Modulated instabilities and the AdS$_2$ point in dense holographic matter

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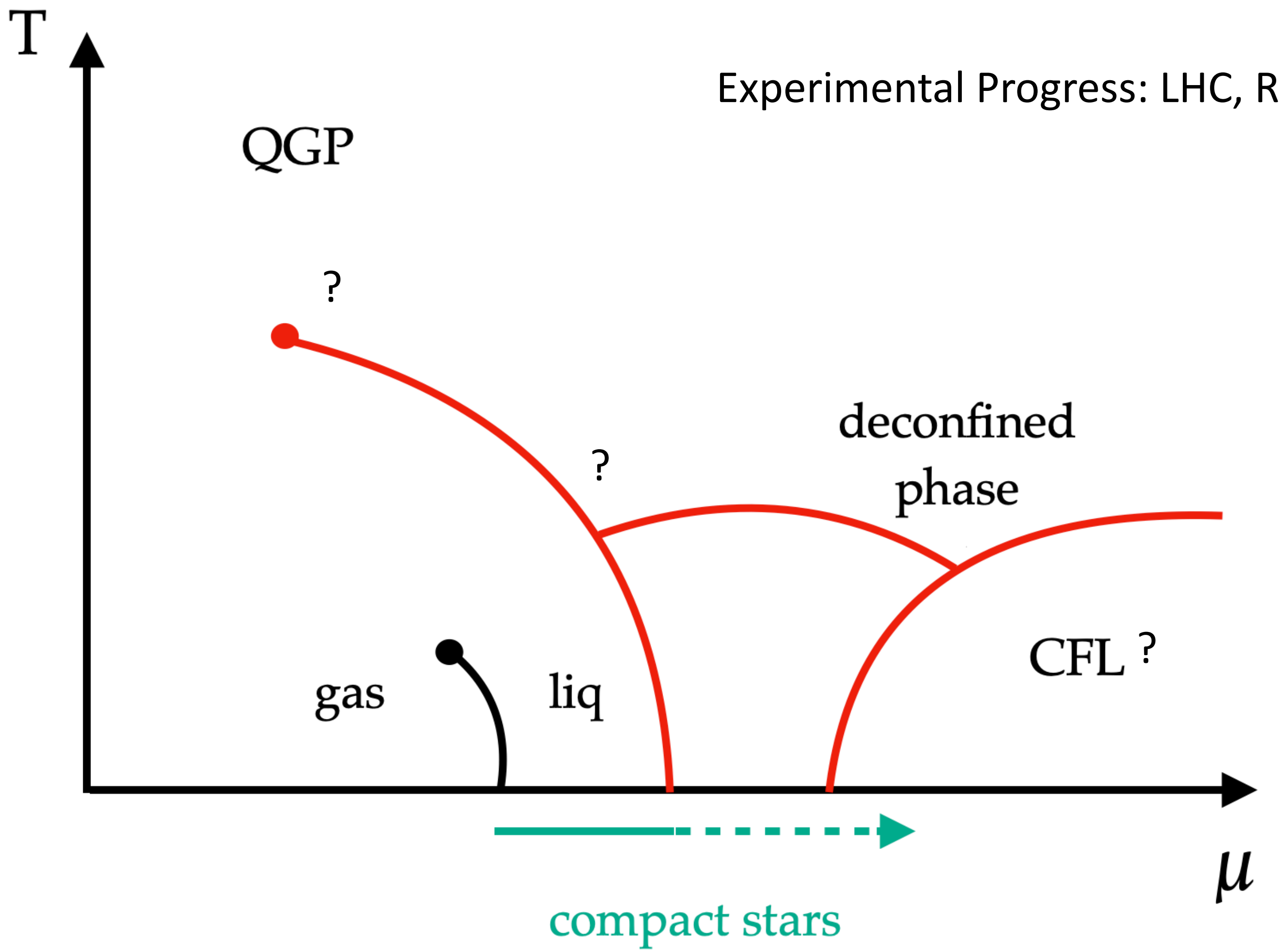
Plan

- Motivation
- Introduction to V-QCD model
- Fluctuation analysis
- Striped instabilities
- Approach to the AdS_2 point
- Summary and future work

Description of the phase diagram

We are interested in study the strongly coupled regime of QCD at low temperature and high densities at the deconfined phase.

Experimental Progress: LHC, RHIC, FAIR, NICA



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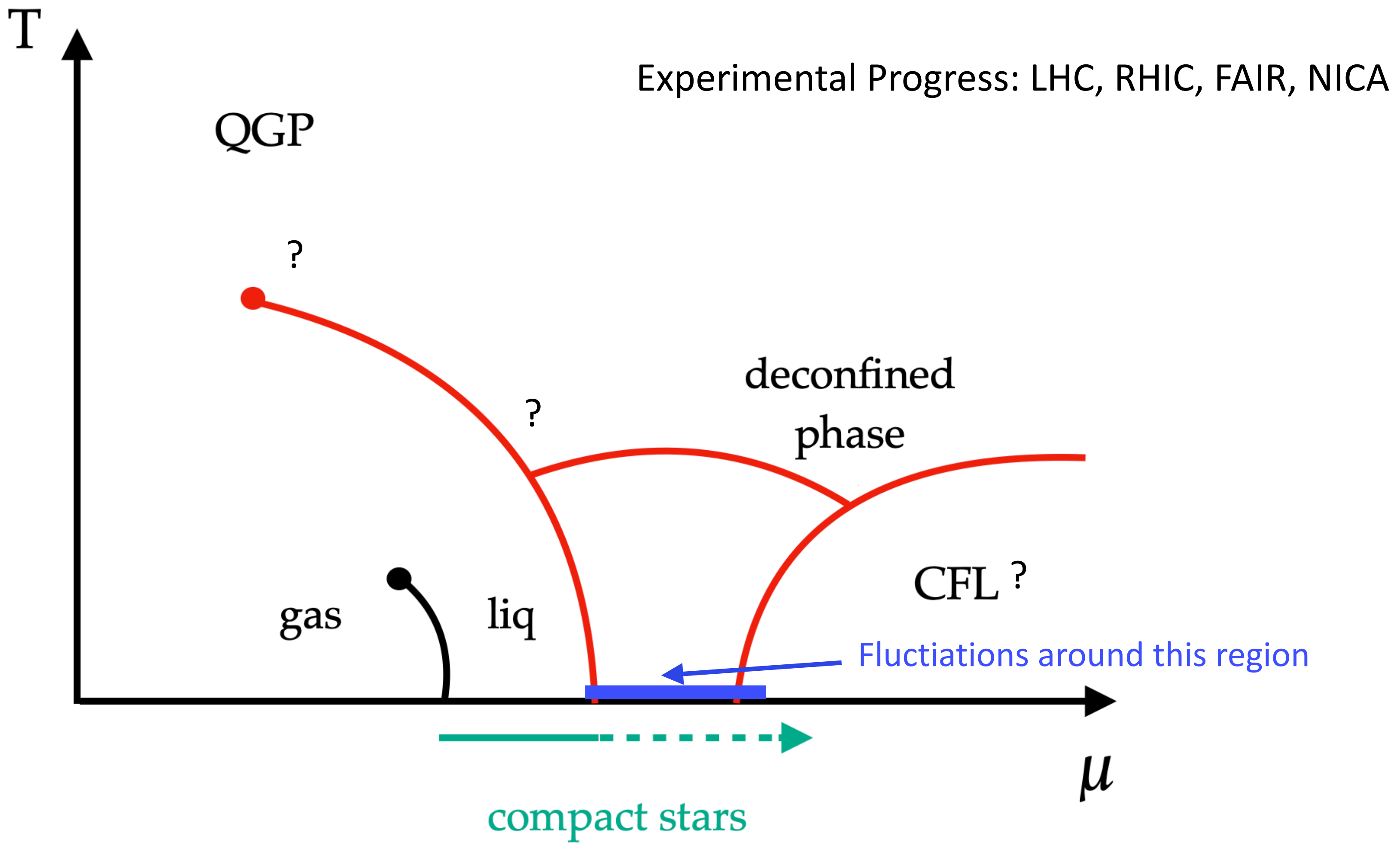


Holographic description of QCD

Holography has emerged as a new tool to study strongly coupled gauge theories.

It provides the ability to rigorously work in theories close to large N_c $\mathcal{N} = 4$ SYM.

Bottom-up approaches have more room to incorporate several real QCD constrains coming from Lattice QCD, pQCD and gravitational wave measurements. It is natural to try to use it to study the stability in different regions of the phase diagram

We want to describe a QCD-like theory: $SU(N_c)$ Yang-Mills coupled to fundamental matter

Bottom-Up vs Top Down approaches

Top-down: models directly based on string theory.

- Concrete, fixed string models in 10/11 d with branes
- Less control on what dual field theory is (it’s not QCD)

Bottom-up: models constructed “by hand”

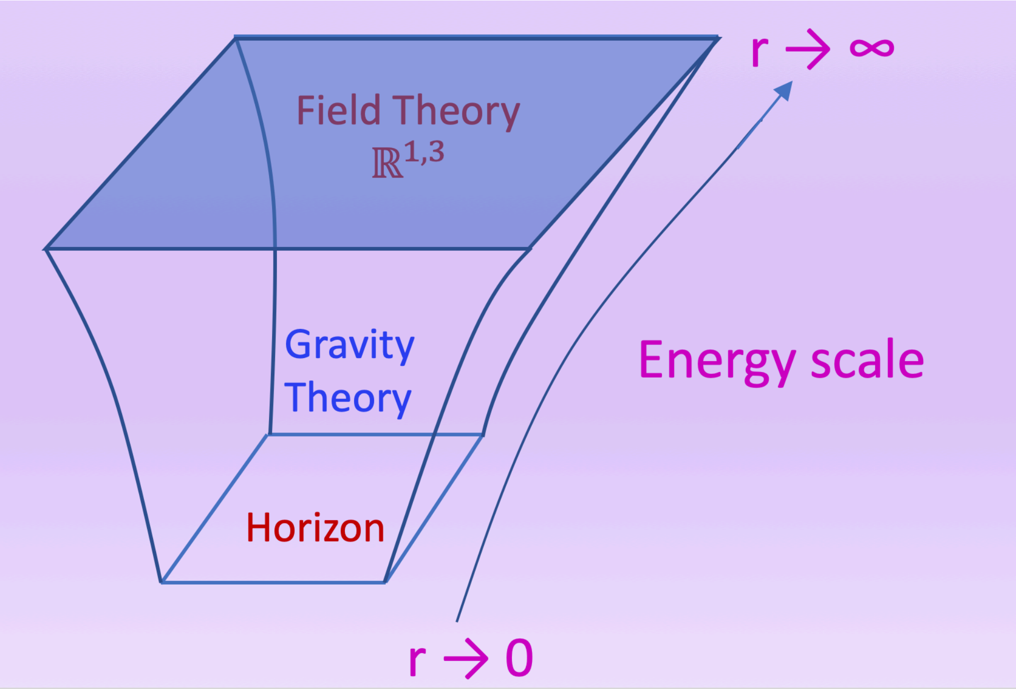
- Follow generic ideas of holography, inspiration from top-down
- Introduce fields for most important operators
- Lots of freedom → effective 5d description, no link to specific dual theory, (comparison with QCD data essential)
- Either a fixed geometry (AdS) or dynamical gravity

Gauge/Gravity duality for QCD

The additional dimension (the holographic coordinate) is identified with the energy scale

UV $\longleftrightarrow$ boundary, IR $\longleftrightarrow$ center/horizon

Thermodynamics of QCD $\longleftrightarrow$ thermodynamics of a planar bulk black hole



Gauge/Gravity duality for QCD

For the study of QCD we loss conformality and SUSY. We end up with a non-AdS/non-CFT duality.

We introduce bulk fields for relevant and marginal operators:

$$\text{Operators } O_i\left(x^\mu\right) \leftrightarrow \text{classical bulk fields } \phi_i\left(x^\mu, r\right)$$

$$Z_{\text{grav}}\left(\phi_i|_{\text{bdry}} = J_i\left(x^\mu\right)\right) = \int \mathcal{D}e^{iS_{QCD}+i\int d^4x J^i(x^\mu)O_i(x^\mu)}$$

$$\phi \leftrightarrow \mathrm{tr}\, G^2$$

Source is the coupling

$$\lambda_{'t} = g^2 N_c$$

$$T^{ij} \leftrightarrow \bar{\psi}^i \psi^j, \text{ with } i,j = 1,\ldots N_f$$

$$A^{ij}_{L\,\mu} \leftrightarrow \bar{\psi}^i(1+\gamma_5)\gamma_\mu\psi^j$$

$$A^{ij}_{R\,\mu} \leftrightarrow \bar{\psi}^i(1-\gamma_5)\gamma_\mu\psi^j$$

Stability at large densities

Our main motivation: instabilities in QCD

- At low temperatures, there is a new quantum critical regime at T = 0, with exotic properties which realize the symmetries of $AdS_2 \times \mathbb{R}^3$

Alho, Järvinen , Kajantie, Kiritsis, Rosen, Tuominen

- Such AdS_2 solutions are unstable as AdS_2 has a more restrictive BF bound.

- May compete with color superconducting/color-flavor locked phases

Spatial modulation often appears in condensed matter

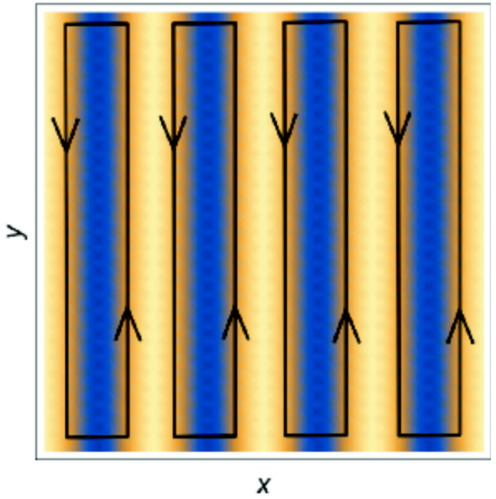
- Basic examples: **charge density wave** in which electrons form a standing wave pattern and can collectively carry an electric current and causing the charge density to become spatially modulated.

Modulated instabilities in holography

Typical example (Ooguri-Park): instability of gauge fields in the bulk

Domokos, Harvey 0704.1604
Nakamura, Ooguri, Park 0911.0679; Ooguri, Park 1007.3737

- Induced by a Chern-Simons (CS) term (needed to implement axial and flavour anomalies on the field theory)
- Field theory ground state has modulated persistent currents



Also appear in top-down constructions:

- Witten-Sakai-Sugimoto model

Chuang, Dai, Kawamoto, Lin, Yeh 1004.0162; Ooguri, Park

- D3-D7 and D2-D8 models, with 2+1 dimensional dual

Bergman, Jokela, Lifschytz, Lippert. ; Jokela, Järvinen, Lippert

AdS_2 regions in holography

Generic phenomenon: $AdS_2 \times \mathbb{R}^n$ appears in the extremal limit of charged black holes

- AdS_2 point $\leftrightarrow$ Emergence of a quantum critical region of the field theory, controlled by a (0+1) dimensional IR CFT

Faulkner, Liu, McGreevy, Vegh 0907.2694

Universal behavior of hydrodynamics at the AdS_2 points

- Convergence radius of hydrodynamic series set by crossing of quasinormal modes

- Near AdS_2 points, as $T \to 0$ one obtains universal results for the equilibration rates

Grozdanov, Kovtun, Starinets, Tadic 1904.01018, 1904.12862

$$\omega_{\rm eq} = 2\pi \Delta T \ , \qquad k_{\rm eq}^2 = \frac{\omega_{\rm eq}}{D}$$

Areán, Davison, Goutéraux, Suzuki 2011.12301

The V-QCD framework

- A bottom-up holographic model for QCD, but trying to follow principles from string theory closely.
- Full backreaction from flavor sector by Keeping $\frac{N_f}{N_c}$ fixed.
- The aim is to provide a general framework to modeling the physics of QCD as closely as possible with holography.
- Apart from confinement and chiral symmetry breaking, it describes, among other things, the hadron spectra, as well as physics at finite T and μ.
- In particular the equation of state at finite T and μ. A good description of the thermodynamics means a major improvement over simpler bottom-up models

Demircik, Ecker, Järvinen, *Phys.Rev.X* 12 (2022) 4, 041012

Our Set-up

We can define: $\mathbf{A}_{L\,MN} = g_{MN} + w(\phi) F^{(L)}_{MN}$ · A_L and A_R are N_f x N_f matrices in flavor space.

The action of the model is: $S = S_g + S_f + S_{\rm CS}$ with:

$$S_g = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R - \frac{4}{3} g^{MN} \partial_M \phi \partial_N \phi + V_g(\phi) \right]$$

$$S_f = -\frac{1}{32\pi G_5 N_c} \mathrm{Tr} \int d^4x\, dr \left(V_f(\lambda, T^\dagger T) \sqrt{-\det \mathbf{A}_L} + V_f(\lambda, T T^\dagger) \sqrt{-\det \mathbf{A}_R} \right)$$

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Comparison to data

V-QCD is constrained by fixing the potentials requiring qualitative agreement with QCD

- With a choice of Vg, with large λ asymptotics similar to what is found from noncritical string theory, the glue sector, IHQCD, confines and produces a mass gap for glueballs

- Fit to lattice data (Equation of State and baryon number susceptibility) near $\mu=0$

Gürsoy, Kiritsis, Mazzanti, Nitti; Järvinen, Jokela Remes

- UV expansions of potentials should match pQCD

Gürsoy, Kiritsis ; Järvinen, Kiritsis

- Extrapolated V-QCD for cold quark matter agrees with known constraints for the equation of state from astrophysical measurements

Järvinen, Jokela Remes

The Chern-Simons term

$$S_{\rm CS} = \frac{iN_c}{24\pi^2}\int \mathrm{Tr}\Big[-iA_L\wedge F_L\wedge F_L+\frac{1}{2}A_L\wedge A_L\wedge A_L\wedge F_L$$
$$+\frac{i}{10}A_L\wedge A_L\wedge A_L\wedge A_L\wedge A_L-(L\leftrightarrow R)\Big]$$

Writing $V=A_L+A_R/2$, and as the only non zero component in the background is $V_t(r)=\Phi(r)$ the CS term can be written as:

$$S_{\rm CS} = \frac{N_c}{2\pi^2}\int \Phi dt\wedge \mathrm{Tr}\left[d\tilde{A}\wedge d\tilde{V}\right]$$

Quasinormal modes in Holography

Small perturbations of a BH on the form of damped oscillations (QNM)

$$\sim e^{-i\omega t+iqx}$$

- QNM describe the late time evolution of any dynamical process that results in a BH at equilibrium
- Can also become unstable by showing exponential increase.
- The holographic duality identifies QNM with the poles of Green's functions supplying information about the theory's transport coefficients.

Quasinormal modes in Holography

Modes classified according to helicity (rotations keeping q fixed)

– helicity 0 –

Abelian

$\delta\hat{V}_t, \delta\hat{V}_z, \delta\hat{A}_t, \delta\hat{A}_z, \delta g_{tz}, \delta g_{tt}, \delta g_{zz}, \delta g_{xx} + \delta g_{yy}, \delta\phi, \delta\tau, \delta\hat{a}$

Non-Abelian

$\delta V_t^a, \delta V_z^a, \delta A_t^a, \delta A_z^a$

GIVs: $\{Z_0, \hat{E}_V, Z_\phi\}$ $\{Z_a, \hat{E}_A\}$ $\{\delta\tau\}$ $\{E_V^a\}$ $\{E_A^a\}$

– helicity ± 1 –

Abelian

$\delta\hat{V}_\alpha, \delta\hat{A}_\alpha, \delta g_{t\alpha}, \delta g_{z\alpha}$

Non-Abelian

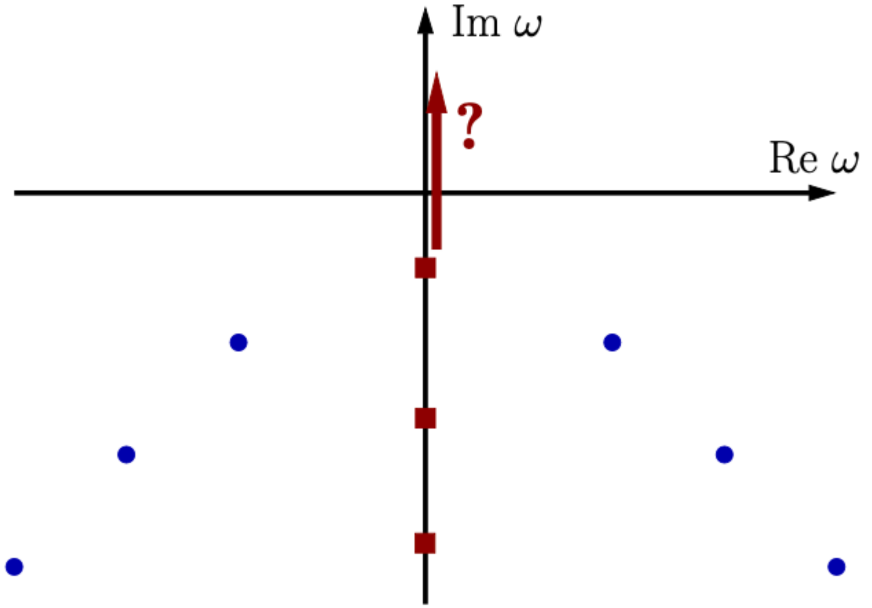
$\delta V_\alpha^a, \delta A_\alpha^a$

GIVs: $\{Z_1^\pm, \delta\hat{V}^\pm, \delta\hat{A}^\pm\}$ $\{\delta A_{L\alpha}{}^a\}$ $\{\delta A_{R\alpha}{}^a\}$

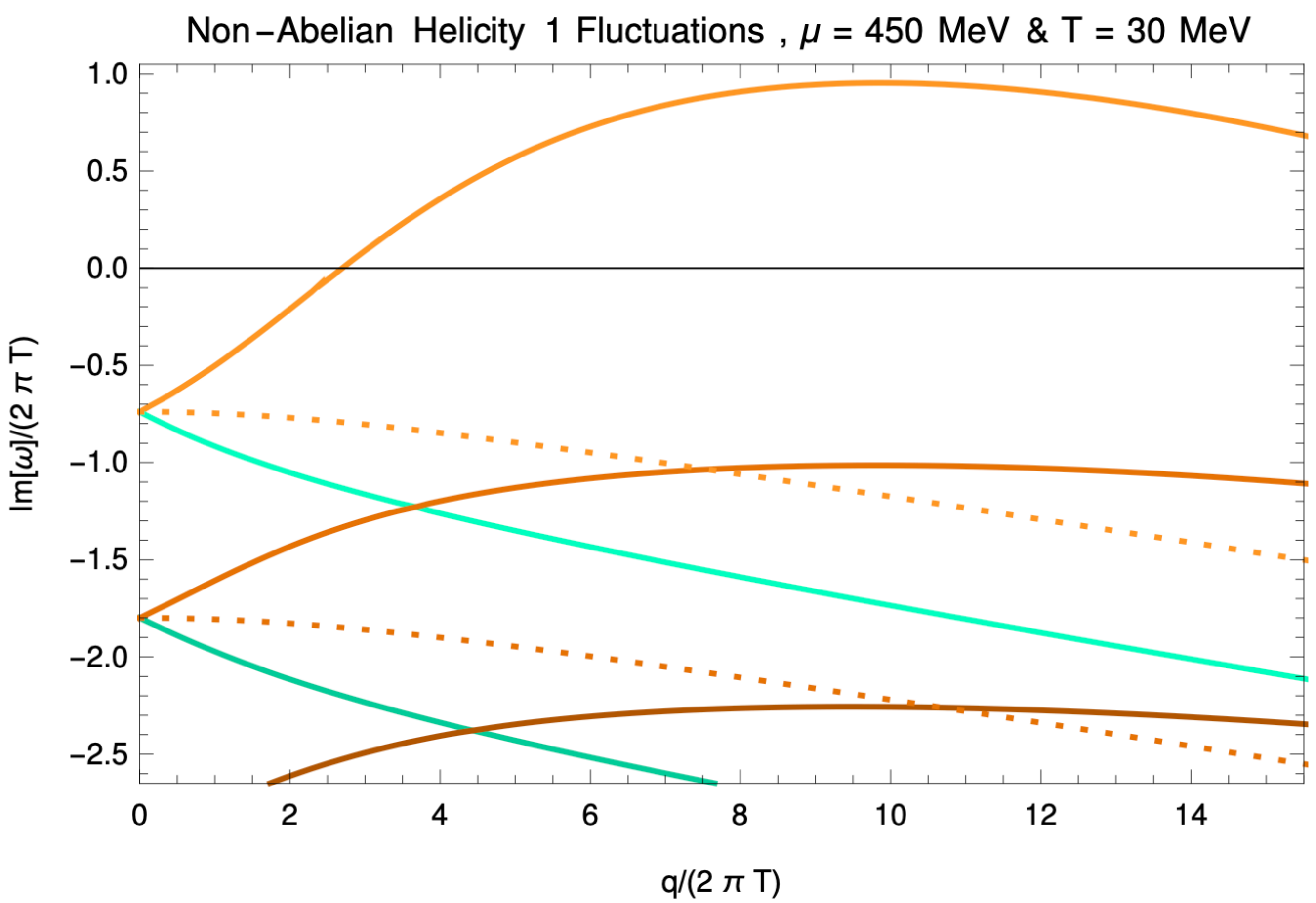
– helicity ± 2 –

$\delta g_{xy}, \delta g_{xx} - \delta g_{yy}$

GIVs: $\{Z_2^\pm\}$



Instability



Result in the non-Abelian sector:

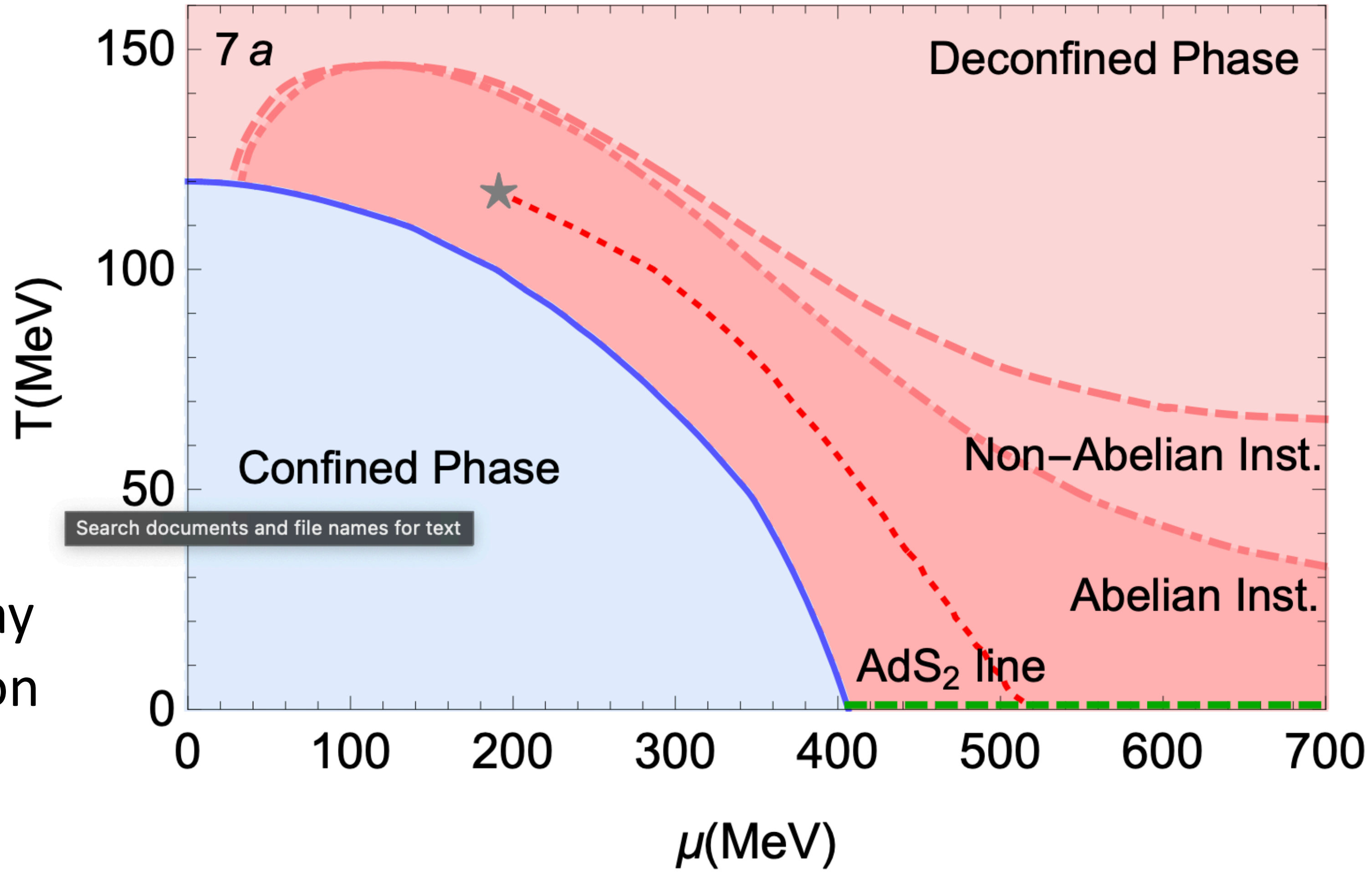
- Plain Ooguri-Park instability
- Disappears if the CS term is turned off (dotted curves)

Region with instability

- Strong instability when parameters fitted to QCD

- Surprise: extends to very low densities.

- Caveat: holographic description may be unreliable in the transition region



Approach to AdS_2

Geometric description of the flow:

Large T: “regular” black holes

- AdS_5 (UV) $\rightarrow$ horizon (IR)

Small T:

- AdS_5 (UV) $\rightarrow$ Small AdS_2 BH $\times R^3$ (IR)

Zero T:

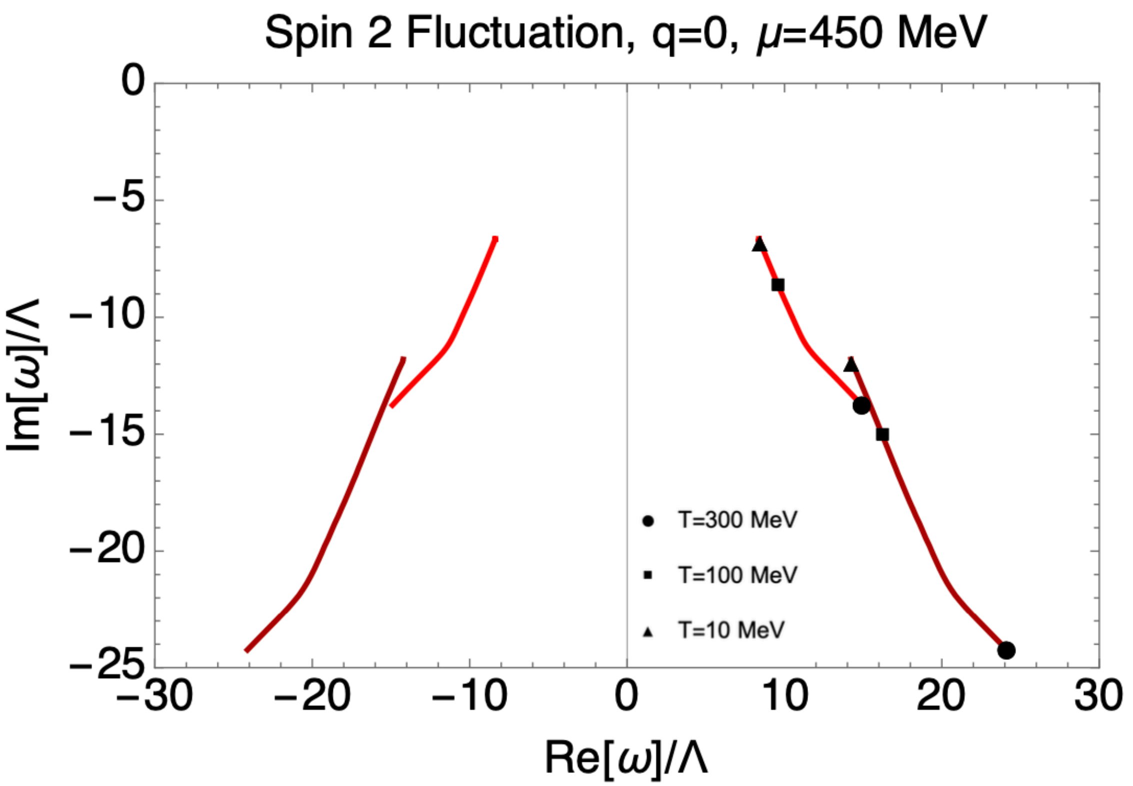
- AdS_5 (UV) $\rightarrow AdS_2 \times R^3$ (IR)

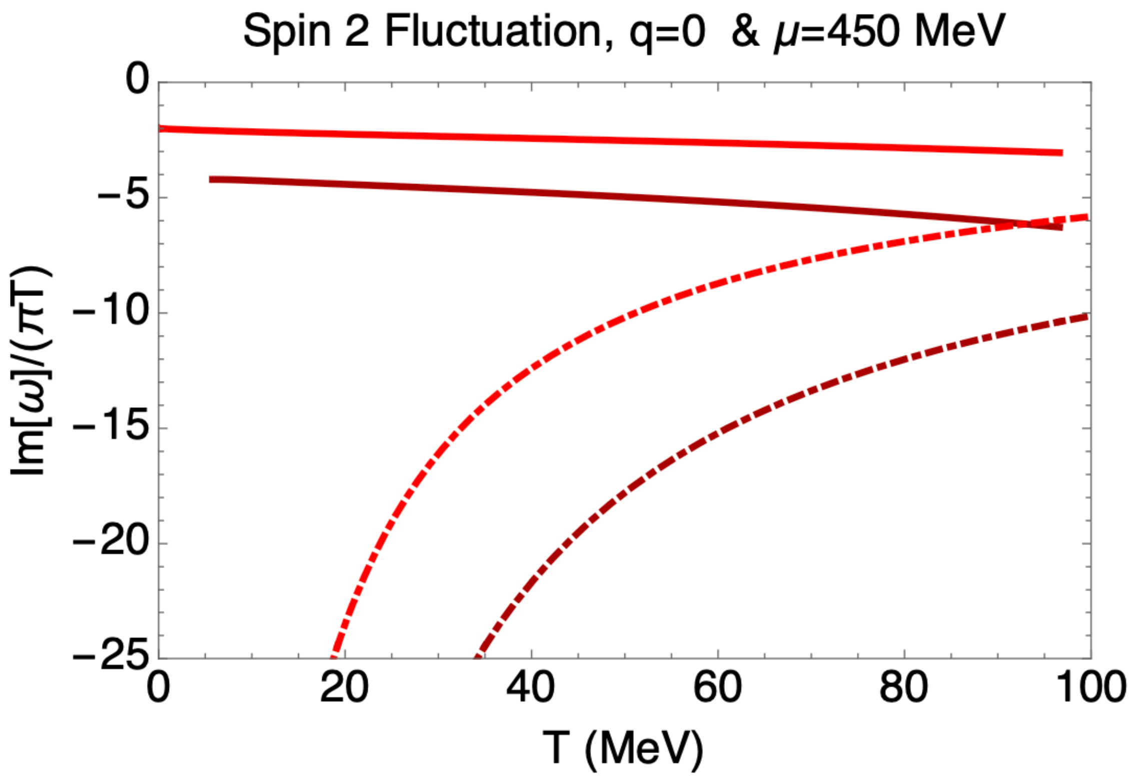
  The endpoint of the flow (AdS_2) is μ-independent

Faulkner, Liu, McGreevy, Vegh 0907.2694
Alho, Jarvinen, Kajantie, Kiritsis, Rosen, Tuominen;
Hoyos, Jokela, Jarvinen, Subils, Tarrio, Vuorinen

Expansions around the IR and UV known analytically in all cases

QNM modes as a function of T





As T is lowered:

- Complex modes obey Imω ~ μ ~ Λ (= the UV scale)
- Imaginary "AdS2" modes obey Im ω ~ T
- ⇒ imaginary modes closer to real axis, more important

QNMs in the AdS$_2$ region: zoom into $T \to 0$

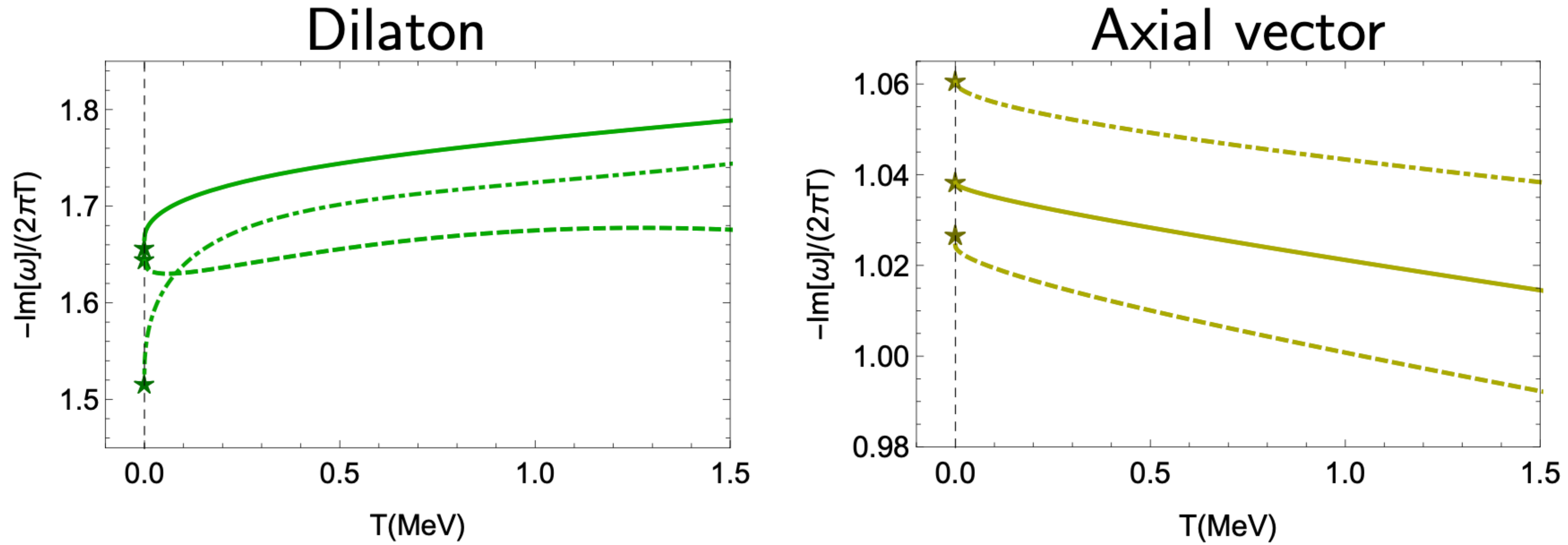
All imaginary "AdS$_2$ modes" arrange into towers as $T \to 0$, (Δ depends on the fluctuation)

$$\frac{\omega}{2\pi T} \longrightarrow -i\,(\Delta + n)\ , \qquad n = 0, 1, 2, \ldots$$

Faulkner, Iqbal, Liu, McGreevy, Vegh 1101.0597

Integer Δ: "universal" modes (metric $\Delta = 0$ or 1, vector $\Delta = 2$)

Δ is known analytically: obtained by considering fluctuations for the AdS$_2$ metric



Summary

- A strong Ooguri-Park instability observed
  - Extends to low density, where the model is determined through fit to QCD lattice data

- We have analyzed the QNM for all sectors as a function of momentum, temperature and for different fixed values of chemical potential and found the behavior at finite momentum of the modes.

- Detailed study of the approach to zero temperature and the AdS_2 region
  - QNMs mostly obey expected zero T limit – dilaton behavior “anomalous” f
  - Universal breakdown of hydrodynamics (in unstable phase)

- Future Projects
  - Study competition with color superconducting phases
  - Study correlators: extract residues at poles near the AdS_2 point

¡Muchas Gracias!

Method for localization of QNM

QNM spectrum comes from solving an ODE eigenvalue problem

We use the pseudospectral method to find the QNM. The solution is approximated as a weighted sum of a set of basis functions –> translate the problem to a system of algebraic equations.

The essence of pseudospectral method is the discretization of differential equations to turn it to numerically solvable matrix equations

We locate the poles of $\ln\left(|\det M| \right)$ where M is a matrix constructed with the coefficient of the fluctuation equations, the field fluctuations and the matrix of differential operators on the grid.

Definition of the scale

Usually in bottom-up models of QCD one makes sure that the leading behavior of the bulk fields near the boundary agrees with the leading UV dimensions of the dual operators.

Here we also require that the first few quantum corrections, and the RG flow imposed by them, agrees with the near-boundary holographic RG flow of the bulk fields

The boundary conditions for T are such that it vanishes near the boundary. It also turns out that the gauge field is irrelevant for the boundary behavior of the metric

Setting T= 0 in the action the geometry is determined by the effective potential $V_{\text{eff}} = V_g - xV_f$

For the geometry to be asymptotically AdS at the boundary we need. $V_{\text{eff}}(\lambda) \to \frac{12}{\ell^2}$ as $\lambda \to 0$

Definition of the scale Λ

Assuming a Taylor series around. λ= 0 we get that the near boundary asymptotics of the geometry is AdS5 with logarithmic corrections

$$A(r) = -\log\frac{r}{\ell_0} + \frac{4}{9\log(r\Lambda)} + \frac{\left(\frac{95}{162} - \frac{32v_2}{81v_1^2}\right) + \left(-\frac{23}{81} + \frac{64v_2}{81v_1^2}\right)\log(-\log(r\Lambda))}{(\log(r\Lambda))^2} + \mathcal{O}\left(\frac{1}{(\log(r\Lambda))^3}\right)$$

$$\frac{v_1\lambda(r)}{\lambda_0} = -\frac{8}{9\log(r\Lambda)} + \frac{\left(\frac{46}{81} - \frac{128v_2}{81v_1^2}\right)\log(-\log(r\Lambda))}{(\log(r\Lambda))^2} + \mathcal{O}\left(\frac{1}{(\log(r\Lambda))^3}\right).$$

The source term of the dilaton has become logarithmically flowing instead of a constant, and the value of the source is now identified with the scale $\Lambda = \Lambda_{UV}$. This scale defines the units for all dimensionful quantities