The neutral pion screening mass as an example of strong-field physics

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[1]A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Overview

- ▶ Why is strong-field physics important?
- ▶ Interplay between strong magnetic fields and QCD.
- ▶ Effective Field Theories (EFTs)
- ▶ Linear sigma model with quarks (LSMq)
- ▶ Analysis of the neutral pion screening mass
- ▶ Results
- ▶ Summary and perspectives

Strong-(Electromagnetic)Field Physics

- ▶ High energy physics (heavy ion collisions)
- ▶ High intensity laser physics
- ▶ Condensed matter physics
- ▶ Astrophysics (neutron stars)

Strong-(Electromagnetic)Field Physics

▶ **High energy physics (heavy ion collisions)**

▶ High intensity laser physics

▶ Condensed matter physics

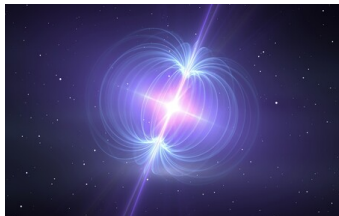▶ **Astrophysics (neutron stars)**

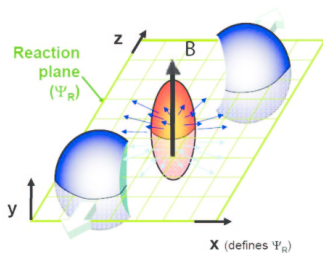# Strong-Field Physics

Figure: magnetar ($10^{13} - 10^{15}$ G.)

Figure: Heavy ion collisions ($10^{18} - 10^{20}$ G.)

Comparison of different magnetic fields

- ▶ Earth's magnetic field: $0.6\ G$
- ▶ Common comercial magnet: $100\ G$
- ▶ Strongest magnetic field produced in labs: $4.5 \times 10^5\ G$
- ▶ Magnetars: $(10^{13} - 10^{15})G$
- ▶ Heavy ion collisions: $(10^{18} - 10^{20})\ G$

Interplay between strong magnetic fields and QCD

- ▶ Magnetic catalysis at zero temperature
- ▶ Inverse magnetic catalysis around T_C
- ▶ Electromagnetic fields provide a powerful probe to explore the properties of the QCD vacuum.

In this work we will study the effect of a constant magnetic field B in the screening mass of neutral pions.

EFT protoype (Fermi Theory for the Weak Interaction)

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} J_\mu^\dagger J^\mu$$

Where,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

$$J^\mu = \overline{e}\gamma^\mu(1 - \gamma_5)\nu_e + \overline{\mu}\gamma^\mu(1 - \gamma_5)\nu_\mu + V_{ud}^* \overline{d}\gamma^\mu(1 - \gamma_5)u + h.c + ...$$

The scale of new physics in this case is the mass of the W boson.

LSMq Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \overrightarrow{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \overrightarrow{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \overrightarrow{\pi}^2)^2 + i\bar{\psi}\gamma^\mu \partial_\mu \psi$$
$$- ig\bar{\psi}\gamma^5 \vec{\psi} \overrightarrow{\tau} \cdot \overrightarrow{\pi} \psi - g\bar{\psi}\psi\sigma \,.$$

- ▶ it implements the SSB of: $SU(2)_L \times SU(2)_R \to SU(2)_V$.
- ▶ $m_\pi(v) = \sqrt{\lambda v^2 - a^2} = 0$ at VEV.
- ▶ $m_f(v) = gv$.
- ▶ $m_\sigma(v) = \sqrt{3\lambda v^2 - a^2}$

$\mathcal{L} \to \mathcal{L} + h(\sigma + v)$ in order to give the correct vacuum pion mass.
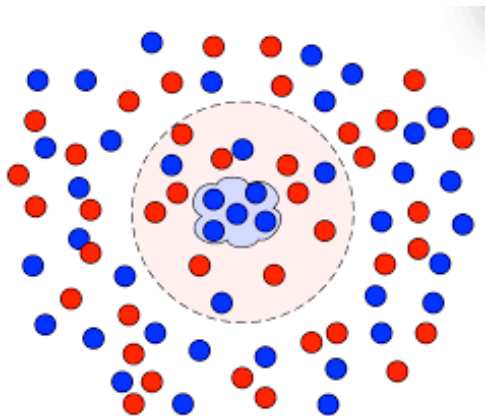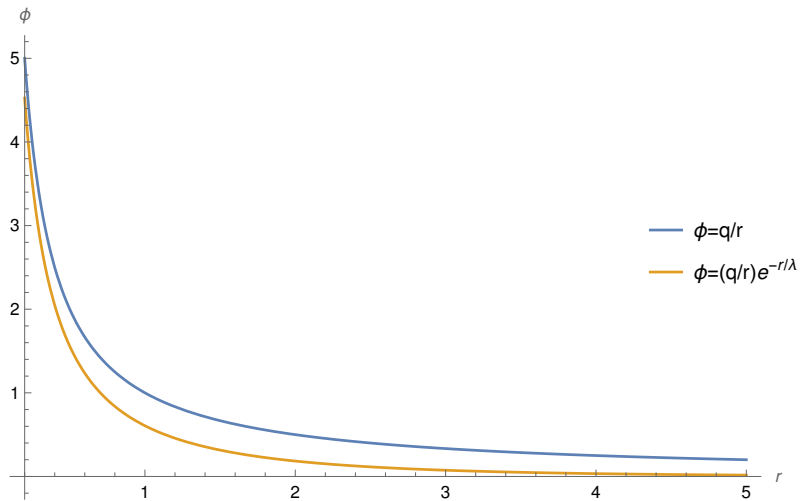
Screening Mass

Figure: Debye mass (inverse of Debye length)

Screening Mass

Pole and screening masses ($B \neq 0$, $T = 0$)

- Pole mass

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_3^2 = p_\perp^2 = 0} = 0$$

- Screening mass B breaks Lorentz invariance and defines $\parallel$ and $\perp$
  - Longitudinal

  $$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_\perp^2 = 0} = 0$$

  - Transverse

  $$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_3^2 = 0} = 0$$
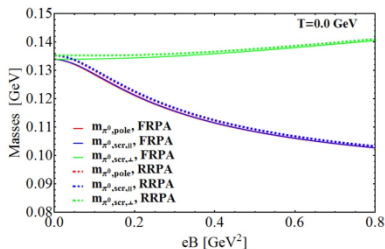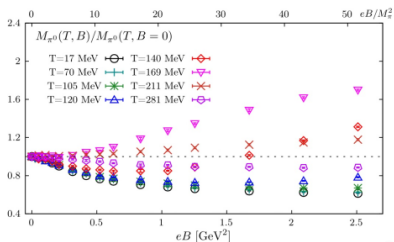
Comparison with the NJL model

Figure: H. T. Ding, S. T. Li, J. H. Liu, and X. D. Wang, Phys. Rev D105, 034514 (2022), and B. Sheng, Y. Wang, X. Wang, and L. Yu, Phys. Rev. D103 (2021) 9, 094001.
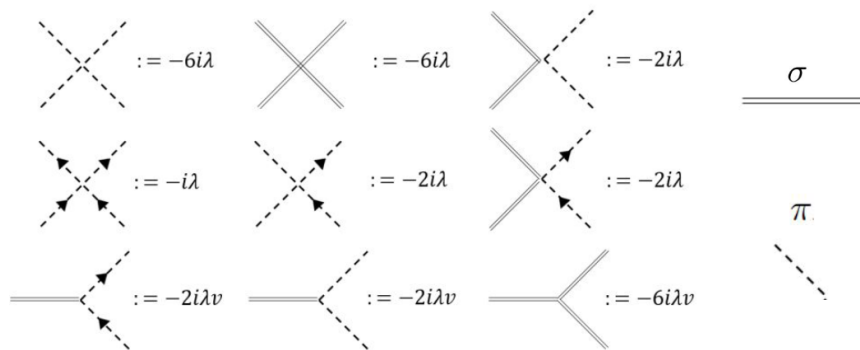
LSMq Lagrangian and features

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \overrightarrow{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \overrightarrow{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \overrightarrow{\pi}^2)^2 + i\bar{\psi}\gamma^\mu \partial_\mu \psi$$
$$- ig\bar{\psi}\gamma^5 \vec{\psi} \overrightarrow{\tau} \cdot \overrightarrow{\pi}\psi - g\bar{\psi}\psi\sigma \,.$$
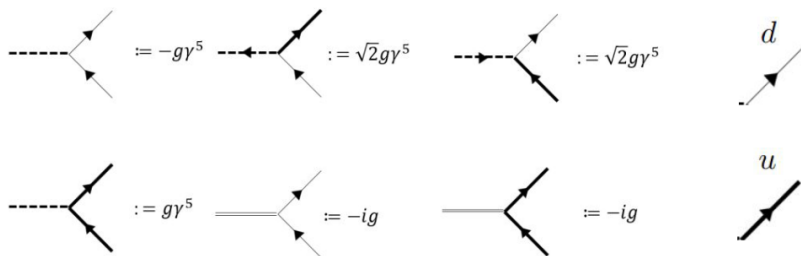
- ▶ it implements the SSB of: $SU(2)_L \times SU(2)_R \to SU(2)_V$.
- ▶ $m_\pi(v) = \sqrt{\lambda v^2 - a^2} = 0$ at VEV.
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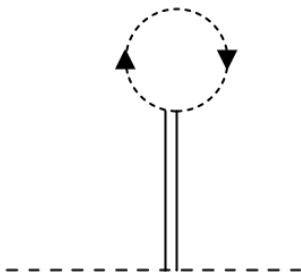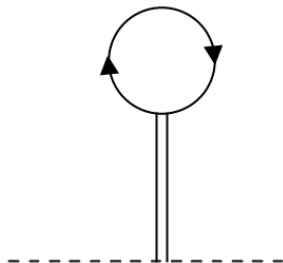$\mathcal{L} \to \mathcal{L} + h(\sigma + v)$ in order to give the correct vacuum pion mass.

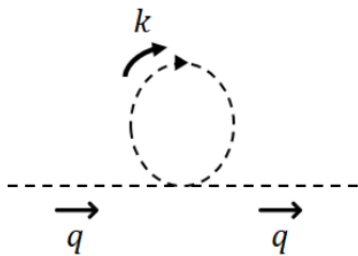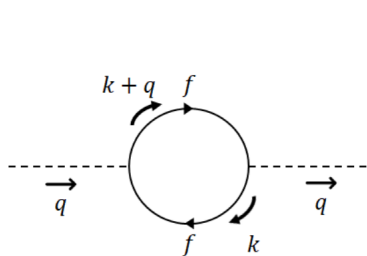# Feynman rules for boson-boson interactions

Feynman rules for boson-fermion interactions

$$:= -g\gamma^5 \qquad := \sqrt{2}g\gamma^5 \qquad := \sqrt{2}g\gamma^5$$

$$d$$

$$:= g\gamma^5 \qquad := -ig \qquad := -ig$$

$$u$$

Relevant Feynman diagrams

Feynman rules for the fermionic contribution to $\pi_{f\bar{f}}$ (vertices and propagator)
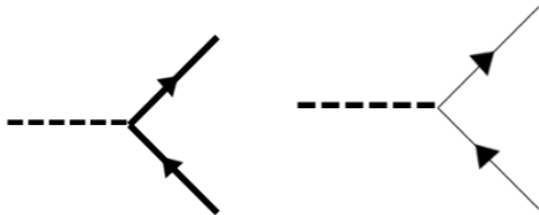

Figure: π_0-u quark vertex=$g\gamma^5$; π_0-d quark vertex=-$g\gamma^5$

$$iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp\left[is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon \right) \right]$$
$$\times \left\{ (m_f + \not{p}_\parallel)\left(\cos(qBs) + \gamma^1\gamma^2 \sin(qBs) \right) - \frac{\not{p}_\perp}{\cos(qBs)} \right\}$$

$$iS(p) \to i\frac{(m_f + \not{p})}{p^2 - m_f^2 + i\epsilon}$$
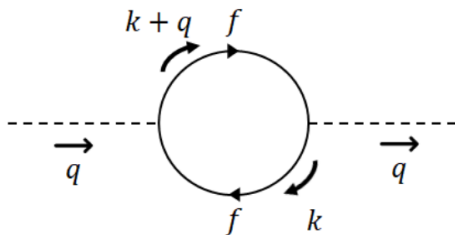
Neutral pion self-energy (fermion contribution)

Figure: Neutral pion self-energy

$$-i\pi_{f\bar{f}} = -g^2 \int \frac{d^4k}{(2\pi)^4}\, Tr\left[\gamma^5 iS(k)\gamma^5 iS(k+q)\right] + c.c..$$

Neutral pion self-energy (fermion contribution)

$$-i\pi_{f\bar{f}} = -4g^2 \int_0^\infty \int_0^\infty \frac{ds\,ds'}{\cos(qBs)\cos(qBs')}$$

$$\times \int \frac{d^4k}{(2\pi)^4} e^{is\left(k_\parallel^2 - k_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon\right)} e^{is'\left((k+p)_\parallel^2 - (k+p)_\perp^2 \frac{\tan(qBs')}{qBs'} - m_f^2 + i\epsilon\right)}$$

$$\left\{ \cos[qB(s+s')][m_f^2 - k_\parallel \cdot (k+p)_\parallel] + \frac{k_\perp \cdot (k_\perp + p_\perp)}{\cos(qBs)\cos qBs'} \right\}$$

Analysis

$$f(p_0, p_\perp, p_\parallel, B) = \pi_{f\bar{f}} - \lim_{B \to 0} \pi_{f\bar{f}}$$

We start with the simplest case ($p_\perp^2 = p_0^2 = 0$). This is the 'longitudinal' screening mass which is found by solving the equation:

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_\perp^2 = 0} = 0$$

Important integrals and convenient change of variables

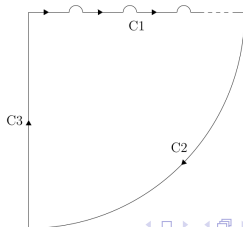$$\int_{-\infty}^{\infty} e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2}dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$u = s + s'$$

$$s = u(1 - v)$$

$$s' = uv$$

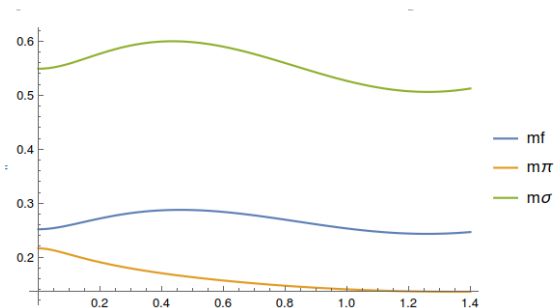$$\frac{\partial(s, s')}{\partial(u, v)} = u$$

Final expression

$$\Re f = lim_{\epsilon \to 0} \Bigg(-\frac{4g^2}{(4\pi)^2} \int_0^1 dv \Bigg\{ \Big(-2v(1-v)p_3^2 \Big)$$

$$\times \left[\frac{\pi}{2} \frac{\sin(\frac{a\pi}{qB})}{\cosh(\frac{\epsilon\pi}{qB}) - \cos(\frac{a\pi}{qB})} - \tan^{-1}\left(\frac{\sin(\frac{a\pi}{qB})e^{\left(\frac{-\epsilon\pi}{qB}\right)}}{1 - e^{\left(\frac{-\epsilon\pi}{qB}\right)}\cos\left(\frac{a\pi}{qB}\right)} \right) \right]$$

$$+ qB\left[\frac{\epsilon\pi}{2qB} - \ln\sqrt{2\cosh\left(\frac{\epsilon\pi}{qB}\right) - 2\cos\left(\frac{a\pi}{qB}\right)} \right]$$

$$- \frac{qB}{\pi}\left[\Re\Big(Li_2\Big[e^{-(ia+\epsilon)\frac{\pi}{qB}} \Big] \Big) \right] \Bigg\} \Bigg)$$

Parameters

Parameters at zero magnetic field

$$g = \frac{m_f}{v} = 2.75; \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2 f_\pi^2} = 15$$

Magnetic field dependence of masses

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[2]S. S. Avancini, R. Farias, M. B. Pinto, W. R. Tavares, Phys. Letters B 767 (2017) 247-252.

Fermionic contribution for different g values

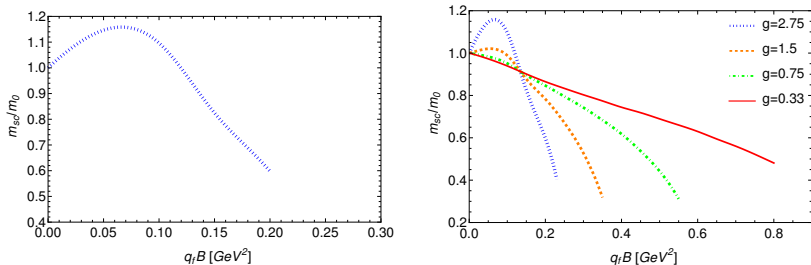Figure: 'Longitudinal' SC mass as function of B for $g = 2.75$ (left). 'Longitudinal' SC mass as function of B for different values of g.

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[3] A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).
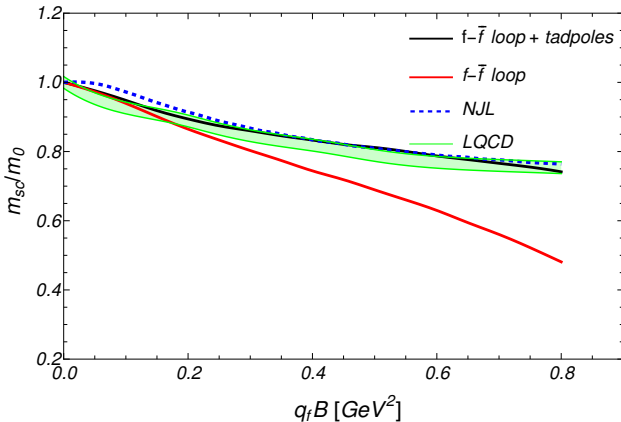
Fermionic contribution + Tadpoles

Figure: 'Longitudinal' Screening mass as a function of B, $g = 0.33, \lambda = 2.5$.

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[4] A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Summary and perspectives

Summary:

- ▶ We have calculated the neutral pion self-energy in the LSMq.
- ▶ We have obtained the 'longitudinal' screening mass as a function of B.
- ▶ We have compared our results with LQCD and NJL, and we have found a nice agreement only when we have a magnetic field dependence on the couplings and masses.

Perspectives:

- ▶ We are studying the 'transverse' screening mass as a function of B.
- ▶ We will study the case where $T \neq 0$.

Thank You

Important integrals and convenient change of variables

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$u = s + s'$$

$$s = u(1 - v)$$

$$s' = uv$$

$$\frac{\partial(s, s')}{\partial(u, v)} = u$$

Neutral pion self-energy (fermion contribution)

$$
\pi_{\bar{f}f} = \frac{-4g^2 qB}{(4\pi)^2} \int_0^1 dv \int_0^\infty du \exp\left(-ix\right) \exp\left(-u\epsilon\right) \Bigg[\frac{m_f^2}{\tan\left(qBu\right)}
$$

$$
- \frac{qB}{\sin^2\left(qBu\right)} \left(\frac{p_\perp^2}{|qB|} \frac{\sin\left(qBu(1-v)\right)\sin\left(qBuv\right)}{\sin\left(qBu\right)} \right) - \frac{v(1-v)(p_3^2 - p_0^2)}{\tan(qBu)}
$$

$$
- \frac{iqB}{\sin\left(qBu\right)} - \frac{i}{u\tan\left(qBu\right)} \Bigg]
$$

where x is given by:

$$
x = \frac{p_\perp^2}{qB} \frac{\sin(qBu(1-v))\sin(qBuv)}{\sin(qBu)} + p_3^2 uv(1-v) - p_0^2 uv(1-v) + m_f^2 u
$$

$B \to 0$ limit of $\pi_{\bar{f}f}$

$$\lim_{B \to 0} \pi_{\bar{f}f} = -\frac{4g^2}{(4\pi)^2} \int_0^1 dv \int_0^\infty \frac{du}{u} e^{-ix_0} e^{-u\epsilon}$$
$$\times \left\{ m_f^2 - v(1-v)(p_\perp^2 - p_3^2) + v(1-v)p_0^2 - \frac{2i}{u} \right\}$$

where

$$x_0 = uv(1-v)(p_\perp^2 + p_3^2) - p_0^2 uv(1-v) + m_f^2$$

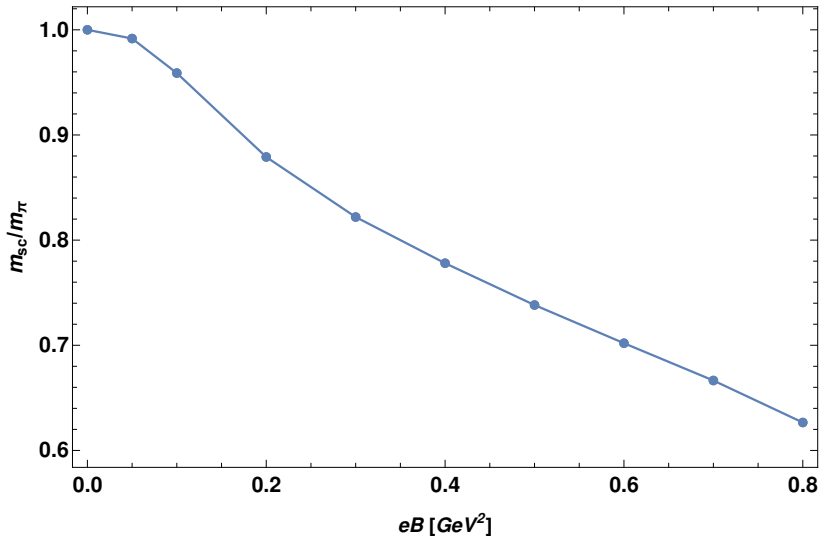# Fermionic contribution with g_{eff} as a function of B

Figure: 'Longitudinal' Screening mass as a function of B,
$g_{eff}(B) = 0.3 + 1.2\exp[-(7B)^2]$.
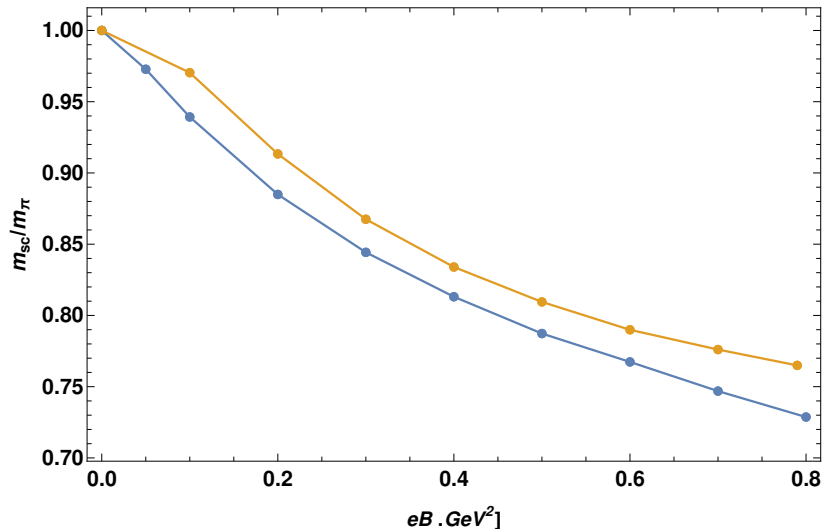
Fermionic contribution + Tadpoles

Figure: 'Longitudinal' Screening mass as a function of B vs NJL
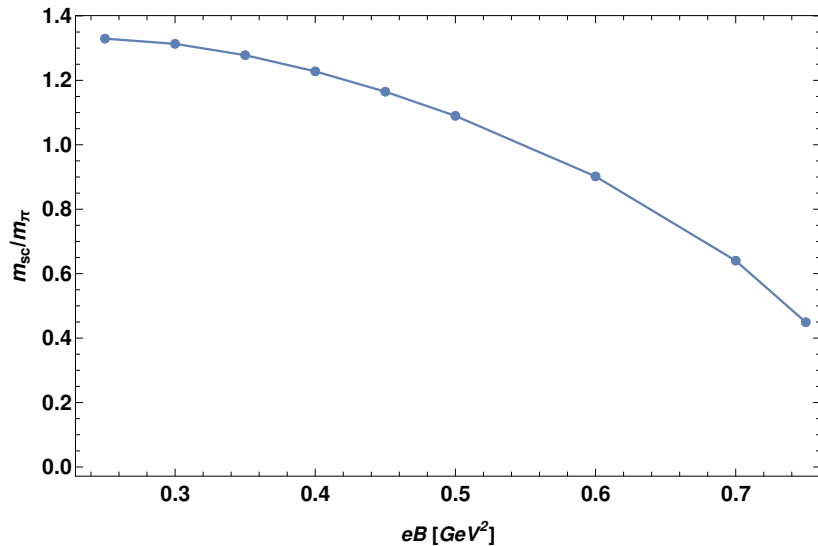$g_{eff}(B) = 0.3, \lambda = 1.$

Preliminary results

Figure: 'Longitudinal' Screening mass as a function of B.