Magnetic corrections to the QCD coupling

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Introduction

The community of high-energy physics has recently focused on the impact of magnetic fields on strongly interacting matter. Neutron stars, relativistic heavy-ion collisions, and the early universe are systems where large magnetic fields are generated. Moreover, such systems have the conditions to find nuclear matter in the deconfined phase. One of the most important elements to understand the phase transition in strongly interacting matter is the behavior of the coupling, which provides information about the interaction strength. Therefore, in this work [1], we show how the coupling is modified due to the presence of a very intense magnetic field; this means that we compute the 1-loop magnetic correction to the QCD vertex, under the strong field approximation.

QCD vertex

The Feynman rules, obtained from the Lagrangian density of QCD provide the vertices and propagators of the degrees of freedom. In order to take into account the magnetic effects on the interaction strength, quantum corrections at the 1-loop order are computed [2, 3].

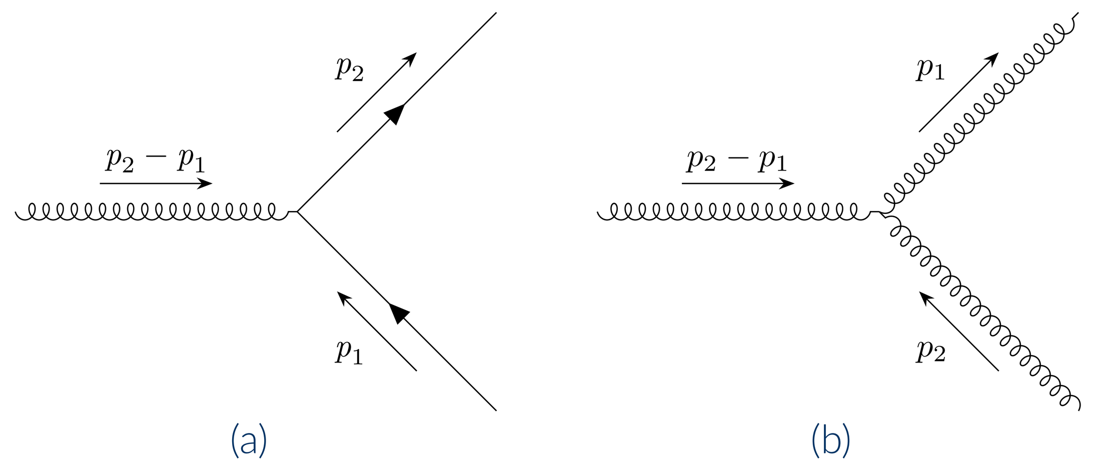

Figure 1. Feynman diagrams of the QCD interaction. (1a) is the quark-gluon vertex and (1b) is the three-gluon vertex.

We proceed to compute the vertex functions at 1-loop order, which consist of two terms.. The first term corresponds to the Feynman diagram in Fig. (2a), named $\Gamma_\mu^{a,1}$, and the second one is the Feynman diagram in Fig. (2b), corresponding to $\Gamma_\mu^{a,2}$. The expressions for the vertex functions are

$$ig\,\Gamma_\mu^{a,1} = \int \frac{d^4k}{(2\pi)^4} igt^b\gamma_\nu iS^{LLL}(p_2-k)igt^a\,\gamma_\mu\, iS^{LLL}(p_1-k)igt^c\gamma_\delta iD^{cb}_{\delta\nu}(k), \qquad (1)$$

$$ig\Gamma_\mu^{a,2} = \int \frac{d^4k}{(2\pi)^4} igt^b\gamma_\nu iS^{LLL}(k)igt^{c_1}\gamma_\alpha iD^{c_1c_2}_{\alpha\beta}(p_2-k)if^{ac_2c_3}ig \times V_{\mu\beta\eta}(p_2-p_1,p_2-k,p_1-k)iD^{c_3b}_{\eta\nu}(p_1-k), \qquad (2)$$

where $igt^a\gamma_\mu$ is the quark-gluon vertex. The fermion propagator for a charged field in the presence of a magnetic field in the *lowest Landau level* approximation is written as follows

$$iS^{LLL}(k) = 2ie^{-\frac{{k_\perp}^2}{|q_fB|}}\frac{\not{k}_\parallel + m_f^2}{{k_\parallel}^2 - m_f^2 + i\epsilon}\mathcal{O}^+, \qquad (3)$$

Furthermore, the gluon propagator in the Feynman gauge, is written as

$$iD^{ab}_{\mu\nu}(p) = \delta^{ab}\frac{-ig_{\mu\nu}}{p^2+i\epsilon}. \qquad (4)$$

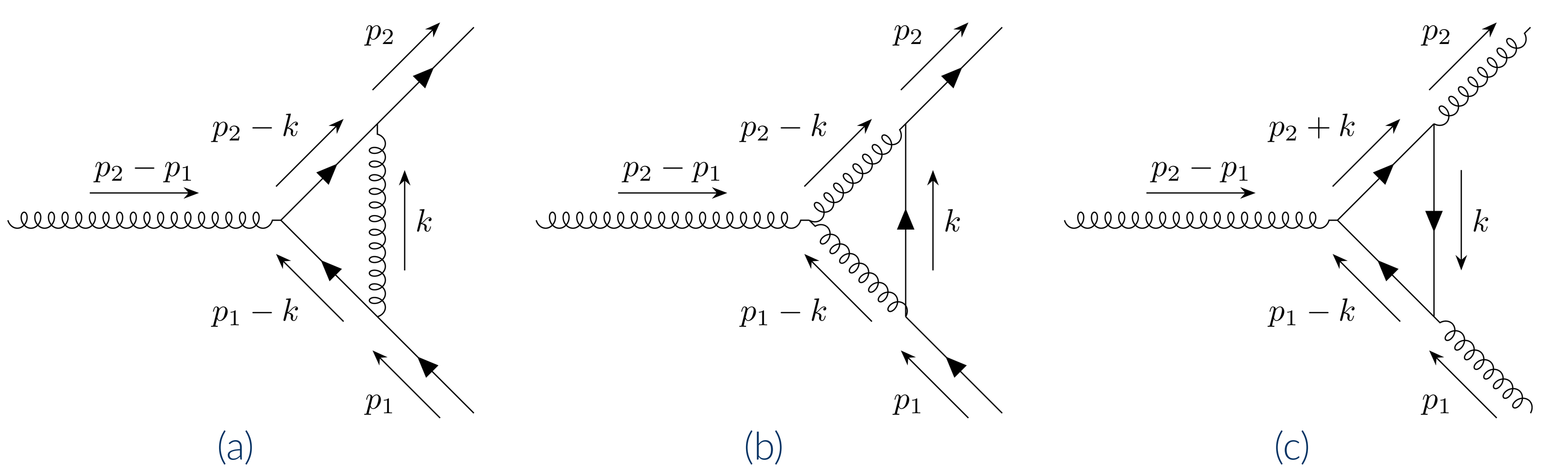

Figure 2. Feynman diagrams which includes the magnetic correction at 1-loop order. The QED-like contribution (2a), the pure QCD contribution (2b) and, the three-gluon vertex (2c).

Substituting Eqs. (3) and (4) in Eqs. (1) and (2), and after some straightforward algebra, the vertex corrections at 1-loop order become

$$ig\,\Gamma_\mu^{a,1} = -16g^3\left(C_F - \frac{C_A}{2}\right)\int_0^1 dx\int_0^{1-x} dy\int\frac{d^2k_\perp}{(2\pi)^2}e^{-\frac{(p_2-k)_\perp^2}{|q_fB|}}e^{-\frac{(p_1-k)_\perp^2}{|q_fB|}}\int\frac{d^2\ell_{\parallel,1}}{(2\pi)^2}\frac{\gamma_\alpha^\parallel\gamma_\mu^\parallel\gamma_\beta^\parallel\ell_\parallel^\alpha\ell_\parallel^\beta + f_{\mu,1}^\parallel}{(\ell_{\parallel,1}^2-\Delta_1)^3}, \qquad (5)$$

$$ig\,\Gamma_\mu^{a,2} = 2C_Ag^3t^a\int_0^1 dx\int_0^{1-x} dy\int\frac{d^2k_\perp}{(2\pi)^2}e^{-\frac{k_\perp^2}{|q_fB|}}\int\frac{d^2\ell_{\parallel,2}}{(2\pi)^2}\frac{\ell_\parallel^\alpha\ell^\nu\gamma_\alpha^\parallel\gamma_\nu\gamma_\mu - \gamma_\mu\gamma_\sigma^\parallel\gamma_\delta\ell_\parallel^\sigma\ell^\delta - f_{\mu,2}}{(\ell_{\parallel,2}^2-\Delta_2)^3}, \qquad (6)$$

with

$$\ell_{\parallel,1} = k_\parallel - (y\,p_2 + y\,p_1) \quad , \quad \ell_{\parallel,2} = k_\parallel - (y\,p_2 + (1-x-y)\,p_1). \qquad (7)$$

$$\Delta_1 = (xp_{2,\parallel} + yp_{1,\parallel})^2 + m^2(x+y) + k_\perp^2(1-x-y) - xp_{2,\parallel}^2 - yp_{1,\parallel}^2, \qquad (8)$$

$$\Delta_2 = (({k_\perp}^2(1-x)) + 2\,k_\perp\,p_{1,\perp}(x+y-1) - 2\,k_\perp\,p_{2,\perp}y + m^2x + p_1^2x + p_1^2y - p_1^2 + (p_{2,\parallel}y - p_{1,\parallel}(x+y-1))^2 - p_2^2y), \qquad (9)$$

and

$$f_{\mu,1}^\parallel = \gamma_\mu^\parallel(\not{p}_2(1-x) - \not{p}_1\,y)(\not{p}_1(1-y) - \not{p}_2\,x) - 2m_f(p_2(1-2x) + p_1(1-2y))_\mu^\parallel + m_f^2\,\gamma_\mu^\parallel), \qquad (10)$$

$$f_{\mu,2} = (4m_f - 2(\not{p}_{1\parallel}(1-x-y) + \not{p}_{2\parallel}\,y))(p_1-p_2)_\mu + \gamma_\mu(\not{p}_{1\parallel}(1-x-y) + \not{p}_{2\parallel}\,y)(\not{p}_{2\parallel}(1+y) - \not{p}_{1\parallel}(1+x+y)) + (\not{p}_{1\parallel}(x+y) - \not{p}_{2\parallel}\,y)(\not{p}_{1\parallel}(1-x-y) + \not{p}_{2\parallel}\,y)\gamma_\mu. \qquad (11)$$

Results and discussion

Once the integrals over all the Feynman parameters and momentum components are performed in Eqs. (5) and (6), and under the approximation where the magnetic field $|q_fB|$ is the highest energy scale, we obtain the final expressions

$$ig\,\Gamma_\mu^{a,1} = ig^3\left(C_F - \frac{C_A}{2}\right)t^a\,\gamma_\mu^\parallel\,\frac{|q_fB|}{Q^2}\left(\ln\left(1-\frac{Q^2}{4m_f^2}\right) + \frac{2Q\tan^{-1}\left(\frac{Q}{\sqrt{4m_f^2-Q^2}}\right)}{\sqrt{4m_f^2-Q^2}}\right), \qquad (12)$$

$$ig\Gamma_\mu^{a,2} = 0. \qquad (13)$$

From Eq. (12), we can extract the effective QCD coupling in the presence of a very large magnetic field

$$g_{eff} = g\left[1 - g^2\frac{3|q_fB|}{2Q^2}\left(\ln\left(1-\frac{Q^2}{4m_f^2}\right) + \frac{2Q\tan^{-1}\left(\frac{Q}{\sqrt{4m_f^2-Q^2}}\right)}{\sqrt{4m_f^2-Q^2}}\right)\right], \qquad (14)$$

where we have used $C_F - \frac{C_A}{2} = -\frac{N_f}{2}$ and $N_f = 3$.

Conclusion

We calculated the 1-loop magnetic correction of the QCD vertex function, which allowed us to construct a function of the effective QCD coupling constant which shows an increasing coupling behavior as the magnetic field intensity increases, it means that the magnetic field favors the intensity of the strong interaction to increase and it can be consider as a signa of the *Magnetic Catalysis*.

References

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