N=1 SUGRA: From constructive and BCFW to KLT formulations

Based on arXiv:2406.11001

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We use a perturbative language to identify particles

- S- Matrix (LSZ) $\longrightarrow$ Feynman rules $\longrightarrow$ Scattering process.
- Feynman Diagrams = external lines + internal lines (Propagator) + vertex.
- Amplitud = $\sum$ (Feynman diagrams) [Space-time approach on QED, R. Feynman, 1949].
- Differential cross section:

$$d\sigma = \frac{1}{2E_A 2E_B |v_A - v_B|} \left(\prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \right) |\mathcal{M}(p_A, p_B \longrightarrow p_f)|^2$$
$$\times\, (2\pi)^4 \delta^4 (p_A + p_B - \sum p_f).$$

Recipe to the calculation of cross sections:

- Draw the diagrams for the desire process
- Use the Feynman rules to write down the amplitude $\mathcal{M}$.
- Square the amplitude and average or sum over spins, using completeness relations: $\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m; \cdots$
- Evaluate traces using the trace theorems; collect terms and simplify the answer as much as possible.
- Choose a particular frame of reference. Express all 4−momentum vectors in terms of variables such as E and θ.
- Plug the resulting expression for $|\mathcal{M}|^2$ into the cross-section formula and integrate over phase-space [An introduction to QFT, Peskin and Schroeder, 1995]

Figure: Perturbative expansion

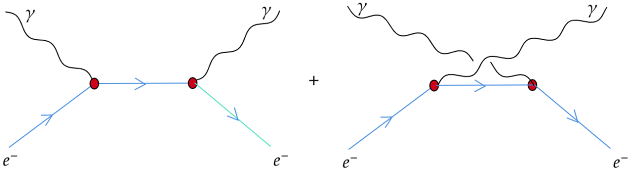


Figure: Tree level Compton effect

Tree level amplitude

$$i\mathcal{M} = -ie^2\epsilon^*_\nu(k_f,\lambda_f)\epsilon_\mu(k_i,\lambda_i)\bar{u}(p_f,s_f)\Gamma^{\nu\mu}u(p_i,s_i) \tag{1}$$

$$\Gamma^{\mu\nu} = \gamma^\nu\frac{(\not{p}_i+\not{k}_i+m)}{(p_i+k_i)^2-m^2}\gamma^\mu + \gamma^\mu\frac{(\not{p}_i-\not{k}_f+m)}{(p_i-k_f)^2-m^2}\gamma^\nu. \tag{2}$$

The average of the squared amplitude

$$|\bar{\mathcal{M}}|^2 = \frac{1}{4}\sum_{spins}|\mathcal{M}|^2 = \frac{1}{4}\sum_{spins}\mathcal{M}\mathcal{M}^\dagger. \tag{3}$$

Traces to compute

$$I = Tr[(\not{p}_f+m)(\not{\epsilon}_f\not{k}_i\not{\epsilon}_i)(\not{p}_i+m)(\not{\epsilon}_i\not{k}_i\not{\epsilon}_f)] \tag{4}$$

$$II = Tr[(\not{p}_f+m)(\not{\epsilon}_i\not{k}_f\not{\epsilon}_f)(\not{p}_i+m)(\not{\epsilon}_f\not{k}_f\not{\epsilon}_i)] \tag{5}$$

$$III = Tr[(\not{p}_f+m)(\not{\epsilon}_f\not{\epsilon}_i)\not{k}_i(\not{p}_i+m)\not{k}_f(\not{\epsilon}_f\not{\epsilon}_i)] \tag{6}$$

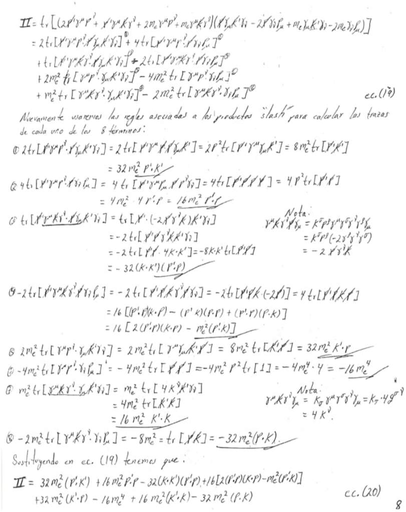


Figure: Some traces of Compton effect. It is possible calculate them using, for instance, Feyncalc.

Final answer when we take the limit $m \to 0$

$$|\bar{\mathcal{M}}|^2 = 2e^4 \left(\frac{p_i \cdot k_f}{p_i \cdot k_i} + \frac{p_i \cdot k_i}{p_i \cdot k_f} \right). \tag{7}$$

The number of Feynman diagrams tends to grow very quicly with the number of particles involved, for instance:

$$\begin{aligned} gg &\longrightarrow gg, \quad 4 \quad diagrams \\ gg &\longrightarrow ggg, \quad 25 \quad diagrams \\ gg &\longrightarrow gggg, \quad 220 \quad diagrams \\ gg &\longrightarrow 8g, \quad \approx 1,000,000 \end{aligned}$$

[See Andres Luna Talk]

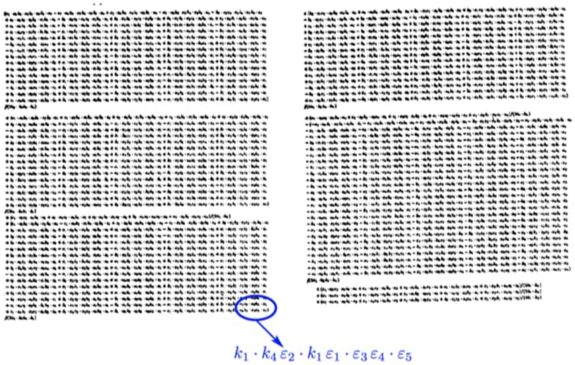


Figure: Part of the result for the 5 gluons amplitude at tree level.
"Magic tricks for scattering amplitudes" by Zvi Bern.

On-Shell methods

The key question here is: What defines a particle? The answer is given by the concept of **Little group**, which is the set of Lorentz transformations that leaves the momentum of the particles invariant: **L p = p** [E.P. Wigner, ´39. Y.S. Kim and E.P. Wigner,´90. S.Weinberg, QFT vol I, ´05.].

- Massive particles are defined by its mass m and spin s. The little group is SO(3). Known particles: $s = 0$ (Higgs), $s = 1/2$ (q y l), $s = 1$ (bosons W y Z)
- Massless particles are defined by **helicity (h)**. The little group is $E(2) = U(1) \times T(2)$. Known particles: $h = \pm 1$ (photon, gluon), $h = \pm 2$ (Graviton).

Spinor helicity formalism: Massless case

- In the massless case, **helicity spinors** are defined as real or complex doublets transforming in the $(\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$ representations of the Lorentz group.
  $\langle\lambda\chi\rangle = \epsilon^{ab}\lambda_a\chi_b = -\langle\chi\lambda\rangle$, $\quad [\lambda\chi] = \epsilon_{\dot{a}\dot{b}}\tilde{\lambda}^{\dot{a}}\tilde{\chi}^{\dot{b}} = -[\chi\lambda]$.
  [Weyl-Van der Waerden notation]
- To represent momenta as bispinors, we use sigma Pauli matrices:

$$p^{\dot{a}a} = (\bar{\sigma}^{\mu})^{\dot{a}a}p_{\mu} = \begin{pmatrix} p^0 - p^3 & -p^1 + ip^2 \\ -p^1 - ip^2 & p^0 + p^3 \end{pmatrix} \tag{8}$$
$$= \lambda_a\tilde{\lambda}_{\dot{a}} \equiv |p\rangle[p|$$

[P. De Causmaecker, R. Gastmans, W. Troost and T. T. Wu, ´81. Kaoru Hagiwara and D. Zeppenfeld, ´86. Henriette Elvang and Yu-Tin Huang, ´2015.]

Dirac spinors can be either left or right handed, in the Weyl basis we can write them as (High energy limit):

$$v_+(p) = u_-(p) = \begin{pmatrix} |p]_a \\ 0 \end{pmatrix} \equiv |p], \quad v_-(p) = u_+(p) = \begin{pmatrix} 0 \\ |p\rangle^{\dot{a}} \end{pmatrix} \equiv |p\rangle,$$

$$\bar{u}_-(p) = \bar{v}_+(p) = (0, \langle p|_{\dot{a}}) \equiv \langle p| \quad , \bar{u}_+(p) = \bar{v}_-(p) = ([p|^a, 0) \equiv [p|.$$

Contraction between two momentum vectors is:

$$q \cdot p = q^\mu p_\mu = \frac{1}{2} q_{\dot{a}a} p^{a\dot{a}} \equiv \frac{1}{2} Tr(|q]\langle qp\rangle[p|) = \frac{1}{2}\langle qp\rangle[pq].$$

Also

$$p^\mu = \frac{1}{2}(\sigma^\mu)^a p_{\dot{a}a}, \quad p^\mu = \frac{1}{2}\bar{\sigma}^\mu_{\dot{a}a} p^{a\dot{a}} \tag{9}$$

[M.D. Schwartz, QFT and The Standard Model ´14]

- Vector boson polarizations in this formalism are written as

$$[\epsilon_p^-(r)]^{\dot{a}a} = \sqrt{2}\frac{|p\rangle[r|}{[pr]}, \quad [\epsilon_p^+(r)]^{\dot{a}a} = \sqrt{2}\frac{|r\rangle[p|}{\langle rp\rangle}, \tag{10}$$

- Products between momentums and polarization vectors in terms of Weyl spinors [M. D Schwartz, QFT and The Standard Model ´14.]

$$\epsilon^+(p_i, r)\cdot p_j = \frac{[ij]\langle jr\rangle}{\sqrt{2}\langle ri\rangle}, \quad \epsilon^-(p_i, r)\cdot p_j = \frac{\langle ij\rangle[jr]}{\sqrt{2}[ir]} \tag{11}$$

$$\epsilon^-(p_i, r)\cdot\epsilon^+(p_j, q) = \frac{\langle iq\rangle[jr]}{[ir]\langle qj\rangle} \tag{12}$$

$$\not{\epsilon}^-(p, r) = \frac{\sqrt{2}}{[rp]}(|p\rangle[r| + |r]\langle p|), \quad \not{\epsilon}^+(p, r) = \frac{\sqrt{2}}{\langle rp\rangle}(|p]\langle r| + |r\rangle[p|) \tag{13}$$

In this formalism Mandelstam variables are defined as:

$$s_{ij} = (p_i + p_j)^2, \quad s_{ijk} = (p_i + p_j + p_k)^2, \quad etc. \tag{14}$$

In particular, we have standard Mandelstam variables as $s = s_{12}, t = s_{13}, u = s_{14}$. In terms of Weyl spinors

$$\begin{aligned} s &= \langle 12\rangle[21] = \langle 34\rangle[43], \\ u &= \langle 14\rangle[41] = \langle 23\rangle[32], \\ t &= \langle 13\rangle[31] = \langle 24\rangle[42]. \end{aligned} \tag{15}$$

[M.D. Schwartz, QFT and The Standard Model ´14].
[Aspectos básicos del método de amplitudes, Jonathan Reyes Pérez. Rev.Mex.Fis.'24]

Compton Effect: The amplitudes in terms of Weyl spinors are

$$\mathcal{M}(1^+2^-3^+4^-) = 2e^2\frac{\langle 24\rangle^2}{\langle 13\rangle\langle 23\rangle}, \quad \mathcal{M}(1^+2^-3^-4^+) = 2e^2\frac{\langle 23\rangle^2}{\langle 14\rangle\langle 24\rangle}.$$

the square averaged amplitude is

$$\begin{aligned}\overline{|\mathcal{M}|^2} &= \frac{1}{4}[2(|\mathcal{M}(1^+2^-3^+4^-)|^2+|\mathcal{M}(1^+2^-3^-4^+)|^2)] \qquad (16)\\ &= 2e^4(|\frac{s_{14}}{s_{13}}|+|\frac{s_{13}}{s_{14}}|)\\ &= 2e^4(\frac{u}{s}+\frac{s}{u}),\end{aligned}$$

where $s_{13} = s, s_{12} = t$ y $s_{14} = u$. [Weyl spinors and helicity formalism, J. Díaz Cruz, B. Larios, O. Meza Aldama, and J. Reyes Pérez, 1511.07477.]

- From $gg \longrightarrow gg$ we have

$$\tilde{\mathcal{M}}(1^-2^-3^+4^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}. \tag{17}$$

- For 5 gluons we have

$$\tilde{\mathcal{M}}_5(1^+2^+3^+4^-5^-) = \frac{\langle 45\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}. \tag{18}$$

- The MHV amplitud for N gluons is

$$\tilde{\mathcal{M}}(1^+2^+\cdots j^-\cdots k^-\cdots n^+) = \frac{\langle jk\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\cdots\langle n1\rangle}, \tag{19}$$

This equation is known as **Parker-Taylor formula**

SUGRA Compton effect

We want to calculate:

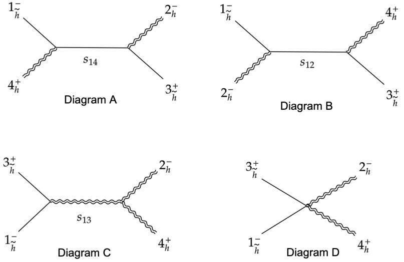


Figure: SUGRA Compton effect: Scattering process between spin-2 graviton h and spin-3/2 gravitino $\tilde{h}$.

- How to calculate the total amplitude?

$$\mathcal{M}_{Total} = ? \tag{20}$$

Tree amplitudes in supergravity theories can be computed using the following action

$$S_{\text{SUGRA}} = S_{EH} + S_{RS}, \\ = -\frac{1}{2\kappa^2}\int d^4x\Big((R^L_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R^L)h^{\mu\nu} + \epsilon^{\mu\nu\rho\sigma}(\widetilde{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma - \\ \chi_\mu\sigma_\nu\partial_\rho\widetilde{\chi}_\sigma)\Big) \tag{21}$$

where $\kappa^2 = 8\pi G_N$. A pure supergravity theory will have to include in addition the supersymmetric partner of the spin-2 graviton, this is spin-$\frac{3}{2}$ gravitino. [D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]

SUGRA Compton effect: A la Feynman

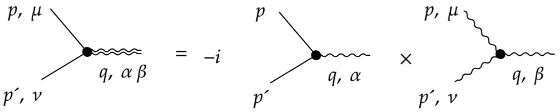
- Kawai-Lewellen-Tye relations from string theory [H. Kawai, D. C. Lewellen and S.H.H. Tye, '86]. In the field theory limit ($\alpha' \longrightarrow 0$), these relations take the form

$$\mathcal{M}_{Gravity} \sim \sum_{ij} K_{ij} \mathcal{A}_{YM}^{iL} \times \mathcal{A}_{YM}^{jR} \tag{22}$$

$$\mathcal{A}_3(1^-, 2^-, 3^+) = \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle}, \quad \mathcal{M}_3 = \frac{\langle 12 \rangle^6}{\langle 23 \rangle^2 \langle 31 \rangle^2} \tag{23}$$

- Amplitude of two gravitinos and two gravitons [Bjerrum-Bohr and Oluf Tang Engelund,´10]

$$\mathcal{M}_4(1^-_{\tilde{h}}, 2^-_h, 3^+_{\tilde{h}}, 4^+_h) = -is_{14} \mathcal{A}_4(1^-_{\tilde{g}}, 4^+_g, 2^-_g, 3^+_{\tilde{g}}) \mathcal{A}_4(1^-_g, 2^-_g, 3^+_g, 4^+_g) \tag{24}$$



$$V^{\mu,\,\alpha\beta,\,\nu}_{\tilde{h}h\tilde{h}}\,(p,\,q,\,p') \;=\; -\,i\,V^{\alpha}_{\tilde{g}g\tilde{g}}\,(p,q,\,p') \times V^{\mu,\,\beta,\,\nu}_{ggg}\,(p,q,p')$$

$$= \; -i\left(\frac{i}{\sqrt{2}}\,\gamma^{\alpha}\right) \times \Big[\frac{i}{\sqrt{2}}\,\big(\,(p-q)^{\nu}\,\eta^{\beta\mu} + (q-p')^{\mu}\,\eta^{\beta\nu} + (p'-p)^{\beta}\,\eta^{\mu\nu}\big)\Big]$$

Figure: Factorization of the gravitino three vertex

[Z. Bern and A. Grant, ´99. Bjerrum-Bohr and Oluf Tang Engelund,´10.] 1002.2279

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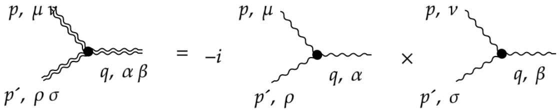


$$V_{hhh}^{\mu\nu,\,\alpha\beta,\,\rho\sigma}(p,\,q,\,p') \;=\; -i\,V_{ggg}^{\mu,\,\alpha,\,\rho}(p,q,\,p') \times V_{ggg}^{\nu,\,\beta,\,\sigma}(p,q,p')$$

$$= -i\left(\frac{i}{\sqrt{2}}\right)^2$$

$$\times\left[\frac{i}{\sqrt{2}}\Big((p-q)^{\rho}\,\eta^{\mu\alpha} + (q-p')^{\mu}\,\eta^{\alpha\rho} + (p'-p)^{\alpha}\,\eta^{\rho\mu}\Big)\right]$$

$$\times\left[\frac{i}{\sqrt{2}}\Big((p-q)^{\sigma}\,\eta^{\beta\nu} + (q-p')^{\nu}\,\eta^{\sigma\beta} + (p'-p)^{\beta}\,\eta^{\nu\sigma}\Big)\right]$$

Figure: The graviton three vertex

- Polarization tensors of the gravitino $\tilde{h}$ field

$$\tilde{\epsilon}_{\tilde{h}}^{+\mu}(p,q) = \tilde{\epsilon}_{\tilde{g}}^{+}(p) \times \epsilon_{g}^{+\mu}(p,q), \quad \tilde{\epsilon}_{\tilde{h}}^{-\mu}(p,q) = \tilde{\epsilon}_{\tilde{g}}^{-}(p) \times \epsilon_{g}^{-\mu}(p,q), \tag{25}$$

$$\epsilon_{\tilde{h}}^{+\mu}(p,q) = \epsilon_{\tilde{g}}^{+}(p) \times \epsilon_{g}^{+\mu}(p,q), \quad \epsilon_{\tilde{h}}^{-\mu}(p,q) = \epsilon_{\tilde{g}}^{-}(p) \times \epsilon_{g}^{-\mu}(p,q). \tag{26}$$

-Polarization tensors of graviton h field:

$$(\epsilon_{h}^{++})^{\mu\nu}(p,q) = \epsilon_{g}^{+\mu}(p,q) \times \epsilon_{g}^{+\nu}(p,q) \tag{27}$$

$$(\epsilon_{h}^{--})^{\mu\nu}(p,q) = \epsilon_{g}^{-\mu}(p,q) \times \epsilon_{g}^{-\nu}(p,q) \tag{28}$$

[N.J. Bjerrum-Bohr and Oluf Tang Engelund,´10. 1002.2279]

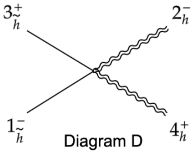
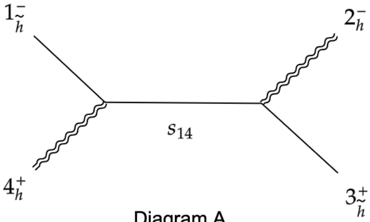


Figure: Diagram D

$$\tilde{\mathcal{M}}_D(1^{-\frac{3}{2}}, 2^{-2}, 3^{+\frac{3}{2}}, 4^{-2}) = V^{\mu,\nu,\alpha\beta,\kappa\lambda}_{\tilde{h}\tilde{h}hh}(p_1, p_3, p_2, p_4) \times \tilde{\epsilon}^{-}_{\tilde{h},\mu}(p_1, q)$$
$$\epsilon^{+}_{\tilde{h},\nu}(p_3, r)\epsilon^{--}_{h,\alpha\beta}(p_2, l)\epsilon^{++}_{h,\kappa\lambda}(p_4, j),$$
$$= 0 \qquad (29)$$

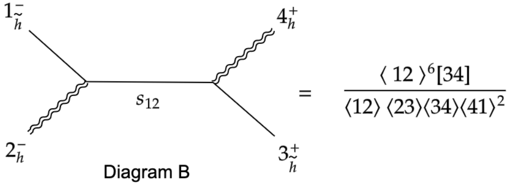
[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]



$$
\begin{aligned}
\tilde{\mathcal{M}}_A = & \, \tilde{\epsilon}^{-}_{\tilde{h},\kappa}(p_1,q) V^{\kappa,\alpha\beta,\mu}_{\tilde{h}h\tilde{h}}(p_1,p_4,p)\epsilon^{++}_{h,\alpha\beta}(p_4,j)\times \\
& P_{\tilde{h}\tilde{h},\mu\nu}(p) \times \epsilon^{+}_{\tilde{h},\tau}(p_3,r) V^{\tau,\rho\sigma,\nu}_{\tilde{h}h\tilde{h}}(p_3,p_2,p)\epsilon^{--}_{h,\rho\sigma}(p_2,l) \\
& = 0
\end{aligned}
\quad (30)
$$

[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]

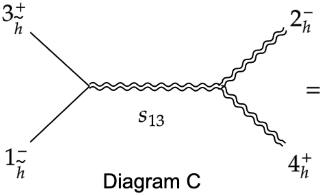
Diagram B



Figure

$$\widetilde{\mathcal{M}}_B = \widetilde{\epsilon}^{-}_{\widetilde{h},\kappa}(p_1,q) V^{\kappa,\alpha\beta,\mu}_{\widetilde{h}h\widetilde{h}}(p_1,p_2,p)\,\epsilon^{--}_{h,\alpha\beta}(p_2,l) \times P_{\widetilde{h}\widetilde{h},\mu\nu}(p) \\ \times \epsilon^{+}_{\widetilde{h},\tau}(p_3,r) V^{\tau,\rho\sigma,\nu}_{\widetilde{h}h\widetilde{h}}(p_3,p_4,p)\,\epsilon^{++}_{h,\rho\sigma}(p_4,j) \tag{31}$$

[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]



$$= \frac{\langle 12 \rangle^6 [34]}{\langle 13 \rangle \langle 32 \rangle \langle 24 \rangle \langle 41 \rangle^2}$$

$$\widetilde{\mathcal{M}}_C = \widetilde{\epsilon}^{-}_{\widetilde{h},\kappa}(p_1,q) V^{\kappa,\mu\theta,\tau}_{\widetilde{h}h\widetilde{h}}(p_1,p,p_3)\epsilon^{+}_{\widetilde{h},\tau}(p_3,r) \times P_{hh,\mu\theta,\nu\gamma}(p)$$
$$\times\, \epsilon^{--}_{h,\rho\sigma}(p_2,l) V^{\rho\sigma,\nu\gamma,\alpha\beta}_{hhh}(p_2,p,p_4)\epsilon^{++}_{h,\alpha\beta}(p_4,j) \qquad (32)$$

[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]

SUGRA Compton effect: A la Constructive approach and BCFW

In the case of Three point amplitudes, we have $p_1 + p_2 + p_3 = 0$. This implies some constrictions

$$(p_1 + p_2)^2 = \langle 12\rangle[12] = 0, \tag{33}$$

$$(p_2 + p_3)^2 = \langle 23\rangle[23] = 0, \tag{34}$$

$$(p_3 + p_1)^2 = \langle 31\rangle[31] = 0. \tag{35}$$

If $\langle 12\rangle \neq 0$ Then $[12] = [23] = [31] = 0$, **(Holomorfico)**.
If $[12] \neq 0$ Then $\langle 12\rangle = \langle 23\rangle = \langle 31\rangle = 0$, **(anti-holomorfico)**.
The variables $\langle 1|$ and $[1|$ are independent. [TASI Lectures on Scattering Amplitudes, C. Cheung, ´17]

A general formula for three-particle amplitude of massless particles in four dimensions; when we have $h = h_1 + h_2 + h_3 < 0$ then

$$\mathcal{M}(1^{h_1}2^{h_2}3^{h_3}) = c_{123}\langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}, \quad (36)$$

in the case of $h = h_1 + h_2 + h_3 > 0$ then

$$\mathcal{M}(1^{h_1}2^{h_2}3^{h_3}) = \tilde{c}_{123}[12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2}, \quad (37)$$

theories using this general expression are known as Constructive approach.
[TASI Lectures on Scattering Amplitudes, C. Cheung, ´17]

Interactions	Input parameters				3-point amplitude
	h_1	h_2	h_3	H	$\mathcal{M}(1^{h_1}, 2^{h_2}, 3^{h_3})$
gluino-gluino-gluon ($\tilde{g}\tilde{g}g$)	$+\frac{1}{2}$	$+1$	$-\frac{1}{2}$	$+1>0$	$\tilde{c}_{123}\frac{[12]^2}{[31]}$
	$-\frac{1}{2}$	-1	$+\frac{1}{2}$	$-1<0$	$c_{123}\frac{\langle 12\rangle^2}{\langle 31\rangle}$
graviton-graviton-graviton (hhh)	$+2$	$+2$	-2	$+2>0$	$\widetilde{G}_{123}\frac{[12]^6}{[23]^2[31]^2}$
	-2	-2	$+2$	$-2<0$	$G_{123}\frac{\langle 12\rangle^6}{\langle 23\rangle^2\langle 31\rangle^2}$
gravitino-graviton-gravitino ($\tilde{h}h\tilde{h}$)	$+\frac{3}{2}$	$+2$	$-\frac{3}{2}$	$+2>0$	$\widetilde{g}_{123}\frac{[12]^5}{[23][31]^2}$
	$-\frac{3}{2}$	-2	$+\frac{3}{2}$	$-2<0$	$g_{123}\frac{\langle 12\rangle^5}{\langle 23\rangle\langle 31\rangle^2}$

Figure: three point amplitudes for different interactions.

[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]

- BCFW allow us to compute higher point partial amplitudes from lower point ones in a recursive way.
- The BCFW transformations are

$$|\hat{i}\rangle = |i\rangle, \quad [\hat{i}| = [i| + z[j|, \tag{38}$$

$$[\hat{j}| = [j|, \quad |\hat{j}\rangle = |j\rangle - z|i\rangle, \tag{39}$$

- When $z = 0$ then $\mathcal{M}_n = \hat{\mathcal{M}}_n(z = 0)$.
- According to Cauchy's Theorem

$$\mathcal{M}_n = -\sum_{z^*} Res_{z=z^*} \frac{\hat{\mathcal{M}}_n(z)}{z} + B_n, \tag{40}$$

Where B_n is the residue of the pole at $z = \infty$. In order to have $B_n = 0$, we prove the statement that

$$\hat{\mathcal{M}}_n(z) \to 0 \quad for \quad z \to \infty. \tag{41}$$

This is the known **Large z behavior**

Then BCFW transformations are

$$|\hat{1}\rangle = |1\rangle, \quad [\hat{1}| = [1| + z^*_{2,3}[3|, \tag{42}$$

$$[\hat{3}| = [3|, \quad |\hat{3}\rangle = |3\rangle - z^*_{2,3}|1\rangle. \tag{43}$$

$$\Rightarrow \tilde{\mathcal{M}}(1^{-\frac{3}{2}}_{\tilde{h}}, 4^{+2}_{h}, 3^{+\frac{3}{2}}_{\tilde{h}}, 2^{-2}_{h}) = \tilde{\mathcal{M}}(1^{-\frac{3}{2}}, 4^{+2}, \hat{P}^{+\frac{3}{2}}) \frac{1}{s_{14}} \tilde{\mathcal{M}}(-\hat{P}^{-\frac{3}{2}}, 3^{+\frac{3}{2}}, 2^{-2})$$

$$= \frac{[\hat{P}4]^5}{[4\hat{1}][\hat{1}\hat{P}]^2} \frac{1}{\langle 14\rangle[41]} \frac{\langle(-\hat{P})2\rangle^5}{\langle 2\hat{3}\rangle\langle\hat{3}(-\hat{P})\rangle^2} = 0.$$

- The pole is

$$z^*_{2,3} = \frac{(p_2+p_3)^2}{\langle 12\rangle[23] + \langle 13\rangle[33]} = \frac{\langle 23\rangle[32]}{\langle 12\rangle[23]} = -\frac{\langle 23\rangle}{\langle 12\rangle}. \tag{44}$$

[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]

BCFW transformations are

$$|\hat{1}\rangle = |1\rangle, \quad [\hat{1}| = [1| + z^*_{3,4}[4|, \quad [\hat{4}| = [4|, \quad |\hat{4}\rangle = |4\rangle - z^*_{3,4}|1\rangle.$$

$$\Rightarrow \tilde{\mathcal{M}}(1^{-\frac{3}{2}}, 2^{-2}, 3^{+\frac{3}{2}}, 4^{+2}) = \tilde{\mathcal{M}}(1^{-\frac{3}{2}}, 2^{-2}, \hat{P}^{+\frac{3}{2}})\frac{1}{s_{12}}\tilde{\mathcal{M}}(-\hat{P}^{-\frac{3}{2}}, 3^{+\frac{3}{2}}, 4^{+2})$$

$$= \frac{\langle\hat{1}2\rangle^5}{\langle 2\hat{P}\rangle\langle\hat{P}\hat{1}\rangle^2}\frac{1}{\langle 34\rangle[43]}\frac{[3\hat{4}]^5}{[\hat{4}(-\hat{P})][(-\hat{P})3]^2}$$

$$= \frac{\langle 12\rangle^6[34]}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle^2}.$$

the pole is

$$z^*_{3,4} = \frac{\langle 34\rangle[43]}{\langle 13\rangle[34]} = -\frac{\langle 34\rangle}{\langle 13\rangle}. \tag{45}$$

[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]

$[i,\ j\rangle$	$[\hat{1},\hat{4}\rangle$	$[\hat{1},\hat{2}\rangle$	$[\hat{3},\hat{4}\rangle$	$[\hat{3},\hat{2}\rangle$
$\widehat{\mathcal{M}}(z) \sim$	$\frac{1}{z}$	$\frac{1}{z}$	$\frac{1}{z^2}$	z^6

Figure: Large z behavior of BCFW Diagram B

[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]

BCFW transformations are

$$|\hat{1}\rangle = |1\rangle, \quad [\hat{1}| = [1| + z^*_{2,4}[4|, \quad [\hat{4}| = [4|, \quad |\hat{4}\rangle = |4\rangle - z^*_{2,4}|1\rangle.$$

$$\Rightarrow \tilde{\mathcal{M}}(1^{-\frac{3}{2}}, 3^{+\frac{3}{2}}, 2^{-2}, 4^{+2}) = \frac{\langle \hat{1}\hat{P}\rangle^5}{\langle \hat{P}3\rangle\langle 3\hat{1}\rangle^2}\frac{1}{s_{13}}\frac{[(-\hat{P})\hat{4}]^6}{[\hat{4}2]^2[2(-\hat{P})]^2} + \frac{[3\hat{P}]^5}{[\hat{P}\hat{1}][\hat{1}3]^2}\frac{1}{s_{13}}\frac{\langle(-\hat{P})2\rangle^6}{\langle 2\hat{4}\rangle^2\langle\hat{4}(-\hat{P})\rangle^2} = \frac{\langle 12\rangle^6[34]}{\langle 13\rangle\langle 32\rangle\langle 24\rangle\langle 41\rangle^2}. \quad (46)$$

The pole is

$$z^*_{2,4} = \frac{\langle 24\rangle[42]}{\langle 12\rangle[24]} = -\frac{\langle 24\rangle}{\langle 12\rangle}. \quad (47)$$

[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]

$[i, j\rangle$	$[\hat{1},\hat{4}\rangle$	$[\hat{1},\hat{2}\rangle$	$[\hat{3},\hat{4}\rangle$	$[\hat{3},\hat{2}\rangle$
$\widehat{\mathcal{M}}(z) \sim$	$\frac{1}{z}$	$\frac{1}{z^2}$	$\frac{1}{z^3}$	z^6

Figure: Large z behavior of BCFW Diagram C

[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]

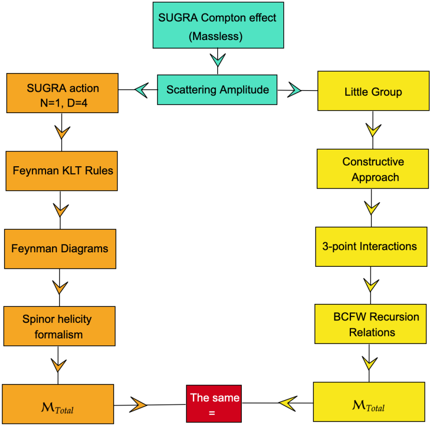


Figure: Feynman Rules vs BCFW

$$
\begin{aligned}
\mathcal{M}_{Total} &= \widetilde{g}_{123} g_{123} \widetilde{\mathcal{M}}_B + g_{123} \widetilde{G}_{123} \widetilde{\mathcal{M}}_C \\
&= i\langle 12\rangle[34] \times \left(g_{123}\widetilde{G}_{123} \frac{\langle 12\rangle^5}{\langle 13\rangle\langle 32\rangle\langle 24\rangle\langle 41\rangle^2} \right. \\
&\left. - \widetilde{g}_{123} g_{123} \frac{\langle 12\rangle^5}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle^2} \right) \qquad (48)
\end{aligned}
$$

[D. Chakraborty, J. Lorenzo Díaz Cruz, P. Ortega Ruíz and J. Reyes Pérez, arXiv:2406.11001]

- **Next step is considering a massive gravitino.**
- **We are exploring a Super-Higgs Mechanism in the contexts of spin-spinors formalism (massive)** [In collaboration with Lorenzo Díaz Cruz and Ivan Pérez Castro (Cinvestav)] Work in progress

We want to compute the next tree-level amplitudes [In collaboration with Manfred Kraus]

$$\begin{aligned} q\bar{q} &\to t\bar{t}H, \\ q\bar{q} &\to t\bar{t}Hg, \\ q\bar{q} &\to t\bar{t}Hgg \\ qq &\to t\bar{t}Hqq, \\ q\bar{q} &\to t\bar{t}HQ\bar{Q}, \end{aligned} \tag{49}$$

and the corresponding gluon amplitudes. These all are necessary for $t\bar{t}H$ at NNLO.

- Using ideas of BCFW recursion relations, (massive) spin-spinors formalism [arXiv:1709.04891], BCJ relation, etc.
- Lower order $\mathcal{M}(g,g,g)$, $\mathcal{M}(q,g,\bar{q})$, $\mathcal{M}(t,g,\bar{t})$, $\mathcal{M}(t,H,\bar{t})$.
- Some work is done in this direction [Top tree amplitudes for higher order calculations,arXiv:2309.03323]

- Supergravity Feynman rules derived via KLT are simpler than results derived from conventional analysis.
- The results obtained via KLT Feynman rules and Constructive approach with BCFW, coincide in the massless case.
- On-shell methods like spinor helicity, BCFM recurtion relations, etc. are more powerful to compute scattering amplitudes of massless particles than the conventional methods of Feynman rules.

Thank you!