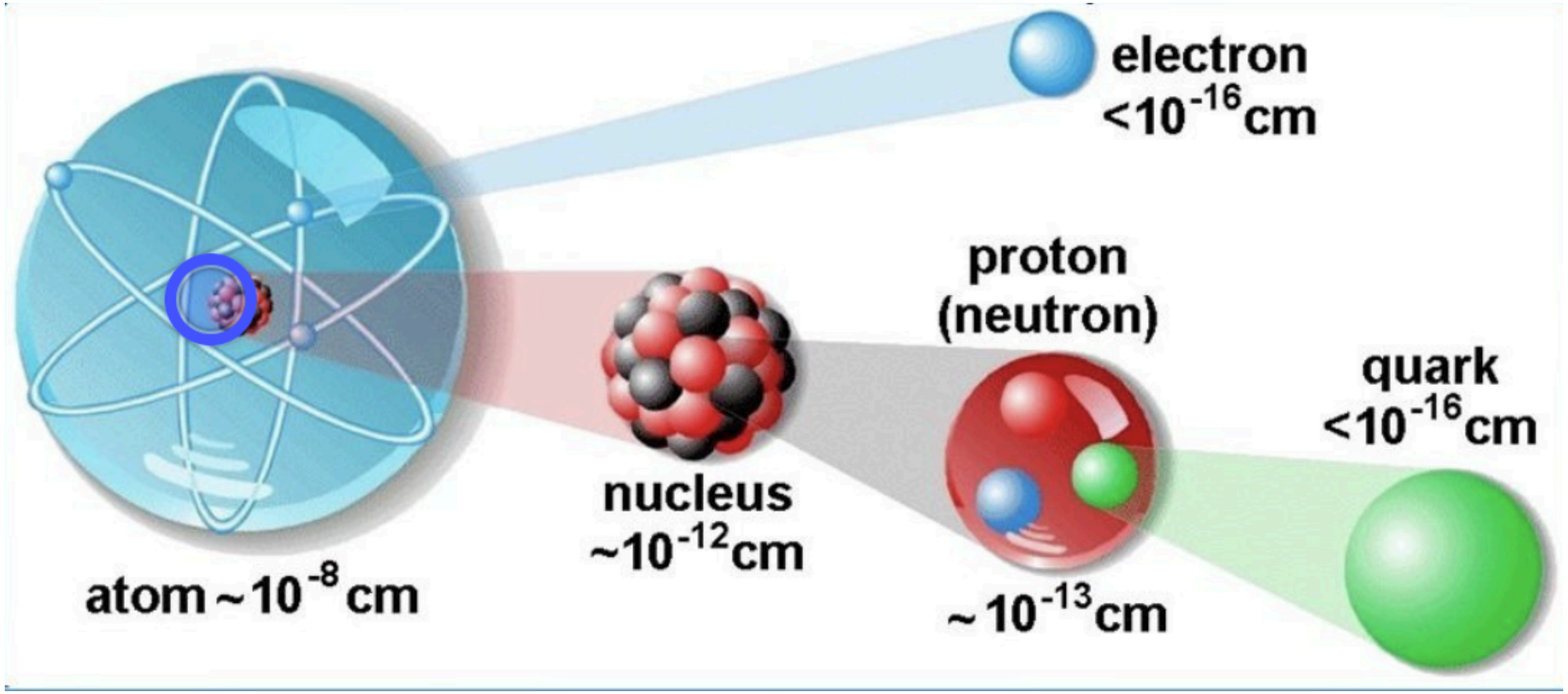
Non perturbative physics using perturbation theory

Learning the power of the Renormalization group with Axel Weber

Pietro Dall'Olio

Morelia 22th November 2023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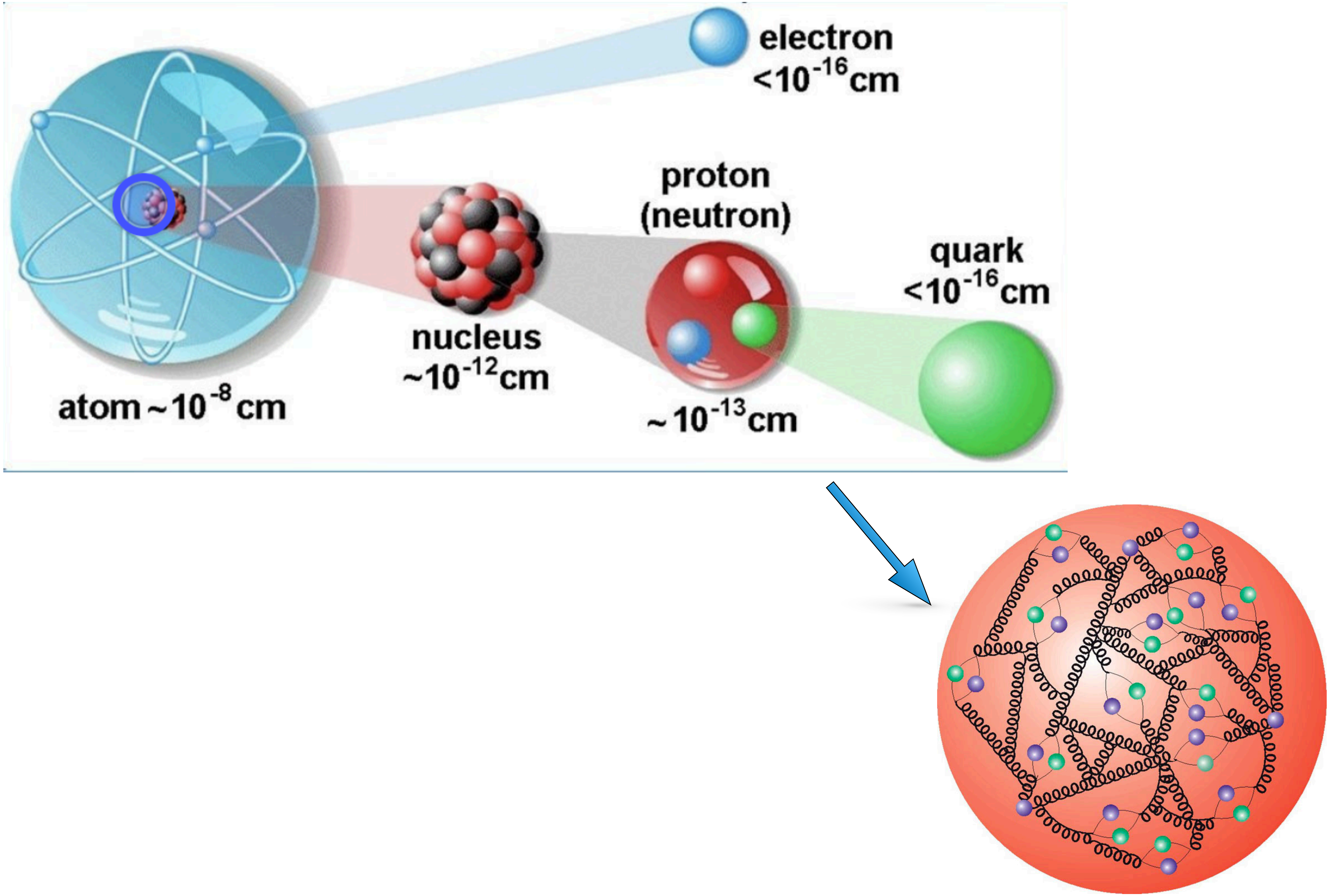
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QUANTUM CHROMODYNAMICS (QCD)

$$S_{FP} = \int d^D x \left[\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi} \left(\gamma_\mu D_\mu + m \right) \psi + i b^a \partial_\mu A^a_\mu + \frac{\xi}{2} b^a b^a + \partial_\mu \bar{c}^a D^{ab}_\mu c^b \right]$$

S_{QCD} S_{gf} FP gauge fixing

Non perturbative features:

- Dynamical colored degrees of freedom (quarks and gluons) are confined inside the hadrons.
- Dynamical generation of mass $\leftrightarrow$ Spontaneous breaking of chiral symmetry.
- Mass gap in Yang-Mills theory.

Infrared (IR) dynamics is perturbatively inaccesible with S_{FP} due to the presence of a Landau pole in the running coupling at Λ_{QCD}

Non perturbative approaches:

- Lattice Monte Carlo simulations.
- Functional equations: DSE, FRG

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Long-standing debate about the IR behavior of the gluon $G_A(p)$ and ghost $G_c(p)$ propagators in Yang-Mills theory (Landau gauge $\xi = 0$):

- scaling solution: $G_A(p) \underset{p\to 0}{\longrightarrow} (p^2)^{-1-\alpha_G}, \quad G_c(p) \underset{p\to 0}{\longrightarrow} (p^2)^{-1-\alpha_F}, \quad \alpha_G + 2\alpha_F = \frac{D-4}{2}$

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★ $\alpha_G = -\frac{D}{2}, \quad \alpha_F = \frac{D-2}{2}, \quad 2 < D < 4$

★★ $\alpha_G \approx -\frac{16-D}{10}, \quad \alpha_F \approx \frac{D-1}{5}, \quad 2 \leq D \leq 4$

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↓ found in DSE imposing $(p^2 G_c(p))^{-1}\Big|_{p^2=0} = 0$

- decoupling solution: $G_A(p) \underset{p\to 0}{\longrightarrow} const\,, \quad G_c(p) \underset{p\to 0}{\longrightarrow} \propto \frac{1}{p^2} \quad \Rightarrow \quad \alpha_G = 1\,, \quad \alpha_F = 0$

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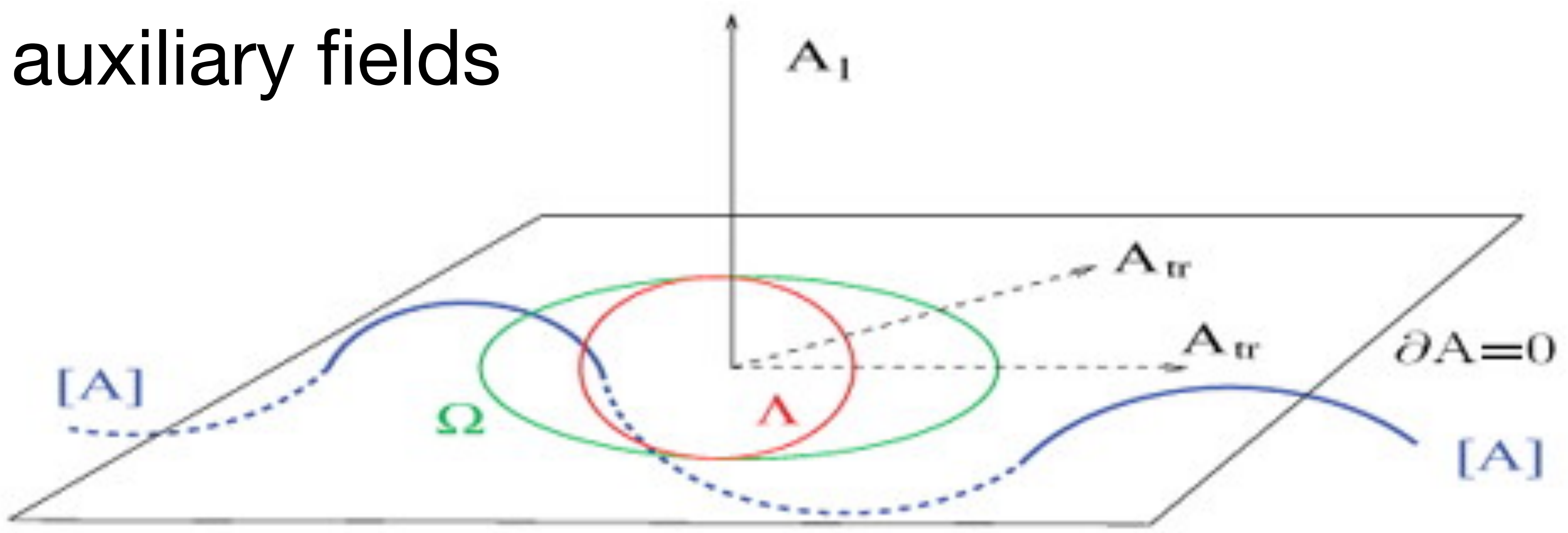
Gribov copies: FP gauge-fixing is not valid non-perturbatively $\rightarrow \partial_\mu A_\mu^U = 0$ has more solutions

GZ scenario: restrict to Gribov region $-\partial_\mu D_\mu[A] > 0$ introducing auxiliary fields



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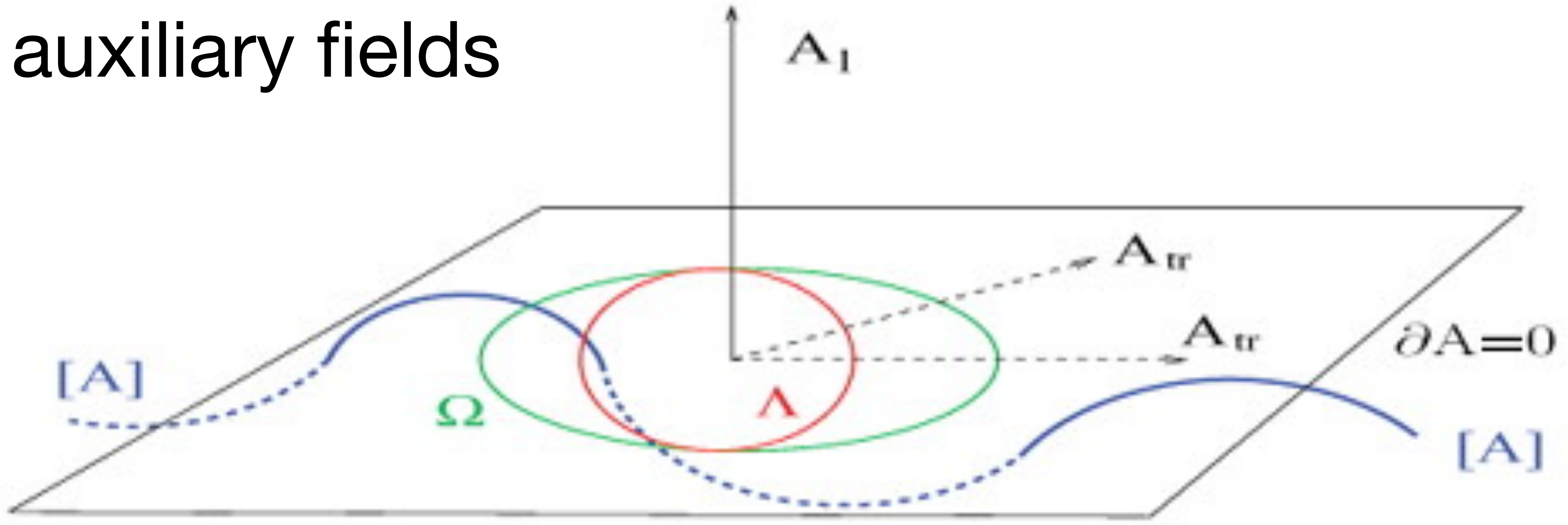

- No Landau pole
- BRST symmetry is softly broken
- Scaling type of propagators ★

$$G_A(p) = \frac{p^2}{p^4 + \lambda^4}\,, \quad G_c(p) \underset{p\ll\lambda}{\propto} \frac{1}{p^4}$$

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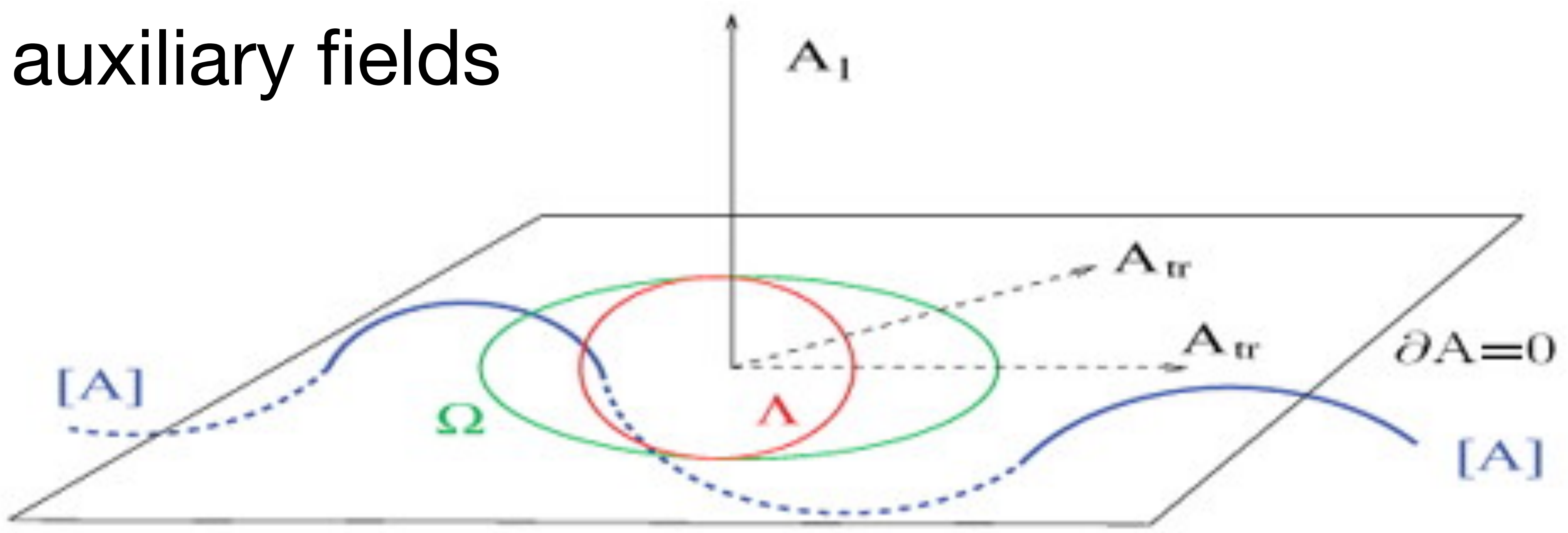

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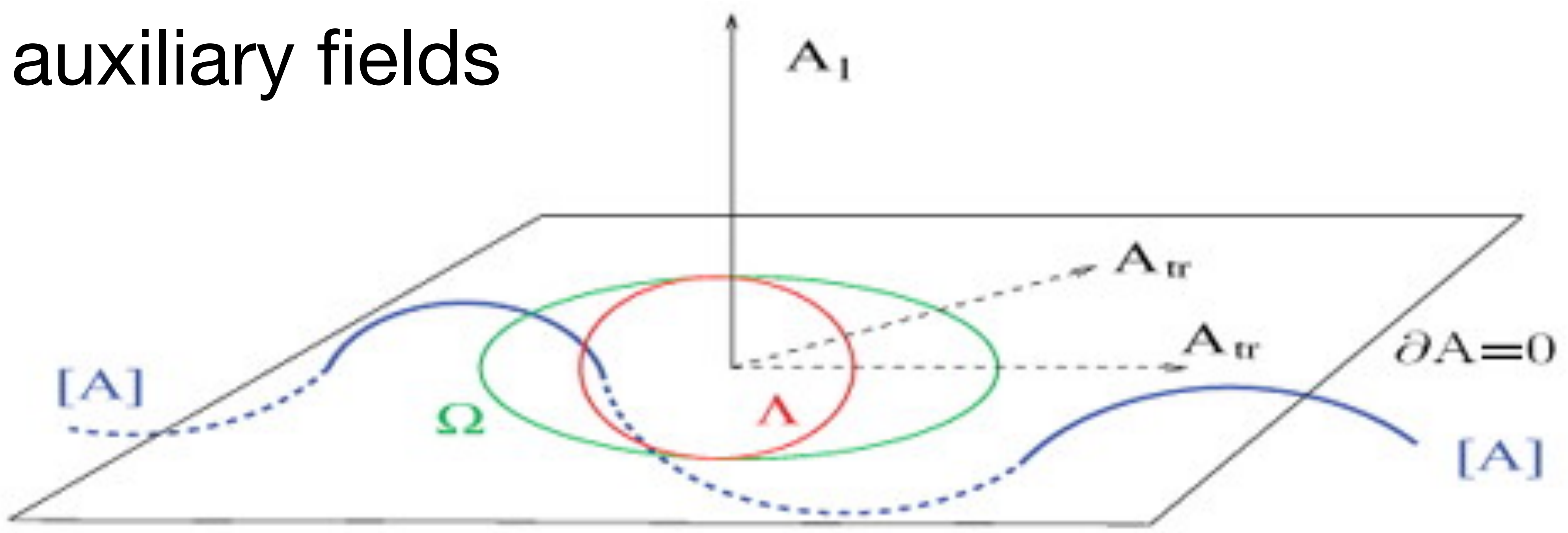

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$\swarrow \langle A_\mu^a A_\mu^a \rangle$ $\quad \searrow \langle \phi_\mu^{ab}\phi_\mu^{ab} - \omega_\mu^{ab}\omega_\mu^{ab} \rangle$

RENORMALIZATION GROUP

Framework that extracts the physical description of a model at different scales.
It is particularly effective in describing the scaling behavior close to a phase transition.

Wilson formulation

- Modes integration: $$e^{-S'[\varphi]} = \int \prod_{\Lambda_0/s \leq |k| \leq \Lambda_0} d\varphi(k)\, e^{-S[\varphi]}\,, \quad s > 1$$
- Rescaling: $$k' = sk\ (x' = x/s)\,, \quad \varphi'(x') = s^{d_\varphi}\varphi(x)\,, \quad d_\varphi = \frac{D-2+\eta}{2}$$

Callan-Symanzik formulation

$$\varphi_B = Z_\varphi^{1/2}\varphi\,, \quad g_B = Z_g g \quad \longrightarrow \quad \mu\frac{d}{d\mu}\Gamma_{\varphi_1\cdots\varphi_n}(p_1,\cdots,p_n,g(\mu),\mu) = \frac{n}{2}\gamma(\mu)\,\Gamma_{\varphi_1\cdots\varphi_n}(p_1,\cdots,p_n,g(\mu),\mu)$$

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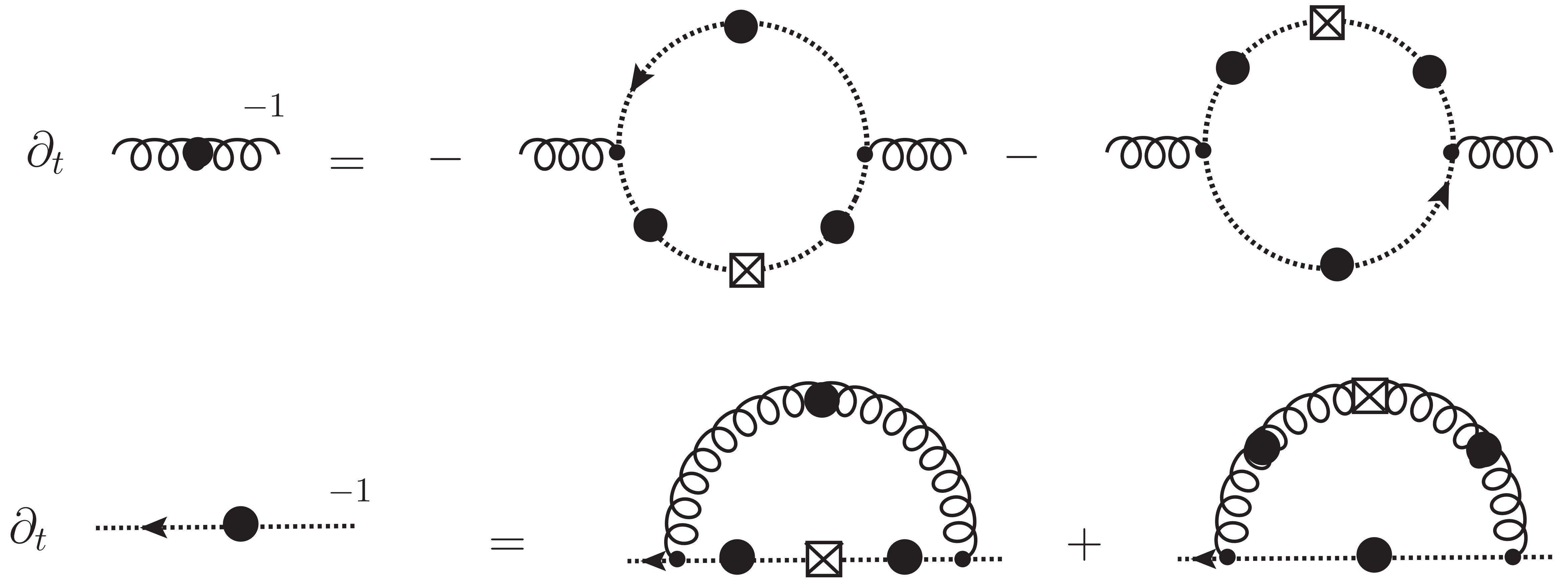
$$\gamma(\mu) = \frac{\mu}{Z_\varphi}\frac{dZ_\varphi}{d\mu}$$

FUNCTIONAL RENORMALIZATION GROUP

IR modes $p < k$ suppressed by a regulator $\Delta S_k = \frac{1}{2}\int_p \varphi(-p)\, R_k(p)\, \varphi(p)$, $\lim_{p/k\to 0} R_k(p) = \infty\,,\quad \lim_{k/p\to 0} R_k(p) = 0$

Flux equation: $\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\, \dot{R}_k \left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + R_k\right)^{-1}$ $\qquad t = \ln(k/k_0)\,,\quad \phi \equiv \langle\varphi\rangle_J$

Truncated equations for the propagators (IR ghost dominance)



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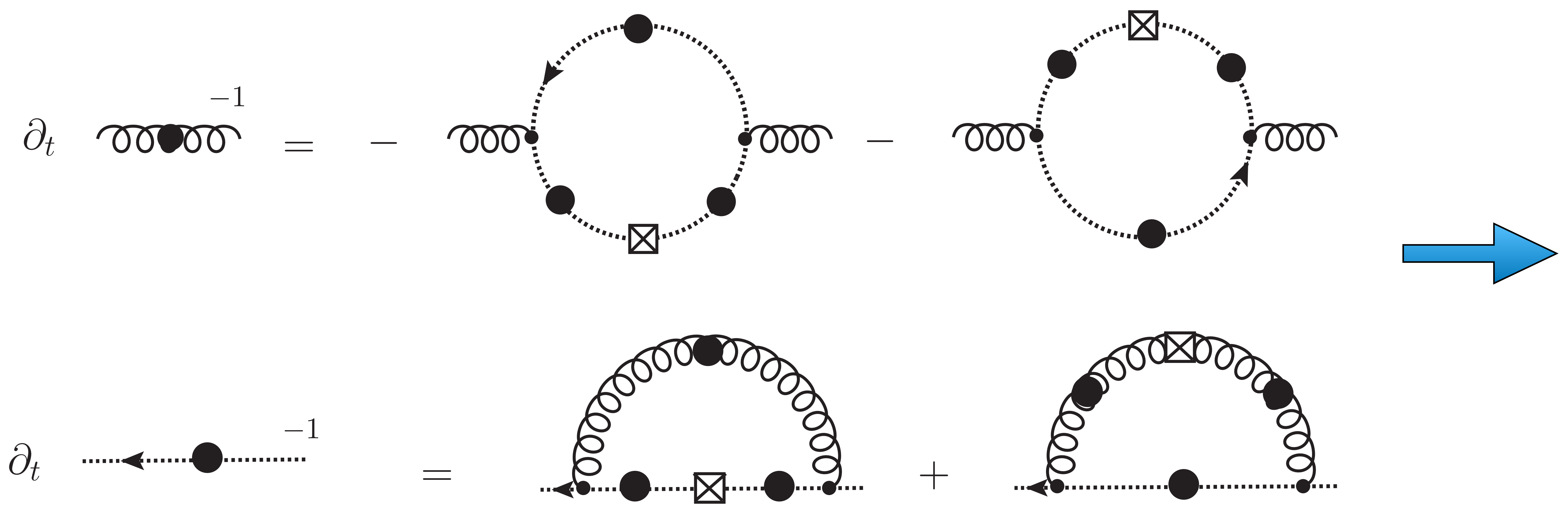

Converges to decoupling solution.

If $(p^2 G_c(p))^{-1}\Big|_{p^2=0} = 0$ is imposed, it converges to scaling solution ★★

IR Renormalization group analysis

A. Weber, *Epsilon expansion for infrared Yang-Mills theory in Landau gauge,* Phys. Rev. D **85** (2012)

Motivations:

- *M. Tissier, N. Wschebor*, Phys. Rev. D **84** (2011)

  Curci-Ferrari model in Landau gauge (action augmented with massive gluon term $\frac{m^2}{2} A_\mu^a A_\mu^a$) is IR-safe (no Landau pole). One-loop order resummed propagators in agreement with Lattice data.

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IR fixed point ($m^2 \gg p^2$):

$$S_{quad}^{IR} = \int d^D x \left[\frac{m^2}{2} A_\mu^a A_\mu^a + i b^a \partial_\mu A_\mu^a + \partial_\mu \bar{c}^a \partial_\mu c^a \right]$$

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→ Change in the scaling of $A^a_\mu(x) \to s^{D/2} A^a_\mu(sx)$ → Pure gluonic interactions become irrelevant:

$$g_{A^3} \to s^{-1-D/2}\, g_{A^3}\,, \quad g_{A^4} \to s^{-D}\, g_{A^4}$$

Ghost-gluon coupling becomes irrelevant at $D > 2$ $\qquad g_{\bar{c}Ac} \to s^{1-D/2}\, g_{\bar{c}Ac}$

Epsilon expansion at $D = 2$ (upper critical dimension) $\rightarrow D = 2 + \epsilon$

One-loop perturbation theory imposing the normalization conditions:

$$G_A(p^2 = \mu^2) = \frac{1}{m^2} \quad \Rightarrow \quad \gamma_A(\mu) = \mu^2 \frac{d}{d\mu^2} \ln Z_A = \frac{1}{2} \frac{N_c \bar{g}^2}{4\pi}$$

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Taylor NR theorem

$$\beta_{\bar{g}} = \mu^2 \frac{d\bar{g}}{d\mu^2} = \frac{1}{2} \bar{g} \left(\frac{\epsilon}{2} + \gamma_A + 2\gamma_c \right) = \frac{1}{2} \bar{g} \left(\frac{\epsilon}{2} - \frac{1}{2} \frac{N_c \bar{g}^2}{4\pi} \right)$$

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$$= \frac{1}{2} \bar{g} \left(\frac{\epsilon}{2} - \frac{1}{2} \frac{N_c \bar{g}^2}{4\pi} \right)$$

IR stable trivial fixed point $\frac{N_c \bar{g}^2(\mu)}{4\pi} = \frac{\left(\mu^2/\Lambda^2\right)^{\epsilon/2}}{1 + \left(\mu^2/\Lambda^2\right)^{\epsilon/2}} \epsilon$

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$$G_A(p) = \frac{1}{m^2} \frac{1 + \left(p^2/\Lambda^2\right)^{\epsilon/2}}{1 + \left(\mu^2/\Lambda^2\right)^{\epsilon/2}} \xrightarrow[\text{reinserting } p^2 A_\mu A_\mu]{D=4} G_A(p) = \left(p^2 + m^2 \frac{1 + \left(\mu^2/\Lambda^2\right)}{1 + \left(p^2/\Lambda^2\right)} \right)^{-1} \rightarrow \text{GZR without } \langle A_\mu^a A_\mu^a \rangle$$

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What about $D = 2$ and scaling solution ★★ ?

Locally imposing horizon condition $\left.(p^2 G_c(p))^{-1}\right|_{p^2=0} = 0$: $\partial_\mu \bar{c}(x)\partial_\mu c(x) \longrightarrow \frac{1}{b^2}\partial_\mu \bar{c}(x)(-\partial^2)\partial_\mu c(x)$

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→ $c(x) \to s^{D/2-2} c(x)$ → $D = 6$ upper critical dimension (expansion at $D = 6 - \epsilon$)

IR stable fixed point → Scaling solution with $\alpha_G = -\frac{18-D}{12}, \quad \alpha_F = \frac{5D-6}{24}$

close to (= at $D = 6$) ★★ $\alpha_G \approx -\frac{16-D}{10}, \quad \alpha_F \approx \frac{D-1}{5}$

- In a massive IR regime the decoupling solution is the stable one (physically realized). Confirmed by lattice results.

- Gribov type of scaling solution is found as IR unstable.

- An approximation of the second scaling solution is found as IR attractive only if the horizon condition is imposed. It is observed in the lattice for $D = 2$.

- Both decoupling and scaling solutions violate spectral positivity. In the case of the decoupling solution it is due to the IR growing of the gluon propagator (also observed in the lattice).

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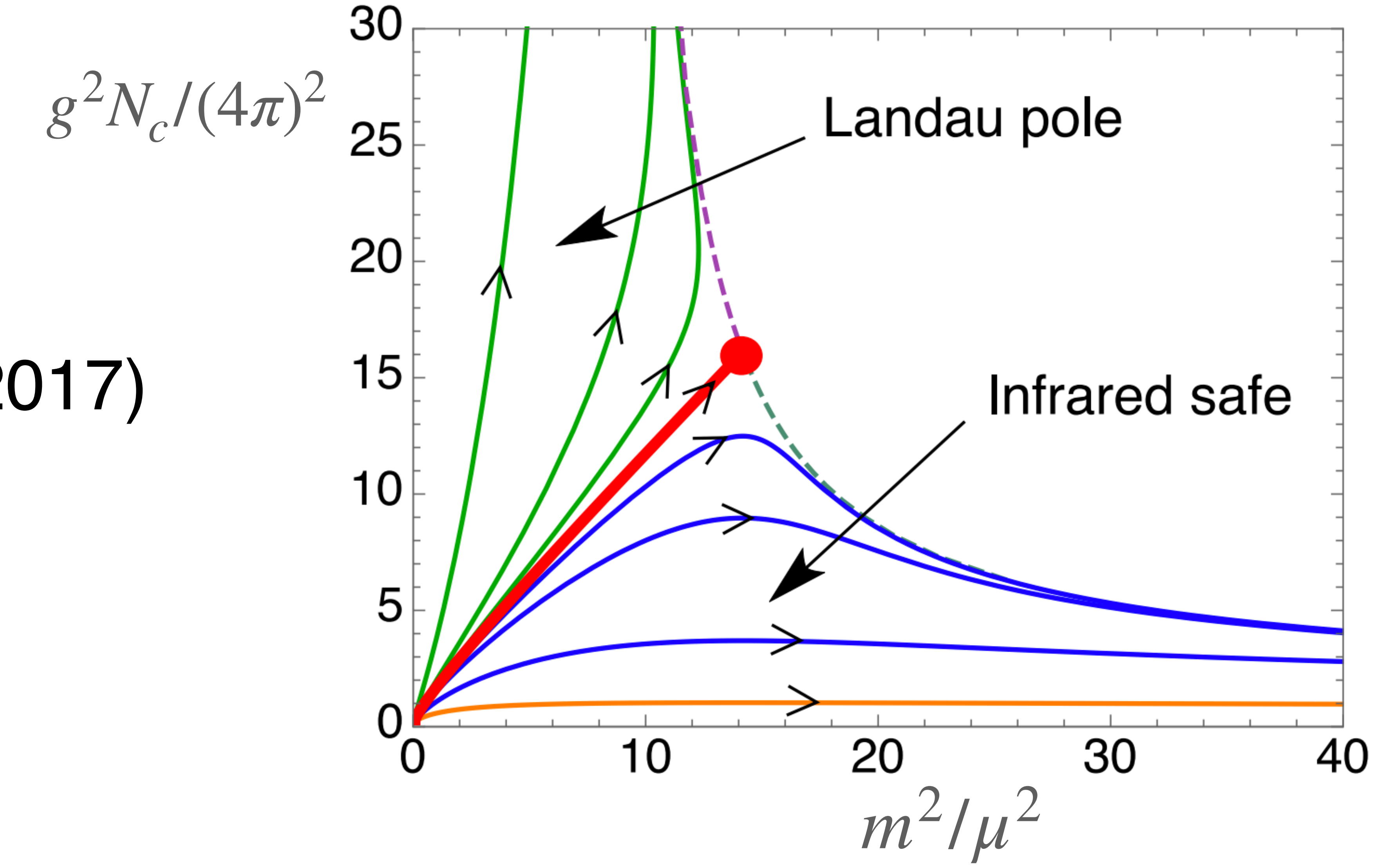
- Both decoupling and scaling solutions are found within the same IR massive regime (No BRST symmetry restoration for the scaling solution).

U. Reinosa et al., Phys. Rev. D **96**, 014005 (2017)



IR - UV Renormalization group analysis

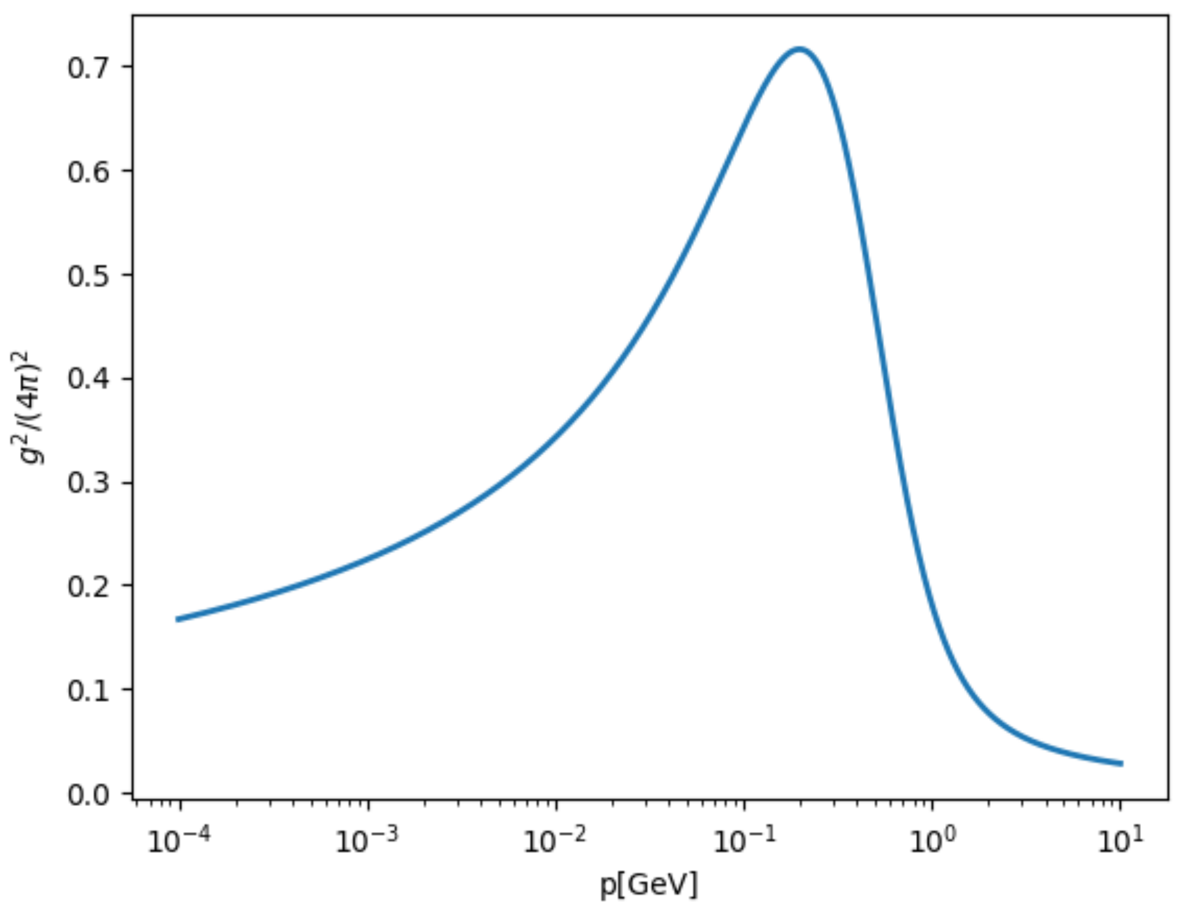
M. Tissier, N. Wschebor, Phys. Rev. D **84** (2011)

$$S_{CF} = \int d^D x \left[\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{m^2}{2} A^a_\mu A^a_\mu + i b^a \partial_\mu A^a_\mu + \partial_\mu \bar{c}^a D^{ab}_\mu c^b \right]$$

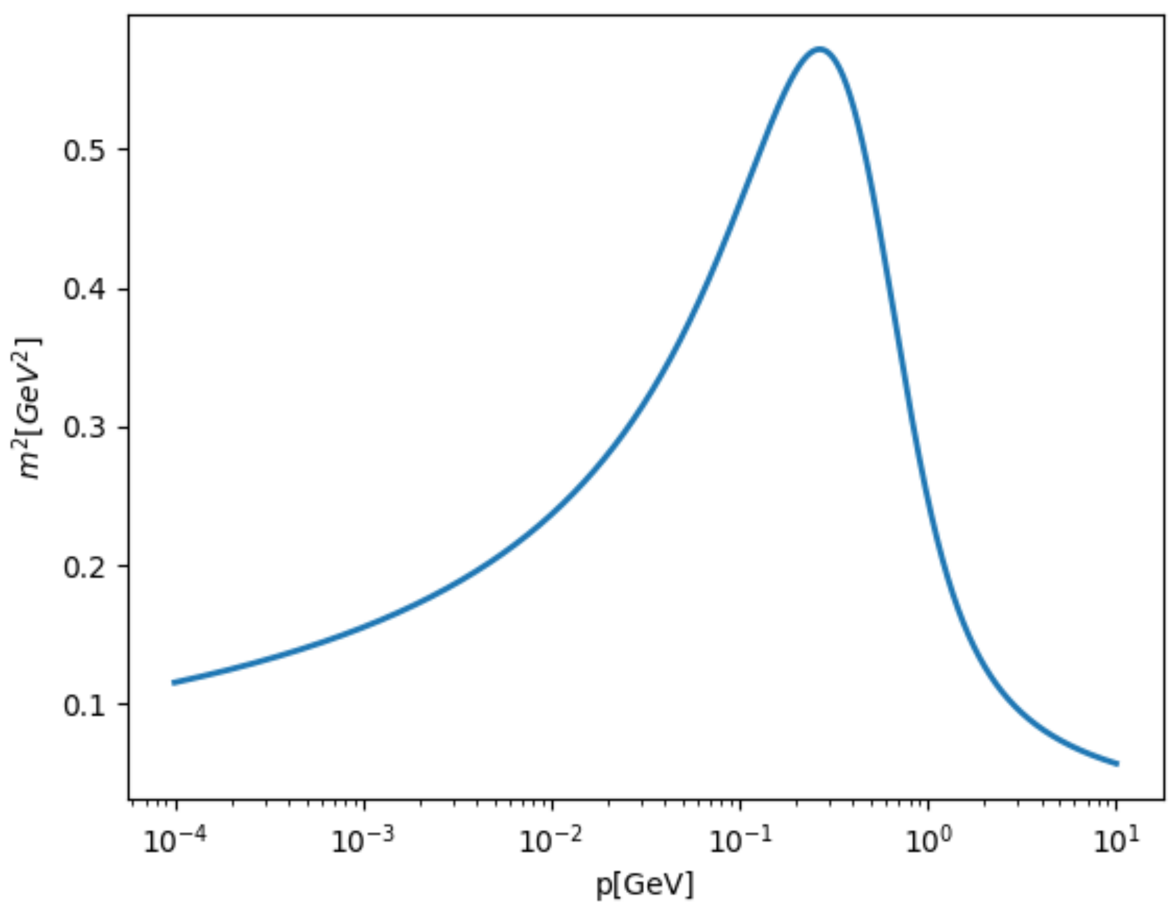
IR safe renormalization scheme

$$\Gamma^{\perp}_{AA}(\mu) - \Gamma^{||}_{AA}(\mu) = \mu^2, \qquad \Gamma_{c\bar{c}}(\mu) = \mu^2, \qquad m^2(\mu) = \Gamma^{||}_{AA}(\mu) \longrightarrow$$ beta function changes sign

$g^2/(4\pi)^2$



m^2



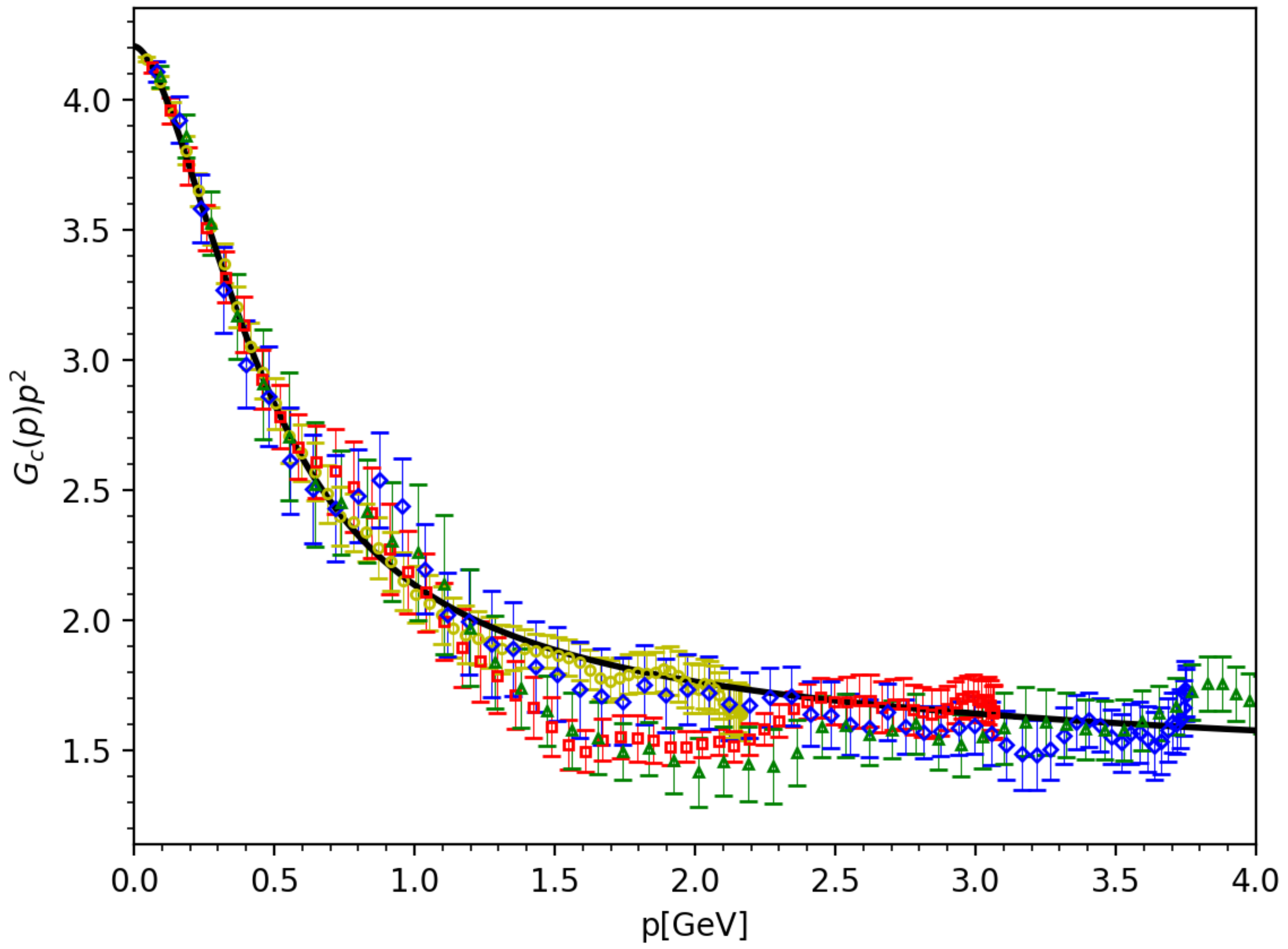
Flow equations:

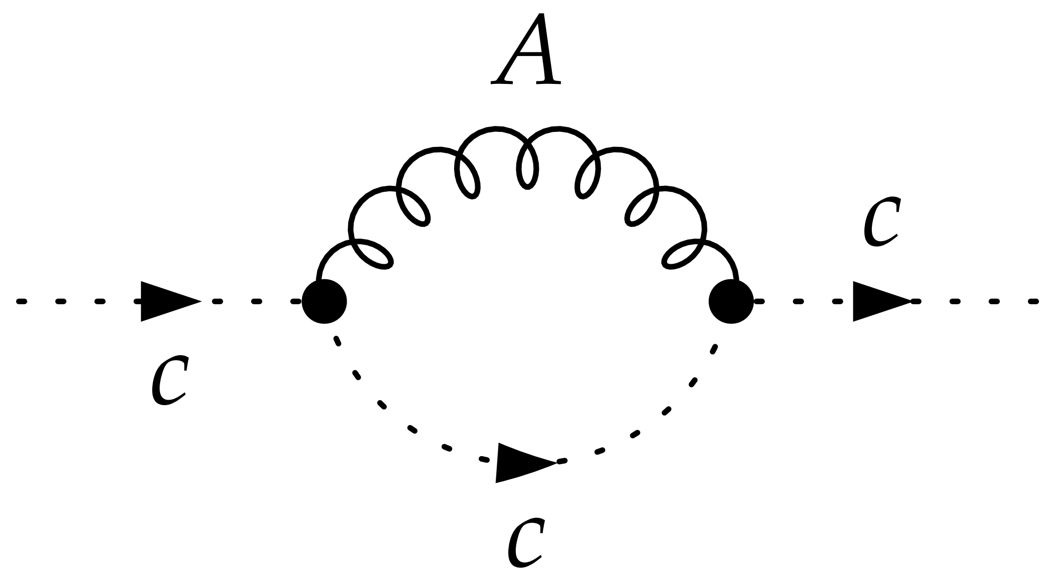
$$\mu \frac{dg(\mu)}{d\mu} = g \left(\frac{\gamma_A(\mu, g, m)}{2} + \gamma_c(\mu, g, m) \right),$$

$$\mu \frac{dm^2}{d\mu} = m^2 \left(\gamma_A(\mu, g, m) + \gamma_c(\mu, g, m) \right)$$

$$G_c(p,\mu) = \frac{1}{p^2} e^{\int_\mu^p d\mu' \gamma_c(\mu', g(\mu'), m(\mu'))}$$

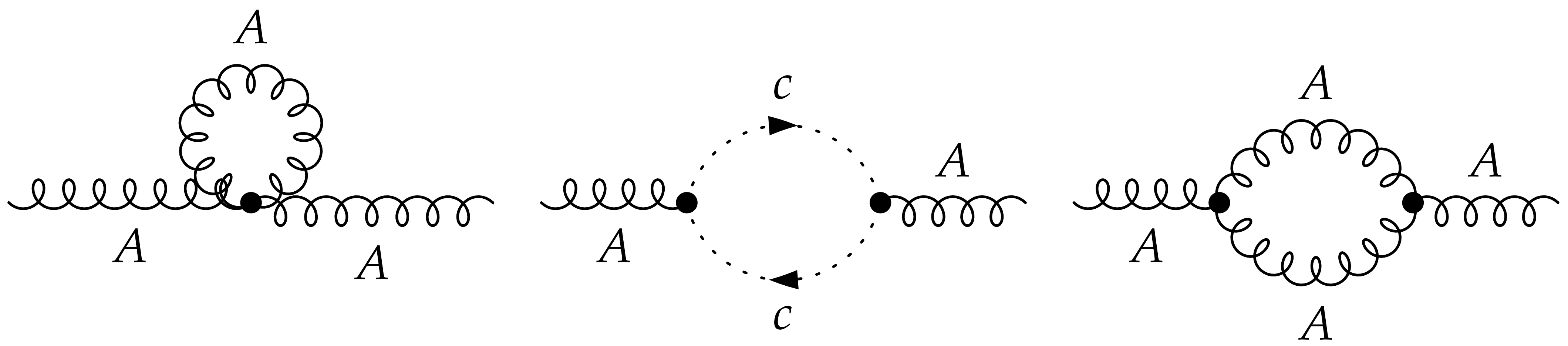
ghost dressing function



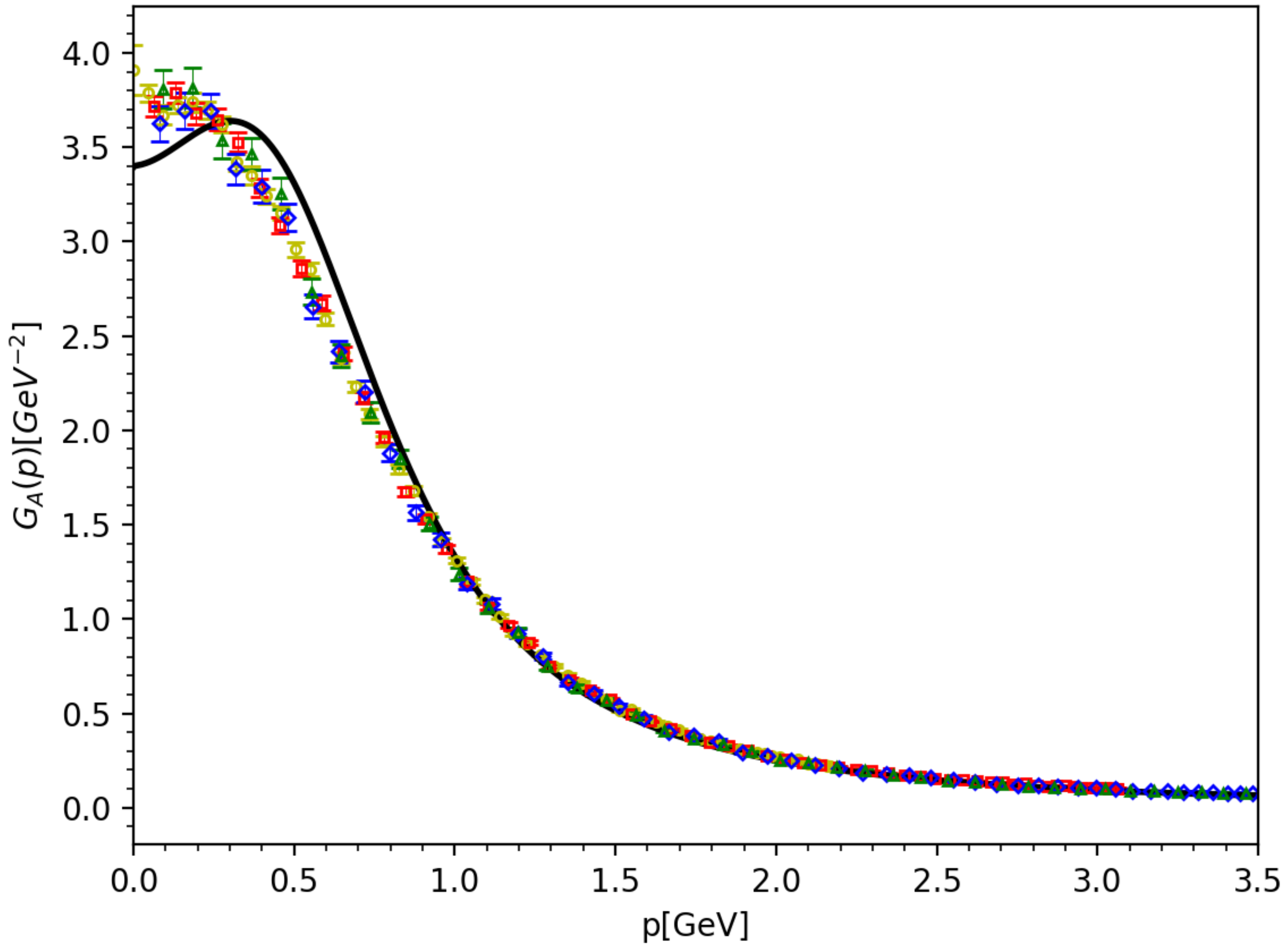


$g = 2.9\,, \quad m = 0.31$GeV en $\mu = 3$ GeV

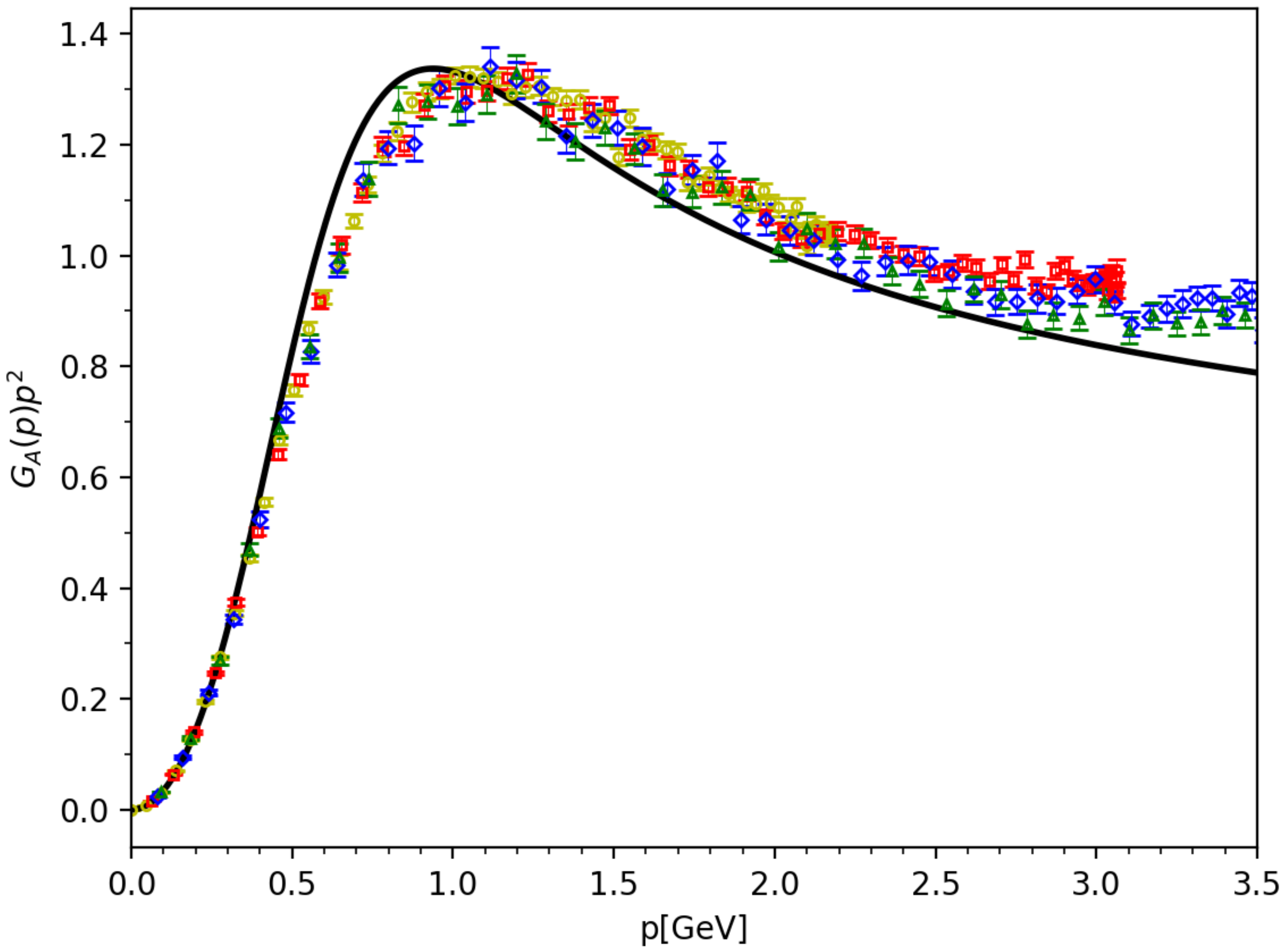
$$G_A(p,\mu) = \frac{1}{p^2 + m^2(p)} e^{\int_\mu^p d\mu' \gamma_A(\mu', g(\mu'), m(\mu'))}$$



gluon propagator



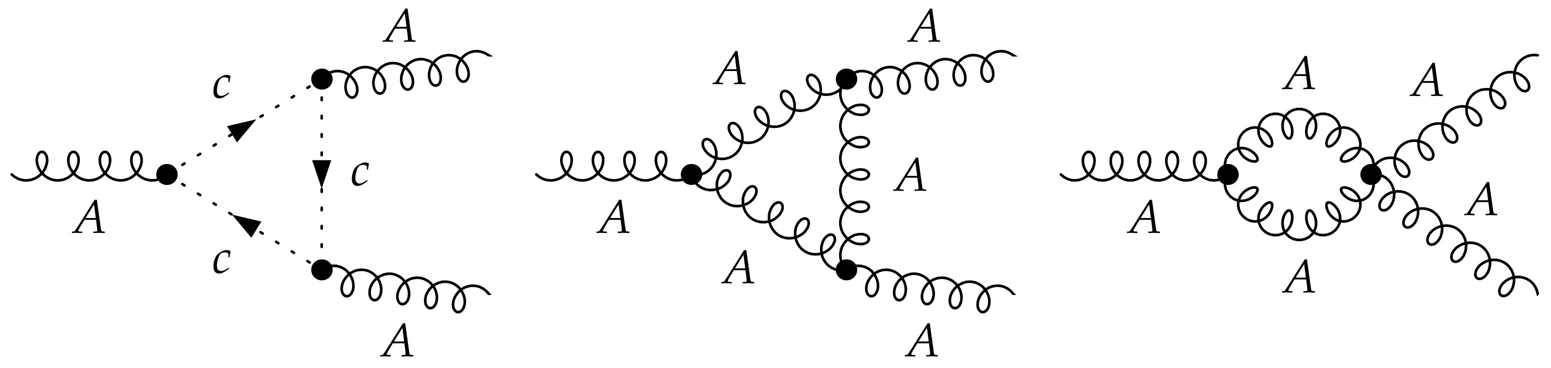
gluon dressing function



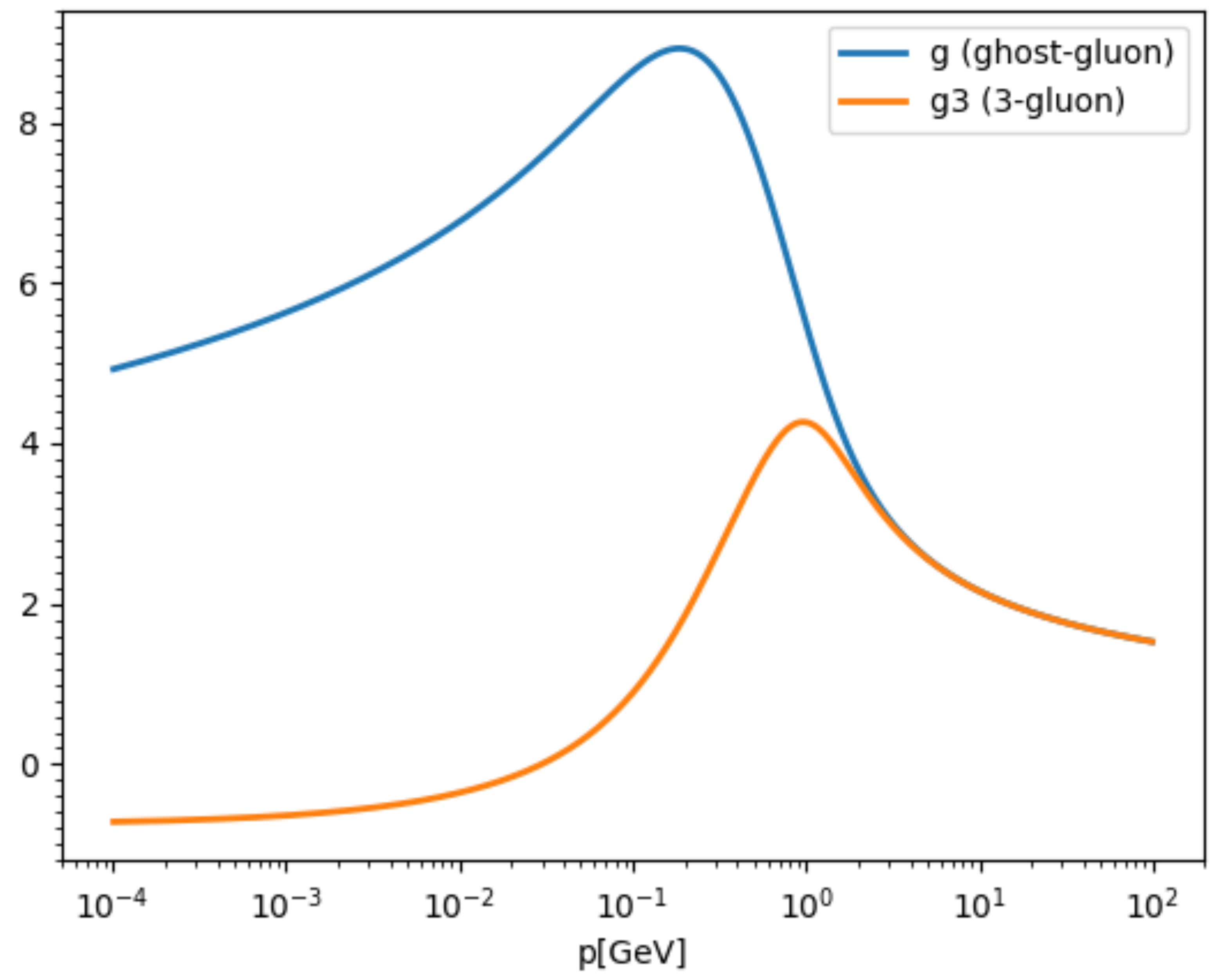
P.D. and A. Weber, *Annals.Phys.* **439** (2022)

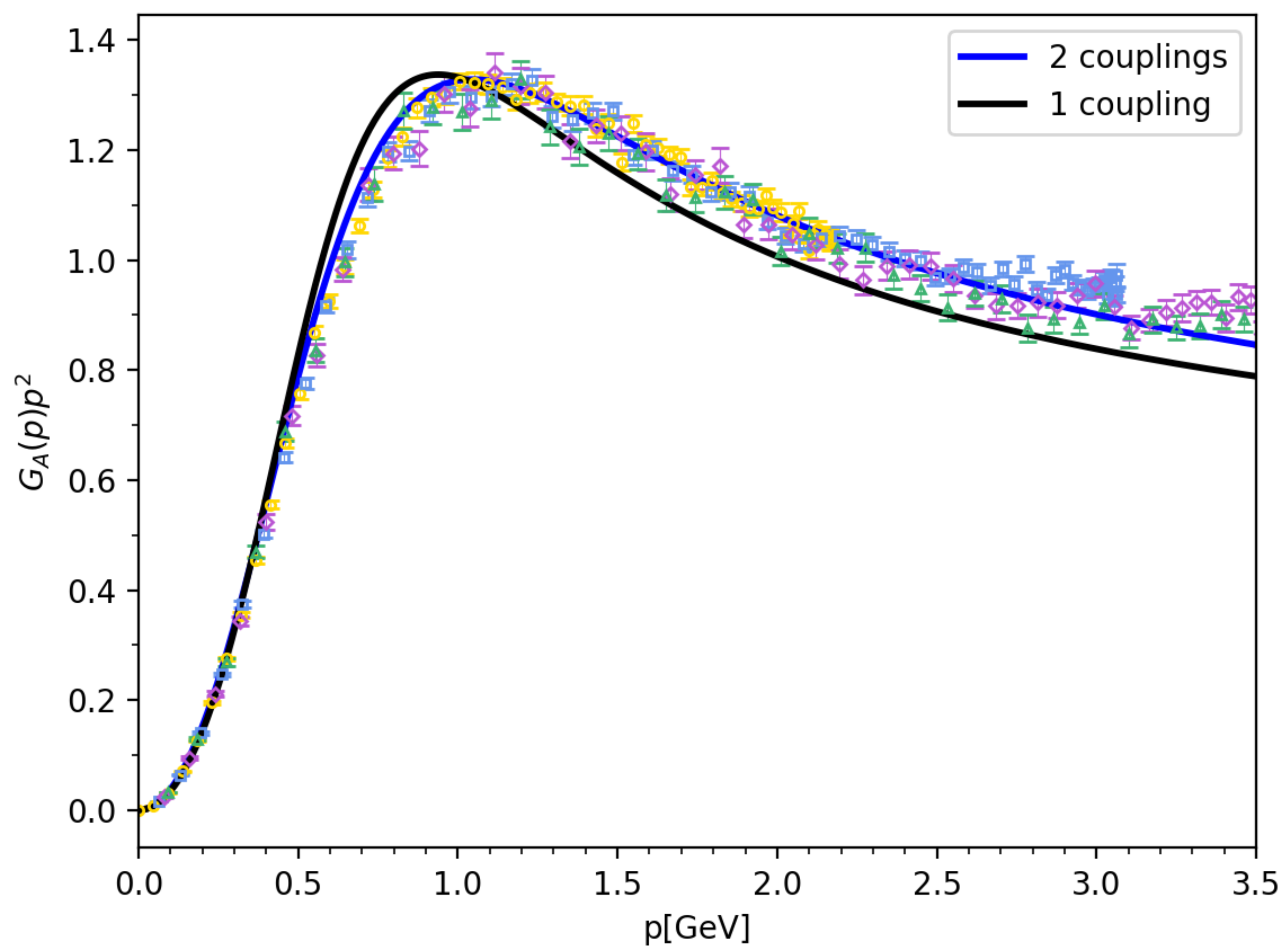
- We scrutinized the features that a sensible renormalization scheme must possess to be IR safe.
- We analyzed (analytically in IR) the properties of different renormalization schemes that better match lattice data.
- We found a renormalization scheme that yields a family of non-trivial IR fixed points associated to decoupling solutions.
- We found better results renormalizing separately the ghost-gluon and 3-gluon couplings.

$g_3 \equiv$

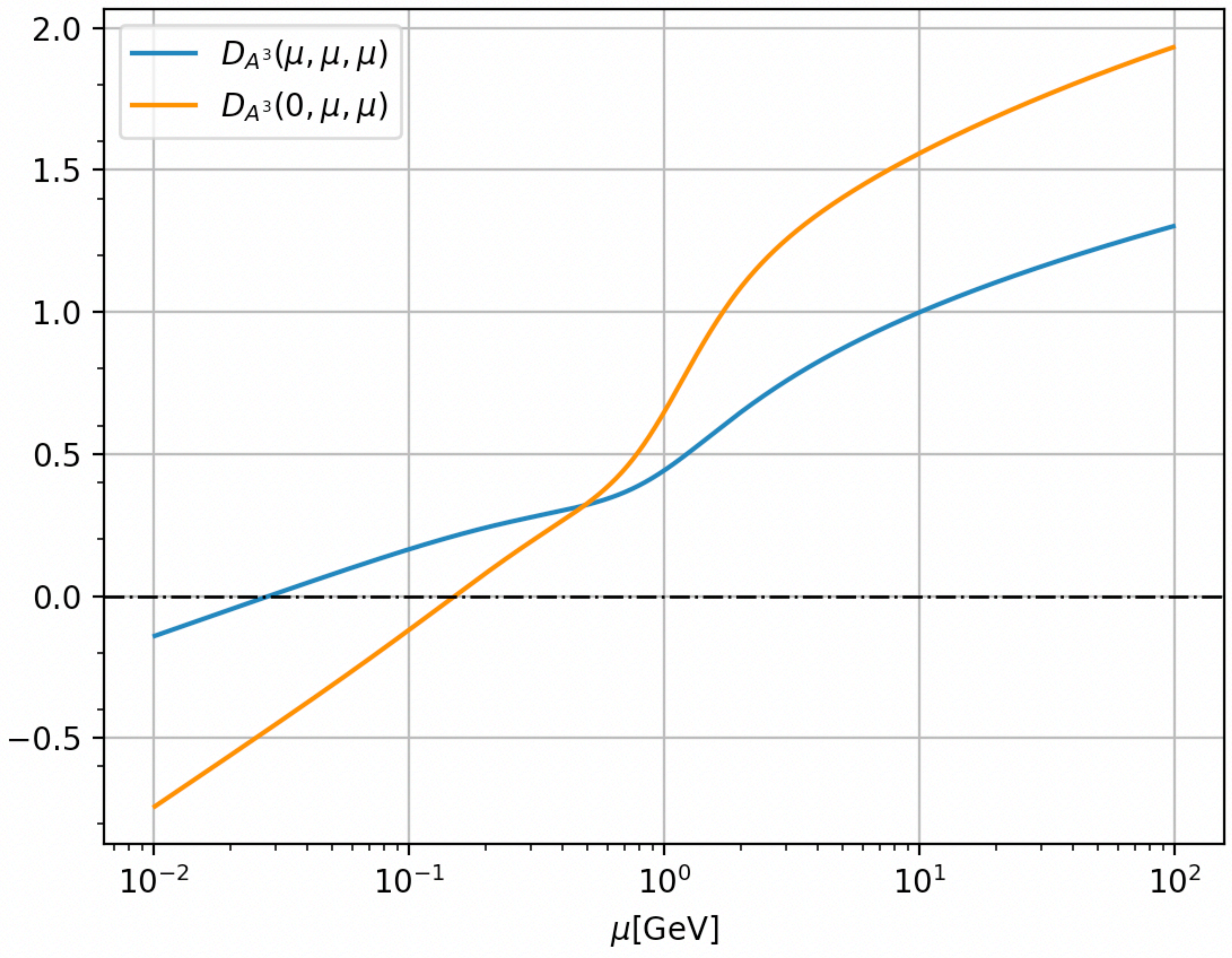


$p_1^2 = p_2^2 = p_3^2 = \mu^2$

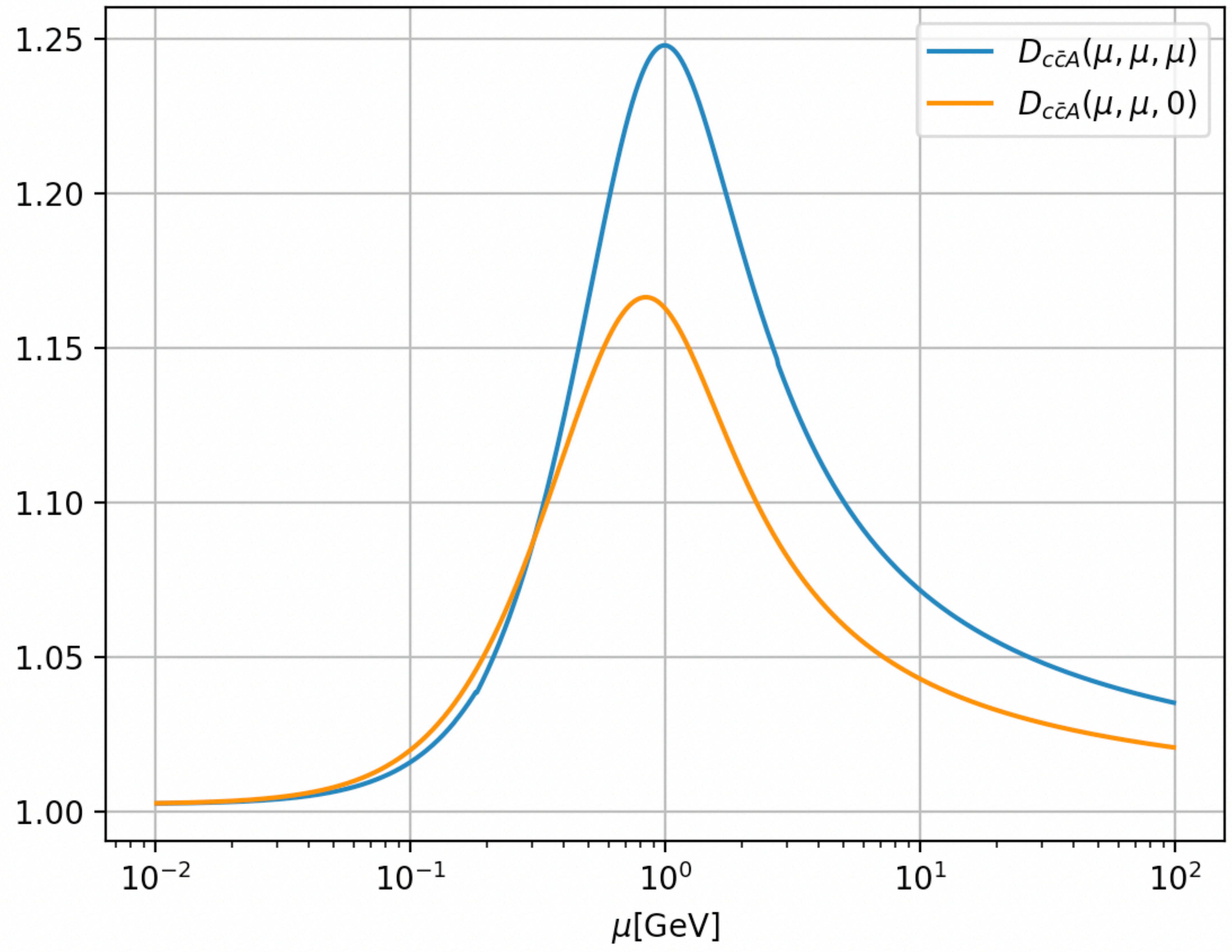




3-gluon dressing function



ghost-gluon dressing function

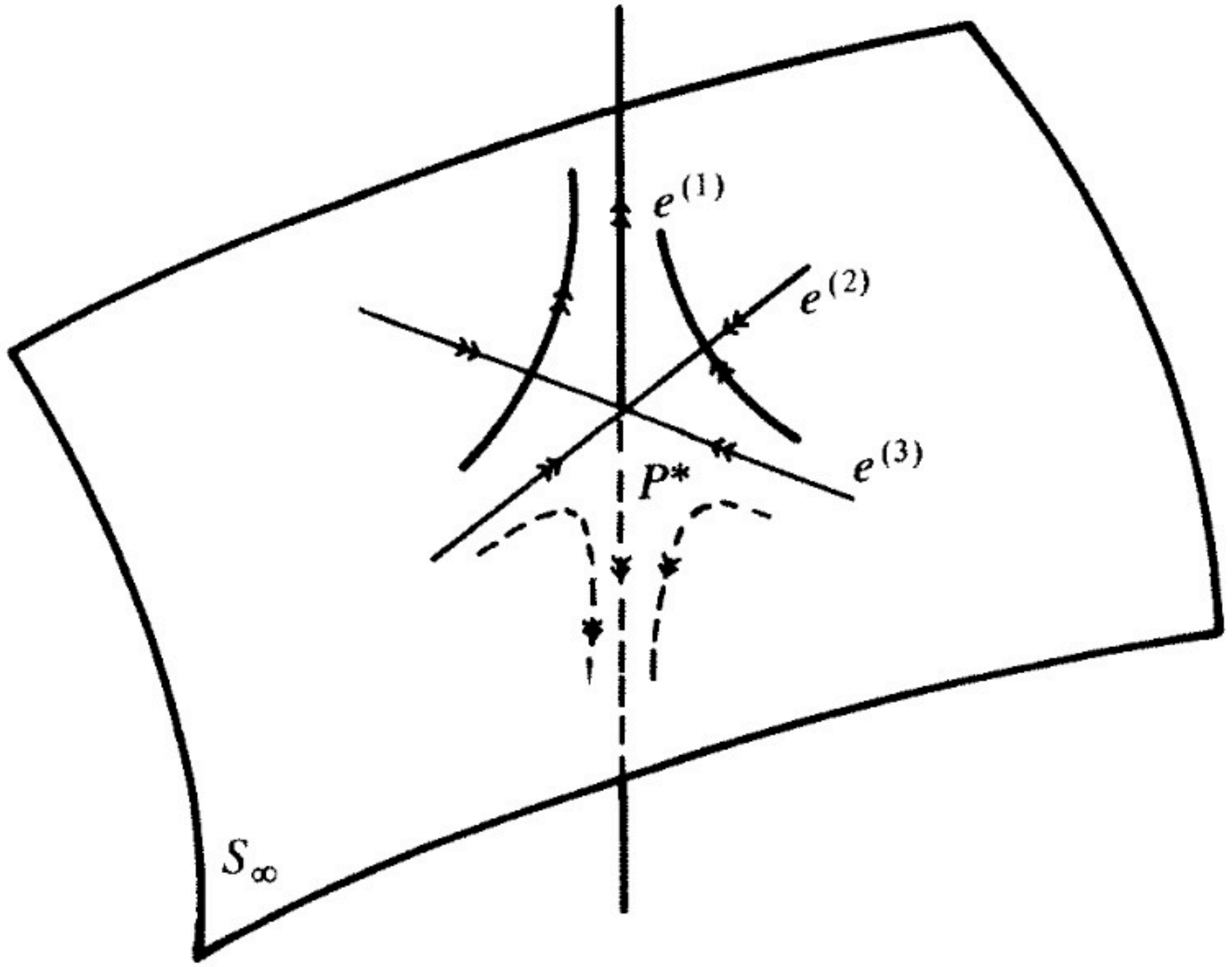


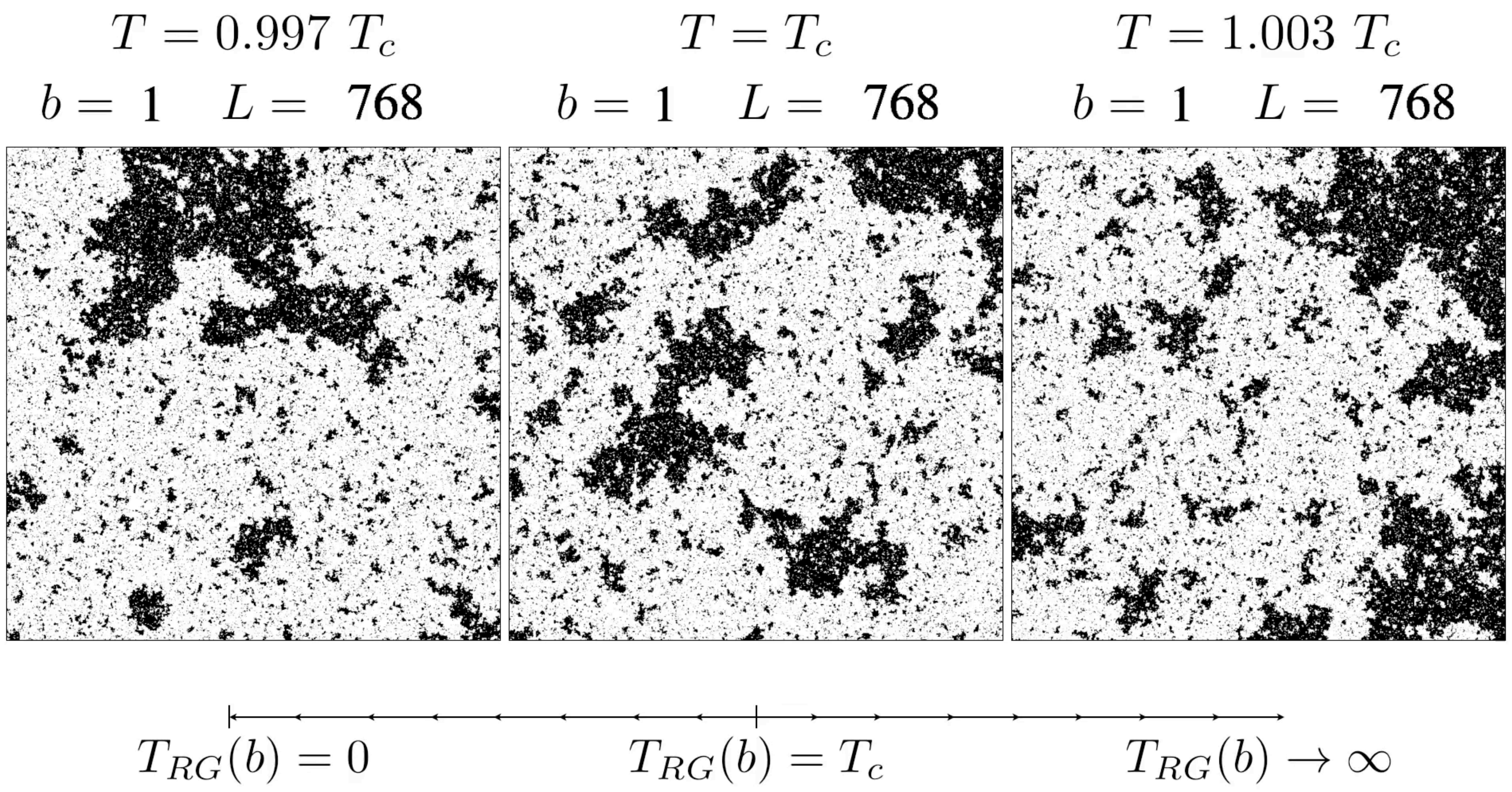
POSSIBLE FUTURE WORK

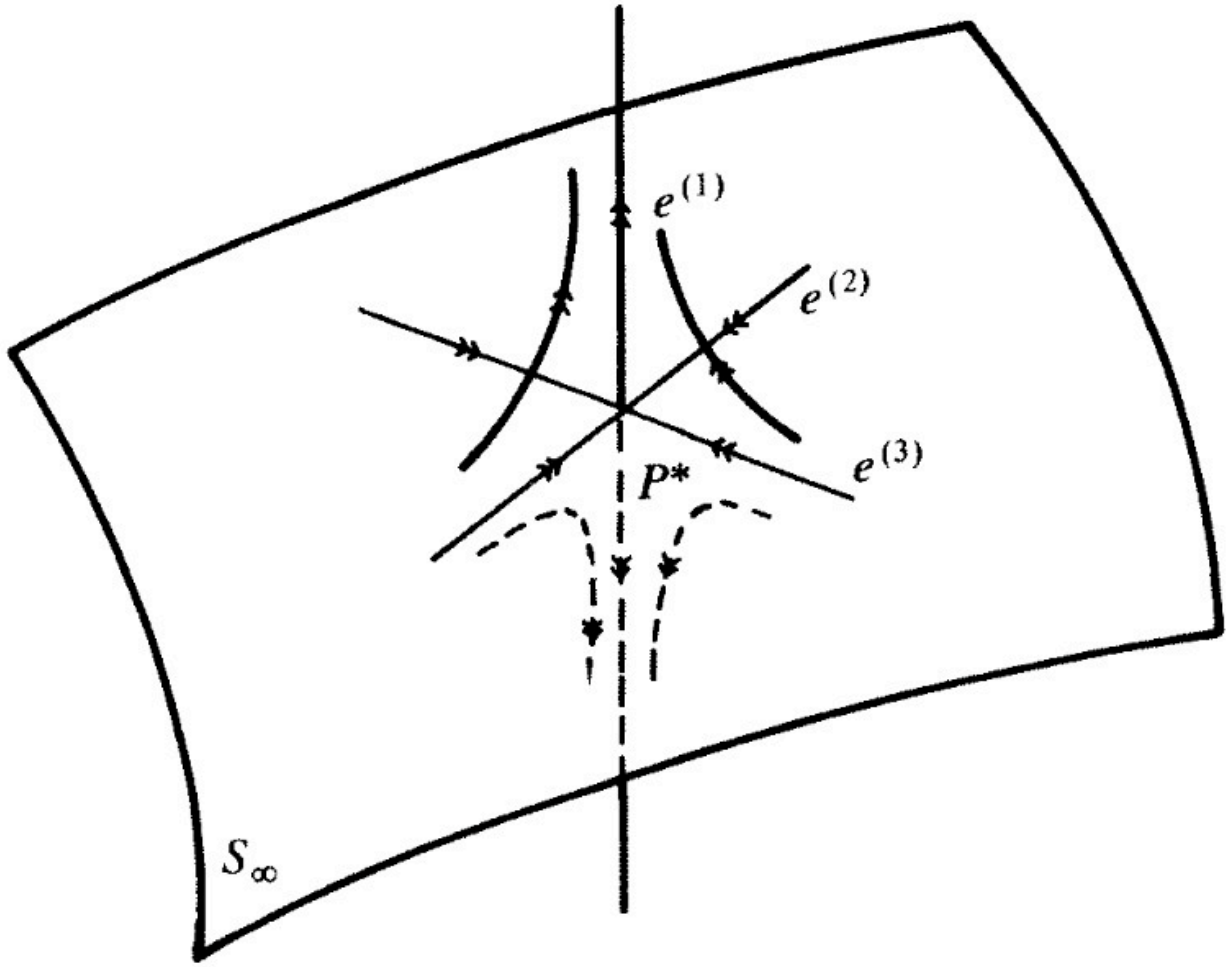
- Redo the IR Epsilon-expansion introducing massless quark fields (upper critical dimension is still $D = 2$).
- Introduce the chiral condensate as a perturbation and study its IR flow.
- Generalize the IR-UV study to linear covariant gauges:

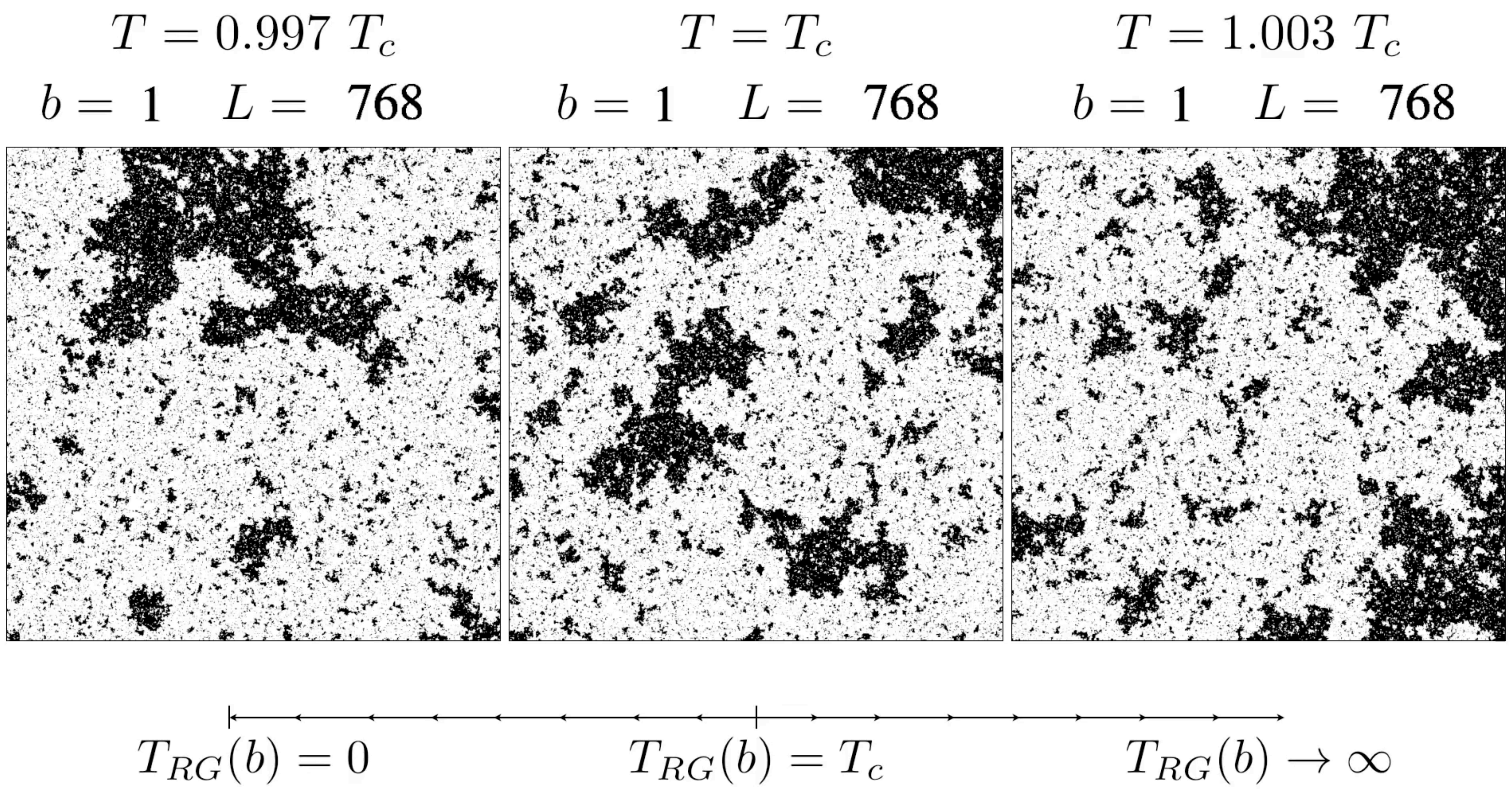
    A. Introduce the mass term as a gauge invariant composite operator $m^2 A_\mu^h A_\mu^h$ that contains a Stueckelberg field.

    B. Use the Curci-Ferrari model in linear covariant gauges.


$e^{(1)}$
$e^{(2)}$
$e^{(3)}$
P^*
S_∞


$T = 0.997\ T_c$
$b = 1 \quad L = 768$
$T = T_c$
$b = 1 \quad L = 768$
$T = 1.003\ T_c$
$b = 1 \quad L = 768$
$T_{RG}(b) = 0$
$T_{RG}(b) = T_c$
$T_{RG}(b) \to \infty$


$e^{(1)}$
$e^{(2)}$
$e^{(3)}$
P^*
S_∞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