FROM VACUUM AMPLITUDES TO PHYSICAL OBSERVABLES IN THE CAUSAL LOOP-TREE DUALITY

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CONFÍE
CIENCIA PARA EL PROGRESO

INTRODUCTION

- The description of the fundamental interactions rely on unitary and local quantum field theories.

- Multi-loop scattering amplitudes describe the quantum fluctuations at high-energy scattering processes are the main bottleneck.

- Accurate theoretical predictions in High Energy Physics require to deal with multi-loop and multi-leg scattering amplitudes.

- The precision phenomenology requires to re-think the way of doing the calculations.

HANDLING INFINITIES

- The standard definition for observables rely on QFTs.
- The basic algorithm are the Feynman rules which provides the theoretical predictions of nature though complicated expressions.
- First, build the scattering amplitude $|\mathscr{M}\rangle$, and then

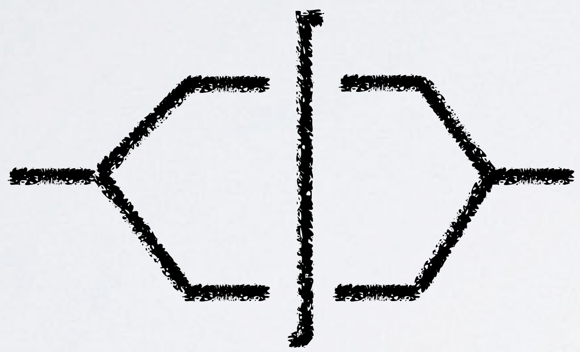
$$\sigma \sim \int d\mathrm{PS}\,|\mathscr{M}|^2$$

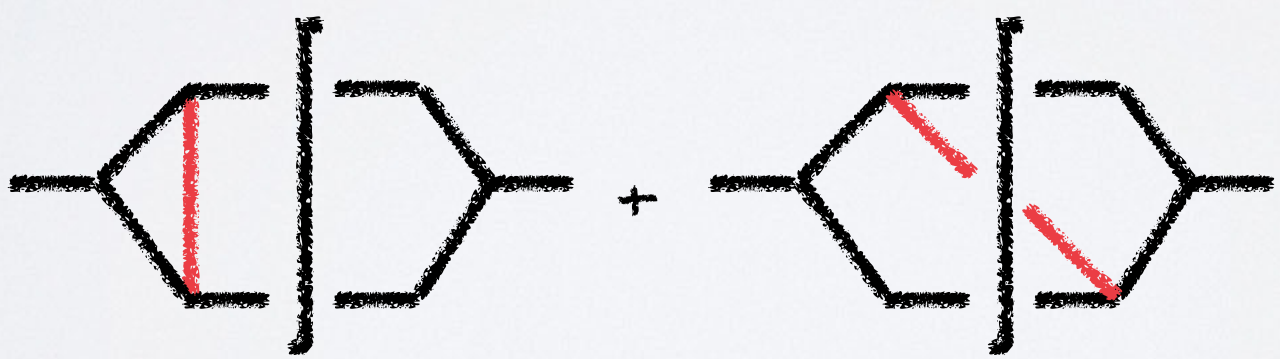
- However, the validity of QFT is extrapolated to *infinity energy,* when loops are computed, and also to *zero energy* when parallel particles mimic the behavior to a single particle emitted.

WHY D-DIMENSIONS ?

- Why divergences if the cross section is always finite ?

- Real and virtual amplitudes are not defined in the same integration domain.

$$\sigma^{\mathrm{LO}} =$$ 

$$\sigma^{\mathrm{NLO}} =$$ 

- Is there any way to mix both real and virtual contributions at integral level ?

- FDU takes the Loop-Tree Duality (LTD) theorem to merge both contributions at once.

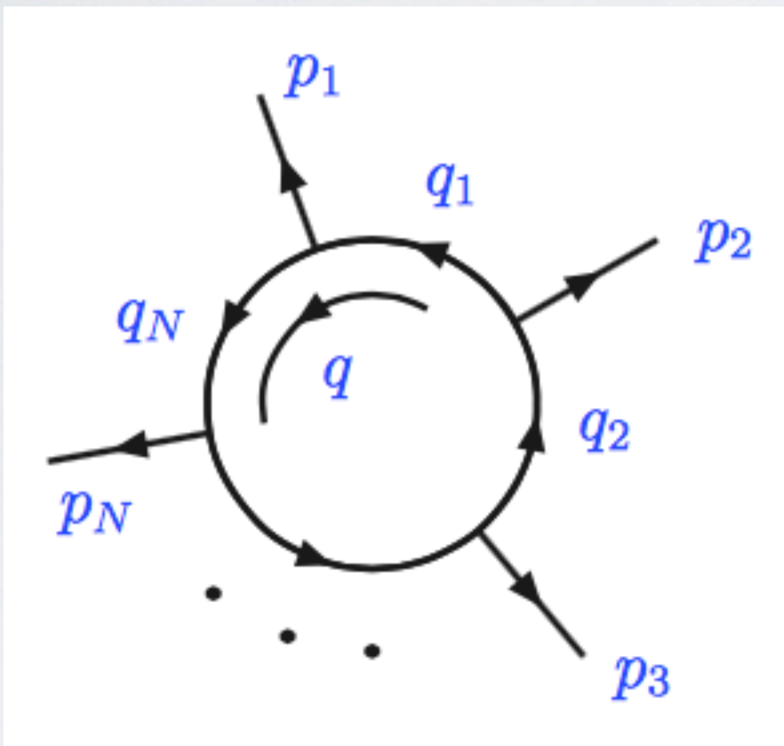
THE LTD PATH

The Four Dimensional Unsubtracion (FDU) formalism is an attempt to formulate QFT in four dimensions.

- Loop-Tree Duality. Catani et al, JHEP 09 (2008) 065
  Rodrigo et al, Nucl. Phys. Proc. Suppl. 183:262-267 (2008)
- Applications of the LTD. JHEP 11(2014) 014, JHEP 02(2016) 044
- FDU formalism. JHEP 08 (2016) 160, JHEP 10 (2016) 162
  Eur. Phys. J. C 78 no.3, 231, JHEP 02 (2019) 143
- Singular structures. JHEP 1912 (2019) 163
- Two-loop numerical approach. Phys. Rev. D 105, 016012
- Multiloop analytical results. Phys. Rev. Lett. 124 (2020) 21, JHEP 01 (2021) 069
  JHEP 04 (2021) 129, JHEP 02 (2021) 112
- Geometry and Causal representations. Phys. Rev. D 104, 036014, JHEP 04 (2021) 183
- Automation & UV in the LTD. Eur. Phys. J. C 81 (2021) 6, 514, JHEP 06 (2021) 089
- LTD Causal Unitary. e-Print: 2404.05492, e-Print: 2404.05491, PoS LL2024 (2024) 024

GENERALITIES OF THE LTD

- Massive one-loop scalar integrals are,



$$= -\imath \int \frac{d^d q}{(2\pi)^d} \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + \imath 0}$$

- where the +i0 prescription establishes that particles are going forward in time.

Catani et al, JHEP 09 (2008) 065, Rodrigo et al, Nucl. Phys. Proc. Suppl. 183:262-267 (2008)

- LTD at one loop establishes then

$$L^{(1)}(p_1, \cdots, p_N) = -\sum \int_{\ell_1} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j\neq i}}^{N} G_D(q_i; q_j)$$

- where Feynman propagators are transformed to dual propagators.

$$G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta \cdot (q_j - q_i)}$$

- $\tilde{\delta}(q_i) = 2\pi i \theta(q_{i,0}) \delta(q_i^2 - m_i^2)$ and sets internal lines on-shell and in the positive energy mode.

- LTD modify the +i0 prescription, instead of having multiple cuts like in the Feynman Tree Theorem.

- η^μ is a future-like vector, for simplicity we take $\eta^\mu = (1, \mathbf{0})$. In fact, the only relevance is the sign in the prescription.

NUMERICAL IMPLEMENTATION

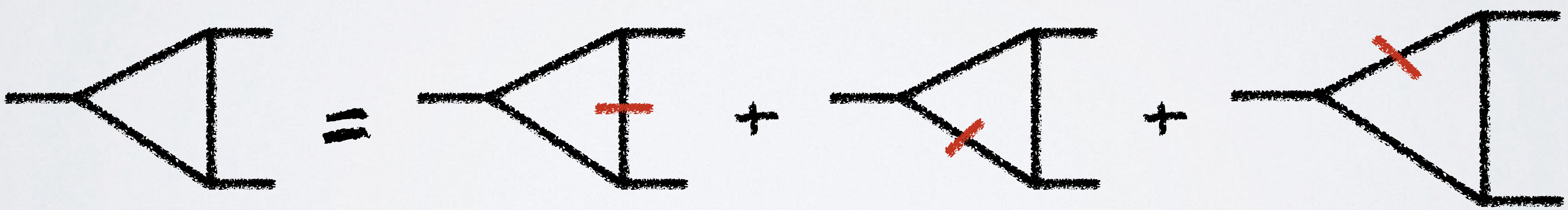
- Faster computations are needed for the Montecarlo simulations for the LHC observables.
- Using LTD the standard methods become time consuming

	Rank	Tensor Pentagon	Real Part	Imaginary Part	Time [s]
P16	2	LoopTools	-1.86472×10^{-8}		
		SecDec	$-1.86471(2) \times 10^{-8}$		45
		LTD	$-1.86462(26) \times 10^{-8}$		1
P17	3	LoopTools	1.74828×10^{-3}		
		SecDec	$1.74828(17) \times 10^{-3}$		550
		LTD	$1.74808(283) \times 10^{-3}$		1
P18	2	LoopTools	-1.68298×10^{-6}	$+i\ 1.98303 \times 10^{-6}$	
		SecDec	$-1.68307(56) \times 10^{-6}$	$+i\ 1.98279(90) \times 10^{-6}$	66
		LTD	$-1.68298(74) \times 10^{-6}$	$+i\ 1.98299(74) \times 10^{-6}$	36
P19	3	LoopTools	-8.34718×10^{-2}	$+i\ 1.10217 \times 10^{-2}$	
		SecDec	$-8.33284(829) \times 10^{-2}$	$+i\ 1.10232(107) \times 10^{-2}$	1501
		LTD	$-8.34829(757) \times 10^{-2}$	$+i\ 1.10119(757) \times 10^{-2}$	38

S. Buchta, et al. , Eur. Phys. J. C 77, 274 (2017)

IR SINGULARITIES

- The scalar one-loop three-point function contains a single pole, the IR.
- The first analysis towards the understanding of the LTD was done in this simple diagram.
- The application of the LTD, diagrammatically means that,



- It means that the full integrals is the sum over three phase-space integrals,

$$L^{(1)}(p_1, p_2, -p_3) = -\sum_{i=1}^{3} I_i$$

H.-P. et al., JHEP 02(2016) 044

IR REGULARISATION

- By KLN, the cancellation must be done through the addition of the real contribution. The matching condition is a key point in the cancellation of IR singularities, by segmenting the phase space as,

$$\tilde{\sigma}_{i,R} = \sigma_0^{-1} 2\mathrm{Re} \int d\Phi_{1\to 3} \langle \mathcal{M}_{2r}^{(0)} | \mathcal{M}_{1r}^{(0)} \rangle \theta(y'_{jr} - y'_{ir})$$

$$\tilde{\sigma}_{i,V} = \sigma_0^{-1} 2\mathrm{Re} \int d\Phi_{1\to 2} \langle \mathcal{M}^{(0)} | \mathcal{M}_{i}^{(1)} \rangle \theta(y'_{jr} - y'_{ir})$$

- where the virtual and real matrix elements are given by,

$$\langle \mathcal{M}^{(0)} | \mathcal{M}_{i}^{(1)} \rangle = -g^4 s_{12} I_i$$

$$\langle \mathcal{M}_{2r}^{(0)} | \mathcal{M}_{1r}^{(0)} \rangle = g^4 s_{12} / (s'_{1r} s'_{2r})$$

$$y'_{ir} = \frac{s_{12}}{s'_{ir}}$$

- By momentum conservation, $p_1 + p_2 = p'_1 + p'_2 + p'_r$

H.-P. et al., JHEP 02(2016) 044

UV SINGULARITIES

- The scalar bubble diagram contains only UV divergences.
- UV renormalisation requires local cancellation of divergences.
- In general, counterterms are obtained by expanding the propagator around a UV propagator

$$G_F(q_i) = \frac{1}{q_{UV}^2 - \mu_{UV}^2 + \imath 0} + \cdots \,, \qquad q_{UV} = \ell + k_{UV}$$

- For the bubble integral, the counterterm is

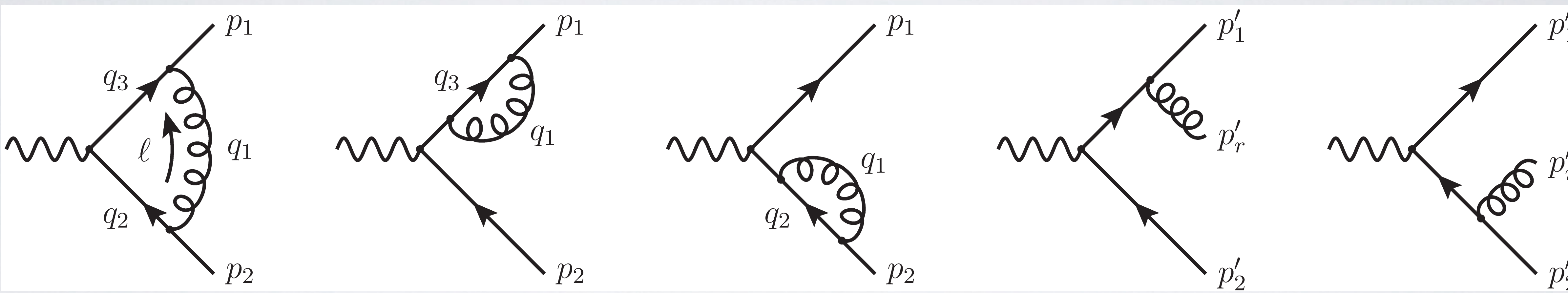
$$I_{UV}^{cnt} = \int_\ell \frac{1}{(q_{UV}^2 - \mu_{UV}^2 + \imath 0)^2} \xrightarrow{\text{LTD}} \quad I_{UV}^{cnt} = \int_\ell \frac{\tilde{\delta}(q_{UV})}{2(q_{UV,0}^{(+)})^2}$$

$$q_{UV,0}^{(+)} = \sqrt{\mathbf{q}_{UV}^2 + \mu_{UV}^2 - \imath 0}$$

H.-P. et al., JHEP 02(2016) 044

CROSS SECTION IN 4D

- As in Dimensional methods, we study $\gamma^* \to q\bar{q}$ @ NLO in QCD



- Using the FDU formalism, we find,

$$\tilde{\sigma}_1^{(1)} = \sigma^{(0)} \frac{\alpha_S}{4\pi} C_F \left(19 - 32\log(2)\right),$$

$$\tilde{\sigma}_2^{(1)} = \sigma^{(0)} \frac{\alpha_S}{4\pi} C_F \left(-\frac{11}{2} + 8\log(2) - \frac{\pi^2}{3}\right),$$

$$\bar{\sigma}_V^{(1)} = \sigma^{(0)} \frac{\alpha_S}{4\pi} C_F \left(-\frac{21}{2} + 24\log(2) + \frac{\pi^2}{3}\right).$$

Each piece has been computed in 4D

The sum coincides with the result with Dimensional methods

Sborlini et al., JHEP 08 (2016) 160

AMPLITUDES

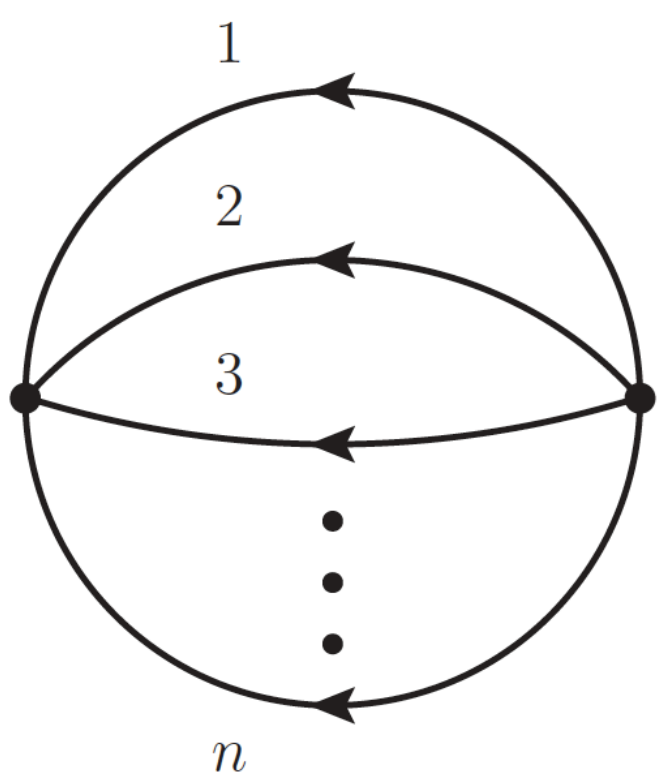
AMPLITUDES IN QFT

- Amplitudes contains the information of the theory that is tested in Particle Physics Phenomenology.
- In general, it is a rational function which has issues always in the denominator. The numerator can be handle to reduce the complexity of the denominator.
- The maximal complexity is, then, when the numerator is unity.
- Since amplitudes will be integrated, the computation of analytic functions has been a bottleneck in phenomenology.

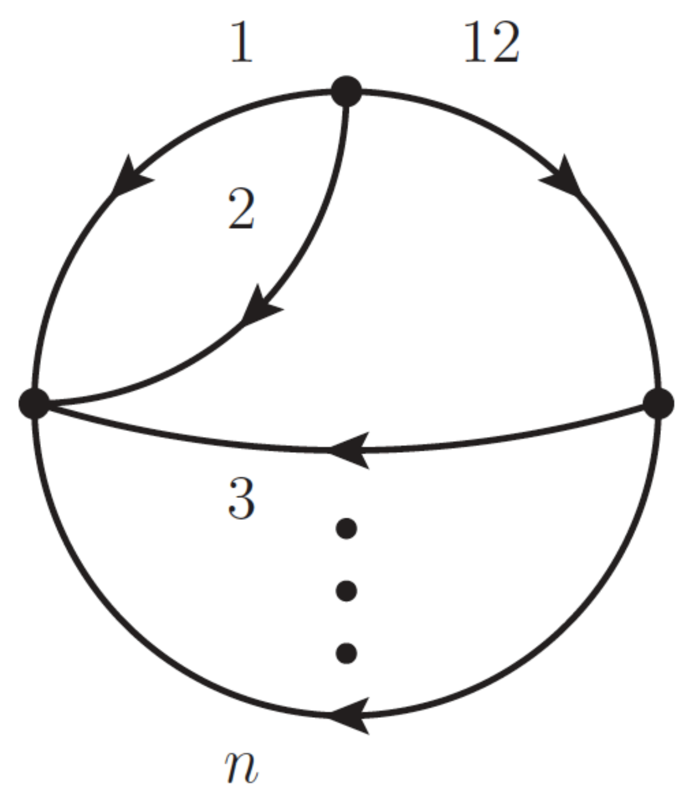
MULTI-LOOP DIAGRAMS

- We were interested in the application of the LTD to some topologies.

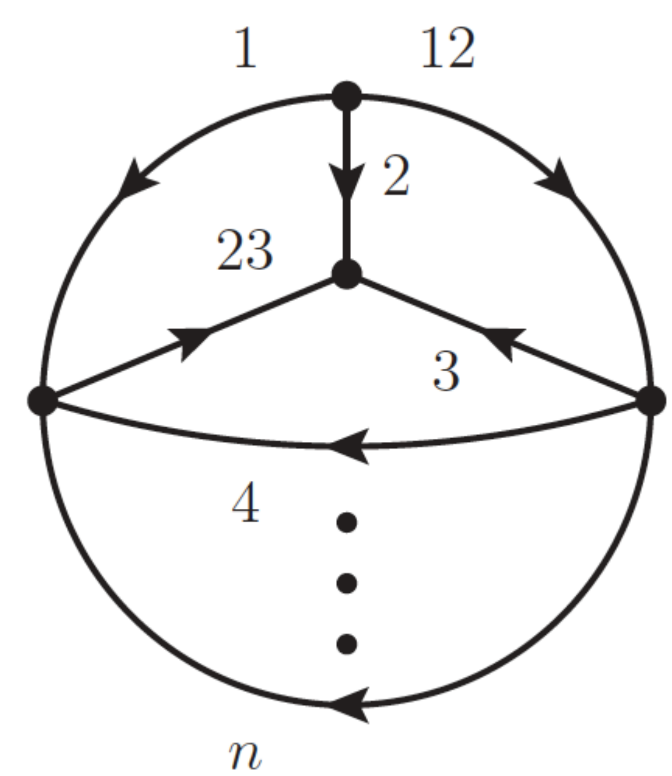
- The most simplest arrangements are, up to three-loops,



Maximal Loop Topology
MLT



Next-to-Maximal Loop Topology
NMLT

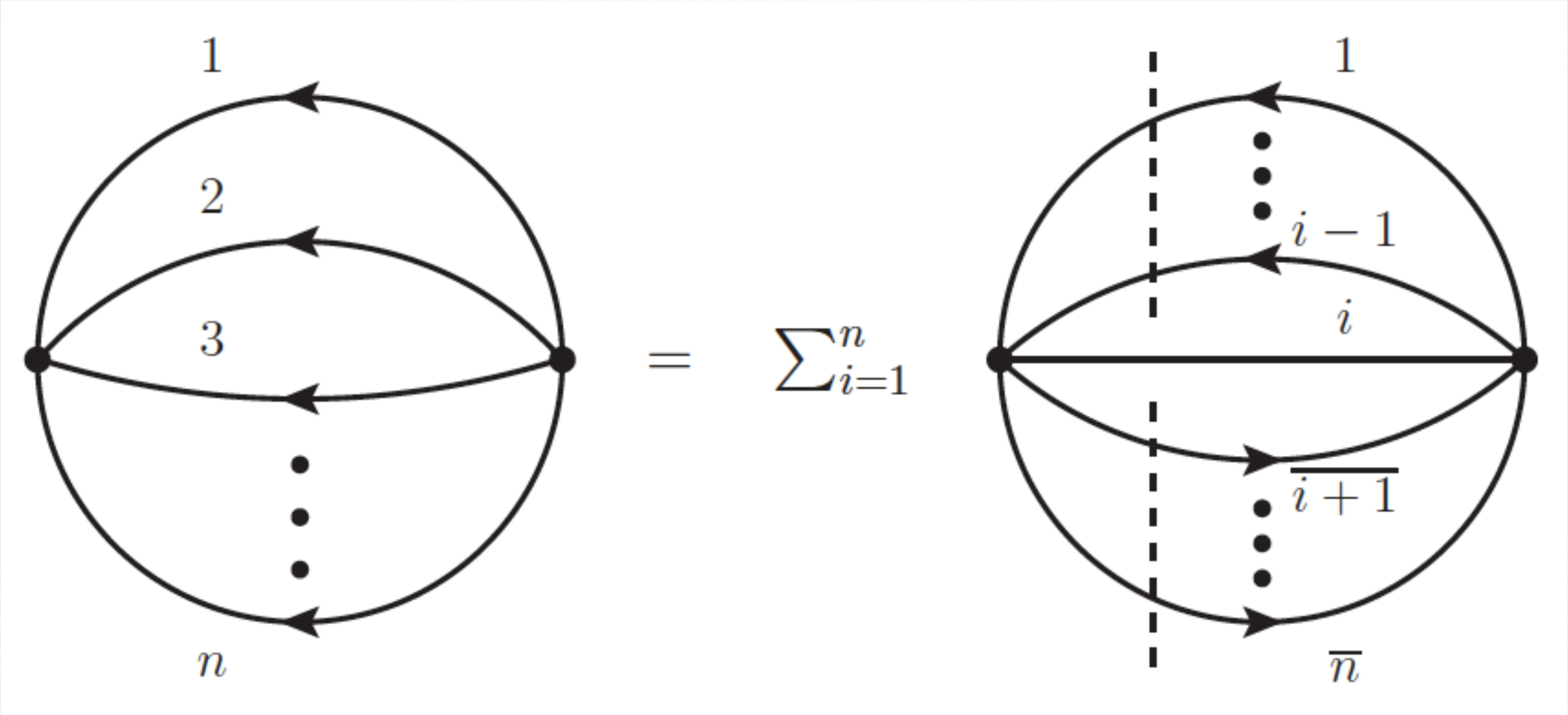


Next-to-Next-Maximal Loop Topology
NNMLT

Aguilera-Verdugo et al., Phys. Rev. Lett. 124 (2020) 21

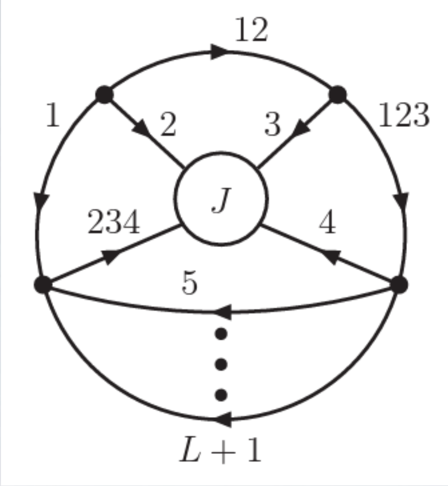
DUAL REPRESENTATIONS

- It is possible to find, dual representations for each topology. For instance,

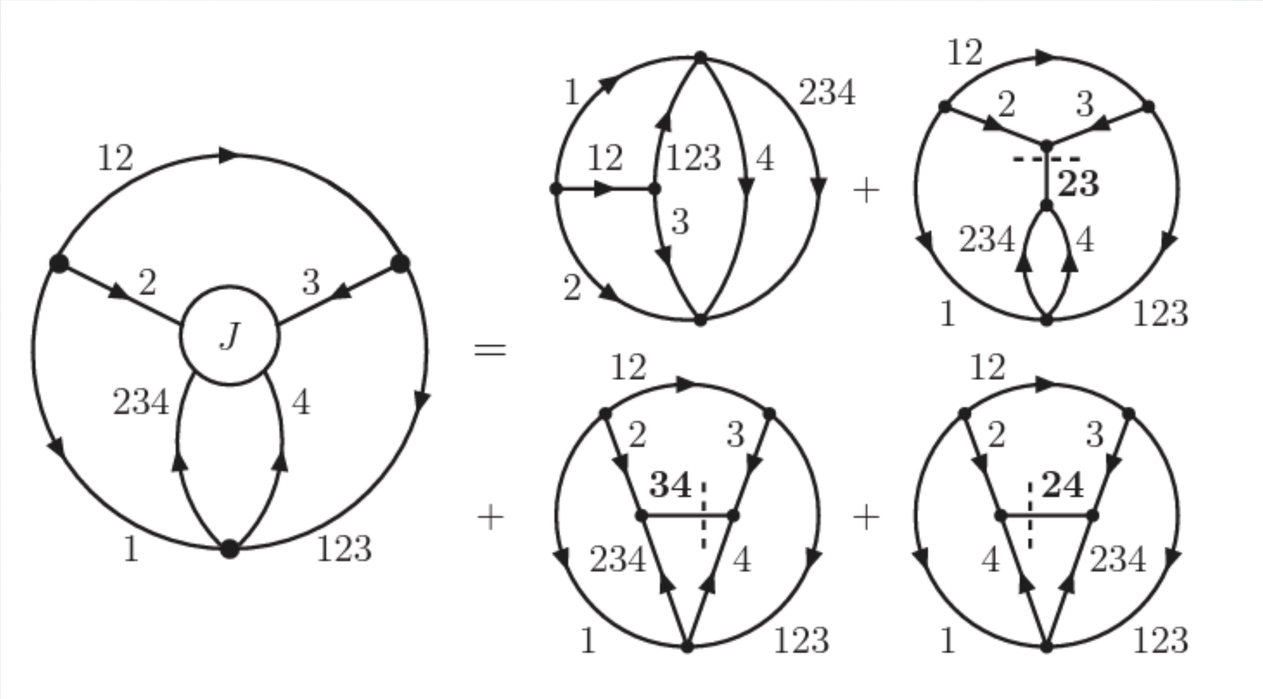


$$\mathcal{A}^{(L)}_{\mathrm{MLT}}(1,\cdots,n) = \int_{\ell_1,\cdots,\ell_L} \sum_{i=1}^{n} \mathcal{A}^{L}_{D}(1,\cdots,i-1,\overline{i+1},\cdots,\overline{n};i)$$

LTD FOREST



- At four loops, we find that four main topologies appear. They can be casted in terms of the well know s, t and u kinematical variables.



- Nevertheless, the number of trees depends on the number of loops.

s	15(3L-7)
t	5(8L-7)
u	9(5L-11)

Ramírez-Uribe et al., JHEP 04 (2021) 129

CAUSAL REPRESENTATIONS

- Denominators are important in the representations since the complexity of the integrals are explicitly there.

- Combinatorics in the dual representations shows that there are denominators that could present a non-physical singularity, such as,

$$+q_{1,0}^{(+)} + \cdots + q_{i-1,0}^{(+)} - q_{i,0}^{(+)} + q_{i+1,0}^{(+)} + \cdots + q_{N,0}^{(+)}$$

- However, causal representation do not have such non-physical divergences. The most general expression is given by,

$$\sum_{i=1} \pm q_{i,0}^{(+)} \to 0$$

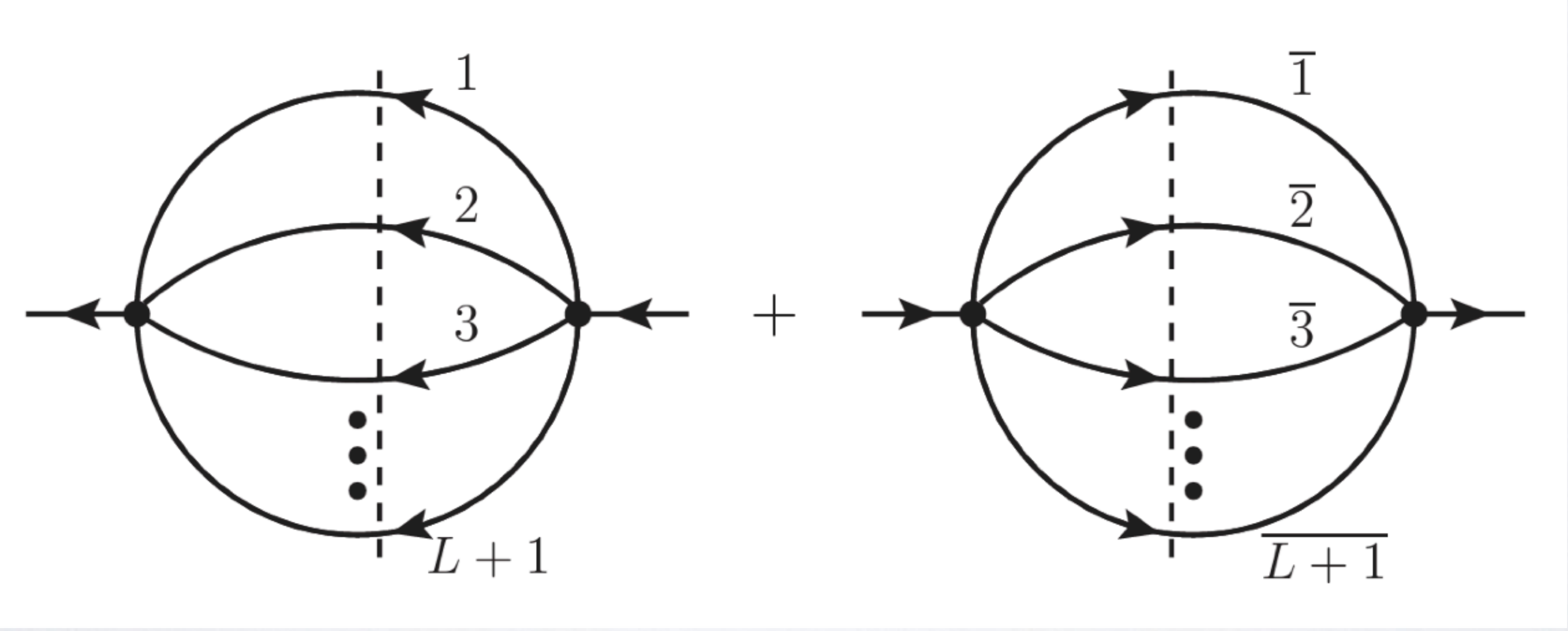
Aguilera-Verdugo et al., JHEP 01 (2021) 069

CAUSAL REPRESENTATIONS

- The most simple causal diagram is in the MLT topology.The MLT contains two different propagators,

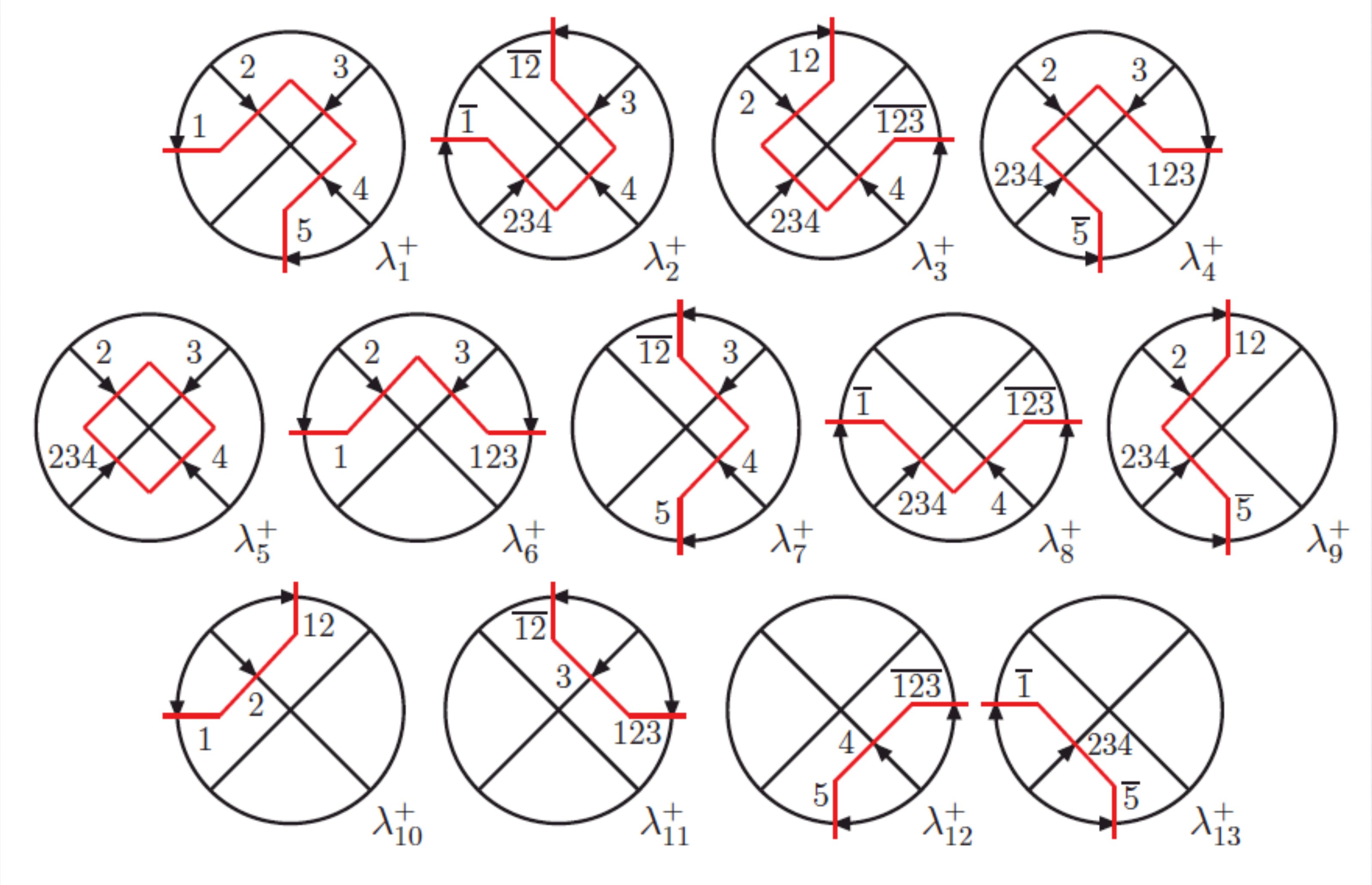
$$\sum_{i=1} q_{i,0}^{(+)} \pm p_{1,0}$$

- which can be interpreted diagrammatically.



CAUSAL NMLT

- For instance, the N3MLT requires 13 causal propagators to describe the full amplitude.

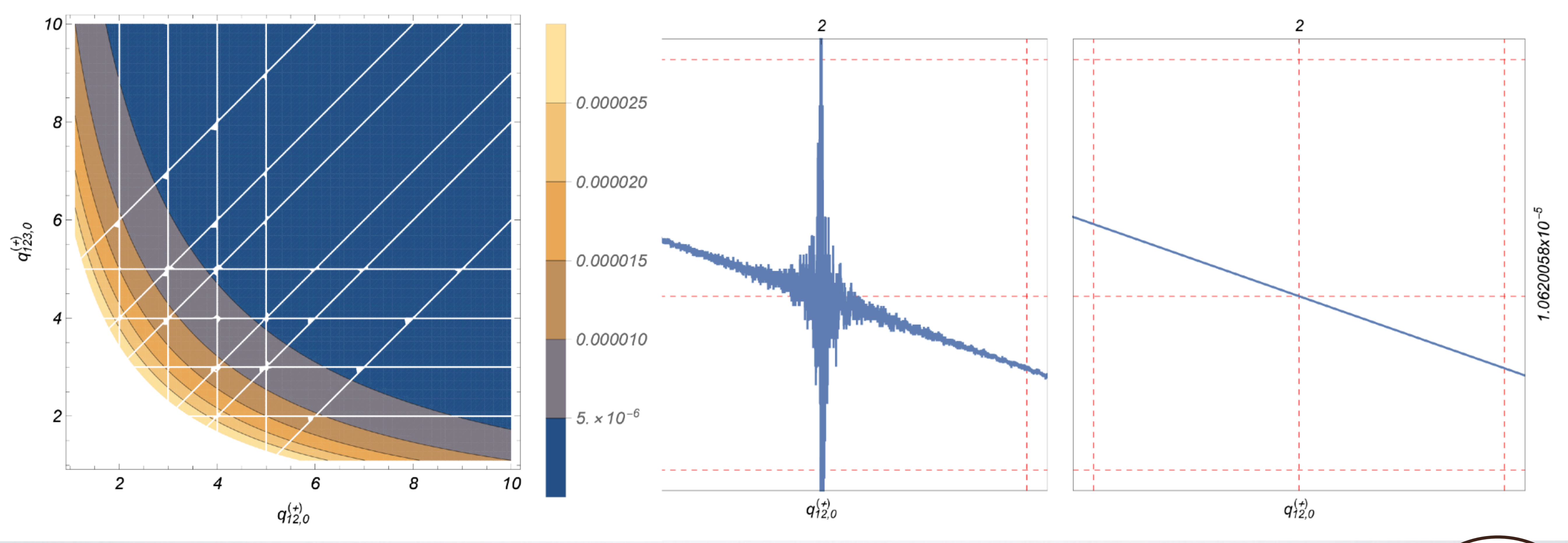


- Poles are fixed in physical singularities, are they easy to integrate numerically ?

Ramírez-Uribe et al., JHEP 04 (2021) 129

DUAL VS CAUSAL

- Dual representations have still threshold singularities; however, Causal representation has only physical divergences.



- Smooth integrands in the Causal representation.

Ramírez-Uribe et al., JHEP 04 (2021) 129

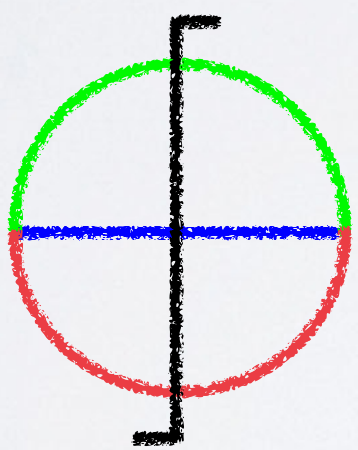
LTD CAUSAL UNITARY

FROM VACUUM TO OBSERVABLES

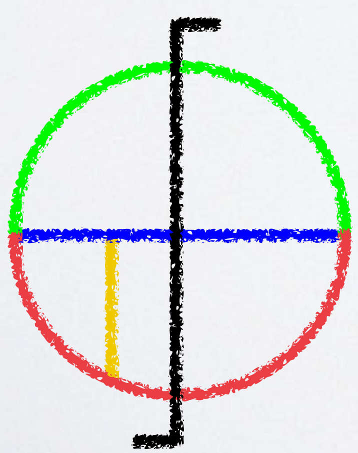
- All observables are computed from squared amplitudes and summing over final states and, in general, summing over initial polarization states.

- Therefore, in general for a decay process at NLO, $1 \to 23$,

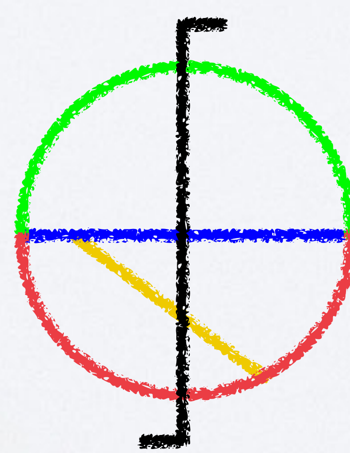
$\Gamma^{LO} =$



$\Gamma^{NLO} =$



$+$



$-$ UV counterterms

Ramírez-Uribe et al, e-Print: 2404.05492

- Therefore, the differential contribution for a process at $k^{th}-$order in the perturbation theory in the LTD causal unitary has the form,

$$d\Gamma_a^{(k)} = \frac{d\Lambda}{2m_a} \sum_{(i_1\cdots i_k)\in\Sigma} \mathscr{A}_D^{(\Lambda,R)}(i_1\cdots i_n a)\tilde{\Delta}_{i_1\cdots i_k \bar{a}}$$

- where Σ denotes the set of all phase-space configurations, then

$$d\Gamma_a^{N^kLO} = \sum_{j=0}^{k} d\Gamma_a^{(j)}$$

- where $d\Gamma_a^{N^kLO}$ denotes the differential decay rate up to (next-to)^k -leading order, and the integration measure is given by,

$$d\Lambda = \prod_{j=1}^{\Lambda-1} d\Phi_{\vec{\ell}_j} = \prod_{j=1}^{\Lambda-1} \mu^{4-d} \frac{d^{d-1}\vec{\ell}}{(2\pi)^{d-1}}$$

- with $\{\ell_j\}_{j=1,\cdots,\Lambda}$ the loop momenta.

- The renormalized vacuum amplitud is given by,

$$\mathcal{A}_D^{(\Lambda,R)}(i_1\cdots i_n a) = \mathcal{A}_D^{(\Lambda)}(i_1\cdots i_n a) - \mathcal{A}_{UV}^{(\Lambda)}(i_1\cdots i_n a)$$

- with the causal amplitude is defined as,

$$\mathcal{A}_D^{(\Lambda)}(i_1\cdots i_n a) = Res\left(\frac{x_a}{2}\mathcal{A}_D^{(\Lambda)}, \lambda_{i_1\cdots i_n a}\right)$$

- where the $\lambda-$structures are in general,

$$\lambda_{i_1\cdots i_n a} = \sum_{s=1}^{n} q_{i_s,0}^{(+)} + q_{a,0}^{(+)}$$

- with the on-shell energies defined as

$$q_{a,0}^{(+)} = \sqrt{\vec{q}_a^{\,2} + m_a^2 - i0}$$

- and $x_a = 2q_{a,0}^{(+)}$.

- The complete definition of the decay rate requires de definition of the phase-space as a function of,

$$\tilde{\Delta}_{i_1 \cdots i_n \bar{a}} = 2\pi\delta(\lambda_{i_1 \cdots i_n \bar{a}})$$

- with

$$\lambda_{i_1 \cdots i_n \bar{a}} = \sum_{s=1}^{n} q_{i_s,0}^{(+)} - q_{a,0}^{(+)}$$

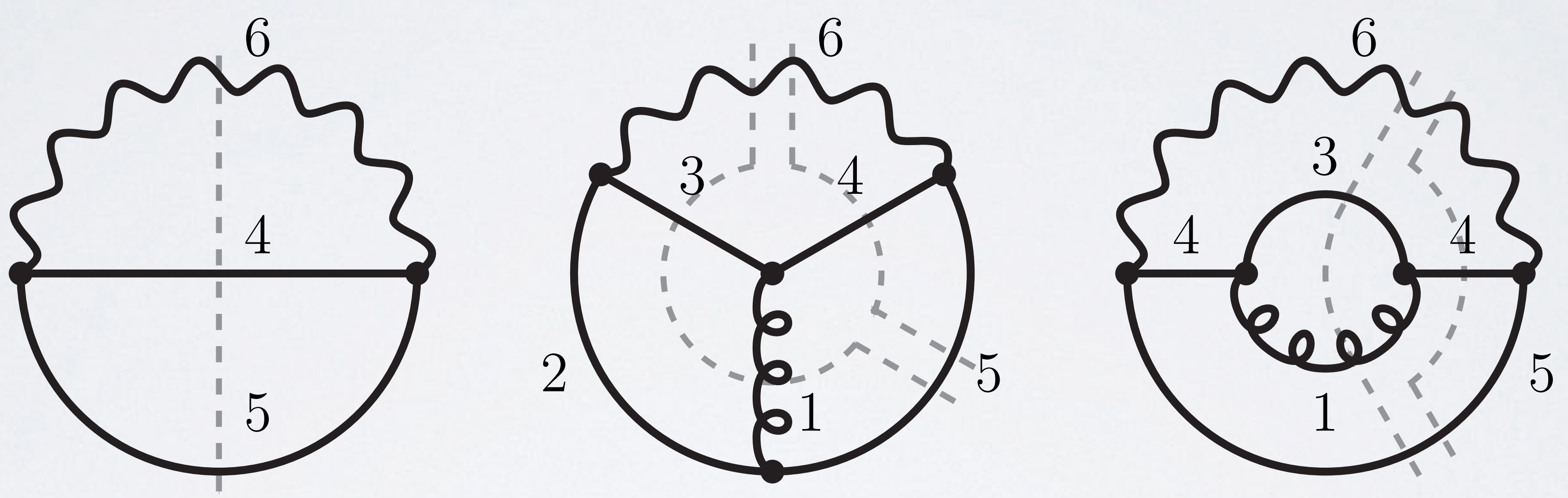
- Finally, it is important to mention that the causal propagators shall appears as,

$$\frac{1}{\lambda_{i_1 \cdots i_m}} = \left(\sum_{s=1}^{m} q_{i_s,0}^{(+)} \right)^{-1}$$

- It is important to mention that in general the numerator shall be a function of on-shell energies and internal masses.

$f \to q\bar{q}$ IN THE LTD CAUSAL UNITARY

- Let us consider the standard process, $f \to q\bar{q}$ at NLO in QCD with $f \in \{\gamma^*, H\}$. The vacuum amplitudes in the LTD causal unitary are,



- where the momenta labelling are,

$$q_1 = \ell_1 + \ell_2, q_2 = \ell_1 + \ell_3, q_3 = \ell_1$$

$$q_4 = \ell_2\,, q_5 = \ell_2 - \ell_3\,, q_6 = \ell_3$$

Ramírez-Uribe et al, e-Print: 2404.05492

- Applying the procedure described before, we define de LO decay rate as,

$$d\Gamma^{LO}_{f\to q\bar{q}} = \frac{d\Phi_{\vec{\ell}_2}}{2\sqrt{s}}\mathscr{A}^{(2,f)}_{D}(456)\tilde{\Delta}_{45\bar{6}}$$

- where,

$$\mathscr{A}^{(2,f)}_{D}(456) = \frac{2g_f^{(0)}}{x_{456}}\left(\frac{\overline{|\mathscr{M}^{(0)}_{f\to q\bar{q}}|^2}}{\lambda_{456}} + 2\lambda_{45\bar{6}}\right)$$

- and

$$\tilde{\Delta}_{45\bar{6}} = \frac{\pi}{\beta}\delta\left(|\vec{\ell}_2| - \frac{\beta\sqrt{s}}{2}\right) \quad ,$$

- with $\beta = \sqrt{1-4m^2/s}$ and the standard results,

$$\overline{|\mathscr{M}^{(0)}_{\gamma^*\to q\bar{q}}|^2} = 2s\left(1 + \frac{1-\beta^2}{d-2}\right) \, , \qquad \overline{|\mathscr{M}^{(0)}_{H\to q\bar{q}}|^2} = 2s\beta^2$$

- The NLO decay rate is now,

$$d\Gamma^{(1)}_{f\to q\bar{q}} = \frac{d\Phi_{\vec{\ell}_1\vec{\ell}_2}}{2\sqrt{s}}\left(\left(\mathscr{A}^{(3,f,R)}_{D}(456)\tilde{\Delta}_{45\bar{6}} + \mathscr{A}^{(3,f)}_{D}(1356)\tilde{\Delta}_{135\bar{6}}\right) + (5 \leftrightarrow 2, 4 \leftrightarrow 3)\right)$$

- where the renormalized amplitude is given in 4 dimensions is,

$$\mathscr{A}^{(3,f,R)}_{D}(456) = \left(\mathscr{A}^{(3,f)}_{D}(456) - \mathscr{A}^{(3,f)}_{UV}(456)\right)\Big|_{d=4}$$

- with the subtraction counterterms,

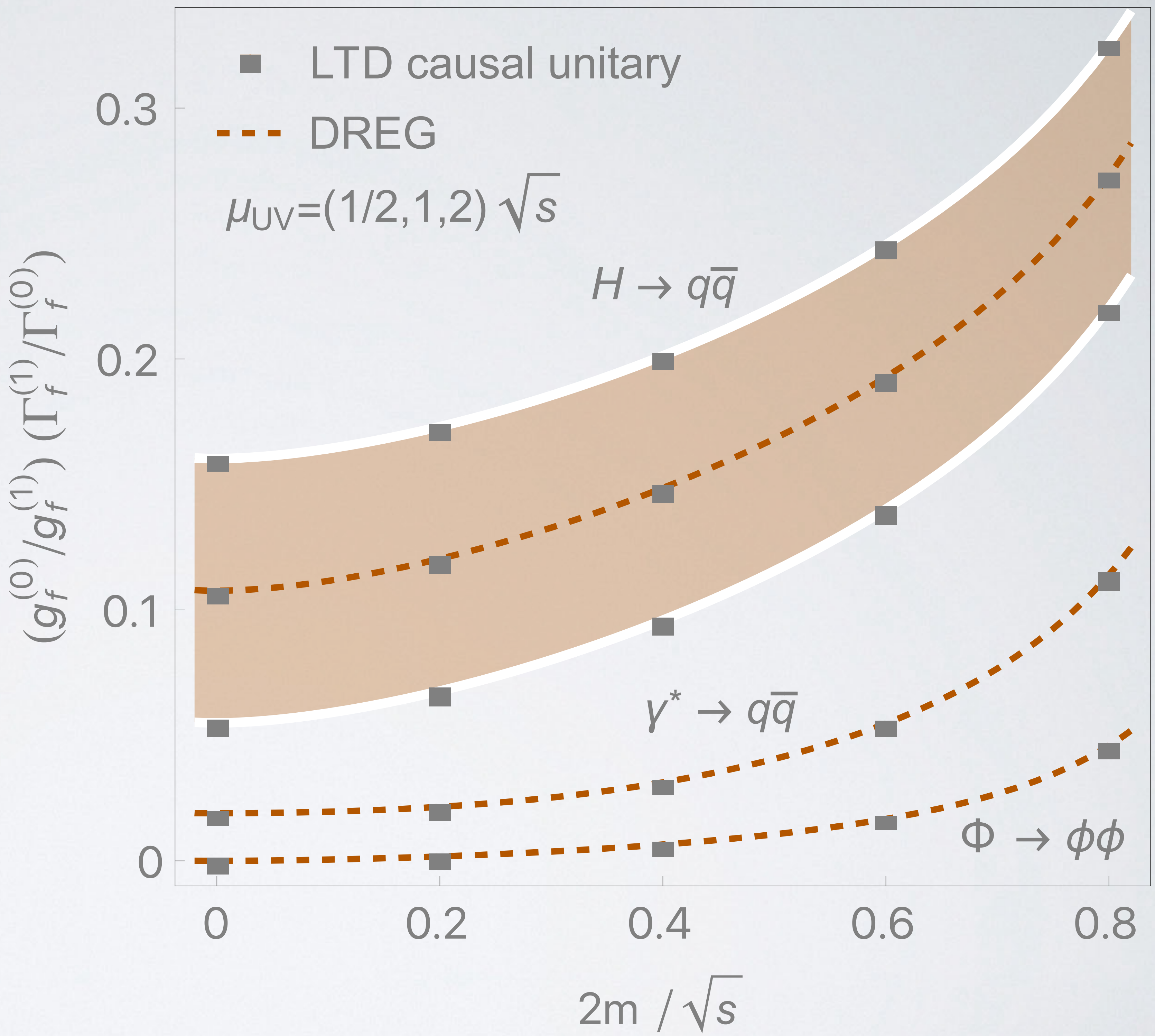
$$\mathscr{A}^{(3,H)}_{UV}(456) = \frac{2g^{(1)}_{H}}{x_{45}}\left(\Delta Z^{(UV)}_{H}\overline{|\mathscr{M}^{(0)}_{f\to q\bar{q}}|^2} - \Delta Z^{(UV)}_{m} 8m^2(1+\beta^2) + \Delta^{(UV)}_{H}\right)$$

- and the renormalization constants in the LTD causal unitary.

- Finally, we compare our analytical results with numerical evaluations.

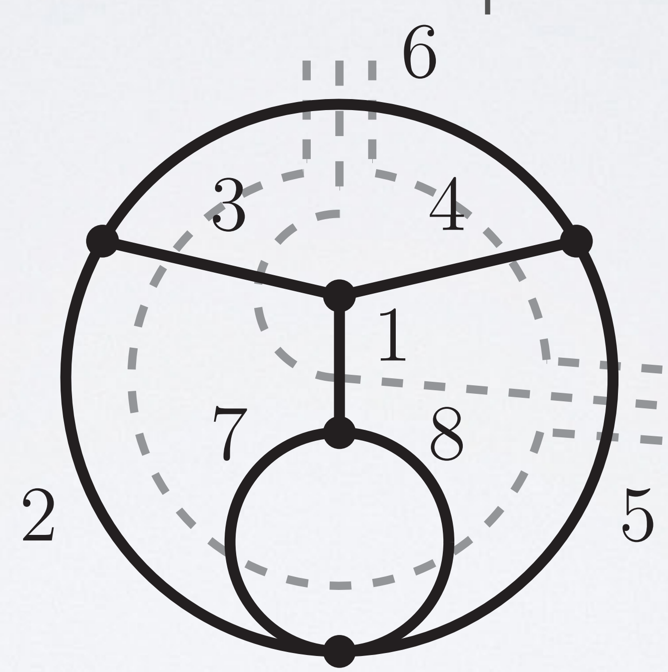
- We find perfect agreement with DREG for both Higgs and virtual photon decay.

- We present also the numerical evaluation for the heavy scalar decay into light scalars.



$\Phi \to \phi\phi$ AT NNLO

- The vacuum amplitud contributing to this particular process is given by the four loop amplitude,

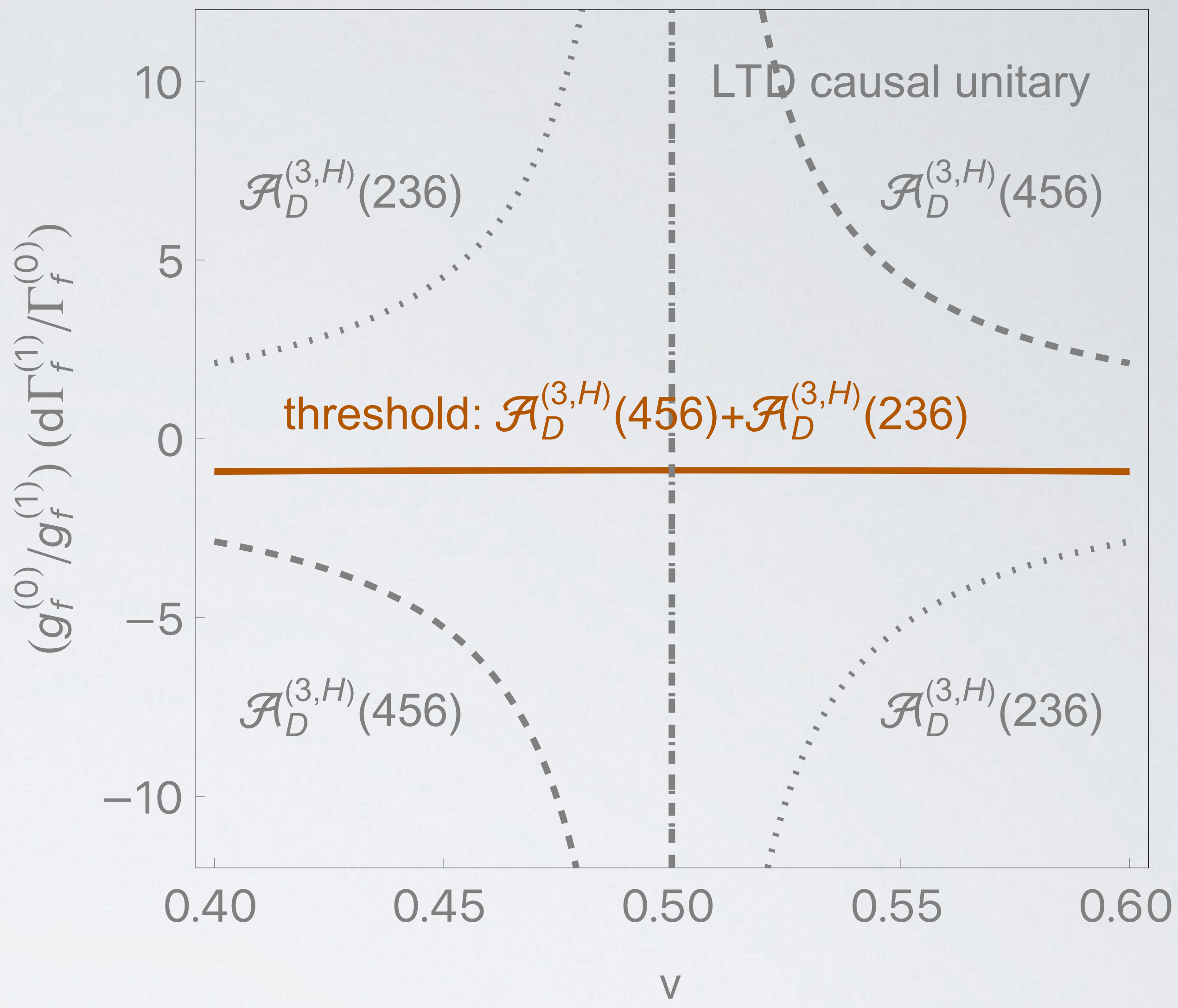


- where the extra four-momenta given by,

$$q_7 = \ell_4 \text{ and } q_8 = \ell_4 + \ell_1 + \ell_2$$

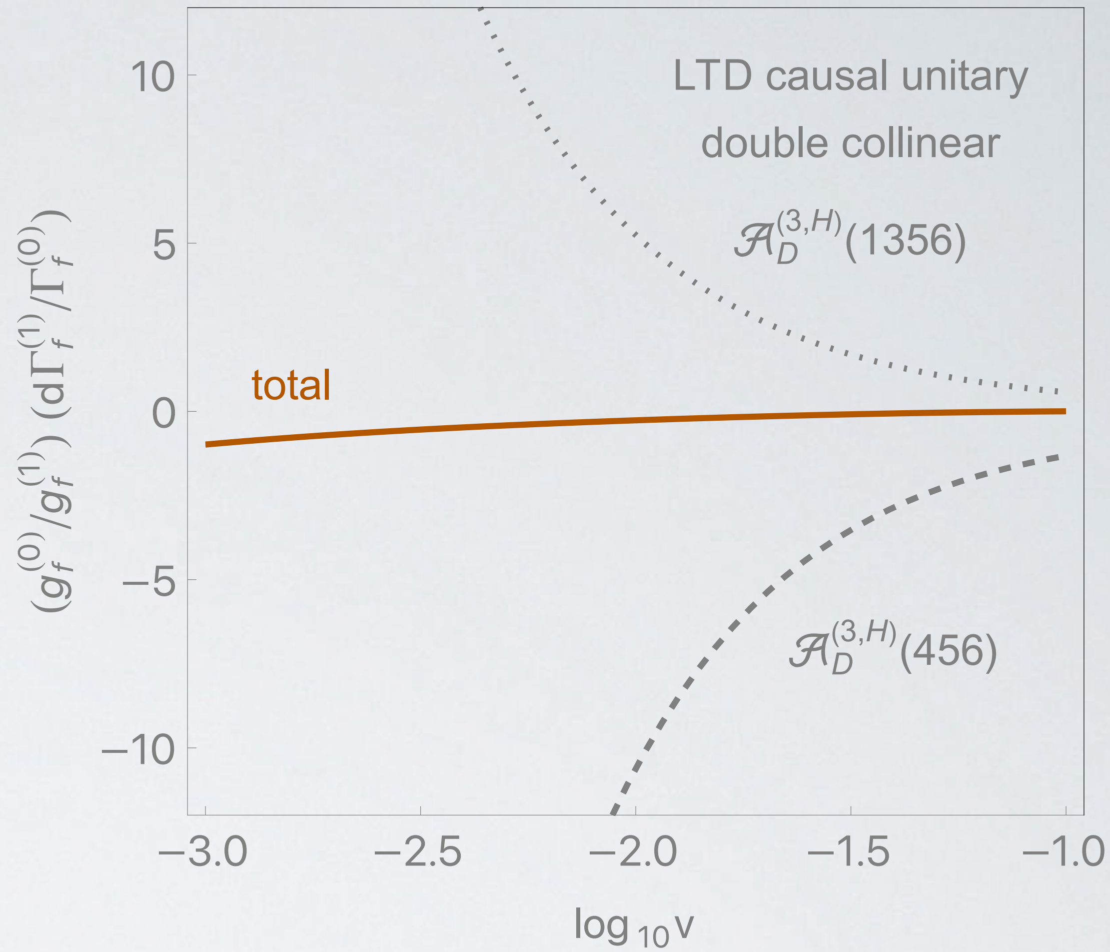
Ramírez-Uribe et al, e-Print: 2404.05492

- The unintegrated amplitude remains finite in the LTD causal unitary.

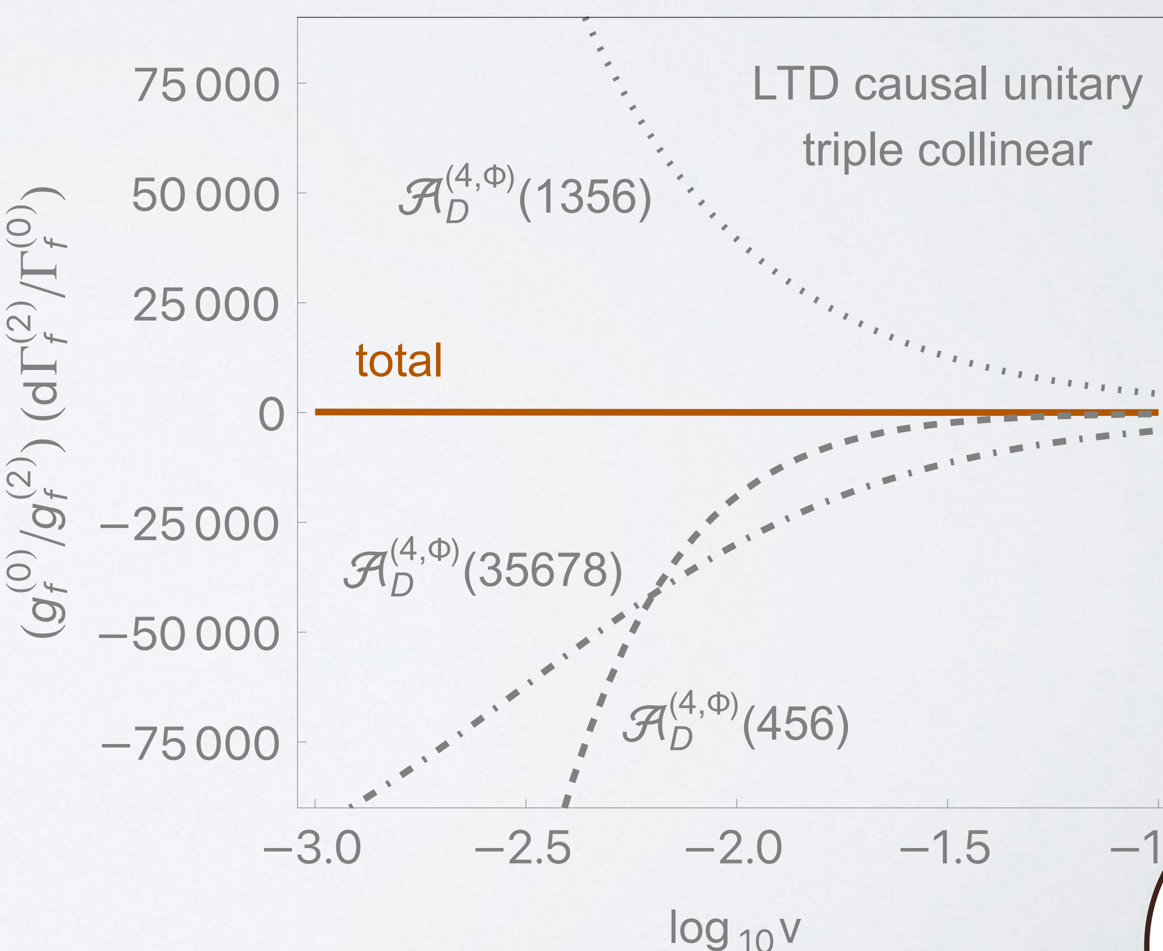


- The unintegrated amplitude remains finite in the LTD causal unitary.

- In addition, local cancellation of collinear singularities are present at NLO and NNLO



NLO



NNLO

CONCLUSIONS

- New mathematical methods for computing higher order corrections are needed for upcoming LHC and the FCC observables.
- The LTD has opened a new methodology to tackle only physical divergences through causal representation of Feynman integrals.
- We find the first proof-of concept of the LTD causal unitary at NLO and NNLO free of unphysical singularities in the processes $H \to q\bar{q}, \gamma^* \to q\bar{q}$ at NLO and $\Phi \to \phi\phi$ at NLO and NNLO.
- The results presented constitute a solid confirmation of the unique capabilities and advantages of LTD causal unitary at higher perturbative orders.

THANK YOU