The flavor problem and their symmetries

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Contents

The Standard Model

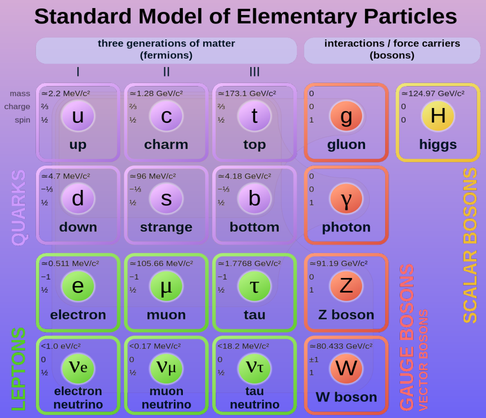

Figure: The Standard Model of Elementary Particles (SM).

Background

1. Cabibbo (1963): Quarks d and s decay into u.
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = V_{\text{Cabibbo}} \begin{pmatrix} d \\ s \end{pmatrix}, \; V_{\text{Cabibbo}} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}.$$
2. Glashow–Iliopoulos–Maiani (1970): Flavor changing neutral currents (Z).
$\begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}$ are an $SU(2)$ doublet.
3. Cronin-Fitch (1964): Kaon decays.
$gV_{\text{Cabibbo}} \neq gV^*_{\text{Cabibbo}}$.
4. Cabibbo-Kobayashi-Maskawa (1973).
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \; V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

The CKM matrix

$$V_{\text{CKM}} =$$

$$\begin{pmatrix} \cos\theta_{12}\cos\theta_{13} & \sin\theta_{12}\cos\theta_{13} & \sin\theta_{13}e^{-i\delta} \\ -\sin\theta_{12}\cos\theta_{23} - \sin\theta_{13}\sin\theta_{23}\cos\theta_{12}e^{i\delta} & \cos\theta_{12}\cos\theta_{23} - \sin\theta_{12}\sin\theta_{13}\sin\theta_{23}e^{i\delta} & \sin\theta_{23}\cos\theta_{13} \\ \sin\theta_{12}\sin\theta_{23} - \sin\theta_{13}\cos\theta_{12}\cos\theta_{23}e^{i\delta} & -\sin\theta_{23}\cos\theta_{12} - \sin\theta_{12}\sin\theta_{13}\cos\theta_{23}e^{i\delta} & \cos\theta_{13}\cos\theta_{23} \end{pmatrix}.$$

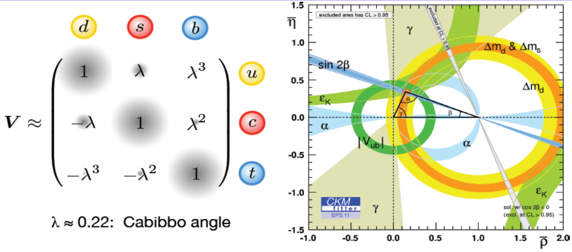

Figure: Parametrization of the CKM matrix.

The PMNS matrix

1. Pontecorvo–Maki–Nakagawa–Sakata (1962).

$$\begin{pmatrix} \nu'_e \\ \nu'_\mu \\ \nu'_\tau \end{pmatrix} V_{\text{PMNS}} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad V_{\text{PMNS}} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix}.$$

2. Fukuda (1998): Neutrino oscillation.
3. If neutrinos are Majorana, the parametrization includes additional phases,

$$A = \begin{pmatrix} e^{\alpha_1/2} & 0 & 0 \\ 0 & e^{\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

What is the flavor puzzle?

1. The origin of the 3 fermion generations.
2. Hierchical pattern of the charged lepton masses $m_e << m_\mu << m_\tau$. $m_e/m_\mu \approx 1/200$ and $m_\mu/m_\tau \approx 1/17$.
3. Lightness of neutrinos $m_{\nu i} \lesssim 0.5\text{eV}$, $m_l \gtrsim 0.511\text{MeV}$ and $m_q \gtrsim 2\text{MeV}$.
4. Masses and mixings of fermions from V_{CKM} and U_{PMNS}.

Vector-Valued Modular Forms

The Modular Group

$$\Gamma := \mathrm{SL}(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \,\middle|\, ad - bc = 1\,, \quad a,b,c,d \in \mathbb{Z} \right\}. \tag{1}$$

Γ has two generators S and T obeying

$$S^4 = (ST)^3 = 1\,, \quad S^2 T = T S^2\,, \tag{2}$$

which can be representated as

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \tag{3}$$

VVMF

The group acts on $\mathcal{H} = \{\tau \in \mathbb{C} \mid \mathrm{i}\tau > 0\}$ as the transformations

$$\tau \to \gamma\tau := \frac{a\tau + b}{c\tau + d}, \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma. \tag{4}$$

We consider the holomorphic functions $Y(\tau) = (Y_1(\tau), \ldots, Y_d(\tau))^T$ in $\mathcal{H}$ transforming as [2112.14761]

$$Y(\gamma\tau) = (c\tau + d)^{k_Y} \rho_Y(\gamma) Y(\tau), \tag{5}$$

where ρ_Y is a d-dimensional representation of $\gamma \in \Gamma$ with finite image.

VVMF

We define the functions

$$E_4(\tau) = 1 + 240 \sum_{n=1}^{\infty} \sigma_3(n) q^n \, ,$$

$$E_6(\tau) = 1 - 504 \sum_{n=1}^{\infty} \sigma_5(n) q^n \, ,$$

$$\eta^{2p}(\tau) = q^{p/12} \prod_{n=1}^{\infty} (1 - q^n)^{2p} \, . \quad q \equiv e^{2\pi i \tau} \, , \tag{6}$$

with $\sigma_k(n) = \sum_{d|n} d^k$,

$\eta(\tau)$ the Dedekind-eta function $\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)$.

VVMF

Under the irrep ρ, all VVMFs constitute a free module $\mathcal{M}(\rho)$ over the ring $\mathcal{M}(1) = \mathbb{C}[E_4, E_6]$. It is possible to generate a basis through the operators

$$D_k^n := D_{k+2(n-1)} \circ D_{k+2(n-2)} \circ \cdots \circ D_k \,, \tag{7}$$

acting on the VVMFs with minimal weight, where

$$D_k := \frac{1}{2\pi \mathrm{i}} \frac{d}{d\tau} - \frac{kE_2(\tau)}{12} \,, \quad k \in \mathbb{N}^+ \,, \tag{8}$$

and $E_2(\tau)$ is the quasi-modular Eisenstein series

$$E_2(\tau) = 1 - 24 \sum_{n=1}^{\infty} \sigma_1(n) q^n \,. \tag{9}$$

MLDE

A desirable basis of $\mathcal{M}(\rho)$ is $\{Y^{(k_0)}, D_{k_0} Y^{(k_0)}, \ldots, D_{k_0}^{d-1} Y^{(k_0)}\}$.
For a VVMF with weight $k_0 + 2d$, the ring $\mathcal{M}(1) = \mathbb{C}[E_4, E_6]$ is a linear combination of the previous basis such that

$$(D_{k_0}^d + M_4 D_{k_0}^{d-2} + \cdots + M_{2(d-1)} D_{k_0} + M_{2d}) Y^{(k_0)} = 0\,, \tag{10}$$

where $M_k \in \mathbb{C}[E_4, E_6]$ is is the scalar modular form of weight k.

In summary, $\mathcal{M}(\rho)$ only depends on the VVMFs under the irrep ρ with minimal weight.

Binary dihedral group

$2D_3$

$$\Gamma(N) := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mod N \right\} . \quad (11)$$

$$\Gamma'_N := \Gamma / \Gamma(N) . \quad (12)$$

$$2D_3 \cong \mathrm{SL}(2, Z) \,/\, \langle\, S^2 T^2 \,\rangle . \quad (13)$$

$2D_3$

The binary dihedral group $2D_3$ is the preimage of the dihedral group D_3 under the maping $\mathrm{SU}(2) \cong \mathrm{Spin}(3) \to \mathrm{SO}(3)$.
$2D_3$ has genetrators S y T such that

$$S^4 = (ST)^3 = S^2T^2 = 1, \quad S^2T = TS^2 \,. \tag{14}$$

$2D_3$ has 12 elements and its GAP ID is $[12, 1]$.
Its center is $Z_2^{S^2} := \langle 1, S^2 \rangle$.
$2D_3 \cong Z_3 \rtimes Z_4$.

Character table of $2D_3$

Classes	$1C_1$	$1C_2$	$3C_4$	$2C_3$	$3C_4$	$2C_6$
Representative	1	S^2	T	TS	TS^2	TS^3
$\mathbf{1}$	1	1	1	1	1	1
$\mathbf{1}'$	1	1	-1	1	-1	1
$\hat{\mathbf{1}}$	1	-1	$-i$	1	i	-1
$\hat{\mathbf{1}}'$	1	-1	i	1	$-i$	-1
$\mathbf{2}$	2	2	0	-1	0	-1
$\hat{\mathbf{2}}$	2	-2	0	-1	0	1

Table: The character table of the binary dihedral group $2D_3 \cong Z_3 \rtimes Z_4$.

Irreducible representations of $2D_3$

$$
\begin{aligned}
&\mathbf{1}:\ \rho_{\mathbf{1}}(S)=1\,, \qquad \rho_{\mathbf{1}}(T)=1\,,\\
&\mathbf{1}':\ \rho_{\mathbf{1}'}(S)=-1\,, \qquad \rho_{\mathbf{1}'}(T)=-1\,,\\
&\hat{\mathbf{1}}:\ \rho_{\hat{\mathbf{1}}}(S)=i\,, \qquad \rho_{\hat{\mathbf{1}}}(T)=-i\,,\\
&\hat{\mathbf{1}}':\ \rho_{\hat{\mathbf{1}}'}(S)=-i\,, \qquad \rho_{\hat{\mathbf{1}}'}(T)=i\,,\\
&\mathbf{2}:\ \rho_{\mathbf{2}}(S)=\frac{1}{2}\begin{pmatrix}-1 & \sqrt{3}\\ \sqrt{3} & 1\end{pmatrix}, \qquad \rho_{\mathbf{2}}(T)=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\\
&\hat{\mathbf{2}}:\ \rho_{\hat{\mathbf{2}}}(S)=\frac{i}{2}\begin{pmatrix}-1 & \sqrt{3}\\ \sqrt{3} & 1\end{pmatrix}, \qquad \rho_{\hat{\mathbf{2}}}(T)=\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}.
\end{aligned}
\tag{15}
$$

Products in $2D_3$

$$1'\otimes 1' = 1\,,\ 1'\otimes\hat{1} = \hat{1}'\,,\ 1'\otimes\hat{1}' = \hat{1}\,,\ \hat{1}\otimes\hat{1} = 1'\,,\ \hat{1}\otimes\hat{1}' = 1\,,\ \hat{1}'\otimes\hat{1}' = 1'\,. \tag{16}$$

$$\begin{cases} 1'\otimes 2 = 2\,, & \hat{1}'\otimes 2 = \hat{2} \\ 1'\otimes\hat{2} = \hat{2}\,, & \hat{1}\otimes\hat{2} = 2 \end{cases} \quad : \quad \alpha_1\begin{pmatrix}\beta_2\\ -\beta_1\end{pmatrix}, \tag{17}$$

$$\begin{cases} \hat{1}\otimes 2 = \hat{2}\,, & \hat{1}'\otimes\hat{2} = 2 \end{cases} \quad : \quad \alpha_1\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}. \tag{18}$$

$$\begin{cases} 2\otimes 2 = 1\oplus 1'\oplus 2 \\ 2\otimes\hat{2} = \hat{1}\oplus\hat{1}'\oplus\hat{2} \end{cases} : (\alpha_1\beta_1+\alpha_2\beta_2)\oplus(\alpha_1\beta_2-\alpha_2\beta_1)\oplus\begin{pmatrix}\alpha_2\beta_2-\alpha_1\beta_1\\ \alpha_1\beta_2+\alpha_2\beta_1\end{pmatrix}, \tag{19}$$

$$\begin{cases} \hat{2}\otimes\hat{2} = 1\oplus 1'\oplus 2 \end{cases} : (\alpha_1\beta_2-\alpha_2\beta_1)\oplus(\alpha_1\beta_1+\alpha_2\beta_2)\oplus\begin{pmatrix}\alpha_1\beta_2+\alpha_2\beta_1\\ \alpha_1\beta_1-\alpha_2\beta_2\end{pmatrix}. \tag{20}$$

VVMFs in $2D_3$ with minimal weight

We can express the module $\mathcal{M}(2D_3)$ as

$$\mathcal{M}(2D_3) = \mathcal{M}(1)\oplus\mathcal{M}(1')\oplus\mathcal{M}(\hat{1})\oplus\mathcal{M}(\hat{1}')\oplus\mathcal{M}(2)\oplus\mathcal{M}(\hat{2})\,. \quad (21)$$

Each $\mathcal{M}(\rho) = \bigoplus_{k=0}^{\infty}\mathcal{M}_k(\rho)$ is a graded module over the ring $\mathcal{M}(1) = \mathbb{C}[E_4, E_6]$.
We look for a basis of VVMFs with minimal weight under the irreps of $2D_3$.

VVMFs in $2D_3$ with minimal weight

We use the generators over $\mathcal{M}(1) = \mathbb{C}[E_4, E_6]$.

$$\begin{aligned}
\mathcal{M}(1) &= \langle 1 \rangle \,, \\
\mathcal{M}(1') &= \langle Y_{1'}^{(6)} \rangle \,, \\
\mathcal{M}(\hat{1}) &= \langle Y_{\hat{1}}^{(9)} \rangle \,, \\
\mathcal{M}(\hat{1}') &= \langle Y_{\hat{1}'}^{(3)} \rangle \,, \\
\mathcal{M}(2) &= \langle Y_{2}^{(2)},\ \ D_2 Y_{2}^{(2)} \rangle \,, \\
\mathcal{M}(\hat{2}) &= \langle Y_{\hat{2}}^{(5)},\ \ D_5 Y_{\hat{2}}^{(5)} \rangle \,.
\end{aligned} \tag{22}$$

VVMFs in $2D_3$ with minimal weight

The VVMFs with minimal weight are

$$Y_{1'}^{(6)}(\tau) = \eta^{12}(\tau)\,, \qquad Y_{\hat{1}}^{(9)}(\tau) = \eta^{18}(\tau)\,, \qquad Y_{\hat{1}'}^{(3)}(\tau) = \eta^{6}(\tau)\,,$$
$$Y_{2}^{(2)}(\tau) = \begin{pmatrix} \eta^4(\tau)(\frac{K(\tau)}{1728})^{-\frac{1}{6}}\ {}_2F_1(-\frac{1}{6},\frac{1}{6};\frac{1}{2};K(\tau)) \\ -8\sqrt{3}\eta^4(\tau)(\frac{K(\tau)}{1728})^{\frac{1}{3}}\ {}_2F_1(\frac{1}{3},\frac{2}{3};\frac{3}{2};K(\tau)) \end{pmatrix},$$
$$Y_{\hat{2}}^{(5)}(\tau) = \begin{pmatrix} 8\sqrt{3}\eta^{10}(\tau)(\frac{K(\tau)}{1728})^{\frac{1}{3}}\ {}_2F_1(\frac{1}{3},\frac{2}{3};\frac{3}{2};K(\tau)) \\ \eta^{10}(\tau)(\frac{K(\tau)}{1728})^{-\frac{1}{6}}\ {}_2F_1(-\frac{1}{6},\frac{1}{6};\frac{1}{2};K(\tau)) \end{pmatrix}, \tag{23}$$

where ${}_2F_1$ is the generalized hypergeometric series and $K(\tau) = 1728/j(\tau)$ is the inverse of Klein-j function $j(\tau)$.

VVMFs in $2D_3$

The VVMFs for $2D_3$ are:

$$
\begin{aligned}
k=2: &\quad Y_{2}^{(2)}\,,\\
k=3: &\quad Y_{\hat{1}'}^{(3)}\,,\\
k=4: &\quad Y_{1}^{(4)} \equiv E_4\,, \quad Y_{2}^{(4)} \equiv -6D_2Y_{2}^{(2)}\,,\\
k=5: &\quad Y_{\hat{2}}^{(5)}\,,\\
k=6: &\quad Y_{1}^{(6)} \equiv E_6\,, \quad Y_{1'}^{(6)}\,, \quad Y_{2}^{(6)} \equiv E_4Y_{2}^{(2)}\,,\\
k=7: &\quad Y_{\hat{2}}^{(7)} \equiv 3D_5Y_{\hat{2}}^{(5)}\,, \quad Y_{\hat{1}'}^{(7)} \equiv E_4Y_{\hat{1}'}^{(3)}\,,\\
&\dots\,.
\end{aligned}
\tag{24}
$$

SUSY with $2D_3$ invariance

Supersymmetry

We consider the global supergravity theory $\mathcal{N} = 1$.
The action is given as

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta} \mathcal{K}(\Psi_I, \bar{\Psi}_I, \tau, \bar{\tau}) + \int d^4x d^2\theta \mathcal{W}(\Psi_I, \tau).$$

The superpotential is

$$\mathcal{W}(\Phi_I, \tau) = \sum_n Y_{I_1 \ldots I_n}(\tau) \Phi_{I_1} \ldots \Phi_{I_n} \,. \tag{25}$$

SUSY $2D_3$ invariance

Every term in the superpotential satisfies

$$-k_{I_1} - k_{I_2} - \cdots - k_{I_n} + k_Y = -1\,. \tag{26}$$

We propose

$$\Phi_I \to (c\tau + d)^{-k_I}\rho_I(\gamma)\Phi_I\,, \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma\,, \tag{27}$$

where $k_I \in \mathbb{Q}$ is the modular weight of Φ_I and $\rho_I(\gamma)$ is a $2D_3$ irrep.
We choose

$$k_I \in \left\{\frac{-16}{4}, \frac{-15}{4}, \ldots, \frac{15}{4}, \frac{16}{4}\right\}. \tag{28}$$

SUSY $2D_3$ invariance

We denote the left-handed and right-handed superfields

$$\psi^c \in \{u^c, d^c, E^c, N^c\},$$

$$\psi \in \{Q, L\}.$$

We write

$$\psi_D^c = (\psi_1^c, \psi_2^c)^T, \psi_D = (\psi_1, \psi_2)^T,$$

with $i \in \{1, 2, 3\}$. The modular weights of ψ_i, ψ_i^c, ψ_D and ψ_D^c are k_{ψ_i}, $k_{\psi_i^c}$, k_{ψ_D} and $k_{\psi_D^c}$, respectively.
We propose the Higgs doublets H_u y H_d are such that $k_{H_u} = k_{H_d} = 0$.

Superpotentials

Eg. the superpotential of Dirac type is

$$\mathcal{W} \supset \left[\alpha \left(Y_{DD}^{(k_{DD})} \psi_D^c \psi_D\right)_{\mathbf{s}} + \beta \left(Y_{D3}^{(k_{D3})} \psi_D^c \psi_3\right)_{\mathbf{s}} + \gamma \left(Y_{3D}^{(k_{3D})} \psi_3^c \psi_D\right)_{\mathbf{s}} + \delta \left(Y_{33}^{(k_{33})} \psi_3^c \psi_3\right)_{\mathbf{s}}\right] \left(H_{u/d}\right)_{\tilde{\mathbf{s}}} , \tag{29}$$

where $\mathbf{s} \otimes \tilde{\mathbf{s}} = \mathbf{1}$.

General fermion mass matrices

All the mass matrices have a structure

$$M_{\psi/\psi^c} = \begin{pmatrix} M_{DD} & \vdots & M_{D3} \\ \cdots & \cdot & \cdots \\ M_{3D} & \vdots & M_{33} \end{pmatrix}, \tag{30}$$

Example of a fermion mass matrix

$$M_d = \begin{pmatrix} -\alpha^d Y^{(2)}_{\mathbf{2},1} & \alpha^d Y^{(2)}_{\mathbf{2},2} & -\beta^d Y^{(2)}_{\mathbf{2},2} \\ \alpha^d Y^{(2)}_{\mathbf{2},2} & \alpha^d Y^{(2)}_{\mathbf{2},1} & \beta^d Y^{(2)}_{\mathbf{2},1} \\ -\gamma^d Y^{(4)}_{\mathbf{2},2} & \gamma^d Y^{(4)}_{\mathbf{2},1} & \delta^d Y^{(4)}_{\mathbf{1}} \end{pmatrix} v_d \,, \tag{31}$$

$$M_u = \begin{pmatrix} -\alpha^u_1 Y^{(4)}_{\mathbf{2},1} + \alpha^u_2 Y^{(4)}_{\mathbf{1}} & \alpha^u_1 Y^{(4)}_{\mathbf{2},2} & -\beta^u Y^{(4)}_{\mathbf{2},2} \\ \alpha^u_1 Y^{(4)}_{\mathbf{2},2} & \alpha^u_1 Y^{(4)}_{\mathbf{2},1} + \alpha^u_2 Y^{(4)}_{\mathbf{1}} & \beta^u Y^{(4)}_{\mathbf{2},1} \\ -\gamma^u Y^{(2)}_{\mathbf{2},2} & \gamma^u Y^{(2)}_{\mathbf{2},1} & 0 \end{pmatrix} v_u \,. \tag{32}$$

Phenomenology

Experimental data

Quark sector		Leptons sector	
Observables	Central value and 1σ error	Observables	Central value and 1σ error
m_u/m_c	$(1.9286 \pm 0.6017) \times 10^{-3}$	m_e/m_μ	0.00474 ± 0.00004
m_c/m_t	$(2.8213 \pm 0.1195) \times 10^{-3}$	m_μ/m_τ	0.0586 ± 0.00047
m_d/m_s	$(5.0523 \pm 0.6191) \times 10^{-2}$	$\Delta m^2_{21}/10^{-5}\text{eV}^2$	$7.41^{+0.21}_{-0.20}$
m_s/m_b	$(1.8241 \pm 0.1005) \times 10^{-2}$	$\Delta m^2_{31}/10^{-3}\text{eV}^2$	$2.511^{+0.028}_{-0.027}$
$\delta^q_{CP}/^\circ$	69.2133 ± 3.1146	δ^l_{CP}/π	$1.0944^{+0.2333}_{-0.1389}$
θ^q_{12}	0.22736 ± 0.00073	$\sin^2\theta^l_{12}$	$0.303^{+0.012}_{-0.011}$
θ^q_{13}	0.00349 ± 0.00013	$\sin^2\theta^l_{13}$	$0.02203^{+0.00056}_{-0.00059}$
θ^q_{23}	0.04015 ± 0.00064	$\sin^2\theta^l_{23}$	$0.572^{+0.018}_{-0.023}$

Data of charged lepton mass ratios, quark mass ratios and quark mixing parameters are taken from [1306.6879] with the SUSY breaking scale $M_{\text{SUSY}} = 10$ TeV and $\tan\beta = 10, \bar{\eta}_b = 0$. The lepton mixing parameters are taken from NuFIT 5.2 (2022) [2007.14792].

Successful models

- A quark model with 9 parameters:

We obtain $\chi^2_{\min} \sim 19.9$.
Only m_d/m_s is about the edge of 4σ.
The VEV of the modulus is close to $\omega := e^{\frac{2\pi i}{3}}$.

- A quark model with 10 parameters:

We obtain $\chi^2_{\min} \sim 0.0002$.
All predictions fall within 1σ.
The VEV of the modulus is close to i.

Successful models

- A Dirac neutrino model with 8 parameters:

  We obtain $\chi^2_{\text{min}} \sim 10^{-6}$.
  All predictions fall within 1σ.

- A Majorana neutrino model for Type-I seesaw mechanism with 9 parameters:

  We obtain $\chi^2_{\text{min}} \approx 10^{-6}$.
  All predictions fall within 3σ level.

Successful models

- A Majorana neutrino model for Weinberg operator with 7 parameters:

We obtain $\chi^2_{\text{min}} = 17.14$.
All predictions fall within 3σ level except $\sin^2\theta^l_{23}$.

A complete model of quarks and leptons

For the model

$$d^c \sim u^c \sim , Q \sim E^c \sim L \sim \hat{2} \oplus \hat{1}' \,, H_d \sim H_u \sim \mathbf{1}' \,.$$
$$k_{d^c_D} = k_{u^c_D} = -k_{E^c_3} = 1/2 \,, \; k_{d^c_3} = k_{u^c_D} = k_{Q_D} = k_{Q_3} = 5/2,$$
$$k_{E^c_3} = k_{L_D} = 7/2, \; k_{L_3} = 3/2 \,, k_{H_d} = k_{H_u} = 0 \,.$$

The best-fit input parameters are

$$\begin{gathered}
\langle\tau\rangle = -0.0613689 + 2.68637\mathrm{i} \,, \; \beta^d/\alpha^d = 20.3565 \,, \\
\delta^d/\alpha^d = 309.699 \,, \; \gamma^d/\alpha^d = -1040.08 \,, \; \alpha_1^u/\gamma^u = -6422.49 \,, \\
\alpha_2^u/\gamma^u = -6413.76 \,, \; \beta^u/\gamma^u = 4383.68 \,, \; \delta^l/\alpha^l = -0.0808854 \,, \\
\gamma^l/\alpha^l = 17.0445 \,, \; \alpha_1^\nu/\beta^\nu = 2.3088 \,, \; \alpha_2^\nu/\beta^\nu = -11784.2 \,, \\
\alpha^d v_d = 0.891348\ \mathrm{MeV} \,, \; \gamma^u v_u = 6.44174\ \mathrm{MeV} \,, \\
\alpha^l v_d = 76.207\ \mathrm{MeV} \,, \; \frac{\beta^\nu v_u^2}{\Lambda} = 0.010066\ \mathrm{eV} \,.
\end{gathered} \tag{22}$$

A complete model of quarks and leptons

The predicted flavor observables are

$$\begin{aligned}
&\sin^2\theta^l_{12} = 0.31818\,,\ \sin^2\theta^l_{13} = 0.021746\,,\ \sin^2\theta^l_{23} = 0.527212\,,\\
&\delta^l_{CP} = 1.59044\pi\,, \alpha_{21} = 0.901365\pi\,,\ \alpha_{31} = 0.526527\pi\,,\\
&m_e/m_\mu = 0.00473731\,,\ m_\mu/m_\tau = 0.058568\,,\ \frac{\Delta m^2_{21}}{\Delta m^2_{31}} = 0.030349\,,\\
&m_1 = 5.23272\text{ meV}\,,\quad m_2 = 10.0738\text{ meV}\,,\ m_3 = 49.6888\text{ meV}\,,\\
&\theta^q_{12} = 0.227464\,,\ \theta^q_{13} = 0.00339533\,,\ \theta^q_{23} = 0.0403661\,,\\
&\delta^q_{CP} = 69.1464^\circ\,,\ m_u/m_c = 0.00192463\,,\ m_c/m_t = 0.00282265\,,\\
&m_d/m_s = 0.0505174\,,\ m_s/m_b = 0.0182406\,. \qquad (34)
\end{aligned}$$

We obtain $\chi^2_{\text{min}} = 8.4$.
All predictions fall within 3σ.

Conclusions

Conclusion

We have succesful model best-fit models with $2D_3$ symmetry!!

Conclusions

- We investigated the (yet) unexplored phenomenology of $2D_3$.
- $2D_3$ as a flavor group only allows structures $2+1$ with VVMFs.
- After a exhaustive numerical analysis, benchmark models with new phenomenology were provided.
- Several fermion scenarios were taken into account.
- Higgs fields are accommodated in non-trivial representations od $2D_3$.
- We allowed fractional modular weights for the fields.
- This phenomenology could arise from some $\mathbb{T}^2/\mathbb{Z}_4$ heterotic orbifold [2405.20378].

Future work

Current literature

$$
\begin{aligned}
\Gamma_N &= \langle\, S, T \mid T^N \;=\; S^2 \;=\; (ST)^3 \;=\; 1 \,\rangle\,, && N \leq 5,\\
\Gamma'_N &= \langle\, S, T \mid T^N \;=\; S^4 \;=\; (ST)^3 \;=\; 1\,,\; S^2T = TS^2 \,\rangle\,, && N \leq 5,\\
\Gamma_6 &= \langle\, S, T \mid T^6 \;=\; S^2 \;=\; (ST)^3 \;=\; ST^2ST^3ST^4ST^3 \;=\; 1\,,\; S^2T = TS^2 \,\rangle\,,\\
\Gamma'_6 &= \langle\, S, T \mid T^6 \;=\; S^4 \;=\; (ST)^3 \;=\; ST^2ST^3ST^4ST^3 \;=\; 1\,,\; S^2T = TS^2 \,\rangle\,,\\
\Gamma_7 &= \langle\, S, T \mid T^7 \;=\; S^2 \;=\; (ST)^3 \;=\; (ST^3)^4 \;=\; 1 \,\rangle\,. && (35)
\end{aligned}
$$

Non famous literature

For $N \in 2Z+1$,

$$\Gamma_N = \langle\, S, T \mid T^N \;=\; S^2 \;=\; (TS)^3 \;=\; (T^{(N+1)/2} S T^4 S)^2 \;=\; 1 \,\rangle\,, \tag{36}$$

$$\Gamma'_N = \langle\, S, T \mid T^N S^2 \;=\; S^4 = (TS)^3 (T^{(N+1)/2} S T^4 S)^2 \;=\; 1 \,\rangle\,.$$

[J.G. Sunday, Presentations of the Groups SL$(2,m)$ And PSL$(2,m)$]

Future work

$$\Gamma_8' = \langle\, S, T \mid T^8 = S^4 = (ST^7)^3 S^2 = (ST^3)^3 S (ST^3)^3 S^3 = T (ST^3)^3 T^7 (ST^3)^{-3} = 1 \,\rangle\,, \tag{37a}$$

$$\Gamma_8 = \langle\, S, T \mid T^8 = S^2 = (ST^7)^3 S^2 = (ST^3)^3 S (ST^3)^3 S^3 = T^9 (ST^3)^3 T^7 (ST^3)^{-3} = 1 \,\rangle\,. \tag{37b}$$

Future work

$$\Gamma_8' = \langle\, S, T \mid S^4 = T^8 = (ST)^3 = ST^2ST^{-3}S^{-1}T^2ST^{-3} = 1,\; S^2T = TS^2 \,\rangle\,,$$
$$\Gamma_8 = \langle\, S, T \mid S^2 = T^8 = (ST)^3 = (ST^2ST^{-3})^2 = 1 \,\rangle\,. \tag{38}$$

Gracias por su atención.