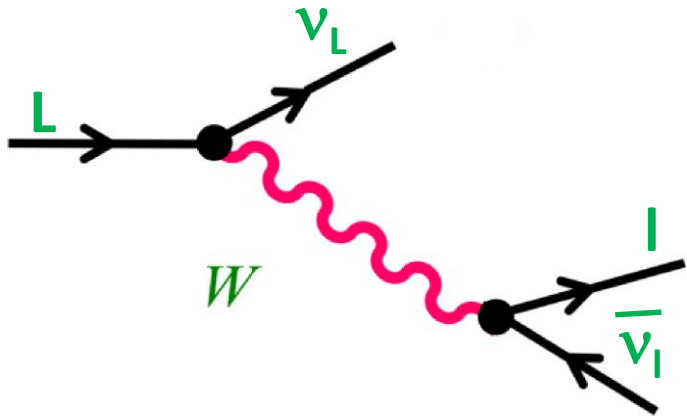
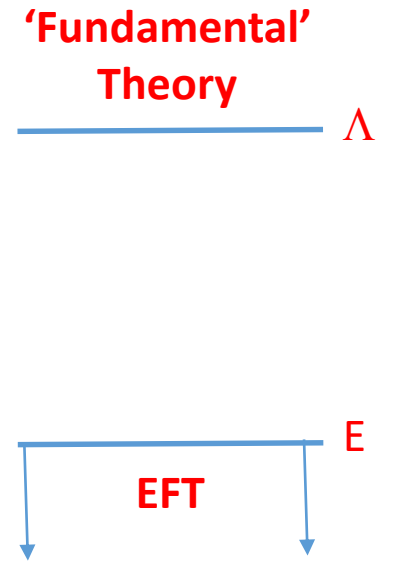
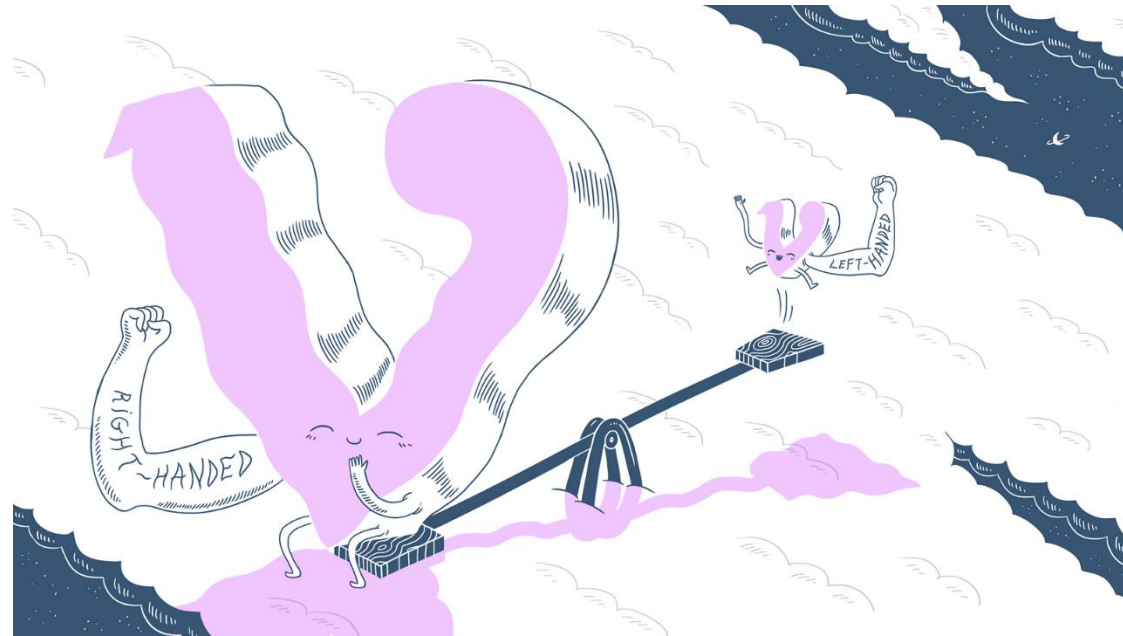


Naturaleza del neutrino masivo: análisis de teorías efectivas cuánticas de campo



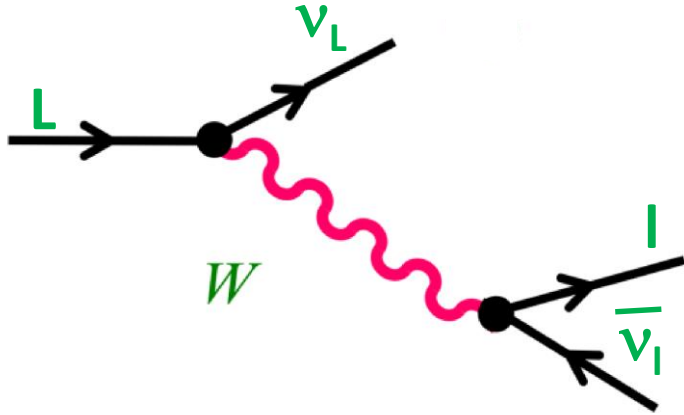
(and related processes)



Seminario conjunto ICN-UNAM IF-UNAM de Física de Altas Energías
ICN 10 de Abril de 2024

Pablo Roig
Cinvestav (Mexico)

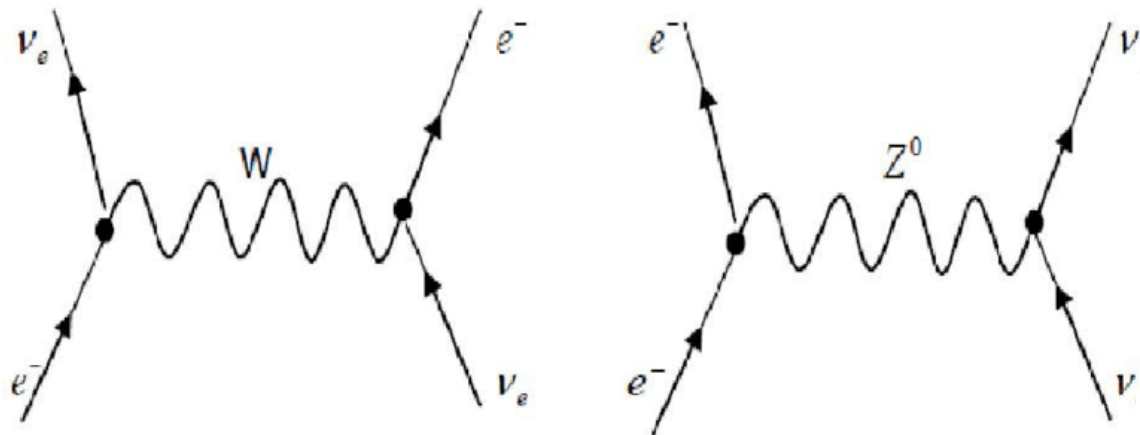
CONTENTS



Michel parameters in the presence of massive Dirac and Majorana neutrinos

In collaboration with Juanma Márquez & Gabriel López Castro.
JHEP11(2022)117. arXiv:2208.01715

INTERMISSION



(a)

(b)

$\nu e \rightarrow \nu e$ scattering with massive Dirac or Majorana neutrinos and general interactions

In collaboration with Juanma Márquez & Mónica Salinas.
Under review. arXiv: 2401.14305

Pequeñez de la masa de los neutrinos ($10^{-7} \sim m_\nu/m_e < m_e/m_t \sim 3 \times 10^{-6}$)

$\bar{\psi}_L \psi_R \phi + \bar{\psi}_R \psi_L \phi^* \rightarrow m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) = m \bar{\psi} \psi$. Si se genera m_e/m_t , ¿por qué no m_ν/m_e ?
SSB

Una vez se introduce ψ_R , el principio de gauge permite tener términos de masa de Majorana $m_M(\bar{\psi}_R^c \psi_R + \bar{\psi}_R \psi_R^c)$



See-saw mechanism

$$m_L \sim m^2/m_M$$

$$m_H \sim m_M$$

$$m_L \sim 50 \text{ meV} \Rightarrow m_M \sim 10^{[10,15]} \text{ GeV}$$

Las desintegraciones de m_H producen leptogénesis

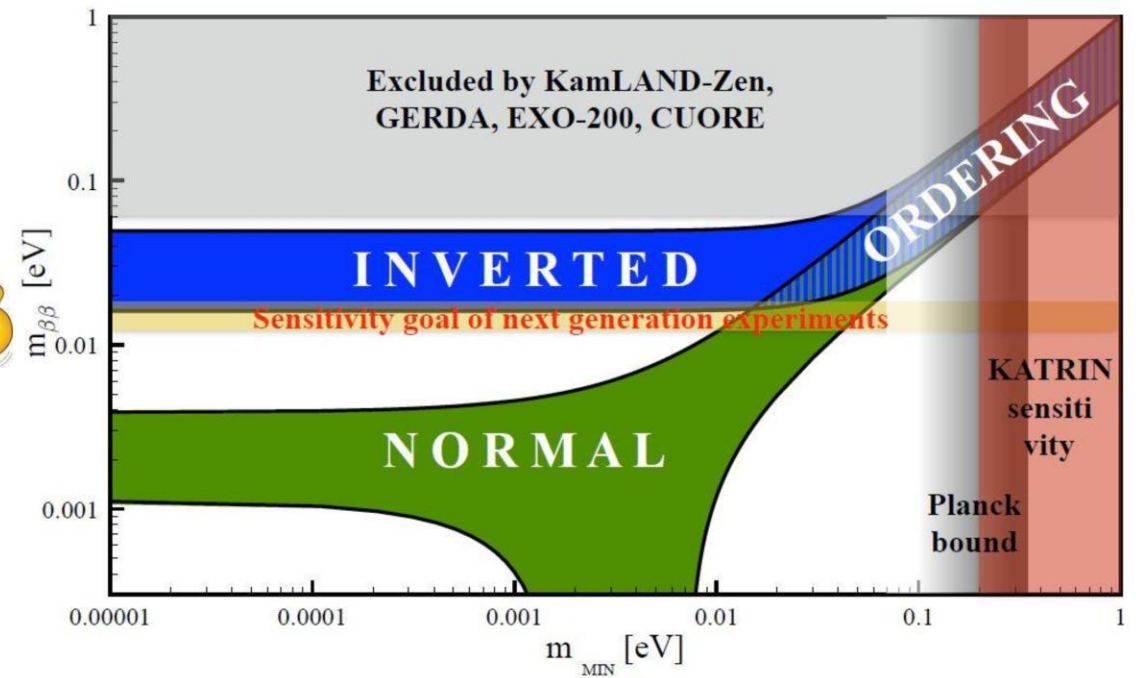
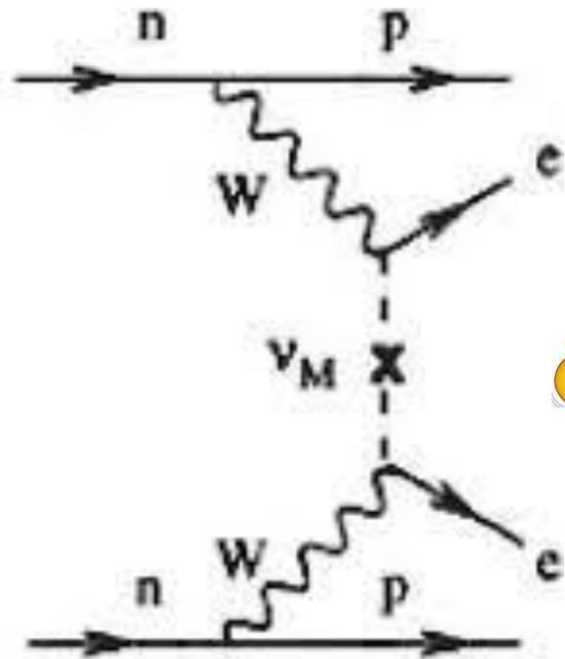
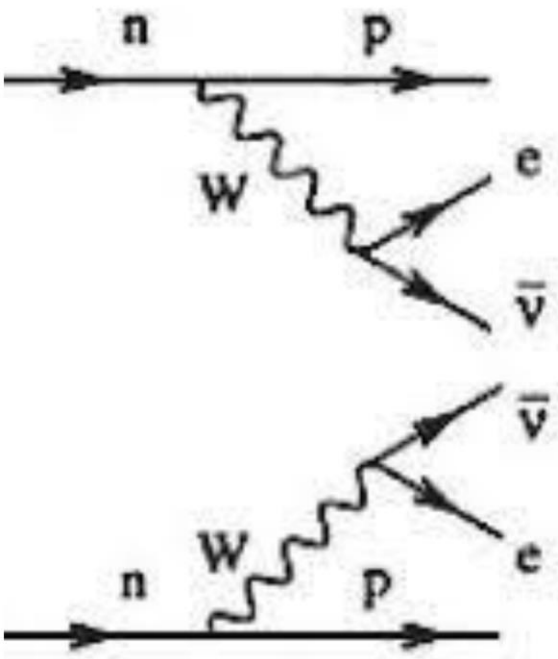
Pero probablemente sin que se pueda medir nada (cLFV, etc.) en experimentos de esta o la siguiente generación.

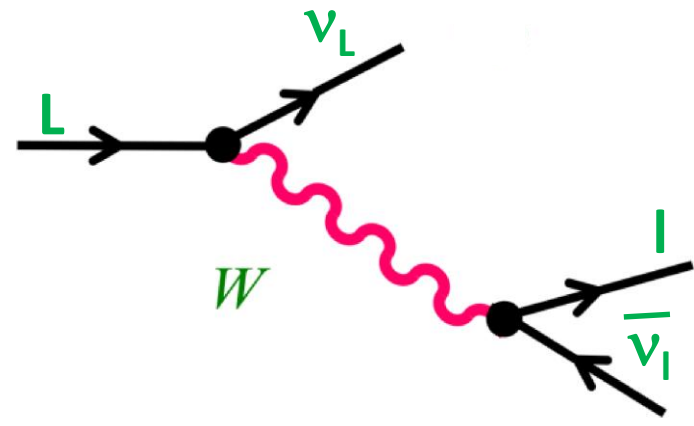


Pequeñez de la masa de los neutrinos ($10^{-7} \sim m_\nu/m_e < m_e/m_t \sim 3 \times 10^{-6}$)

Si los neutrinos son de **Majorana**
Podemos tener desintegración doble β sin ν s...

Si medimos desintegración doble β sin ν s
 \Rightarrow Los ν s son de **Majorana**
(el inverso no es cierto)





Michel parameters in the presence of massive Dirac and Majorana neutrinos



Cinvestav

Pablo Roig
Cinvestav (Mexico)

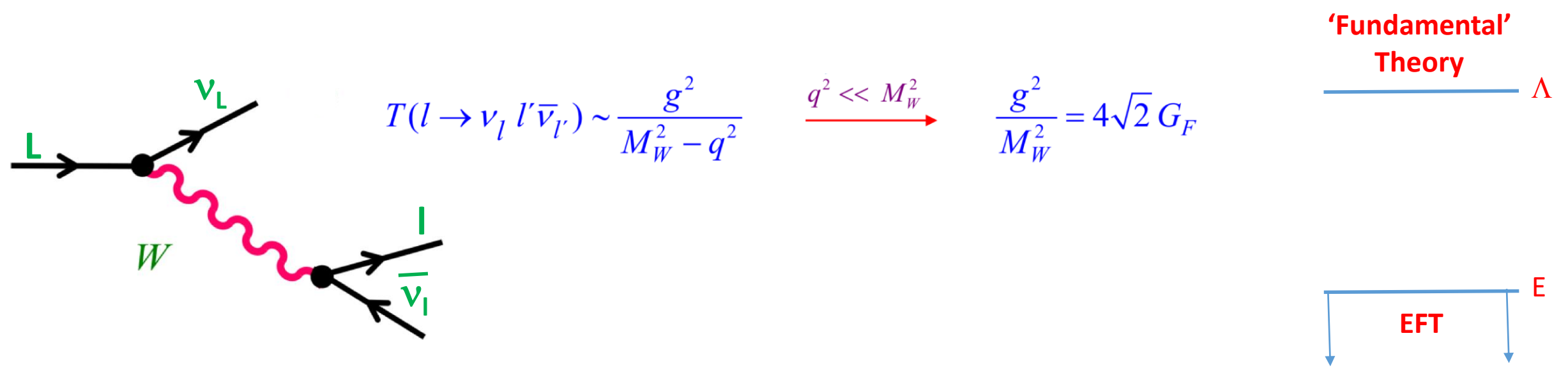
In collaboration with:

J.M. Márquez (Cinvestav, Mexico)

G. López-Castro (Cinvestav, Mexico)

JHEP11(2022)117. arXiv:2208.01715

Pablo Roig
Cinvestav (Mexico)



The most general, derivative-free, four-lepton interaction Hamiltonian, consistent with Lorentz invariance is:

$$\mathcal{H} = 4 \frac{G_{ll'}}{\sqrt{2}} \sum_{n, \epsilon, \omega} g_{\epsilon\omega}^n \left[\bar{l}'_{\epsilon} \Gamma^n (\nu_{l'})_{\sigma} \right] \left[(\bar{\nu}_l)_{\lambda} \Gamma_n l_{\omega} \right] + h.c.$$

Where $\epsilon, \omega, \sigma, \lambda$ label the chiralities (L, R) of fermions, and $n = S, V, T$ the type of interaction: scalar ($\Gamma^S = I$), vector ($\Gamma^V = \gamma^{\mu}$) and tensor ($\Gamma^T = \sigma^{\mu\nu} / \sqrt{2}$).

$$\tau_{\mu} \longrightarrow |g_{\epsilon\omega}^S| \leq 2, |g_{\epsilon\omega}^V| \leq 1 \text{ and } |g_{\epsilon\omega}^T| \leq 1/3.$$

For the case of massless neutrinos, the differential decay rate is:

$$\frac{d\Gamma}{dx d\cos\theta} = \frac{m_1}{4\pi^3} \omega^4 G_{II'}^2 \sqrt{x^2 - x_0^2} (F(x) - \frac{\xi}{3} \mathcal{P} \sqrt{x^2 - x_0^2} \cos\theta A(x)) \times [1 + \hat{\zeta} \cdot \vec{\mathcal{P}}_{I'}(x, \theta)],$$

where \mathcal{P} is the degree of the initial lepton polarization, θ is the angle between the l^- spin and the final charged-lepton momenta, $\omega \equiv (m_1^2 + m_4^2)/2m_1$, $x \equiv E_4/\omega$ is the reduced energy and $x_0 \equiv m_4/\omega$, $\hat{\zeta}$ is an arbitrary direction parallel to the final charged-lepton spin and the polarization vector $\vec{\mathcal{P}}_{I'}$ is:

$$\vec{\mathcal{P}}_{I'} = P_{T_1} \cdot \hat{x} + P_{T_2} \cdot \hat{y} + P_L \cdot \hat{z}.$$

The components of $\vec{\mathcal{P}}_{I'}$ are, respectively:

$$P_{T_1} = \mathcal{P} \sin\theta \cdot F_{T_1}(x) / \{F(x) - \frac{\xi}{3} \mathcal{P} \sqrt{x^2 - x_0^2} \cos\theta A(x)\},$$

$$P_{T_2} = \mathcal{P} \sin\theta \cdot F_{T_2}(x) / \{F(x) - \frac{\xi}{3} \mathcal{P} \sqrt{x^2 - x_0^2} \cos\theta A(x)\},$$

$$P_L = \frac{-F_{IP}(x) + \mathcal{P} \cos\theta \cdot F_{AP}(x)}{F(x) - \frac{\xi}{3} \mathcal{P} \sqrt{x^2 - x_0^2} \cos\theta A(x)}.$$

These functions are written in terms of the well-known **Michel Parameters**

$(\rho, \eta, \delta, \xi, \eta'', \xi', \xi'', \alpha', \beta')$:

$$F(x) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x),$$

$$A(x) = 1-x + \frac{2}{3}\delta(4x - 4 + \sqrt{1-x_0^2}),$$

$$F_{T_1}(x) = \frac{1}{12} \left[-2\left(\xi'' + 12\left(\rho - \frac{3}{4}\right)\right)(1-x)x_0 - 3\eta(x^2 - x_0^2) + \eta''(-3x^2 + 4x - x_0^2) \right],$$

$$F_{T_2}(x) = \frac{1}{3} \sqrt{x^2 - x_0^2} \left[3\frac{\alpha'}{\mathcal{A}}(1-x) + 2\frac{\beta'}{\mathcal{A}} \sqrt{1-x_0^2} \right],$$

$$F_{IP}(x) = \frac{1}{54} \sqrt{x^2 - x_0^2} \left[9\xi' \left(-2x + 2 + \sqrt{1-x_0^2} \right) + 4\xi \left(\delta - \frac{3}{4} \right) \left(4x - 4 + \sqrt{1-x_0^2} \right) \right],$$

$$F_{AP}(x) = \frac{1}{6} \left[\xi''(2x^2 - x - x_0^2) + 4\left(\rho - \frac{3}{4}\right)(4x^2 - 3x - x_0^2) + 2\eta''(1-x)x_0 \right].$$

As an example:

$$\eta = \frac{1}{2} \operatorname{Re}[g_{LL}^V g_{RR}^{S*} + g_{RR}^V g_{LL}^{S*} + g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*}) + g_{RL}^V (g_{LR}^{S*} + 6g_{LR}^{T*})].$$

In the SM, $\rho = \delta = 3/4$, $\eta = \eta'' = \alpha' = \beta' = 0$ and $\xi = \xi' = \xi'' = 1$.

	$\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$	$\tau^- \rightarrow e^- \nu_\tau \bar{\nu}_e$	$\tau^- \rightarrow \mu^- \nu_\tau \bar{\nu}_\mu$
ρ	0.74979 ± 0.00026	0.747 ± 0.010	0.763 ± 0.020
η	0.057 ± 0.034	—	0.094 ± 0.073
ξ	$1.0009^{+0.0016}_{-0.0007}$	$\eta = 0.016 \pm 0.013$ 0.994 ± 0.040	1.030 ± 0.059
$\xi\delta$	$0.7511^{+0.0012}_{-0.0006}$	0.734 ± 0.028	0.778 ± 0.037
ξ'	1.00 ± 0.04	—	—
ξ''	0.65 ± 0.36	—	—

The total decay rate is:

$$\Gamma_{I \rightarrow I'} = \frac{\hat{G}_{II'}^2 m_1^5}{192\pi^3} f(m_4^2/m_1^2) \left(1 + \delta_{RC}^{II'}\right),$$

where

$$\hat{G}_{II'} \equiv G_{II'} \sqrt{1 + 4\eta \frac{m_4}{m_1} \frac{g(m_4^2/m_1^2)}{f(m_4^2/m_1^2)}}$$

$f(x) = 1 - 8x - 12x^2 \log(x) + 8x^3 - x^4$, $g(x) = 1 + 9x - 9x^2 - x^3 + 6x(1+x)\log(x)$
and the SM radiative correction $\delta_{RC}^{II'}$ has been included.

$$\delta_{RC}^{II'} = \frac{\alpha}{2\pi} \left[\frac{25}{4} - \pi^2 + \mathcal{O}\left(\frac{m_4^2}{m_1^2}\right) \right] + \dots$$

$$G_{II'}^2 = \left[\frac{g^2}{4\sqrt{2}M_W^2} (1 + \Delta r) \right]^2 \left[1 + \frac{3}{5} \frac{m_1^2}{M_W^2} + \frac{9}{5} \frac{m_4^2}{M_W^2} + \mathcal{O}\left(\frac{m_4^4}{m_1^2 M_W^2}\right) \right]$$

The current neutrino ($\nu_{L,R}$) is assumed to be the superposition of the mass-eigenstate neutrinos (N_j) with the mass m_j , that is,

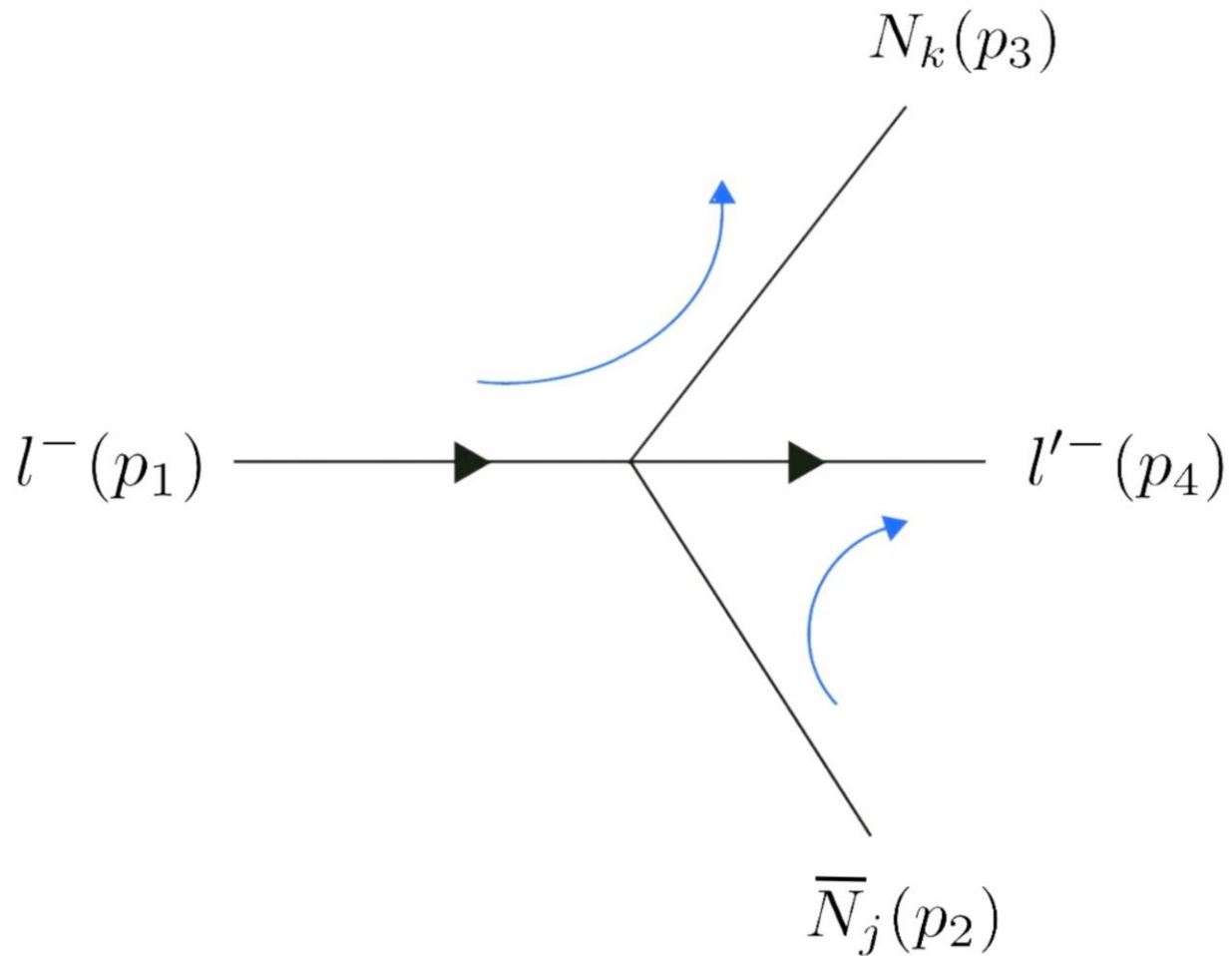
$$\nu_{iL} = \sum_j \underbrace{U_{lj}} N_{jL}, \quad \nu_{iR} = \sum_j \underbrace{V_{lj}} N_{jR},$$

where $j = \{1, 2, \dots, n\}$ with n the number of mass-eigenstate neutrinos. Thus, we can write the effective Hamiltonian in the mass basis, for the process $l^- \longrightarrow l'^- \bar{N}_j N_k$.

$$\begin{aligned}
\mathcal{H} = & 4 \frac{G_{II'}}{\sqrt{2}} \sum_{j,k} \left\{ g_{LL}^S \left[\vec{l}_L V_{l'j} N_{jR} \right] \left[\bar{N}_{kR} V_{Ik}^* I_L \right] + g_{LL}^V \left[\vec{l}_L \gamma^\mu U_{l'j} N_{jL} \right] \left[\bar{N}_{kL} U_{Ik}^* \gamma_\mu I_L \right] \right. \\
& + g_{RR}^S \left[\vec{l}_R U_{l'j} N_{jL} \right] \left[\bar{N}_{kL} U_{Ik}^* I_R \right] + g_{RR}^V \left[\vec{l}_R \gamma^\mu V_{l'j} N_{jR} \right] \left[\bar{N}_{kR} V_{Ik}^* \gamma_\mu I_R \right] \\
& + g_{LR}^S \left[\vec{l}_L V_{l'j} N_{jR} \right] \left[\bar{N}_{kL} U_{Ik}^* I_R \right] + g_{LR}^V \left[\vec{l}_L \gamma^\mu U_{l'j} N_{jL} \right] \left[\bar{N}_{kR} V_{Ik}^* \gamma_\mu I_R \right] \\
& + g_{LR}^T \left[\vec{l}_L \frac{\sigma^{\mu\nu}}{\sqrt{2}} V_{l'j} N_{jR} \right] \left[\bar{N}_{kL} U_{Ik}^* \frac{\sigma^{\mu\nu}}{\sqrt{2}} I_R \right] + g_{RL}^S \left[\vec{l}_R U_{l'j} N_{jL} \right] \left[\bar{N}_{kR} V_{Ik}^* I_L \right] \\
& \left. + g_{RL}^V \left[\vec{l}_R \gamma^\mu V_{l'j} N_{jR} \right] \left[\bar{N}_{kL} U_{Ik}^* \gamma_\mu I_L \right] + g_{RL}^T \left[\vec{l}_R \frac{\sigma^{\mu\nu}}{\sqrt{2}} U_{l'j} N_{jL} \right] \left[\bar{N}_{kR} V_{Ik}^* \frac{\sigma^{\mu\nu}}{\sqrt{2}} I_L \right] \right\}.
\end{aligned}$$

Note that $\boxed{\bar{N}}$ represents an antineutrino for the Dirac neutrino case, but should be identified with N for the Majorana neutrino case $(N=N^c=C\bar{N}^T)$.

Dirac Neutrinos



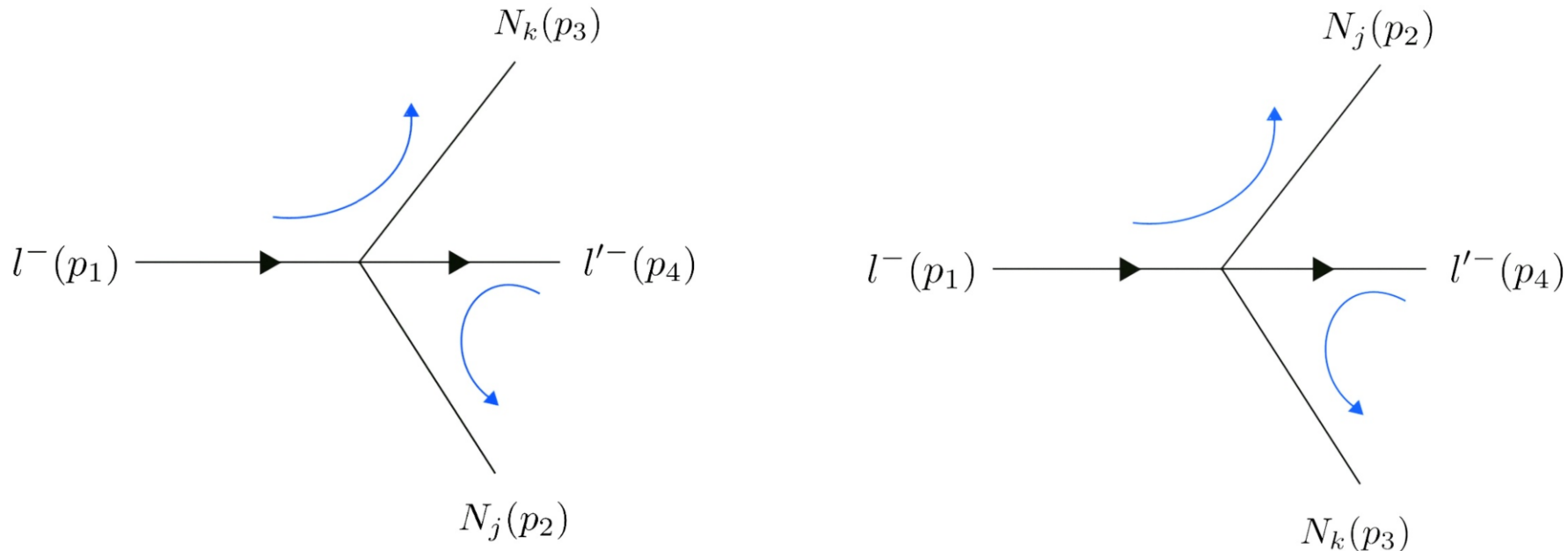
- Neutrino \neq Antineutrino.
- One possible first-order Feynman diagram.
- Well defined fermionic flux.

The Hamiltonian for the case of Majorana neutrinos is

$$\begin{aligned}
\mathcal{H} = & 4 \frac{G_{II'}}{\sqrt{2}} \sum_{j,k} \left\{ g_{LL}^S \left[\vec{I}_L V_{I'j} N_{jR} \right] \left[N_{kR} V_{Ik}^* I_L \right] + g_{LL}^V \left[\vec{I}_L \gamma^\mu U_{I'j} N_{jL} \right] \left[N_{kL} U_{Ik}^* \gamma_\mu I_L \right] \right. \\
& + g_{RR}^S \left[\vec{I}_R U_{I'j} N_{jL} \right] \left[N_{kL} U_{Ik}^* I_R \right] + g_{RR}^V \left[\vec{I}_R \gamma^\mu V_{I'j} N_{jR} \right] \left[N_{kR} V_{Ik}^* \gamma_\mu I_R \right] \\
& + g_{LR}^S \left[\vec{I}_L V_{I'j} N_{jR} \right] \left[N_{kL} U_{Ik}^* I_R \right] + g_{LR}^V \left[\vec{I}_L \gamma^\mu U_{I'j} N_{jL} \right] \left[N_{kR} V_{Ik}^* \gamma_\mu I_R \right] \\
& + g_{LR}^T \left[\vec{I}_L \frac{\sigma^{\mu\nu}}{\sqrt{2}} V_{I'j} N_{jR} \right] \left[N_{kL} U_{Ik}^* \frac{\sigma_{\mu\nu}}{\sqrt{2}} I_R \right] + g_{RL}^S \left[\vec{I}_R U_{I'j} N_{jL} \right] \left[N_{kR} V_{Ik}^* I_L \right] \\
& \left. + g_{RL}^V \left[\vec{I}_R \gamma^\mu V_{I'j} N_{jR} \right] \left[N_{kL} U_{Ik}^* \gamma_\mu I_L \right] + g_{RL}^T \left[\vec{I}_R \frac{\sigma^{\mu\nu}}{\sqrt{2}} U_{I'j} N_{jL} \right] \left[N_{kR} V_{Ik}^* \frac{\sigma_{\mu\nu}}{\sqrt{2}} I_L \right] \right\}.
\end{aligned}$$

Majorana Neutrinos

The possible first order Feynman diagrams for the $l^- \longrightarrow l'^- N_j N_k$ decay are:



The first diagram leads to the same matrix element as the Dirac case, while the second diagram is only possible in the Majorana neutrino case and we already defined the orientation for each fermion chain.

Majorana Neutrinos

Then, after integrating over the neutrinos momenta, the decay rate will have the following dependence on the amplitude:

$$\begin{aligned}d\Gamma &\propto \frac{1}{2} \sum_{j,k} |\mathcal{M}_{jk}^D - \mathcal{M}_{jk}^M|^2 \\ &= \frac{1}{2} \sum_{j,k} \left\{ |\mathcal{M}_{jk}^D|^2 + |\mathcal{M}_{jk}^M|^2 - 2 \operatorname{Re}(\mathcal{M}_{jk}^D \mathcal{M}_{jk}^{M*}) \right\} \\ &= \sum_{j,k} |\mathcal{M}_{jk}^D|^2 - \underbrace{\sum_{j,k} \operatorname{Re}(\mathcal{M}_{jk}^D \mathcal{M}_{jk}^{M*})}_{\text{Majorana term}}.\end{aligned}$$

The interference term distinguishes between Dirac and Majorana cases, which is sometimes called the **Majorana term**.

The differential decay rate taking into account finite Dirac or Majorana neutrino masses is:

$$\frac{d\Gamma}{dx d\cos\theta} = \sum_{j,k} \frac{m_1}{4\pi^3} \omega^4 G_{ll'}^2 \sqrt{x^2 - x_0^2} \times \left((F_{IS}(x) + F'_{IS}(x) + F''_{IS}(x)) - \mathcal{P} \cos\theta (F_{AS}(x) + F'_{AS}(x) + F''_{AS}(x)) \right) \times [1 + \hat{\zeta} \cdot \vec{\mathcal{P}}_{l'}(x, \theta)],$$

Linear in ν masses
Quadratic in ν masses

where

$$\vec{\mathcal{P}}_{l'} = P_{T_1} \cdot \hat{x} + P_{T_2} \cdot \hat{y} + P_L \cdot \hat{z}.$$

and the components of $\vec{\mathcal{P}}_{l'}$ are, respectively,

' is linear, '' is quadratic in ν masses

$$P_{T_1} = \mathcal{P} \sin\theta \cdot (F_{T_1}(x) + F'_{T_1}(x) + F''_{T_1}(x)) / N,$$

$$P_{T_2} = \mathcal{P} \sin\theta \cdot (F_{T_2}(x) + F'_{T_2}(x) + F''_{T_2}(x)) / N,$$

$$P_L = \left(- (F_{IP}(x) + F'_{IP}(x) + F''_{IP}(x)) + \mathcal{P} \cos\theta \cdot (F_{AP}(x) + F'_{AP}(x) + F''_{AP}(x)) \right) / N.$$

with N the normalization factor:

$$N = (F_{IS}(x) + F'_{IS}(x) + F''_{IS}(x)) - \mathcal{P} \cos\theta (F_{AS}(x) + F'_{AS}(x) + F''_{AS}(x)).$$

Total Decay Rate

Finally, integrating over all energy and angular configurations we obtained:

$$\Gamma_{l \rightarrow l'} = \sum_{j,k} \frac{\hat{G}_{ll'}^2, m_1^5}{192\pi^3} f(m_4^2/m_1^2) \left(1 + \delta_{RC}'' \right),$$

where

$$\hat{G}_{ll'} \equiv G_{ll'} \left\{ (I)_{jk} + 4(\eta)_{jk} \frac{m_4}{m_1} \frac{g(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} - 2 \frac{m_j}{m_1} \left[(\kappa_L^+)_{jk} \frac{f'(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} + (\kappa_R^+)_{kj} \frac{m_4}{m_1} \frac{g'(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} \right] - 4 \frac{m_j m_k}{m_1^2} \left[(C^+)_{jk} \frac{f''(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} + 3(H^+)_{jk} \frac{m_4}{m_1} \frac{g''(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} \right] \right\}^{1/2},$$

Linear in v masses
Quadratic in v masses

with the functions defined as:

$$f'(x) = -1 + 6x - 2x^3 + 3x^2 \left(4 \operatorname{arctanh} \left(\frac{x-1}{x+1} \right) - 1 \right),$$

$$f''(x) = 1 - 3x + 3x^2 - x^3,$$

$$g'(x) = 2 - 6x^2 + x^3 + 3x \left(4 \operatorname{arctanh} \left(\frac{x-1}{x+1} \right) + 1 \right),$$

$$g''(x) = 1 - x^2 + 2x \log(x).$$

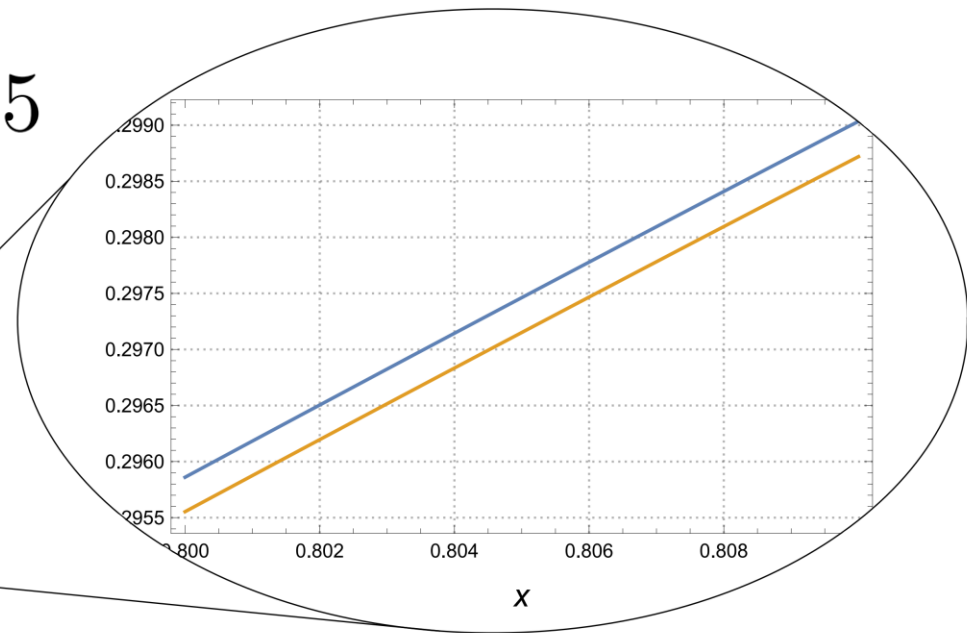
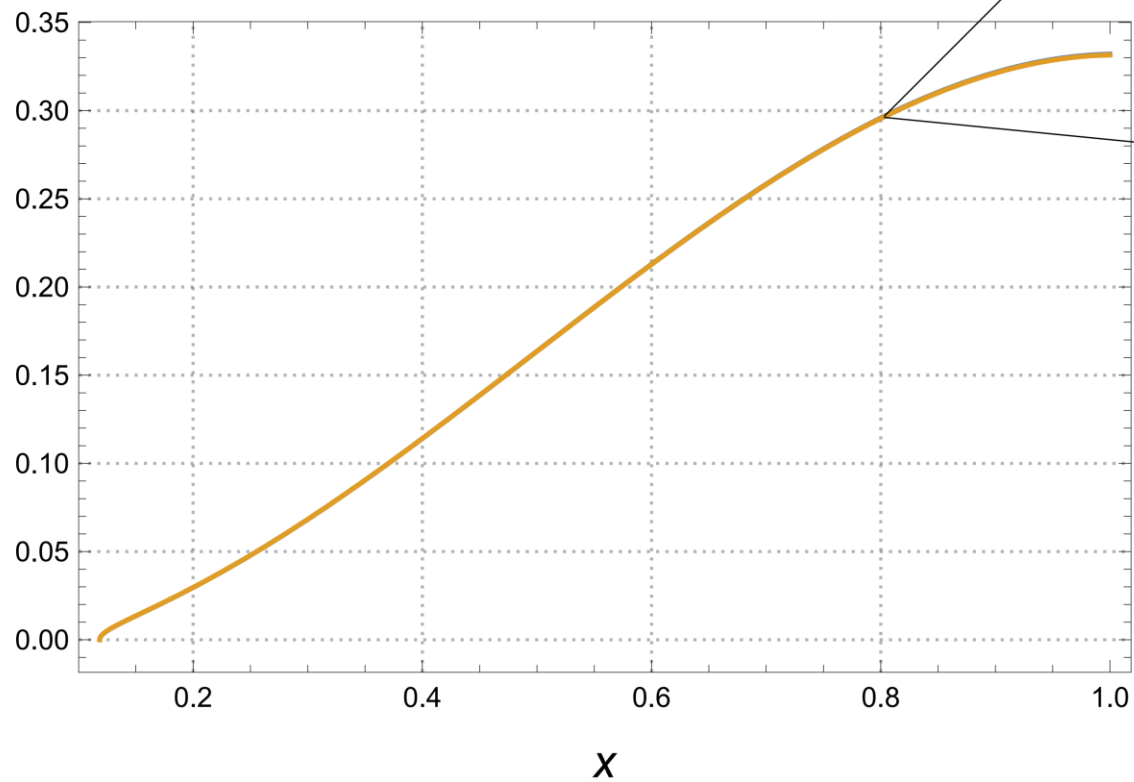
Considering the constraints on an invisible heavy neutrino¹, we can estimate the suppression of the neutrino mass dependent terms compared with the ones without this dependence (standard Michel distribution).

Neutrino	Mass (MeV)	Mixing $ U_{l4} ^2$	Process
Heavy ($l = e$)	0.001 - 0.45	10^{-3}	$n \rightarrow p + e + \nu_4$
	10 - 55	10^{-8}	$\pi \rightarrow e\nu_4$
	135 - 350	10^{-6}	$k \rightarrow e\nu_4$
Heavy ($l = \mu$)	10 - 30	10^{-4}	$\pi \rightarrow \mu\nu_4$
	70 - 300	10^{-5}	$k \rightarrow \mu\nu_4$
	175 - 300	10^{-8}	$k \rightarrow \mu\nu_4$
Heavy ($l = \tau$)	$100 - 1.2 \times 10^3$	$10^{-7} - 10^{-3}$	$\tau \rightarrow \nu_4 + 3\pi$
	$1 \times 10^3 - 60 \times 10^3$	$10^{-5} - 10^{-3}$	$Z \rightarrow \nu\nu_4$

¹A. de Gouvea and A. Kobach, Phys.Rev.D 93 (2016).

Neutrino	Mass (MeV)	Mixing Suppression	Linear Term Suppression (m_ν)	Quadratic Term Suppression (m_ν^2)
Light (2)	1×10^{-6}	—	10^{-9}	10^{-18}
Heavy (1) ($l = e$)	0.001 - 0.45	10^{-3}	$10^{-9} - 10^{-7}$	$10^{-18} - 10^{-16}$
	10 - 55	10^{-8}	10^{-10}	10^{-19}
	135 - 350	10^{-6}	10^{-7}	10^{-16}
Heavy (1) ($l = \mu$)	10 - 30	10^{-4}	10^{-6}	10^{-15}
	70 - 300	10^{-5}	$10^{-7} - 10^{-6}$	$10^{-16} - 10^{-15}$
	175 - 300	10^{-8}	10^{-9}	10^{-18}
Heavy (1) ($l = \tau$)	100 - 1.2×10^3	$10^{-7} - 10^{-3}$	$10^{-8} - 10^{-3}$	$10^{-18} - 10^{-12}$
	$1 \times 10^3 - 60 \times 10^3$	$10^{-5} - 10^{-3}$	$10^{-5} - 10^{-3}$	$10^{-14} - 10^{-12}$
Heavy (2) ($\mu \rightarrow eNN$)	10 - 30	10^{-12}	10^{-14}	10^{-16}
	175 - 300	$10^{-14} - 10^{-11}$	$10^{-15} - 10^{-12}$	$10^{-16} - 10^{-13}$
Heavy (2) ($\tau \rightarrow eNN$)	135 - 350	$10^{-13} - 10^{-9}$	$10^{-14} - 10^{-10}$	$10^{-14} - 10^{-10}$
Heavy (2) ($\tau \rightarrow \mu NN$)	100 - 300	$10^{-12} - 10^{-8}$	$10^{-13} - 10^{-9}$	$10^{-14} - 10^{-10}$
	175 - 350	$10^{-15} - 10^{-11}$	$10^{-16} - 10^{-12}$	$10^{-16} - 10^{-12}$

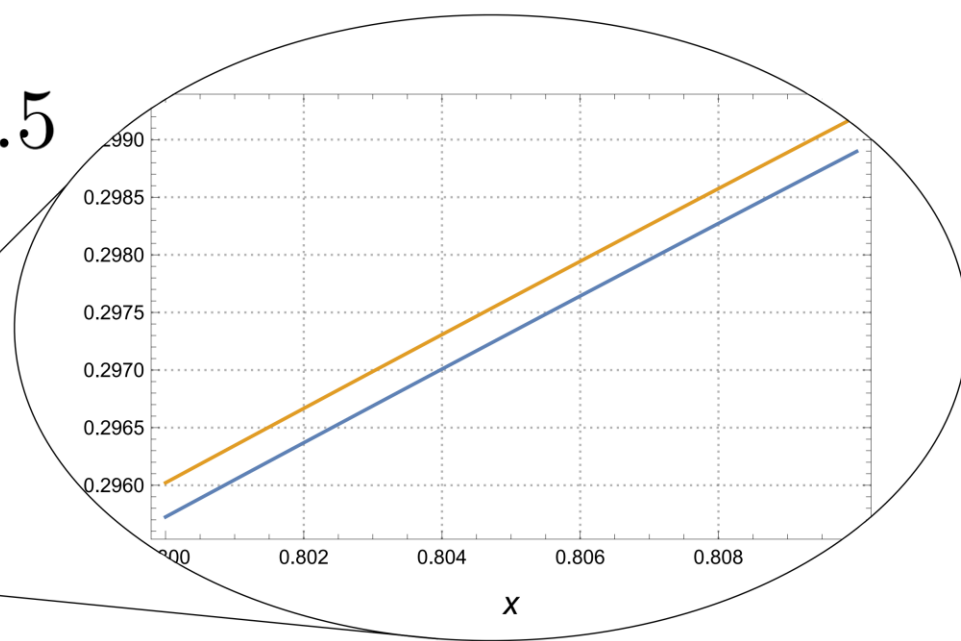
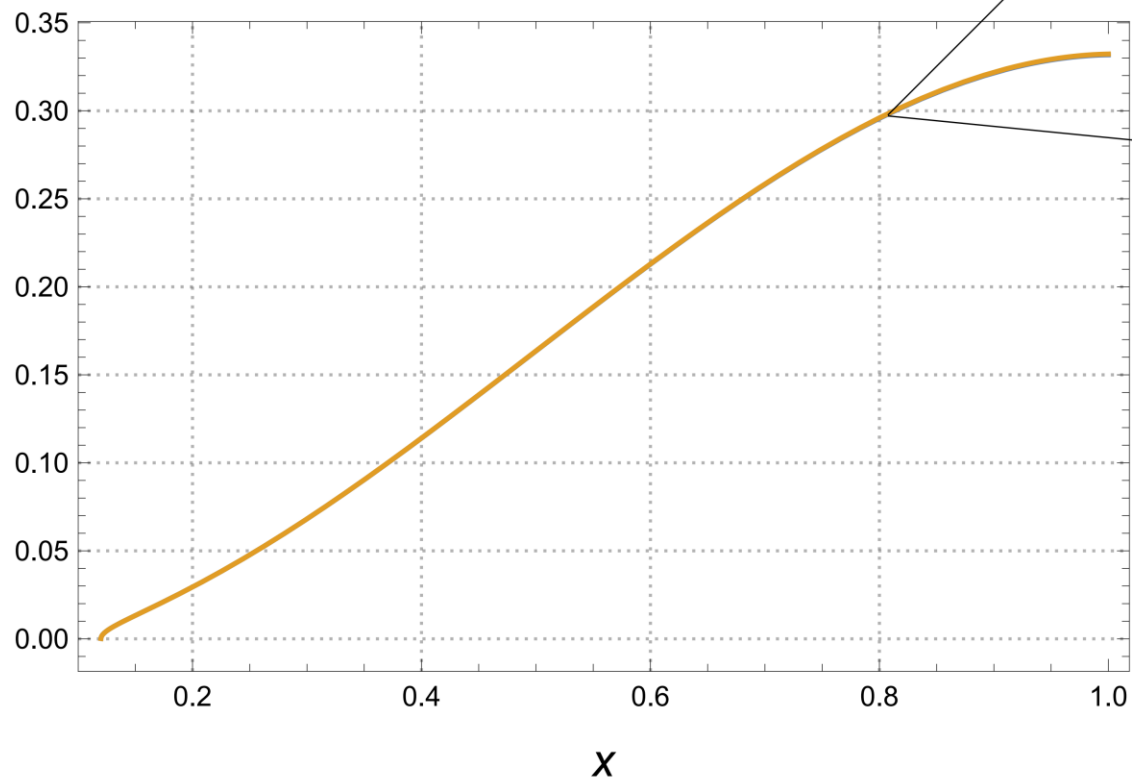
$$g_{LL}^V = 0.96, g_{RR}^S = 0.25 \text{ and } g_{LR}^S = 0.5$$



- Without m_ν contribution.
- With m_ν contribution.

(a) Dirac neutrinos.

$$g_{LL}^V = 0.96, g_{RR}^S = 0.25 \text{ and } g_{LR}^S = 0.5$$



- Without m_ν contribution.
- With m_ν contribution.

(b) Majorana neutrinos.

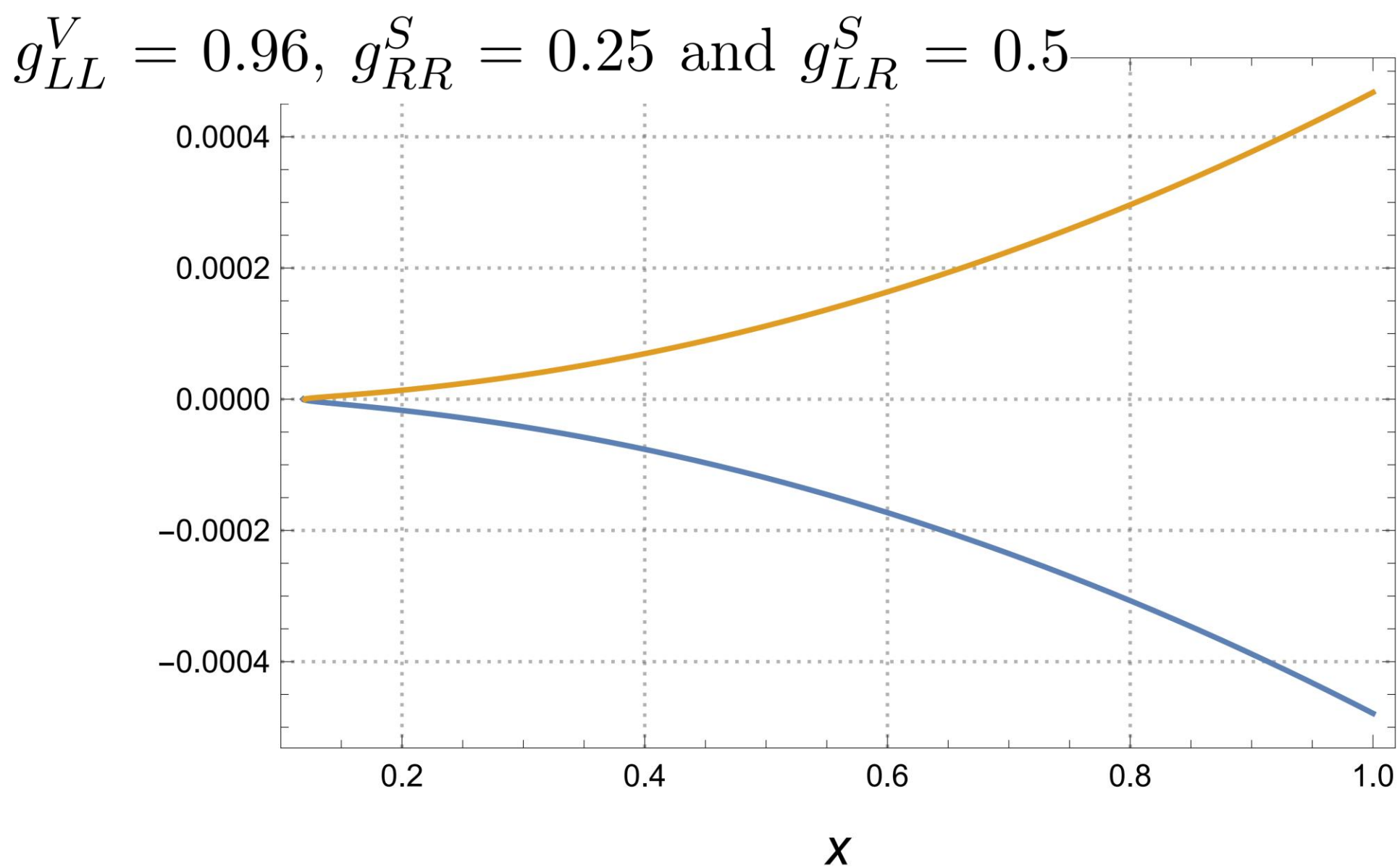


Figure 6: Neutrino mass contribution to Dirac and Majorana distributions.

- In this work we have studied the leptonic decay $l^- \longrightarrow l'^- N_j N_k$, where N_j and N_k are mass-eigenstate neutrinos.
- We have constructed its matrix element by using the most general four-lepton effective interaction Hamiltonian and obtained the specific energy and angular distribution of the final charged lepton, complemented with the decaying and final charged-lepton polarization and the effects of Dirac and Majorana neutrino masses.
- We have introduced generalized Michel parameters, that arise due to considering finite neutrino masses and a specific neutrino nature.
- We discuss their properties and main differences, together with some examples of its application to model-dependent theories.
- Specifically, for the case of τ -decay with one heavy final-state neutrino with a mass around $10^2 - 10^3 \text{ MeV}$ the linear term suppression could be of order 10^{-3} , low enough to be measured in current and forthcoming experiments.
- Finally, it would also be interesting to analyze other type of leptonic decays, such as radiative muon and tau decay with Dirac and Majorana neutrinos, where new information could be obtained.

What is new:

- We write our expressions in the PDG parametrization form, in a way that complements all previous results, facilitating their application to model-dependent scenarios.
- We classify the Dirac and Majorana contributions with the help of a flag parameter $\epsilon = 0, 1$, making easier to distinguish between Dirac and Majorana nature of neutrinos.
- We also introduced and discussed the leading W -boson propagator correction to the differential decay rate including the final charged-lepton polarization.

Previous work on Michel parameters: Michel'50, Bouchiat-Michel'57, Shrock'82, Doi-Kotani-Takasugi'85, Mursula-Scheck'85, Fetscher-Gerber-Johnson'86, Langacker-London'89, Fetscher'94, Stahl-Voss'97, Flores Tlalpa-López Castro-Roig'16, Arbuzov-Kopylova'16,... And of course all the essential work on the required RadCors and the precise measurements.

Inferring the nature of active neutrinos: Dirac or Majorana

(C.S. Kim, M.V.M. Murthy & D. Sahoo, PRD105(2022)11,113006)

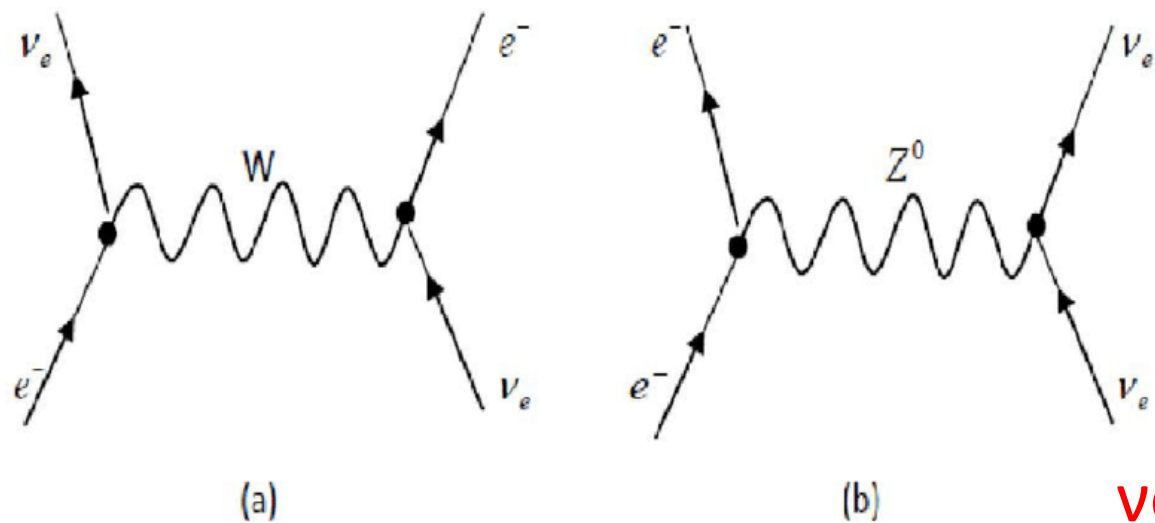
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The idea is to use 4-body decays including a pair of ν s and another pair of particles, and go to the back-to-back configuration for these pairs, in which the properties of the neutrinos can be inferred without actually measuring them, thus avoiding Kaiser's Th.

[Phys.Rev.D 109 \(2024\) 3, 033005](#)

When looked in depth (Juanma Márquez, Diego Portillo & P. R., to appear soon), there is a loophole in their derivation. When corrected, it yields observables which are orders or magnitude smaller than initially thought, possibly preventing the observation of this effect.





$\nu_e \rightarrow \nu_e$ scattering with massive Dirac or Majorana neutrinos and general interactions



Cinvestav

Pablo Roig
Cinvestav (Mexico)

In collaboration with:
J.M. Márquez (Cinvestav, Mexico)
M. Salinas (Cinvestav, Mexico)

See also Rodejohan, Xu & Yaguna JHEP05(2017)024

Under review. arXiv:2401.14305

Pablo Roig
Cinvestav (Mexico)

ve→ve scattering with massive Dirac or Majorana ν s and general interactions (with Juanma-Mónica)

$$\mathcal{L} \supset \frac{G_F}{\sqrt{2}} \sum_{a=S,P,V,A,T} \bar{\nu} \Gamma^a \nu \left[\bar{l} \Gamma^a (C_a + \bar{D}_a i \gamma^5) l \right]$$

Hermiticity =>
(with C_a & D_a real)

$$D_a \equiv \bar{D}_a, \quad a = S, P, T,$$

$$D_a \equiv i \bar{D}_a, \quad a = V, A,$$

ve→ve scattering with massive Dirac or Majorana vs
and general interactions (with Juanma-Mónica)

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Majorana condition $\nu_j = \nu_j^c = C \bar{\nu}_j^T \Rightarrow$

$$C_V = D_V = C_T = D_T = 0$$

ve→ve scattering with massive Dirac or Majorana ν s and general interactions (with Juanma-Mónica)

$$\mathcal{L} \supset \frac{G_F}{\sqrt{2}} \sum_{a=S,P,V,A,T} \bar{\nu} \Gamma^a \nu \left[\bar{l} \Gamma^a (C_a + \bar{D}_a i \gamma^5) l \right]$$

SM =>

$$\mathcal{L}_{\text{NC}} = \frac{G_F}{\sqrt{2}} 2 \left[\bar{\nu} \gamma^\mu (g_V^\nu - g_A^\nu \gamma^5) \nu \right] \left[\bar{l} \gamma^\mu (g_V^l - g_A^l \gamma^5) l \right]$$

$$g_V^\nu = g_A^\nu = \frac{1}{2}, \quad g_V^l = -\frac{1}{2} + 2s_w^2, \quad g_A^l = -\frac{1}{2}.$$

With CC (same flavor), $g_{V,A}^l \rightarrow g_{V,A}^l + 1$

ve → ve scattering with massive Dirac or Majorana vs and general interactions (with Juanma-Mónica)

$$\text{SM} \Rightarrow \left\{ \begin{array}{ll} C_V^{\text{SM}} = 2g_V^\nu g_V^l, & D_V^{\text{SM}} = -2g_V^\nu g_A^l, \\ C_A^{\text{SM}} = 2g_A^\nu g_A^l, & D_A^{\text{SM}} = -2g_A^\nu g_V^l, \\ C_S^{\text{SM}} = 0, & D_S^{\text{SM}} = 0, \\ C_P^{\text{SM}} = 0, & D_P^{\text{SM}} = 0, \\ C_T^{\text{SM}} = 0, & D_T^{\text{SM}} = 0, \end{array} \right. \quad \text{Dirac}$$

$$\left\{ \begin{array}{ll} C_V^{\text{SM}} = 0, & D_V^{\text{SM}} = 0, \\ C_A^{\text{SM}} = 4g_A^\nu g_A^l, & D_A^{\text{SM}} = -4g_A^\nu g_V^l, \\ C_S^{\text{SM}} = 0, & D_S^{\text{SM}} = 0, \\ C_P^{\text{SM}} = 0, & D_P^{\text{SM}} = 0, \\ C_T^{\text{SM}} = 0, & D_T^{\text{SM}} = 0. \end{array} \right. \quad \text{Majorana}$$

$\nu e \rightarrow \nu e$ scattering with massive Dirac or Majorana ν s and general interactions (with Juanma-Mónica)

Neglecting neutrino masses, incoming neutrinos are LH:

$$\frac{d\sigma}{dT}(\nu + e) = \frac{G_F^2 M}{2\pi} \left[A + 2B \left(1 - \frac{T}{E_\nu}\right) + C \left(1 - \frac{T}{E_\nu}\right)^2 + D \frac{MT}{4E_\nu^2} \right],$$

Recoil E_e M_e

$$\frac{d\sigma}{dT}(\bar{\nu} + e) = \frac{G_F^2 M}{2\pi} \left[C + 2B \left(1 - \frac{T}{E_\nu}\right) + A \left(1 - \frac{T}{E_\nu}\right)^2 + D \frac{MT}{4E_\nu^2} \right],$$

No mixing of V/A interactions with S,P or T. D is only relevant in non-relativistic scattering.

$$B \equiv -\frac{1}{8} (C_P^2 + C_S^2 + D_P^2 + D_S^2 - 8C_T^2 - 8D_T^2)$$

$\nu e \rightarrow \nu e$ scattering with massive Dirac or Majorana ν s and general interactions (with Juanma-Mónica)


$$(A, B, C, D)^{\text{SM}} = \left((1 - 2s_w^2)^2, 0, 4s_w^4, 1 - (1 - 4s_w^2)^2 \right)$$

which lead to a cross section that has the same value for Dirac and Majorana neutrinos. Then, other interactions are needed in order to distinguish the Dirac and Majorana cases in this process.

Or observables

$\nu e \rightarrow \nu e$ scattering with massive Dirac or Majorana ν s and general interactions (with Juanma-Mónica)

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For instance, as pointed out by Rosen in 1982, the ratio of the forward to backward scattering cross-sections, $R_\rho > 2 \Rightarrow$ Dirac neutrinos (but both natures are possible if it is ≤ 2 , complementary to the non-observation vs. observation of $0\nu 2\beta$).

$$R_\rho \equiv \frac{2(A + 2B + C)}{A + C}$$

$$0 \leq R_\rho \leq 4 \text{ (Dirac),}$$

$$0 \leq R_\rho \leq 2 \text{ (Majorana).}$$

$\nu e \rightarrow \nu e$ scattering with massive Dirac or Majorana ν s and general interactions (with Juanma-Mónica)

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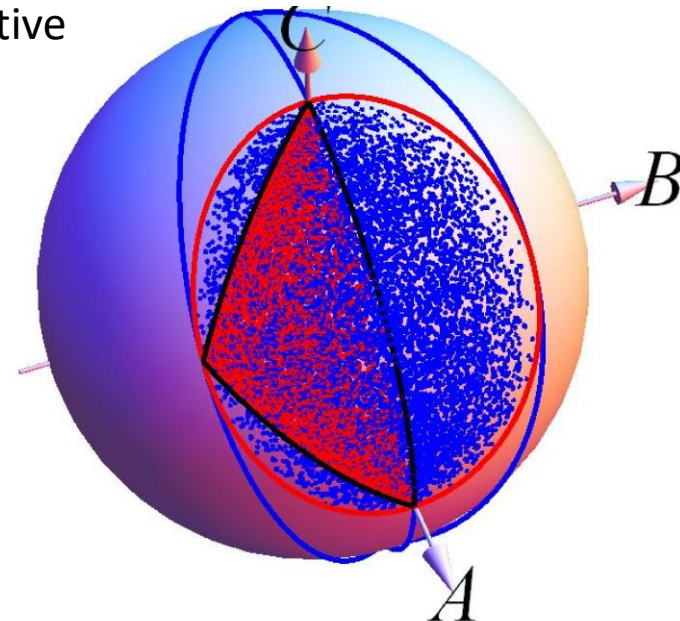
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As shown by Rodejohan, Xu & Yaguna, actual bounds are more restrictive

- Dirac: Blue points
- Majorana: Red points
- Blue curves: $R_\rho = 0, 2, 4$.
- Actual ranges surrounded by red (Dirac) or black (Majorana) curves



$\nu e \rightarrow \nu e$ scattering with massive Dirac or Majorana vs general interactions (with Juanma-Mónica)

Another interesting result from Rodejohan, Xu & Yaguna paper:

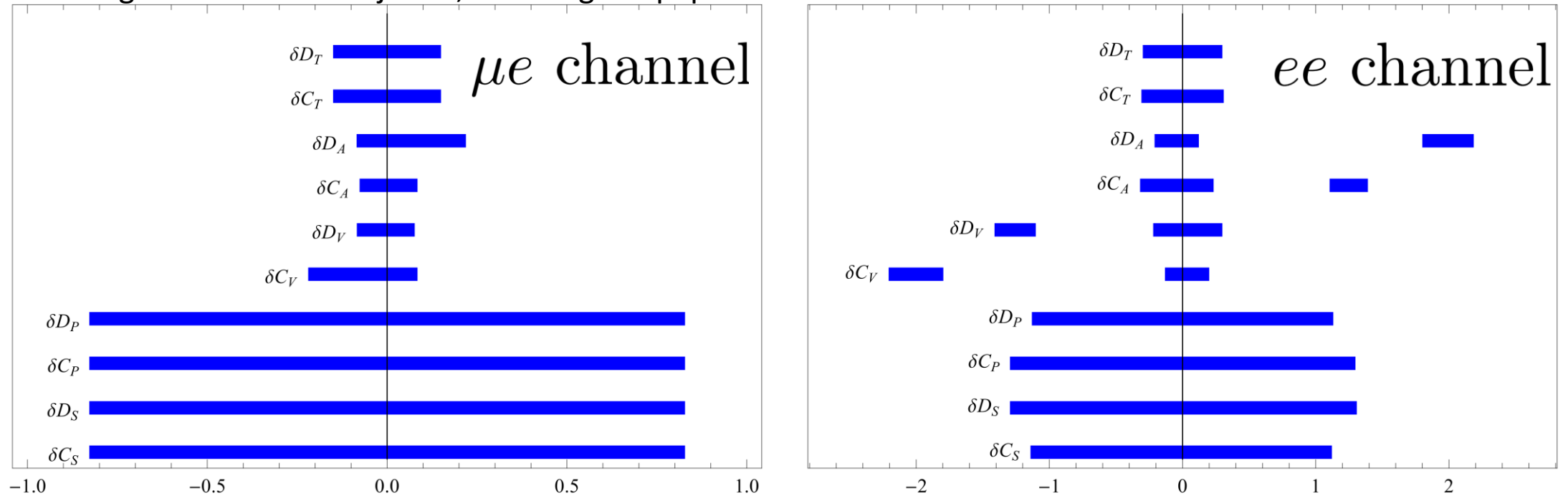


Figure 7. Constraints on $\delta C_a \equiv C_a - C_a^{\text{SM}}$ and $\delta D_a \equiv D_a - D_a^{\text{SM}}$ from one-parameter fitting of CHARM-II (left panel) and TEXONO (right panel). Blue bars represent 90% C.L. allowed values for δC_a and δD_a . There are two local minima for $C_{A,V}$ and $D_{A,V}$ in the fit of TEXONO. The slightly more minimal global minimum is around -2 for C_V , 0 for D_V , 0 for C_A and $+2$ for D_A . In the SM, $C_{V,A}$ and $D_{V,A}$ are non-zero for Dirac neutrinos, while only C_A and D_A are non-zero for Majorana neutrinos. The plot assumes Dirac neutrinos. (For Majorana vs the results look very similar)

ve → ve scattering with massive Dirac or Majorana vs and general interactions (with Juanma-Mónica)

$$\frac{d\sigma}{dT}(\nu + e) = \sum_{i,f} |U_{\ell i}|^2 \frac{G_F^2 M}{2\pi} \frac{E_\nu^2}{E_\nu^2 - m_{\nu_i}^2} \left\{ A + 2B \left(1 - \frac{T}{E_\nu}\right) + C \left(1 - \frac{T}{E_\nu}\right)^2 \right.$$

Keeping neutrino masses, incoming neutrinos are LH:

$$\nu_{\ell L} = \sum_j U_{\ell j} N_{jL}, \quad \nu_{\ell R} = \sum_j V_{\ell j} N_{jR}$$

$$+ D \frac{MT}{4E_\nu^2} + \frac{(m_{\nu_i}^2 - m_{\nu_f}^2)}{2ME_\nu} \left[(A + 2B) + C \left(1 - \frac{T}{E_\nu}\right) + F \frac{m_{\nu_f}}{E_\nu} \right] - B \frac{m_{\nu_i}^2 T}{ME_\nu^2} + \frac{m_{\nu_f}}{E_\nu} \left[G + F \left(1 - \frac{T}{E_\nu}\right) \right] + D \frac{m_{\nu_i}^2 + m_{\nu_f}^2}{8E_\nu^2} \left. \right\},$$

$$B \equiv -|V_{\ell f}|^2 \left[\frac{1}{8} (C_P^2 + C_S^2 + D_P^2 + D_S^2 - 8C_T^2 - 8D_T^2) \right]$$

$$A \leftrightarrow C, \quad F \leftrightarrow G, \quad m_{\nu_i} \leftrightarrow -m_{\nu_i} \text{ and } m_{\nu_f} \leftrightarrow -m_{\nu_f}$$

$$\frac{d\sigma}{dT}(\bar{\nu} + e) = \sum_{i,f} |U_{\ell i}|^2 \frac{G_F^2 M}{2\pi} \frac{E_\nu^2}{E_\nu^2 - m_{\nu_i}^2} \left\{ C + 2B \left(1 - \frac{T}{E_\nu}\right) + A \left(1 - \frac{T}{E_\nu}\right)^2 \right.$$

Mixing of V/A interactions with S,P or T.

$$F \equiv \text{Re} [U_{\ell f} V_{\ell f}^*] \frac{1}{4} [(C_S + 6C_T)(C_V - D_A) + (C_P - 6C_T)(C_A - D_V)],$$

$$G \equiv \text{Re} [U_{\ell f} V_{\ell f}^*] \frac{1}{4} [(C_S - 6C_T)(C_V - D_A) - (C_P + 6C_T)(C_A - D_V)].$$

$$+ D \frac{MT}{4E_\nu^2} + \frac{(m_{\nu_i}^2 - m_{\nu_f}^2)}{2ME_\nu} \left[(C + 2B) + A \left(1 - \frac{T}{E_\nu}\right) - G \frac{m_{\nu_f}}{E_\nu} \right] - B \frac{m_{\nu_i}^2 T}{ME_\nu^2} - \frac{m_{\nu_f}}{E_\nu} \left[F + G \left(1 - \frac{T}{E_\nu}\right) \right] + D \frac{m_{\nu_i}^2 + m_{\nu_f}^2}{8E_\nu^2} \left. \right\},$$

$\nu e \rightarrow \nu e$ scattering with massive Dirac or Majorana vs and general interactions (with Juanma-Mónica)

Neutrino flavor	m_{ν_f} (MeV)	$ U_{\ell 4} ^2$	Linear term suppression		Quadratic term suppression	
			$ U_{\ell 4} ^2 m_{\nu_f} / E_\nu$	$ U_{\ell 4} ^2 m_{\nu_f}^2 / E_\nu^2$	$ U_{\ell 4} ^2 m_{\nu_f} / E_\nu$	$ U_{\ell 4} ^2 m_{\nu_f}^2 / E_\nu^2$
			E_ν	E_ν	E_ν	E_ν
			500 MeV	2500 MeV	500 MeV	2500 MeV
$l = e$ [25]	150-375	10^{-8}	10^{-9}	10^{-10} - 10^{-9}	10^{-10} - 10^{-9}	10^{-10}
	375-440	10^{-9}	10^{-10}	10^{-10}	10^{-10}	10^{-11}
$l = \mu$ [26]	10	10^{-3}	10^{-5}	10^{-6}	10^{-7}	10^{-8}
	20	10^{-4}	10^{-6}	10^{-7}	10^{-7}	10^{-9}
	50	10^{-5}	10^{-6}	10^{-7}	10^{-7}	10^{-9}
	100	10^{-6}	10^{-7}	10^{-8}	10^{-8}	10^{-9}
$l = \tau$ [27]	100-200	10^{-3}	10^{-4}	10^{-4}	10^{-4}	10^{-5}
	300-400	10^{-4}	10^{-4}	10^{-5}	10^{-4}	10^{-6} - 10^{-5}
	500-800	10^{-5}	10^{-5}	10^{-6}	10^{-5}	10^{-6}
	900-1100	10^{-5}	10^{-5}	10^{-6}	10^{-5}	10^{-6}

$\nu e \rightarrow \nu e$ scattering with massive Dirac or Majorana vs and general interactions (with Juanma-Mónica)

SM+ C_S case.

SM

Parameter	Dirac and Majorana
A	$ U_{\ell f} ^2 (1 - 2s_w^2)^2$
B	0
C	$ U_{\ell f} ^2 4s_w^4$
D	$ U_{\ell f} ^2 (1 - (1 - 4s_w^2)^2)$
F	0
G	0

Parameter	Dirac and Majorana
A	$ U_{\ell f} ^2 (1 - 2s_w^2)^2 + V_{\ell f} ^2 \frac{1}{8} C_S^2$
B	$- V_{\ell f} ^2 \frac{1}{8} C_S^2$
C	$ U_{\ell f} ^2 4s_w^4 + V_{\ell f} ^2 \frac{1}{8} C_S^2$
D	$ U_{\ell f} ^2 (1 - (1 - 4s_w^2)^2) + V_{\ell f} ^2 C_S^2$
F	$-\frac{1}{4} \text{Re} [U_{\ell f} V_{\ell f}^*] C_S (1 - 4s_w^2)$
G	$-\frac{1}{4} \text{Re} [U_{\ell f} V_{\ell f}^*] C_S (1 - 4s_w^2)$

ve → ve scattering with massive Dirac or Majorana vs general interactions (with Juanma-Mónica)

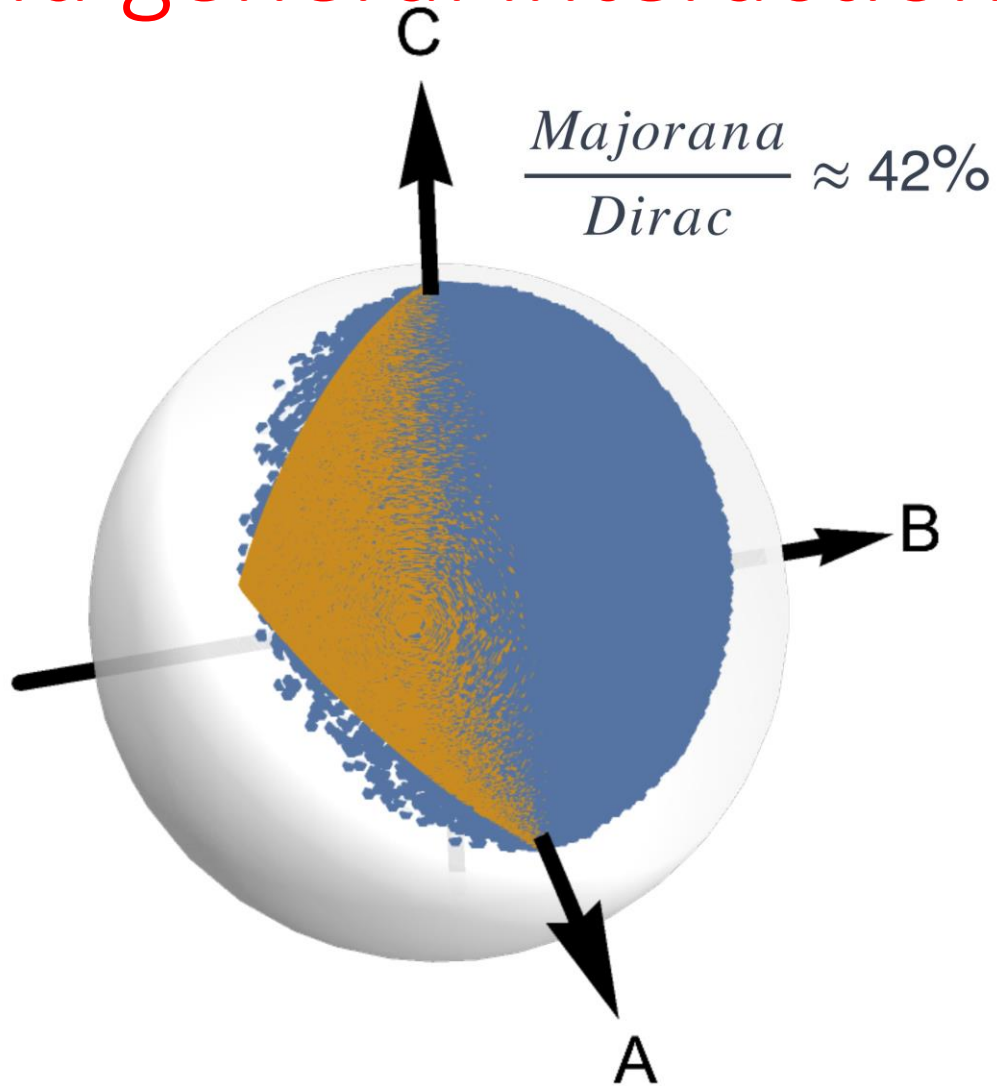
SM

SM+ C_T case.

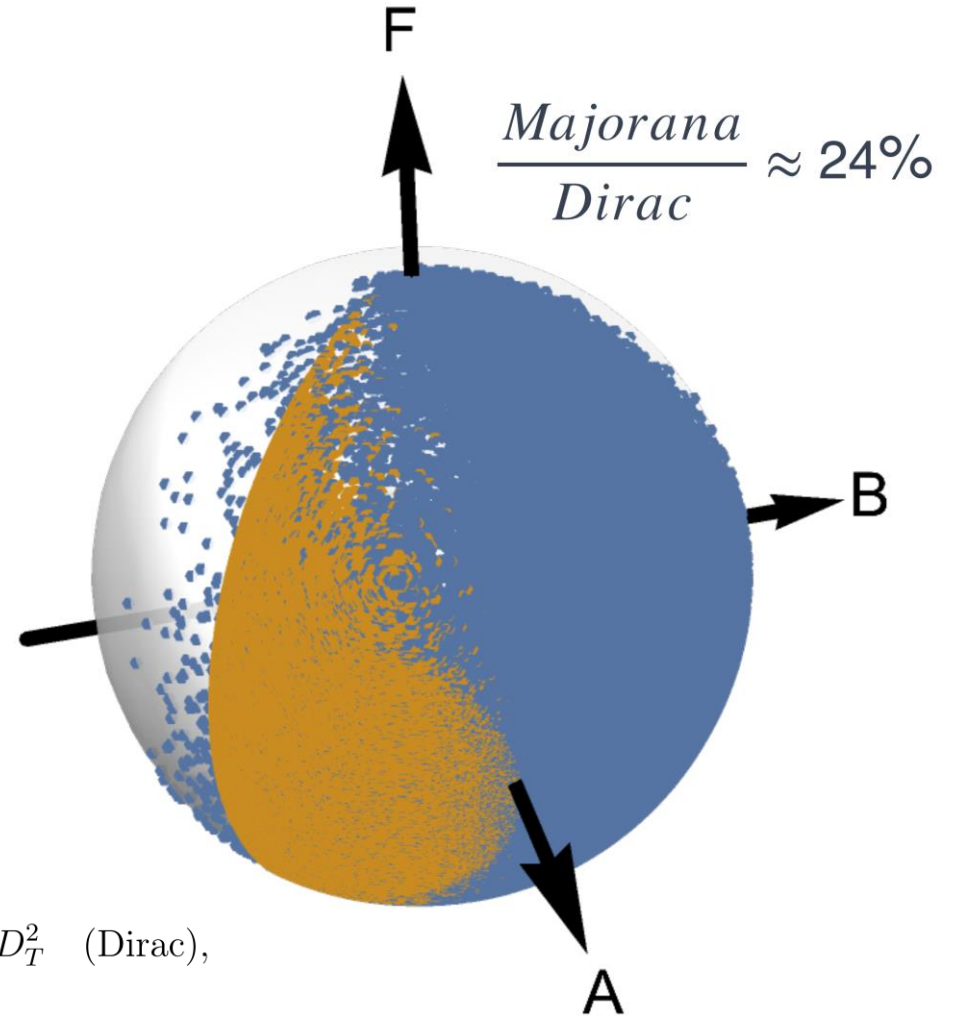
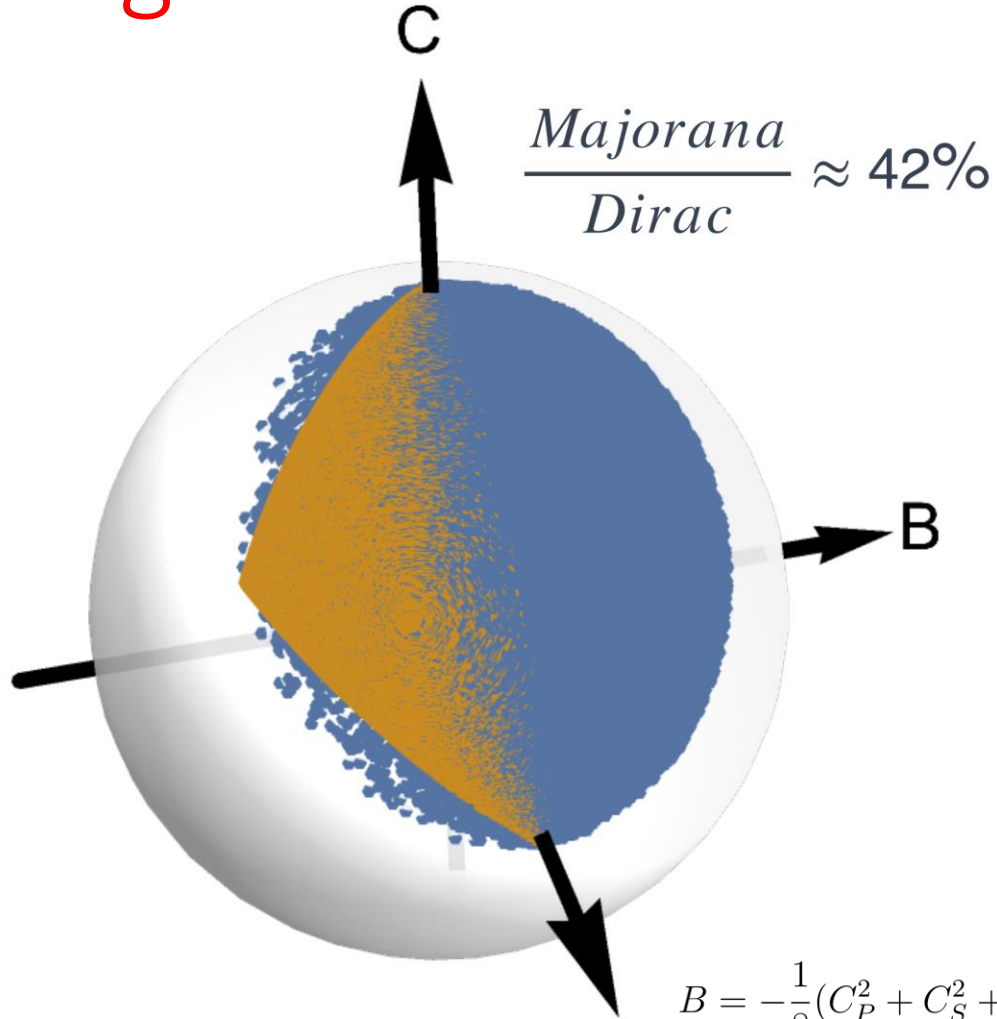
Parameter	Dirac and Majorana
A	$ U_{\ell f} ^2(1 - 2s_w^2)^2$
B	0
C	$ U_{\ell f} ^2 4s_w^4$
D	$ U_{\ell f} ^2 (1 - (1 - 4s_w^2)^2)$
F	0
G	0

Parameter	Dirac	Majorana
A	$ U_{\ell f} ^2(1 - 2s_w^2)^2 + V_{\ell f} ^2 C_T^2$	$ U_{\ell f} ^2(1 - 2s_w^2)^2$
B	$ V_{\ell f} ^2 C_T^2$	0
C	$ U_{\ell f} ^2 4s_w^4 + V_{\ell f} ^2 C_T^2$	$ U_{\ell f} ^2 4s_w^4$
D	$ U_{\ell f} ^2 (1 - (1 - 4s_w^2)^2) - 4 V_{\ell f} ^2 C_T^2$	$ U_{\ell f} ^2 (1 - (1 - 4s_w^2)^2)$
F	$6 \operatorname{Re} [U_{\ell f} V_{\ell f}^*] C_T s_w^2$	0
G	$3 \operatorname{Re} [U_{\ell f} V_{\ell f}^*] C_T (1 - 2s_w^2)$	0

$\nu e \rightarrow \nu e$ scattering with massive Dirac or Majorana ν s and general interactions (with Juanma-Mónica)



$\nu e \rightarrow \nu e$ scattering with massive Dirac or Majorana vs general interactions (with Juanma-Mónica)



$$B = -\frac{1}{8}(C_P^2 + C_S^2 + D_P^2 + D_S^2) + C_T^2 + D_T^2 \quad (\text{Dirac}),$$

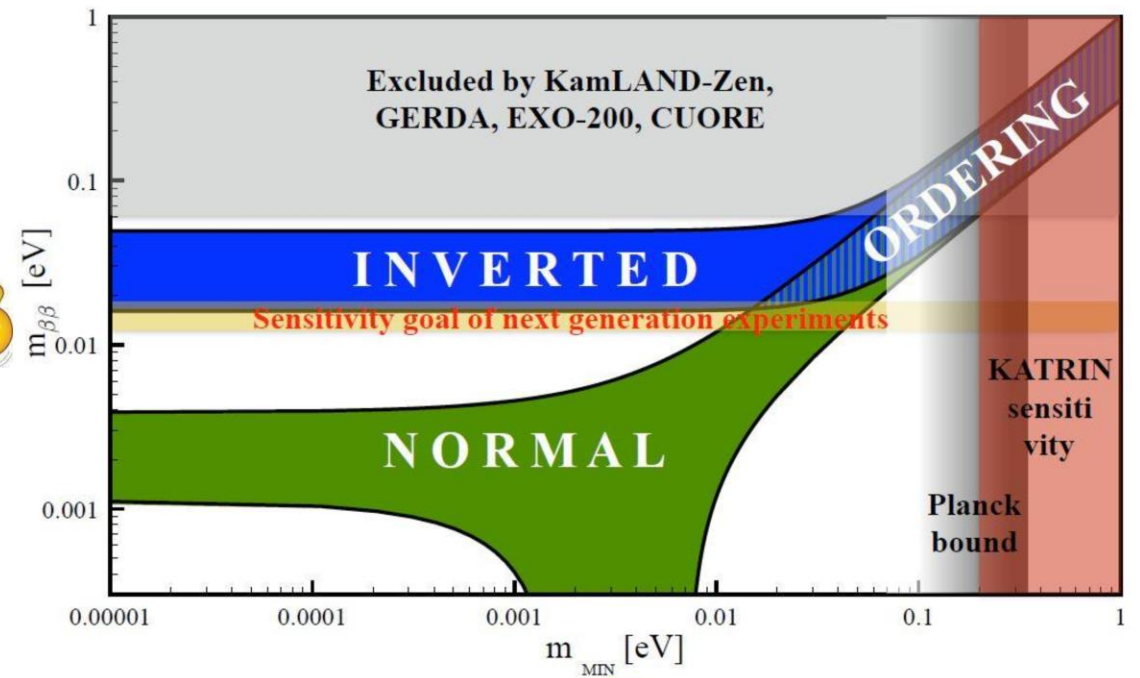
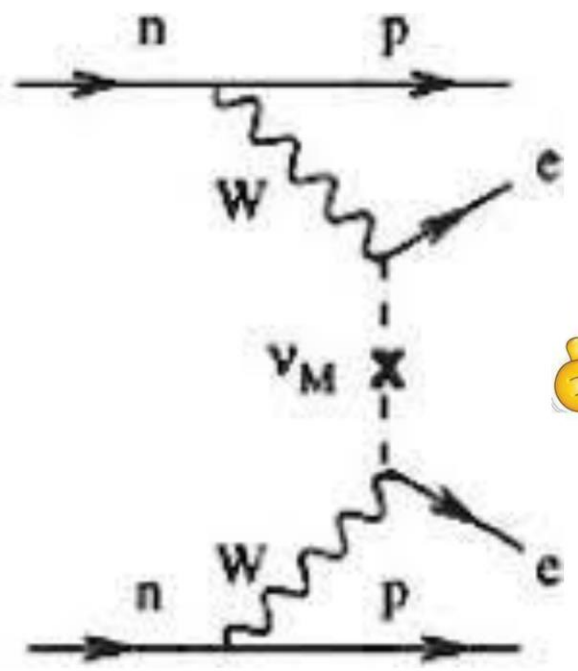
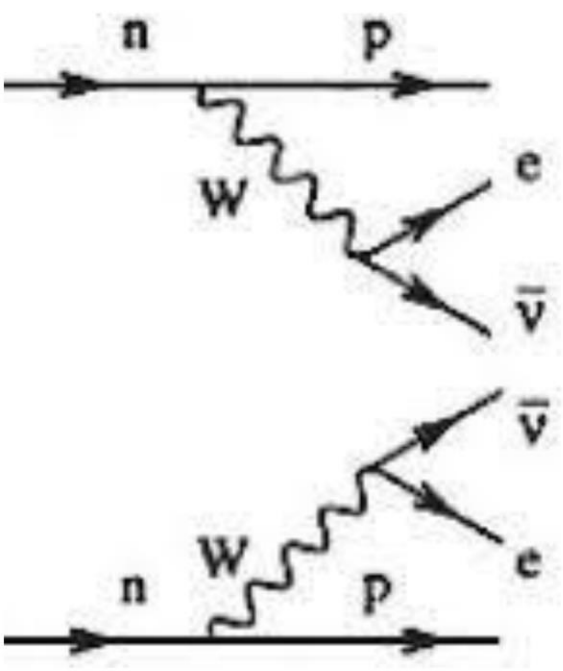
$$B = -\frac{1}{8}(C_P^2 + C_S^2 + D_P^2 + D_S^2) \quad (\text{Majorana}),$$

CONCLUSIÓN: Aunque el neutrino sea de Majorana puede que no podamos medir nunca $0\nu 2\beta$...

Pequeñez de la masa de los neutrinos ($10^{-7} \sim m_\nu/m_e < m_e/m_t \sim 3 \times 10^{-6}$)

Si los neutrinos son de **Majorana**
Podemos tener desintegración doble β sin ν s...

Si medimos desintegración doble β sin ν s
 \Rightarrow Los ν s son de **Majorana**
(el inverso no es cierto)



CONCLUSIÓN: Aunque el neutrino sea de Majorana, quizá no podamos medir nunca $0\nu 2\beta$. En ese caso debemos tener otras opciones, para que también puedan fallar



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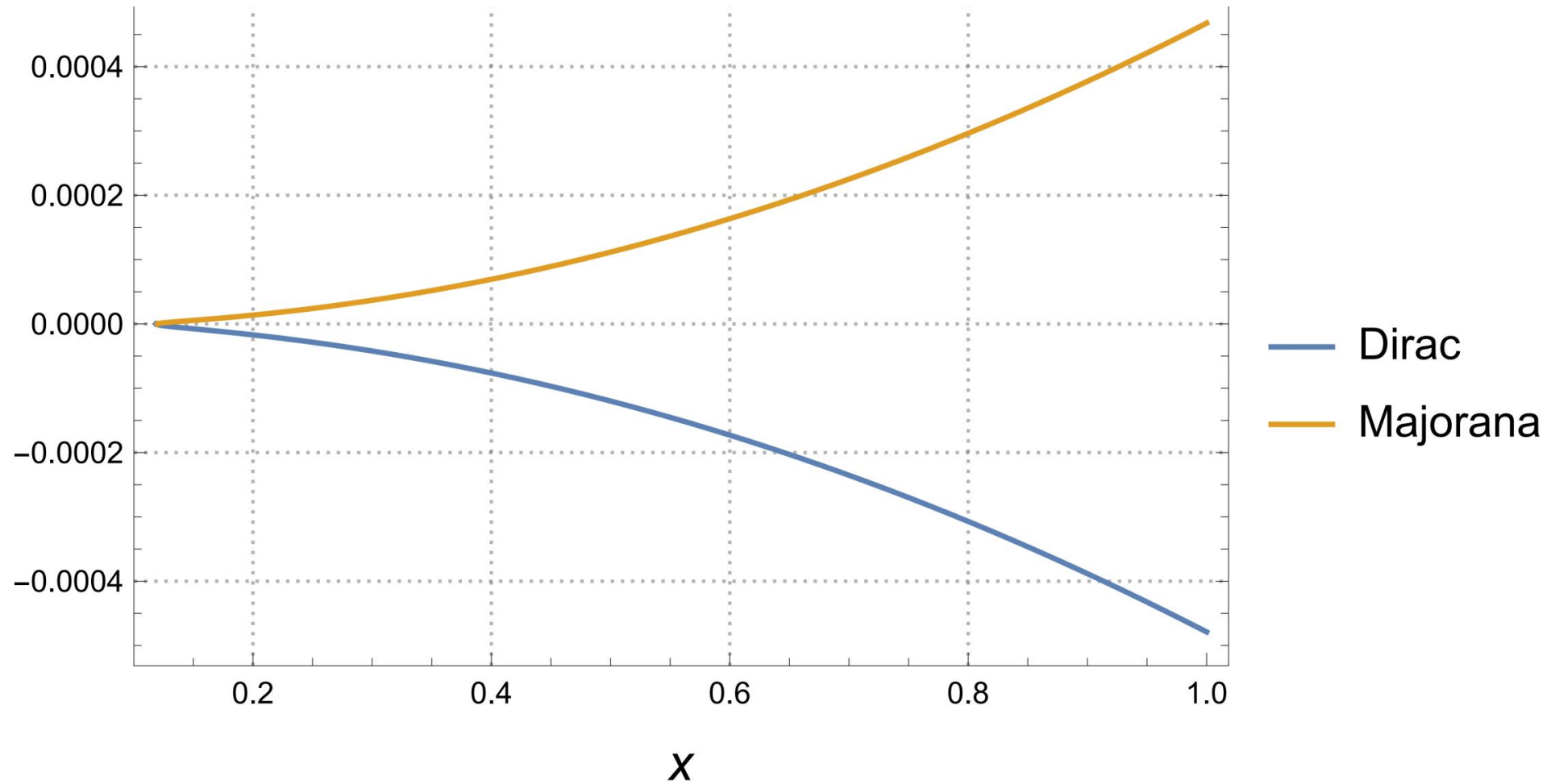
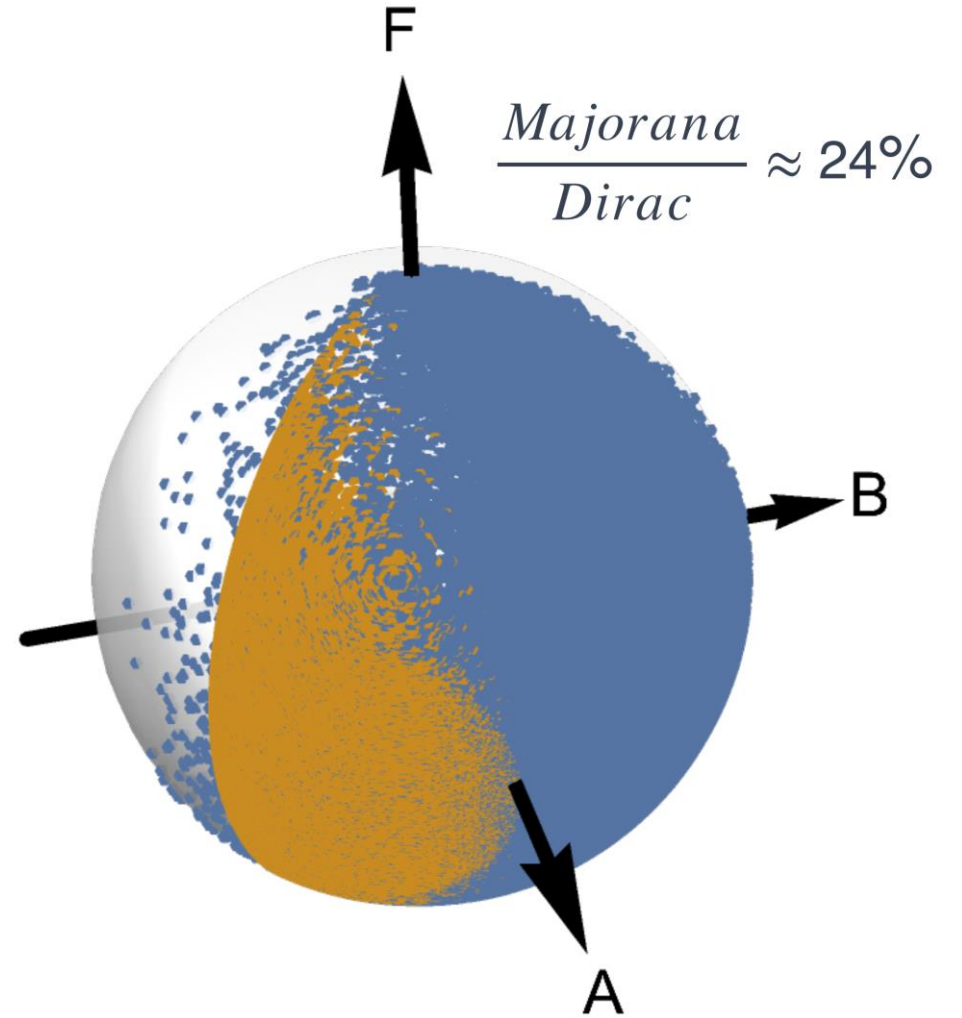
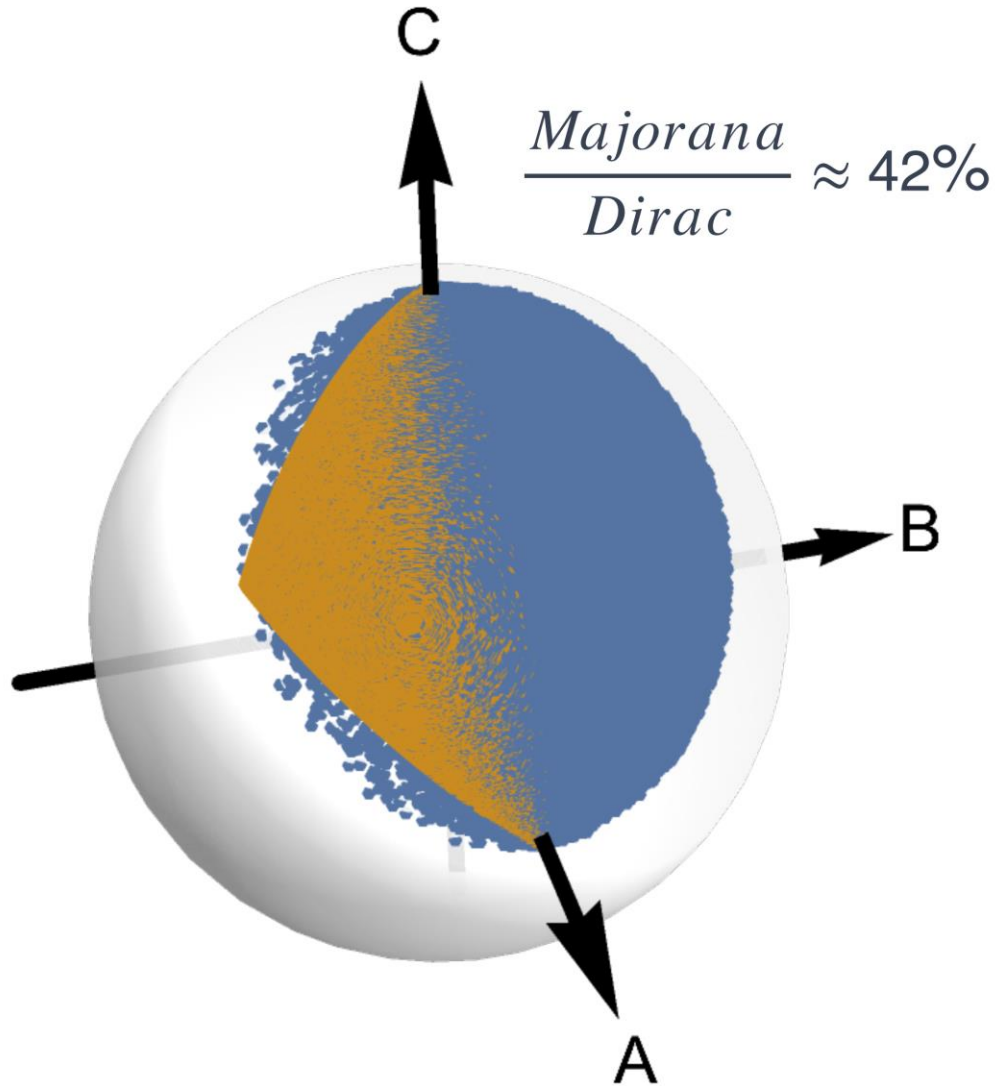


Figure 6: Neutrino mass contribution to Dirac and Majorana distributions.

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Inferring the nature of active neutrinos: Dirac or Majorana

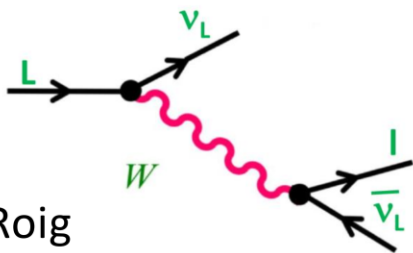
(C.S. Kim, M.V.M. Murthy & D. Sahoo, PRD105(2022)11,113006)

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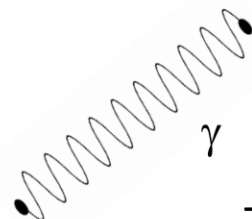
The idea is to use 4-body decays including a pair of ν s and another pair of particles, and go to the back-to-back configuration for these pairs, in which the properties of the neutrinos can be inferred without actually measuring them, thus avoiding Kaiser's Th.

[Phys.Rev.D 109 \(2024\) 3, 033005](#)

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Taller IF-UNAM