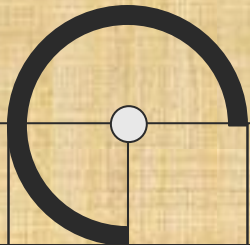


One-loop contributions to WWZ from a seesaw variant with radiatively induced light neutrino masses

H. Novales, **Mónica Salinas**, H. Vázquez,



Cinvestav



Calculation, estimation and analysis
of the one loop contributions from
Majorana neutrinos



Lorentz-covariant WWZ
parametrization



BSM description characterized by $\mathcal{L}_{\text{BSM}}$ [1]

Two phases of spontaneous symmetry breaking



Symmetry breaking at some high-energy scale w

Brout-Englert-Higgs mechanism
 $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_e$

at $v = 246 \text{ GeV}$

[1] Pilaftsis, A. (1992). Radiatively induced neutrino masses and large Higgs-neutrino couplings in the Standard Model with Majorana fields. Zeitschrift für Physik C Particles and Fields, **55**(2), 275-282.

Asume that previous chain of events gives rise to the
Lagrangian

$$\mathcal{L}_{\text{BSM}} = \mathcal{L}_{\text{mass}}^{\nu} + \mathcal{L}_{\text{CC}}^{\text{SM}} + \mathcal{L}_{\text{NC}}^{\text{SM}} + \mathcal{L}_{\text{HC}}^{\text{SM}} + \dots$$

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$$\mathcal{L}_{\text{mass}}^{\nu} = - \sum_{j=1}^3 \sum_{k=1}^3 \left(\overline{\nu_{j,L}^0} (m_{\text{D}})_{jk} \nu_{k,R}^0 + \frac{1}{2} \overline{\nu_{j,R}^{0c}} (m_{\text{M}})_{jk} \nu_{k,R}^0 \right) + \text{H.c.} \quad (1)$$

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$$\nu_k^c = C \bar{\nu}_k^T$$

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3×3 matrix assumed to
emerge from EW symmetry
breaking

3×3 matrix assumed to
emerge from symmetry
breaking at w

Defining

$$f_L = \begin{pmatrix} \nu_{1,L}^0 \\ \nu_{2,L}^0 \\ \nu_{3,L}^0 \end{pmatrix}$$

$$F_L = \begin{pmatrix} \nu_{1,R}^{0c} \\ \nu_{2,R}^{0c} \\ \nu_{3,R}^{0c} \end{pmatrix}$$

$$f_R = C \overline{f_L}^T$$

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$$\Rightarrow \mathcal{L}_{\text{mass}}^\nu = -\frac{1}{2} (\overline{f_L} \quad \overline{F_L}) \mathcal{M} \begin{pmatrix} f_R \\ F_R \end{pmatrix} + \text{H.c.} \quad (2)$$

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$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & m_M \end{pmatrix} \quad \begin{array}{l} 6 \times 6 \text{ matrix} \\ \text{complex} \\ \text{symmetric} \end{array}$$

✓ Unitary diagonalization transformation

Define the mass eigenspinors base

Assume 6×6 unitary matrix

$$\mathcal{U}_\nu = \begin{pmatrix} \mathcal{U}_{11} & \mathcal{U}_{12} \\ \mathcal{U}_{21} & \mathcal{U}_{22} \end{pmatrix}$$

3×3 matrix blocks $\mathcal{U}_{kj}$

Unitary transformation

$$\mathcal{U}_\nu^T \mathcal{M} \mathcal{U}_\nu = \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix}$$

3×3 real-valued diagonal matrix

$$\begin{aligned} (m_\nu)_{jk} &= m_{\nu_j} \delta_{jk} \\ (m_N)_{jk} &= m_{N_j} \delta_{jk} \end{aligned}$$

✓ Unitary diagonalization transformation

Define the mass eigenspinors base

$$\mathcal{L}_{\text{mass}}^{\nu} = \sum_{j=1}^3 \left(-\frac{1}{2} m_{\nu_j} \bar{\nu}_j \nu_j - \frac{1}{2} m_{N_j} \bar{N}_j N_j \right) \quad (3)$$

Majorana fields

$$\nu_j^c = \nu_j \quad N_j^c = N_j$$

Light neutrinos

Heavy neutrinos

$$\mathcal{L}_{\text{BSM}} = \mathcal{L}_{\text{mass}}^{\nu} + \mathcal{L}_{\text{CC}}^{\text{SM}} + \mathcal{L}_{\text{NC}}^{\text{SM}} + \mathcal{L}_{\text{HC}}^{\text{SM}} + \dots$$

$$\mathcal{L}_{\text{CC}}^{\text{SM}} = \sum_{\alpha} \sum_{j=1}^3 \left(\frac{g}{\sqrt{2}} \mathcal{B}_{\alpha\nu_j} W_{\rho}^{-} \bar{l}_{\alpha} \gamma^{\rho} P_L \nu_j + \frac{g}{\sqrt{2}} \mathcal{B}_{\alpha N_j} W_{\rho}^{-} \bar{l}_{\alpha} \gamma^{\rho} P_L N_j \right) + \text{H.c.} \quad (4)$$

$$\mathcal{B}_{\alpha\nu_j} = \sum_{k=1}^3 V_{\alpha k}^{\ell} (\mathcal{U}_{11}^{*})_{kj}$$

$$\mathcal{B}_{\alpha N_j} = \sum_{k=1}^3 V_{\alpha k}^{\ell} (\mathcal{U}_{12}^{*})_{kj}$$

$$\mathcal{L}_{\text{BSM}} = \mathcal{L}_{\text{mass}}^{\nu} + \mathcal{L}_{\text{CC}}^{\text{SM}} + \mathcal{L}_{\text{NC}}^{\text{SM}} + \mathcal{L}_{\text{HC}}^{\text{SM}} + \dots$$

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V^{ℓ}



3×3 matrix
Lepton mixing

$$\mathcal{L}_{\text{BSM}} = \mathcal{L}_{\text{mass}}^{\nu} + \mathcal{L}_{\text{CC}}^{\text{SM}} + \mathcal{L}_{\text{NC}}^{\text{SM}} + \mathcal{L}_{\text{HC}}^{\text{SM}} + \dots$$

$$\mathcal{L}_{\text{CC}}^{\text{SM}} = \sum_{\alpha} \sum_{j=1}^3 \left(\frac{g}{\sqrt{2}} \mathcal{B}_{\alpha\nu_j} W_{\rho}^{-} \bar{l}_{\alpha} \gamma^{\rho} P_L \nu_j + \frac{g}{\sqrt{2}} \mathcal{B}_{\alpha N_j} W_{\rho}^{-} \bar{l}_{\alpha} \gamma^{\rho} P_L N_j \right) + \text{H.c.} \quad (4)$$

Unitary block parametrization [2,3]

$$\mathcal{B}_{\alpha\nu_j} = \sum_{k=1}^3 V_{\alpha k}^{\ell} (\mathcal{U}_{11}^*)_{kj}$$

$$\mathcal{B}_{\alpha N_j} = \sum_{k=1}^3 V_{\alpha k}^{\ell} (\mathcal{U}_{12}^*)_{kj}$$

$$\mathcal{U} = \begin{pmatrix} (\mathbf{1}_3 + \xi^* \xi^{\text{T}})^{-\frac{1}{2}} & \xi^* (\mathbf{1}_3 + \xi^{\text{T}} \xi^*)^{-\frac{1}{2}} \\ -\xi^{\text{T}} (\mathbf{1}_3 + \xi^* \xi^{\text{T}})^{-\frac{1}{2}} & (\mathbf{1}_3 + \xi^{\text{T}} \xi^*)^{-\frac{1}{2}} \end{pmatrix} \quad (5)$$

✓ 3×3 complex matrix ξ

Assumption: $|\xi_{jk}|$ are small $\longrightarrow \xi = m_{\text{D}} m_{\text{M}}^{-1}$

- [2] J. G. Korner, A. Pilaftsis, and K. Schilcher, Leptonic CP asymmetries in flavor-changing H^0 decays, Phys. Rev. D **47**, 1080 (1993).
 [3] P. S. B. Dev and A. Pilaftsis, Minimal radiative neutrino mass mechanism for inverse seesaw models, Phys. Rev. D **86**, 113001 (2012).

$$\mathcal{L}_{\text{BSM}} = \mathcal{L}_{\text{mass}}^{\nu} + \mathcal{L}_{\text{CC}}^{\text{SM}} + \mathcal{L}_{\text{NC}}^{\text{SM}} + \mathcal{L}_{\text{HC}}^{\text{SM}} + \dots$$

$$\begin{aligned} \mathcal{L}_{\text{NC}}^{\text{SM}} = & \sum_{k=1}^3 \sum_{j=1}^3 \frac{g}{4c_W} \left(Z_{\rho} \bar{\nu}_k \gamma^{\rho} (i\mathcal{C}_{\nu_k \nu_j}^{\text{Im}} - \mathcal{C}_{\nu_k \nu_j}^{\text{Re}} \gamma_5) \nu_j \right. \\ & + \left(Z_{\rho} \bar{\nu}_k \gamma^{\rho} (i\mathcal{C}_{\nu_k N_j}^{\text{Im}} - \mathcal{C}_{\nu_k N_j}^{\text{Re}} \gamma_5) N_j + \text{H.c.} \right) \\ & \left. + Z_{\rho} \bar{N}_k \gamma^{\rho} (i\mathcal{C}_{N_k N_j}^{\text{Im}} - \mathcal{C}_{N_k N_j}^{\text{Re}} \gamma_5) N_j \right) \end{aligned} \quad (6)$$

$$\begin{aligned}
 \mathcal{C}_{\nu_i \nu_l} &= \sum_{j=1}^3 (\mathcal{U}_{11})_{ji} (\mathcal{U}_{11}^*)_{jl} \\
 \mathcal{C}_{\nu_i N_l} &= \sum_{j=1}^3 (\mathcal{U}_{11})_{ji} (\mathcal{U}_{12}^*)_{jl} \\
 \mathcal{C}_{N_i \nu_l} &= \sum_{j=1}^3 (\mathcal{U}_{12})_{ji} (\mathcal{U}_{11}^*)_{jl} \\
 \mathcal{C}_{N_i N_l} &= \sum_{j=1}^3 (\mathcal{U}_{12})_{ji} (\mathcal{U}_{12}^*)_{jl}
 \end{aligned}$$

6 × 6 hermitian matrix



$$\mathcal{C} = \begin{pmatrix} \mathcal{C}_{\nu\nu} & \mathcal{C}_{\nu N} \\ \mathcal{C}_{N\nu} & \mathcal{C}_{NN} \end{pmatrix}$$

3 × 3 matrix
blocks



$$\mathcal{C} = \begin{pmatrix} (\mathbf{1}_3 + \xi \xi^\dagger)^{-1} & (\mathbf{1}_3 + \xi \xi^\dagger)^{-1} \xi \\ \xi^\dagger (\mathbf{1}_3 + \xi \xi^\dagger)^{-1} & \xi^\dagger (\mathbf{1}_3 + \xi \xi^\dagger)^{-1} \xi \end{pmatrix} \quad (7)$$

$$\mathcal{L}_{\text{BSM}} = \mathcal{L}_{\text{mass}}^{\nu} + \mathcal{L}_{\text{CC}}^{\text{SM}} + \mathcal{L}_{\text{NC}}^{\text{SM}} + \mathcal{L}_{\text{HC}}^{\text{SM}} + \dots$$

$$\begin{aligned} \mathcal{L}_{\text{HC}}^{\text{SM}} = & \sum_{k=1}^3 \sum_{j=1}^3 \frac{g}{4m_W} \left(h\overline{\nu}_k \left((m_{\nu_k} + m_{\nu_j}) \mathcal{C}_{\nu_k \nu_j}^{\text{Re}} - i\gamma_5 (m_{\nu_k} - m_{\nu_j}) \mathcal{C}_{\nu_k \nu_j}^{\text{Im}} \right) \nu_j \right. \\ & + h\overline{\nu}_k \left((m_{\nu_k} + m_{N_j}) \mathcal{C}_{\nu_k N_j}^{\text{Re}} - i\gamma_5 (m_{\nu_k} - m_{N_j}) \mathcal{C}_{\nu_k N_j}^{\text{Im}} \right) N_j \\ & \left. + h\overline{N}_k \left((m_{N_k} + m_{N_j}) \mathcal{C}_{N_k N_j}^{\text{Re}} - i\gamma_5 (m_{N_k} - m_{N_j}) \mathcal{C}_{N_k N_j}^{\text{Im}} \right) N_j \right) \end{aligned} \quad (8)$$

Consider the neutrino mass model [1]



Light neutrinos:

$$(\mathcal{M}\mathcal{U}_\nu)_{jk} = 0 \quad j = 1, 2, 3, 4, 5, 6$$

the k – th neutrino mass vanishes at tree level.



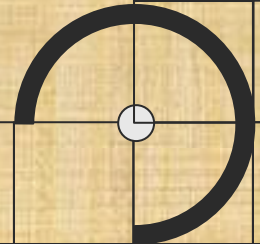
Heavy neutrinos:

Tree-level mass are left untouched

$$m_N \simeq m_M \left(1 + \frac{1}{2} m_M^{-1} (\xi^\dagger m_D + m_D^T \xi^*) \right) \quad (9)$$

Smallness of light-neutrino masses comes about if heavy neutrino masses are nearly-degenerate

One loop effects from
Majorana neutrinos on the
vertex **WWZ**



The WWZ vertex

Lagrangian

$$\mathcal{L}_{\text{eff}}^{WWZ} = \mathcal{L}_{WWZ}^{\text{even}} + \mathcal{L}_{WWZ}^{\text{odd}}$$

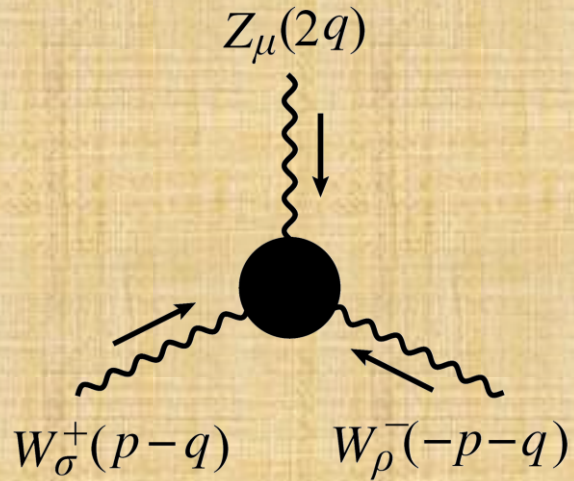
Refer to CP-
transformation
properties

Where

$$\begin{aligned} \mathcal{L}_{WWZ}^{\text{even}} = & -ig_Z \left(g_1 (W_{\mu\nu}^+ W^{-\mu} - W_{\mu\nu}^- W^{+\mu}) Z^\nu \right. \\ & \left. + \kappa W_\mu^+ W_\nu^- Z^{\mu\nu} + \frac{\lambda}{m_W^2} W_{\mu\nu}^+ W^{-\nu}{}_\rho Z^{\rho\mu} - ig_2 \epsilon^{\rho\mu\lambda\nu} Z_\rho (W_\nu^+ \partial_\lambda W_\mu^- - W_\mu^- \partial_\lambda W_\nu^+) \right) \end{aligned} \quad (10)$$

$$\mathcal{L}_{WWZ}^{\text{odd}} = -ig_Z \left(\tilde{\kappa} W_\mu^+ W_\nu^- \tilde{Z}^{\mu\nu} + \frac{\tilde{\lambda}}{m_W^2} W_{\mu\nu}^+ W^{-\nu}{}_\rho \tilde{Z}^{\rho\mu} - i\tilde{g}_1 W_\mu^+ W_\nu^- (\partial^\mu Z^\nu + \partial^\nu Z^\mu) \right) \quad (11)$$

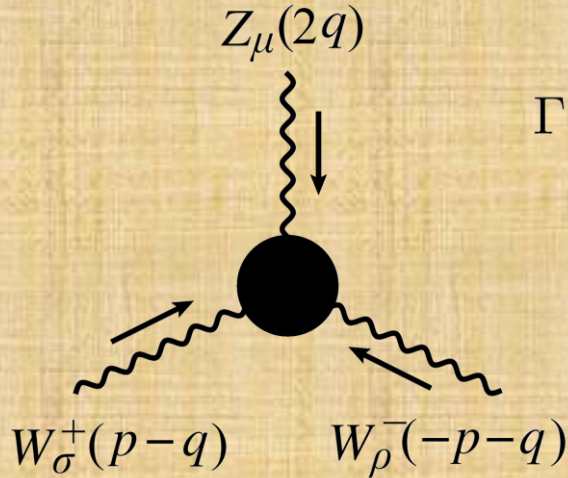
WWZ vertex [4]



SM W bosons on-shell
 Z boson off-shell

[4] W. A. Bardeen, R. Gastmans, and B. Lautrup, Static quantities in Weinberg's model of weak and electromagnetic interactions, Nucl. Phys. B **46**, 319 (1972).

CP-even WWZ vertex function



$$\begin{aligned} \Gamma_{\sigma\rho\mu}^{\text{even}} = & ig_Z \left(g_1 (2p_\mu g_{\sigma\rho} + 4(q_\rho g_{\sigma\mu} - q_\sigma g_{\rho\mu})) \right. \\ & + \frac{4\Delta Q}{m_W^2} p_\mu \left(q_\sigma q_\rho - \frac{q^2}{2} g_{\sigma\rho} \right) \\ & \left. + 2\Delta\kappa (q_\rho g_{\sigma\mu} - q_\sigma g_{\rho\mu}) + if_1 \epsilon_{\sigma\rho\mu\alpha} p^\alpha \right) \quad (12) \end{aligned}$$

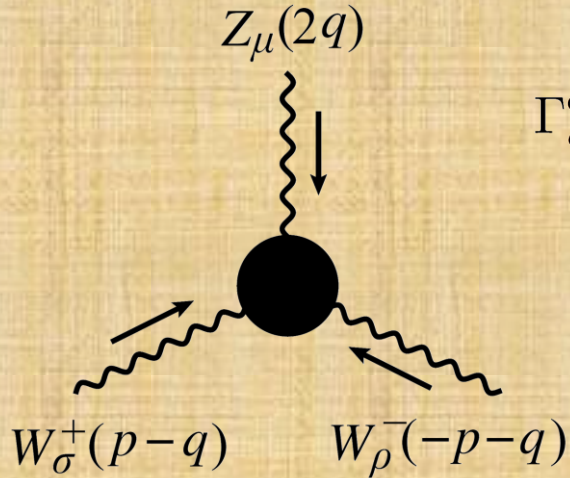
$$\Delta\kappa = -g_1 + \kappa + \lambda$$

$$\Delta Q = -2\lambda$$

Anomalous couplings

$$f_1 = -g_2$$

CP-odd WWZ vertex function



$$\Gamma_{\sigma\rho\mu}^{\text{odd}} = ig_Z \left(2\Delta\tilde{\kappa}\epsilon_{\sigma\rho\mu\alpha}q^\alpha + \frac{4\Delta\tilde{Q}}{m_W^2}q_\rho\epsilon_{\sigma\mu\alpha\beta}p^\alpha q^\beta \right. \\ \left. + i\tilde{f}_1(q_\rho g_{\sigma\mu} + q_\sigma g_{\rho\mu}) \right. \\ \left. + \tilde{f}_2 p^\lambda\epsilon_{\sigma\rho\lambda\alpha}(q^2\delta^\alpha_\mu - q^\alpha q_\mu) \right), \quad (13)$$

$$\Delta\tilde{\kappa} = \tilde{\kappa} + \frac{m_W^2 - 2q^2}{m_W^2}\tilde{\lambda}$$

$$\Delta\tilde{Q} = -2\tilde{\lambda}$$

$$\tilde{f}_1 = 2\tilde{g}_1$$

$$\tilde{f}_2 = -\frac{4\tilde{\lambda}}{m_W^2}$$

Anomalous couplings

One-loop contributions to WWZ

Vertex function $\Gamma_{\sigma\rho\mu}^{WWZ} = \Gamma_{\sigma\rho\mu}^{lln} + \Gamma_{\sigma\rho\mu}^{nnl} + \Gamma_{\sigma\rho\mu}^{nnh}$ (14)

$$\Gamma_{\sigma\rho\mu}^{lln} = \sum_{k=1}^6 \sum_{\alpha} \text{Diagram}$$

$$\Gamma_{\sigma\rho\mu}^{nnl} = \sum_{k=1}^6 \sum_{j=1}^6 \sum_{\alpha} \left(\text{Diagram 1} + \text{Diagram 2} \right)$$

$$\Gamma_{\sigma\rho\mu}^{nnh} = \sum_{k=1}^6 \sum_{j=1}^6 \left(\text{Diagram 1} + \text{Diagram 2} \right)$$

One-loop contributions to WWZ

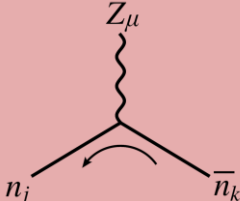
Vertex function $\Gamma_{\sigma\rho\mu}^{WWZ} = \Gamma_{\sigma\rho\mu}^{lln} + \Gamma_{\sigma\rho\mu}^{nnl} + \Gamma_{\sigma\rho\mu}^{nnh}$ (14)

$$\Gamma_{\sigma\rho\mu}^{lln} = \sum_{k=1}^6 \sum_{\alpha} \text{Diagram}$$

n_k represents both ν_k and N_k

One-loop contributions to WWZ

Vertex function $\Gamma_{\sigma\rho\mu}^{WWZ} = \Gamma_{\sigma\rho\mu}^{lln} + \Gamma_{\sigma\rho\mu}^{nnl} + \Gamma_{\sigma\rho\mu}^{nnh}$ (14)

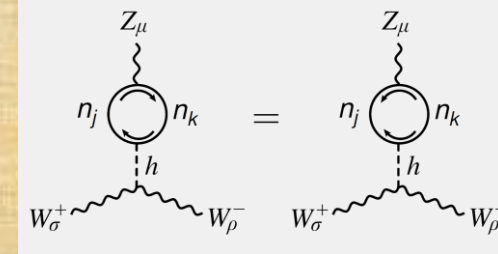


$$= \Gamma'_{kj}{}^\mu = C \Gamma_{jk}^{\mu T} C^{-1}$$

$$\Gamma_{\sigma\rho\mu}^{nnl} = \sum_{k=1}^6 \sum_{j=1}^6 \sum_{\alpha} \left(\begin{array}{c} Z_{\mu} \\ \nearrow n_j \quad \nwarrow n_k \\ \leftarrow l_{\alpha} \\ W_{\sigma}^{+} \quad W_{\rho}^{-} \end{array} + \begin{array}{c} Z_{\mu} \\ \nearrow n_j \quad \nwarrow n_k \\ \bullet \\ \leftarrow l_{\alpha} \\ W_{\sigma}^{+} \quad W_{\rho}^{-} \end{array} \right)$$

One-loop contributions to WWZ

Vertex function $\Gamma_{\sigma\rho\mu}^{WWZ} = \Gamma_{\sigma\rho\mu}^{lln} + \Gamma_{\sigma\rho\mu}^{nnl} + \Gamma_{\sigma\rho\mu}^{nnh} \quad (14)$

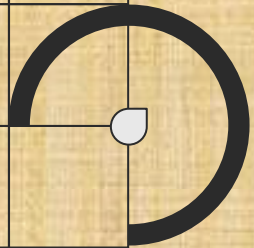


$$\Gamma_{\sigma\rho\mu}^{nnh} = \sum_{k=1}^6 \sum_{j=1}^6 \left(\text{Diagram 1} + \text{Diagram 2} \right)$$

Diagram 1: A Z boson (Z_μ) is connected to a fermion loop (n_j, n_k) via a wavy line. The loop is connected to a W boson pair (W_σ^+, W_ρ^-) via a dashed line labeled h .

Diagram 2: A Z boson (Z_μ) is connected to a fermion loop (n_j, n_k) via a wavy line. The loop is connected to a W boson pair (W_σ^+, W_ρ^-) via a wavy line.

Do not contribute to ΔQ and $\Delta\kappa$



Estimations and discussion




$$\xi = \hat{\rho} X$$

Considering the matrix texture

$$X = e^{i\varphi} \cdot \mathbf{1}_3$$


$$V^\ell = U_\nu$$

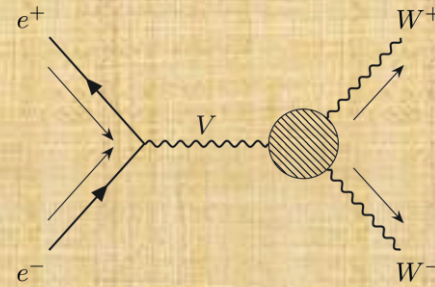
$$m_{\nu_j} \approx 0.8 \text{eV} \quad [5]$$

Nearly degenerated
Neutrino spectrum

$$m_{N_1} \approx m_{N_h}$$

$$m_{N_2} \approx m_{N_h}$$

$$m_{N_3} \approx m_{N_h}$$



The International Linear Collider technical design
report estimated sensitivity [6]

Coupling	$\sqrt{s} = 500 \text{ GeV}$	$\sqrt{s} = 800 \text{ GeV}$
$ \Delta\kappa_{\text{ILC}} $	3.20×10^{-4}	1.90×10^{-4}
$ \Delta Q_{\text{ILC}} $	1.34×10^{-3}	6.00×10^{-4}
$ \Delta\tilde{\kappa}_{\text{ILC}} $	5.33×10^{-2}	5.77×10^{-2}
$ \Delta\tilde{Q}_{\text{ILC}} $	1.5×10^{-3}	6.00×10^{-4}

[5] M. Aker et al. (The KATRIN Collaboration), Direct neutrino-mass measurement with sub-electronvolt sensitivity, Nature Phys. 18, 160 (2022)

[6] H. Baer et al. (ILC Collaboration), The international linear collider technical design report-Volume 2: Physics, arXiv:1306.6352.

CP-Even contributions

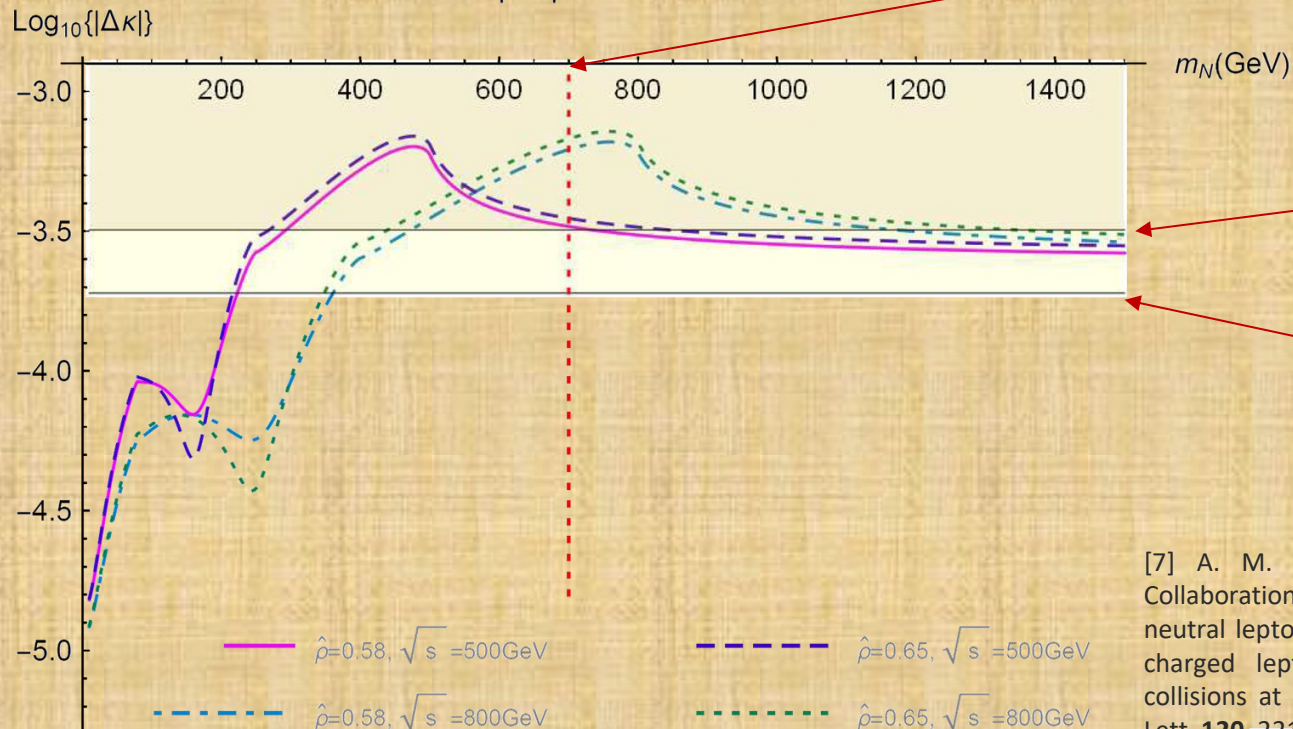
$$\begin{aligned}\Delta\kappa = \sum_{\alpha} & \left(\frac{1}{1 + \hat{\rho}^2} (\Delta\kappa_{\alpha\nu}^{(1)} + \hat{\rho}^2 \Delta\kappa_{\alpha N}^{(1)}) \right. \\ & + \frac{1}{(1 + \hat{\rho}^2)^2} (\Delta\kappa_{\alpha\nu\nu}^{(2)} + \Delta\kappa_{\alpha\nu\nu}^{(3)}) + \frac{\hat{\rho}^4}{(1 + \hat{\rho}^2)^2} (\Delta\kappa_{\alpha NN}^{(2)} + \Delta\kappa_{\alpha NN}^{(3)}) \\ & \left. + \frac{2\hat{\rho}^2}{(1 + \hat{\rho}^2)^2} (\Delta\kappa_{\alpha\nu N}^{(2)} + \cos(2\varphi) \Delta\kappa_{\alpha\nu N}^{(3)}) \right)\end{aligned}\tag{16}$$

Contributions to the light + heavy neutrinos to the CP-even form factor $\Delta\kappa$

$|\Delta\kappa|$ CP-even

Allowed minimal heavy-neutrino mass according to this work by CMS[7]

Accessible values to the
ILC

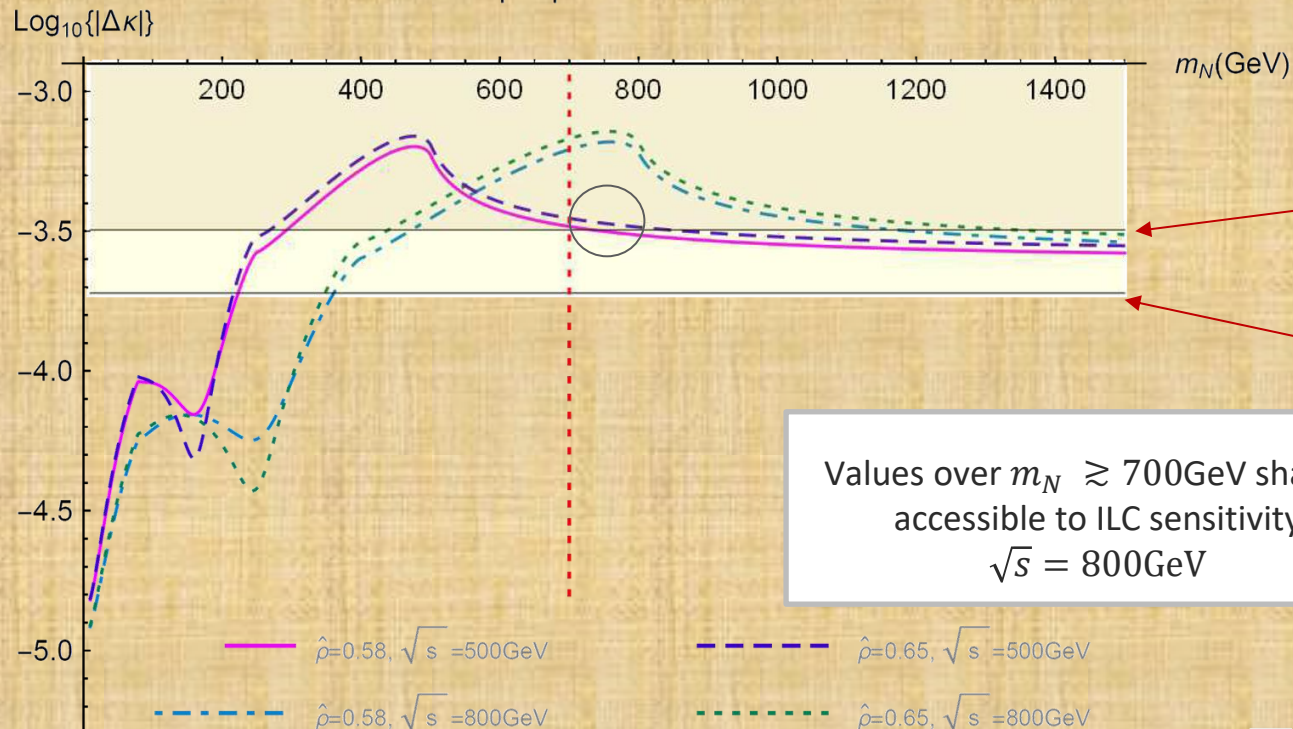


[7] A. M. Sirunyan et al. (CMS Collaboration), Search for heavy neutral leptons in events with three charged leptons in proton-proton collisions at $\sqrt{s}=13\text{TeV}$, Phys. Rev. Lett. **120**, 221801 (2018).

Contributions to the light + heavy neutrinos to the CP-even form factor $\Delta\kappa$

$|\Delta\kappa|$ CP-even

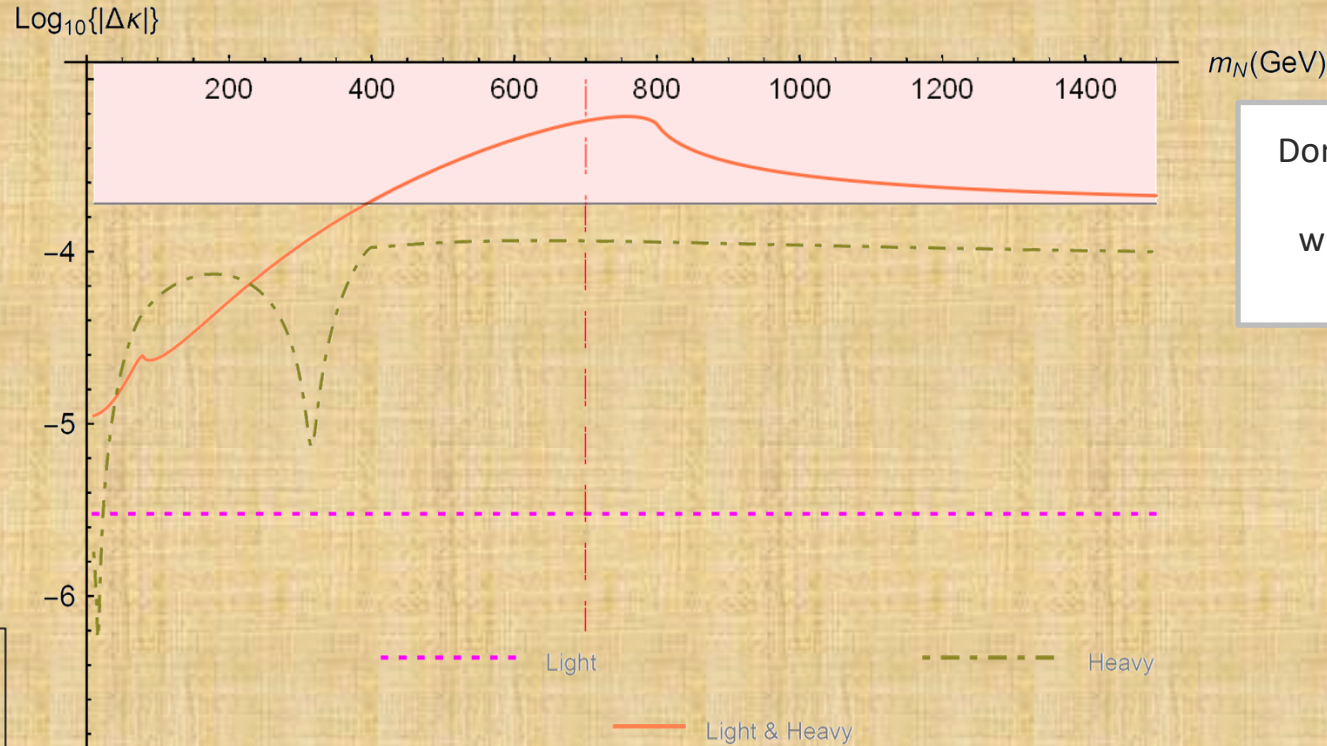
Accessible values to the
ILC



Values over $m_N \gtrsim 700\text{GeV}$ shall be
accessible to ILC sensitivity
 $\sqrt{s} = 800\text{GeV}$

Contributions to the light + heavy neutrinos to the CP-even form factor $\Delta\kappa$

$|\Delta\kappa|$ CP-even for $\hat{\rho}=0.65$ $\sqrt{s} = 800$ GeV



Dominant effect is produced by
the diagrams
with heavy- and light- virtual
neutrinos.

CP-Odd contributions

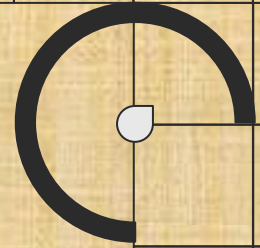
Implementing the X -matrix texture

$$\Delta\tilde{\kappa} = 3(\tilde{\Omega} + \tilde{\mathcal{J}}) + \frac{2i\hat{\rho}^2 \sin 2\varphi}{(1 + \hat{\rho}^2)^2} \sum_{\alpha} \Delta\tilde{\kappa}_{\alpha\nu N}^{(3)} \quad (17)$$



$$|\Delta\tilde{\kappa}| = 1.19 \times 10^{-3}$$

Summary and conclusions



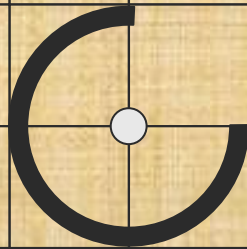
- BSM physics effects on the WWZ with virtual Majorana neutrinos at one-loop level.

- SM W bosons on-shell and Z boson off-shell.

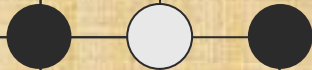
$$\begin{matrix} \Delta\kappa \\ \Delta Q \end{matrix} \begin{matrix} \swarrow \\ \searrow \end{matrix} \text{CP-even + CP-odd effects} \longrightarrow \Delta\tilde{\kappa}$$

- Ultraviolet finite.
- Analytical expressions in terms of three parameters: $m_N, s, \hat{p}$.

- Our study is carried out in the context of some future electron-positron collider, perhaps ILC. Sensitive to the CME of the collision.
- $\Delta\kappa$  Contributions $\mathcal{O}(10^{-3})$
We could reach ILC sensitivity at CME of 800 GeV due to the effects from virtual heavy and light neutrinos.
- ΔQ  At least one order of magnitude below the estimated sensitivity for ILC.
- $\Delta\tilde{\kappa}$  Contributions $\mathcal{O}(10^{-3})$.
One order of magnitude beyond expected ILC sensitivity.



Thanks!



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[1] Pilaftsis, A. (1992). Radiatively induced neutrino masses and large Higgs-neutrino couplings in the Standard Model with Majorana fields. Zeitschrift für Physik C Particles and Fields, **55**(2), 275-282.

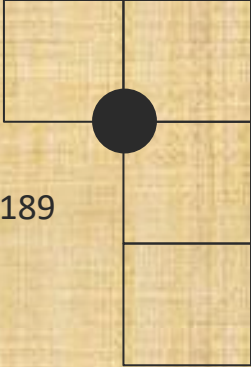
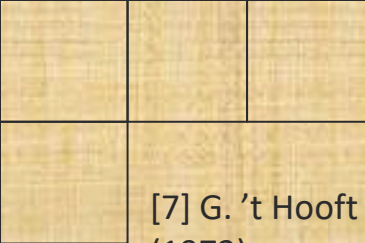
[2] J. G. Korner, A. Pilaftsis, and K. Schilcher, Leptonic CP asymmetries in flavor-changing H^0 decays, Phys. Rev. D **47**, 1080 (1993).

[3] P. S. B. Dev and A. Pilaftsis, Minimal radiative neutrino mass mechanism for inverse seesaw models, Phys. Rev. D **86**, 113001 (2012).

[4] W. A. Bardeen, R. Gastmans, and B. Lautrup, Static quantities in Weinberg's model of weak and electromagnetic interactions, Nucl. Phys. B **46**, 319 (1972).

[5] H. Baer et al. (ILC Collaboration), The international linear collider technical design report-Volume 2: Physics, arXiv:1306.6352.

[6] A. M. Sirunyan et al. (CMS Collaboration), Search for heavy neutral leptons in events with three charged leptons in proton-proton collisions at $\sqrt{s} = 13\text{TeV}$, Phys. Rev. Lett. **120**, 221801 (2018).



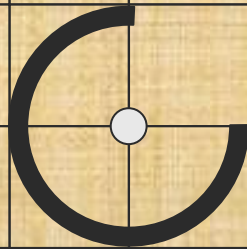
[7] G. 't Hooft and M. Veltman, Regularization and renormalization of gauge fields, Nucl. Phys. B **44**, 189 (1972).

[8] F. Jegerlehner, Facts of life with 5, Eur. Phys. J. C **18**, 67 (2001).

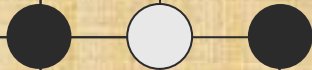
[9] G. Weiglein et al., Physics interplay of the LHC and the ILC, Phys. Rept. **426**, 47 (2006).

[10] L. Bian, J. Shu, and Y. Zhang, Prospects for triple gauge coupling measurements at future lepton colliders and the 14 TeV LHC, JHEP **09**, 206 (2015).





Thanks!



Properties

$$\mathcal{B}\mathcal{B}^\dagger = \mathbf{1}_3,$$

or

$$\sum_{j=1}^6 \mathcal{B}_{\alpha j} \mathcal{B}_{\beta j}^* = \delta_{\alpha\beta},$$

$$\mathcal{B}^\dagger \mathcal{B} = \mathcal{C},$$

or

$$\sum_{\alpha=e,\mu,\tau} \mathcal{B}_{\alpha j}^* \mathcal{B}_{\alpha k} = \mathcal{C}_{jk}$$

$$\mathcal{C}\mathcal{C}^\dagger = \mathcal{C},$$

or

$$\sum_{i=1}^6 \mathcal{C}_{ji} \mathcal{C}_{ki}^* = \mathcal{C}_{jk}$$