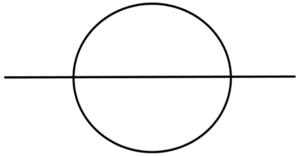
proton interpolating function (QCD sector)

$$\eta_N(x) = \varepsilon^{abc}[(u^a)^T(x) C\gamma_\mu u^b(x)]\gamma^\mu \gamma_5 d^c(x),$$

Neutrons: u ⟺ d

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0 | T\eta(x)\bar{\eta}(0) | 0 \rangle \qquad = \not{q}\Pi_1(q^2) + \Pi_2(q^2)$$

Nucleon – nucleon correlator



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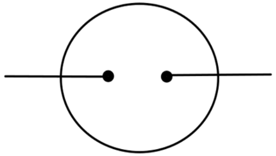
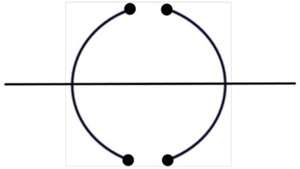
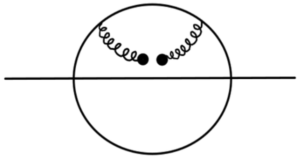


$\sim \langle \bar{q}q \rangle$

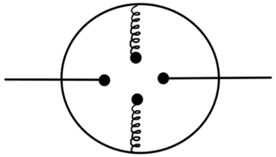


$\sim \langle \bar{q}q\bar{q}q \rangle$

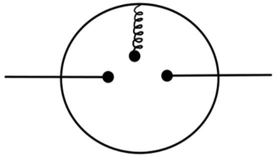
$\sim \Pi_2$



$\sim \langle \alpha_s G^2 \rangle$



$\sim \langle \alpha_s G^2 \bar{q}q \rangle$



$\sim \langle g_s \bar{q} G q \rangle$

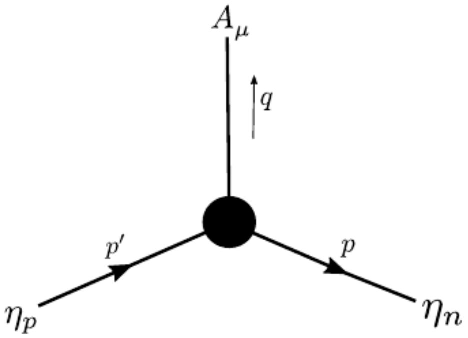
$\sim \Pi_1$

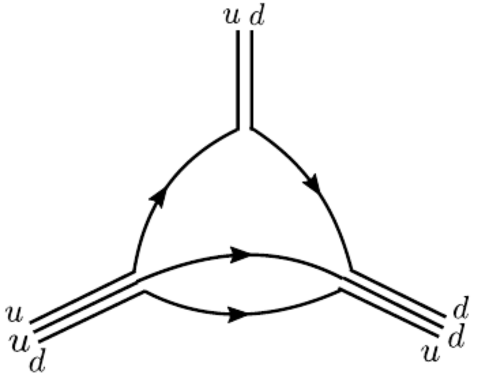
FESR

$$\lambda_N^2 = \frac{s_0^3}{192\pi^4} + \frac{s_0}{32\pi^3}\langle \alpha_s G^2 \rangle + \frac{2}{3}\langle \bar{q}q\bar{q}q \rangle$$

$$\lambda_N^2 m_N = -\frac{s_0^2}{8\pi^2}\langle \bar{q}q \rangle + \frac{1}{12\pi}\langle \alpha_s G^2 \bar{q}q \rangle$$

Nucleon – Axial-vector correlator





double FESR

$$\int_0^{s_p} \frac{ds'}{\pi}\,\mathrm{Im}_{s'}\int_0^{s_n} \frac{ds}{\pi}\,\mathrm{Im}_s\Pi^{\mathrm{had}}(s,s',t) = \oint_{s_p} \frac{ds'}{2\pi i}\oint_{s_n} \frac{ds}{2\pi i}\,\Pi^{\mathrm{QCD}}(s,s',t)$$

Finite temperature and density effects

Propagators

$$\sim \frac{1}{(i\omega_n + \mu)^2 - \boldsymbol{p}^2 - m^2} \qquad \text{finite } T \text{ and } \mu \text{ (Matsubara frequencies)}$$

$$\sim \frac{i}{p_0^2 - \boldsymbol{p}^2 - m^2 + i\epsilon p_0(p_0 - \mu)} \qquad \text{finite } \mu \text{ and } T = 0 \text{ (time ordered propagator)}$$

$$= \frac{i}{p_0^2 - \boldsymbol{p}^2 - m^2 + i\epsilon} - 2\pi\theta(p_0(\mu - p_0))\delta(p_0^2 - \boldsymbol{p}^2 - m^2)$$

Scattering with the thermal bath

First detected by

Bochkarev, Shaposhnikov, NPB **268**, 220 (1986)

Formalized in the quiral limit

Dominguez, Loewe, PLB **233**, 201 (1989)

Scattering with the thermal bath

In the axial-axial correlator

$$\omega \Pi_{\mathrm{sc}} = \int \frac{d^3k}{(2\pi)^3} f(\boldsymbol{p},\boldsymbol{k}) \frac{n_F(E_d-\mu) - n_F(E_u-\mu)}{\omega[\omega - (E_d - E_u)]}$$

Vanishes in the limit $\boldsymbol{p} \to 0$ and $m_u = m_d$

$$E_d = \sqrt{(\boldsymbol{k}+\boldsymbol{p})^2 + m_d^2}, \qquad E_u = \sqrt{\boldsymbol{k}^2 + m_u^2}$$

After contour integration, the residue gives

$$\sim \int \frac{d^dk}{(2\pi)^d} f(\boldsymbol{p},\boldsymbol{k}) \frac{n_F(E_d-\mu) - n_F(E_u-\mu)}{E_d - E_u} \to \frac{d^dk}{(2\pi)^d} f(\boldsymbol{p},\boldsymbol{k}) n_F'(E-\mu)$$

pQCD are calculated with medium dependent propagators

OPE $\rightarrow$ only condensates medium dependent

Operator mixing? $\rightarrow$ not well established in-medium

Prescriptions $\rightarrow$ pQCD medium independent (baryon density)
$\rightarrow$ pQCD medium dependent

our findings $\rightarrow$ Operator mixing contribution to pQCD is medium dependent
$\rightarrow$ Operator mixing contribution to OPE is considered in vacuum

$$\langle : \bar{q}\gamma_\mu iD_\nu q : \rangle = \langle \bar{q}\gamma_\mu iD_\nu q \rangle + c_{\mu\nu}^{(0)}(T, \mu, \dots) + \sum_{n>0} c_{\mu\nu}^{(n)}(0)\langle O_n \rangle$$

Condensates

Quark condensate

Example A-A correlator

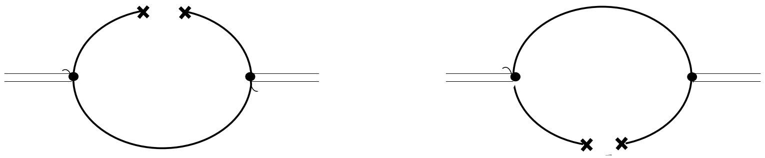
Step 1

$$\langle\Omega|T\,A_\mu(x)A_\nu^\dagger(0)|\Omega\rangle = \langle\Omega|T\,\bar{d}(x)\gamma_\mu\gamma_5 u(x)\,\bar{u}(0)\gamma_\mu\gamma_5 d(0)|\Omega\rangle$$

$$= \Big\{\langle\Omega|:\,\bar{d}_i(x)u_j(x)\,\bar{u}_k(0)d_l(0)\,:|\Omega\rangle$$

$$+\langle\Omega|:\bar{d}_i(x)d_j(0):\,S^u_{jk}(x)|\Omega\rangle\;+\langle\Omega|S^d_{li}(-x)\;:\bar{u}(0)_k u_j(x):|\Omega\rangle$$

$$-\langle\Omega|S^d_{li}(-x)\,S^u_{jk}(x)|\Omega\rangle\Big\}[\gamma_\mu\gamma_5]_{ij}[\gamma_\mu\gamma_5]_{kl}$$



$$\sim \frac{m_q \langle \bar{q} q \rangle}{s^2}$$

Step 2: Small-x expansion

$$q(x) = q(0) + x^\alpha D_\alpha q(0) + \frac{1}{2} x^\alpha x^\beta D_\alpha D_\beta q(0) + \dots$$

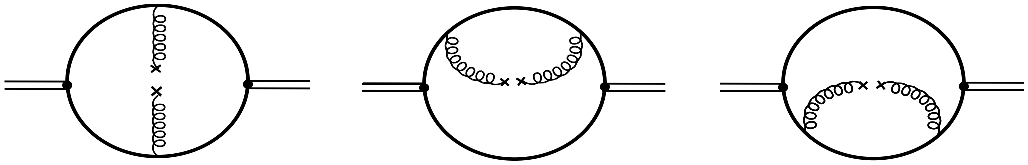
Step 3: Operator mixing → non-normal ordered condensates

Gluon condensate

Expansion in backroun gluon field

$$S(x,y) = \langle x | \frac{i}{i\not{\partial} - g\not{G} - m} | y \rangle$$

Example A-A correlator



$$\sim \frac{\langle \alpha G^2 \rangle}{s^2}$$

Background gluon field small-x expansion

Fock-Schwinger gauge: $x^\alpha G^a_\alpha(x) = 0$

$$G^a_\alpha(x) = \int_0^1 dt\, t\, x^\beta G^a_{\alpha\beta}(tx) \approx \frac{1}{2} x^\beta G^a_{\alpha\beta}(0) + \frac{1}{3} x^\rho x^\beta D_\rho G^a_{\alpha\beta}(0) + \ldots$$

In vacuum

$$\langle \bar{q}_i q_j \rangle \sim \langle \bar{q} q \rangle \delta_{ij}$$

$$\langle \bar{q}_i D_\mu q_j \rangle \sim m_q \langle \bar{q} q \rangle [\gamma_\mu]_{ij}$$

Finite temperature and density → $u_\mu = (1, 0, 0, 0)$

$$\langle \bar{q}_i q_j \rangle \sim \langle \bar{q} q \rangle \delta_{ij} + \langle q^\dagger q \rangle [\gamma_0]_{ij}$$

$$\langle \bar{q}_i i D_\mu q_j \rangle \sim m_q \langle \bar{q} q \rangle [\gamma_\mu]_{ij} + \langle \bar{q} i D_0 q \rangle u_\mu \delta_{ij}$$

In vacuum

$$\langle \alpha_s G^a_{\mu\nu} G^a_{\alpha\beta} \rangle \sim \langle \alpha_s G^2 \rangle (g_{\mu\alpha} g_{\nu\beta} - g_\beta g_{\nu\alpha})$$

Finite temperature and density → $u_\mu = (1, 0, 0, 0)$

$$\langle \alpha_s G^a_{\mu\nu} G^a_{\alpha\beta} \rangle \sim -\big[u_\mu u_\alpha \eta_{\nu\beta} - u_\mu u_\beta \eta_{\nu\alpha} + u_\nu u_\beta \eta_{\mu\alpha} - u_\nu u_\alpha \eta_{\mu\beta} \big] \langle \alpha_s E^2 \rangle$$
$$+ \big[\eta_{\mu\alpha} \eta_{\nu\beta} - \eta_{\mu\beta} \eta_{\nu\alpha} \big] \langle \alpha_s B^2 \rangle$$

$$\eta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

If we consider external electromagnetic effects $\rightarrow$ $F_{\mu\nu}$

Structures include all possible combinations of

$$g_{\mu\nu} \qquad \epsilon_{\alpha\beta\mu\nu} \qquad u_\mu \qquad F_{\mu\nu}$$

A large amount of condensates!

How to calculate condensates?

- By the sum rules
- through your favorite model
- lattice

Example: finite baryon density

$$\langle \mathcal{O}_n \rangle = \langle 0|\mathcal{O}_n|0\rangle + \langle N|\mathcal{O}_n|N\rangle + \langle NN|\mathcal{O}_n|NN\rangle + \dots$$

Gluon condensate:

Vacuum → with FESR

temperature and density contribution → through the traceanomaly (gauge only)

Hadronic sector

$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s+i\epsilon) \qquad \Pi(p) = \int_0^\infty ds\, \frac{\rho(s)}{s-p^2},$$

Example: Axial-vector correlator $\langle 0|TA_\mu(x)A_\nu^\dagger(0)|0\rangle$

$$1 = |\pi\rangle\langle\pi| + |\pi\pi\rangle\langle\pi\pi| + \ldots$$

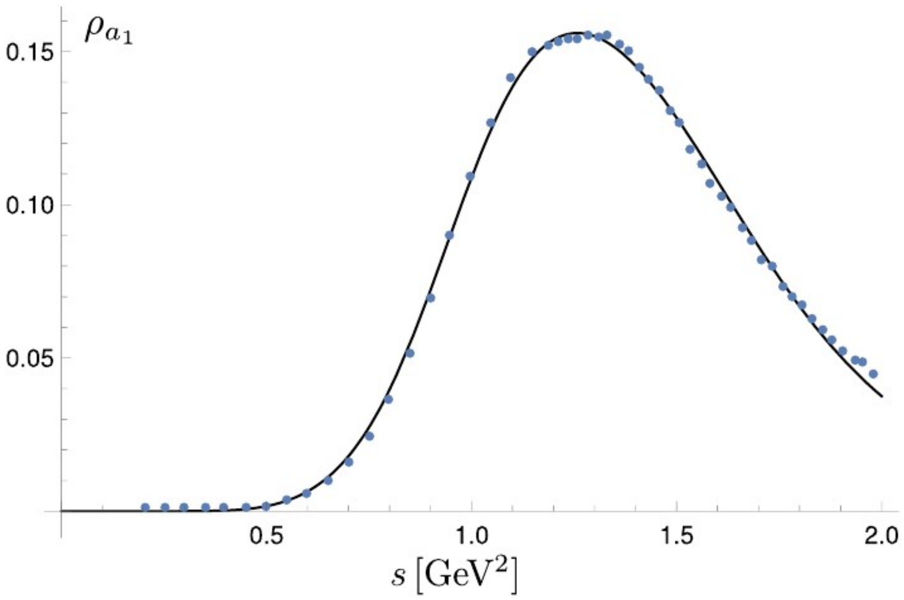
Saturation with one pion

$$\langle 0|A_\mu(x)|\pi(p)\rangle = \sqrt{2}f_\pi e^{-ip\cdot x} \quad \Longrightarrow \quad A_\mu(x) = -\sqrt{2}f_\pi\partial_\mu\pi^+(x)$$

Example: Nucleon-nucleon correlator

$$\langle 0|\eta_N(x)|N(p,s)\rangle = \lambda_N u(p,s)e^{ip\cdot x} \quad \Longrightarrow \quad \eta_N(x) = \lambda_N\psi_N(x)$$

Example: a1



$$\Pi_{a_1}(s) = \frac{-2f_{a_1}^2}{s - m_{a_1}^2 + i\sqrt{s}\Gamma(s)},$$

$$\Gamma(s) = \Gamma_{a_1} \exp\left[-a\left(\frac{s - m_{a_1}^2}{\sqrt{s}\Gamma_{a_1}}\right)^2\right]$$

Or use some model, calculate the self-energy.. as you wish

Finite temperature and density

$$\Pi_i^{\rm had}(\omega, \boldsymbol{p}) = \int_0^\infty dp_0^2 \, \frac{\rho_i(p_0, \boldsymbol{p})}{p_0^2 - \omega^2}$$

$$A_0(x) = -f_t \partial_0 \pi^+(x)$$

$$A_i(x) = -f_s \partial_i \pi^+(x)$$

$$\Pi_{\mu\nu}^A(p) = -2 f_\pi^2 \frac{P_\mu P_\mu}{p_0^2 - v_\pi^2 \boldsymbol{p}^2 - M_\pi^2}$$

$$P_0 = p_0, \qquad P_i = v_\pi^2 p_i$$

$$f_\pi = \sqrt{f_s f_t}, \qquad v_\pi = \sqrt{f_s / f_t}$$ → Imposed by Ward identity

Screening mass: $m_{sc} = M_\pi / v_\pi$

External electromagnetic effects

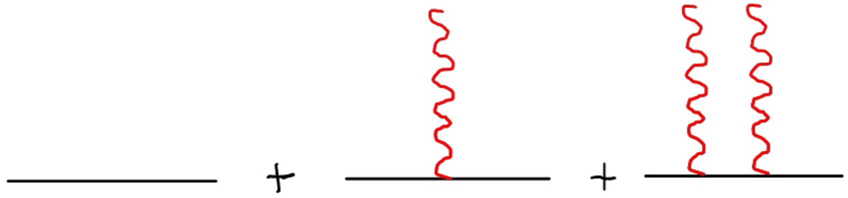
QCD sector

In QCD, we can just do the same as with the gluons

→ expansion in electromagnetic fields

$$S = \frac{i}{i\partial\!\!\!/ + e_f A\!\!\!/ + g G\!\!\!/ - m_f}$$

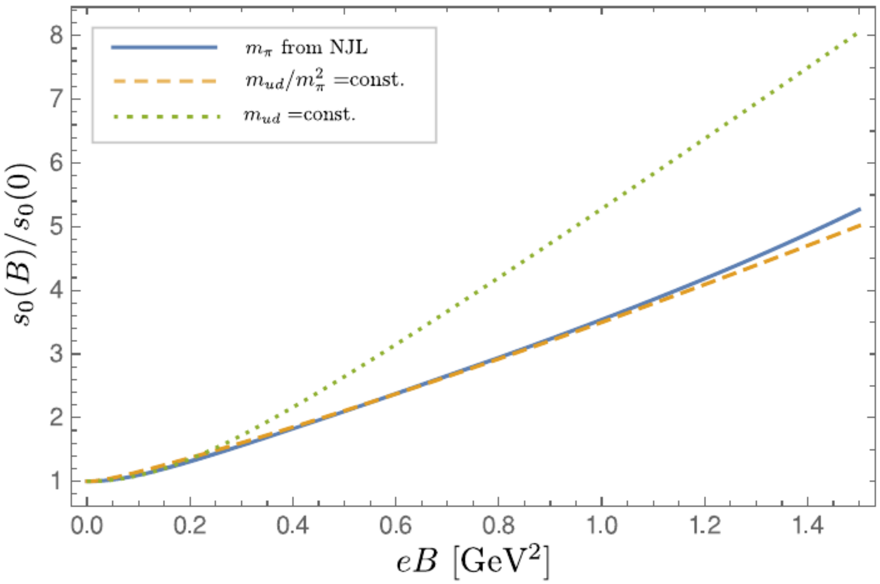
➡ Expansion in powers of $\frac{F_{\mu\nu}F^{\mu\nu}}{(s_0 - m_f^2)^2}$

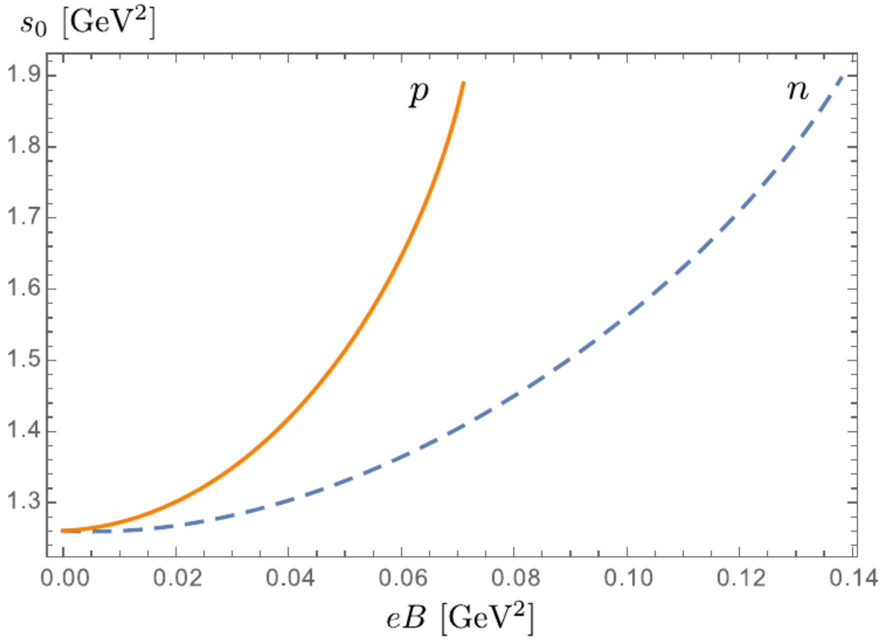


Example: magnetic field $s_0 \sim 1 \text{ GeV}^2 \to |eB| < 0.1 \text{ GeV}^2 \sim 5m_\pi^2$ Big enough!

QCD sector

Magnetic field can reach larger values





Schwinger proper time or Landau levels?

→ complicated if we have different flavors (orthogonality breaks)

Hadronic sector

field expansion

Notice: $$\frac{1}{s-m^2} = \frac{1}{s-(m^2-a)-a} = \sum_n \left(\frac{a}{s-(m^2-a)}\right)^n$$

Produces the same FESR

Condition: $0 < m^2 - a < s_0$

Hadronic sector

Expanded magnetic field propagator is good enough

$$G(x,x') = e^{i\phi(x,x)} S(x-x')$$

Example: fermions

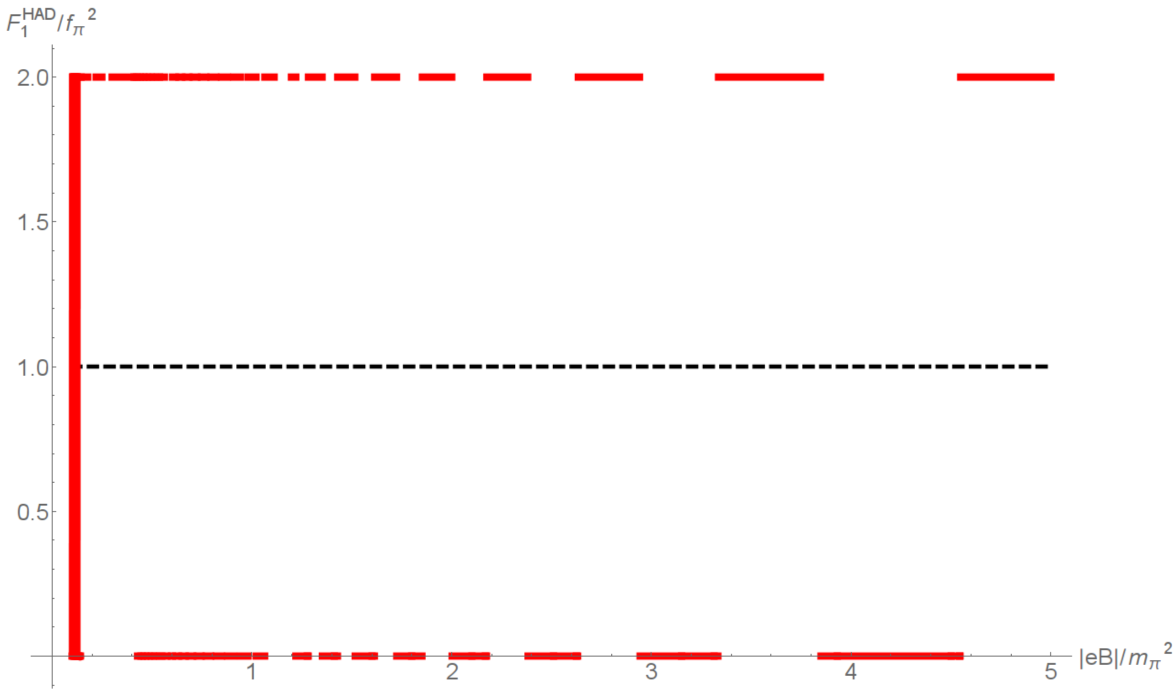
$$\tilde{S}(p) = \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon} - (\kappa B)\frac{i(\not{p}+m)\sigma_{12}(\not{p}+m)}{(p^2-m^2+i\epsilon)^2}$$
$$- (qB)\frac{i\sigma_{12}(\not{p}_{\parallel}+m)}{(p^2-m^2+i\epsilon)^2} + 2i(qB)^2\frac{(\not{p}_{\parallel}+m)\left[p_{\perp}^2 - \not{p}_{\perp}(\not{p}_{\parallel}-m)\right]}{(p^2-m^2+i\epsilon)^4} + \dots$$

$$p = (p^0, v_{\perp}\boldsymbol{p}_{\perp}, v_{\parallel}p^3)$$

Hadronic sector

Propagator expanded in Landau levels: No problem

$$\mathrm{Disc}\,\frac{1}{p_{\|}^2 - 2|eB|n - m^2} = i\pi\delta(p_{\|}^2 - 2|eB|n - m^2)$$ ➡ Discrete results



Hadronic sector

Schwinger proper time → Not recommended

Technical problems with the discontinuity

$$\mathrm{Disc} \int d\tau e^{ip_{\parallel}^2 \tau + \cdots} f(\tau, B) = ?$$

Charged pion correlator

$$A_\mu(x) = -f_\pi D_\mu \pi^+(x)$$

$$\Pi^{AA}_{\mu\nu}(x,y) = i\langle 0|T[A_\mu(x)A^\dagger_\nu(y)]|0\rangle$$

$$\Pi_{5\nu}(x,y) = i\langle 0|T[iD\cdot A(x)A^\dagger_\nu(y)]|0\rangle$$

$$\psi_5(x,y) = i\langle 0|T[iD\cdot A(x)[iD\cdot A(y)]^\dagger]|0\rangle$$

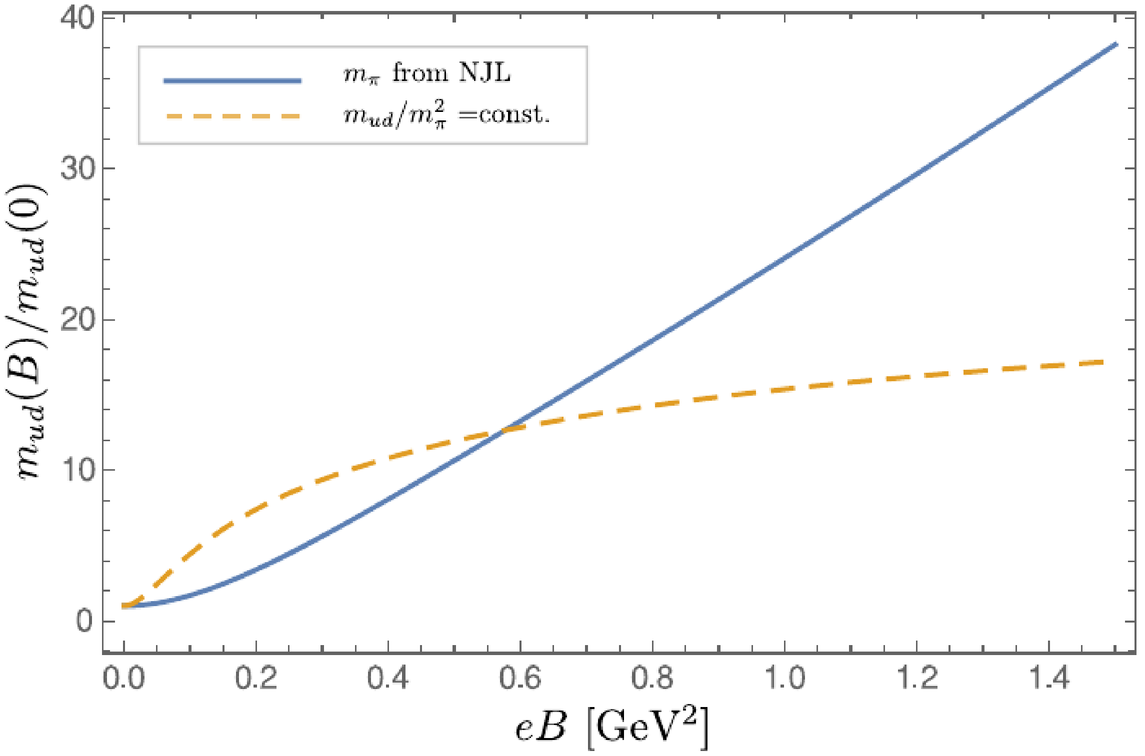
Ward identity

$$Q^\mu \Pi^{AA}_{\mu\nu}(q^2) = \Pi_{5\nu}(q^2) - \Delta_\nu(q^2)$$

$$Q^{*\nu}\Pi_{5\nu}(q^2) = \psi_5(q^2) + \Delta_5(q^2),$$

$$Q_\mu = q_\mu + \frac{ie}{2}F_{\mu\nu}\frac{\partial}{\partial q_\nu}$$

Alejandro's request



No summary, just some references

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