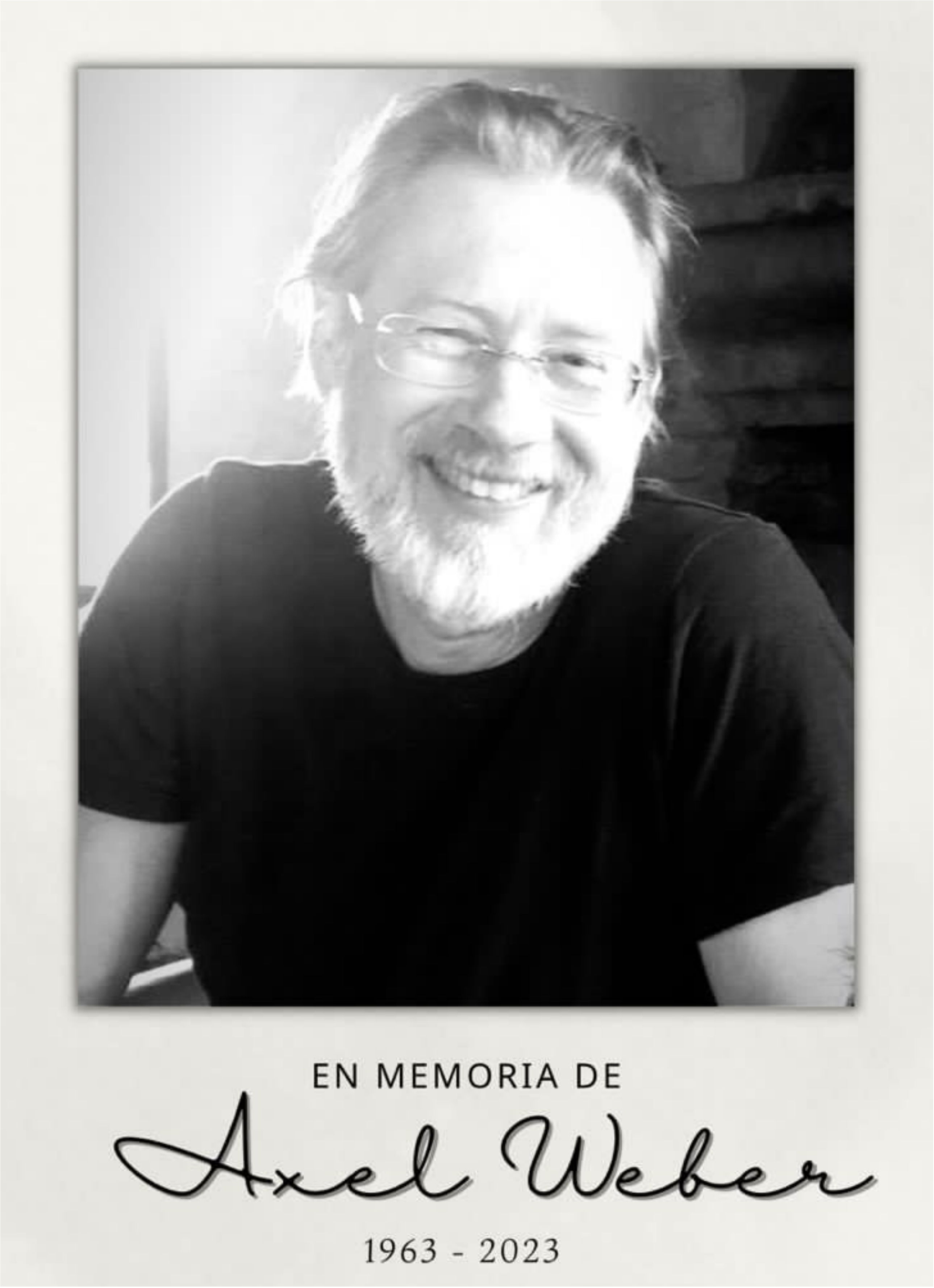
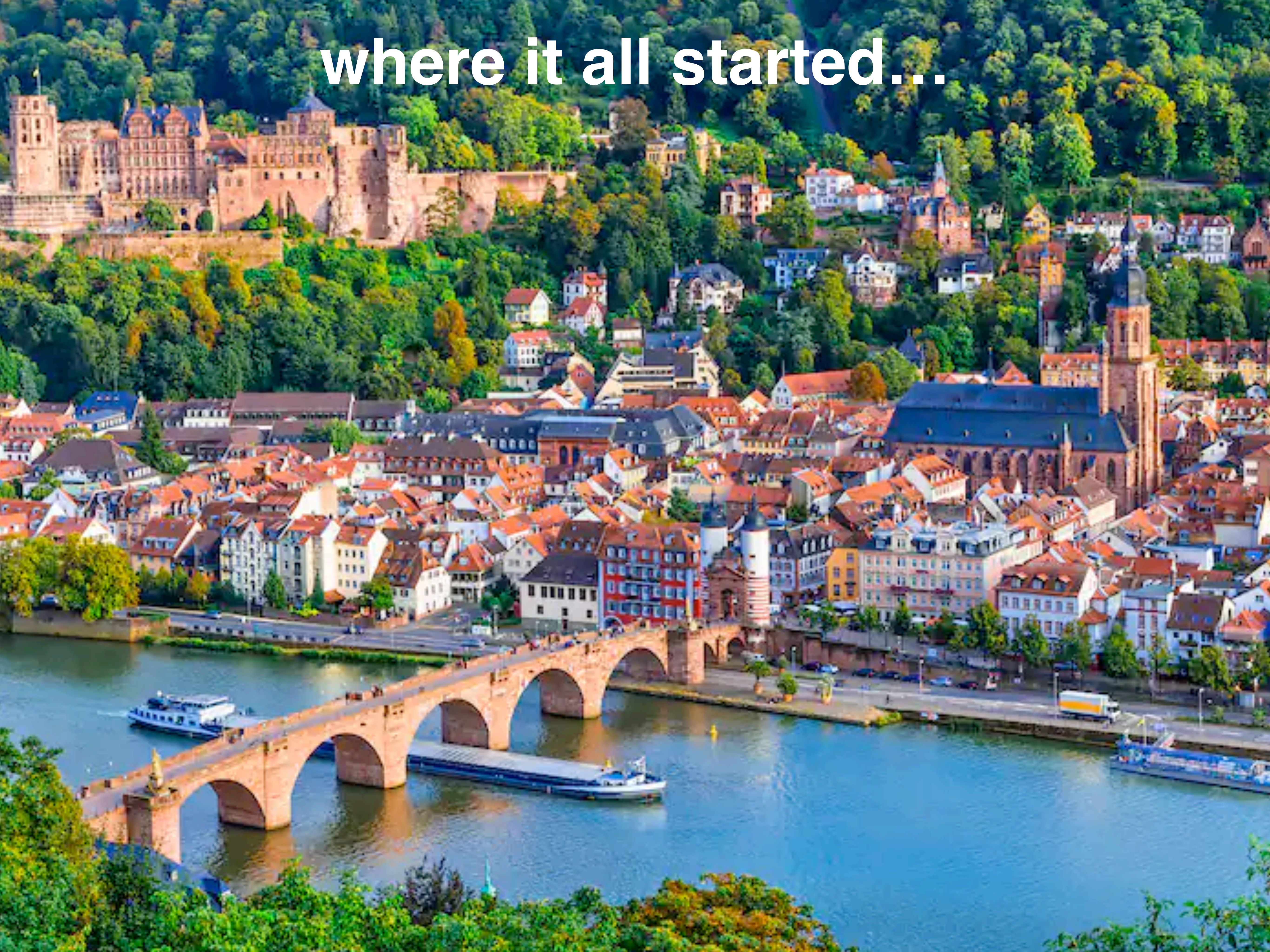


Celebrating Half a Century of QCD

Memorial to Axel Weber

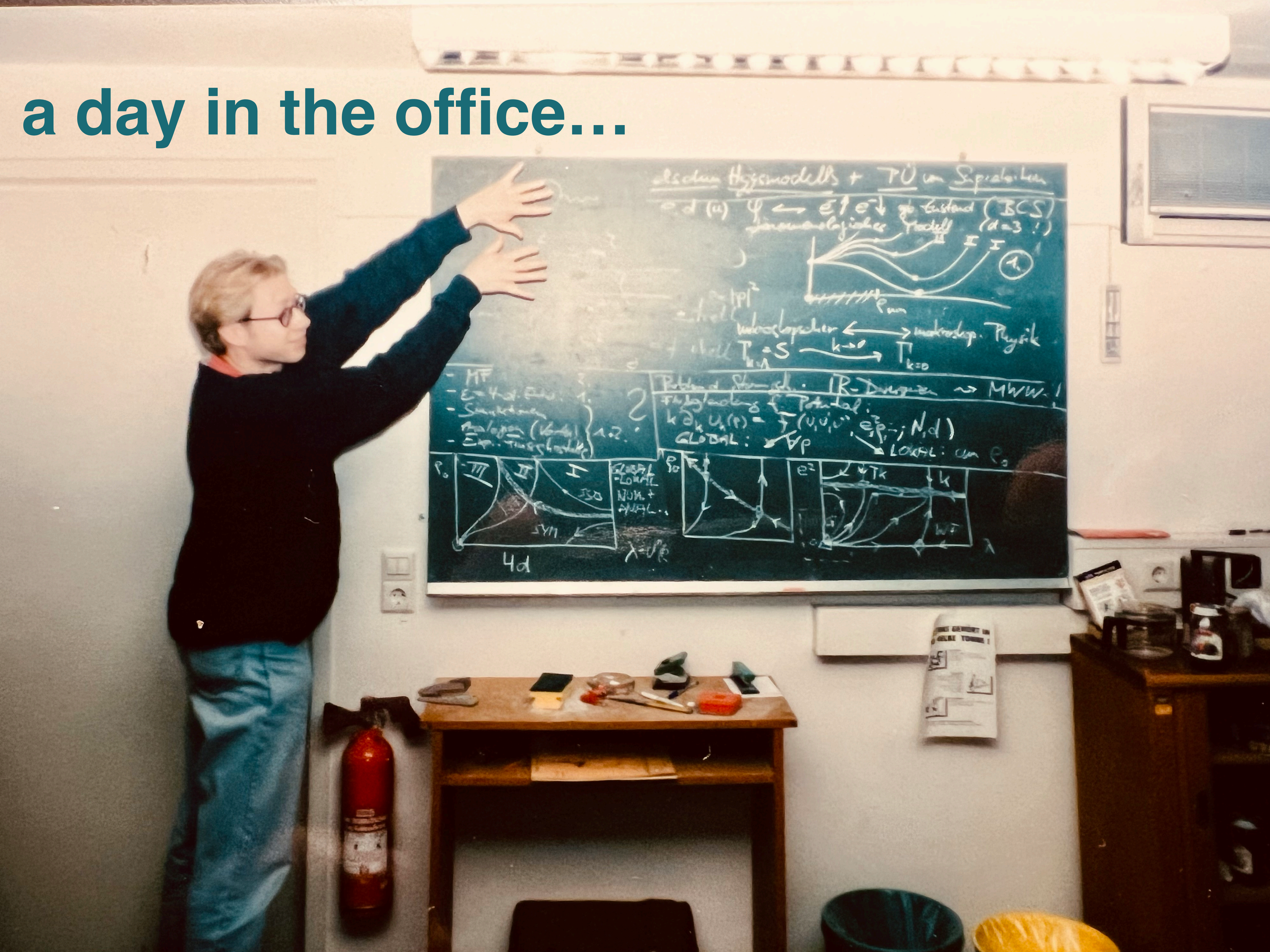

where it all started...

how it all started...

QCD headquarter


QCD requires
heavy lifting

a day in the office…




ice cream always works!
12.95


time for play


hiding in plain sight


who else was there?
Rauchen
verboten!


Rauchen

May 1995 LPTHE Orsay 95–39

FLOW EQUATIONS FOR THE RELEVANT PART OF THE PURE YANG–MILLS ACTION

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May 1995 LPTHE Orsay 95–39

FLOW EQUATIONS FOR THE RELEVANT PART OF THE PURE YANG–MILLS ACTION

Abstract

Wilson's exact renormalization group equations are derived and integrated for the relevant part of the pure Yang–Mills action. We discuss in detail how modified Slavnov–Taylor identities controle the breaking of BRST invariance in the presence of a finite infrared cutoff k through relations among different parameters in the effective action. In particular they imply a nonvanishing gluon mass term for nonvanishing k. The requirement of consistency between the renormalization group flow and the modified Slavnov–Taylor identities allows to controle the self–consistency of truncations of the effective action.

FLOW EQUATIONS

OF TH

The Heavy Quark Potential from Wilson's Exact Renormalization Group

Ulrich ELLWANGER[1]
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aylor
ic ... e infrared cutoff
k t ... action. In particular they
impr ... ng k. The requirement of consistency
betwe ... the modified Slavnov–Taylor identities allows
to cont ... truncations of the effective action.

FLOW EQUATIONS

OF TH

The Heavy Quark Potential from Wilson's Exact Renormalization Group

Abstract

We perform a calculation of the full momentum dependence of the gluon and ghost propagators in pure SU(3) Yang–Mills theory by integrating Wilson's exact renormalization group equations with respect to an infrared cutoff k. The heavy quark potential in the quenched approximation can be expressed in terms of these propagators. Our results strongly indicate a $1/p^4$–behaviour of the heavy quark potential for $p^2 \to 0$. We show in general that effective actions which satisfy Schwinger–Dyson equations, correspond to (quasi–) fixed points of Wilson's exact renormalization group equations.

ic … cutoff

k … particular they

impr… requirement of consistency

betwe… ied Slavnov–Taylor identities allows

to cont… of the effective action.


theory
YangMills
renormalization
Gauge
Hamiltonian
infrared
Coulomb
group
approach
equations
potential
states
integral
Functional
Flow
Landau
equaltime
point
CallanSymanzik
Duality
gluon
bound
path
photons
lattice
quantum
Yukawa
Mass
spatial
fixed
U1
nonrelativistic
DysonSchwinger
quantized
Epsilon
relevant
thermal
different
GellMannLow
Carlo
hyperfine
beyond
effective
revisited
mechanical
screening
Applications
analysis
critical
energy
systems
structure
relativistic
hard
theories
perturbative
Fine
exact
shadow
limit
action
Compact
improvement
Monte
Exploiting
dependence
correlation
Generalized
FRG
scheme
Flows
Dimensional
technique
function
transforms
formalism
Expansion
spectrum
Wilsons
condensates
Similarity
hit
Color
state
loops
Heavy
averaged
part
theorem
worldline
Muonium
quark
ground
pure
mechanics
transformation
functions
approaches
perspective
Confinement
reduction

Epsilon expansion for infrared Yang-Mills theory in Landau gauge

Axel Weber

Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo,

Edificio C-3, Ciudad Universitaria,

A. Postal 2-82, 58040 Morelia, Michoacán, Mexico

(Dated: November 19, 2018)

The study of the Dyson-Schwinger equations of Landau gauge Yang-Mills theory has revealed two types of solutions for the gluon and ghost propagators, with a scaling and a massive (decoupling) behavior in the extreme infrared, respectively. We show that both types of solutions are quantitatively reproduced by applying renormalization group equations of Callan-Symanzik type in an epsilon expansion to the infrared limit of Landau gauge Yang-Mills theory when a mass term for the gluons is added to the action. Only the decoupling solution corresponds to an infrared-stable fixed point in three and four space-time dimensions and is hence expected to be physically realized, in agreement with the results of recent lattice calculations.

Epsilon expansion for infrared Yang-Mills theory in Landau gauge

Axel Weber

Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo,

$$\beta(\bar{g}_R) = \mu^2 \frac{d}{d\mu^2}\,\bar{g}_R = \frac{1}{2}\,\bar{g}_R \left(\frac{\epsilon}{2} - \frac{1}{2}\,\frac{N\bar{g}_R^2}{4\pi} \right)$$

The stud … vealed two types of solutions for the gluon and ghost propagators, with a scaling and a massive (decoupling) behavior in the extreme infrared, respectively. We show that both types of solutions are quantitatively reproduced by applying renormalization group equations of Callan-Symanzik type in an epsilon expansion to the infrared limit of Landau gauge Yang-Mills theory when a mass term for the gluons is added to the action. Only the decoupling solution corresponds to an infrared-stable fixed point in three and four space-time dimensions and is hence expected to be physically realized, in agreement with the results of recent lattice calculations.

weakly-coupled fixed points in QCD-like theories

Daniel F Litim

University of Sussex

running couplings

quantum fluctuations modify interactions
couplings depend on energy

$$\mu \frac{d\alpha}{d\mu} = \beta(\alpha)$$

fluctuations	$\hbar$
energy scale	μ
couplings	$\alpha(\mu)$

QFT provides us with

predictions into regions where we cannot (yet) make measurements

running couplings

quantum fluctuations modify interactions
couplings depend on energy

$$\mu \frac{d\alpha}{d\mu} = 0$$

fluctuations $\hbar$

energy scale μ

couplings $\alpha(\mu)$

weakly coupled **fixed points**
provide us with

(unitary) **conformal field theories**

Polchinski '88
Luty, Polchinski, Rattazzi, '12

4d QFTs

fields **vectors** A^a_μ , **fermions** ψ_I , **scalars** ϕ^A

path integral

$$Z[J] = \exp -i \int d^4x (L + L_{\text{gf}} + L_{\text{gh}} + J^i \, \Phi_i)$$

action

$$L = \frac{1}{4g_a^2} \mathrm{Tr}\, F^a_{\mu\nu} F_a^{\mu\nu} + i\psi_I \not{D} \psi_I + \frac{1}{2}(D_\mu \phi^A)^2$$
$$+ \frac{1}{2} Y^A{}_{IJ} \phi^A \, \psi_I \, \xi \, \psi_J + \frac{1}{4!} \lambda_{ABCD} \phi^A \phi^B \phi^C \phi^D$$

4d QFTs

fields — vectors A_μ^a , fermions ψ_I , scalars ϕ^A

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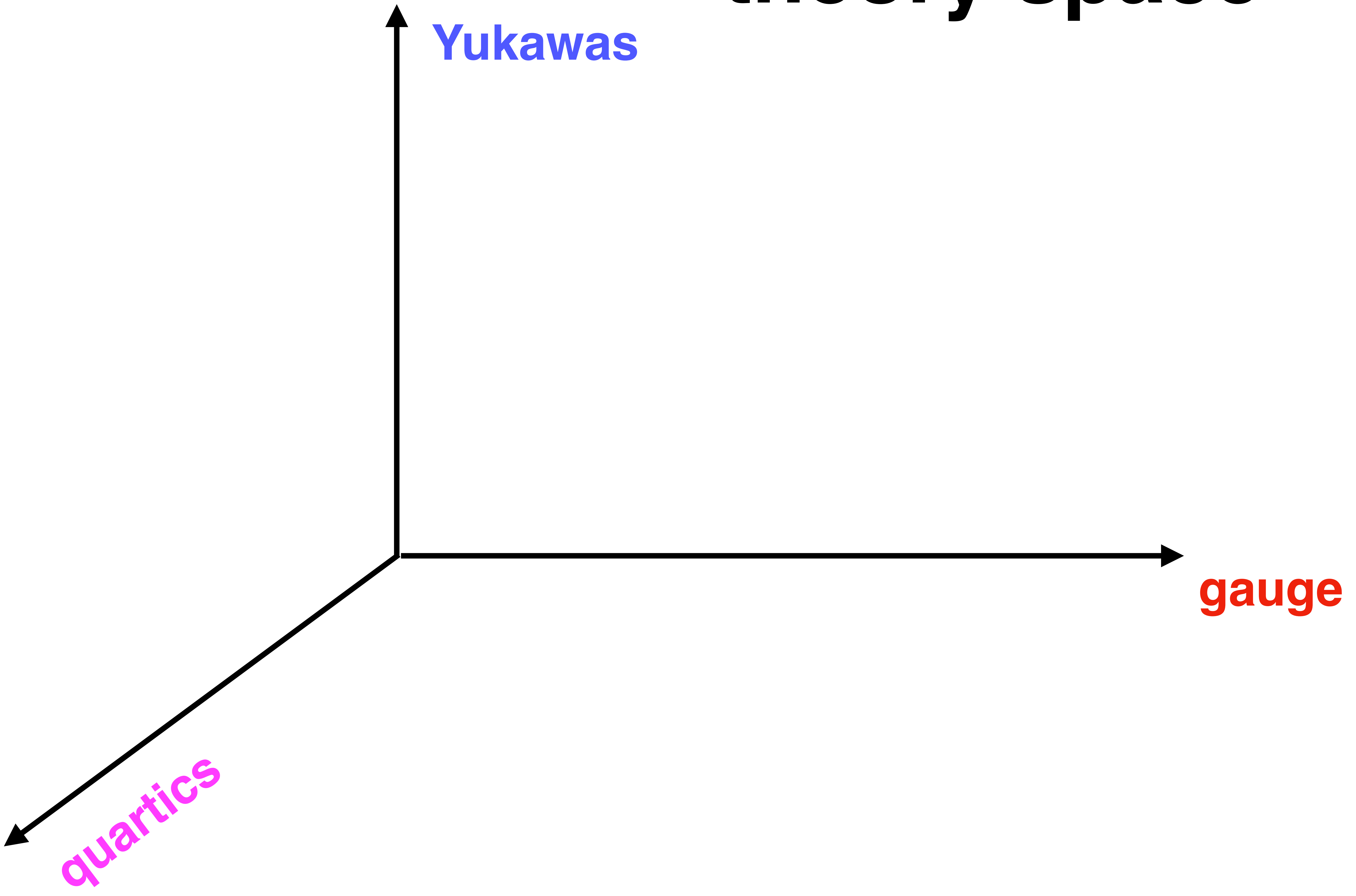
$$+ \frac{1}{2} Y^A{}_{IJ} \phi^A\, \psi_I\, \xi\, \psi_J + \frac{1}{4!} \lambda_{ABCD} \phi^A \phi^B \phi^C \phi^D$$

gauge → $\frac{1}{4g_a^2}$

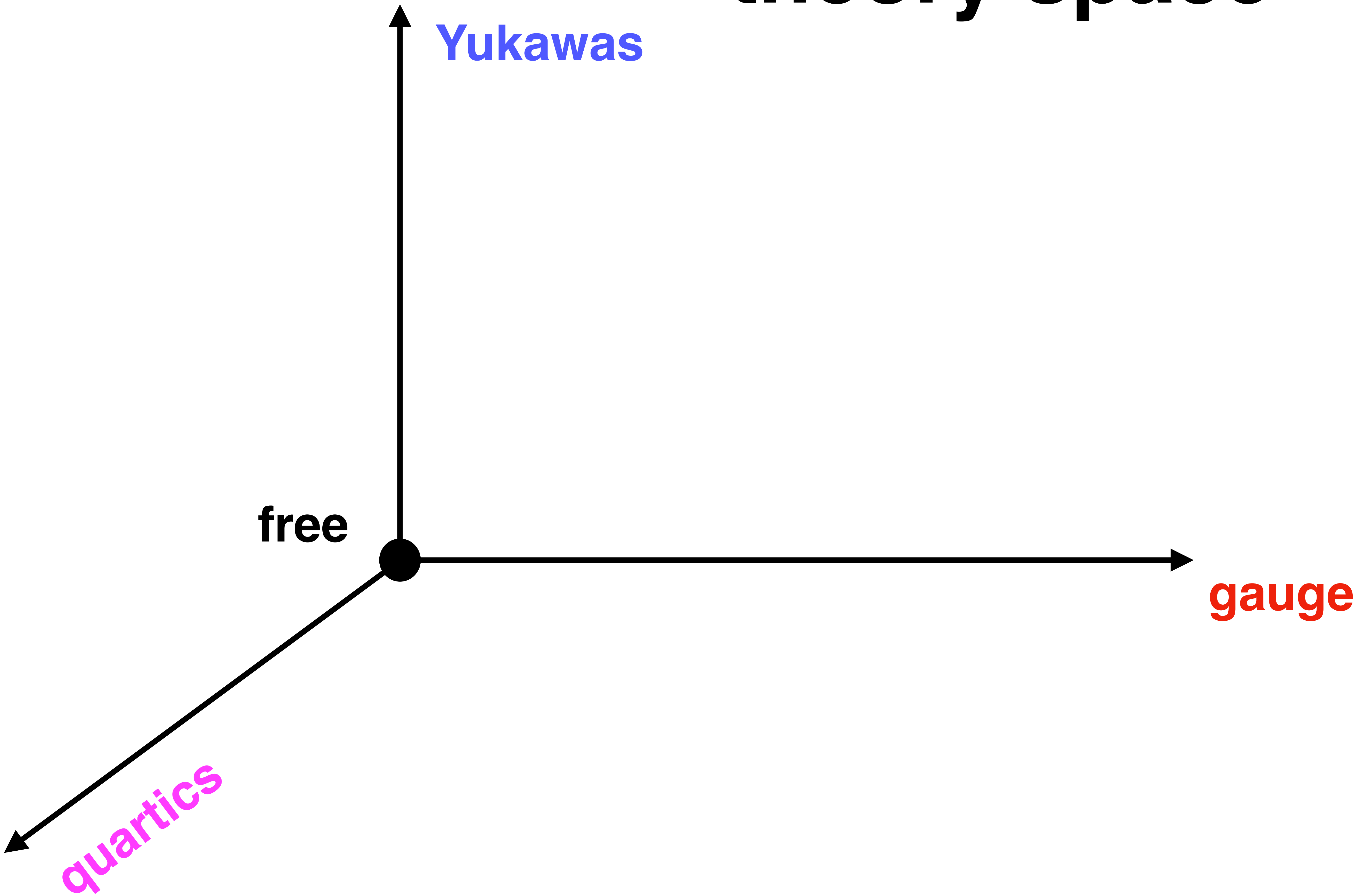
Yukawa → $Y^A{}_{IJ}$

quartics → λ_{ABCD}

“theory space”



“theory space”



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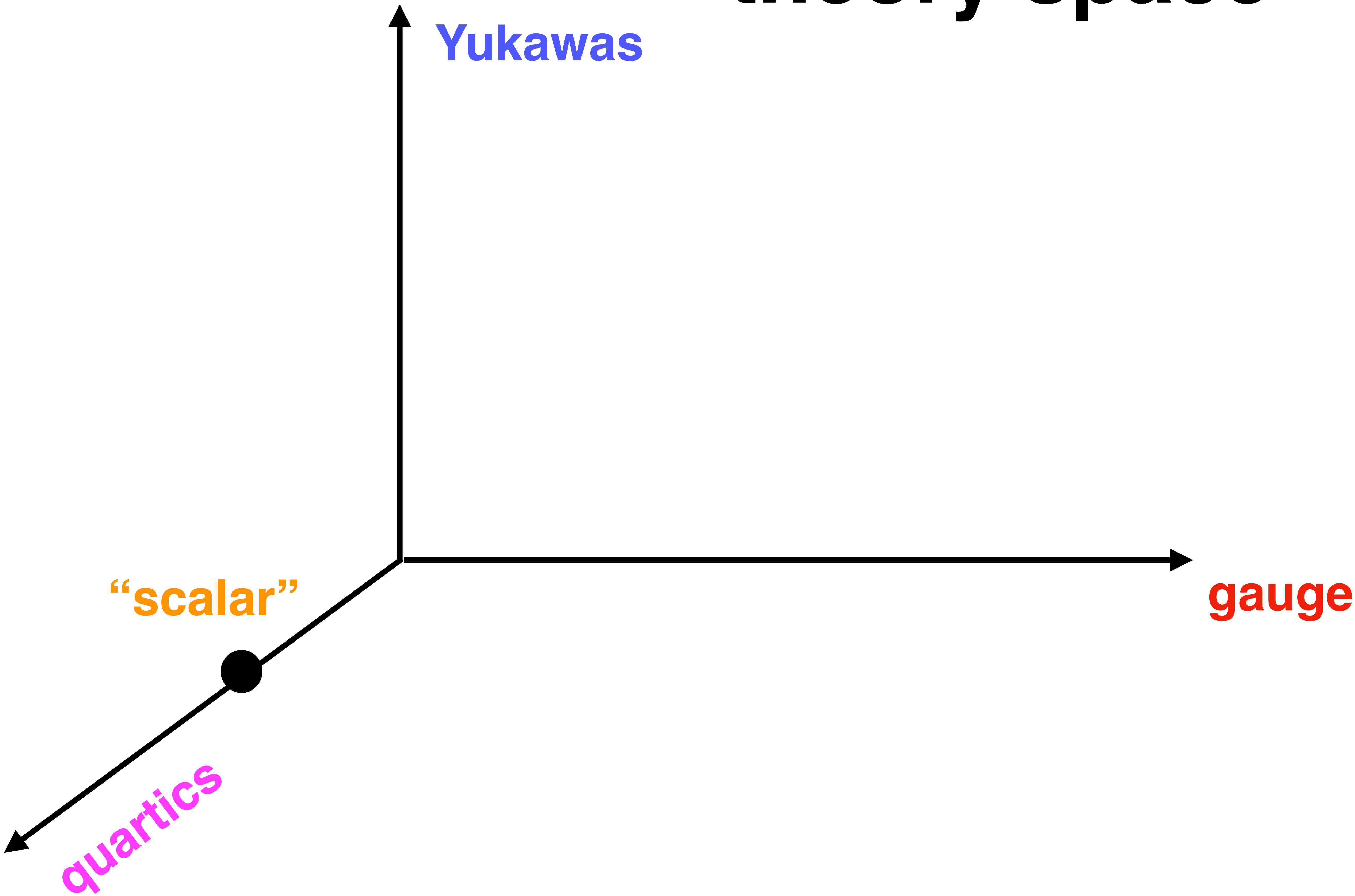

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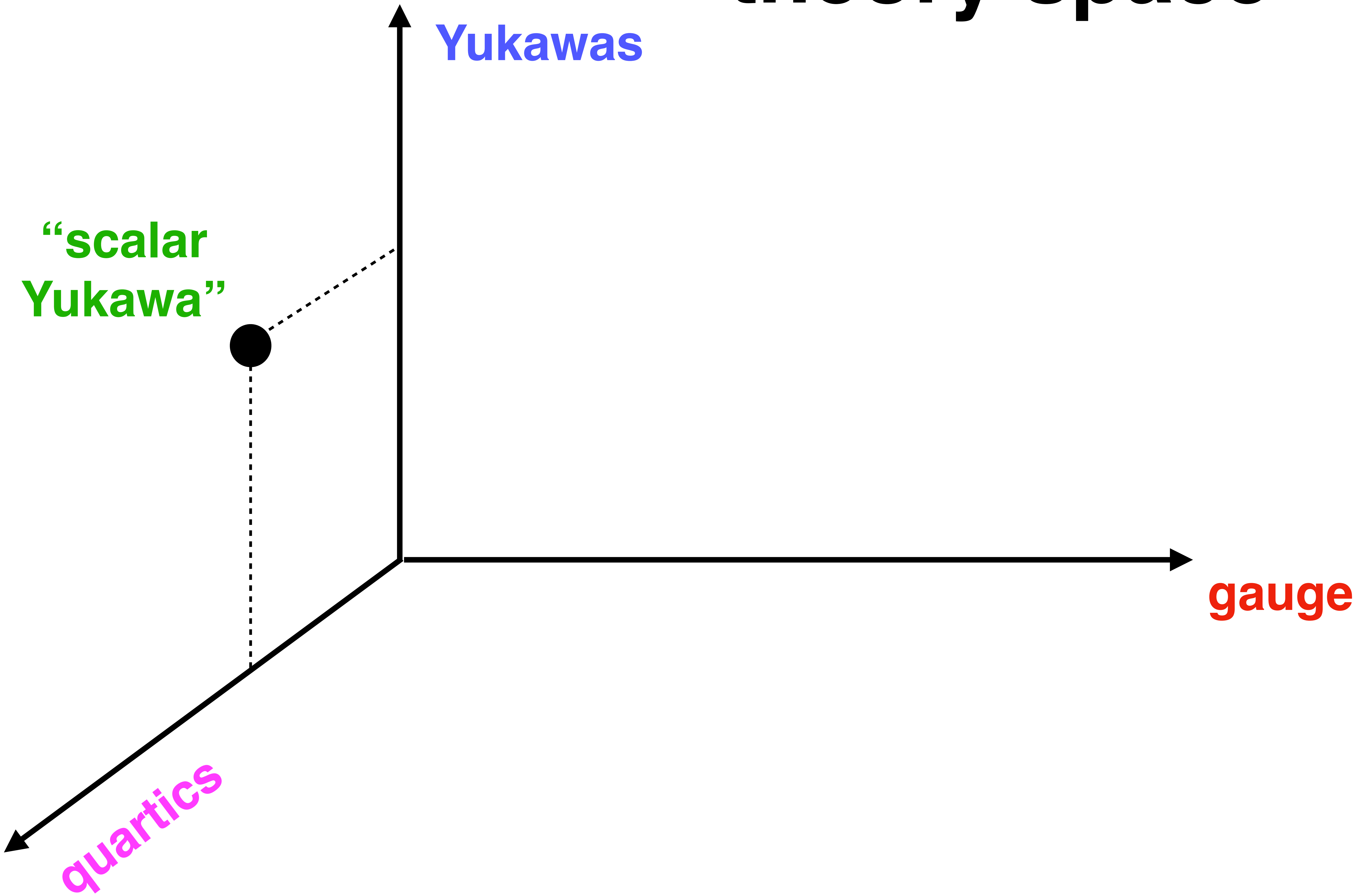

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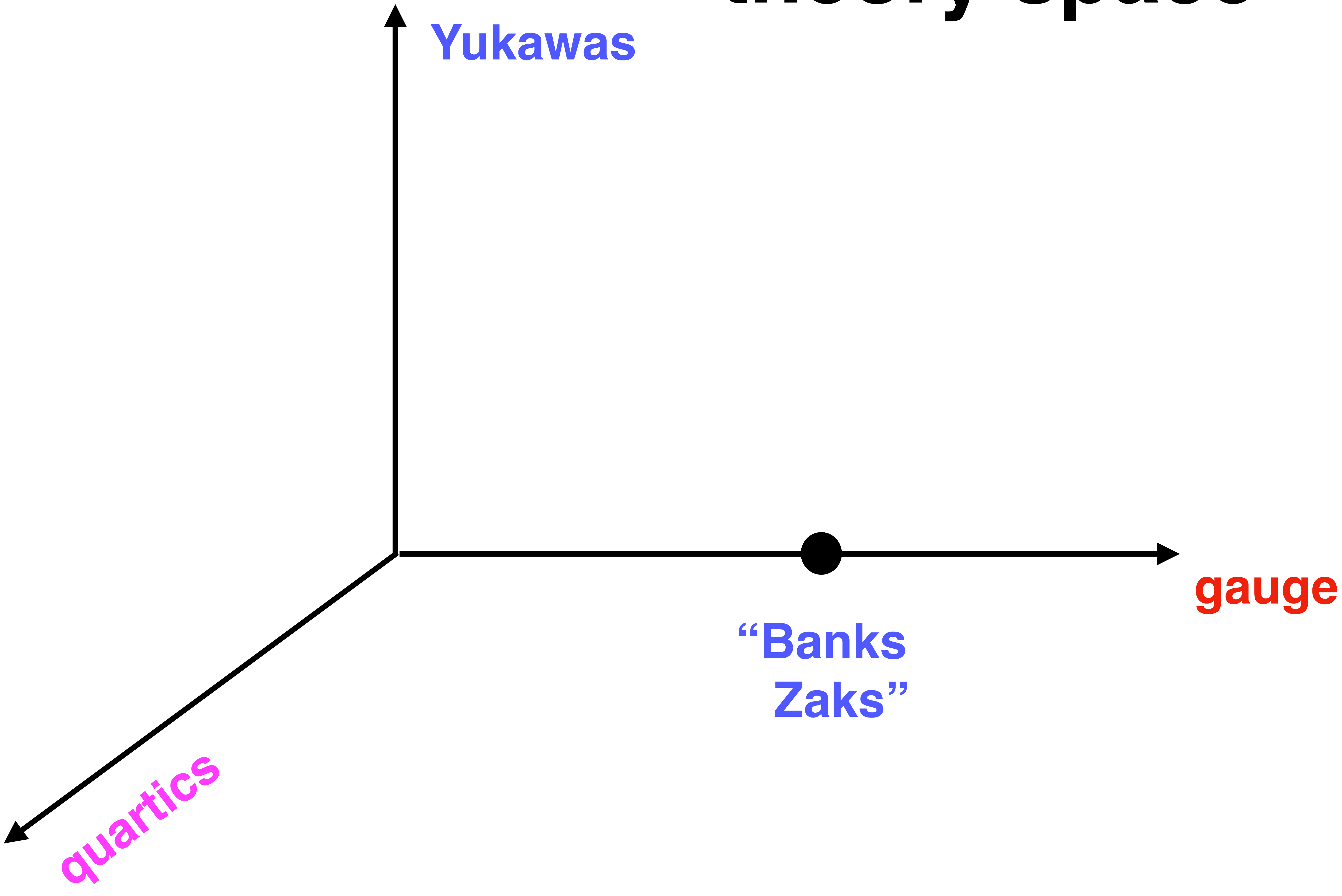

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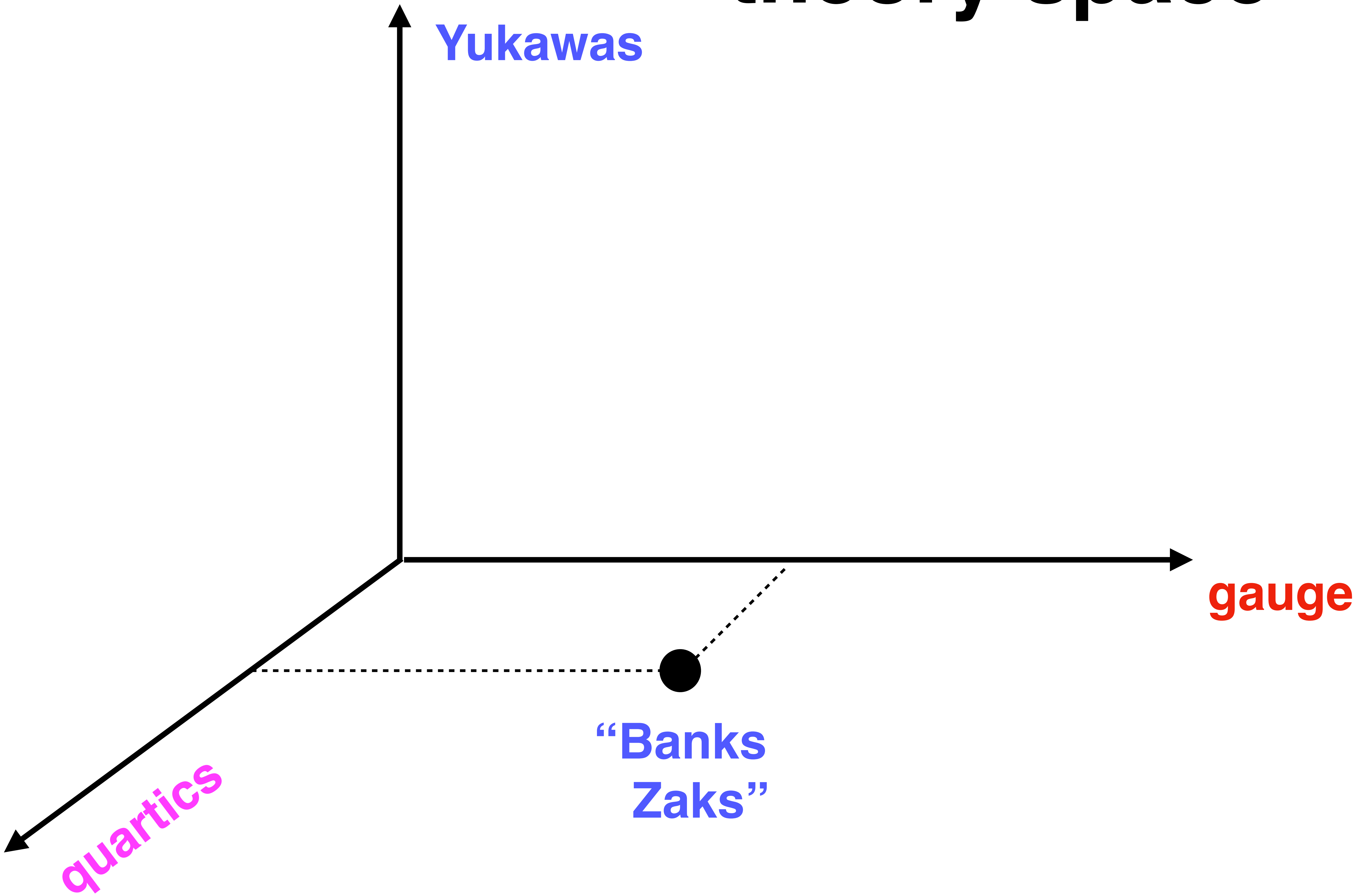

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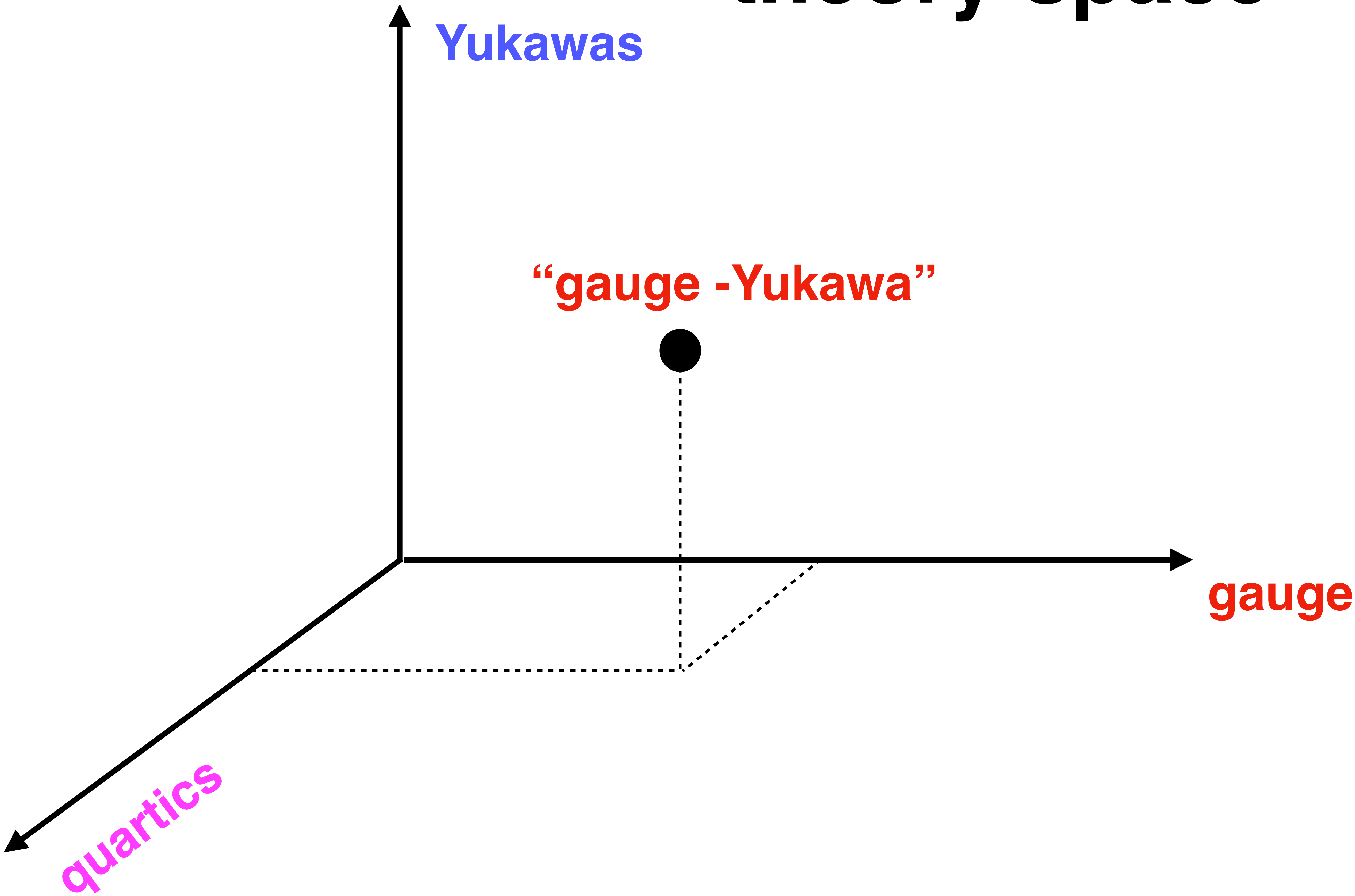

“theory space”

couplings	interacting fixed points				
gauge	Y	Y	Y	N	N
Yukawas	N	N	Y	N	Y
quartics	N	Y	Y	Y	Y

“Banks Zaks”

“gauge Yukawa”

“scalar”

“scalar Yukawa”

“theory space”

couplings	interacting fixed points				
gauge	Y	Y	Y	N	N
Yukawas	N	N	Y	N	Y
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	“Banks Zaks”	“Banks Zaks”	“gauge Yukawa”	“scalar”	“scalar Yukawa”

“theory space”

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“Banks Zaks”

“gauge Yukawa”

“scalar”

“scalar Yukawa”

4d critical points

	YES			NO				
couplings	non-abelian					abelian		
gauge	Y	Y	Y	N	N	Y	Y	Y
Yukawas	N	N	Y	N	Y	N	N	Y
quartics	N	Y	Y	Y	Y	N	Y	Y
weak FPs	IR	IR	IR UV	no	no	no	no	no

Banks Zaks

gauge Yukawa

scalar

scalar Yukawa

Banks Zaks

gauge Yukawa

AD Bond, DF Litim, **Price of Asymptotic Safety,** PRL 122 (2019) 211601 [1801.08527]

4d critical points

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quartics	N	Y	Y	Y	Y	N	Y	Y
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quartics	N	Y	Y	Y	Y	N	Y	Y
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Banks Zaks

gauge Yukawa

scalar

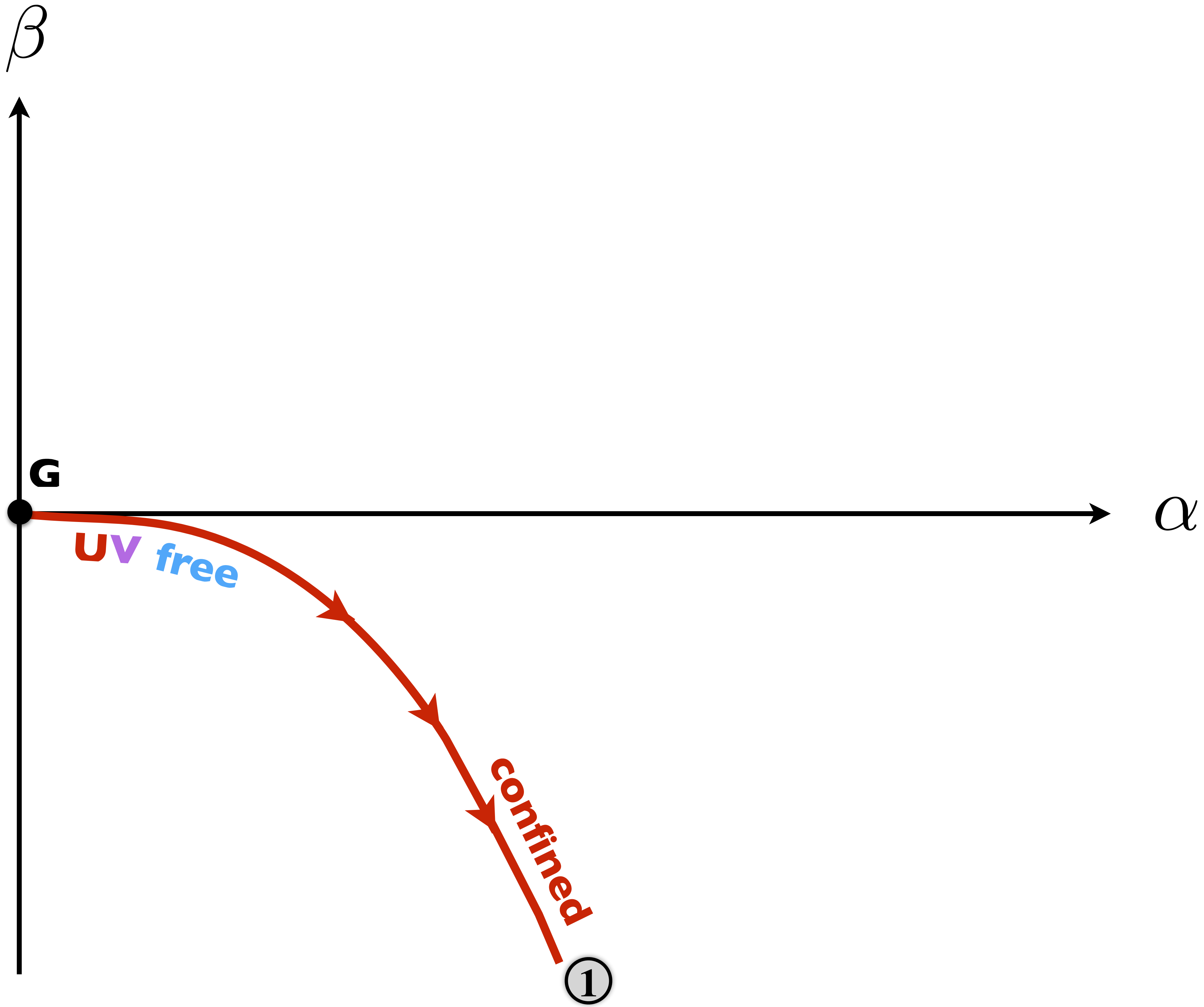
scalar Yukawa

Banks Zaks

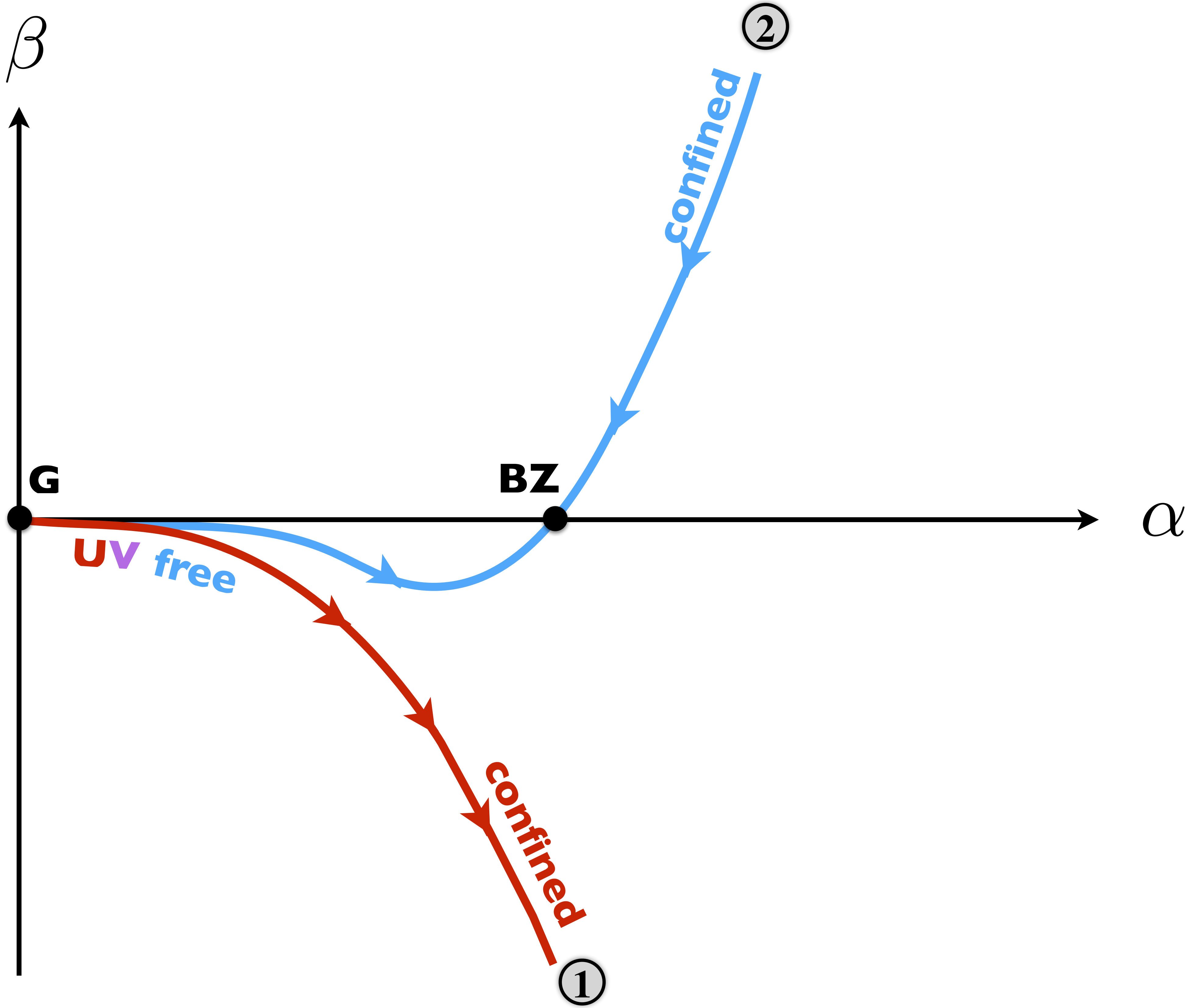
gauge Yukawa

AD Bond, DF Litim, **Price of Asymptotic Safety,** PRL 122 (2019) 211601 [1801.08527]

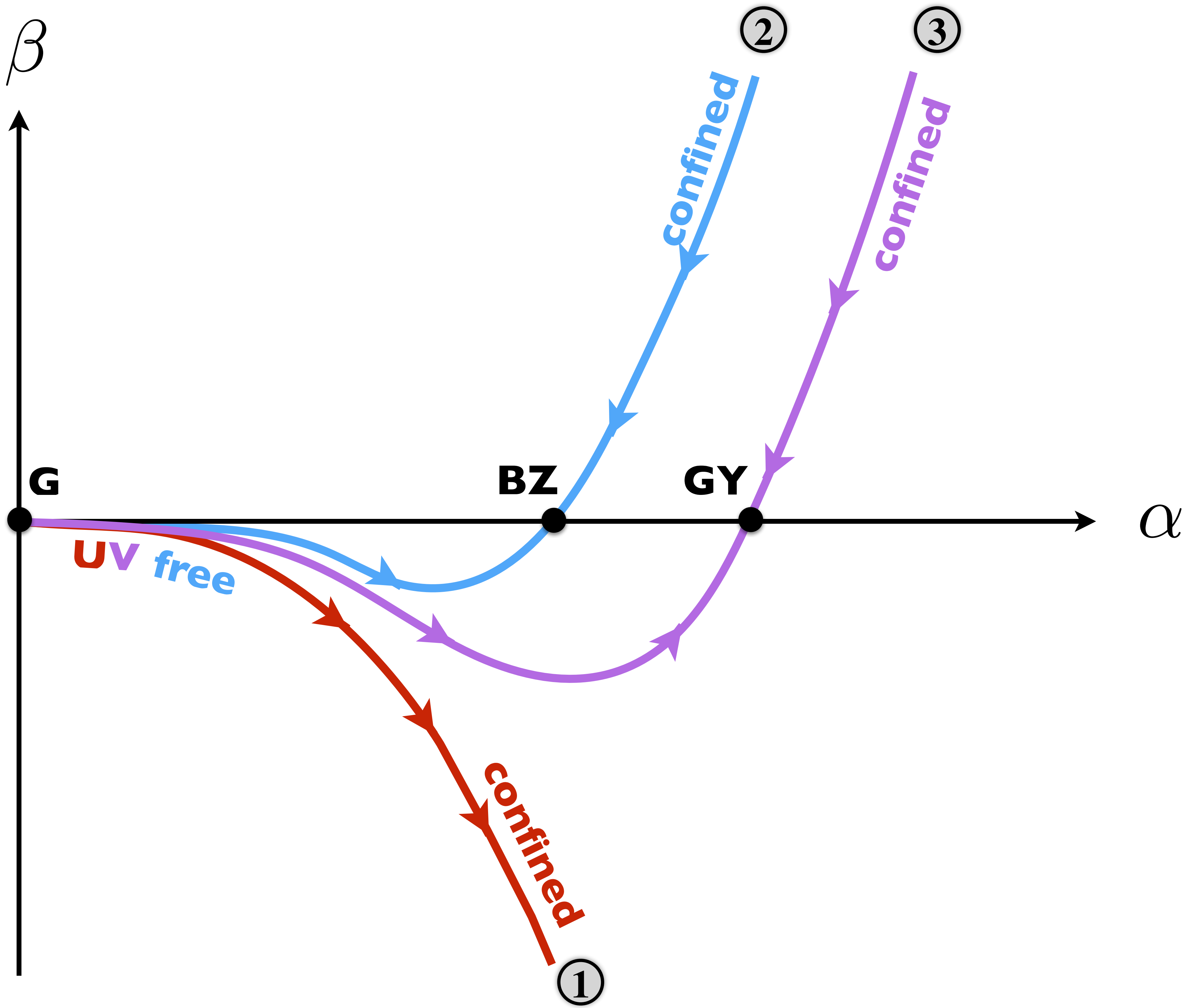
renormalisation group



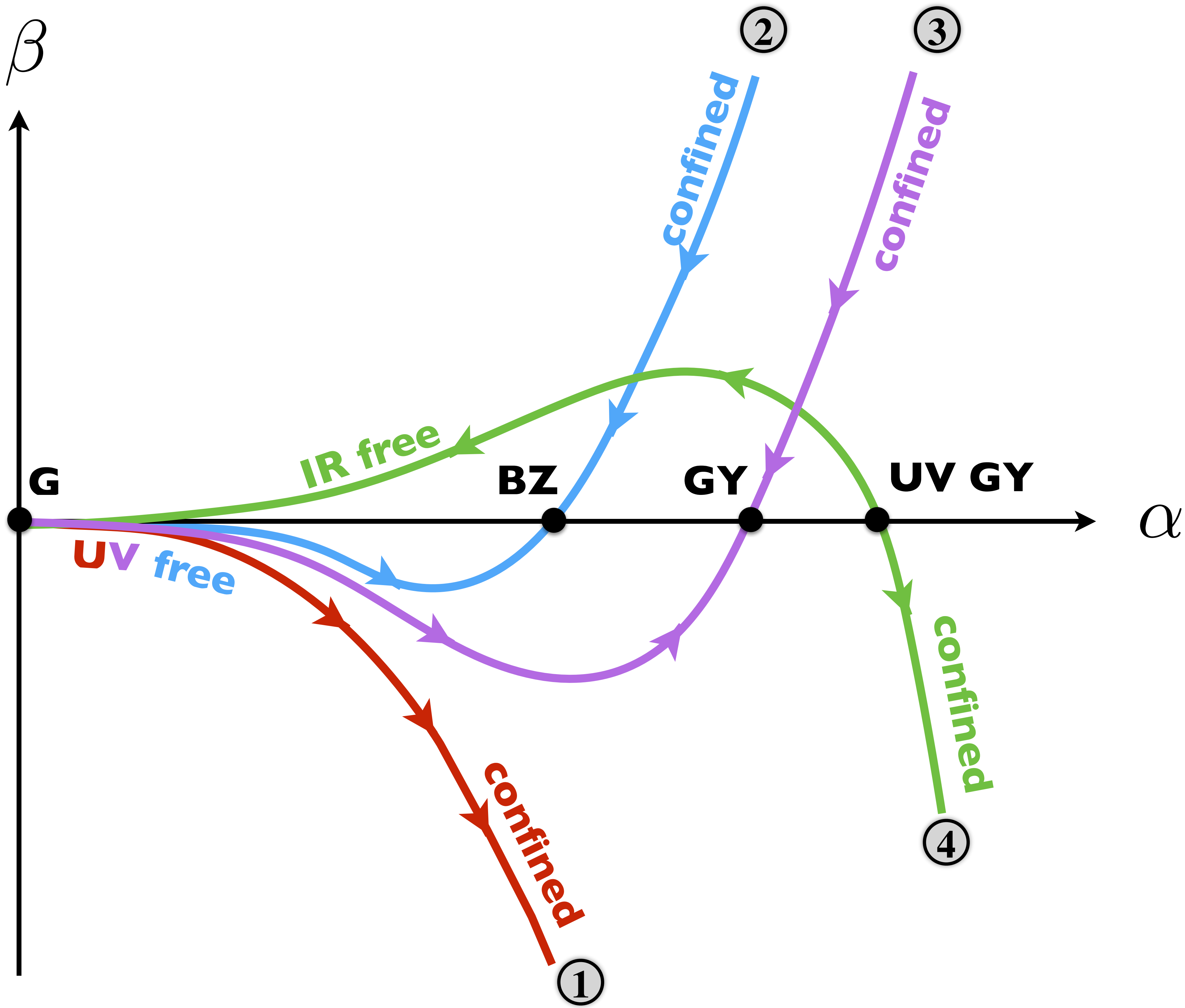
renormalisation group



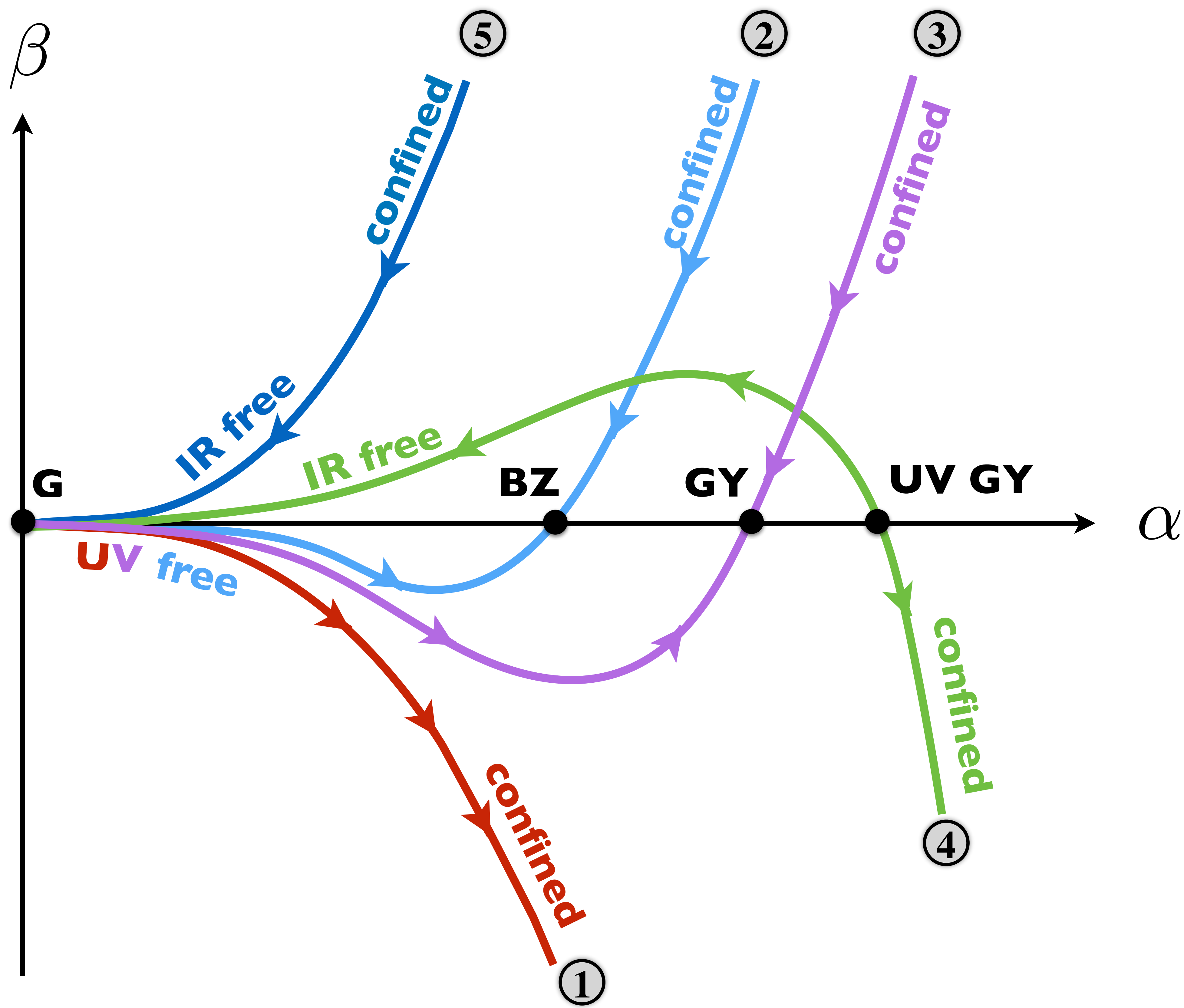
renormalisation group



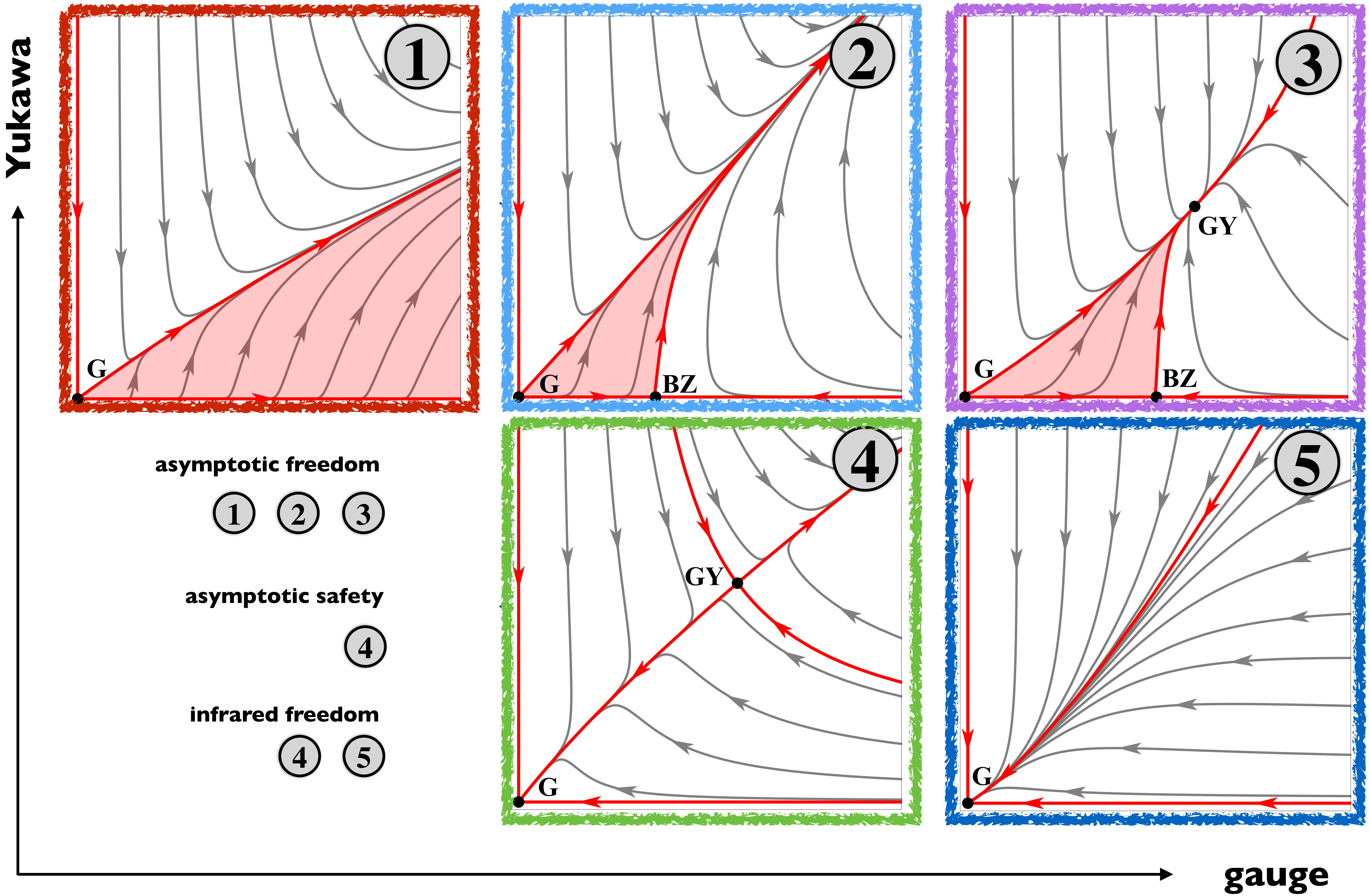
renormalisation group



renormalisation group



phase diagrams



some refs

general **theorems** for fixed points

AD Bond, DF Litim, **Theorems for Asymptotic Safety of Gauge Theories,** 1608.00519 (EPJC)
AD Bond, DF Litim, **Price of Asymptotic Safety,** 1801.08527 (PRL)

simple gauge theories with matter

DF Litim, F Sannino, **Asymptotic Safety Guaranteed,** 1406.2337 (JHEP)
AD Bond, DF Litim, G Medina Vazquez, T Steudtner, **Conformal window for asymptotic safety**, 1710.07615 (PRD)
AD Bond, DF Litim, T Steudtner, **Asymptotic safety with Majorana fermions and new large N equivalences** 1911.11168

semi-simple gauge theories with matter

AD Bond, DF Litim, **More Asymptotic Safety Guaranteed,** 1707.04217 (PRD)

supersymmetric gauge theories with matter

AD Bond, DF Litim, **Asymptotic Safety Guaranteed in Supersymmetry,** 1709.06953 (PRL)

higher order interactions in gauge theories with matter

T Buyukbese, DF Litim, **Asymptotic Safety Beyond Marginal Interactions,** PoS LATTICE2016 (2017) 233

phenomenology and models beyond BSM

A Bond, G Hiller, K Kowalska, DF Litim, **Directions for model building from asymptotic safety,** JHEP1708 (2017) 004
G Hiller, C Hermigos-Feliu, DF Litim, T Steudtner, **Asymptotically safe extensions of the Standard Model and their flavour phenomenology** 1905.11020, **Anomalous magnetic moments from asymptotic safety** 1910.14062
Model building from asymptotic safety with Higgs and flavour portals 2008.08606

4d critical points

couplings	YES			NO				
	non-abelian					abelian		
gauge	Y	Y	Y	N	N	Y	Y	Y
Yukawas	N	N	Y	N	Y	N	N	Y
quartics	N	Y	Y	Y	Y	N	Y	Y
weak FPs	IR	IR	IR UV	no	no	no	no	no

Banks Zaks

gauge Yukawa

scalar

scalar Yukawa

Banks Zaks

gauge Yukawa

AD Bond, DF Litim, **Price of Asymptotic Safety,** PRL 122 (2019) 211601 [1801.08527]

Caswell—Banks-Zaks

gauge coupling

$$\alpha = \frac{g^2}{(4\pi)^2}$$

$$\beta = -B\,\alpha^2 + C\,\alpha^3 + \mathcal{O}(\alpha^4)$$

1-loop (B) 2-loop (C)

weakly coupled IR fixed point $0 < \alpha^* = B/C \ll 1$

competition between matter and gauge fields

Caswell '74
Banks, Zaks, '82

Can this be UV? $B < 0, C < 0$

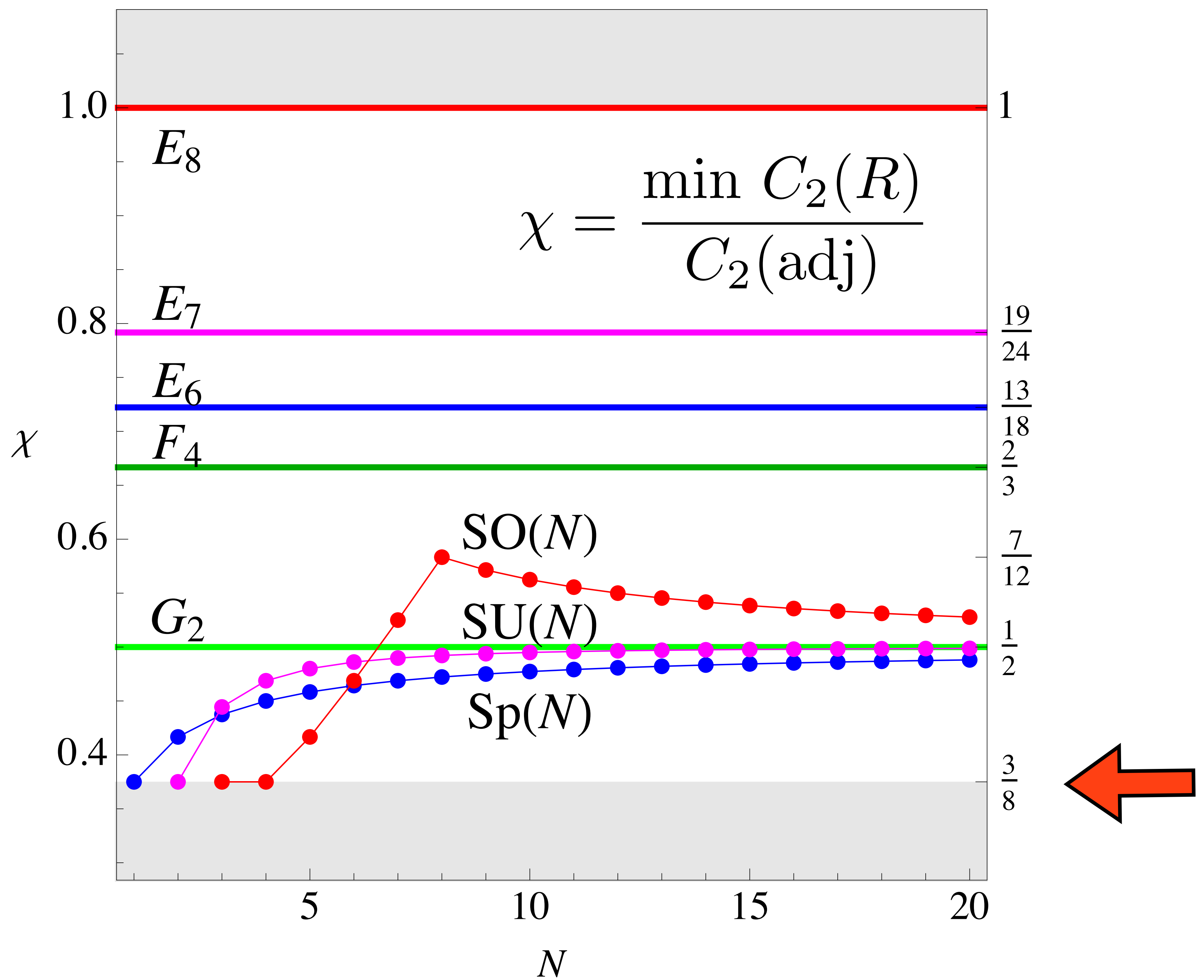
necessary condition

$$\chi = \frac{\min\, C_2(R)}{C_2(\text{adj})} < \frac{1}{11}$$

No.

Bond, DL '16

here's why.



Bond, DL '16

can other couplings help?

more gauge couplings **No** (same sign)

scalar self-couplings **No** (start at 3- or 4-loop)

Yukawa couplings **Yes!** (start at 2-loop)

Bond, DL '16

why Yukawas?

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

one loop　　gauge　　Yukawa

University of Sussex

here's why.

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

one loop — gauge — Yukawa

→ Yukawas ***uniquely*** slow down the running of gauge couplings

here's why.

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y \qquad t = \ln \mu/\Lambda$$

$$\partial_t \alpha_y = E\,\alpha_y^2 - F\,\alpha_g\,\alpha_y \qquad \alpha_* \ll 1$$

→ **interacting UV fixed point** provided that

$$C' = C - \frac{D\,F}{E} < 0$$

$$B < 0$$

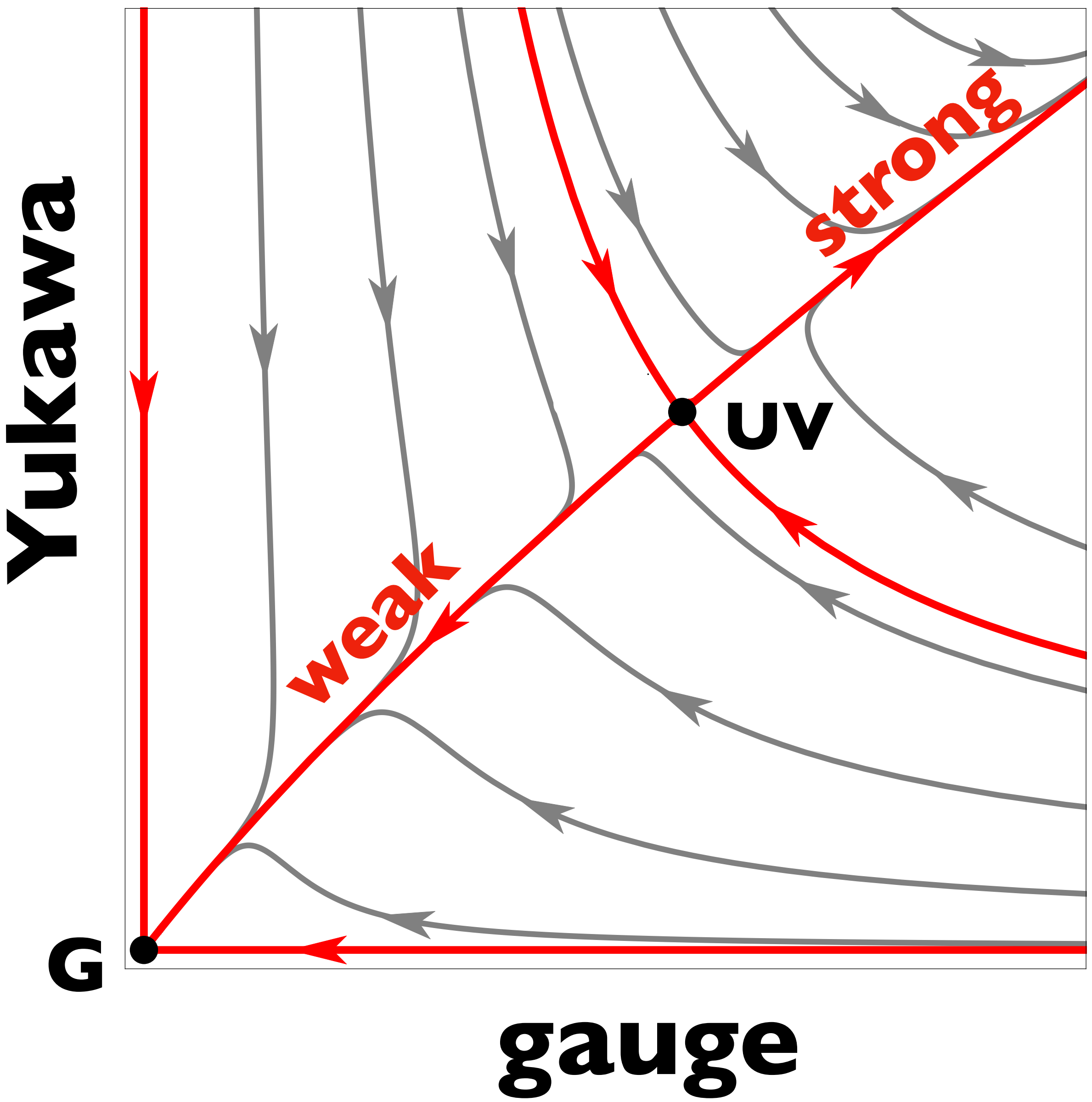
template

Lagrangean	Fields	Interactions
$L_{\rm YM} = -\frac{1}{2}{\rm Tr}\, F^{\mu\nu} F_{\mu\nu}$	gauge	gauge
$L_F = {\rm Tr}\left(\overline{Q}\, i \not{D}\, Q\right)$	fermions	
$L_Y = y\, {\rm Tr}\left(\overline{Q}\, H\, Q\right)$		Yukawas
$L_H = {\rm Tr}\,(\partial_\mu H^\dagger\, \partial^\mu H)$	scalars	
$L_U = -u\, {\rm Tr}\,(H^\dagger H)^2$		quartics
$L_V = -v\, ({\rm Tr}\, H^\dagger H)^2\,.$		

scalars are “meson-like”

Can do all types of UV and IR fixed points

templates with SU/SO/Sp



SU(N) + Diracs
+ mesons

SO(N) + Majoranas
+ mesons

Sp(N) + Majoranas
+ mesons

DF Litim, F Sannino, **Asymptotic safety guaranteed,** 1406.2337
AD Bond, DF Litim, G Medina Vazquez, T Steudtner, **Conformal window for asymptotic safety**, 1710.07615
AD Bond, DF Litim, T Steudtner, **Asymptotic safety with Majorana fermions and new large N equivalences** 1911.11168

4d critical points

Case	Condition	Fixed Point
$i)$	$g_i = \mathbf{Y}^A_{JK} = \lambda_{ABCD} = 0$	Gaussian
$ii)$	some $g_i \neq 0$, all $\mathbf{Y}^A_{JK} = 0$	Banks-Zaks
$iii)$	some $g_i \neq 0$, some $\mathbf{Y}^A_{JK} \neq 0$	gauge-Yukawa

AD Bond, DF Litim, **Price of Asymptotic Safety,** PRL 122 (2019) 211601 [1801.08527]

implications

- particle physics: UV complete 4D theories (free or safe) require non-abelian gauge fields

➡ asymptotic freedom and asymptotic safety are two sides of one and the same medal

implications

- particle physics: **UV complete 4D theories** (free or safe) require non-abelian gauge fields

- statistical physics: **universality class** of any weakly coupled 4D critical point contains non-abelian gauge fields

→ systematic classification of weak critical points in 4D

implications

- particle physics: **UV complete 4D theories** (free or safe) require non-abelian gauge fields

- statistical physics: **universality class** of any weakly coupled 4D critical point contains non-abelian gauge fields

- conformal field theory: **any** weakly-coupled **4D CFT** contains non-abelian gauge fields

"any QFT under perturbative control in the deep UV or IR asymptotes to a conformal field theory"

Polchinski '88
Luty, Polchinski, Rattazzi, '12

implications

- <u>particle physics:</u> **UV complete 4D theories** (free or safe) require non-abelian gauge fields

- <u>statistical physics:</u> **universality class** of any weakly coupled 4D critical point contains non-abelian gauge fields

- <u>conformal field theory:</u> **any** weakly-coupled **4D CFT** contains non-abelian gauge fields

→ infinitely many 4D CFTs
access to CFT data
complementary to bootstrap

outlook: strong coupling

supersymmetry

N=1: fixed points exist non-perturbatively

Bond, DL ‘22

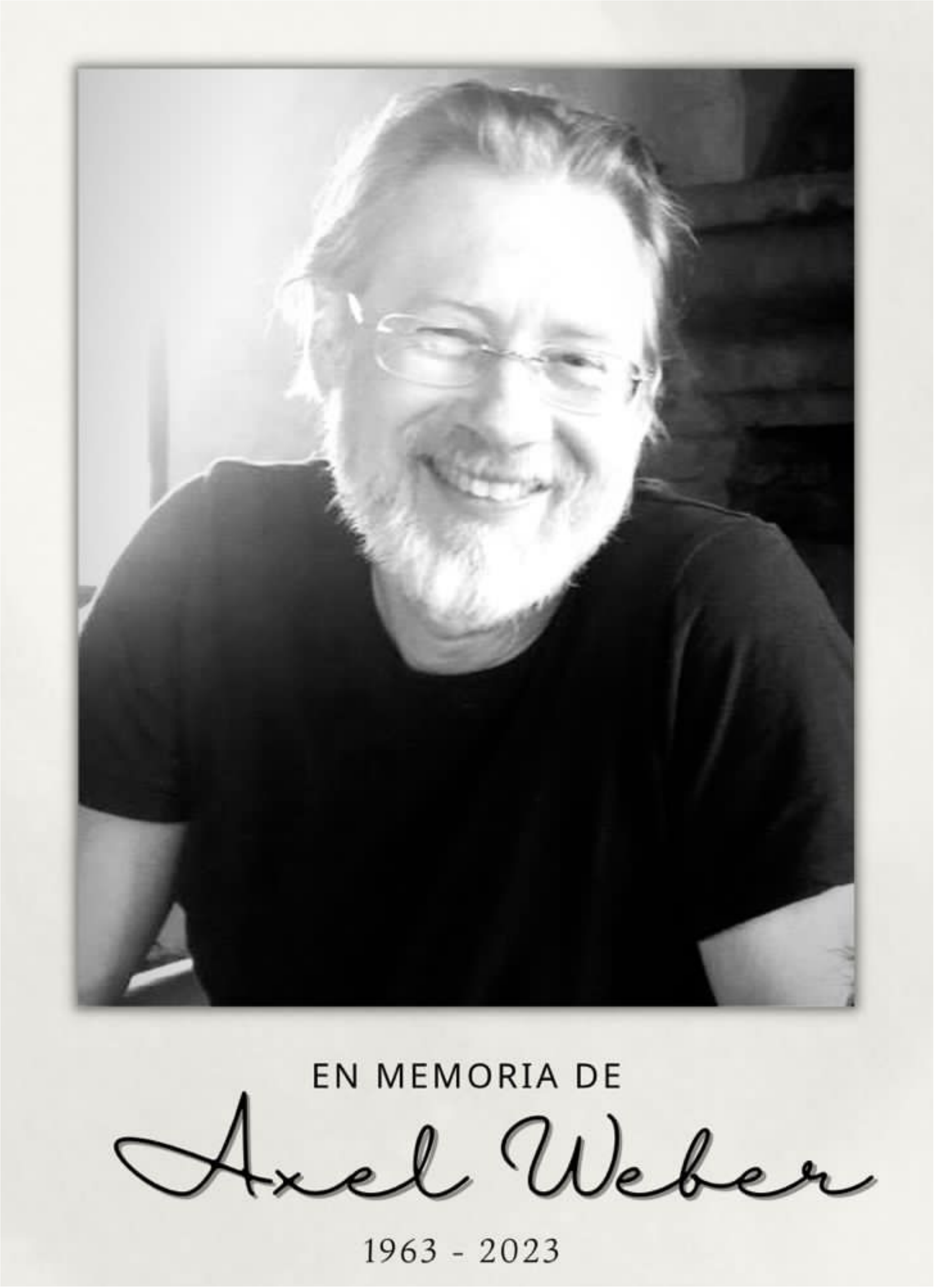
general particle theories

fixed points at strong coupling?

chiral symmetry breaking?

scaling vs decoupling?

confinement?



Celebrating Half a Century of QCD

Memorial to Axel Weber