Probing Maximal Entanglement in Deep Inelastic Scattering

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In collaboration with

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Outline

1 Entanglement: What is it?

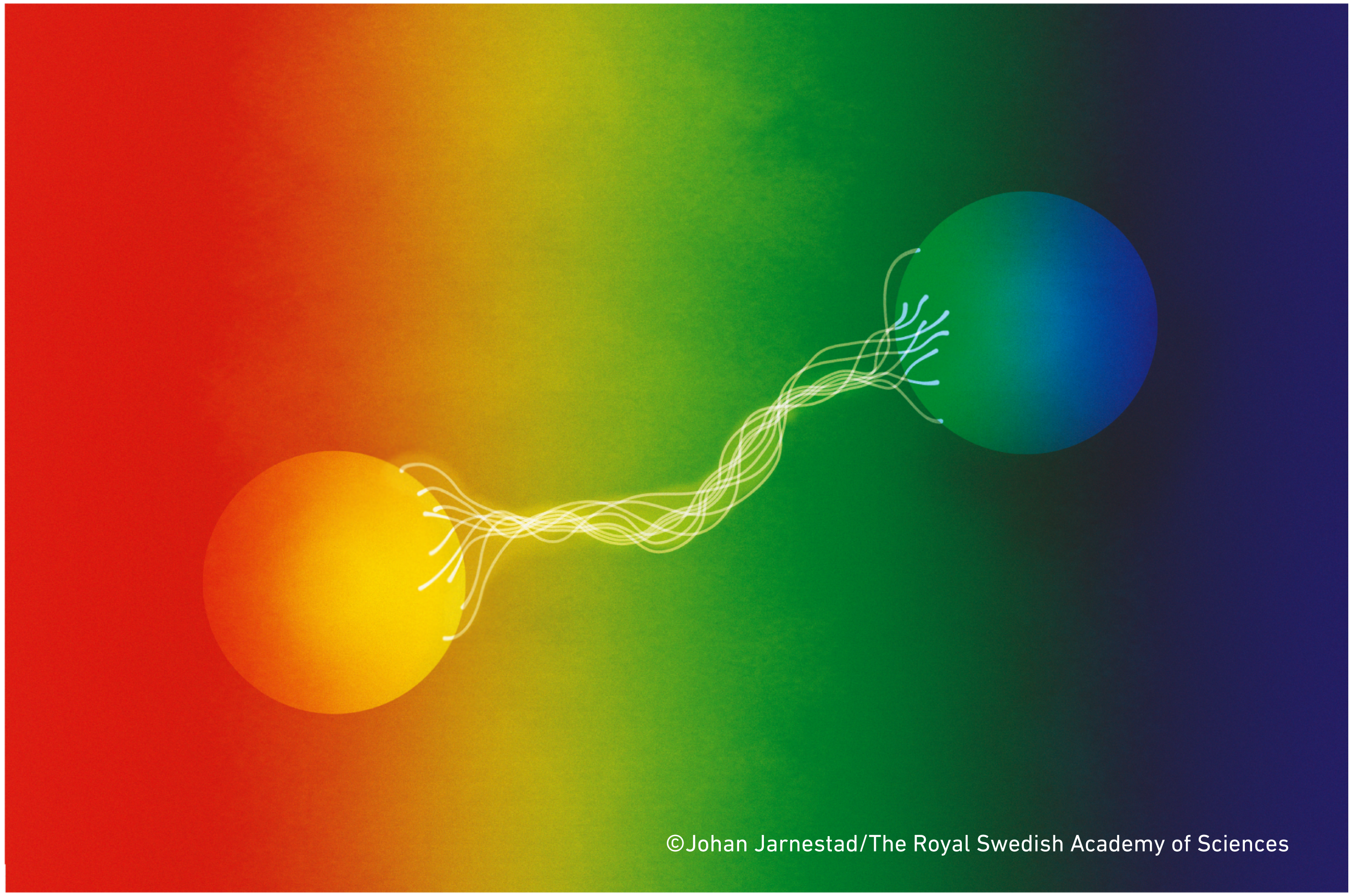
2 Entanglement entropy and the DIS reaction

3 Entanglement entropy in inclusive DIS

4 Entanglement entropy in Diffractive DIS

5 Discussion

Entanglement - What is it?



Source: https://www.nobelprize.org/uploads/2022/10/press-physics2022-figure1.pdf

Entanglement - What is it?

Wikipedia:

An entangled system is defined to be one whose quantum state cannot be factored as a product of states of its local constituents; that is to say, they are not individual particles but are an inseparable whole. In entanglement, one constituent cannot be fully described without considering the other(s). The state of a composite system is always expressible as a sum, or superposition, of products of states of local constituents; it is entangled if this sum cannot be written as a single product term.

Source: https://en.wikipedia.org/wiki/Quantum_entanglement

Entanglement — What is it?

More formal:

2 Hilbert spaces H_A, H_B; consider $H_A \otimes H_B$

$$|\psi\rangle_A \in H_A, \quad |\phi\rangle_B \in H_B$$

Not entangled:

$$|\psi\rangle_A \otimes |\phi\rangle_B$$

A product state

Entangled:

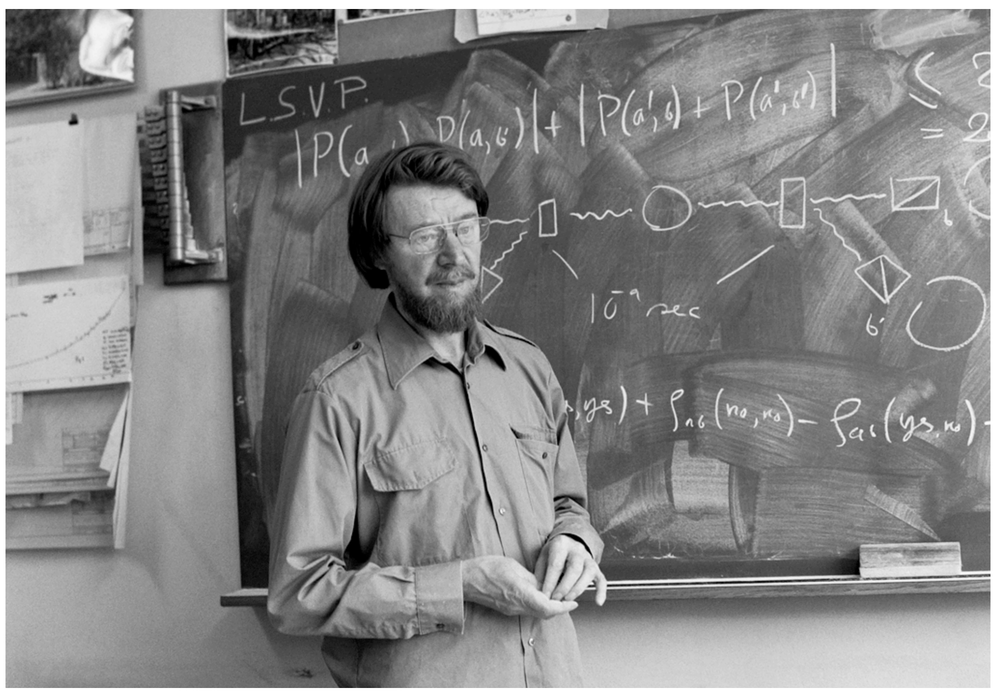
$$|\psi\rangle_{AB} = \sum_{i,j} c_{i,j} |i\rangle_A \otimes |j\rangle_B$$

- for some basis $\{|i\rangle_A\}$ for H_A and some basis $\{|j\rangle_B\}$ for H_B
- If the state is *inseparable* i.e. impossible to find $c_{i,j} = c_i^A c_j^B$

Entanglement and tests of Quantum Mechanics

1935: Einstein, Rosen Podolsky: *"Can Quantum-Mechanical Description of Physical Reality be Considered Complete?"*

- thought experiment with 2 entangled particles
- argued for the existence of "elements of reality" that were not part of quantum theory,



1964: Bell's inequality: local hidden variable theories and quantum mechanics yield different predictions for certain correlations of 2 spin 1/2 particles

Experiment: Bell's inequalities are violated, local hidden variable theories cannot reproduce observed quantum mechanics result

2022: Nobel Prize for John Clauser, Alain Aspect, and Anton Zeilinger

Bell type inequalities in high energy physics

- Spin correlations of Λ-hyperons

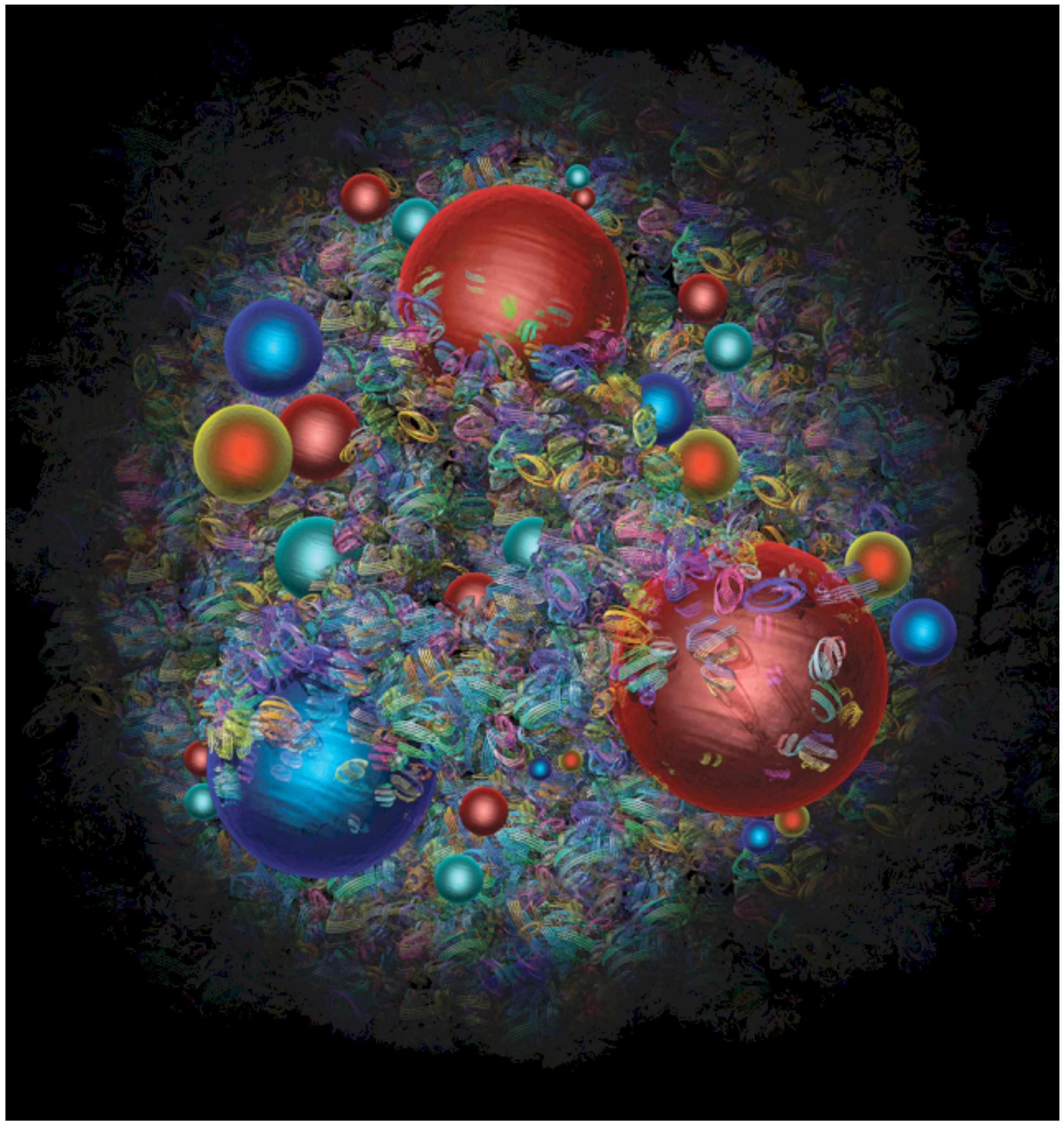
  Wenjie Gong, Ganesh Parida, Zhoudunming Tu, Raju Venugopalan; 2107.13007
  João Barata, Wenjie Gong, Raju Venugopalan; 2308.13596

- Bell Inequalities at the LHC with Top-Quark Pairs

  M. Fabbrichesi, R. Floreanini, G. Panizzo; 2102.11883
  ATLAS collaboration; 2311.07288

- Many more ….

Entanglement and Confinement

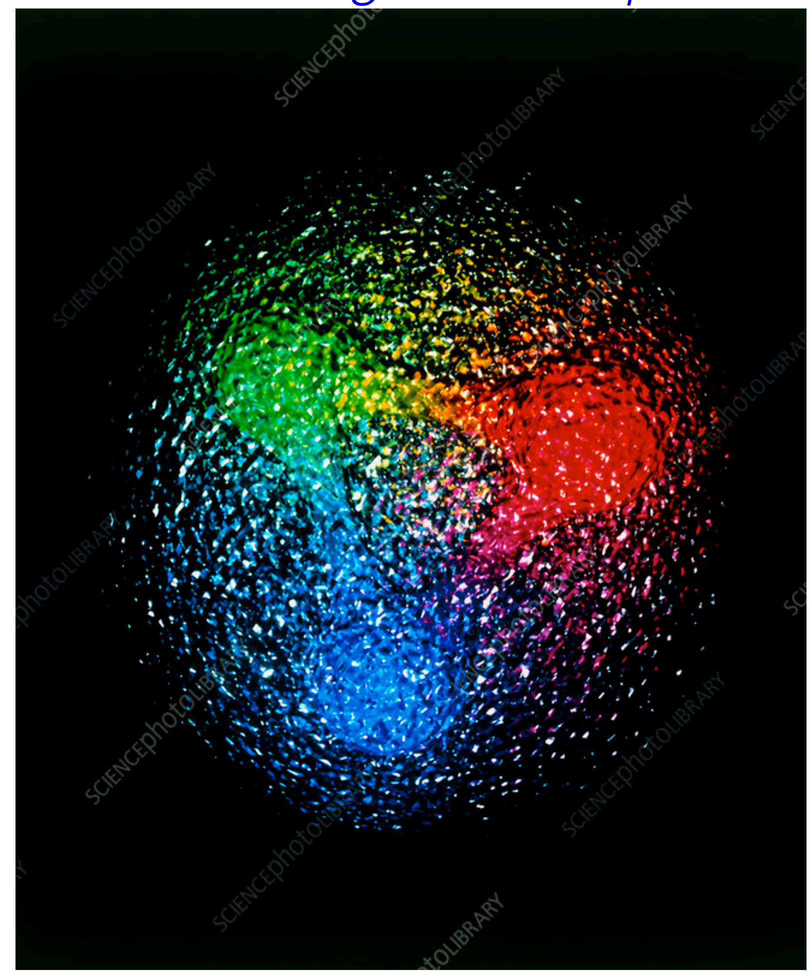


Source: https://cds.cern.ch/record/2747741

Entanglement and Confinement

Wikipedia: *color-charged particles (such as quarks and gluons) cannot be isolated, and therefore cannot be directly observed in normal conditions below the Hagedorn temperature*

- Color Confinement = one of the most important unsolved problems in modern physics
- We know it exists but we don't know why, we don't know the mechanism behind it

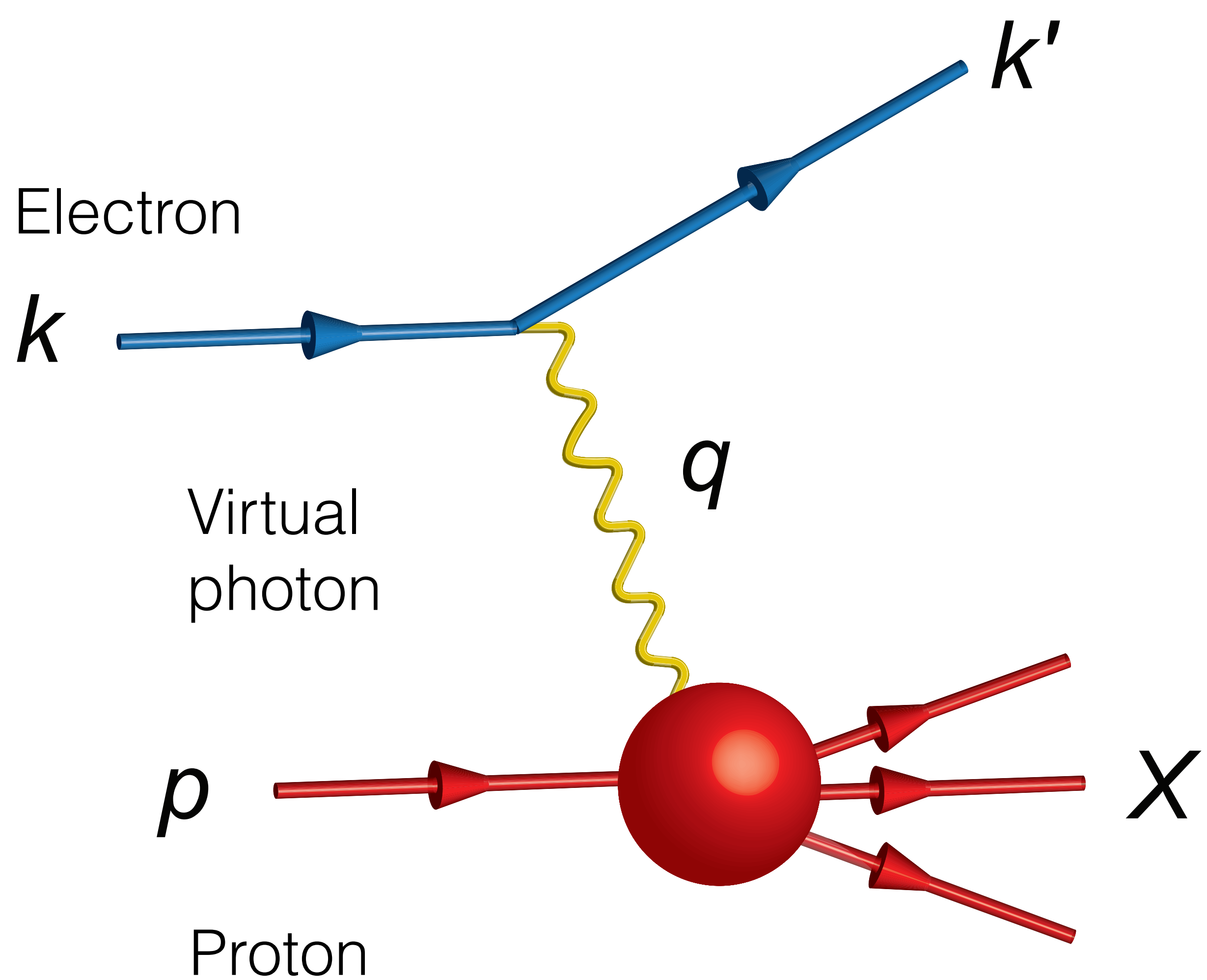


Source: https://www.sciencephoto.com/media/2093/view/visualisation-of-quark-structure-of-proton

A new perspective

- color confinement = limit of maximal entanglement of microscopic degrees of freedom
- Quarks and gluons are not just correlated; they cannot exist in isolation

Experiment to explore the proton: Deep Inelastic electron-proton Scattering (DIS)



Photon virtuality (=resolution)

$Q^2 = -q^2, \quad \lambda \sim \frac{1}{Q}$

Bjorken x

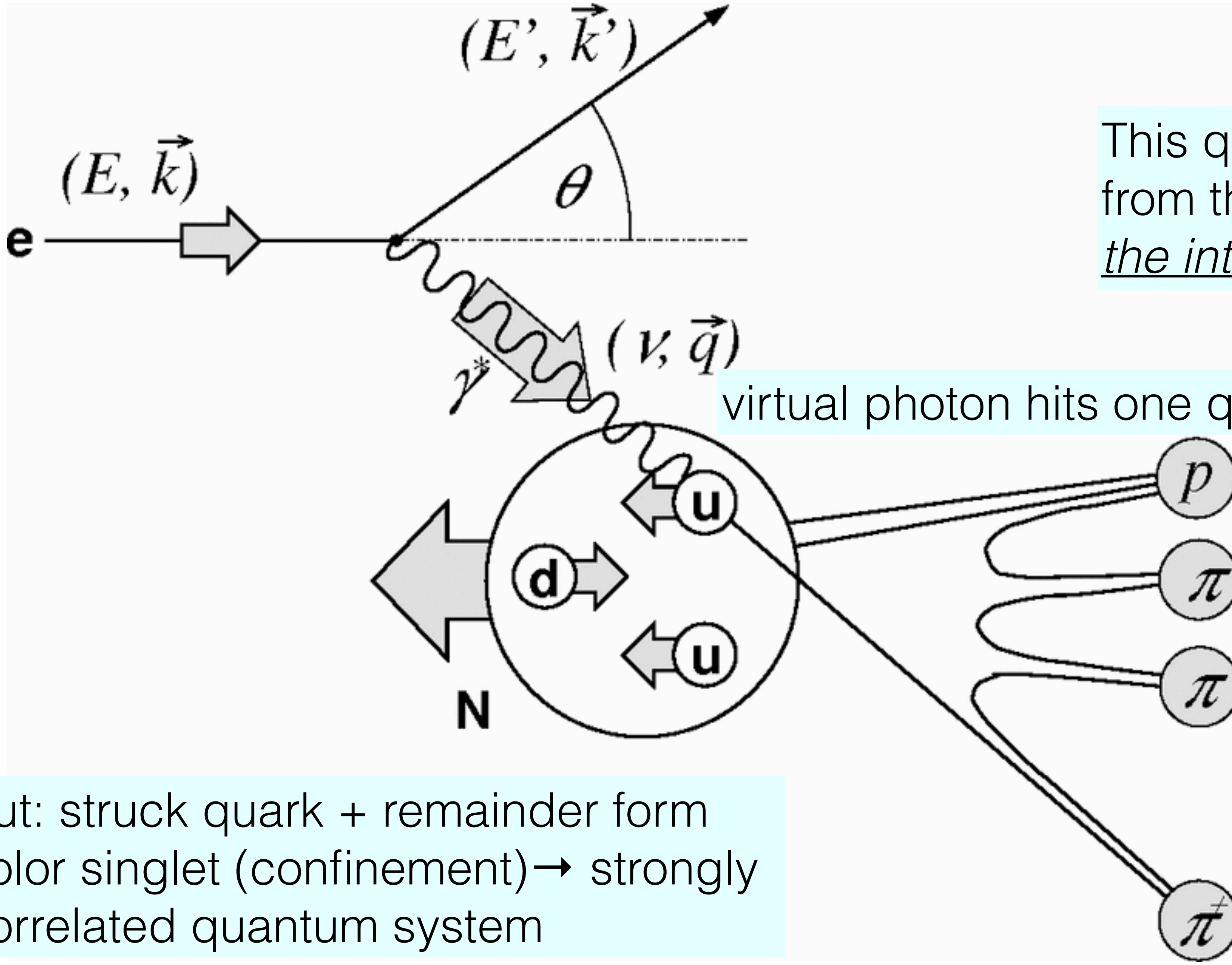
$x_{Bj.} = \frac{Q^2}{2p \cdot q}$

“Mass" of the system X

$W^2 = (p+q)^2 = M_p^2 + \frac{1-x}{x} Q^2$

Elastic scattering: either $Q = 0$ or $x = 1$
Inelastic requires $x < 1$

What happens during DIS?



This quark is casually disconnected from the rest of the proton, *during the interaction* $\tau \sim 1/Q$

virtual photon hits one quark

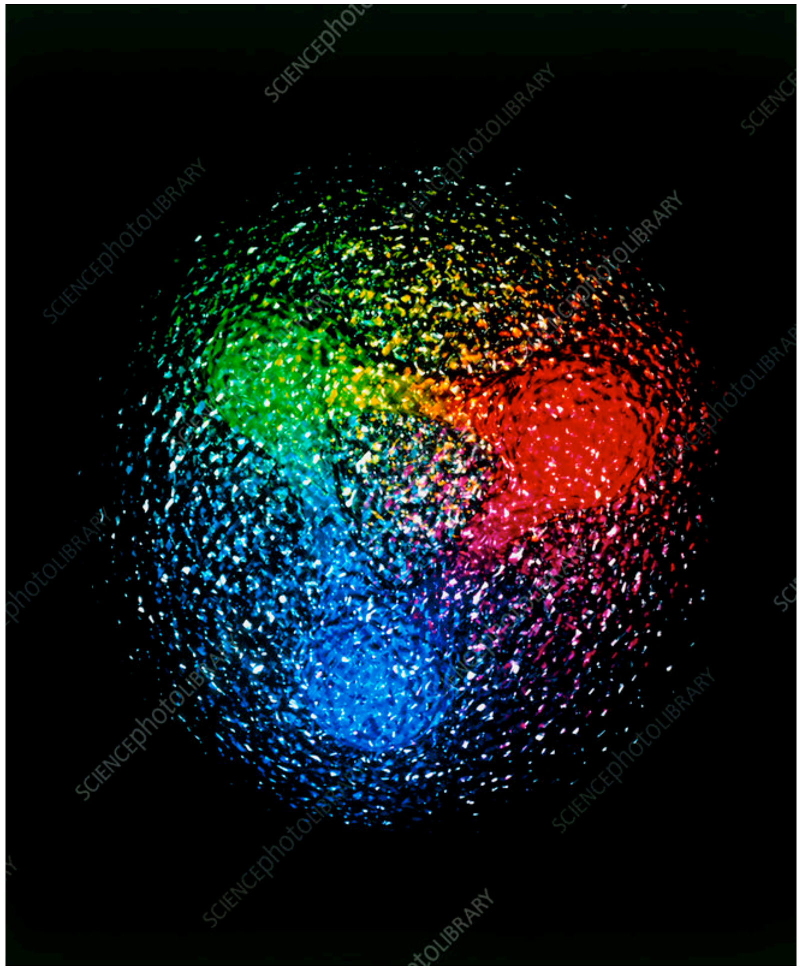
But: struck quark + remainder form color singlet (confinement)→ strongly correlated quantum system

After $\tau \sim 1/\Lambda_{QCD}$, color string/flux tube forms, breaks up, multi particle state forms

Source: arxiv:hep-ex/0407032

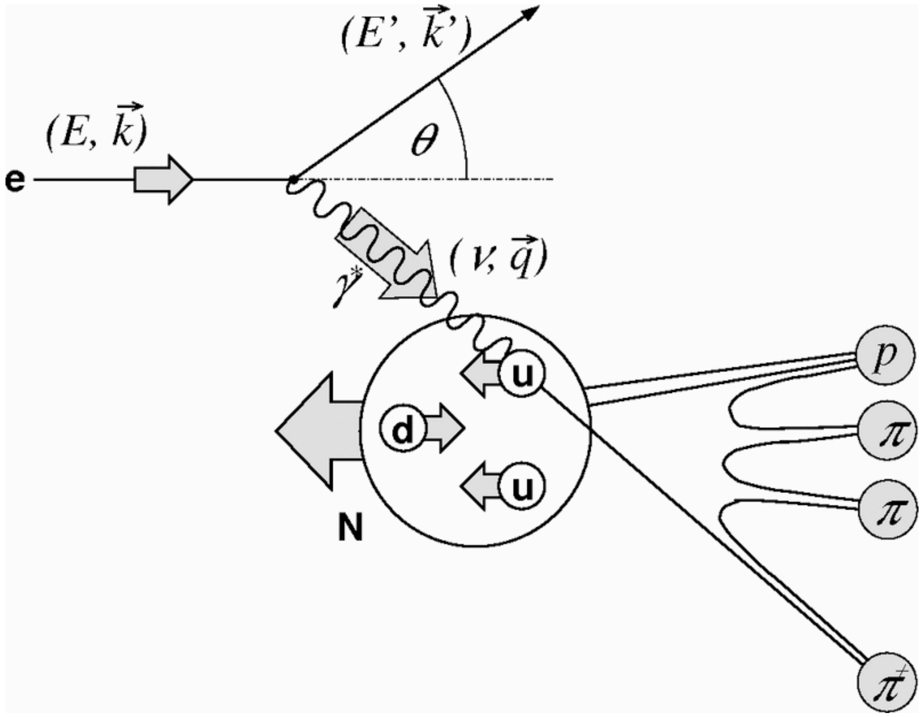
[Tu, Kharzeev, Ullrich; 1904.11974] Einstein-Rosen-Podolsky scenario at subatomic scales: strongly correlated, but casually disconnected

Entropy, the proton and DIS



isolated proton = pure quantum state
zero von Neumann entropy

DIS: proton a collection of quasi-free partons



entropy associated with different ways to distribute partons in phase space

Entropy, starting from information theory …

Basic question: how much information is there in a certain (observed) event?

Claude Shannon (1948), Edwin Jaynes (1957)

- Low probability event: high information (surprising)
- High probability event: low information (boring)

p_i: probability to observe event i with $\sum_i p_i = 1$

Shannon information: $h_i = -\ln(p_i)$,

$h_i \in [0,\infty)$ with $h(0) = \infty$ and $h(1) = 0$

entropy = mean value of information of a certain ensemble

$$S = \langle h(p) \rangle = \sum_i p_i h(p_i) = -\sum_i p_i \ln p_i = S$$

Entropy etc within Quantum Mechanics:

Density matrix: $\hat{\rho} = \sum_{i=1}^{N} p_i |\psi_i\rangle\langle\psi_i|$

System allows for N possible states $|\Psi_i\rangle$ with probability p_i

If $N = 1$, we have $\hat{\rho} = |\Psi\rangle\langle\Psi|$

system in a pure state

$N > 1$

the quantum state $|\Psi_i\rangle$ appears with probability p_i

= system in a mixed state

von Neumann entropy: $S = -\text{tr}\left[\hat{\rho}\ln\hat{\rho}\right]$

quantum generalization of $S = -\sum_i p_i \ln p_i$

- Pure state: $S = 0$
- Mixed state $S > 0$

From a pure to a mixed state

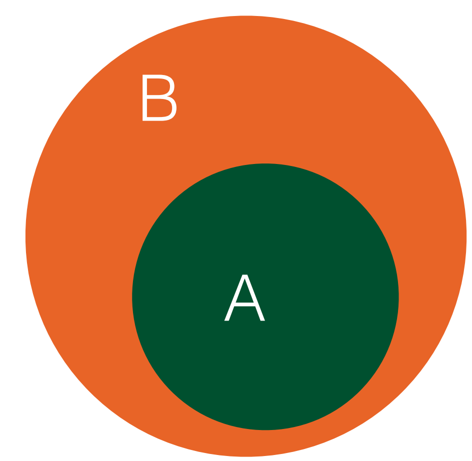
Hilbert space: $\mathscr{H}_{AB} = \mathscr{H}_A \otimes \mathscr{H}_B,$

$$|\Psi\rangle_A \in \mathscr{H}_A, \quad |\Psi\rangle_B \in \mathscr{H}_B$$

$$|\Psi\rangle_{AB} = \sum_{j,k} \alpha_{jk} |\Psi_j\rangle_A \otimes |\Psi_k\rangle_B$$ entangled state, but pure state

$$\rightarrow S_{AB} = -\mathrm{tr}\hat{\rho}_{AB} \ln \hat{\rho}_{AB} = 0$$

Now: do not observe system B (=everything that is not A)



QM → sum over all possibilities that can occur in the system B

To obtain density matrix of observed system A: sum/trace over all unobserved states of $\mathscr{H}_B$

$$\hat{\rho}_A = \mathrm{tr}_B \hat{\rho}_{AB}$$

density matrix of observed system A

Entanglement entropy

Mathematical trick (Schmidt decomposition) = rearrange basis:

$$|\Psi\rangle_{AB} = \sum_{j,k} \alpha_{jk} |j\rangle_A \otimes |k\rangle_B = \sum_i \beta_i |i\rangle_A \otimes |i\rangle_B$$

Density matrix of the subsystem A:

$$\hat{\rho}_A = \mathrm{tr}_B \hat{\rho}_{AB} = \sum_j p_j |\Psi_{A,j}\rangle\langle\Psi_{A,j}|, \qquad p_j = |\beta|_j^2$$

Yields density matrix of a **mixed** system, if state $|\Psi_{AB}\rangle$ was entangled

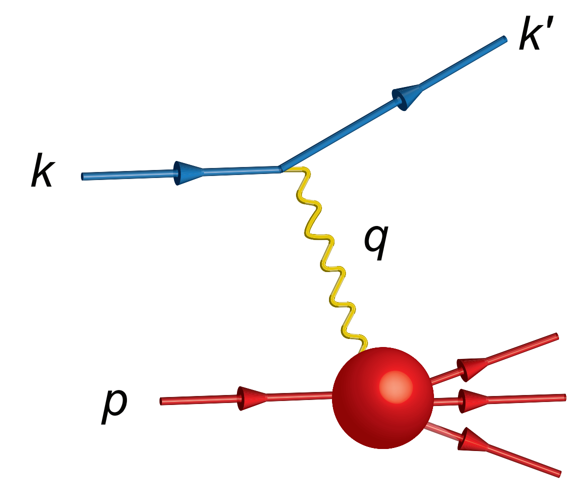
not entangled: $p_1 = 1, \quad p_{i\geq 1} = 0$
pure state

Entropy of system A:

$$S_A = -\,\mathrm{tr}_A \hat{\rho}_A \ln(\hat{\rho}_A) = -\sum_j p_j \ln p_j, \qquad p_j = |\beta_j|^2$$

Note this is symmetric: $S_A = S_B$

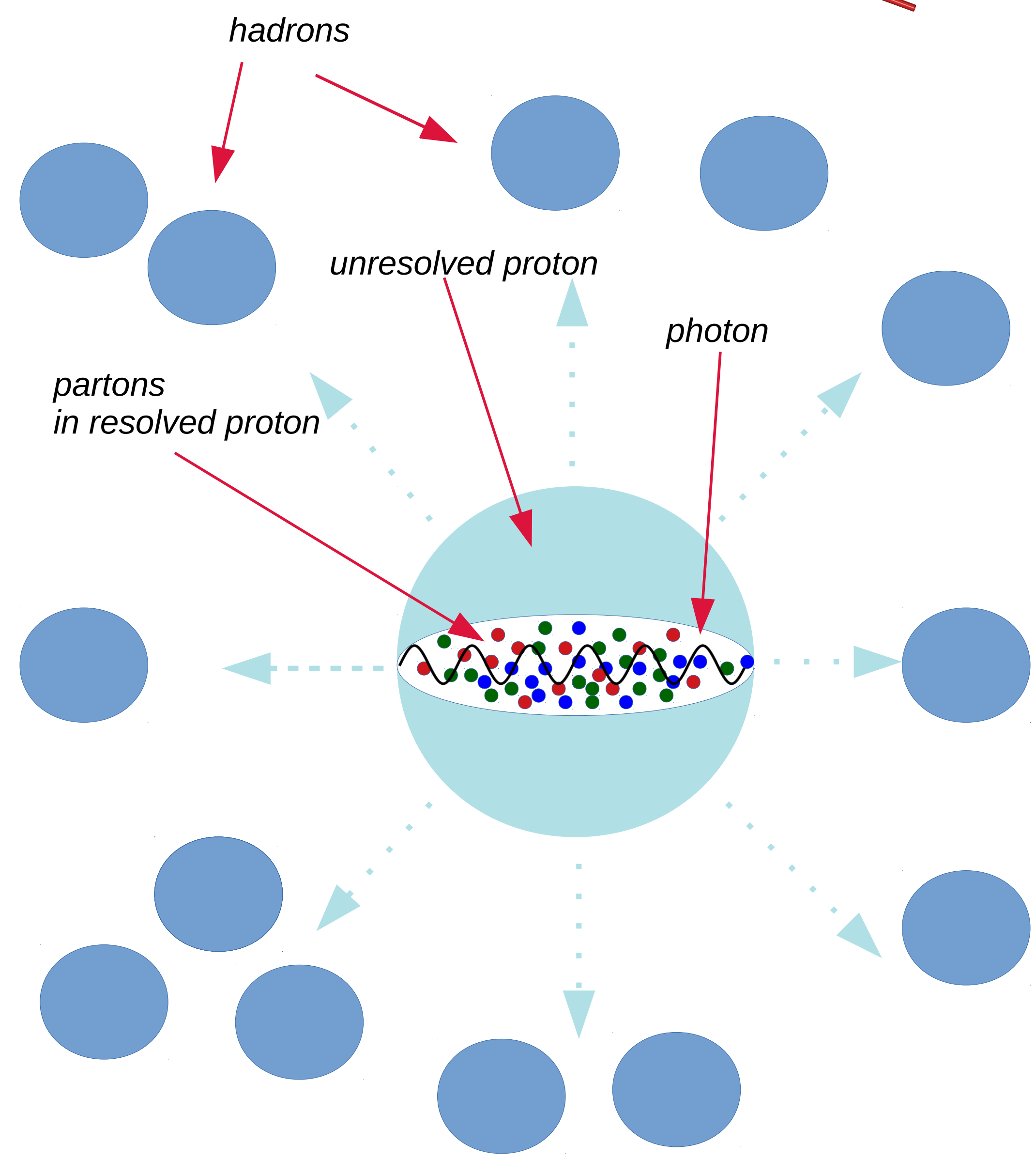
Partial Observation of the Proton in Deep Inelastic Scattering (DIS)



DIS: do not observe the entire proton, but only parts of it

resolved area $\sim 1/Q^2$

[Gribov, Ioffe, Pomeranchuk, SJNP, 2, 549 (1966)];
[Ioffe, PLB 30B, 123, (1969)]



Entropy of final state hadrons = entanglement entropy

determined by the initial proton wave function

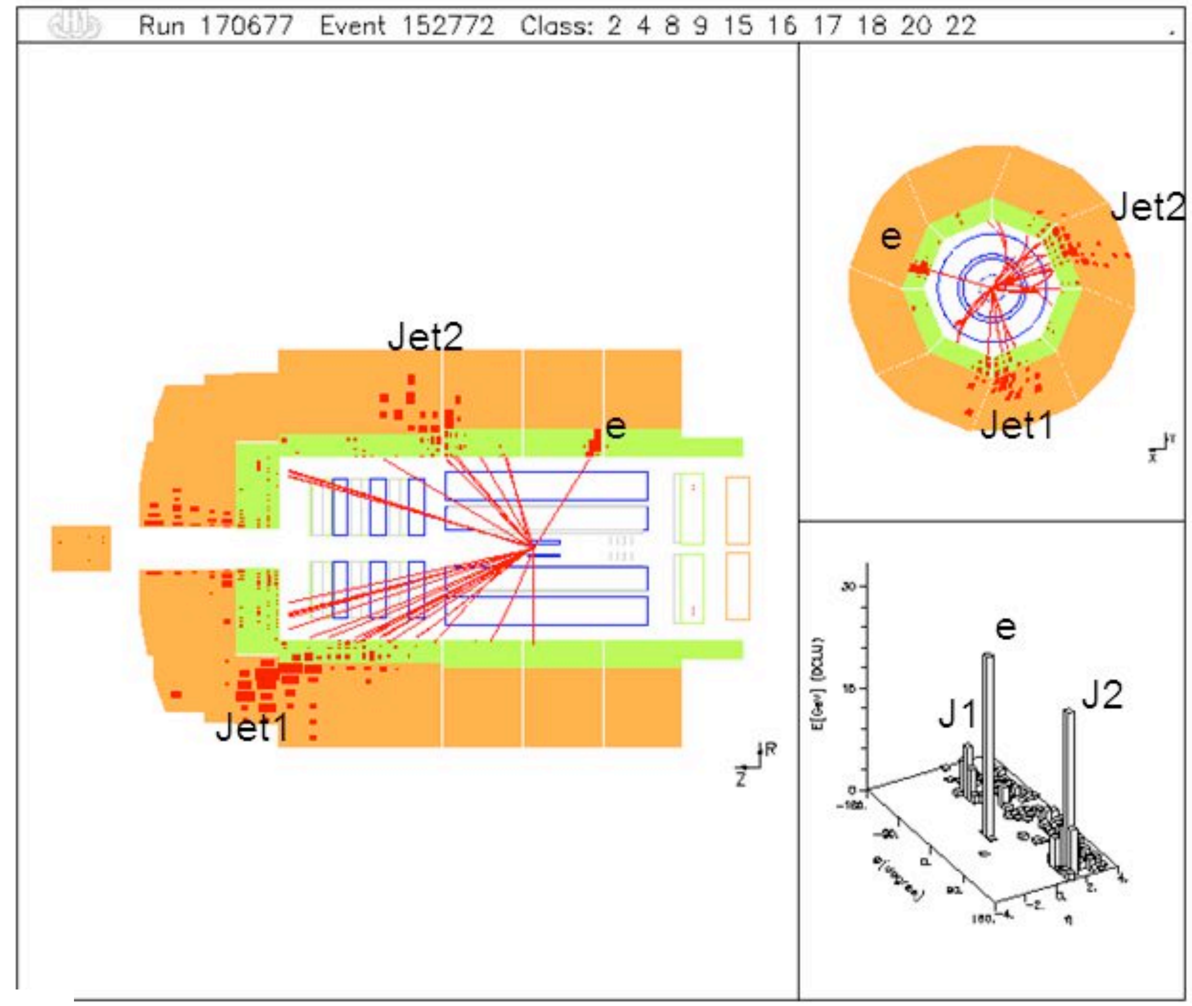
[Kharzeev, Levin; 1702.03489]

Entropy of final state hadrons = entanglement entropy

different DIS events → different # of final state hadrons

A NC-DIS event with two jets $ep \rightarrow e' Jet_1 Jet_2$

$$P(N) = \frac{\#\text{of events with } N \text{ hadrons}}{\text{total \# of events}}$$



Joachim Meyer DESY 2005

hadronic entropy

$$S_{hadron} = -\sum P(N) \ln P(N)$$

[Kharzeev, Levin; 1702.03489]

Entropy of final state hadrons = entanglement entropy

caused by partial observation of the proton

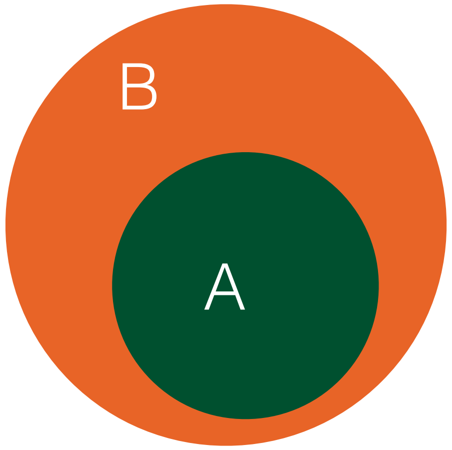
value determined by the initial state proton wave function

The entangled proton wave function

To arrive at an explicit expression for entanglement entropy, need to express entangled wave function as

$$|\Psi\rangle_{AB} = \sum_{j,k} \alpha_{jk} |j\rangle_A \otimes |k\rangle_B = \sum_i \beta_i |i\rangle_A \otimes |i\rangle_B$$

A: observed states
B: unobserved states



then: $$S_A = S_B = \sum_i p_i \ln p_i, \quad p_i = |\beta_i|^2$$

the photon wave function is highly non-perturbative → so far, cannot obtain this basis from first principles (even the concept of quarks and gluons is difficult)

But:

Can use our understanding of DIS at low x to model the proton wave function & get close to reality

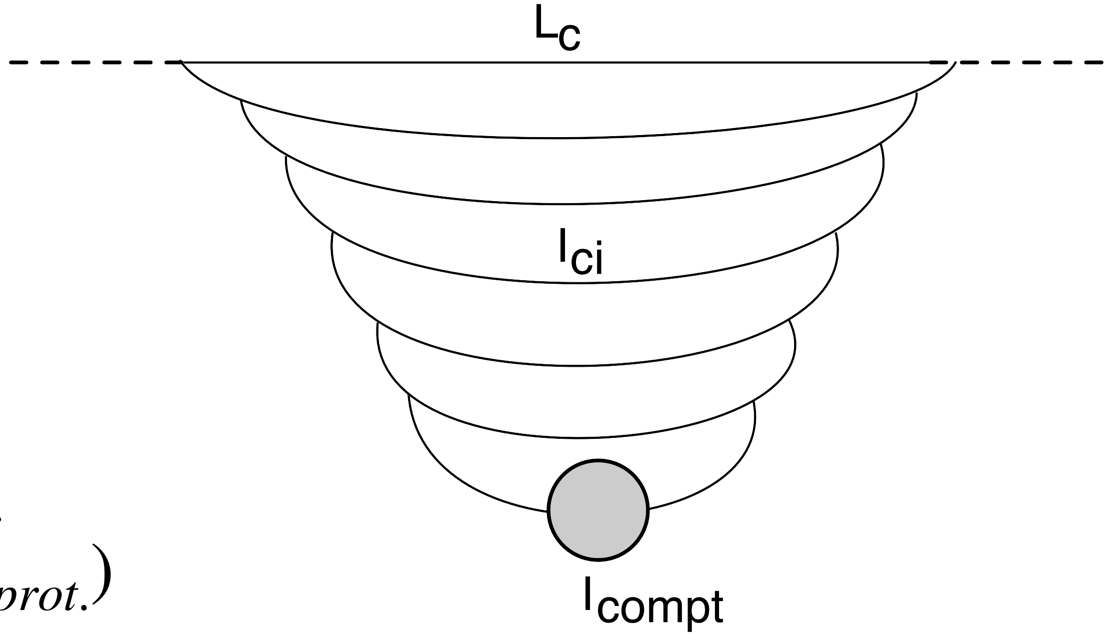
A result from 2D conformal field theories

[Holzhey, Larsen, Wilczek; 1994], [Calabrese, Cardi; 2006]

$$S = \frac{c}{3} \ln \frac{L}{\epsilon}$$

L : extension of studied region
ϵ: regularization scale = resolution
c: central charge

low x limit, proton rest frame: photon fluctuates into many parton state
(time dilation of gluon life time due to large relative boost factor)



yields 2 length scale:
- Compton wave length of the proton $1/m_{prot.}$
- extension of the photon $e^Y/m_{prot.} = 1/(xm_{prot.})$

[Kharzeev, Levin; 1702.03489] identify

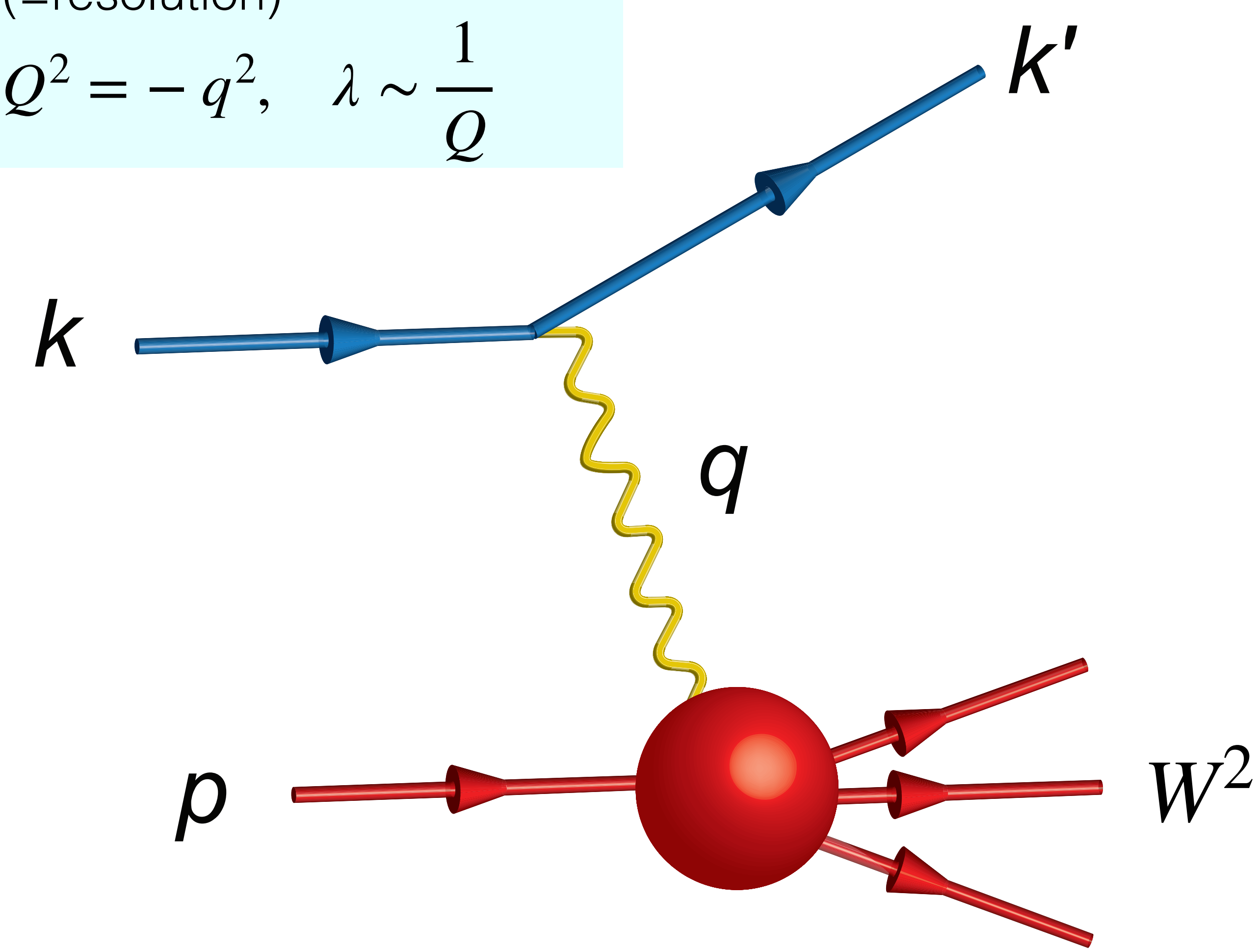
- ϵ with Compton wave length of the proton $\epsilon = 1/m_{prot.}$
- extension of studied region $L = \epsilon/x$

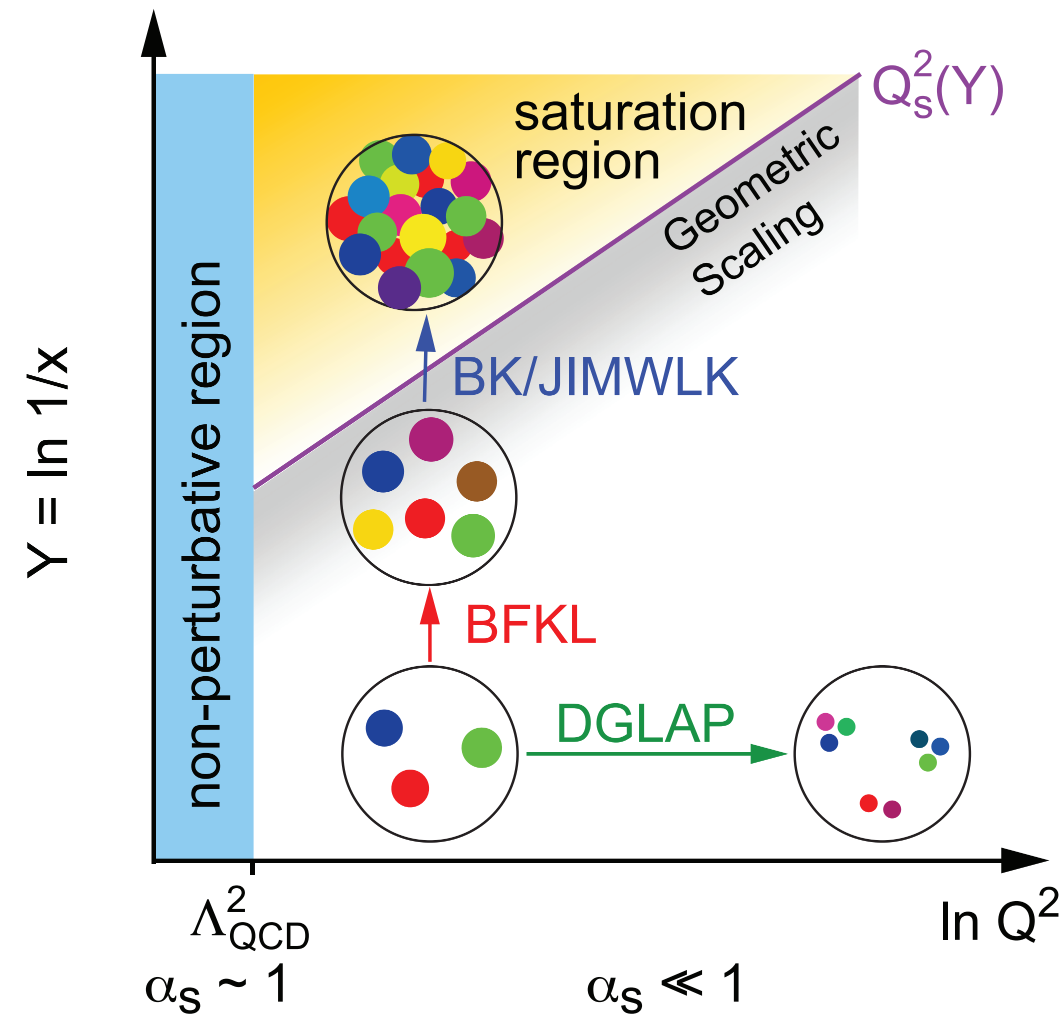
yields

$$S = \frac{c}{3} \ln 1/x$$

QCD evolution equations in DIS

Photon virtuality
(=resolution)
$Q^2 = -q^2, \quad \lambda \sim \frac{1}{Q}$



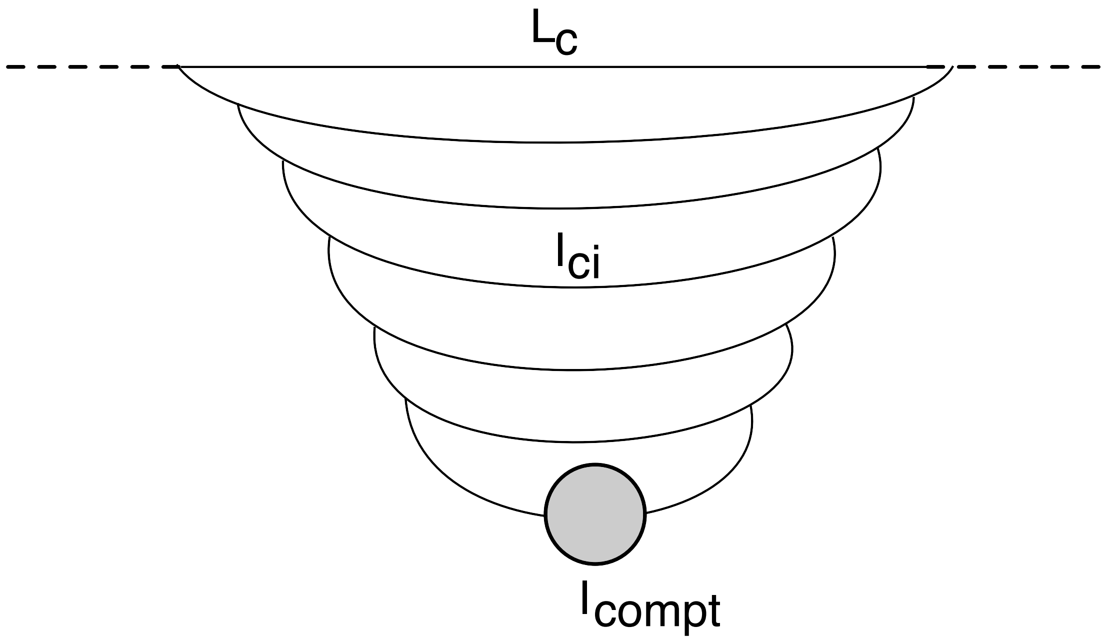


Bjorken $x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{W^2 + Q^2}$
low x ↔ large W at fixed Q

The Mueller Dipole Model

[A. Mueller, 1994-1998]

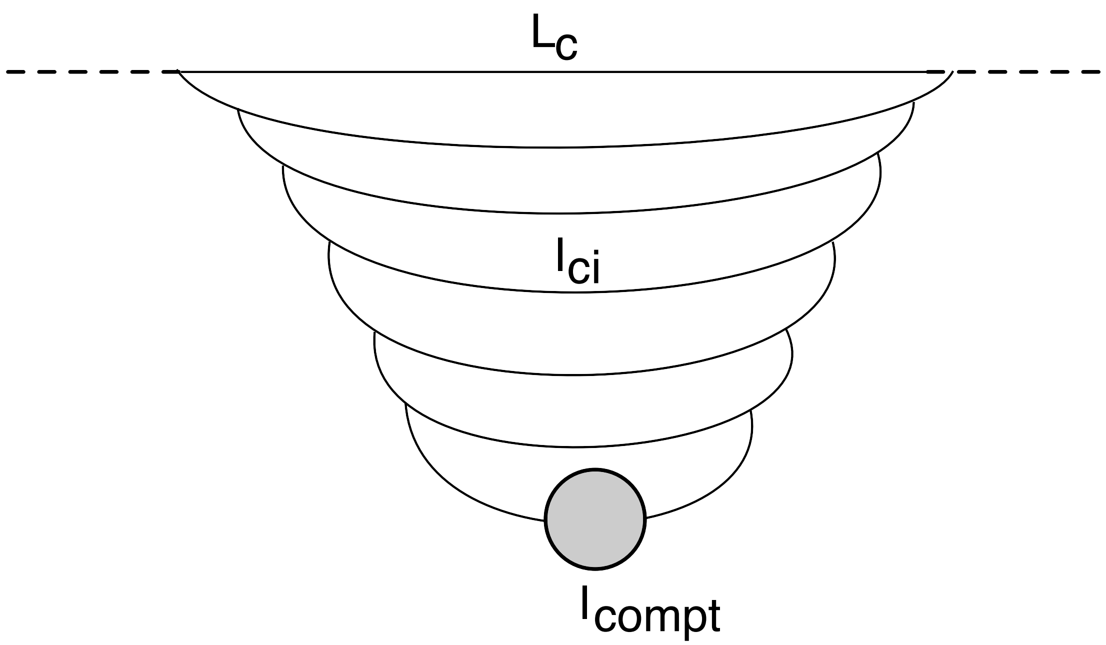
reformulation of BFKL (=low x) evolution as subsequent splitting of color dipoles
(gluon emission by initial $q\bar{q}$ from photon)



- allows to formulate evolution equation for probabilities to find n dipoles in the DIS reaction

- can be solved in approximation where transverse dynamics is ignored
  [A. Mueller; Nucl. Phys. B 415, 373–385 (19

- recently solved in the double logarithmic limit for full description
  [Liu, Nowak, Zahed; 2211.05169]



1+1 non-linear model of non-linear QCD evolution in $Y = \ln(1/x)$

$p_n(Y)$ probability to encounter n color dipoles (~gluons) in the proton

Δ: probability to emit another dipole; phenomenology $\Delta = 0.2 - 0.35$

p_n subject to cascade equation:

$$\frac{d}{dY} p_n(Y) = -\Delta n p_n(Y) + \Delta(n-1) p_{n-1}(Y)$$

initial condition: only 1 dipole at $Y = 0 \leftrightarrow x = 1 \leftrightarrow W^2 = m^2_{prot.}$ (elastic limit)

$$p_1(0) = 1; \quad p_{n>1}(0) = 0$$

p_n at $Y \neq 0$ from solution to cascade equation:

$$p_1 = e^{-\Delta Y}, \quad p_{n>1} = e^{-\Delta Y}\left(1 - e^{-\Delta Y}\right)^{n-1}$$

= our distribution $p_n = |\beta_n|^2$

Properties and Interpretation of the Solution

2 important quantities:

a) Mean number of dipoles = mean number of gluons or partons

$$\langle n \rangle = \sum_n n p_n(Y) = e^{\Delta Y} = \left(\frac{1}{x}\right)^{\Delta} = xg(x)$$

matches

- phenomenological observed powerlike growth of gluon & seaquark distribution at low x
- BFKL predicts such a powerlike rise

b) entropy $S = -\sum_n p_n \ln p_n = (1-Z)\ln\frac{Z-1}{z} + \ln Z, \quad Z = \langle n \rangle = e^{\Delta Y}$

- thermodynamic limit $\lim_{Y \gg 1} S = \ln\langle n \rangle = \Delta \ln(1/x)$ and $p_n = 1/\langle n \rangle$
- state of maximal (entanglement) entropy
- agrees with exact result for entanglement entropy obtain for 2D conformal field theories [Holzhey, Larsen, Wilczek; 1994], [Calabrese, Cardi; 2006] after proper identification of parameters

[Kharzeev, Levin; 1702.03489]

Entanglement entropy = entropy of n parton state = entropy of final state hadrons in DIS $S_{gluon} = \ln xg(x, Q^2)$

[H1 collaboration, 2011.01812]

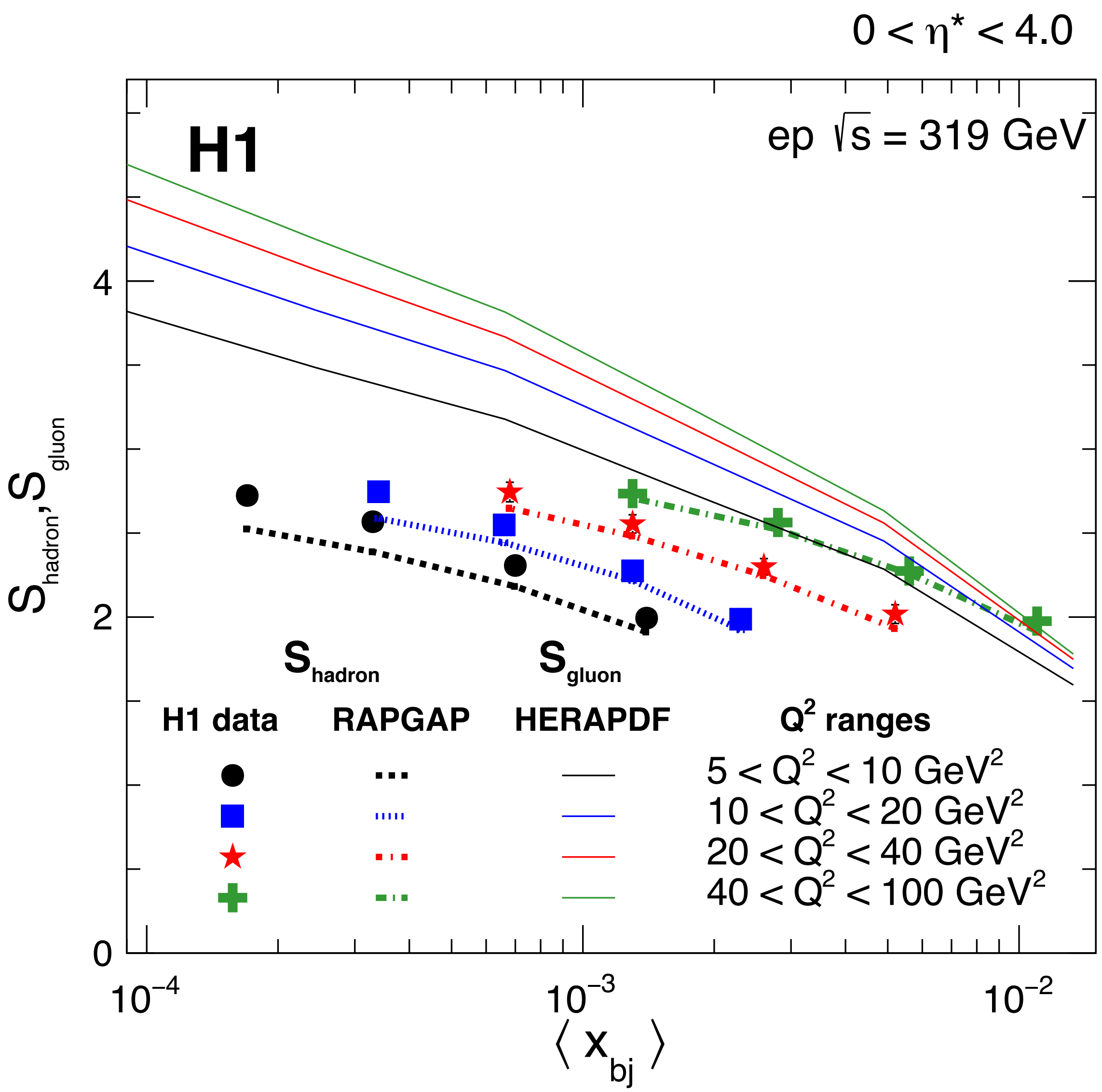
$$P(N) = \frac{\text{\#of events with } N \text{ hadrons}}{\text{total \# of events}} \qquad S_{hadron} = -\sum P(N) \ln P(N)$$

hadronic entropy for given bins of Q^2 and $x = \dfrac{Q^2}{2p \cdot q}$

can we determine correctly S_{hadron} from the model result?

Yes, but …. at first a series of (small) errors

H1 collaboration: results [arXiv:2011.01812]



- [Kharzeev, Levin; 1702.03489] $n_{dipoles} = xg(x, Q^2)$, yields $S_{gluon} = \ln\left[xg(x, Q^2)\right]$
- Reason: glue dominates at low x
- H1 collaboration: LO HERAPDF
- "The predictions from the entanglement approach based on the gluon density again fail to describe S_{hadron} in magnitude. However, at low Q the slope of S_{gluon} has some similarities with that observed for S_{hadron}, while it becomes steeper than observed with increasing Q"

[Kharzeev, Levin; 2102.09773]: try something based seaquarks

Gluon and seaquark PDF from unintegrated gluon

[MH, Kutak; 2110:06156]

this is not working, since H1 uses PDFs
dipole model in DIS = BFKL
let's calculate PDFs from BFKL unintegrated gluon

$$xg(x, Q^2) = \int_0^{Q^2} d\boldsymbol{k}^2 G(x, \boldsymbol{k}^2, Q^2),$$

$$x\Sigma(x, Q^2) = \int_0^\infty \frac{d\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2} \int_0^\infty d\boldsymbol{k}^2 \int_0^1 dz \Theta \left(Q^2 - \frac{\boldsymbol{\Delta}^2}{1-z} - z\boldsymbol{k}^2 \right) \tilde{P}_{qg}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2} \right) G(x, \boldsymbol{k}^2, Q^2)$$

For seaquark: TMD splitting function [Catani, Hautmann, NPB 427 (1994) 475] + many others afterwards

$$\tilde{P}_{qg}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2} \right) = \frac{\alpha_s 2 n_f}{2\pi} T_F \frac{\boldsymbol{\Delta}^2}{[\boldsymbol{\Delta}^2 + z(1-z)\boldsymbol{k}^2]^2} \left[z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2} \right],$$

Bottom line:

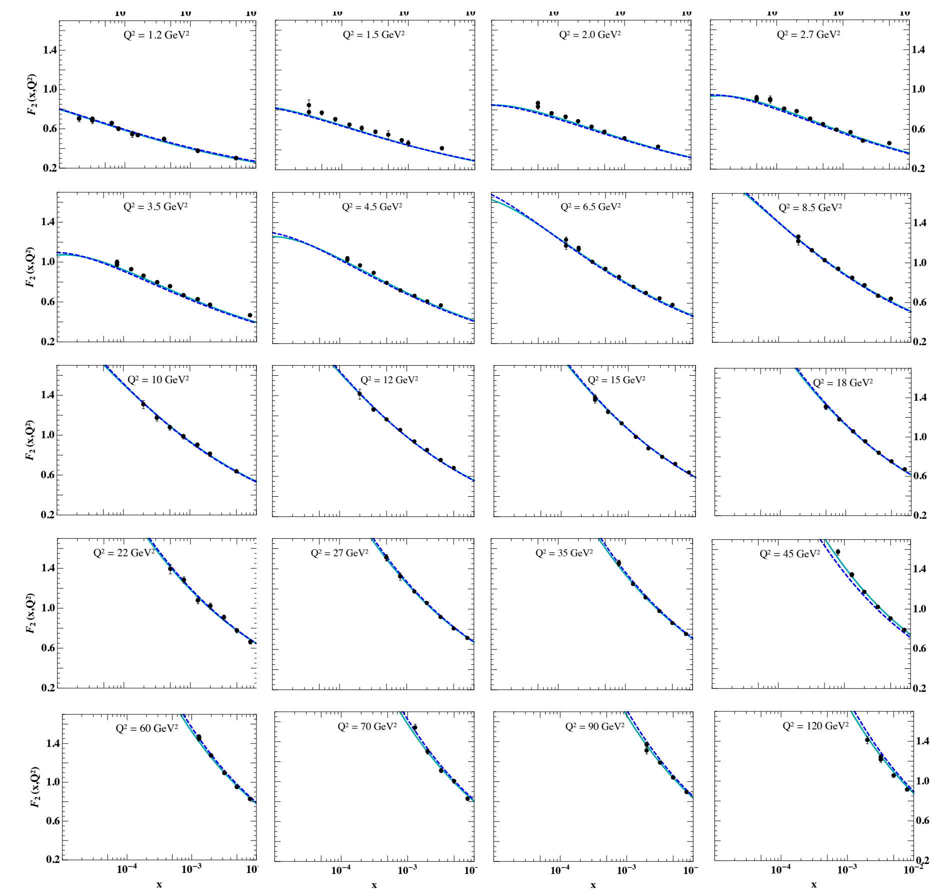
- we can calculate PDFs (and therefore entropy) using a low x formalism
- Low x evolution contained in $G(x, \mathbf{k}^2)$

underlying unintegrated gluon: the HSS fit

use: HSS NLO BFKL fit [MH, Salas, Sabio Vera; 1301.5283]

- uses NLO BFKL kernel
  [Fadin, Lipatov; PLB 429 (1998) 127]
  + resummation of collinear logarithms

- initial kT distribution from fit to combined HERA data



[H1 & ZEUS collab. 0911.0884]

$$F_2(x, Q^2) = \int_0^\infty d\boldsymbol{k}^2 \int_0^\infty \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \Phi_2\left(\frac{\boldsymbol{k}^2}{Q^2}\right) \mathcal{F}^{\text{DIS}}_{\text{BFKL}}(x, \boldsymbol{k}^2, \boldsymbol{q}^2) \Phi_p\left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)$$

Proton impact factor $\Phi_p\left(\frac{\boldsymbol{q}^2}{Q_0^2}, \delta\right) = \frac{\mathcal{C}}{\pi\Gamma(\delta)}\left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)^\delta e^{-\frac{\boldsymbol{q}^2}{Q_0^2}}$

Unintegrated gluon distribution

[Chachamis, M. Deak, MH, Rodrigo, Sabio Vera; 1507.05778],
[Bautista, Fernandez Tellez, MH; 1607.05203]

$$G(x, \boldsymbol{k}^2, Q_0^2) = \int \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \mathcal{F}^{\rm DIS}(x, \boldsymbol{k}^2, \boldsymbol{q}^2) \Phi_p \left(\frac{\boldsymbol{q}^2}{Q_0^2} \right)$$

$$G\left(x, \boldsymbol{k}^2, M\right) = \frac{1}{\boldsymbol{k}^2} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \; \hat{g}\left(x, \frac{M^2}{Q_0^2}, \frac{\overline{M}^2}{M^2}, \gamma\right) \; \left(\frac{\boldsymbol{k}^2}{Q_0^2}\right)^{\gamma}$$

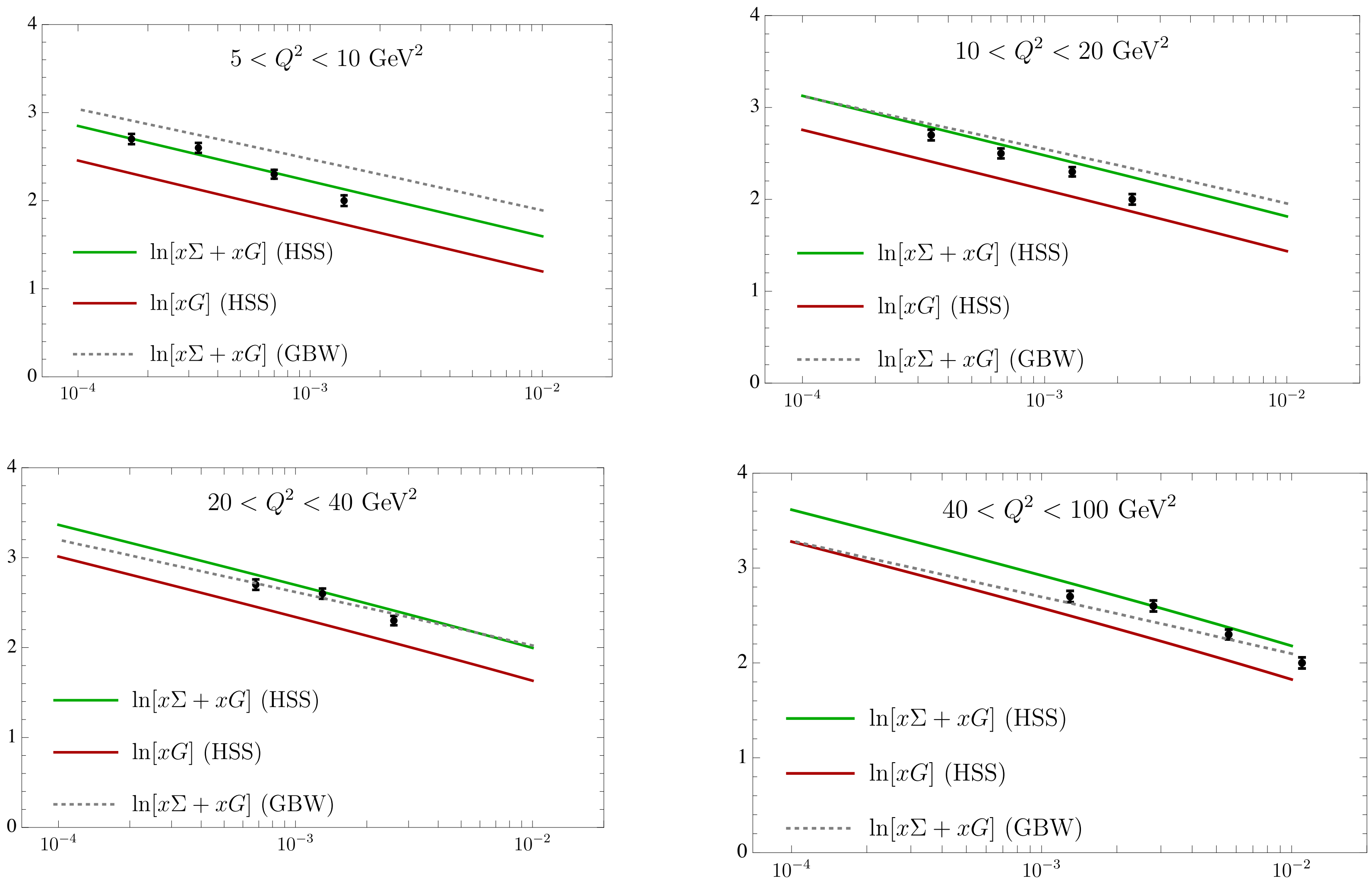
$$\hat{g}\left(x, \frac{M^2}{Q_0^2}, \frac{\overline{M}^2}{M^2}, \gamma\right) = \frac{\mathcal{C} \cdot \Gamma(\delta-\gamma)}{\pi \Gamma(\delta)} \cdot \left(\frac{1}{x}\right)^{\chi\left(\gamma, \frac{\overline{M}^2}{M^2}\right)} \cdot$$

$$\left\{1 + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0\left(\gamma\right)}{8 N_c} \log\left(\frac{1}{x}\right) \left[-\psi\left(\delta-\gamma\right) + \log\frac{M^2}{Q_0^2} - \partial_\gamma \right] \right\}$$

NLO BFKL kernel + collinear resummation + optimal scale setting

First results:

[MH, Kutak; 2110:06156]



- First attempt (inspired by [Kharzeev, Levin; 2102.09773]) only seaquark → not even close to data (now we know it was never meant to describe this data set, but this was our original idea …)
- Gluon only: gets closer
- If # of observed hadrons ≃ # of partons, why not use quarks + gluons? Turns out to work pretty well …

We also did some comparison with NNLO NNPDF and NNLO low x resumed NNPDF; please see [MH, Kutak; 2110:06156]

All good?

No, there's a bunch of mistakes

- us (me in this case) used a wrong normalization constant for HSS gluon → correct constant overshoots data [MH, Kutak; Eur.Phys.J.C 83 (2023) 12, 1147 (erratum)]
- H1 collaboration measures charged hadron multiplicity, yet we calculate entropy for all hadrons roughly related by a factor 2/3
- luckily (?) both effects cancel in magnitude approximately

Erroneous interpretation by H1

probably taken from [Tu, Kharzeev, Ullrich; 1904.11974]

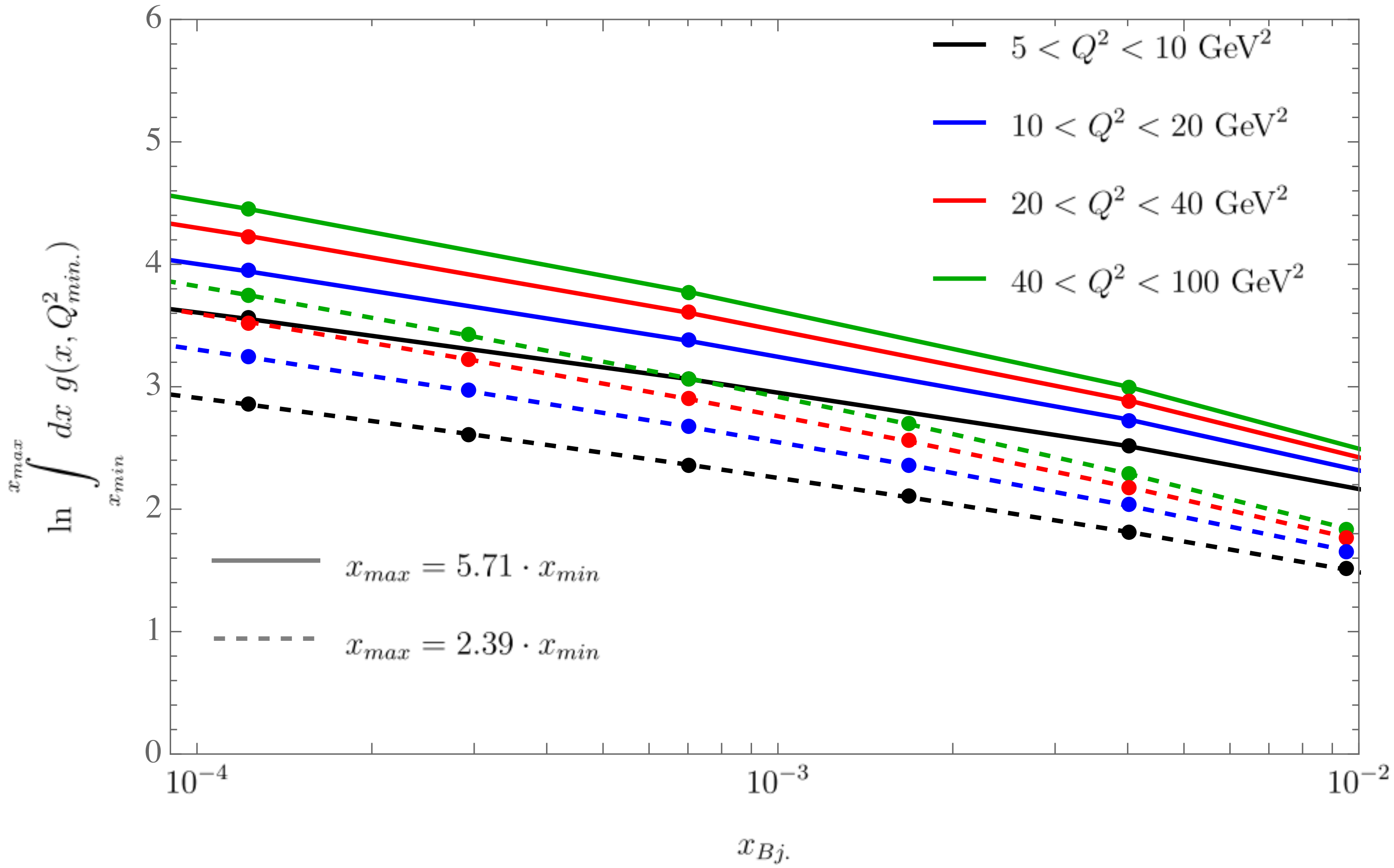
Integrate PDF (somehow) number of patrons

$$n_g(Q^2) = \int_0^1 dx\, g(x, Q^2),$$

H1: (seems) # of partons in a certain bin

$$n_g(\bar{x}) = \int_{x_{\text{min}}}^{x_{\text{max}}} dx g(x, Q^2),$$

Problem: depends obviously on bin size



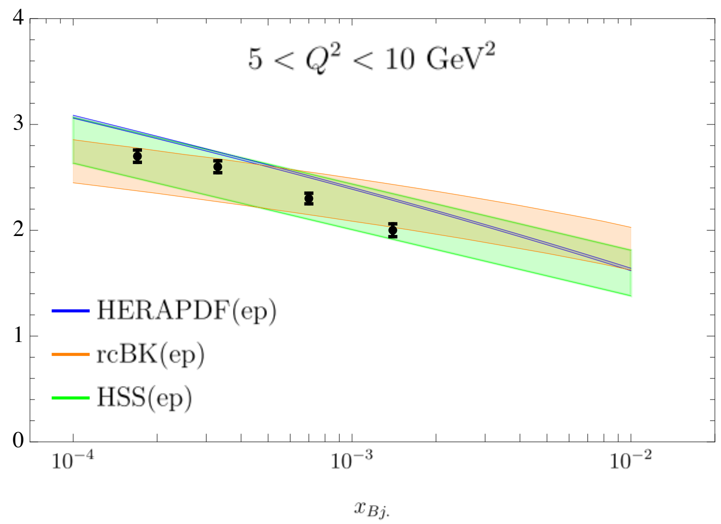
# of partons/bin size (and infinitesimal limit)

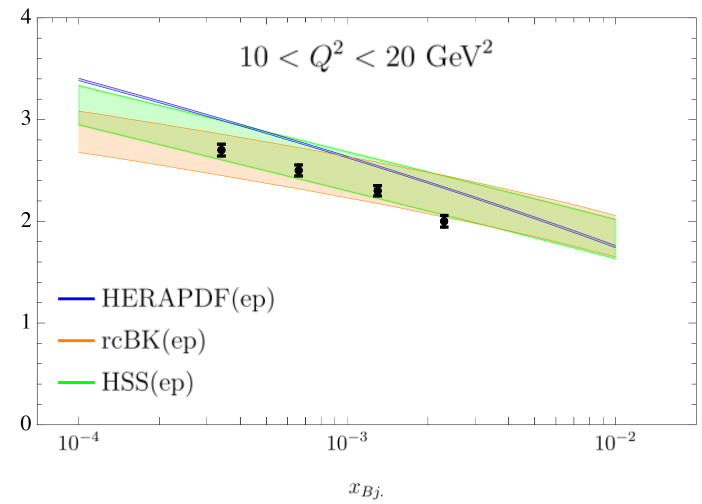
$$\bar{n}_g(x, Q^2) = \frac{dn_g}{d\,\ln(1/x)} = xg(x, Q^2)\,.$$

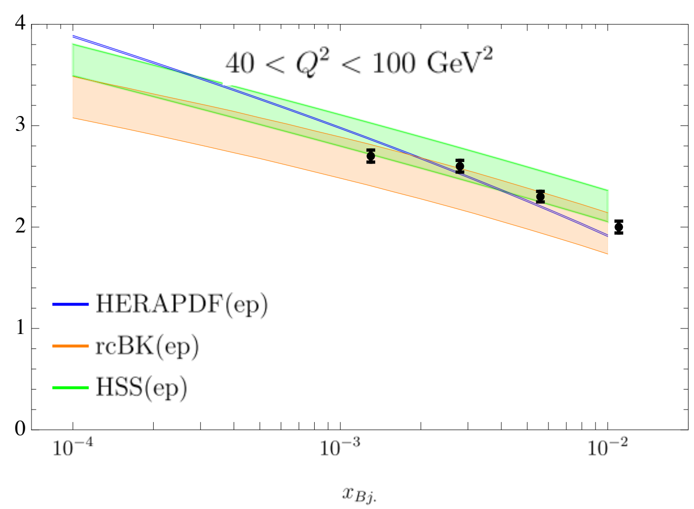
(was already like this in [Kharzeev, Levin; 1702.03489])

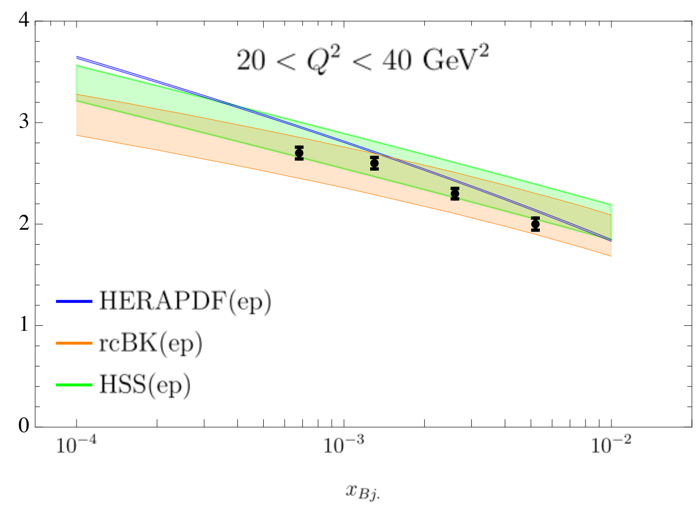
Updated plots

[MH, Kutak, Straka; 2207.09430]







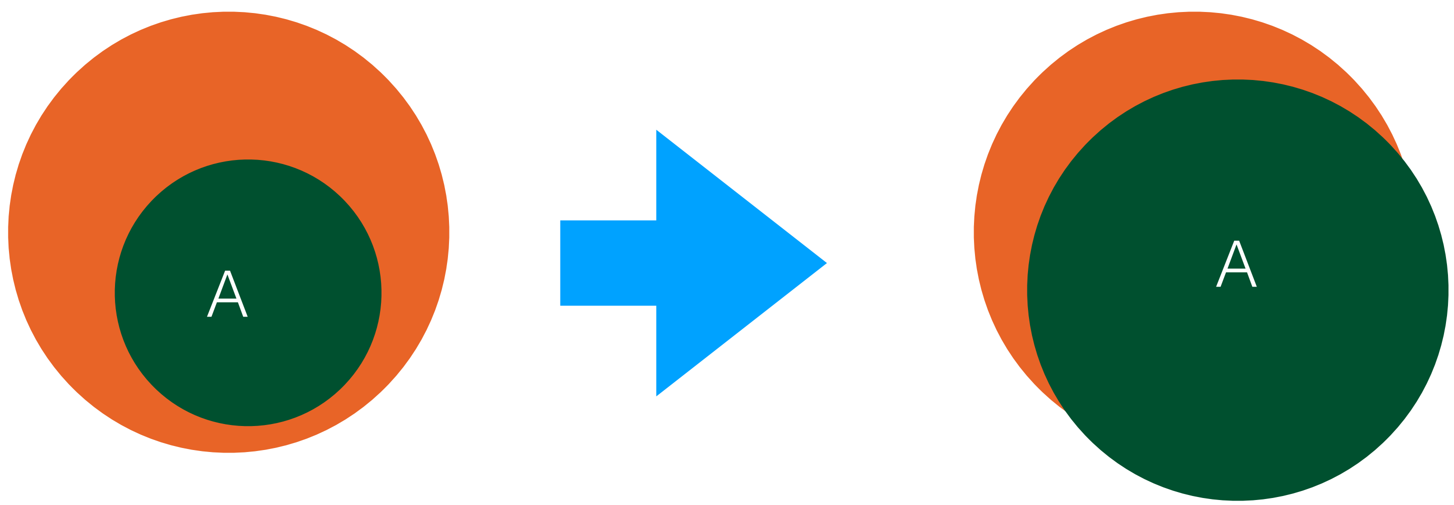


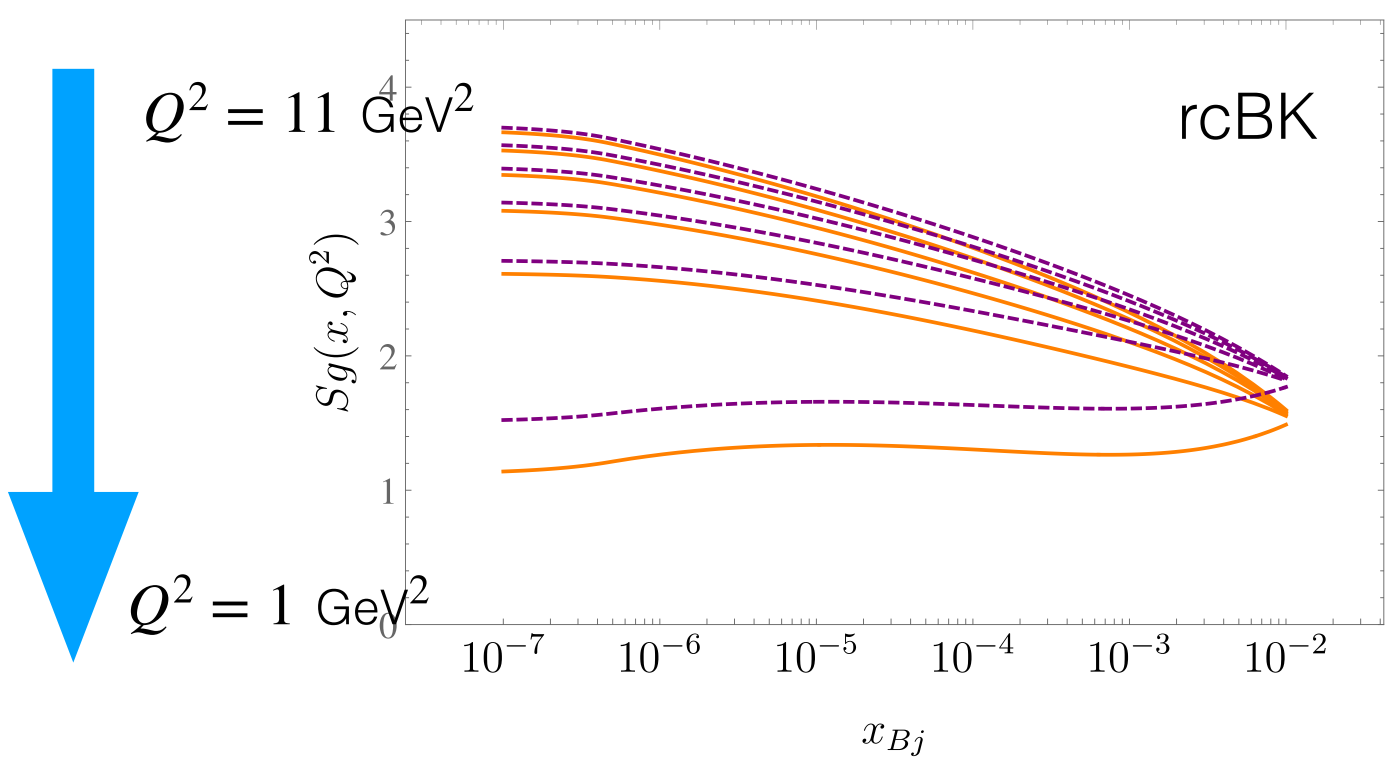
Include now LO HERAPDF (as H1 collab.)
+ corrected HSS description
+ parton distributions subject to non-linear Baltisky-Kovchegov (BK) evolution
+ estimate of uncertainty (HERAPDF only experimental uncertainty)

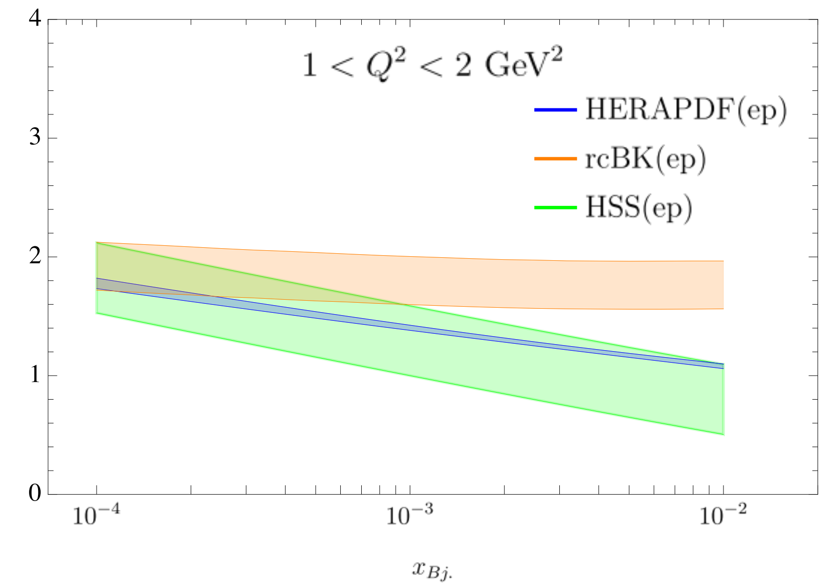
— all work pretty well!

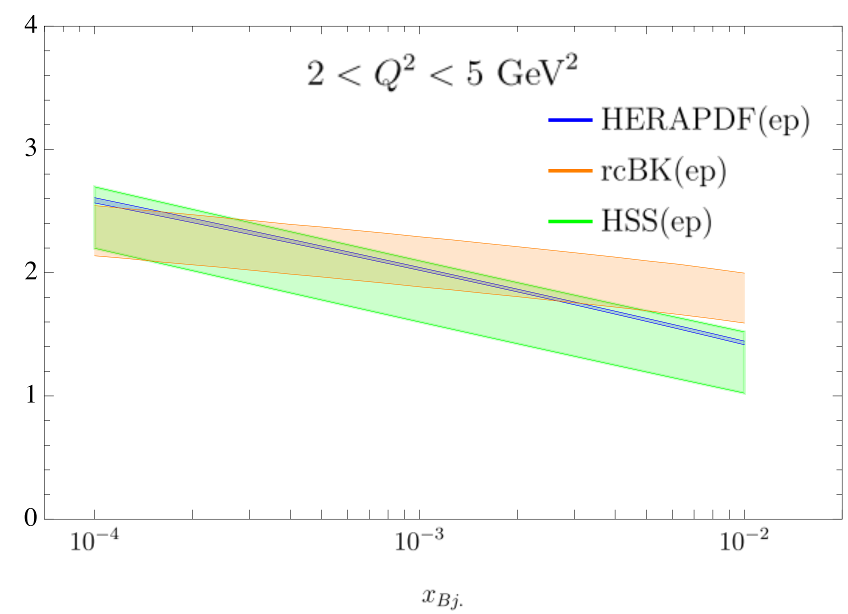
First steps towards the real photon limit

For $Q^2 \to 0$: observe entire proton









[MH, Kutak, Straka; 2207.09430]

[MH, Kharzeev, Kutak, Tu; 2305.03069]

Can we test this further?
Predict: DIS at low x probes a maximally entangled state with maximal entanglement entropy

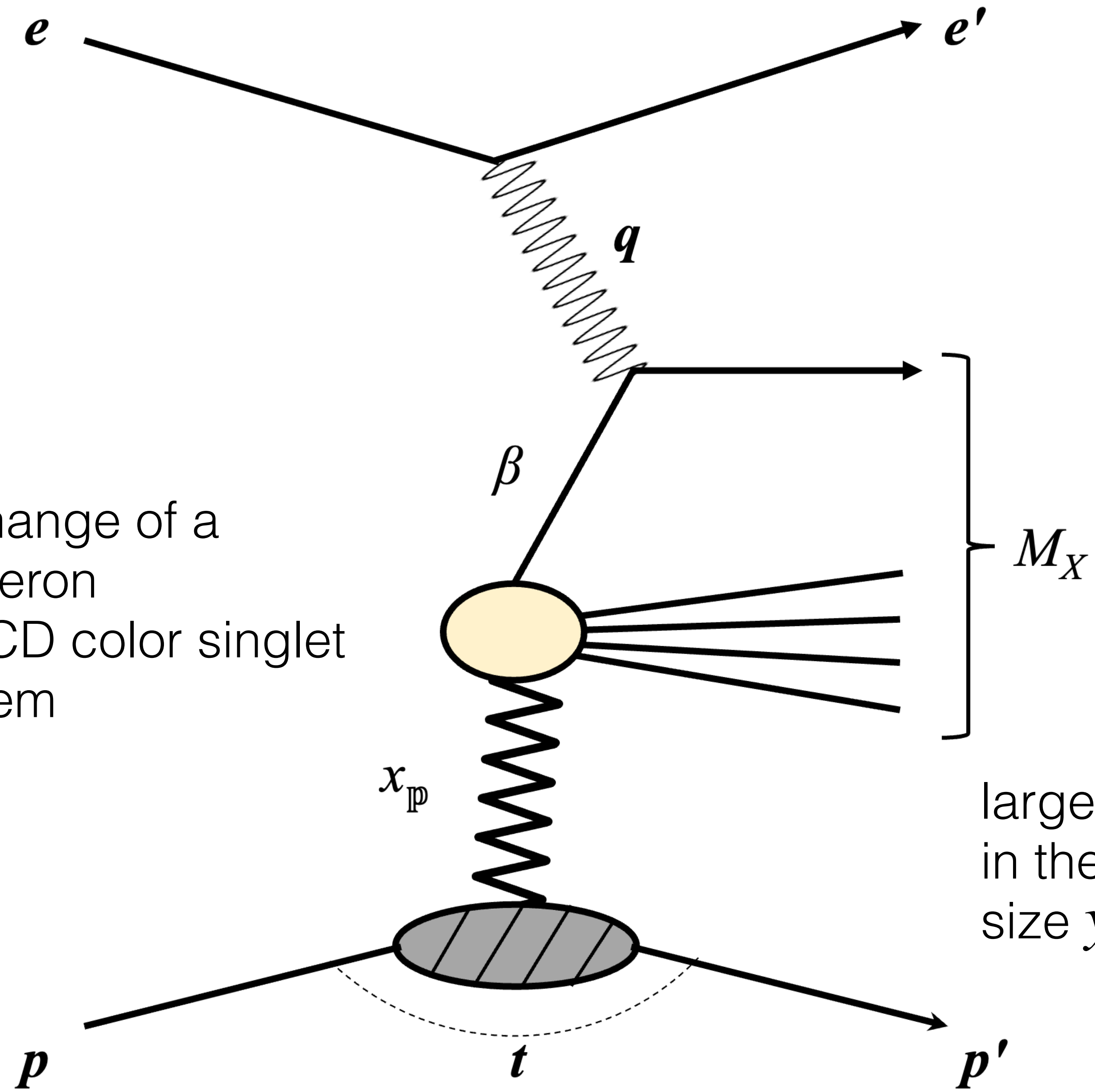
all probabilities for n-parton state are equal $p_n = 1/\langle n \rangle$
homogenous distribution

within the model: this is reached for $x \to 0$

What about reactions where entanglement entropy is **not** maximal where the distribution is **not** homogenous?

A candidate: diffractive DIS

all kinematic relation for collinear kinematics



exchange of a Pomeron = QCD color singlet system

the diffractive system of the dissociated photon
$y_X = Y - y_0 \simeq \ln 1/\beta$

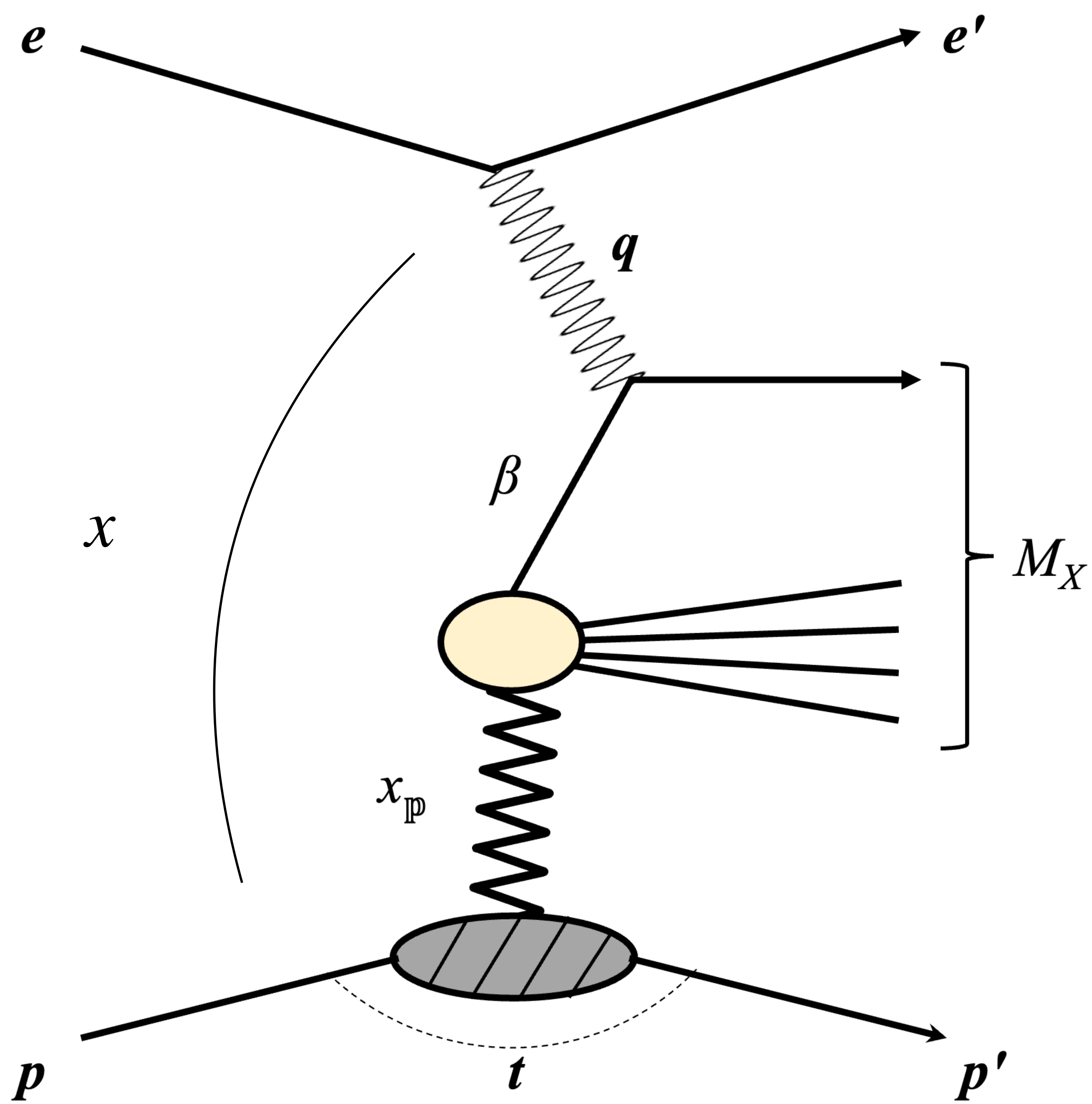
"β" plays the role of inclusive "x" for the diffractive system

large rapidity gap = empty region in the detector rapidity size $y_0 = \ln 1/x_{\mathbb{P}}$

proton scatters elastically or dissociates

- roughly 10-15% of the total HERA cross-section
- described by low x models/evolution & diffractive PDFs

Expectations:



- expect to probe different components of the proton wave function → photon interact with the Pomeron
- for given $Y = \ln 1/x$, the onset of the configuration with maximal entanglement entropy is delayed due to the gap
- Pomeron = QCD color singlet system, grows like $\sim (1/x_{\mathbb{P}})^{\lambda_P}$
- Pomeron = source for partons in diffractive system M_X → size depends on $x_{\mathbb{P}}$

+ there exists data from HERA run 1 (in KNO form), which allow to extract the charged hadron multiplicity distribution

[H1 collab.; hep- ex/9804012]

Theory input: diffractive PDFs

leading order PDFs appear to be preferable (in particular if fitted to the same data set)

- scale uncertainty controlled due to using same data set
- LO DGLAP evolution is scheme independent
- problem: hard to find LO PDFs …

Here: use leading order GKG18-DPDFs
(provided with the code for NLO DPDFs)

[Goharipour, Khanpour, Guzey; 1802.01363]

for details:

[MH, Kharzeev, Kutak, Tu; 2305.03069]

number of partons:

$$\left\langle \frac{dn(\beta)}{d\ln 1/\beta} \right\rangle = \frac{1}{Q^2_{\rm max} - Q^2_{\rm min}} \int_{Q^2_{\rm min}}^{Q^2_{max}} dQ^2 \int_{x_{\mathbb{P},\rm min}}^{x_{\mathbb{P},\rm max}} dx_{\mathbb{P}} \beta \left[f^D_{\Sigma/p}\left(\beta, x_{\mathbb{P}}, Q^2\right) + f^D_{g/p}\left(\beta, x_{\mathbb{P}}, Q^2\right)\right]$$

quark flavor singlet
(sum over all quarks)

gluon

$$Q^2_{\rm min} = 7.5 \text{ GeV}^2, \quad Q^2_{\rm max} = 100 \text{ GeV}^2$$

$$x_{\mathbb{P},min} = 0.0003, \quad x_{\mathbb{P},min} = 0.05$$

reproduce phase space of HERA data (rather inclusive)

A modified description

if we plug $\langle dn/d\beta \rangle$ directly into the entropy formula, the description fails

reason: H1 data at $\beta = 0.05 - 0.5$
In this region, $\langle dn/d\beta \rangle < 1$
leads to probabilities > 1 and the entire setup fails

solution: return to the original model, but introduce additional constant C

allows to include contribution of Pomeron

$$p_n^D(y_X) = \frac{1}{C} e^{-\Delta y_X} \left(1 - \frac{1}{C} e^{-\Delta y_X}\right)^{n-1}$$

average # of dipoles

$$\left\langle \frac{dn(\beta)}{d\ln 1/\beta} \right\rangle = \sum_n n p_n^D(y_X) = C \left(\frac{1}{\beta}\right)^{\Delta}$$

note:

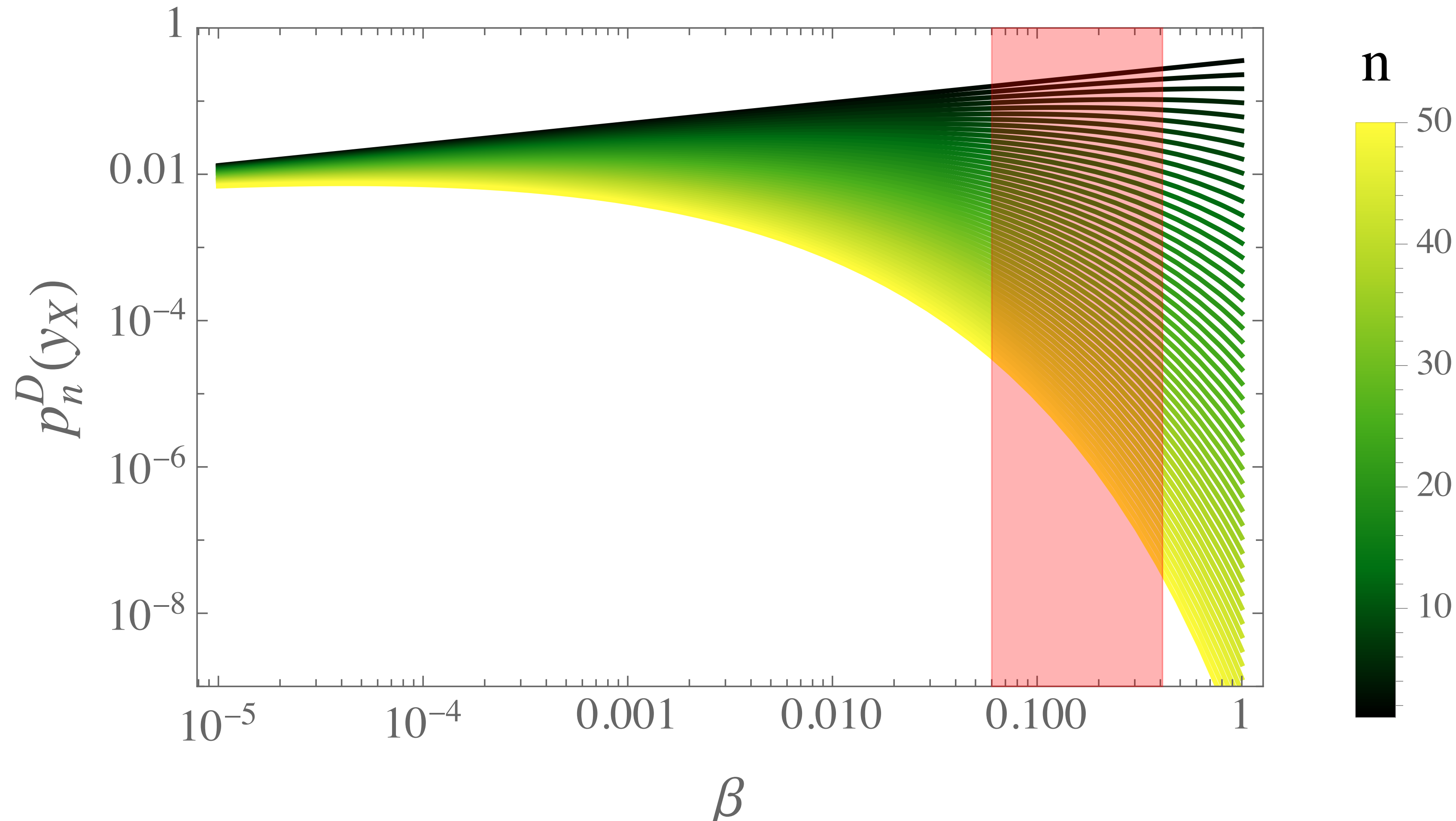
$$p_{n>1}(y_X = 0) \neq 0$$

justification: Pomeron = source for several dipoles at $y_X = 0$

The probability distribution

parameters from fit to $\langle dn/d\beta \rangle$ at $\beta \in [10^{-5}, 10^{-4}]$ → power like growth $\beta^{-\Delta}$ of partons

also: rescale $C \to C' = 2/3C$ (charged hadrons only)



homogenous distribution
maximal entanglement entropy

pre-asymptotic region
probed by H1 data

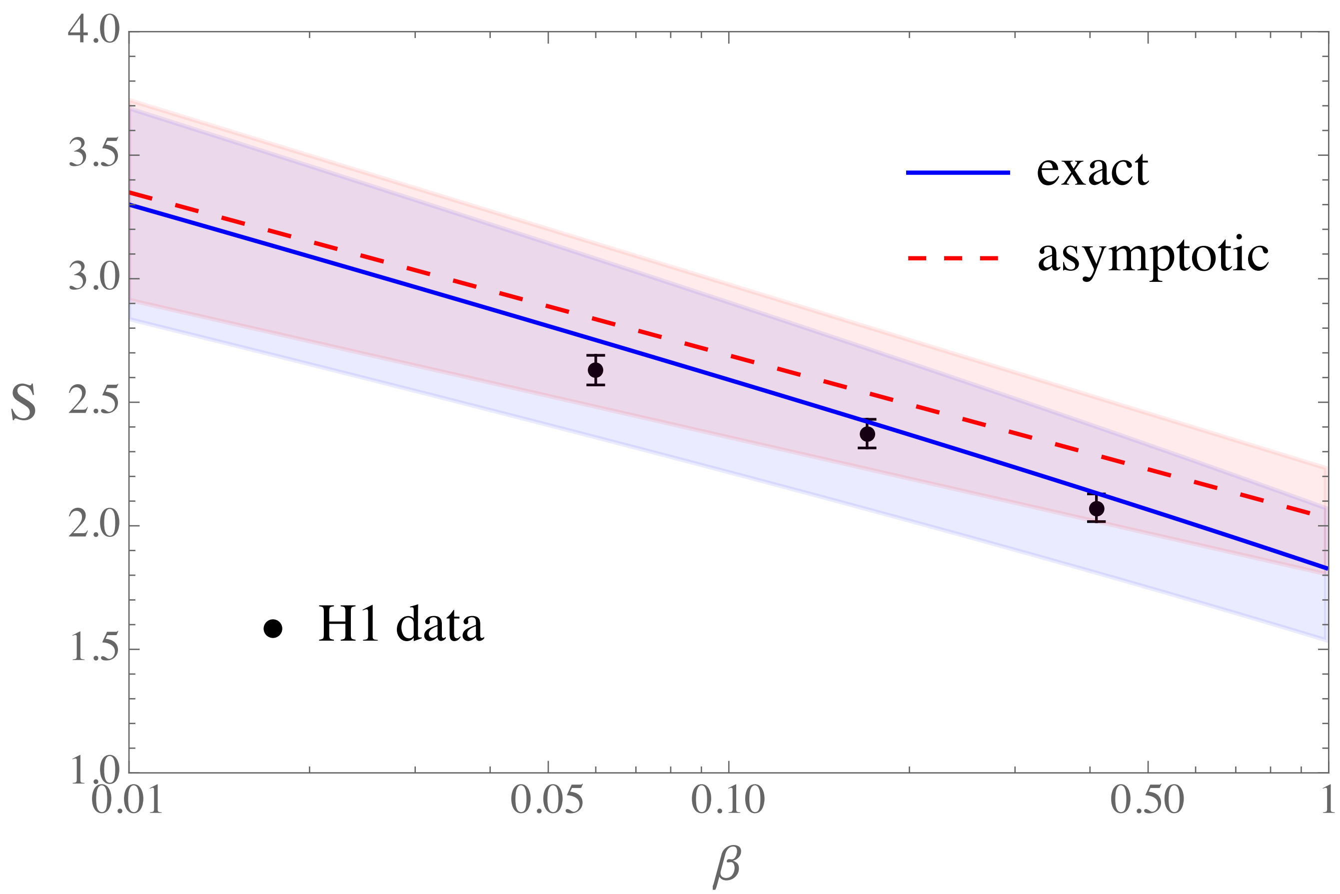
Comparison with data

use two expression for the comparison with data

exact: $$S = -\sum_n p_n \ln p_n = (1-Z)\ln\frac{Z-1}{z} + \ln Z$$

$$Z = C' e^{\Delta y_X}$$

asymptotic: $$S \simeq \ln Z + 1$$



- uncertainty = PDF uncertainty + scale uncertainty + variation in the region where parameters were fitted
- data prefer exact over asymptotic, but both are consistent with data

unpublished: a similar setup can be used for inclusive data

Discussion

quantitative description of H1 data by diffractive entanglement entropy model a coincide

cannot be excluded with certainty, but we don't think so

Quarks and gluons inside the proton are strongly entangled

entanglement entropy at initial stage of reaction
reason: partial measurement of the hadronic density matrix.

allows, at least in principle, to directly relate PDFs and final-state hadron production without the use of fragmentation functions (FFs) or other fragmentation frameworks, such as the Lund string model (used in e.g Pythia). [Sjostrand, Mrenna, Skands; hep-ph/0603175]

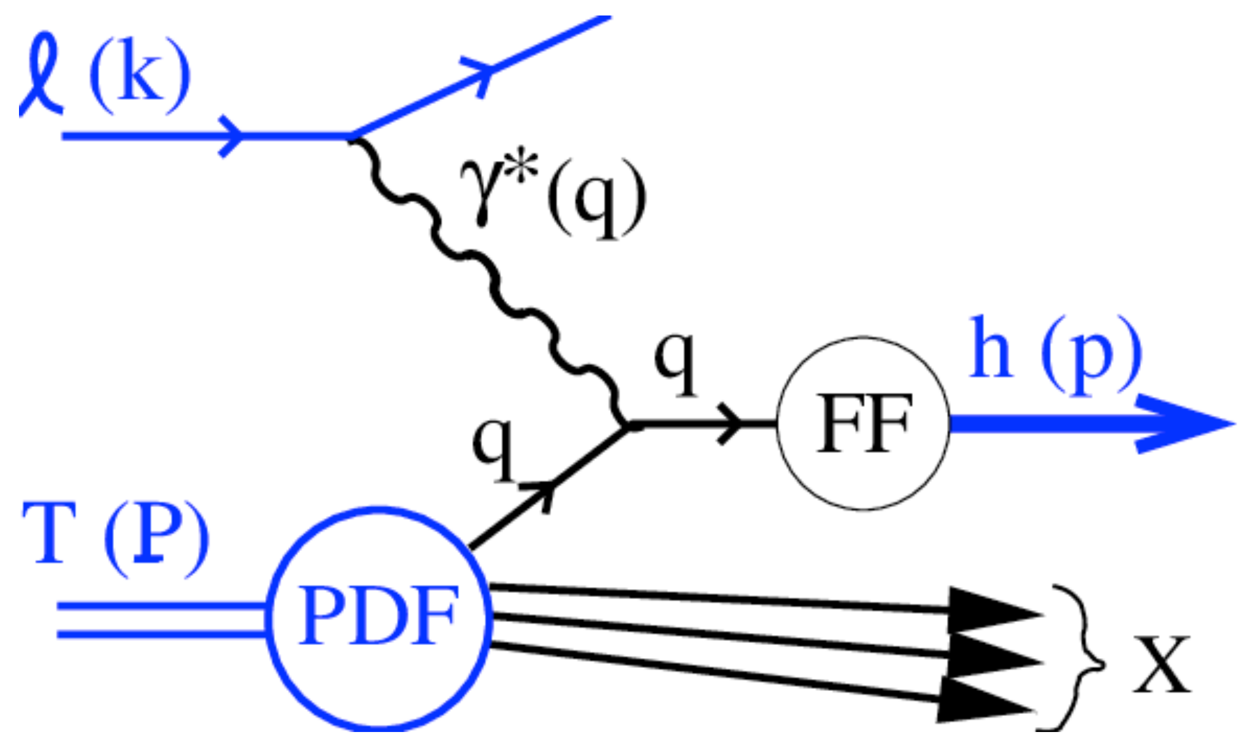
Conventional description

inclusive hadron production in hard reactions requires fragmentation functions (FF), based on factorization theorems

[Collins, Soper, Sterman; hep-ph/0409313], [Bjorken, Paschos; 1969]

$$\begin{aligned}\sigma(e^+e^- \to hX) &= \hat{\sigma} \otimes FF,\\ \sigma(l^\pm N \to hX) &= \hat{\sigma} \otimes PDF \otimes FF,\\ \sigma(p_1 p_2 \to hX) &= \hat{\sigma} \otimes PDF_1 \otimes PDF_2 \otimes FF.\end{aligned}$$

FF = non-perturbative input fitted to data + DGLAP evolution

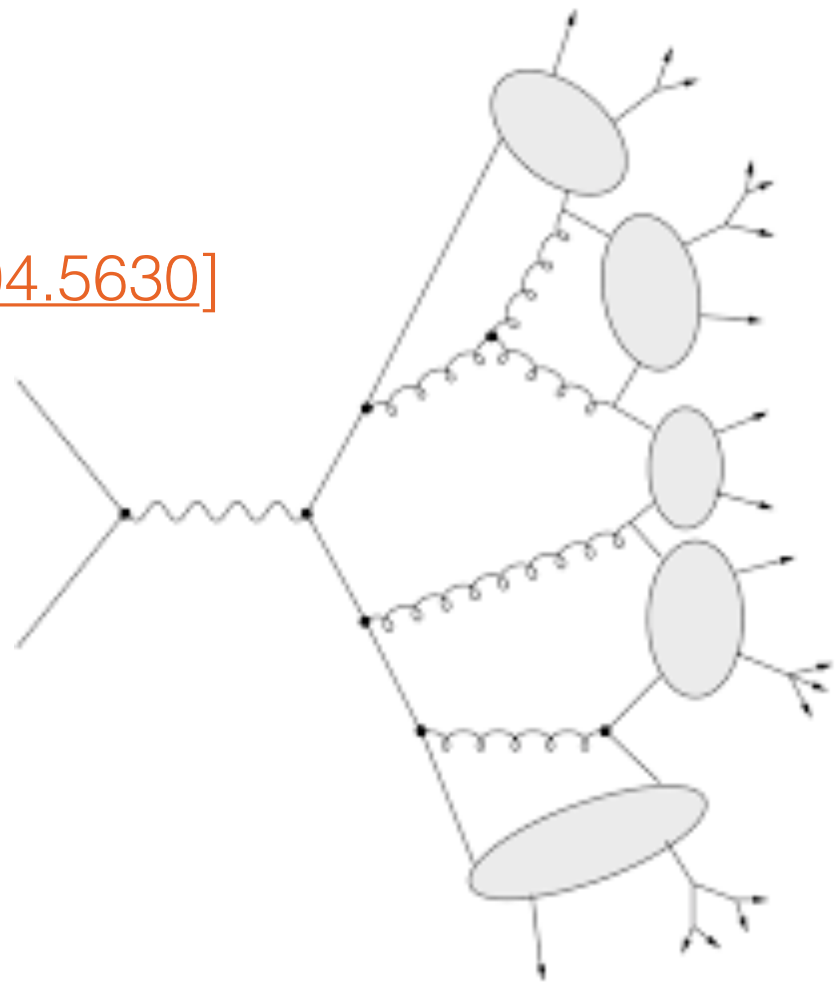


source: hep-ph/0311279

Multiple Hadron Production

- semi-classical models like the Lund string fragmentation model
- not trivial to describe charged hadron multiplicities without significant tuning

e.g. [Skands, Carazza, Rojo; 1404.5630]



In both approaches, no direct relation between parton distribution function and the measured hadron multiplicity

Entanglement entropy

conjecture that entropy of the charged hadron multiplicity is fixed at the initial stages of the collision formulated in [Kharzeev, Levin; 1702.03489]

first experimental test using LHC data and Monte Carlo data

[Tu, Kharzeev, Ullrich; 1904.11974]

DIS data, without ambiguity of initial state hadron

[H1 collaboration; 2011.01812]

Description of inclusive data [MH, Kutak; 2110:06156]

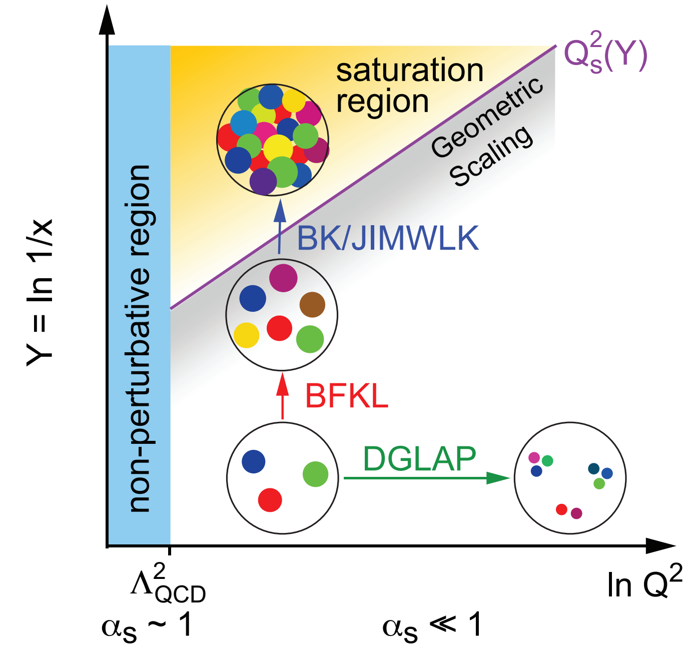
[MH, Kutak, Straka; 2207.09430]

Now: again confirmed in diffractive reactions

[MH, Kharzeev, Kutak, Tu; 2305.03069]

Coincidence cannot be excluded so far, but unlikely

Outlook



- low x drives us into a overoccupied and saturated system of gluons ↔ quantum

  bounds on entropy, Bekenstein bound etc.?

- what happens in the non-perturbative e.g. photo production limit; can one also explore this in UPCs?

- first principle, more field theoretic treatment desirable

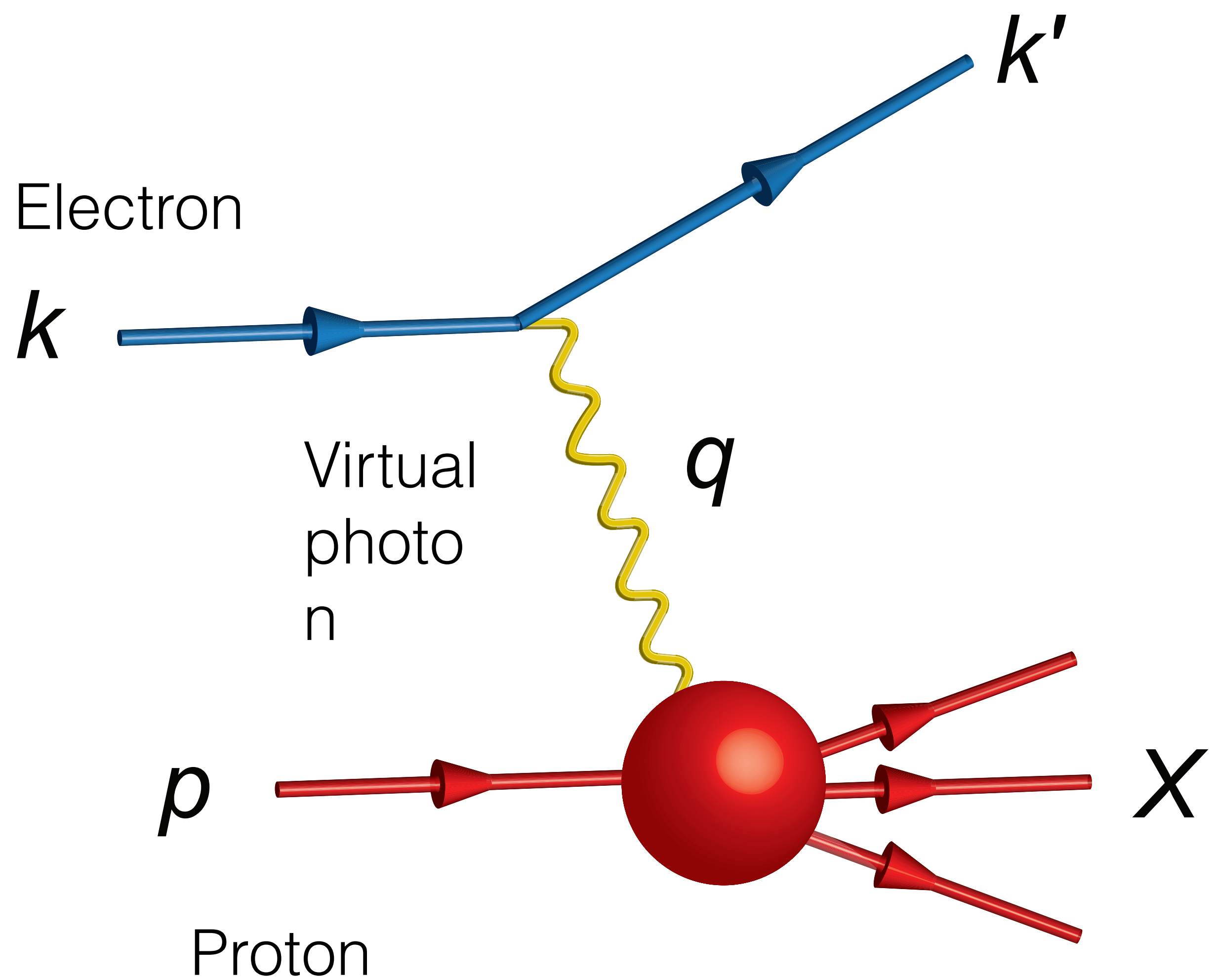
in general: some considerable activity in this direction
but: it's not easy; still time of models, approximations, simplified (conformal etc.) theories

but can provide relevant input

for some attempts to understand things in the context of the BFKL Green's function see [Chachamis, MH, Sabio Vera; 2312.16743]

Appendix

Deep Inelastic electron-proton Scattering (DIS)



Photon virtuality (=resolution)

$$Q^2 = -q^2, \quad \lambda \sim \frac{1}{Q}$$

- Idea: resolve an area of size $A \sim 1/Q^2$
- Remaining region B: unobserved sum/trace over this unobserved ration
- Overall color singlet → expect proton wave function, which entangles both regions

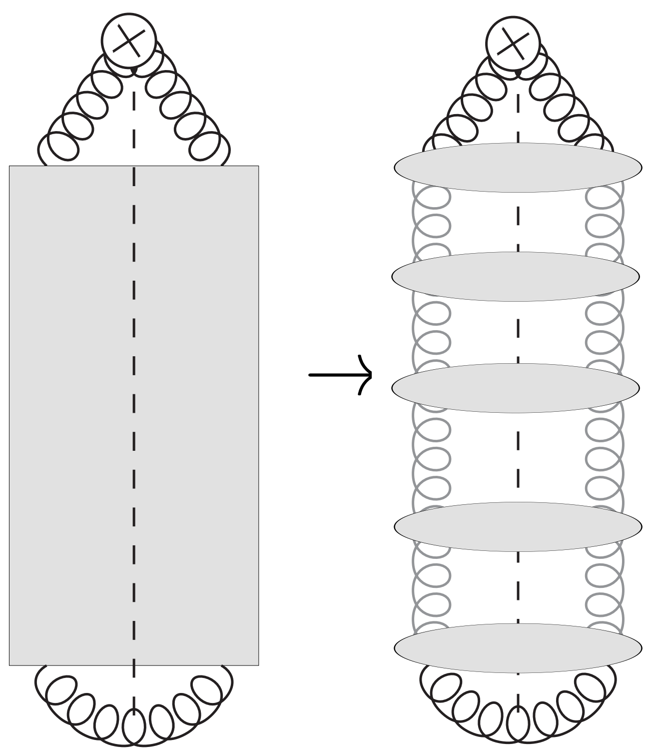
Demonstrating this, is a challenge …

- Pure state at $Q^2 \to 0$ = observe entire proton
- But this is the region, where $\alpha_s(Q)$ is not small ≠ perturbation theory; concept of quarks and gluons as degrees of freedom at least difficult
- Unobserved region subject to non-perturbative dynamics

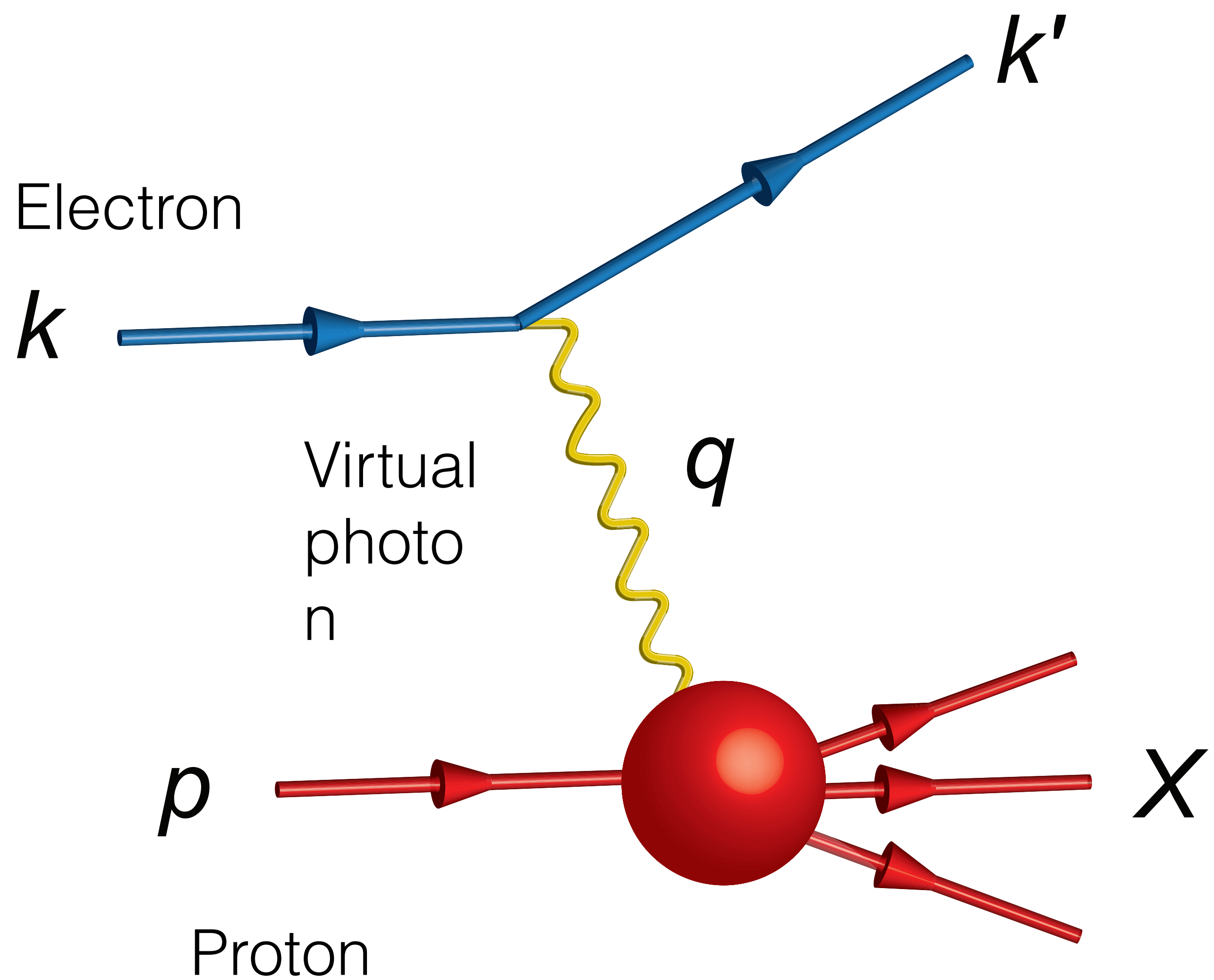
Our approach: PDF from unintegrated gluon

[Catani, Hautmann, NPB 427 (1994) 475]:
idea: use collinear factorization in light-cone gauge
[Curci, Furmanski, Petronzio; NPB 175 (1980) 27]
→ calculate all order low x resumed DGLAP splitting functions

- Yields Transverse Momentum splitting function for gluon - quark splitting
- Splitting = collinear PDF with partonic initial state
- Can calculate PDFs from unintegrated gluon distribution, subject to $\ln(1/x)$ evolution
  see also [Hautmann, MH, Jung; 1205.1759]



Before we proceed: different DIS evolutions



Photon virtuality (=resolution)

$Q^2 = -q^2, \quad \lambda \sim \frac{1}{Q}$

Bjorken x

$x_{Bj.} = \frac{Q^2}{2p \cdot q}$

“Mass" of the system X

$W^2 = (p+q)^2 = M_p^2 + \frac{1-x}{x} Q^2$

Elastic scattering: either $Q = 0$ or $x = 1$

A different picture

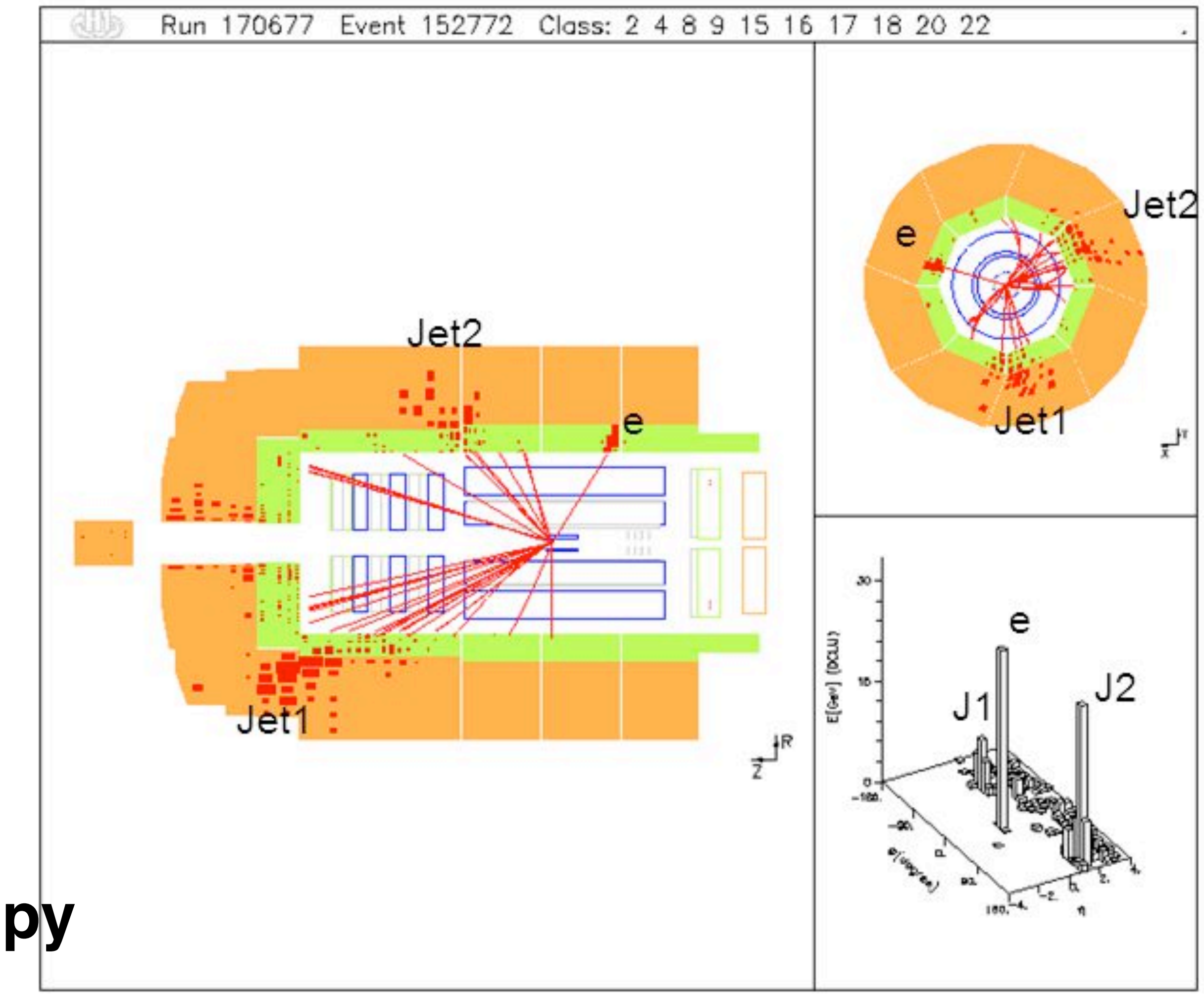
- Production of certain # of particles in DIS → non-zero entropy

- Von Neumann entropy of a proton (=pure quantum state) = 0

- Obviously we're missing something …

(Possible) answer: **entanglement entropy**

A NC-DIS event with two jets $ep \rightarrow e' Jet_1 Jet_2$



H1 Events Joachim Meyer DESY 2005

Entropy as expectation value of information

Expectation value of some function $f(p)$

$$\langle f(p) \rangle = \sum_i p_i f(p_i)$$

For information: $\langle h(p) \rangle = \sum_i p_i h(p_i) = -\sum_i p_i \ln p_i = S$ = entropy

Why entropy?

- microcanonical ensemble: $p_i = \dfrac{1}{\Omega(E)}, \quad \Omega(E)$ = # of states with energy E, obtain $S = \ln \Omega$
- Same for canonical ensemble with $p_i = \dfrac{e^{-E_i/(k_B T)}}{Z}$ etc.

The probed region

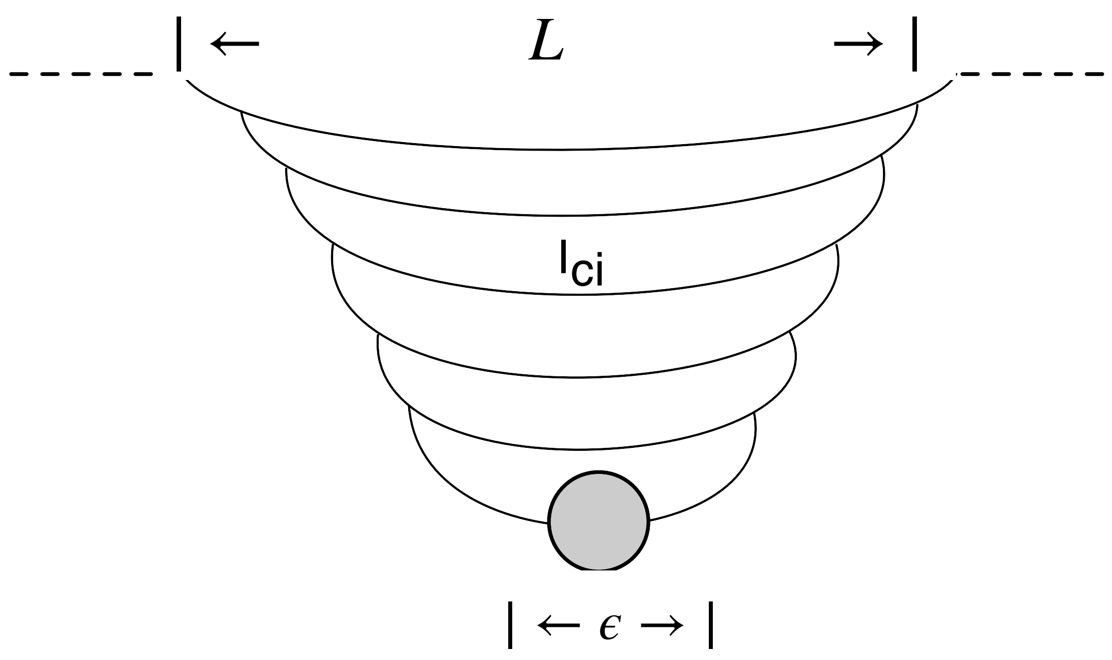


Figure taken from [Kharzeev, Levin; 1702.03489]

In the proton **rest frame**:

- parton (of the the photon) fluctuation over long. distance $L = \dfrac{1}{m_p x}$
- Proton probes partonic fluctuation with resolution $\epsilon = \dfrac{1}{m} \ll L = \dfrac{1}{x}\epsilon$
- Proton probes only region $\epsilon \ll L$ of the entire interaction