Isospin breaking in $\eta' \to \eta\pi\pi$ decays

Adolfo Guevara*

In collaboration with Feng-Kun Guo and Hao-Jie Jing

Universidad Autónoma del Estado de Hidalgo.

* adolfo_guevara@uaeh.edu.mx

November 5 2024

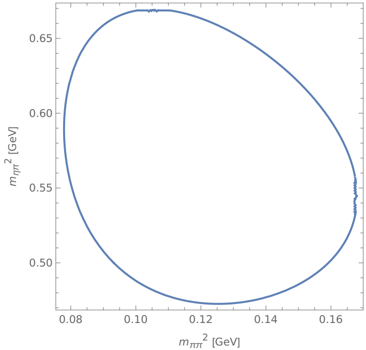
Objectives

- We want to compute isospin breaking contributions to $\eta' \to \eta\pi\pi$ decays.
- Map Dalitz plots to unitary disks.
- Compute these Dalitz disks for $\eta' \to \eta\pi^+\pi^-$ and $\eta' \to \eta\pi^0\pi^0$ decays.
- **Subtract thus both disks bin by bin.**
- We use χPT as theoretical framework.[1]

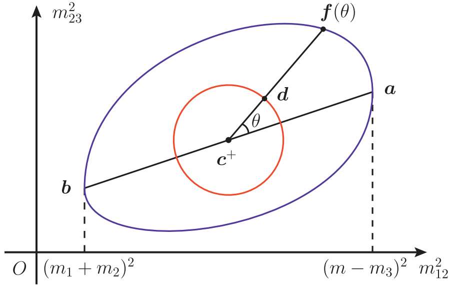
[1] Weinberg, Physica **A96** (1979) 327; Gasser & Leutwyler, Ann. Phys. **158** (1984) 142; Nucl. Phys. **B250** (1985) 465.

Dalitz plot

- Any three-body decay can be represented using a Dalitz plot.
- A Dalitz plot is a two-dimensional histogram depending on $m_{ij}^2 = (p_i + p_j)^2$.
- Nevertheless, its form depend on the masses of the particles.



Why should I care about a disk?



- It's shape does not depend on kinematics.
- We can use the same binnig for two different disks.
- $\Rightarrow$ we can directly compare two disks.
- We can compute $d\Gamma_{\pi^\pm} - d\Gamma_{\pi^0}$ in each bin.

Isospin breaking contributions to χPT

- The total amplitude depends on two terms

$$\mathcal{M} = \mathcal{M}_{IC} + \mathcal{M}_{IB},$$

- where $\mathcal{M}_{IB}$ contains isospin breaking insertions, while $\mathcal{M}_{IC}$ does not.

- Since $\eta' \to \eta\pi\pi$ conserves isospin, each diagram in $\mathcal{M}_{IB}$ has two $\Delta I \neq 0$ vertices.

- The differential decay width is

$$d\Gamma_{\eta' \to \eta\pi\pi} \propto |\mathcal{M}_{IC}|^2 + 2\Re(\overline{\mathcal{M}}_{IC}\ \mathcal{M}_{IB}) + |\mathcal{M}_{IB}|^2 ,$$

Isospin breaking parameter Q

- In χPT, the mass term in the Lagrangian gives $\Delta I = 1$ vertices, so

$$\mathcal{M}_{IB} \propto [B_0(m_u - m_d)]^2 \propto \frac{1}{Q^4}.$$

- On the other hand, $\eta \to 3\pi$ involves only one $\Delta I = 1$ vertex [2].

- This is due to the fact that $\eta \to 3\pi$ breaks isospin, therefore

$$d\Gamma_{\eta\to 3\pi} \propto \frac{1}{Q^4}.$$

- However

$$d\Gamma_{\eta'\to\eta\pi\pi} \propto |\mathcal{M}_{IC}|^2 + 2\Re(\overline{\mathcal{M}}_{IC}\ \mathcal{M}_{IB}) + \mathcal{O}\left(\frac{1}{Q^8}\right)$$

[2] H. Osborn and D.J. Wallace, Nucl. Phys. B**20** (1970) 23

Isospin breaking parameter Q

- In χPT, the mass term in the Lagrangian gives $\Delta I = 1$ vertices, so

$$\mathcal{M}_{IB} \propto [B_0(m_u - m_d)]^2 \propto \frac{1}{Q^4}.$$

- On the other hand, $\eta \to 3\pi$ involves only one $\Delta I = 1$ vertex [3].

- This is due to the fact that $\eta \to 3\pi$ breaks isospin, therefore

$$d\Gamma_{\eta\to 3\pi} \propto \frac{1}{Q^4}.$$

- However ↲ Large background.

$$d\Gamma_{\eta'\to\eta\pi\pi} \propto |\mathcal{M}_{IC}|^2 + 2\Re(\overline{\mathcal{M}}_{IC}\ \mathcal{M}_{IB}) + \mathcal{O}\left(\frac{1}{Q^8}\right)$$

[3] H. Osborn and D.J. Wallace, Nucl. Phys. B**20** (1970) 23

The One Disk

One plot to rule them all...

- To construct the Dalitz Disks we must normalise the partial decay width $d\Gamma$.

- If we integrate on the diks variables

$$\Gamma = \int dsdt \frac{d^2\Gamma}{dsdt} = \int rdrd\theta |J| \frac{d^2\Gamma}{dsdt}.$$

- The correct normalization factor is the Jacobian $|J|$.

- This normalization factor will be considered in the experimental data.

One plot to find them...

- The idea behind the subtraction is to suppress the background $|\mathcal{M}_{IC}|^2$. [4]

$$d\Gamma_{\text{diff}} = \left[|\mathcal{M}_{IC}|^2 + 2\Re\left(\overline{\mathcal{M}}_{IC}\mathcal{M}_{IB}^{(\pi^\pm)}\right)\right] - \left[|\mathcal{M}_{IC}|^2 + 2\Re\left(\overline{\mathcal{M}}_{IC}\mathcal{M}_{IB}^{(\pi^0)}\right)\right] + \ldots$$

- For one given bin, the difference $d\Gamma_{\pi^\pm} - d\Gamma_{\pi^0}$ is

$$\boxed{d\Gamma_{\text{diff}} = 2\Re\left[\overline{\mathcal{M}}_{IC}\left(\mathcal{M}_{IB}^{(\pi^\pm)} - \mathcal{M}_{IB}^{(\pi^0)}\right)\right] + \mathcal{O}(p^8).}$$

- We create another disk using $d\Gamma_{\text{diff}}$ depending only on interference terms. $(\propto \frac{1}{Q^4})$.

[4] With subindices $\pi^\pm$ we refer to $\eta' \to \eta\pi^+\pi^-$ and with π^0 to $\eta' \to \eta\pi^0\pi^0$

One plot to fit them all...

- The normalisation for Dalitz plots are arbitrary.
- Therefore, we can construct the disks such that at each bin the value is $d^2\Gamma/dsdt$.
- Since BESIII [5] parametrises the width as

$$d\Gamma_{exp} = N|\mathcal{M}|^2 = N(1 + aY + bY^2 + cX + dX^2)$$

- The quantity to generate the Dalitz disk using the experimental data is

$$\frac{d\Gamma_{exp}}{|J|}$$

[5] Ablikim et al., BESIII collab., Phys.Rev.D 97 (2018) 012003

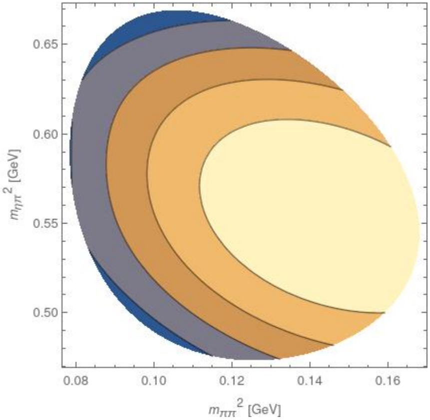
... And in the data, bind them.

- A great advantage is the lack of correlation between different measurements.
- We can fit Q using the $d\Gamma_{\text{diff}}$ disk generated with BESIII data.
- Since $d\Gamma_{\text{diff}} \propto 1/Q^4$, we expect a large precision on Q.
- Athough we first reproduce previous results... [6]

[6] R. Escribano, P. Masjuan and J.J. Sanz Cillero, JHEP **05**(2011) 094

Ordinary Dalitz plots

- Dalitz plot without $\Delta I = 1$ [7].



[7] R. Escribano, P. Masjuan and J.J. Sanz Cillero, JHEP **05**(2011) 094

BESIII data

- The model used by BESIII [8] fitted for their own data for $d^2\Gamma/dsdt$

$$\frac{d\Gamma}{dXdY} = N(1 + aY + bY^2 + cX + dX^2 + ...)$$

$$a = -0.056,\ \ b = -0.049,\ \ d = -0.063 \qquad a = -0.087,\ \ b = -0.073,\ \ d = -0.074$$

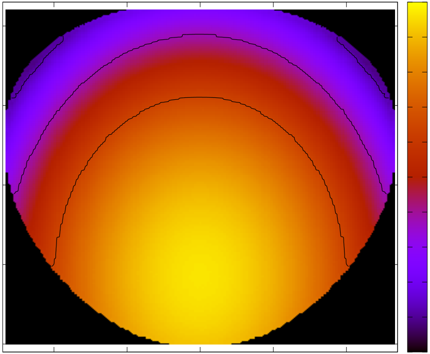
$$C_{\pi^\pm} = \left(\begin{array}{c|cc} & b_{\pi^\pm} & d_{\pi^\pm} \\ \hline a_{\pi^\pm} & -0.417 & -0.239 \\ & & 0.292 \end{array}\right) \qquad C_{\pi^0} = \left(\begin{array}{c|cc} & b_{\pi^0} & d_{\pi^0} \\ \hline a_{\pi^0} & -0.495 & -0.273 \\ & & 0.273 \end{array}\right)$$

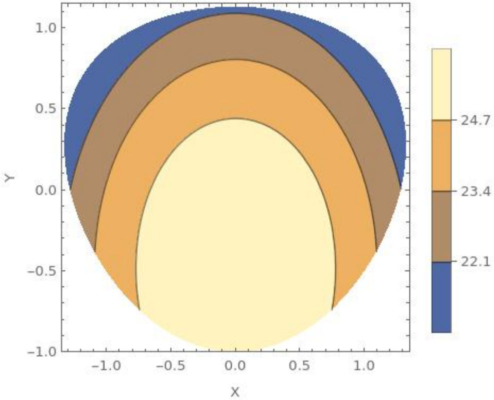
- Using BESIII model, we generate 7,000 points per bin, with 7,837 bins for each plot.

[8] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 97, (2018) 012003

BESIII data

- We then compare the results using BESIII data with the theoretical prediction [9]

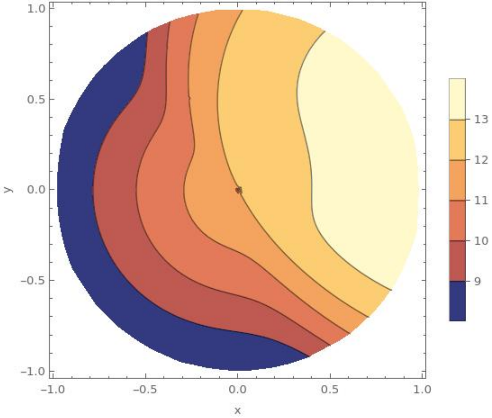


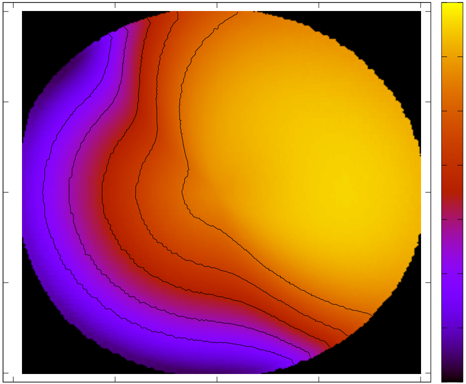


[9] R. Escribano, P. Masjuan and J.J. Sanz Cillero, JHEP **05**(2011) 094

Dalitz disk for $\pi^{\pm}$

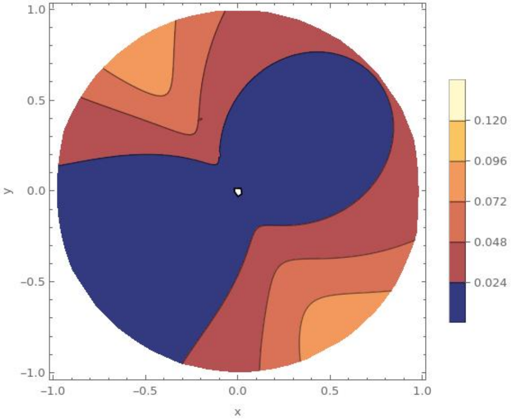
- $\Gamma_{\pi^{\pm}}$ prediction without $\Delta I = 1$ (left) and BESIII data (right).

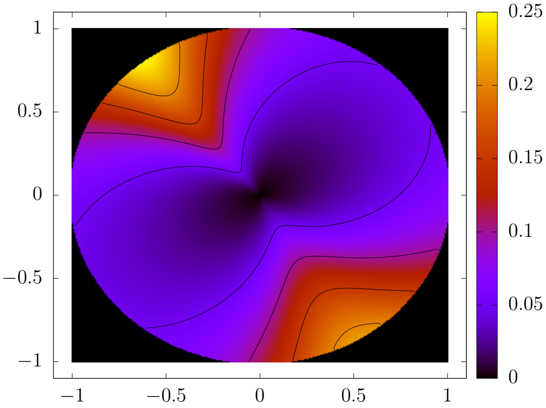




Normalized Dalitz disk for $\pi^{\pm}$

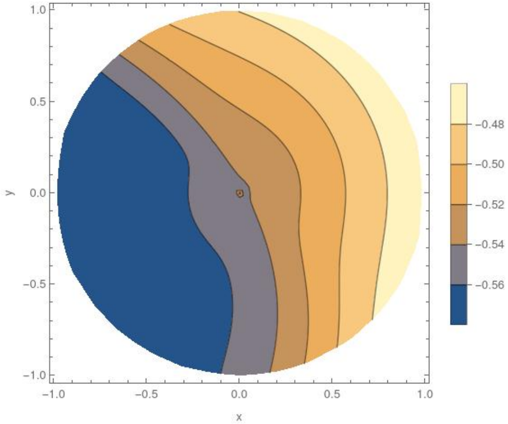
- Γ_{π^0} without $\Delta I = 1$ prediction (left) and BESIII data (right).





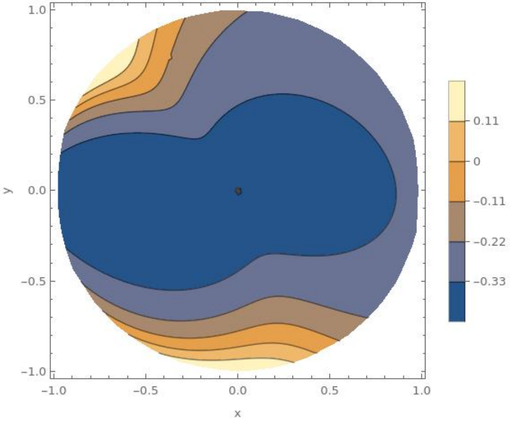
LO subtraction

- The difference disk without $\Delta I = 1$ contributions



Difference including $\Delta I = 1$

- The difference disk considering $\Delta I = 1$ is



Fit of the Isospin-breaking parameter Q.

Q	Ref.
24.3	from Dashen's theorem (1969)
22.7 ± 0.8	A. V. Anisovich & H. Leutwyler (1996)
23.1 ± 0.7	K. Kampf *et al* (2011)
22.1 ± 0.7	G. Collangelo *et al* (2018)
21.50 ± 0.97	M. Albaladejo & B. Moussallam (2017)
24.3 ± 1.5	FLAG $(N_f = 2)$ (2021)
23.3 ± 0.5	FLAG $(N_f = 2 + 1)$ (2021)
22.5 ± 0.5	FLAG $(N_f = 2 + 1 + 1)$ (2021)
22.3 ± 0.7	our fit

Conclusions

- We built a method for fitting parameters of suppressed effects getting rid of the large background intended for three-body decays.
- We computed isospin breaking vertices within χPT due to $m_u - m_d$ mass difference to insert them in isospin conserving processes.
- Applying this method to $\eta' \to \eta\pi\pi$ decays, we obtained Dalitz disks of the difference between charged and neutral channels to fit the isospin breaking parameter Q.
- From BESIII data we were able to use their model for both channels with the correct normalisation to generate the data necessary to generate the Dalitz disks.
- We will send soon the draft to arXiv!

¡Muchas gracias!

Back up!

Fit estimator

- We have used χ^2/dof to estimate the goodness of our fit.
- The $dof = 7,837$ are the number of bins.
- From which we get $\chi^2/dof \approx 2.8$

χPT

- Chiral Perturbation Theory (χPT) se basa en la simetría quiral de la densidad lagrangiana de QCD cuando las masas son muy pequeñas.

$$\mathcal{L}_q = \sum_f \overline{q}_f \left(i\gamma_\mu D^\mu - m_f \right) q_f ,$$

donde $f = u, d, s, \ldots$

- Al proyectar en sus componentes izquierdas o derechas $q_{L/R} = \frac{1}{2}(1 \mp \gamma_5)q$,

$$\overline{q}\gamma_\mu D^\mu q = \overline{q}_L \gamma_\mu D^\mu q_L + \overline{q}_R \gamma_\mu D^\mu q_R ,$$

- siendo

$$-m\bar{q}q = -m\bar{q}_R q_L - m\bar{q}_L q_R .$$

χPT

- Así, la dinámica de los quarks tiene una simetría $G = SU(3)_R \otimes SU(3)_L$.
- Que implica $u_\chi \leftrightarrow d_\chi \leftrightarrow s_\chi \leftrightarrow u_\chi$, para $\chi = R, L$. (simetría de isoespín, $u \leftrightarrow d$)
- Una forma de incluir efectos de rompimiento de isoespín es a través de $m_u - m_d$.
- Siendo $Q_u = 2/3$ y $Q_d = -1/3$, otra forma de romper isoespín es con efectos EM.

χPT

- La realización de G debe ser no lineal[10], $U = \exp\left(\sqrt{2}\phi/f\right)$, siendo

$$\phi = \sum_a \frac{1}{\sqrt{2}}\phi_a\lambda_a = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_3 + \frac{1}{\sqrt{3}}\phi_8) & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{2}}(-\phi_3 + \frac{1}{\sqrt{3}}\phi_8) & K^0 \\ K^- & \overline{K^0} & -\sqrt{\frac{2}{3}}\phi_8 \end{pmatrix},$$

donde $a = 1, ..., 8$ y λ_a son las matrices de Gell-Mann.

- Usando derivadas se obtienen los términos dinámicos

$$\left\langle \partial_\mu U^\dagger \partial^\mu U \right\rangle, \qquad \text{(LO en conteo quiral)}$$

siendo $\langle A \rangle$ la traza de A en el espacio de sabor.

[10] Weinberg, Physica **A96** (1979) 327.

Términos de masa en χPT

- Para el rompimiento de isospín $(m_u - m_d)$ es necesario incluir las masas.

- El término de masas en la densidad lagrangiana de χPT es

$$\mathcal{L}_M = \frac{f^2 B_0}{2} \left\langle M U^\dagger + U^\dagger M \right\rangle ,$$

donde f y B_0 son constantes de bajas energías, $U = \exp\left(\sqrt{2}\phi/f\right)$, y

$$M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} ,$$

es la matriz de masas de quarks, que también puede expresarse en la base λ_a:

Términos de masa

$$M = \frac{2\hat{m}+m_s}{\sqrt{6}}\lambda_0 + \frac{m_u - m_d}{2}\lambda_3 + \frac{\hat{m}-m_s}{\sqrt{3}}\lambda_8,$$

donde $\lambda_0 = \sqrt{\frac{2}{3}}\ \mathbb{1}$ y $\hat{m} = \frac{1}{2}(m_u + m_d)$.

- Al expandir U al orden $\mathcal{O}(\phi^2)$ se obtiene

$$\begin{aligned} \mathcal{L}_M \sim \frac{B_0}{2}\left\langle \phi^2 M \right\rangle \ &= \ B_0(m_u+m_d)\pi^+\pi^- + B_0(m_u+m_s)K^+K^- \\ &\quad + B_0(m_d+m_s)K^0\overline{K^0} + \frac{B_0}{2}(m_u+m_d)\phi_3^2 \\ &\quad + \frac{B_0}{\sqrt{3}}(m_u-m_d)\phi_3\phi_8 - \frac{B_0}{6}(m_u+m_d+4m_s)\phi_8^2. \end{aligned}$$

Términos de masa

- Así, el término de mezcla $\phi_3 - \phi_8$ es responsable del rompimiento de isoespín.
- Se diagonaliza para obtener los eigenestados de masa

$$\begin{pmatrix} \pi^0 \\ \eta \end{pmatrix} = \begin{pmatrix} \cos\varepsilon & \sin\varepsilon \\ -\sin\varepsilon & \cos\varepsilon \end{pmatrix} \begin{pmatrix} \phi_3 \\ \phi_8 \end{pmatrix} = \begin{pmatrix} \phi_3 + \varepsilon\phi_8 \\ \phi_8 - \varepsilon\phi_3 \end{pmatrix} + \mathcal{O}(\varepsilon^2),$$

- la cual se logra con

$$\varepsilon = \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s - \hat{m}} \lesssim 0.01,$$

donde $\hat{m} = \frac{m_u + m_d}{2}$. Lo cual justifica la expansión en series de Taylor.

Términos de masa

- Así las masas de los mesones son

$$m_{\pi^+}^2 = B_0(m_u + m_d), \qquad m_{K^+}^2 = B_0(m_u + m_s), \qquad m_{K^0}^2 = B_0(m_d + m_s)$$

$$m_{\pi^0}^2 = B_0 \left(m_u + m_d + 2\varepsilon \frac{m_u - m_d}{\sqrt{3}} \right), \qquad m_\eta^2 = B_0 \left(\frac{m_u + m_d + 4m_s}{3} - 2\varepsilon \frac{m_u - m_d}{\sqrt{3}} \right).$$

- Que obtienen correcciones de órdenes más altos en la masa de los quarks ($\mathcal{O}(M^2)$),
- así como de interacciones electromagnéticas ($\mathcal{O}(\alpha)$).

Interacciones electromagnéticas (EM) en las masas

- Como Dashen muestra [11] las correcciones $\mathcal{O}(\alpha)$ no inducen una mezcla $\pi^0 - \eta$.

- Más aún, las autoenergías de los mesones neutros se anulan a LO.

$$(m_{\pi^0})_{\text{EM}} = (m_\eta)_{\text{EM}} = (m_{K^0})_{\text{EM}} = (m_{\overline{K^0}})_{\text{EM}} = 0.$$

- Las correcciones de los cargados deben ser iguales[12], lo que implica

$$\boxed{(m^2_{K^+} - m^2_{K^0})_{\text{EM}} = (m^2_{\pi^+} - m^2_{\pi^0})_{\text{EM}}},$$

conocido como el Teorema de Dashen. De las masas físicas

$$m^2_{\pi^+} - m^2_{\pi^0} \sim (37 \text{ MeV})^2$$

[11] R. Dashen, Phys.Rev. **5** (1969) 1245.

[12] Las interacciones EM no distinguen sabor, por lo que debe haber unsa siemtría $s \leftrightarrow d$

Masas en el límite de isoespín

- Siendo $\varepsilon \sim m_d - m_u$,

$$m_{\pi^0}^2 = m_{\pi^+}^2 + \mathcal{O}(\delta^2) \Rightarrow (m_{\pi^+}^2 - m_{\pi^0}^2)_{\text{QCD}} \sim \mathcal{O}(\delta^2),$$

donde $\delta = m_u - m_d \sim 2$ MeV.

- $\Rightarrow$ se pueden expresar las masas en el límite de isoespín como

$$m_\pi^2 = 2B_0\hat{m} + \mathcal{O}(\delta^2), \quad m_K^2 = B_0(\hat{m} + m_s) + \mathcal{O}(\delta), \quad m_\eta^2 = \frac{2}{3}B_0(\hat{m} + 2m_s) + \mathcal{O}(\delta^2).$$

- Las cuales cumplen la relación Gell-Mann–Okubo [13]

$$4m_K^2 = 3m_\eta^2 + m_\pi^2.$$

[13] Gell-Mann, Phys. Rev. 125 (1962) 1067; Okubo, Prog. Theor. Phys. **27** (1962) 949.

Masas físicas vs. masas en el límite de isoespín

- El rompimiento de isoespín en m_π debido a EM es más grande que el debido a M,

$$(m_{\pi^+}^2 - m_{\pi^0}^2)_{\text{QCD}} \ll (m_{\pi^+}^2 - m_{\pi^0}^2)_{\text{EM}}.$$

- El teorema de Dashen dice[14] que los efectos EM a LO son iguales para π y K,

$$(m_{K^+}^2 - m_{K^0}^2)_{\text{QCD}} = (m_{K^+}^2 - m_{K^0}^2) - (m_{\pi^+}^2 - m_{\pi^0}^2).$$

- m_K^2 se puede relacionar con las masas físicas promediando $m_{K^+}^2$ and $m_{K^0}^2$ y sustrayendo EM de m_{K^+}

$$m_K^2 = \frac{1}{2}\left(m_{K^+}^2 + m_{K^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2\right) + \mathcal{O}(M\alpha, \delta^2).$$

[14] Dashen, Phys. Rev. 183 (1969) 1245.

El parámetro de rompimiento de isospín

- De la definición de m_π^2 y m_K^2

$$\frac{m_K^2}{m_\pi^2} = \frac{m_s + \hat{m}}{m_d + m_u}.$$

- De igual manera

$$\frac{m_K^2 - m_\pi^2}{(m_{K^+}^2 - m_{K^0}^2)_{\mathsf{QCD}}} = \frac{m_s - \hat{m}}{m_d - m_u},$$

- con lo cual se define

$$Q^2 := \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2},$$

- tal que

$$\boxed{B_0(m_u - m_d) = -\frac{m_K^2}{m_\pi^2}\frac{m_K^2 - m_\pi^2}{Q^2}}.$$

Dalitz Plot

- ¿Cómo podemos deshacernos de este fondo?
- **Nuestra idea es un plot universal para comparar bin por bin.**
- La parte $|\mathcal{M}_{IC}|^2$ es igual para $\eta' \to \eta\pi^+\pi^-$ y $\eta' \to \eta\pi^0\pi^0$, pero no la parte $\mathcal{M}_{IB}$.
- $\Rightarrow$ transformaremos cada Dalitz plot en un disco unitario.
- De esta forma podemos deshacernos del fondo***.**

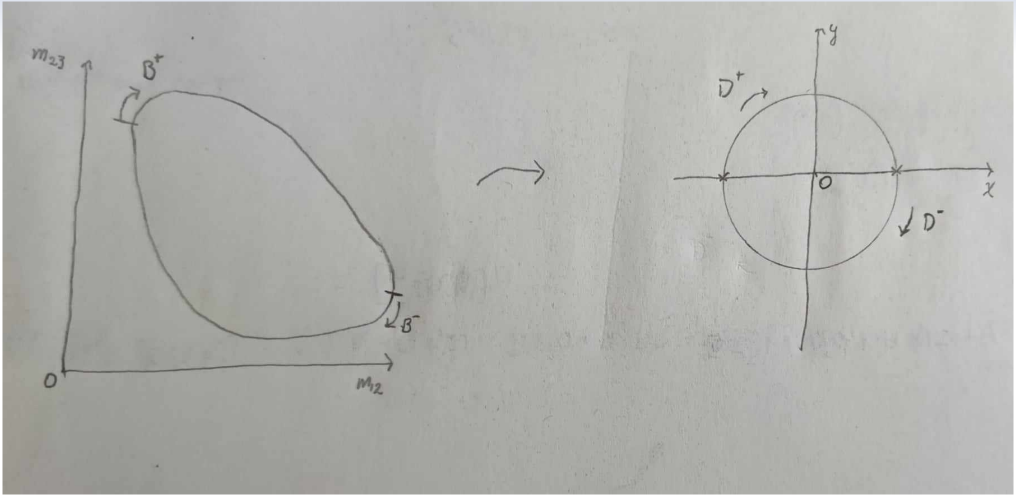
Dalitz Plot

- ¿Cómo podemos deshacernos de este fondo?
- **Nuestra idea es un plot universal para comparar bin por bin.**
- La parte $|\mathcal{M}_{IC}|^2$ es igual para $\eta' \to \eta\pi^+\pi^-$ y $\eta' \to \eta\pi^0\pi^0$, pero no la parte $\mathcal{M}_{IB}$.
- $\Rightarrow$ transformaremos cada Dalitz plot en un disco unitario.
- De esta forma podemos deshacernos del fondo***.**

*** APLICAN RESTRICCIONES**

Bueno, sí. Muy bonito, pero...

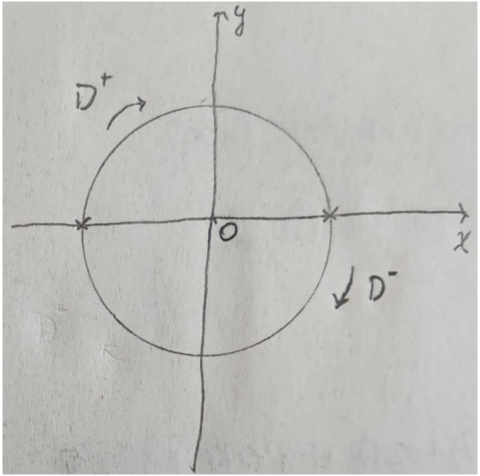
¿Cómo se obtiene tal disco?



- El eje x del disco cruza los puntos donde se unen B^+ y B^- en el Dalitz plot.

Disco Dalitz

- Las fronteras en el disco se describen muy sencillamente.



- Para la frontera con $y \geq 0$

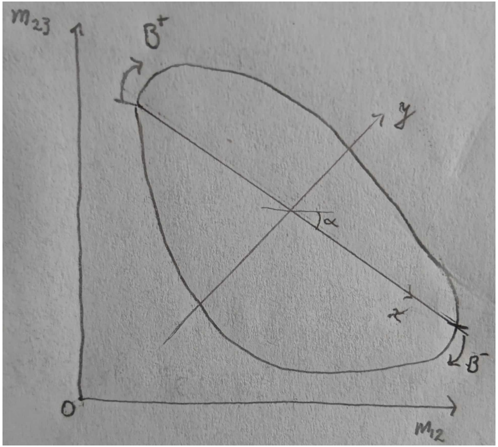
$$D^{+} = \left\{ \frac{y}{\sqrt{1-x^{2}}} = 1 \middle| x \in [-1,1] \right\}.$$

- Para la frontera con $y \leq 0$

$$D^{-} = \left\{ \frac{y}{\sqrt{1-x^{2}}} = -1 \middle| x \in [-1,1] \right\}.$$

Dalitz plot convencional

- Se expresa m_{23} como una función de m_{12} y θ.



- $S(m_{12},\theta) = \left(m_{12} - c_1^+\right)\tan\left(\theta+\alpha\right) + c_2^+.$

- Para la frontera con $y \geq$0 y $y \leq 0$,

$$B^+ = \left\{B\left(m_{12}, S(m_{12},\theta)\right) = 1 |\, m_{12} \in [b_1, a_1]\right\},$$

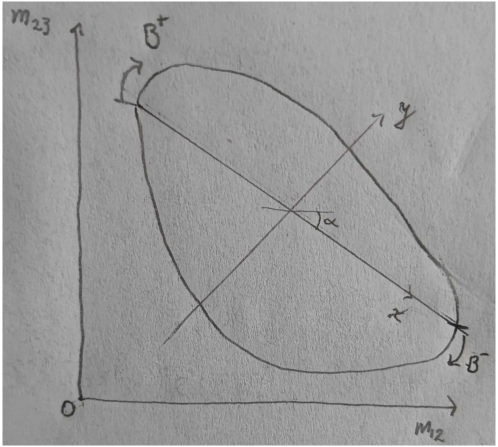
$$B^- = \left\{B\left(m_{12}, S(m_{12},\theta)\right) = -1 |\, m_{12} \in [b_1, a_1]\right\}.$$

- La solución la llamamos $L(\theta)$, esto es

$$\Big(L(\theta),\ S(L(\theta),\theta)\Big) \in B^{\pm}.$$

Dalitz plot convencional

- Se identifica $B(m_{12}, m_{23})$ con el coseno del ángulo entre $\vec{p_1}$ y $\vec{p_3}$.



- De la cinemática se obtiene

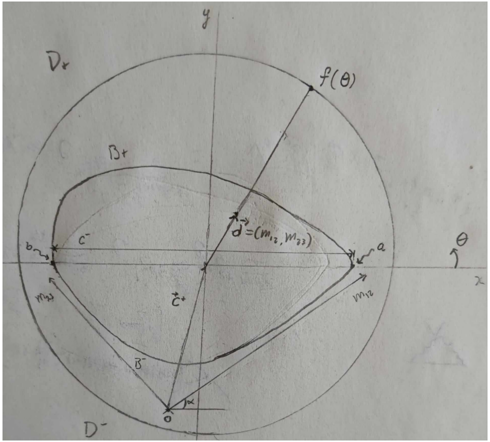
$$\cos\theta_{13} = \frac{\sqrt{h(E_1, E_3)}}{2\sqrt{(E_1^2 - m_1^2)(E_3^2 - m_3^2)}}.$$

- donde

$$h(E_1, E_3) = 2E_1E_3 - 2M(E_1 + E_3) + M^2 + m_1^2 - m_2^2 + m_3^2.$$

Transformación

- Siendo $\vec{c^+}$ el vector de O al centro del disco y $\vec{c^-}$ la distancia entre $\vec{a}$ y $\vec{b}$,



- un punto cualquiera $\vec{d}$ tendrá un ángulo tal que

$$\cos\theta = \frac{(d - c^+)\cdot c^-}{|d - c^+|\,|c^-|},$$

- y un radio

$$r = \frac{|d - c^+|}{|f(\theta) - c^+|},$$

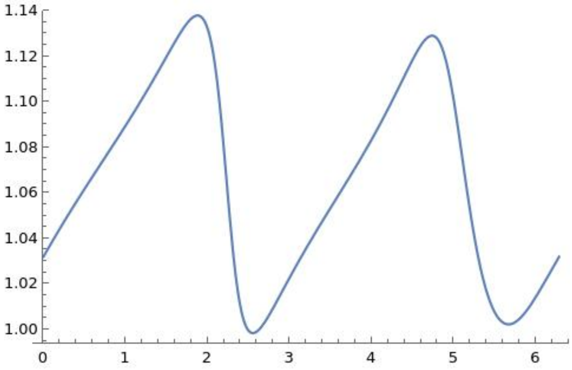
- siendo $f(\theta)$ la solución de $B(m_{12}, m_{23}) = \pm 1$

$$\vec{f}(\theta) = (L(\theta), S(\theta)).$$

* Restricciones

- La función $|J_{\pi^+}|/|J_{\pi^0}|$ depende únicamente del ángulo en el disco unitario.

- Al evaluarla se obtiene



Conexión con las desintegraciones de la η'

- Se necesita $B_0(m_u - m_d)$ para los efectos de rompimiento de isoespín en $\eta' \to \eta\pi\pi$:

- Como se mostró antes

$$\mathcal{L}_{m_f} = \overline{q}Mq = \overline{q}\left(\frac{m_u + m_d + m_s}{\sqrt{6}}\lambda_0 + \frac{m_u - m_d}{2}\lambda_3 + \frac{\hat{m} - m_s}{\sqrt{3}}\lambda_8\right)q,$$

- $\Rightarrow$ los efectos de rompimiento de isoespín vienen del término con λ_3.

- Ya que estos procesos conservan isoespín, se necesitan dos inserciones $\Delta I \neq 0$.

Isospin breaking in χPT

- Por lo tanto, necesitamos la proyección de $\mathcal{L}_M$ en λ_3.

- Expandiendo a orden ϕ^2 se tiene

$$\mathcal{L}_{IB}^{(2)} = \frac{B_0(m_u - m_d)}{2} \left\langle \lambda_3 \phi^2 \right\rangle .$$

- También se tiene a orden ϕ^4

$$\mathcal{L}_{IB}^{(4)} = \frac{B_0(m_u - m_d)}{12 f^2} \left\langle \lambda_3 \phi^4 \right\rangle$$

Isospin structure of mass operator

- $SU(3)_R \otimes SU(3)_L$ breaks spontaneously to $SU(3)_V$, which means q_R transforms exactly as q_L.

- Let's analize the isospin structure of the $\overline{q}\lambda_i q$ operators within $SU(3)_V$.

$$\overline{q}\lambda_i q \xrightarrow{SU(3)_V} \overline{q}U^\dagger \lambda_i U q.$$

- Taking the trace

$$\text{Tr}[U^\dagger \lambda_i U] = \text{Tr}[\lambda_i] = 0,$$

one finds that

$$U^\dagger \lambda_i U = R_{ij}\lambda_j,$$

since any traceless matrix is a linear combination of Gell-Mann matrices.

Isospin structure of mass operator

- On the other hand, isospin transformations form a subgroup $SU(2)_I \subset SU(3)_V$.
- In flavor space this means for $U \in SU(3)_V$

$$U = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix}, \qquad V \in SU(2)_I$$

- Since only λ_1, λ_2 and λ_3 transform thusly under $SU(2)_I$,

$$U^\dagger \lambda_i U = R_{ij} \lambda_j \qquad \text{for } i, j = 1, 2, 3.$$

- Also, since

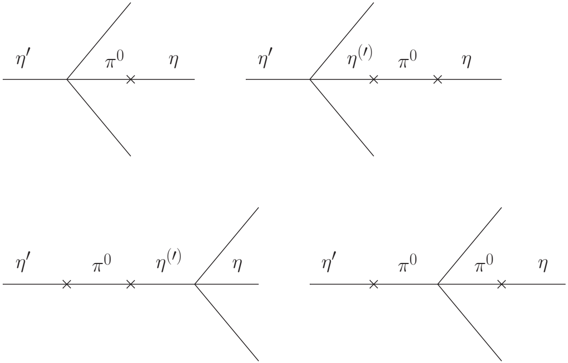
$$2\delta_{ij} = \langle \lambda_i \lambda_j \rangle = \langle (U^\dagger \lambda_i U)(U^\dagger \lambda_j U) \rangle = R_{ik} R_{jl} \langle \lambda_k \lambda_l \rangle = 2(RR^T)_{ij},$$

  R is an orthogonal 3×3 matrix.

Diagramas de Feynman

- De estos operadores se obtienen tres tipos de transiciones $\Delta I = 1$:

$$\eta^{(\prime)} \to \pi^0, \;\; \pi^0 \to \eta \text{ y } \eta^{(\prime)} \to \pi^0\pi\pi$$



Isospin structure of mass operator

- $\Rightarrow \overline{q}\lambda_i q$ for $i = 1, 2, 3$, transform under $SU(3)_V$ exactly as pion fields.
- Thus, $\overline{q}\lambda_3 q$ generates $\Delta I = 1, \Delta I_3 = 0$ transitions.
- Therefore, the $\overline{q}\lambda_3 q$ operator has exactly the same structure as a π^0 field operator.
- We can now use this to compute the $\eta \to \pi^0\pi^+\pi^-$ decay amplitude.

$\eta \to 3\pi$ decay amplitude

- The decay amplitude is defined as

$$i(2\pi)^4\delta^{(4)}(p_1 - p_2 - p_3 - p_4)\mathcal{A}^{ijk}_{\eta\to 3\pi} = \langle \pi^i(p_2)\pi^j(p_3)\pi^k(p_4)|iT|\eta(p_1)\rangle,$$

- where

$$T = -\frac{m_u - m_d}{2}\int d^4x\bar{q}(x)\lambda^3 q(x),$$

- which gives

$$\mathcal{A}^{ijk}_{\eta\to 3\pi} = -\frac{m_u - m_d}{2}\langle \pi^i\pi^j\pi^k|\bar{q}(0)\lambda^3 q(0)|\eta\rangle.$$

- On the other hand, one can define an amplitude for a general isospin index s.t.

$$\mathcal{A}^{ijk,l}_{\eta\to 3\pi} = -\frac{m_u - m_d}{2}\langle \pi^i\pi^j\pi^k|\bar{q}(0)\lambda^l q(0)|\eta\rangle.$$

$\eta \to 3\pi$ decay amplitude

- This means

$$\mathcal{A}^{ijk,3}_{\eta\to 3\pi} = \mathcal{A}^{ijk}_{\eta\to 3\pi}.$$

- $\overline{q}\lambda^l q$ transforms exactly as a pion under isospin transformations.

- Therefore $\mathcal{A}^{ijk,l}_{\eta\to 3\pi}$ must have the exact same isospin structure as $\pi\pi \to \pi\pi$.

- This means that it can be written as $\mathcal{A}^{ijkl}_{\pi\pi\to\pi\pi}$,

$$\mathcal{A}^{ijk}_{\eta\to 3\pi} = \mathcal{A}^{ijk,3}_{\eta\to 3\pi} = A_1(s,t,u)\delta^{ij}\delta^{k3} + A_2(s,t,u)\delta^{ik}\delta^{j3} + A_3(s,t,u)\delta^{i3}\delta^{jk}.$$

$\eta \to 3\pi$ decay amplitude

- Crossing symmetry gives

$$A_1(s,t,u) = A_1(s,u,t), \qquad A_2(s,t,u) = A_1(t,s,u), \qquad A_3(s,t,u) = A_1(u,t,s)$$

- Therefore, the decay amplitude is given by a single function

$$\mathcal{A}^{ijk}_{\eta\to3\pi}(s,t,u) = A(s,t,u)\delta^{ij}\delta^{k3} + A(t,u,s)\delta^{ik}\delta^{j3} + A(u,s,t)\delta^{i3}\delta^{jk}.$$

- There are only two decay channels for physical pions

$$\mathcal{A}^{+-0}_{\eta\to3\pi} = \mathcal{A}^{113}_{\eta\to3\pi} = A(s,t,u),$$

$$\mathcal{A}^{000}_{\eta\to3\pi} = \mathcal{A}^{333}_{\eta\to3\pi} = A(s,t,u) + A(t,u,s) + A(u,s,t).$$

- Thus, the neutral channel can be obtained directly from the charged one.

From η decays to η' decays

- The great advantage of this development is the straightforward use in η' decays.

- Following the previous procedure, we define the decay amplitude

$$\mathcal{A}^{ij}_{\eta'\to\eta\pi\pi} = \langle \pi^i(p_2)\pi^j(p_3)\eta(p_4)|iT|\eta'(p_1)\rangle,$$

- where

$$T = -\frac{m_u - m_d}{2}\int d^4x\bar{q}\lambda^3 q.$$

- So, we construct the amplitude

$$\mathcal{A}^{ij,k}_{\eta'\to\eta\pi\pi} = -\frac{m_u - m_d}{2}\langle \pi^i\pi^j\eta|\bar{q}\lambda^k q|\eta'\rangle,$$

which fulfills $\mathcal{A}^{ij,3}_{\eta'\to\eta\pi\pi} = \mathcal{A}^{ij}_{\eta'\to\eta\pi\pi}$, for which we'll use the same arguments.

$\eta' \to \eta\pi\pi$ decays

- The $\mathcal{A}^{ij,k}_{\eta'\to\eta\pi\pi}$ has the same isospin structure as $\mathcal{A}^{ijk}_{\eta\to 3\pi\pi}$, which means

$$\mathcal{A}^{ij}_{\eta'\to\eta\pi\pi} = \mathcal{A}^{ij,3}_{\eta'\to\eta\pi\pi} = A_1(s,t,u)\delta^{ij} + A_2(s,t,u)\delta^{i3}\delta^{j3} + A_3(s,t,u)\delta^{i3}\delta^{j3}.$$

- Crossing symmetry relates all the previous functions, such that

$$\mathcal{A}^{ij}_{\eta'\to\eta\pi\pi} = A(s,t,u)\delta^{ij} + \left[A(t,u,s) + A(u,s,t)\right]\delta^{i3}\delta^{j3}.$$

- Finally, for the physical pions we have

$$\mathcal{A}^{+-}_{\eta'\to\eta\pi\pi} = \mathcal{A}^{11}_{\eta'\to\eta\pi\pi} = A(s,t,u),$$

$$\mathcal{A}^{00}_{\eta'\to\eta\pi\pi} = \mathcal{A}^{33}_{\eta'\to\eta\pi\pi} = A(s,t,u) + A(t,u,s) + A(u,s,t).$$