Viajando a lo largo de trayectorias cuánticas con Axel Weber

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En colaboración con **Christian Schubert** (FISMAT), **Anabel Trejo** (UAC), **Idrish Huet** (UAC), **Urs Gerber** (UNAM), **Ivan Ahumada** (Plymouth) y **Axel Weber** (IFM)

Trabajo realizado con Axel

The Yukawa potential: ground state energy and critical screening ← christian PM.

James P. Edwards, Urs Gerber, Christian Schubert, Maria Anabel Trejo, Axel Weber
Jun 29, 2017

16 pages
Published in: *PTEP* 2017 (2017) 8, 083A01

Integral transforms of the quantum mechanical path integral: hit function and path averaged potential

James P. Edwards (IFM-UMSNH, Michoacan), Urs Gerber (IFM-UMSNH, Michoacan and Mexico U., ICN), Christian Schubert (IFM-UMSNH, Michoacan), Maria Anabel Trejo (Jena U., TPI and IFM-UMSNH, Michoacan), Axel Weber (IFM-UMSNH, Michoacan)
Sep 14, 2017

7 pages
Published in: *Phys.Rev.E* 97 (2018) 4, 042114

Applications of the worldline Monte Carlo formalism in quantum mechanics

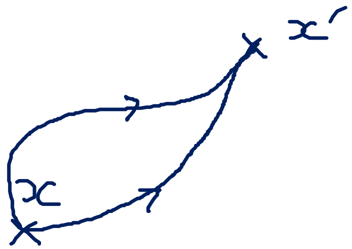
James P. Edwards, Urs Gerber, Christian Schubert, Maria Anabel Trejo, Thomai Tsiftsi, Axel Weber
Mar 1, 2019

13 pages
Published in: *Annals Phys.* 411 (2019) 167966

- Today – just illustrate Axel's ideas – few technical details!

1. Problemas en la mecánica cuántica

- Quantum mechanics – Feynman's formulation:



$$\hat{U}(T) = e^{-i/\hbar\, T} \to K(x',x;T) = \langle x'|\hat{U}(T)|x\rangle$$

$$K(x',x;T) = \int_{x(0)=x}^{x(\tau)=x'} \mathcal{D}x(\tau)\, e^{i/\hbar \int_0^T d\tau\, [m\dot{x}^2/2 - V(x(\tau))]}$$

- Reminiscent of Fermat's principle

$$\sum_{\text{Paths } \{x\}} e^{i/\hbar\, S[x]}$$

- Optical distance ⟺ Classical action

$$K(x',x;T) = \sum\!\!\!\!\!\!\int \Psi_E(x')\,\Psi^*(x)\, e^{i/\hbar\, E T}\, dE$$

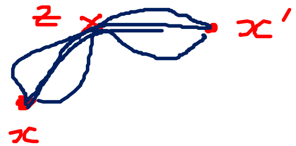
- Spectral representation

$$\hat{H}|\Psi_E\rangle = E|\Psi_E\rangle$$

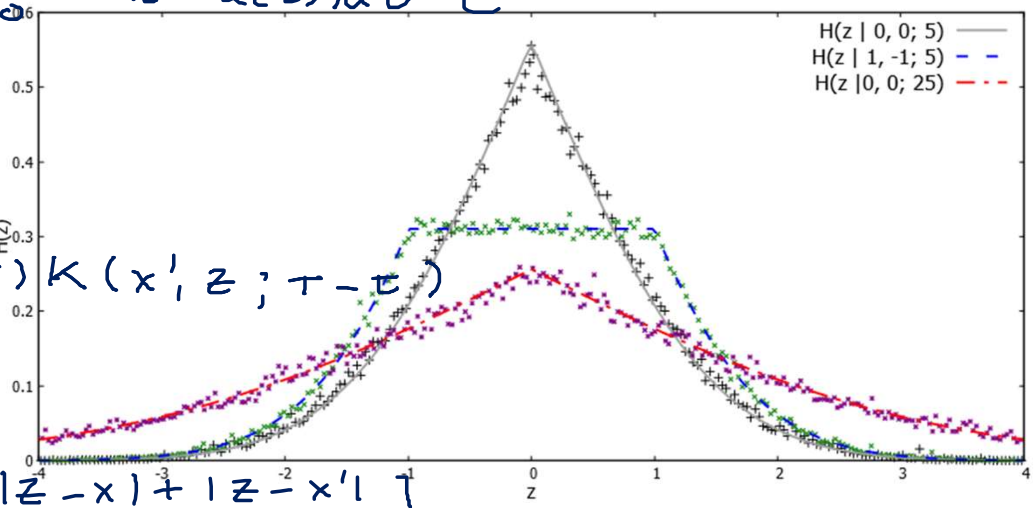
1. Problemas en la mecánica cuántica

- Constrained path integral Phys. Rev. **E 97** (2018) 4, 042114 [1709.04984 [quant-ph]]

"Hit Function"



$$\mathcal{H}(z|x',x;T) = \frac{1}{T}\int_{x(0)=x}^{x(T)=x'} \mathcal{D}x(\tau) \int_0^T \delta^D(z - x(\tau))\, d\tau \; e^{-S[x]}$$



- Integral transform of propagator (kernel)

$$\mathcal{H}(z|x',x;T) = \frac{1}{T}\int_0^T d\tau \, K(z,x;\tau)\, K(x',z;T-\tau)$$

- Example – free particle (V(x) = 0)

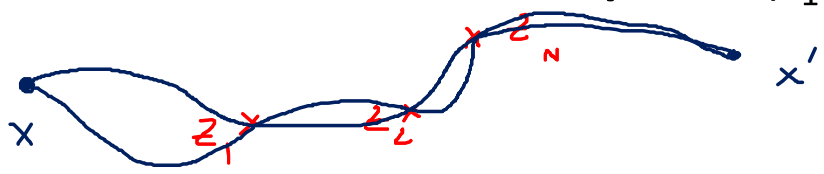
$$\mathcal{H}_0(z|x',x;T) = \frac{\pi}{T}\, \mathrm{Erfc}\left[\frac{|z-x|+|z-x'|}{\sqrt{2T}}\right]$$

- Recovering the kernel $K(x',x;T) = \int d^D z \, \mathcal{H}(z|x',x;T)$

1. Problemas en la mecánica cuántica

- Generalisation – “N-hit Function” Phys. Rev. **E 105** (2022) 6, 064132 [2110.04969 [quant-ph]]

- Trajectories constrained to “**hit**” **N points** (z_1, … z_N)



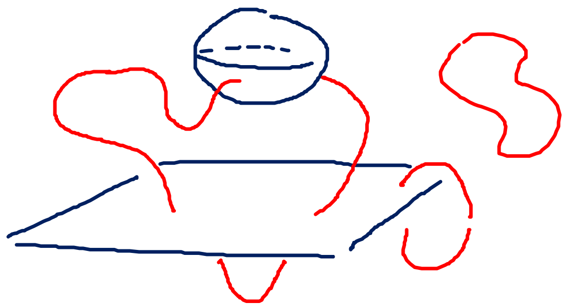
Polyakov

Amplitudes in QFT

- All N-hit Functions calculated (arbitrary N)!

$$\mathcal{H}_0(z_1,\cdots,z_n|y,x;T) = \mathcal{C}_{D,n}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\mathrm{e}^{-i\omega T}}{(-i\omega)^{\frac{(n+1)\nu}{2}}}K_{-\nu}(\sqrt{-i\omega}|x-z_1|)\cdots K_{-\nu}(\sqrt{-i\omega}|z_n-y|)\,.$$

- Related to the **Casimir effect**



1. Problemas en la mecánica cuántica

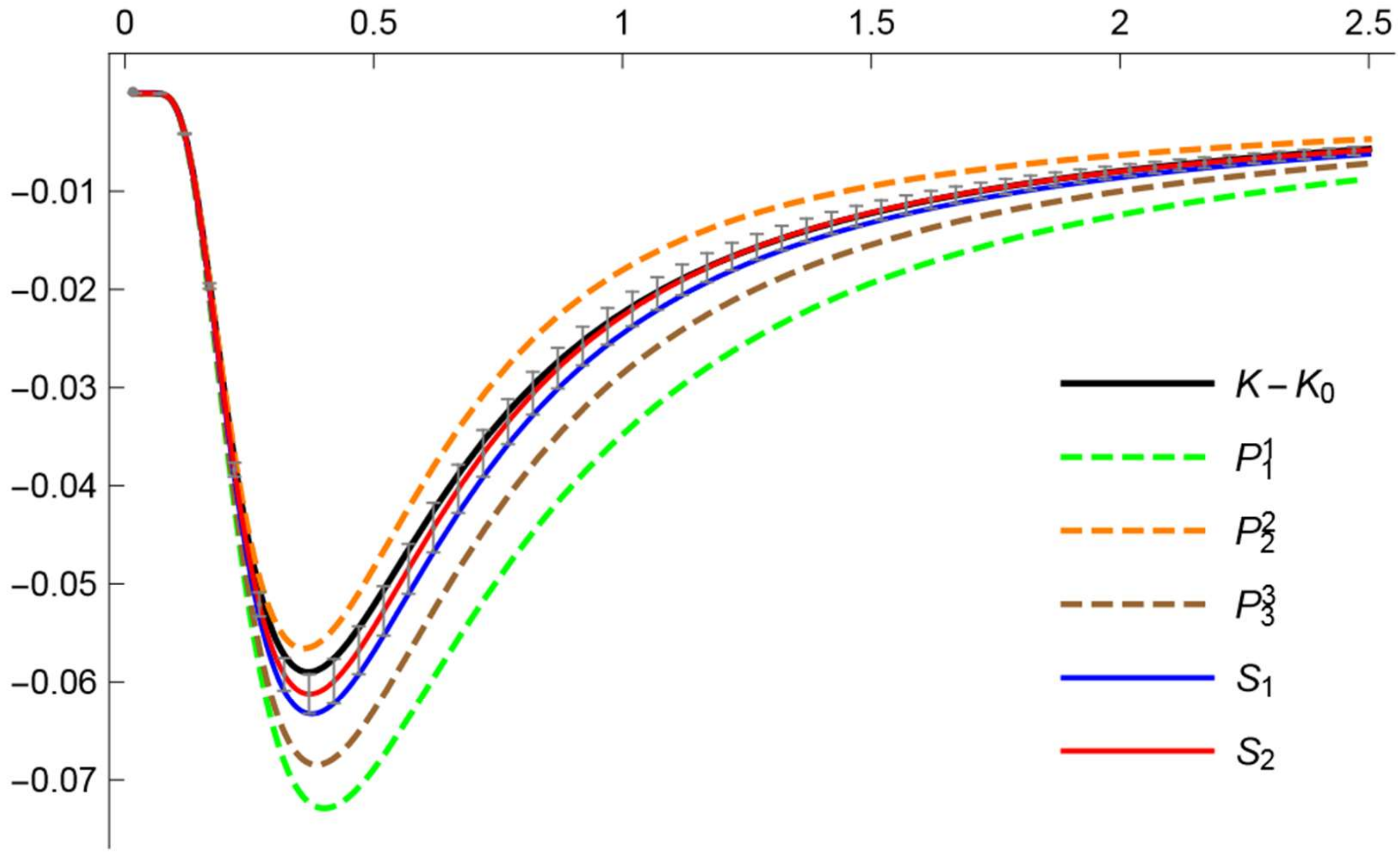
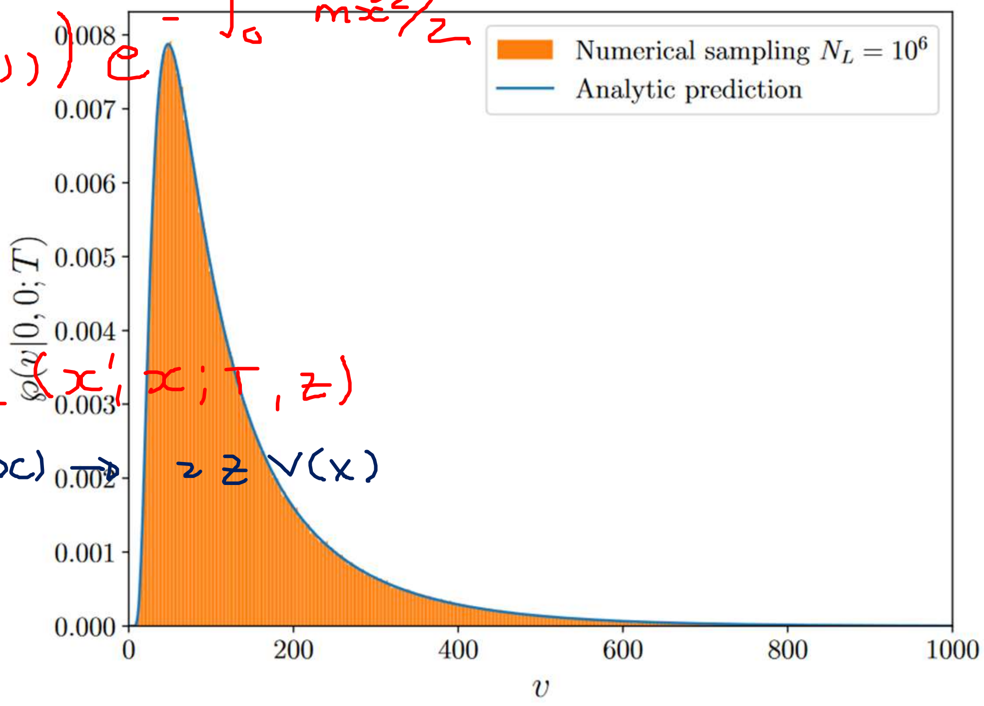


FIG. 3. Center-to-center propagation amplitudes inside a sphere, comparing approximations of various orders against the known analytic results (black solid line)

1. Problemas en la mecánica cuántica

- Constrained path integral Phys. Rev. **E 97** (2018) 4, 042114 [1709.04984 [quant-ph]]

$$\bar{P}(v) = \int \mathcal{D}x(\tau)\, \delta\left(v - \int_0^T d\tau\, V(x(\tau))\right) e^{-\int_0^T m\dot{x}^2/2}$$



- **Integral transform** of the propagator

$$\bar{P}(v) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dz\, e^{ivz}\, \hat{K}(x', x; T, z)$$

$$V(x) \to z\, V(x)$$

- Example – el **oscilador armónico**

$$\mathcal{H}(z|y,x;T) = \int_0^T \frac{d\tau}{T\left(2\pi G_M(\tau,\tau)\right)^{\frac{D}{2}}} e^{-\frac{(z-x_c(\tau))^2}{2G_M(\tau,\tau)}}$$

- Recovering the kernel

$$K = \int_{-\infty}^{\infty} dv\, P(v)\, e^{-v}$$

2. Simulaciones numéricas

- Discretised path integral

$$\int \mathcal{D}x(t) \longrightarrow \boxed{\frac{1}{N_L}\sum_{i=1}^{N_L} e^{-U_i}}$$

$$\int V(x(t)) \longrightarrow \{U_i\} = \frac{1}{N_P}\sum_{k=1}^{N_P} V(x_k)$$

- **Monte Carlo** style estimation

$$K(x', x; T) = K_0(x', x; T) \left\langle e^{-U_i} \right\rangle$$

- Illustration – trajectories generated with one of **Axel's algorithms** - "LSOL"

Annals Phys. 411 (2019) 167966 [1903.00536 [quant-ph]]

- Energía del estado base $\cancel{K(x', x; T)} \cancel{=} \lim_{T \to \infty} -\frac{d}{dT} \ln(K) \sim E_0$

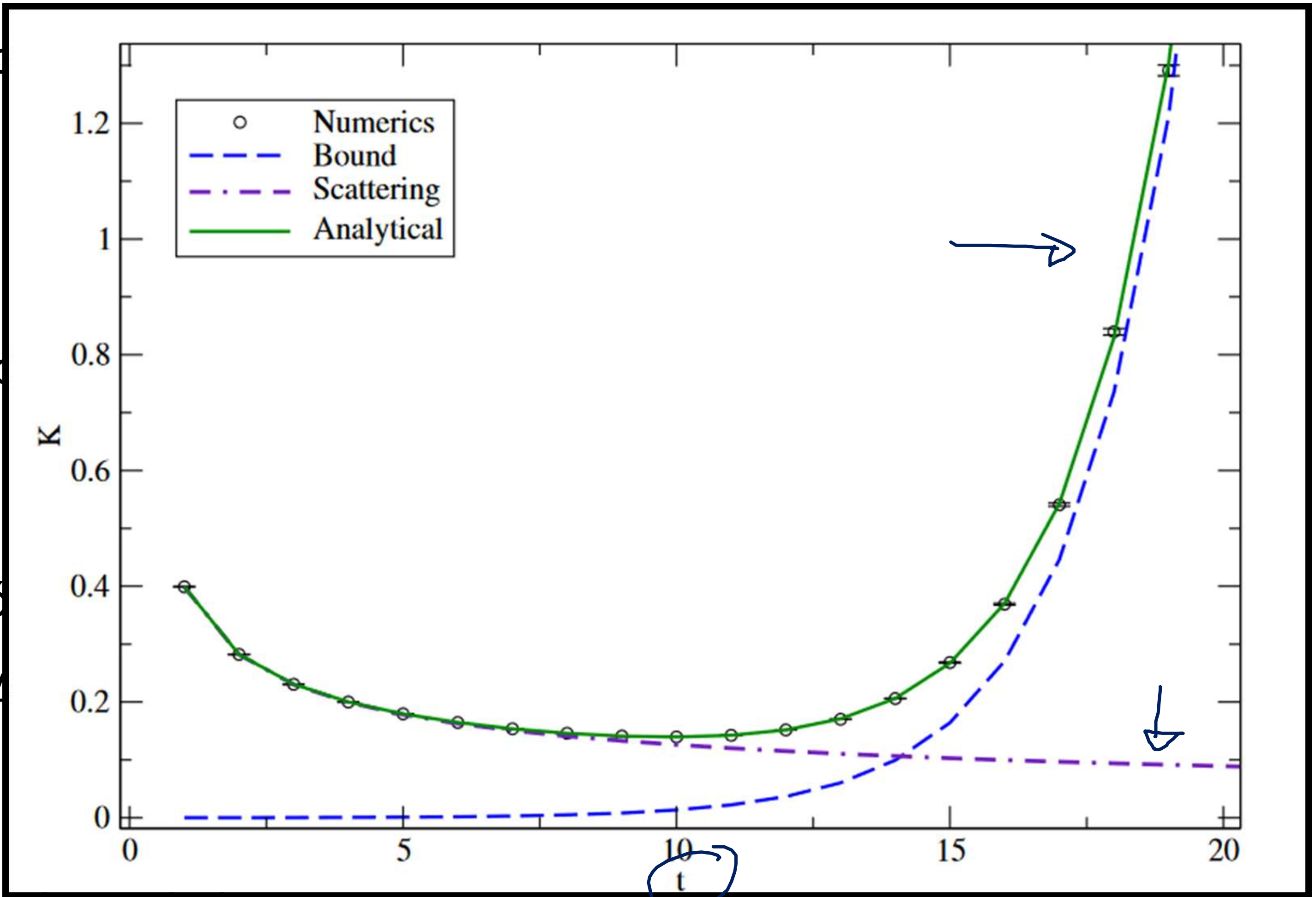
2. Simulaciones numéricas

- Integral d

- Estimació

- Ilustració

  Annals Phy



- Energía del estado base

2. Simulaciones numéricas

- Extension – trajectories generated in a background potential

  2304.10518 [quant-ph] – Aceptado por Phys. Rev. E

- Modifies the spatial support of the trajectories
- Needs to be compensated!

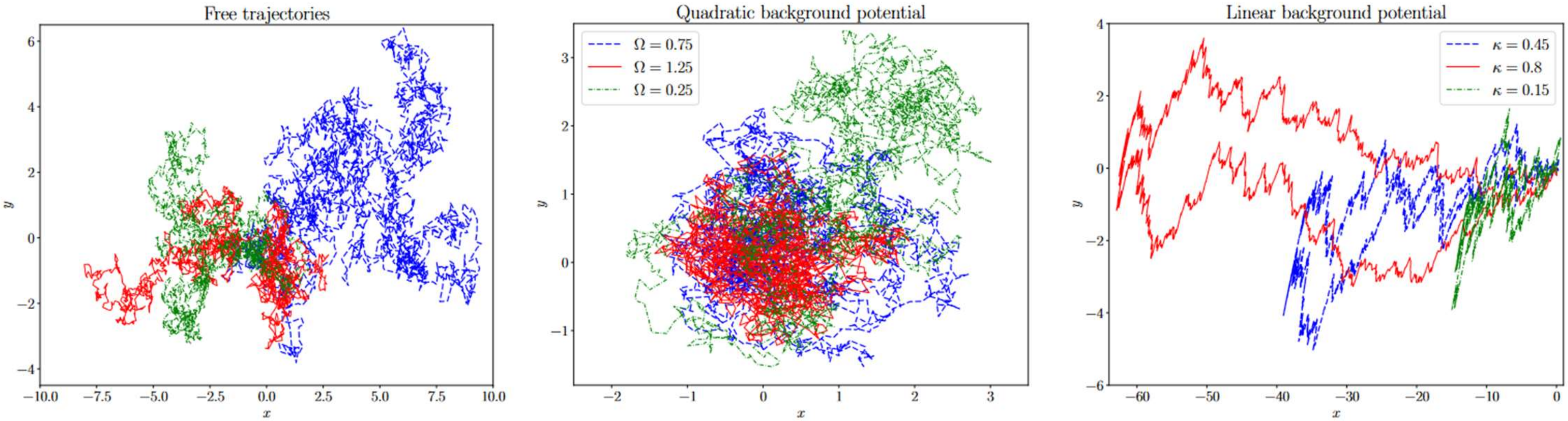
- Analytic compensation

- Numerical compensation

2. Simulaciones numéricas

- Extension – trajectories generated in a background potential

  2304.10518 [quant-ph] – Aceptado por Phys. Rev. E



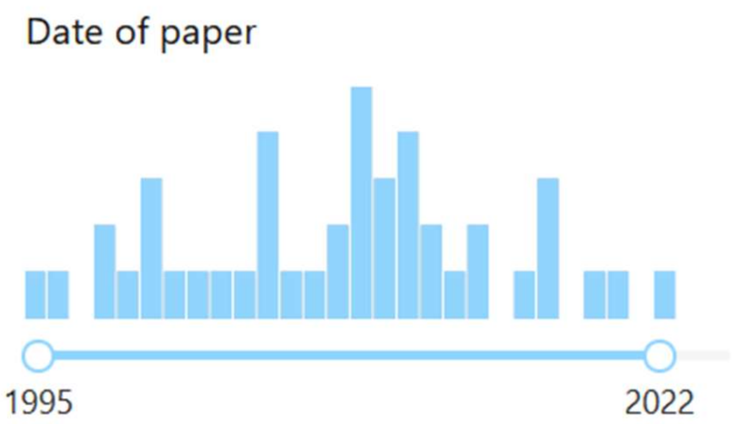
$U(x) = \frac{1}{2} m \Omega^2 x^2$ $\quad$ $U(x) = + \kappa x$

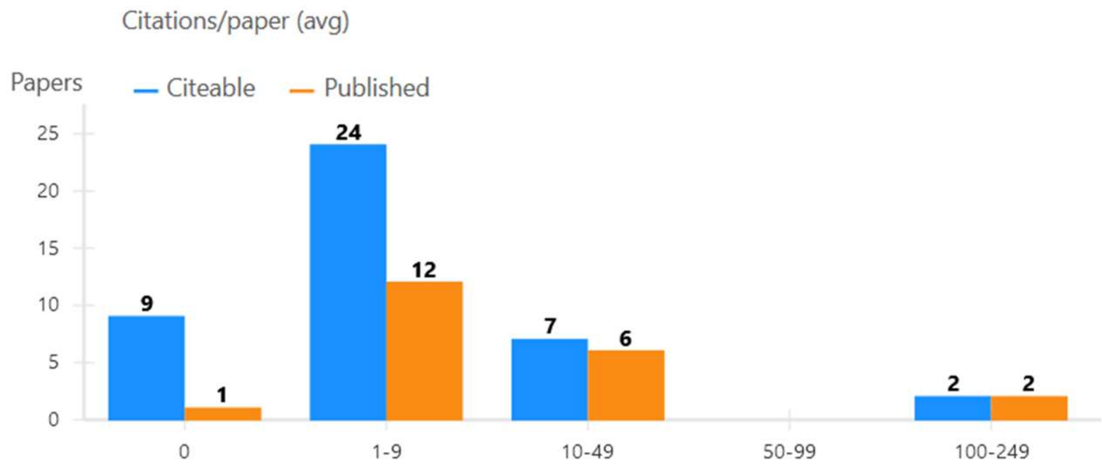
- Trayectorias generadas usando el algoritmos “**Y-loops modificado**”

Y – Loop ← Axel's algorithm.

Conclusiones

- Axel's research – encompasses a variety of topics!





- These problems pertain to fundamental physics – very deep ideas!
- His research continues to generate new questions

Gracias por su atención