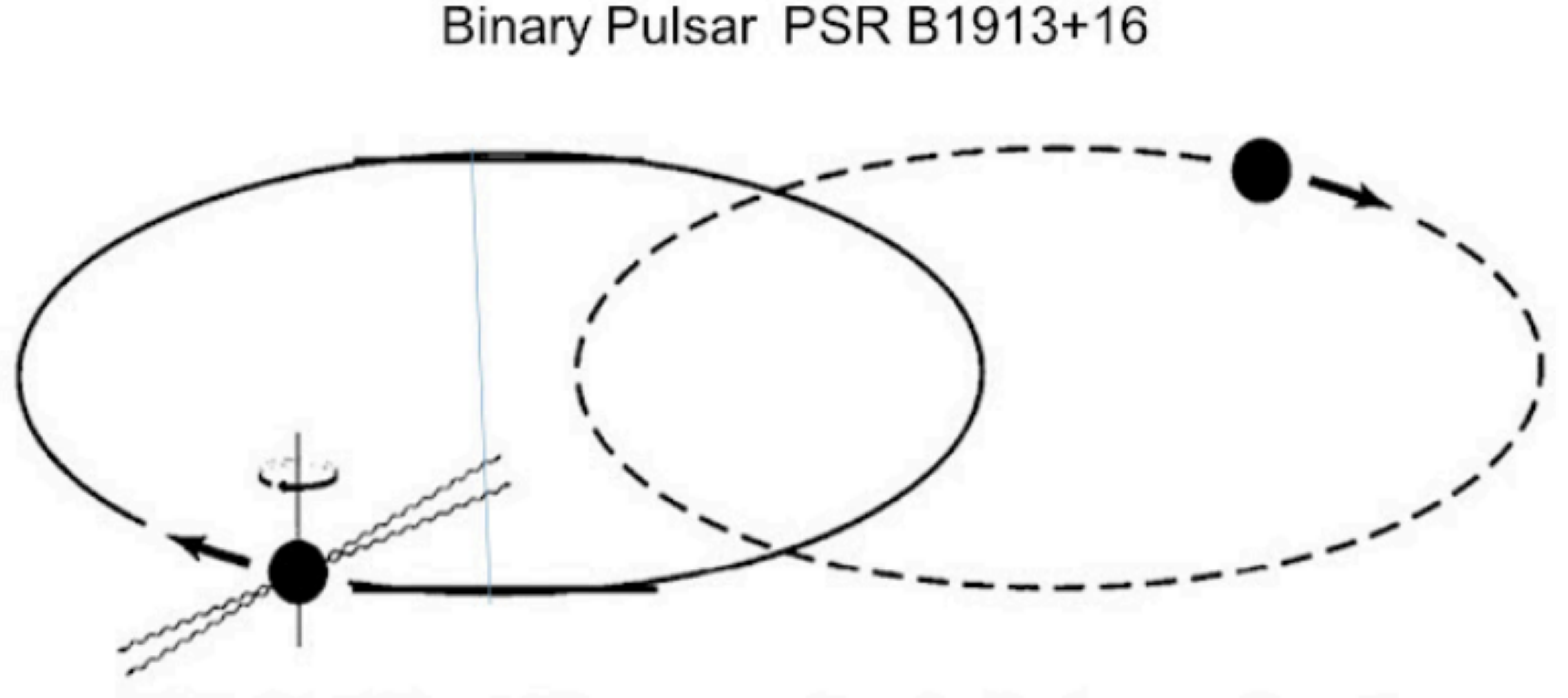
Looking for gravitons in Gravitational Waves

Subhendra Mohanty
IIT Kanpur

Hulse Taylor binary



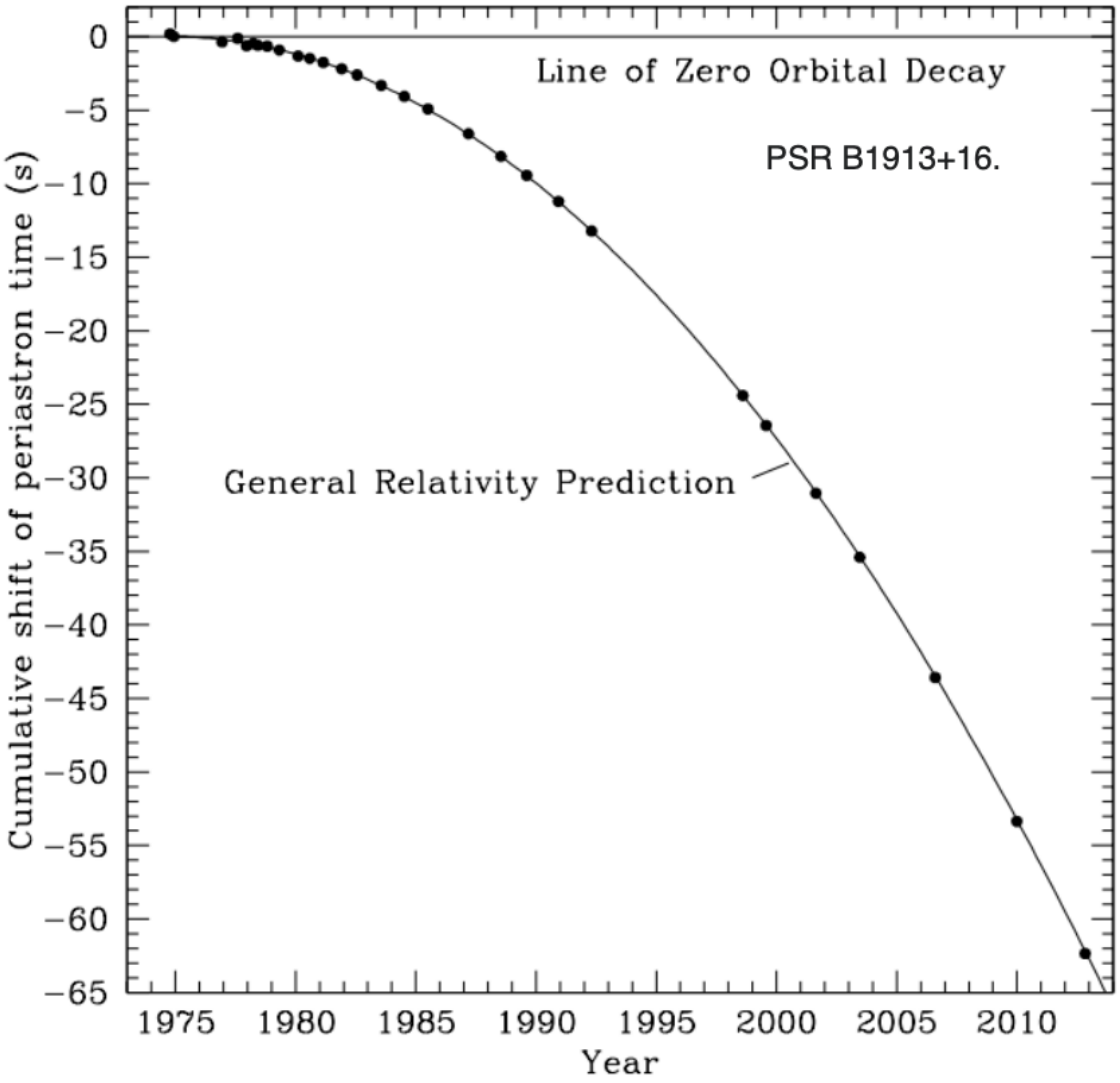
PSR 13 +16 orbiting around a similarly massive star
(picture from the Nobel Lecture – Russell Hulse 1993)

Energy loss by gravitational wave radiation in compact binaries

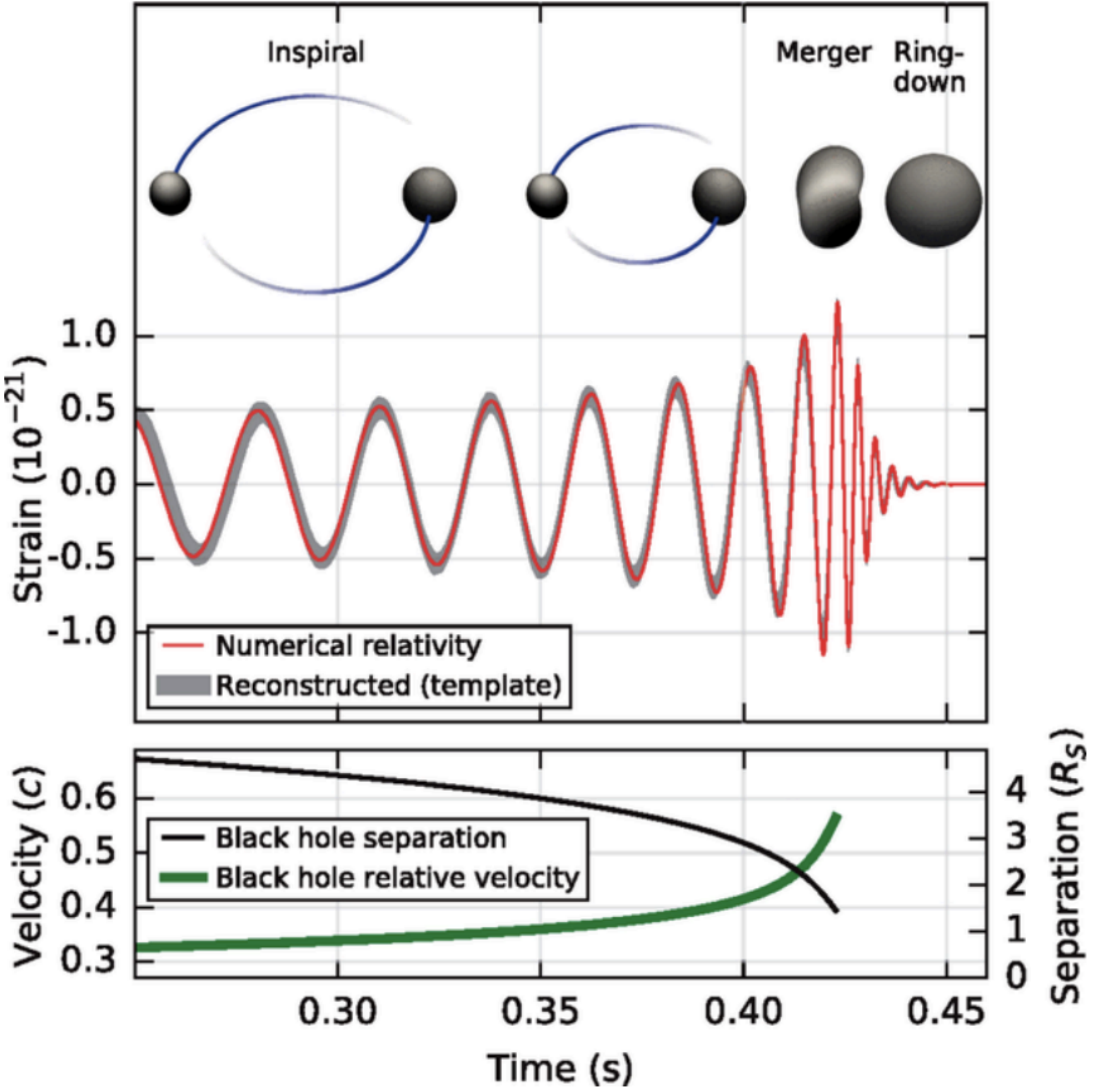
$$\left(\frac{\mathrm{d}E}{\mathrm{d}T}\right)^{\mathrm{GW}} = \frac{32G}{5}\Omega^6\left(\frac{m_1 m_2}{m_1+m_2}\right)^2 a^4(1-e^2)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

P.C Peters and J. Mathews (1963)

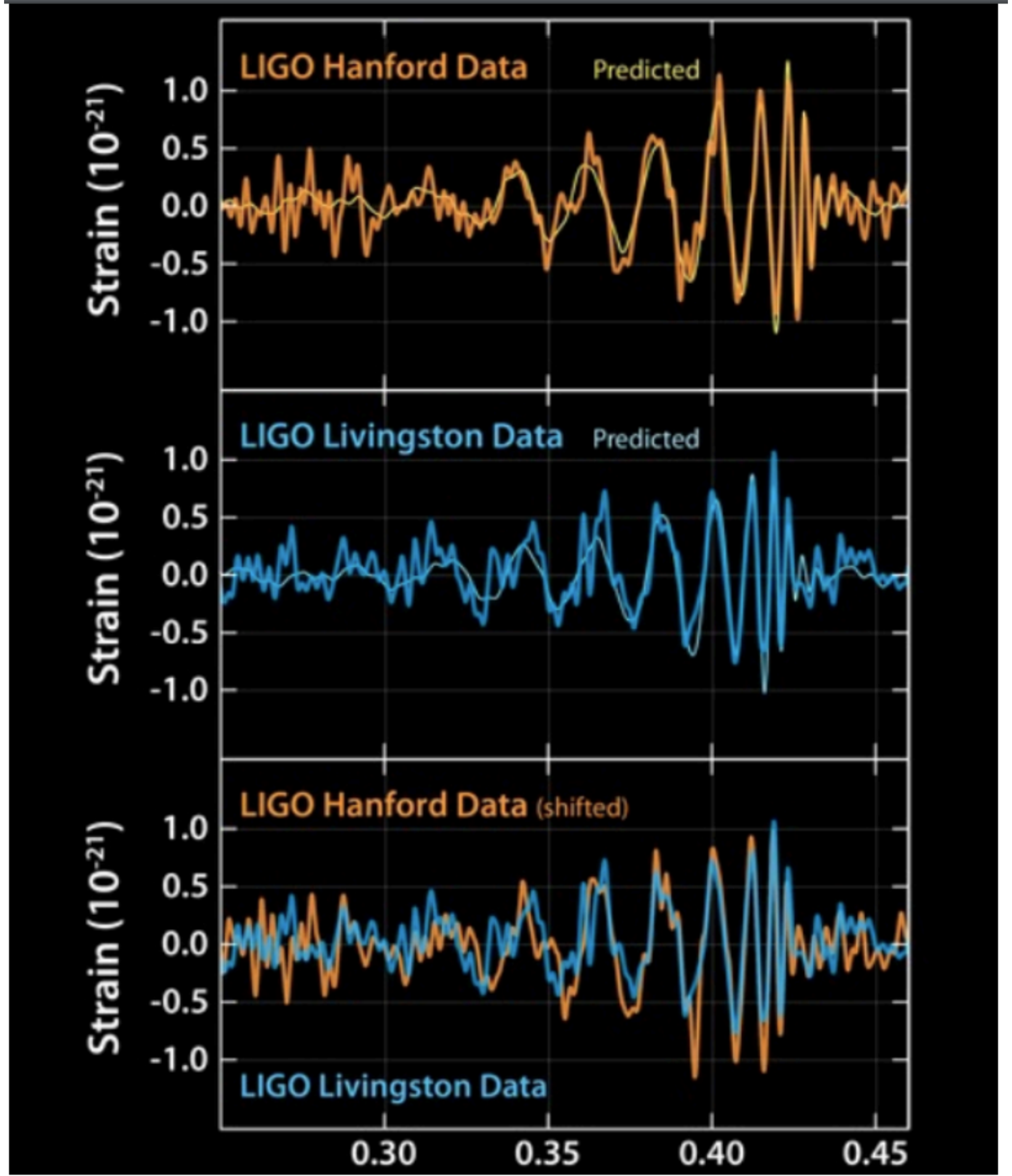
Time period loss of binary pulsars by gravitational wave radiation



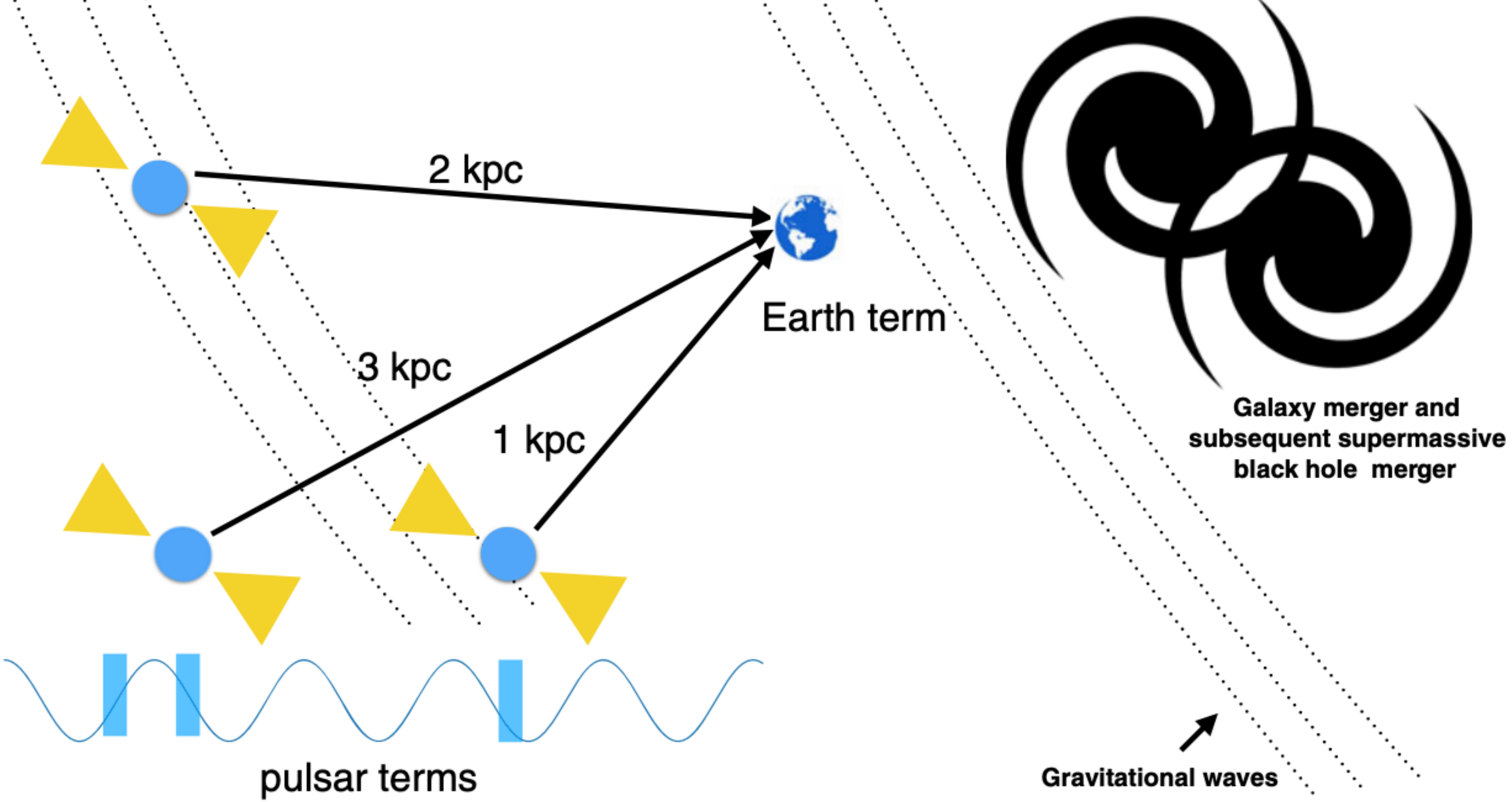
Black-hole mergers



LIGO Event



LIGO Hanford Data
Predicted
Strain (10^-21)
1.0
0.5
0.0
-0.5
-1.0
LIGO Livingston Data
Predicted
Strain (10^-21)
1.0
0.5
0.0
-0.5
-1.0
LIGO Hanford Data (shifted)
Strain (10^-21)
1.0
0.5
0.0
-0.5
-1.0
LIGO Livingston Data
0.30
0.35
0.40
0.45



s. The astrophysics of stochastic gravitational waves, Burke-Spolaor et al.

Modulation of pulsar time period by gravitational waves

$$z_a(t) \equiv \frac{\Delta T_a}{T_a} = \frac{n_a^i n_b^j}{2(1+\mathbf{n}\cdot \mathbf{n}_a)} \left\{ h_{ij}^{TT}[t, \mathbf{x}=\mathbf{0}] - h_{ij}^{TT}[t-\tau_a, \mathbf{x}_a] \right\}$$

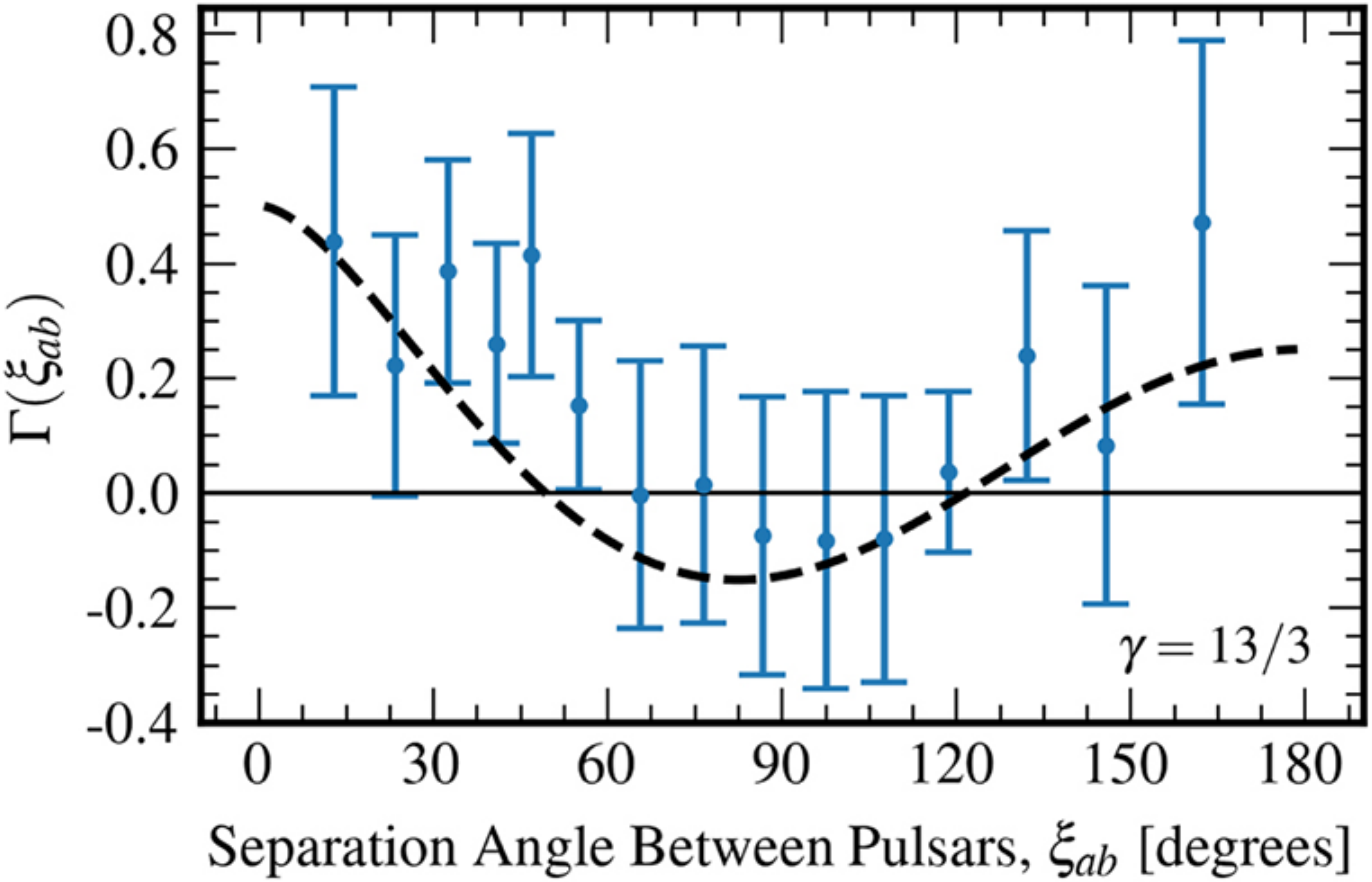
Angular correlation between signals from different pulsars

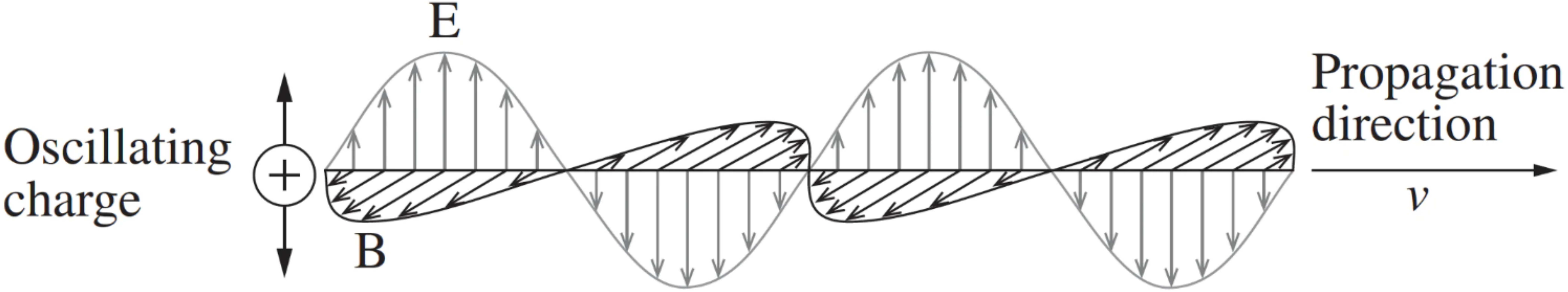
$$\langle z_a(t) z_b(t) \rangle = C(\theta_{ab}) \int_0^\infty df\, S_h(f),$$

Hellings-Downs curve

$$C(\theta_{ab}) = x_{ab} \ln x_{ab} - \frac{1}{6} x_{ab} + \frac{1}{3} \qquad x_{ab} \equiv \sin^2 \frac{\theta_{ab}}{2}$$

NANOGrav observations- Hellings Downs curve

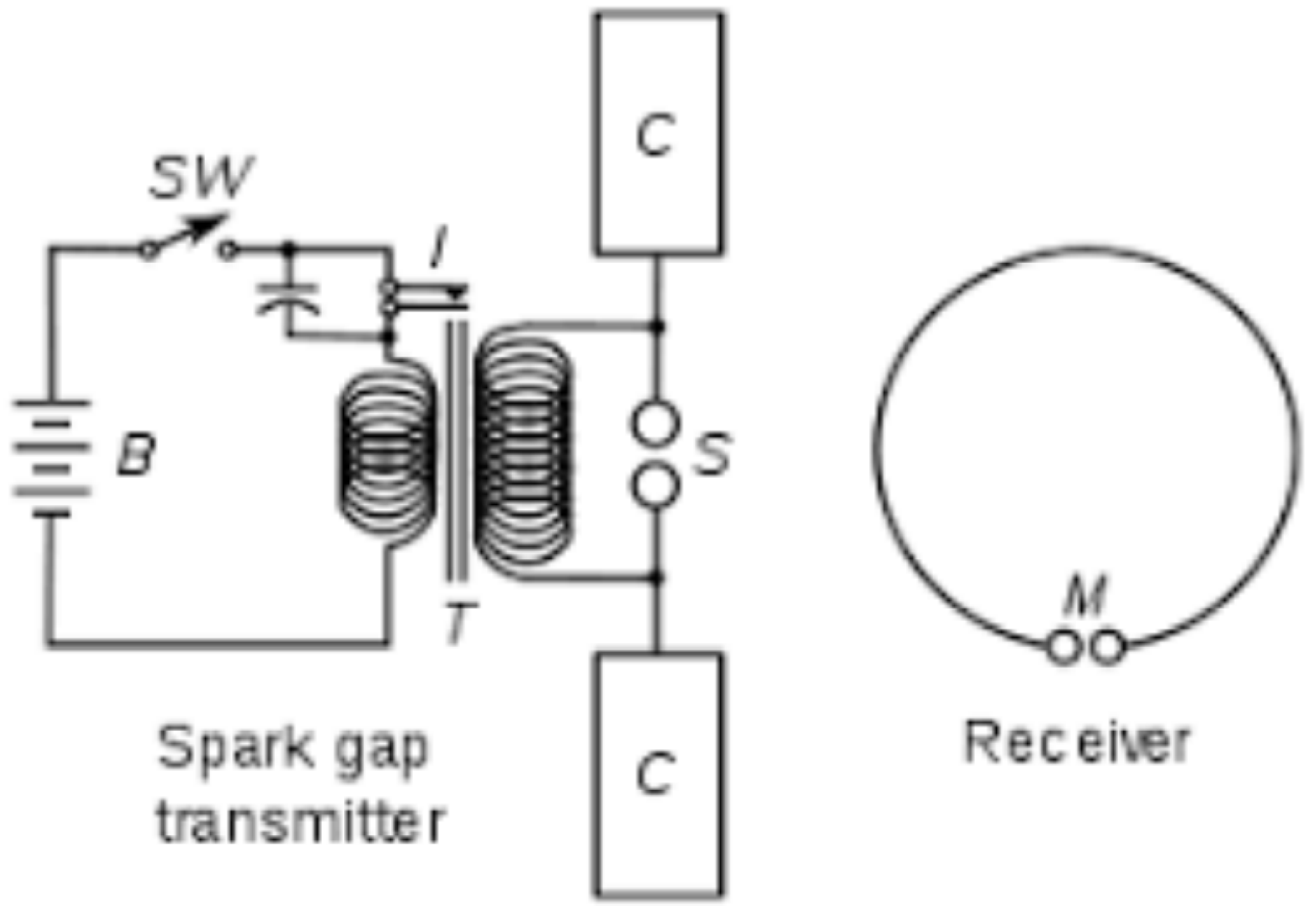




$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

"*We can scarcely avoid the conclusion that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.*"

Maxwell 1865



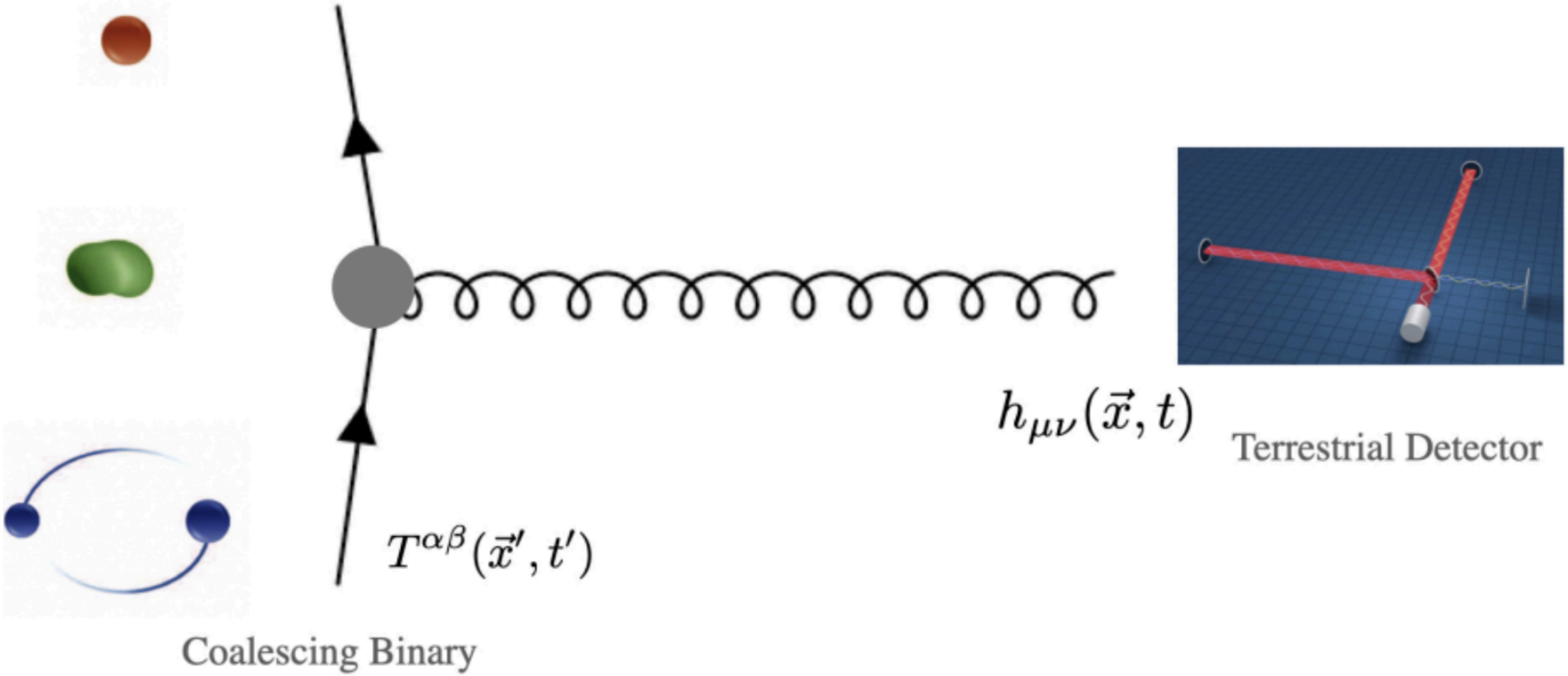
"It's of no use whatsoever… this is just an experiment that proves Maestro Maxwell was right—we just have these mysterious electromagnetic wavesthat we cannot see with the naked eye. But they are there."

Heinrich Hertz, 1887

Einstein's quadrupole formula for energy radiated in gravitational wave (1918)

$$\left(\frac{\mathrm{d}E}{\mathrm{d}T}\right)^{\mathrm{GW}} = \frac{G}{c^5}\left\{\frac{1}{5}\frac{\mathrm{d}^3\mathrm{Q}_{ab}}{\mathrm{d}T^3}\frac{\mathrm{d}^3\mathrm{Q}_{ab}}{\mathrm{d}T^3} + \mathcal{O}\left(\frac{1}{c^2}\right)\right\}$$

Diagramatic calculation of GW emission



Treat the graviton as a Quantum Field

$$\hat{h}_{\mu\nu}(x) = \sum_{\lambda'} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_q}} \left[\epsilon^{\lambda'}_{\mu\nu}(\mathbf{q}) a_{\lambda'}(\mathbf{q}) e^{-iq\cdot x} + \epsilon^{*\lambda'}_{\mu\nu}(\mathbf{q}) a^{\dagger}_{\lambda'}(\mathbf{q}) e^{iq\cdot x} \right]$$

$$\left[a_\lambda(\mathbf{k}), a^{\dagger}_{\lambda'}(\mathbf{k}') \right] = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') \delta_{\lambda\lambda'}$$

$$\epsilon^{\lambda}_{\mu\nu}(\mathbf{k}) \epsilon^{*\lambda'\mu\nu}(\mathbf{k}) = \delta_{\lambda\lambda'}$$

$$\sum_{\lambda=1}^{2} \epsilon^{\lambda}_{\mu\nu}(k) \epsilon^{*\lambda}_{\alpha\beta}(k) = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}$$

Graviton-matter coupling

$$\mathcal{L}_{int} = -\frac{\kappa}{2} T^{\mu\nu} h_{\mu\nu}$$

Scattering amplitude

$$\begin{aligned} S_{if} &= -i \langle \epsilon_\lambda^{\mu\nu}(\mathbf{k}) | \int d^4x \, \mathcal{L}_{int}(x) | 0 \rangle \\ &= -i \frac{\kappa}{2} \int d^4x \, \langle 0 | \sqrt{2\omega_k} \epsilon_\lambda^{\mu\nu}(\mathbf{k}) a_\lambda(\mathbf{k}) T_{\mu\nu}(x) \hat{h}^{\mu\nu}(x) | 0 \rangle \end{aligned}$$

$$S_{if} = -i\langle \epsilon_{\lambda}^{\mu\nu}(\mathbf{k})| \int d^4x\, \mathcal{L}_{int}(x)|0\rangle$$

Probability of graviton emission per unit time

$$\dot{P} = \frac{|S_{fi}|^2}{T} = \frac{\kappa^2}{4}\sum_n \frac{1}{T}\left|T_{\mu\nu}(\mathbf{k},\omega_n')\epsilon_{\lambda}^{*\mu\nu}(\mathbf{k})\,(2\pi)\delta(\omega-\omega_n')\right|^2$$

Rate of graviton emission

$$\Gamma = \sum_{\lambda}\int \frac{|S_{fi}|^2}{T}\frac{d^3\mathbf{k}}{(2\pi)^3\omega_k}$$

$$= \frac{\kappa^2}{4}\sum_n\sum_{\lambda}\int \left|T_{\mu\nu}(\mathbf{k},\omega_n')\epsilon_{\lambda}^{*\mu\nu}(\mathbf{k})\right|^2 (2\pi)\delta(\omega-\omega_n')\,\frac{d^3\mathbf{k}}{(2\pi)^3 2\omega_k}$$

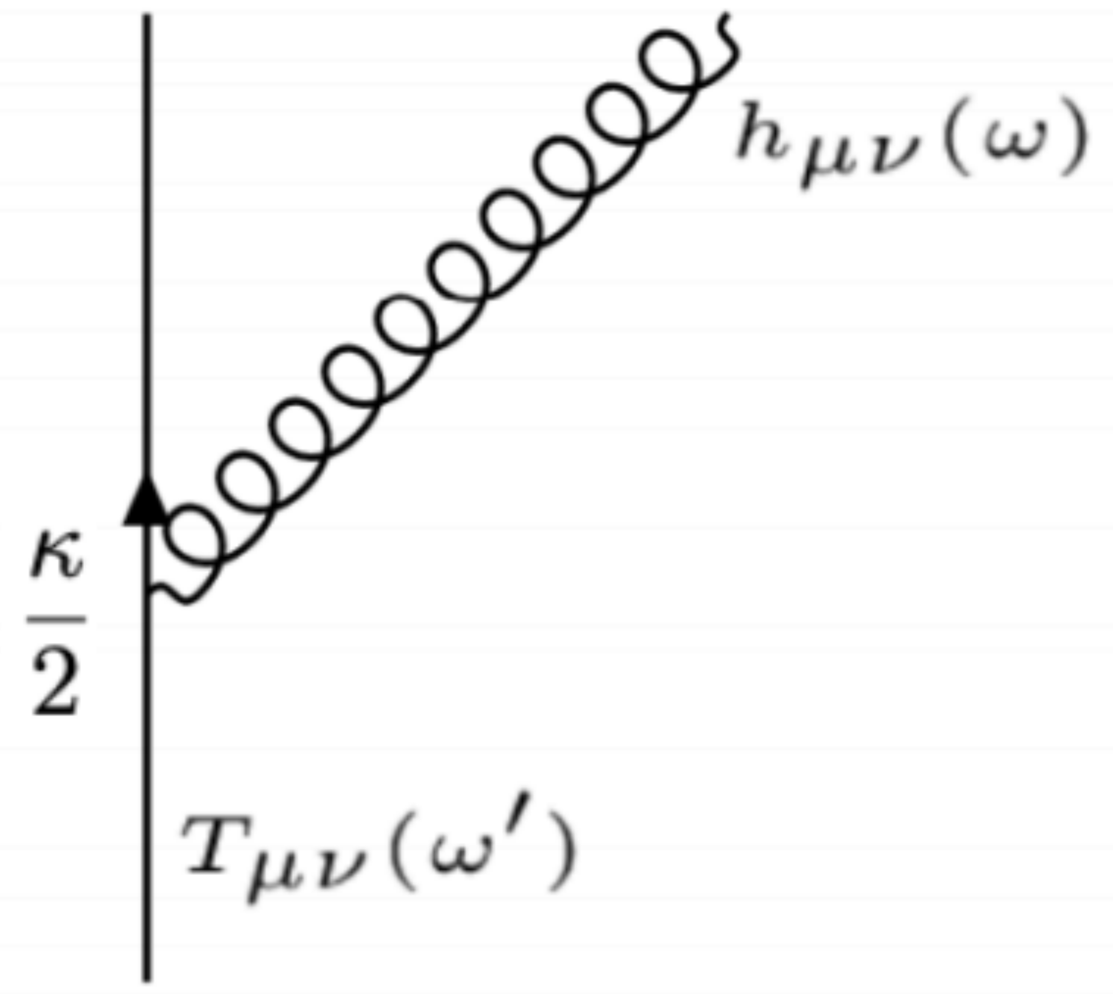
Gravitational radiation from elliptical orbits

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\kappa = \sqrt{32\pi G}$$

Interaction vertex:

$$\frac{\kappa}{2} T_{\mu\nu}(k')\epsilon_\lambda^{\mu\nu}(k)$$



$$T_{\mu\nu}(x') = \mu\delta^3(\mathbf{x}' - \mathbf{x}(t))U_\mu U_\nu$$

$$\mu = \frac{m_1 m_2}{m_1+m_2} \qquad U_\mu = (1, \dot{x}, \dot{y}, 0)$$

Keplerian orbit $\quad x = a(\cos\xi - e), \quad y = a\sqrt{(1-e^2)}\sin\xi, \quad \Omega t = \xi - e\sin\xi, \quad \Omega = \left[G\frac{(m_1+m_2)}{a^3}\right]^{\frac{1}{2}}$

Energy loss from elliptical binaries in gravitational wave radiation

$$\frac{dE}{dt} = \frac{\kappa^2}{8(2\pi)^2}\int \frac{8\pi}{5}\left[T_{ij}(\omega')T^*_{ji}(\omega') - \frac{1}{3}|T^i{}_i(\omega')|^2\right]\delta(\omega-\omega')\omega^2 d\omega$$

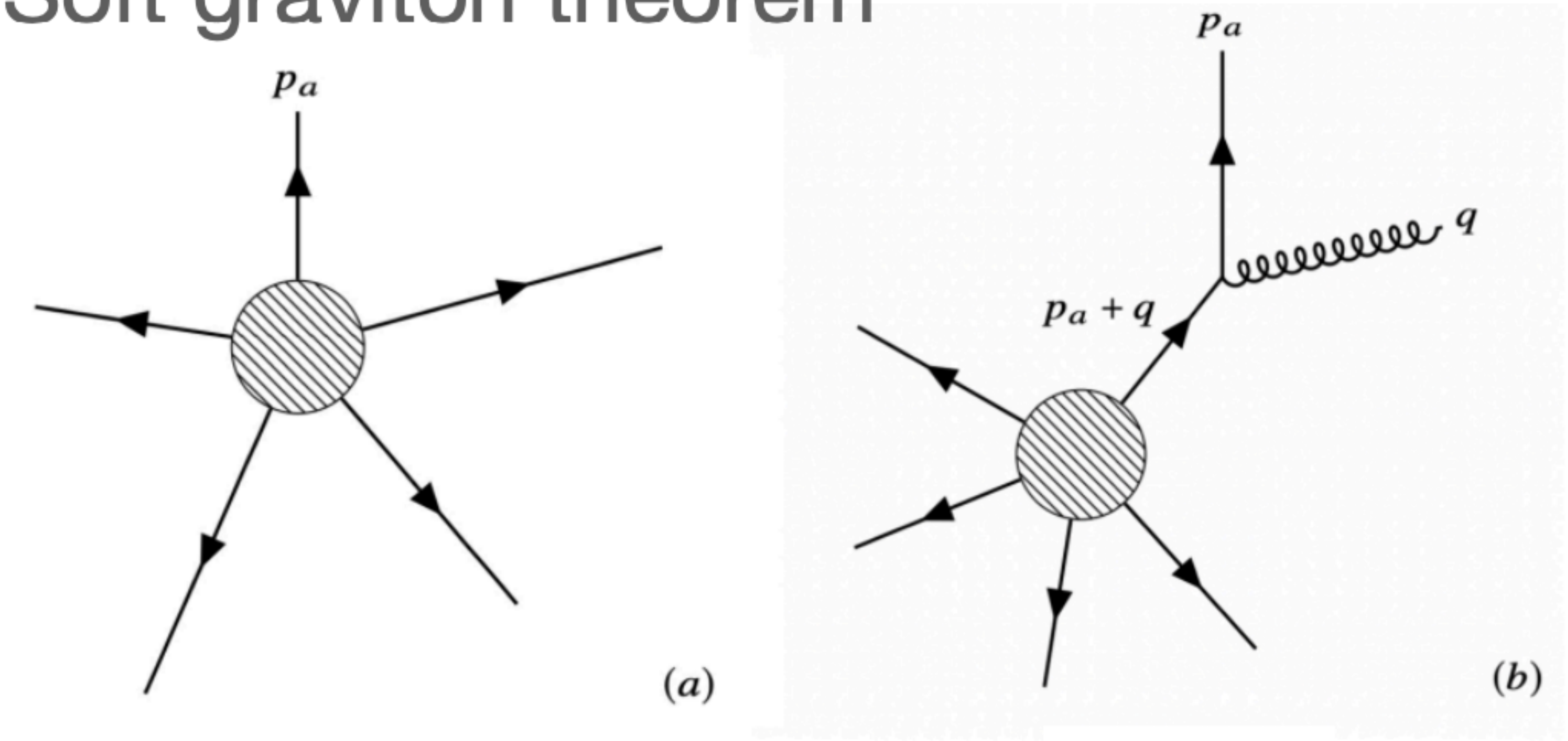
$$= \frac{32G}{5}\Omega^6\mu^2 a^4 \sum_{n=1}^{\infty} n^6 f(n,e)$$

$$= \frac{32G}{5}\Omega^6\left(\frac{m_1 m_2}{m_1+m_2}\right)^2 a^4 \frac{1}{(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

SM & P.K.Panda Phys Rev D (1996)

Matches Peters-Mathews formula (1963) from classical general relativity

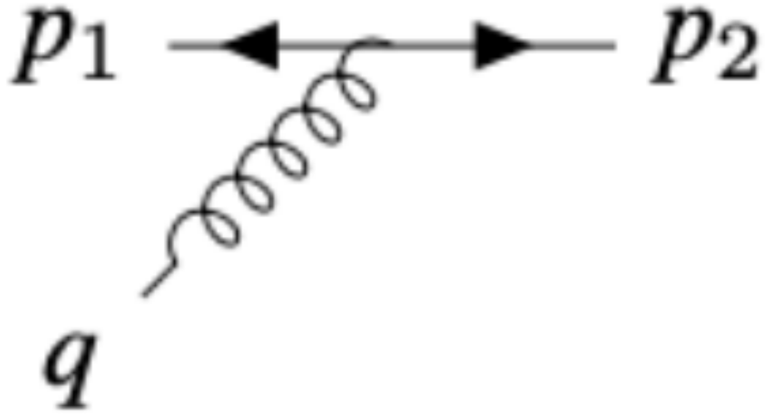
Soft graviton theorem



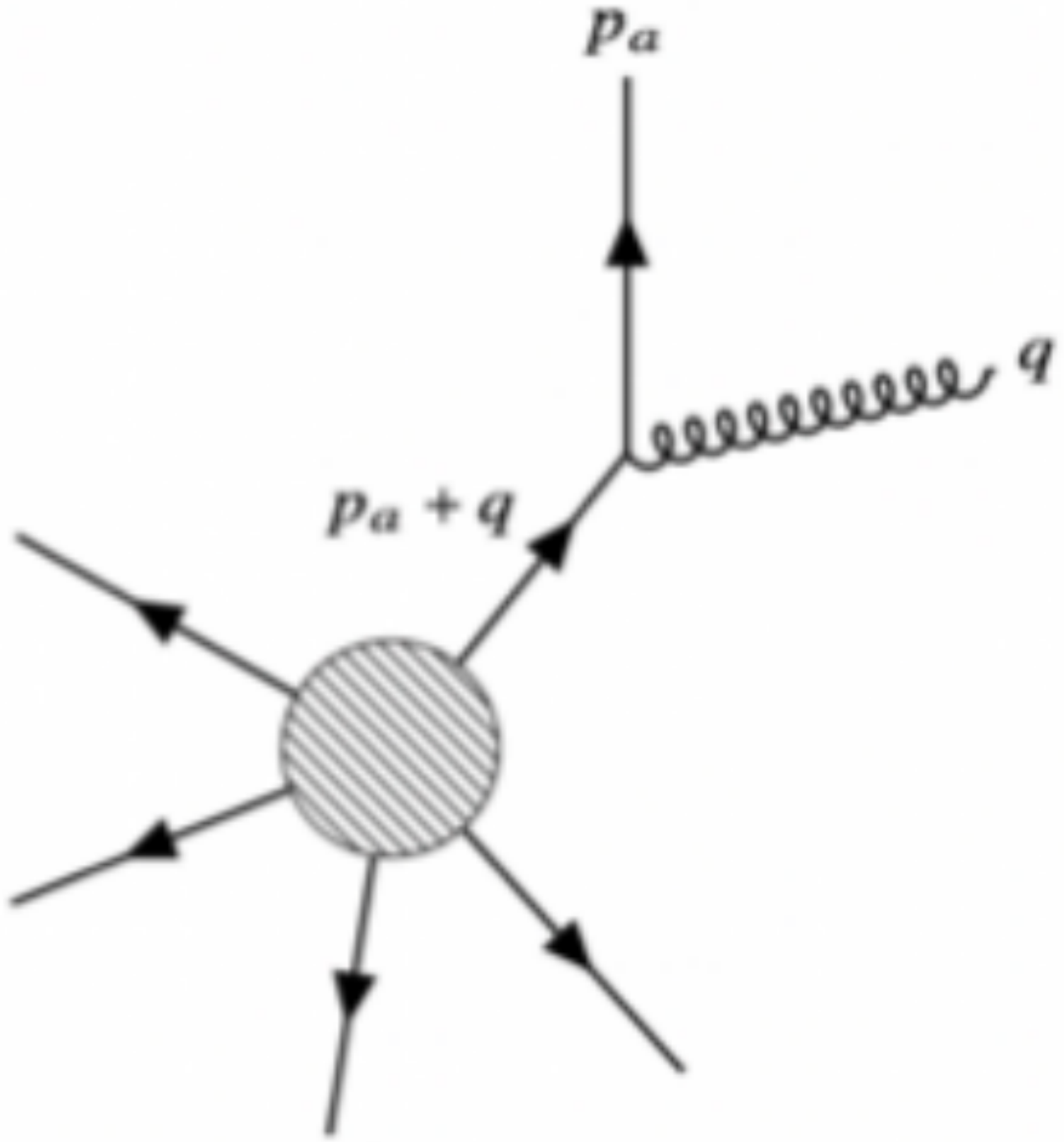
$\mathcal{A}_n(p_a)$ $\mathcal{A}_{n+1}$

$$\mathcal{A}_{n+1}(p_a, q) = \frac{\kappa}{2}\epsilon^{*\mu\nu}(q)\sum_{a=1}^{n}\frac{p_{a\mu}p_{a\nu}}{p_a \cdot q}\mathcal{A}_n(p_a)$$

For each graviton vertex



$$= \frac{i\kappa}{2}\epsilon_{\lambda}^{*\mu\nu}(q)\Big[p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu} - \eta_{\mu\nu}(p_1 \cdot p_2 - m^2)\Big]$$



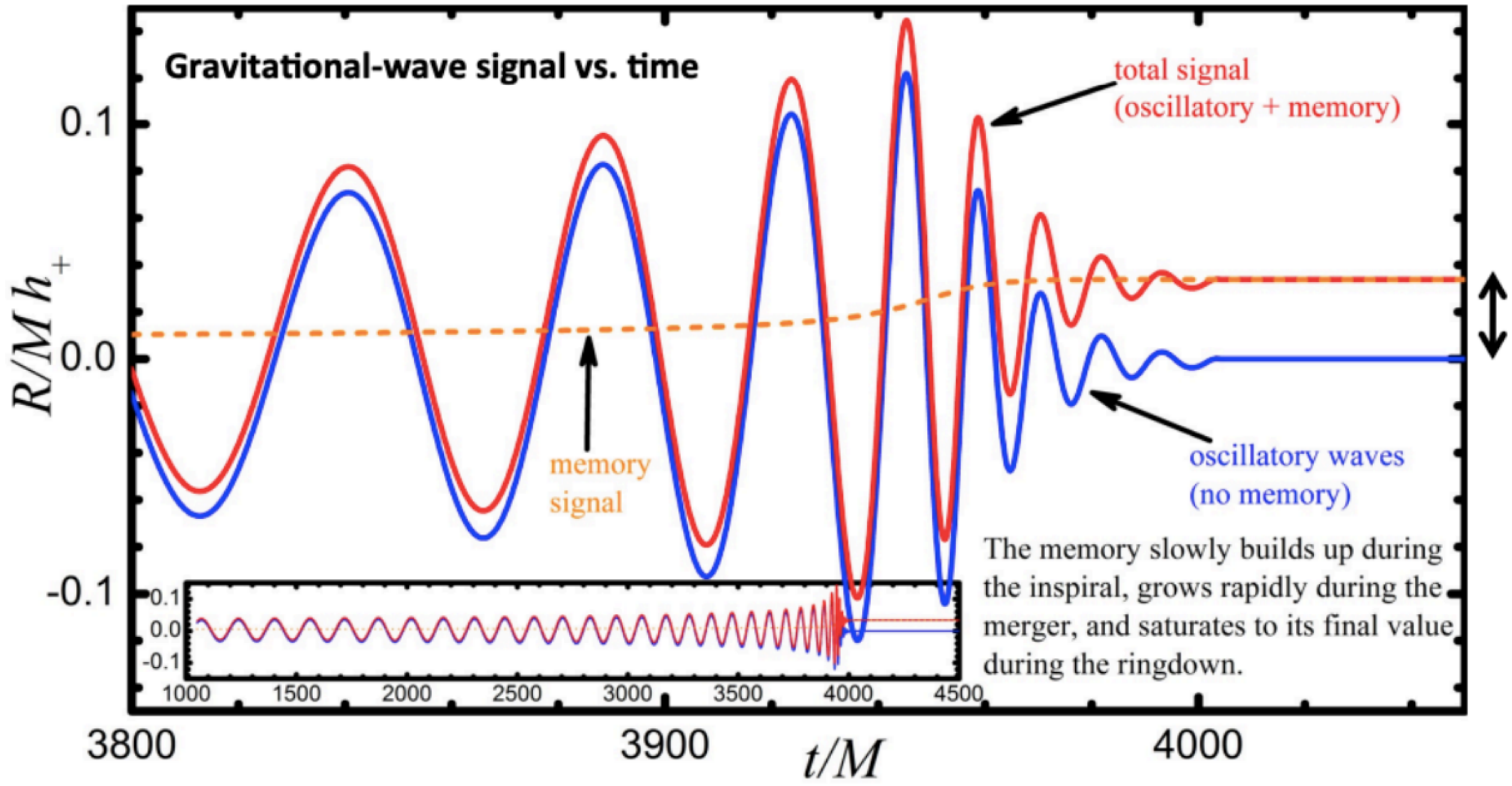
$$= \frac{\kappa}{2}\frac{\epsilon^{*\mu\nu}(q)}{(p_a+q)^2 - m_a^2}\Big[(p_{a\mu} + q_\mu)p_{a\nu} + (p_{a\nu} + q_\nu)p_{a\mu} - \eta_{\mu\nu}\big((p_a + q)\cdot p_a - m_a^2\big)\Big]$$

$$= \frac{\kappa}{2}\frac{\epsilon^{*\mu\nu}(q)}{2p_a \cdot q}\Big[2p_{a\mu}p_{a\nu} + q_\mu p_{a\nu} + q_\nu p_{a\mu} - \eta_{\mu\nu}p_a \cdot q\Big]$$

$$\int \frac{dk_0}{2\pi i k_0} e^{ik_0(t-r)} = \Theta(t-r)$$

The step function gives the DC signal in gravitational waves and this is called the memory effect.

Gravitational memory



Marc Favata (ICTS talk)

Jet Gravitational Waves- Tsvi Piran 2210.02740

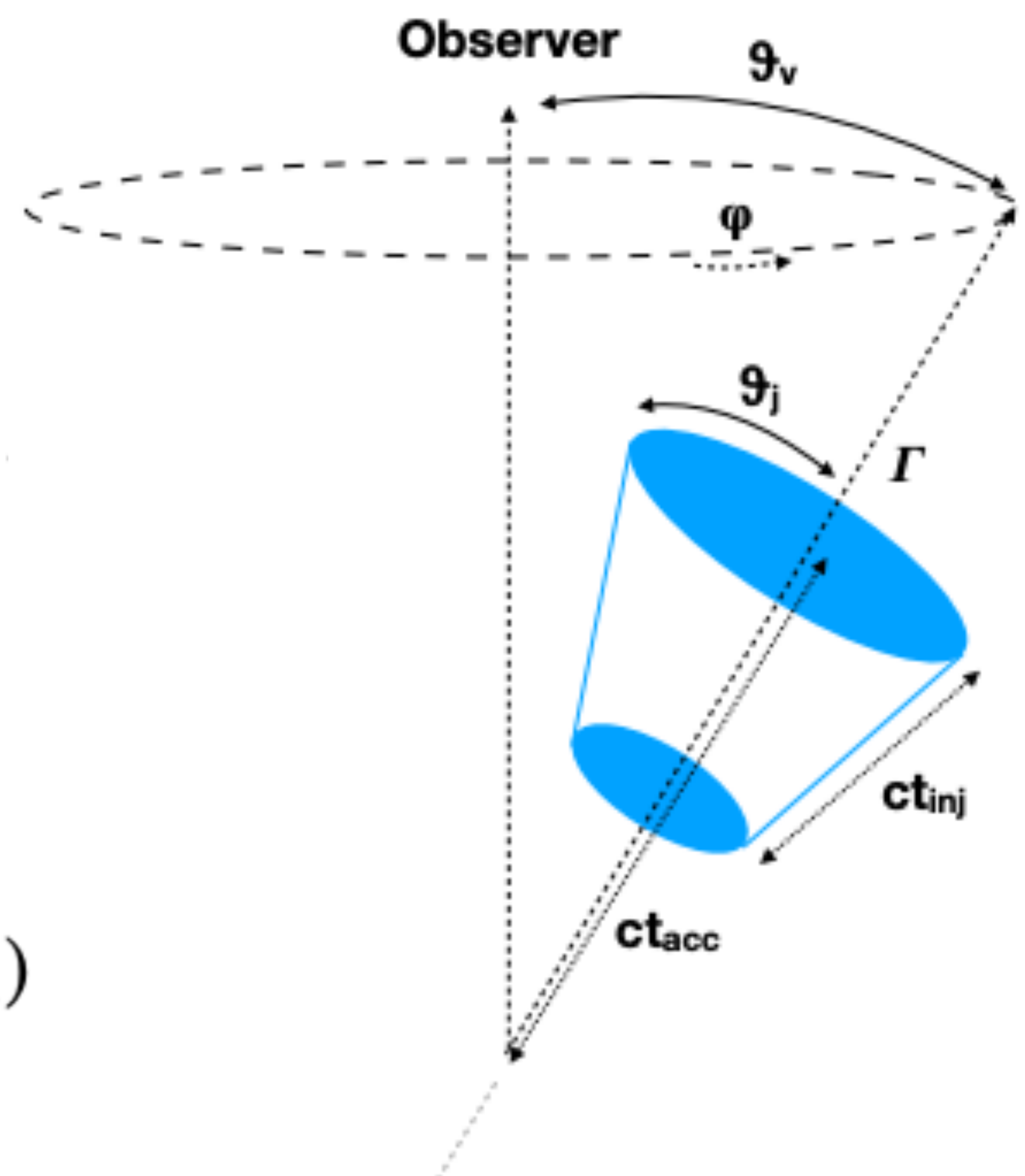
$$h(t,\theta_v) = h(\theta_v)\mathcal{H}(t)$$

$$\tilde{h}(f,\theta_v) = h(\theta_v)/f$$

$$h^{TT}(\theta_v) = h_+ + ih_x = \frac{2\mathcal{E}\beta^2}{r}\frac{\sin^2\theta_v}{1-\beta\cos\theta_v}$$

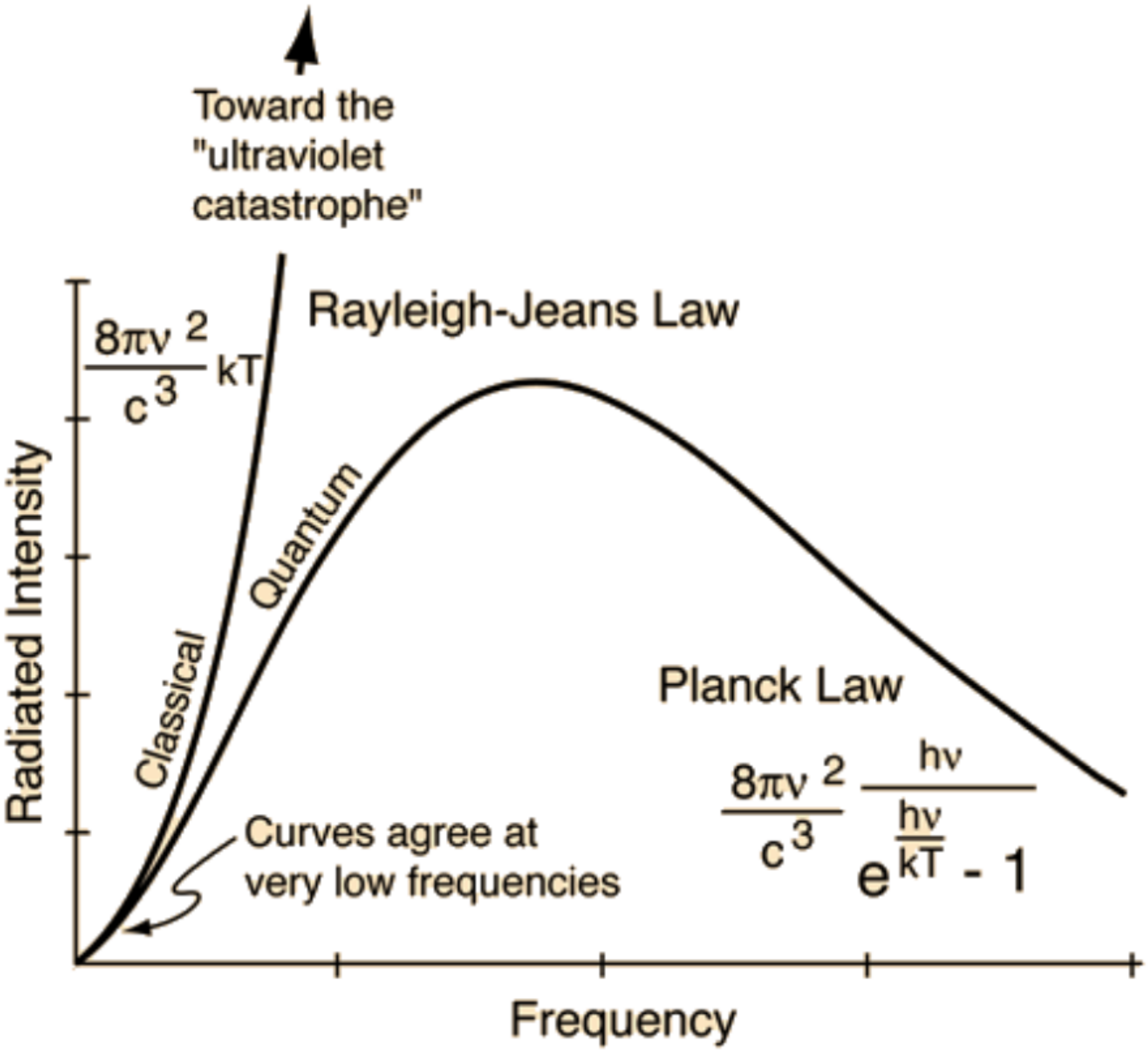
Can be derived from soft-theorem

$$\mathcal{A}_{n+1}(p_a,q) = \frac{\kappa}{2}\epsilon^{*\mu\nu}(q)\sum_{a=1}^{n}\frac{p_{a\mu}p_{a\nu}}{p_a\cdot q}\mathcal{A}_n(p_a)$$

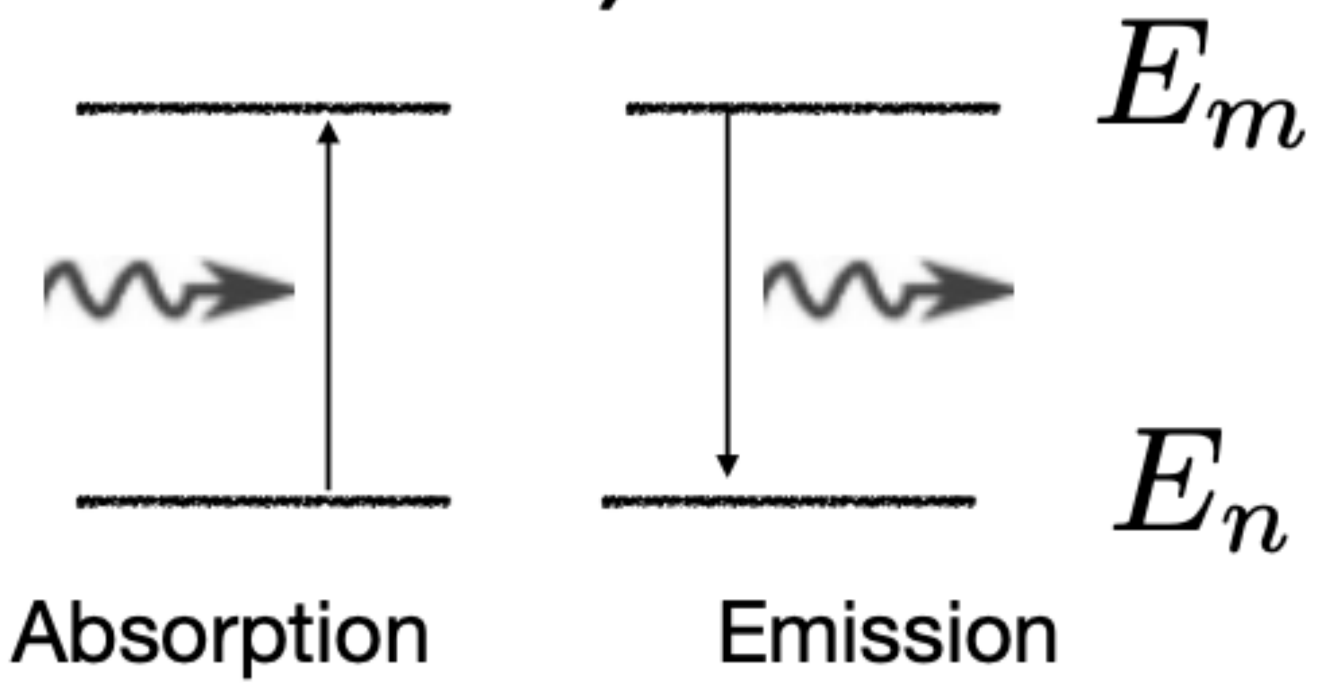


Can we look for the signature of gravitons in gravitational wave observations?

Planck's Radiation Law, 1900



Spontaneous and stimulated emissions (Einstein 1916)



$$\frac{dN_n}{dt} = A_{m\to n} N_m + B_{m\to n} N_m u(\omega) - B_{n\to m} N_n u(\omega)$$

Emission Absorption

Thermal Equilibrium

$$\frac{N_n}{N_m} = e^{-(E_n - E_m)/T} = e^{-\omega/T} \quad \text{and} \quad \frac{dN_n}{dt} = 0$$

$$A_{m\to n} = B_{m\to n}\left(e^{\omega/T-1} - 1\right) u(\omega)$$

$$A_{m\to n} = 8\pi\omega^3 B_{m\to n} \quad \Leftrightarrow \quad u(\omega) = \frac{8\pi\omega^3}{e^{\omega/T} - 1}$$

Net emission rate

$$\Gamma^{emi}_{m\to n} = A_{m\to n} + B_{m\to n} u(\omega)$$
$$= 8\pi\omega^3 B_{m\to n} (1 + n(\omega))$$

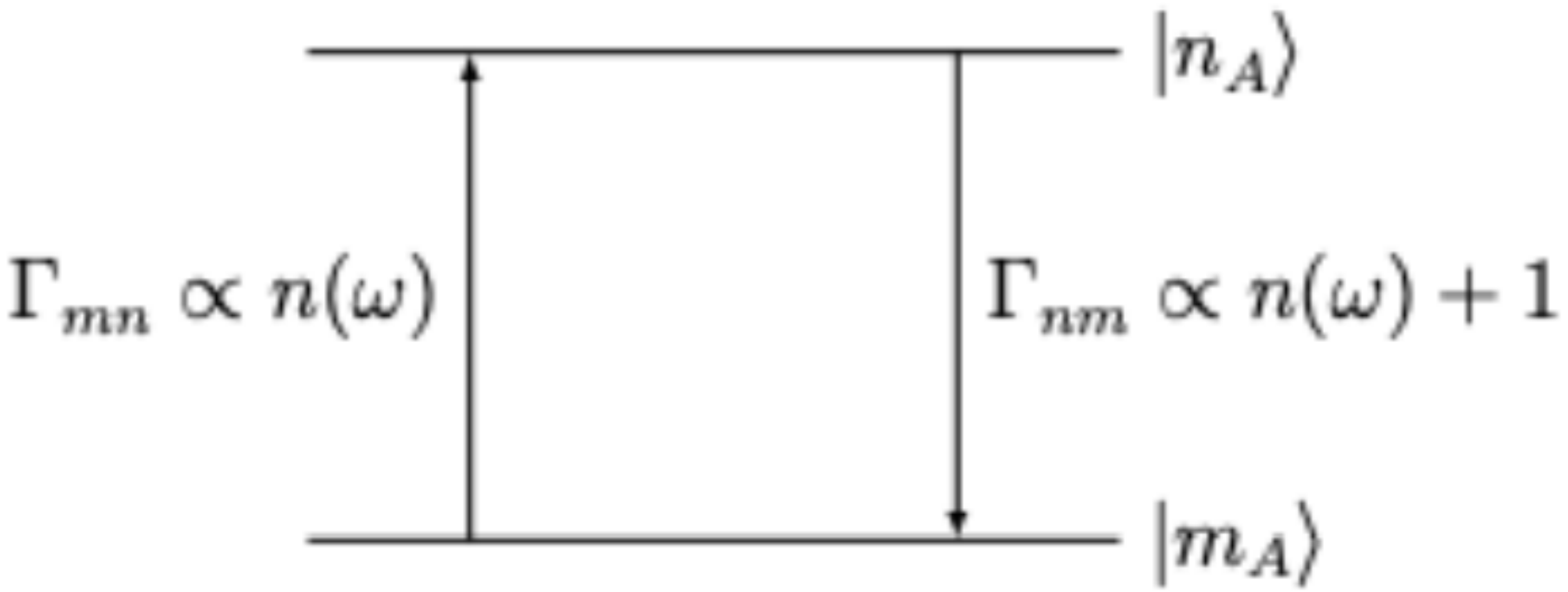
Net absorption rate

$$\Gamma^{abs}_{n\to m} = B_{m\to n} u(\omega)$$
$$= 8\pi\omega^3 B_{m\to n}\, n(\omega)\,.$$

Bose-Einstein distribution

$$n(\omega) = \frac{1}{e^{\omega/T} - 1}$$

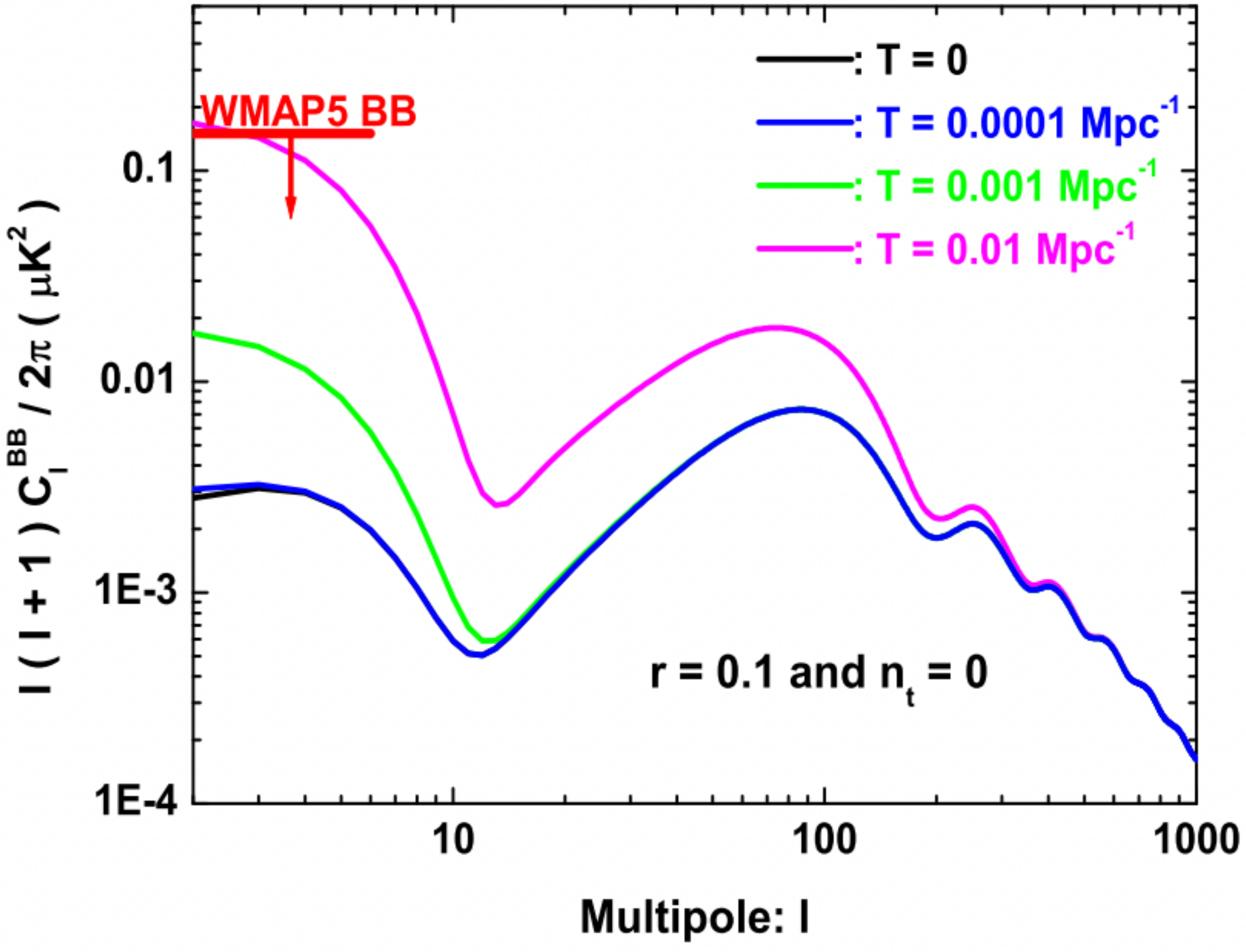
Atomic transitions in a heat bath



$$\Gamma_{nm} = \begin{cases} \dfrac{\omega_{nm}^3 |\boldsymbol{d}_{nm}|^2}{3\pi\varepsilon_0 \hbar c^3} [n(\omega_{nm}) + 1] & \text{Emission rate} \\ \dfrac{\omega_{mn}^3 |\boldsymbol{d}_{nm}|^2}{3\pi\varepsilon_0 \hbar c^3} [n(\omega_{mn})] & \text{Absorption rate} \end{cases}$$

Inflation power spectrum in a thermal background

Bhattacharya, Mohanty, Nautiyal , *Phys.Rev.Lett.* 97 (2006) 251301



Zhao,Baskaran,Coles 2009

During inflation zero-point fluctuations become classical

Zero-point fluctuations in thermal background of gravitons

$$P_t(k) = A_t(k_0)\left(\frac{k}{k_0}\right)^{n_t}\left(1 + \frac{2}{e^{k/T} - 1}\right)$$

Bose-Einstein distribution

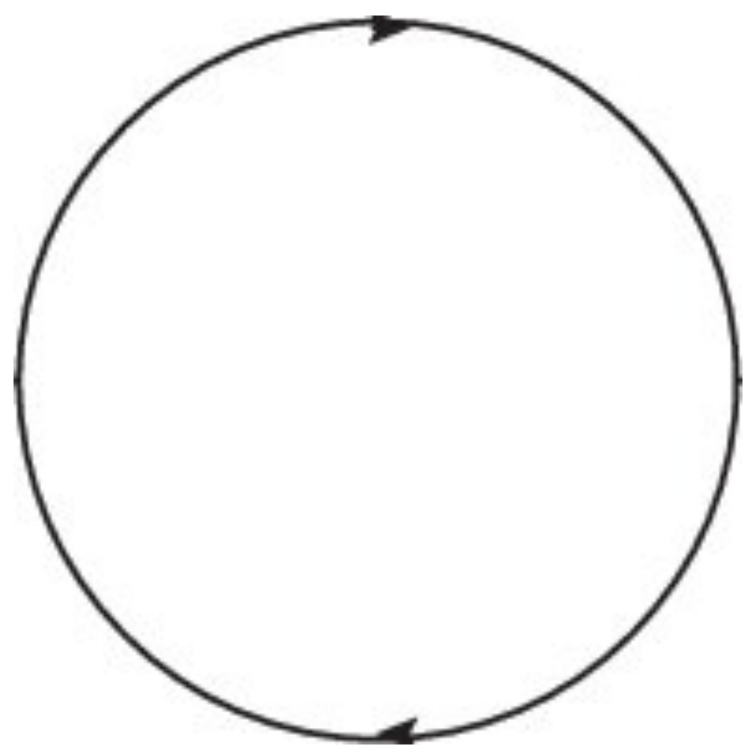
$$n(k) = \frac{1}{e^{k/T} - 1}$$

Creation-annihilation commutation

$$[a_k, a^\dagger_{k'}] = \delta_{kk'}$$

Zero-point fluctuation in vacuum

$$\langle 0|a_k a^\dagger_k + a^\dagger_k a_k|0\rangle = \langle 0|1 + 2a^\dagger_k a_k|0\rangle = 1$$

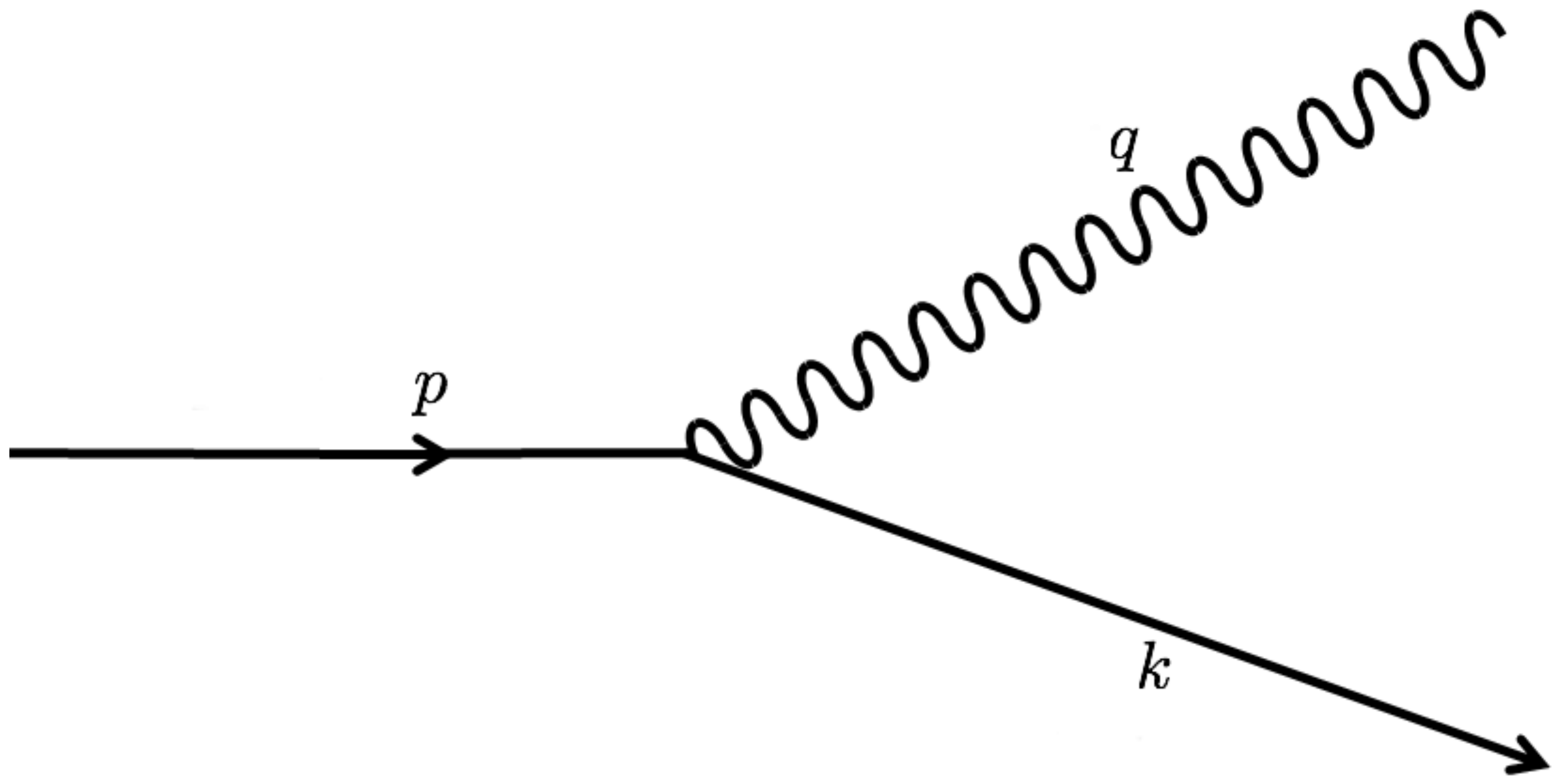


Zero-point fluctuation in a thermal bath

$$\langle n|a_k a^\dagger_k + a^\dagger_k a_k|n\rangle = \langle n|1 + 2a^\dagger_k a_k|n\rangle = 1 + 2n$$

Is there a stimulated emission of gravitational waves in a thermal graviton thermal background?

Graviton emission in vacuum

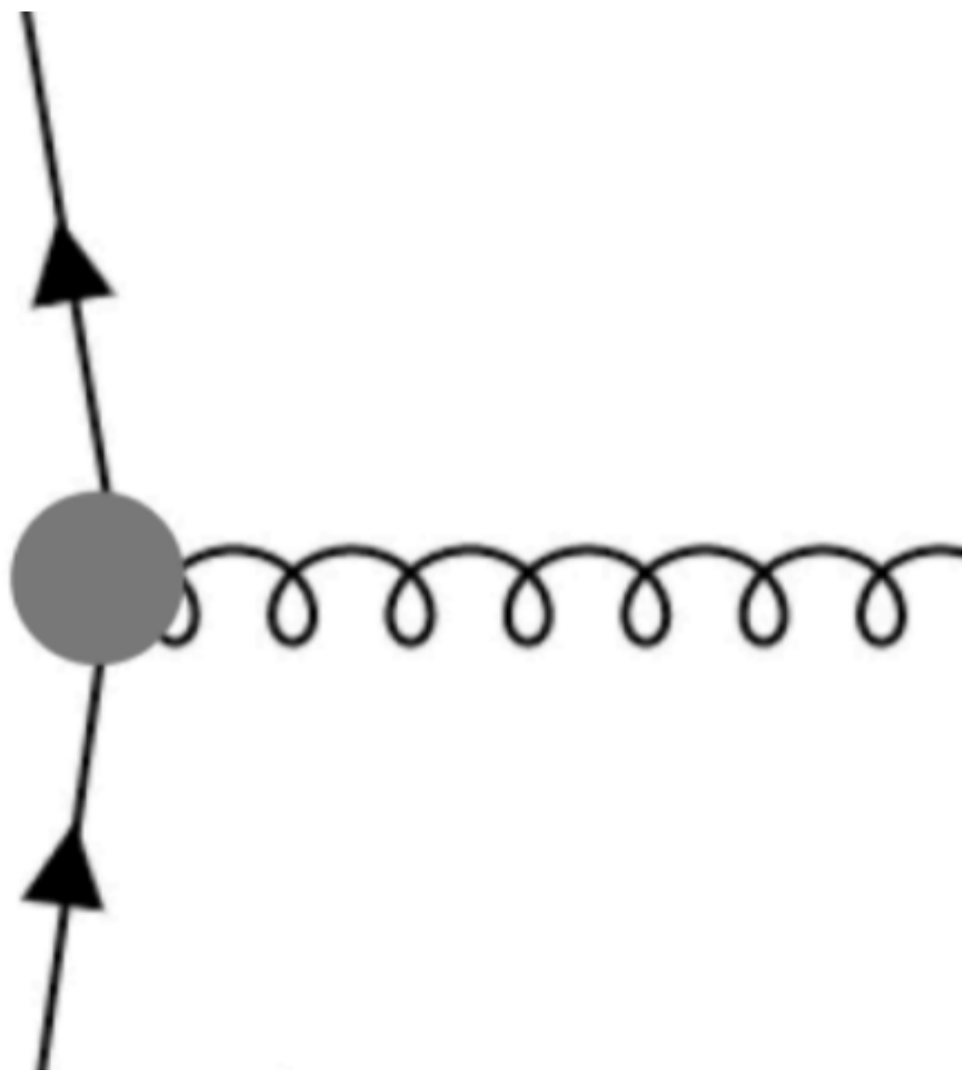


$$\langle 1|a_k a_k^\dagger|0\rangle = 1$$

Graviton radiation in a thermal bath of gravitons- stimulated emission

$$\langle n|a_k a_k^\dagger|n\rangle = \langle n|1 + a_k^\dagger a_k|n\rangle = 1 + n$$

Similar situation exists in QED

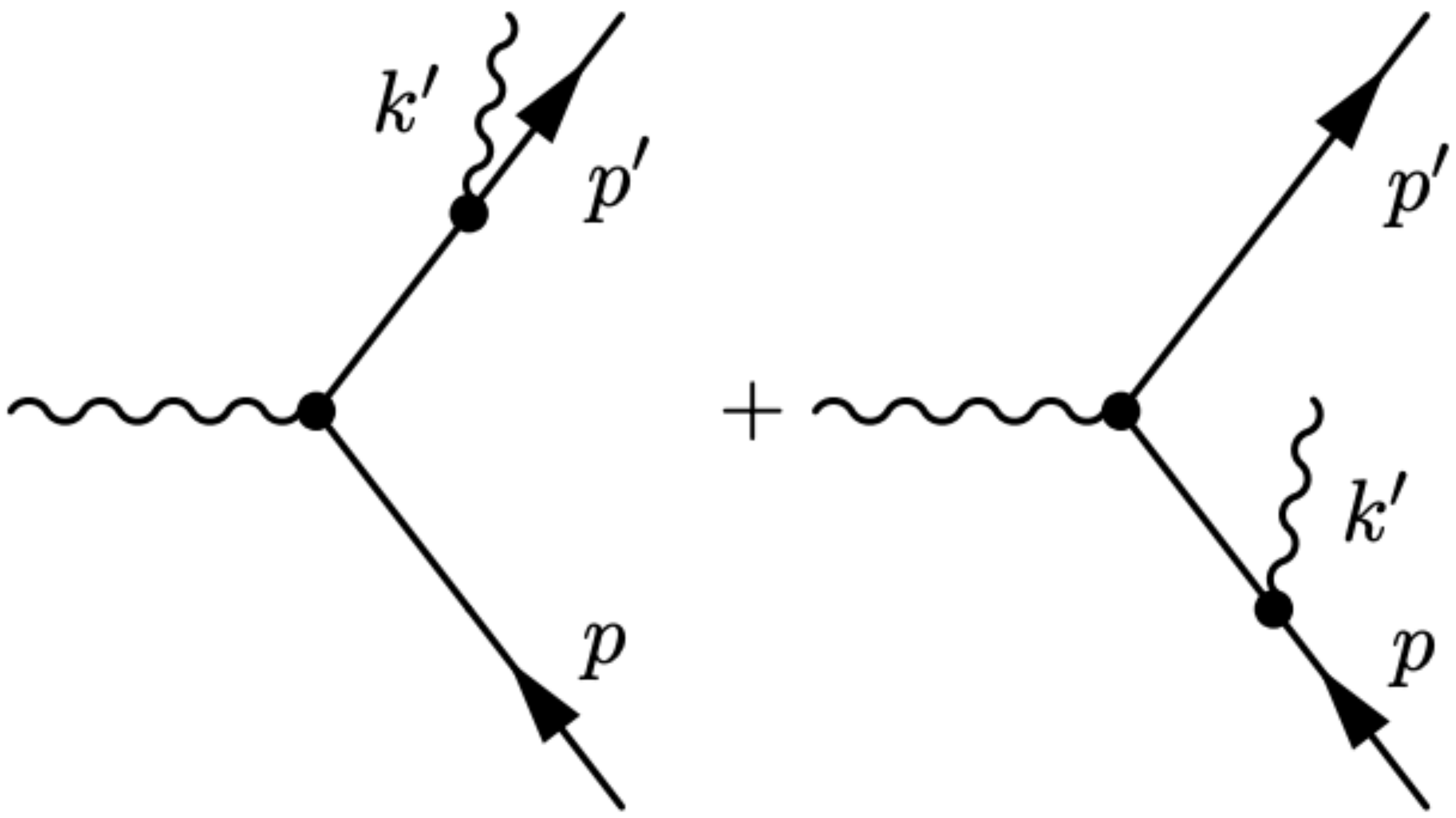


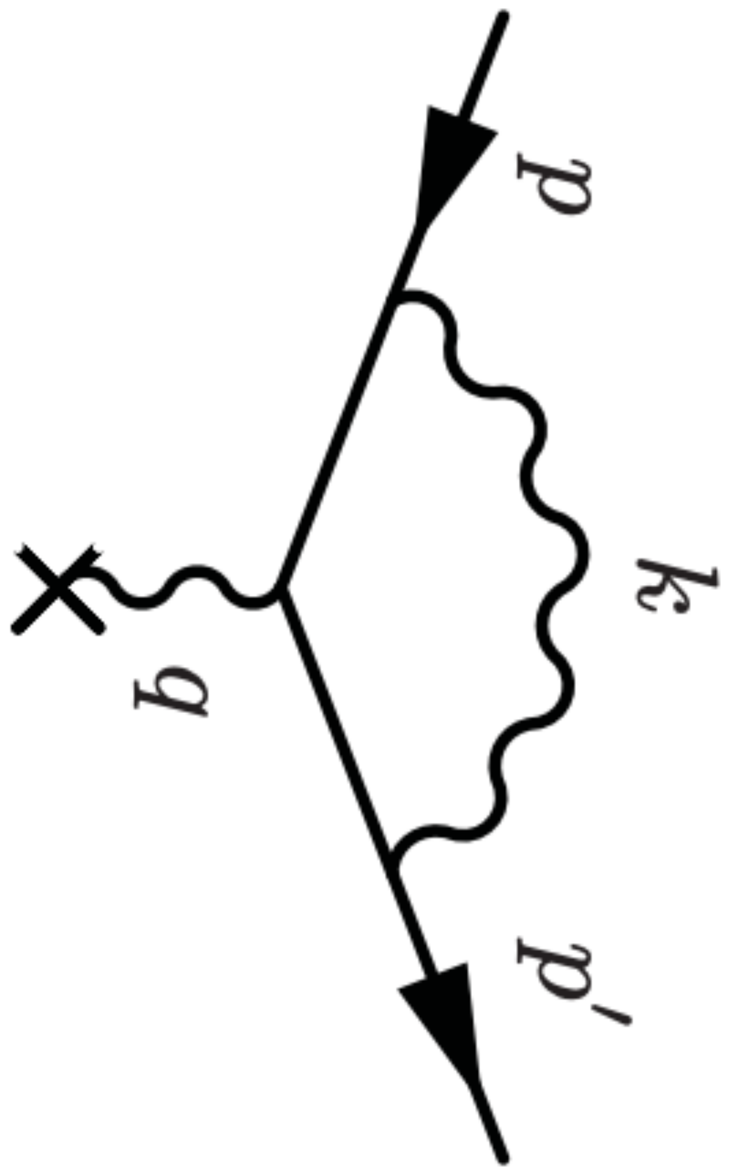
$$\frac{dP}{d\omega} \approx \frac{A}{\omega}[1 + n_B(\omega)] \qquad (A\pi T \ll \omega)$$

$$\frac{dP}{d\omega} \approx \frac{AT}{\omega^2 + (A\pi T)^2} \qquad (\omega \ll T)$$

CANCELLATION OF INFRARED DIVERGENCES IN QED AT NONZERO TEMPERATURE. **Weldon, Phys Rev D 1993**

Bloch-Nordsiek counterpart for soft gravitons?





F. Bloch and A. Nordsieck, "Note on the Radiation Field of the electron," *Phys. Rev.*, vol. 52, pp. 54–59, 1937.

Graviton loop corrections may be needed to correct infrared divergences in gravitational radiation calculations

Thank You