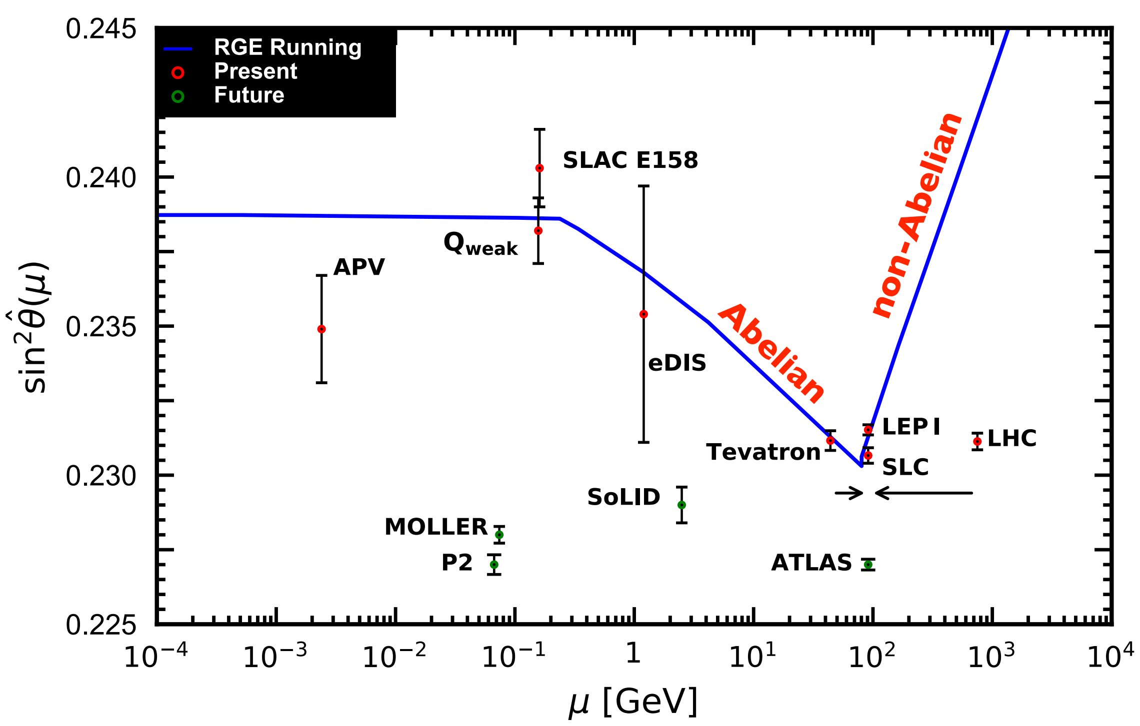
Precision Tests of the SM Running of the Weak Mixing Angle at Low Energy

Misha Gorshteyn
Universität Mainz



Collaborators:
Chuck Horowitz
Jens Erler
Rodolfo Ferro Hernandez
Michael Ramsey-Musolf
Hubert Spiesberger
Nicola Cargioli
Matteo Cadeddu
Jorge Piekarewicz
Xavi Roca Maza

XVth SILAFAE, November 4-8, 2024, CDMX Mexico

Outline

Running weak mixing angle in the Standard Model

Sensitivity to New Physics

Weak mixing angle from PVES

Energy-dependent γZ-box

Summary & Outlook

Weak mixing angle in SM at tree-level and beyond

The E = mc² of the electroweak model

$$\sin^2\theta_W = \frac{g'^2}{g^2+g'^2} = 1 - \frac{M_W^2}{M_Z^2} = \frac{\pi\alpha}{\sqrt{2}G_F M_W^2}$$

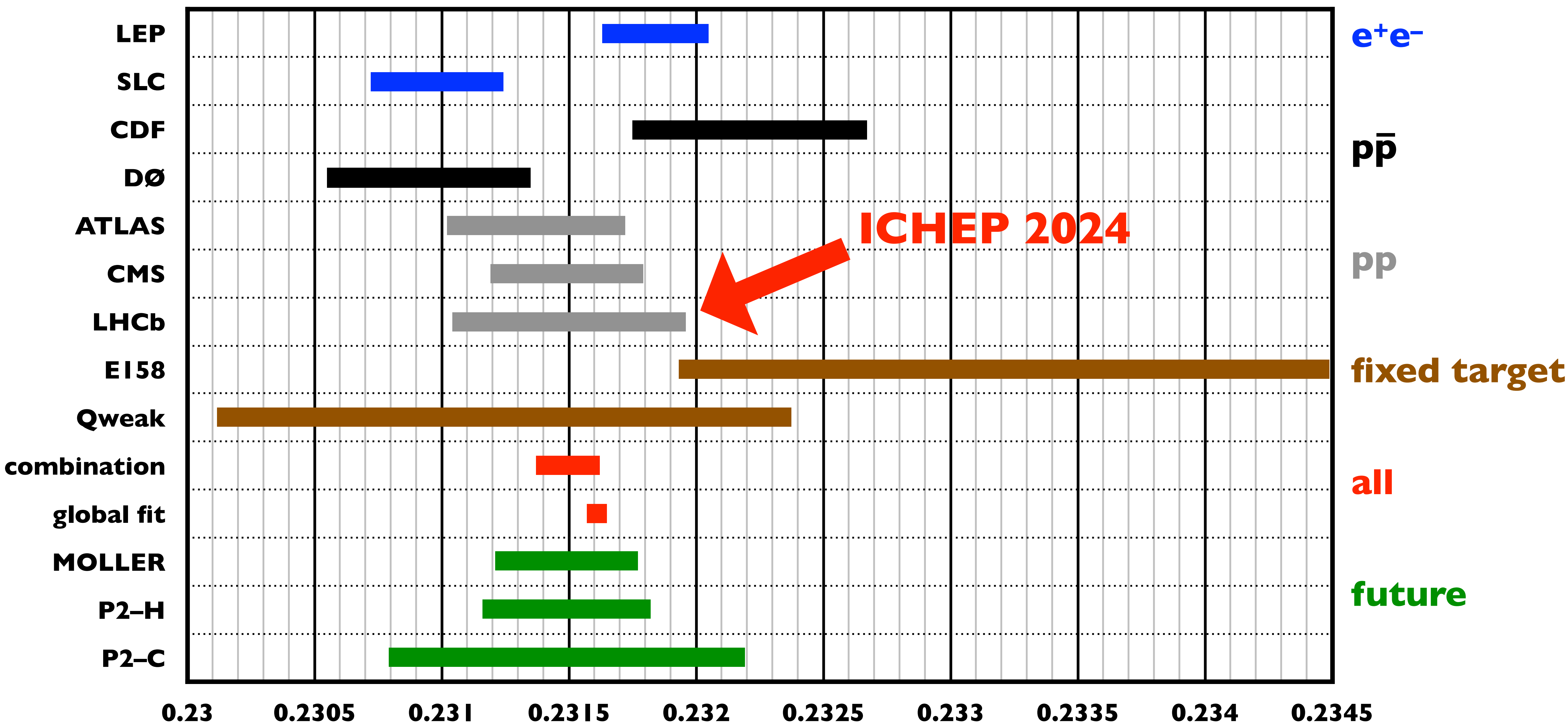
Is modified in presence of radiative corrections

$$\propto \frac{\alpha m_t^2}{M_W^2}$$

$$\Delta\hat{\alpha} + \ldots$$

$$\frac{\sin^2\theta^e_{\text{eff}}}{1+\Delta\hat{k}} = \frac{\hat{g}'^2}{\hat{g}^2+\hat{g}'^2} = 1 - \frac{(1-\Delta\hat{\rho})M_W^2}{M_Z^2} = \frac{\pi\alpha}{(1-\Delta\hat{r})\sqrt{2}G_F M_W^2}$$

$\sin^2 \theta^{\ell}_{eff}$ anno 2024

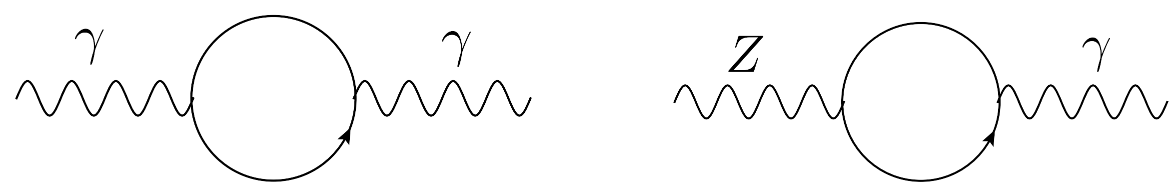


Running $\sin^2\hat{\theta}(\mu)$

MS-bar definition: $\sin^2\hat{\theta} = g'^2/(g^2+g'^2)$

Erler, Ramsey-Musolf, hep-ph/0409169
Erler, Ferro Hernandez, arXiv:1712.09146

Running of WMA with respect to running of α



RG equation for em and weak vector coupling very similar

$$\mu^2\frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi}\left[\frac{1}{24}\sum_i K_i\gamma_i Q_i^2 + \sigma\left(\sum_q Q_q\right)^2\right]$$

$$\mu^2\frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha}Q_f}{24\pi}\left[\sum_i K_i\gamma_i\hat{v}_i Q_i + 12\sigma\left(\sum_q Q_q\right)\left(\sum_q \hat{v}_q\right)\right]$$

$$\hat{v}_f = T_f - 2Q_f\sin^2\hat{\theta}_W$$

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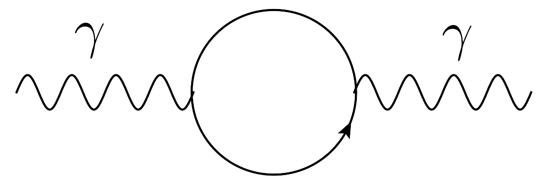

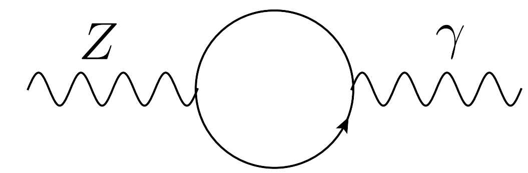

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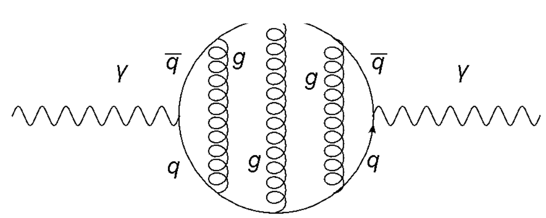
Connected contributions

Q_i, v_i - el. and weak charges

γ_i - field-dependent constants

K_i - h.o. coefficients



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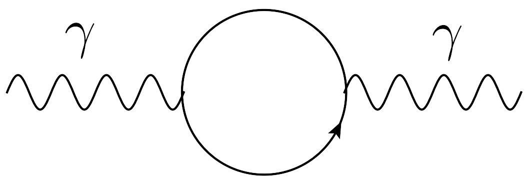

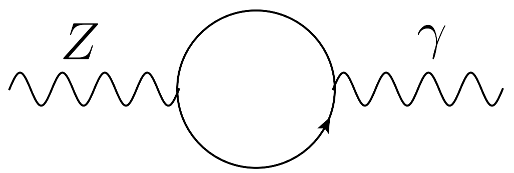

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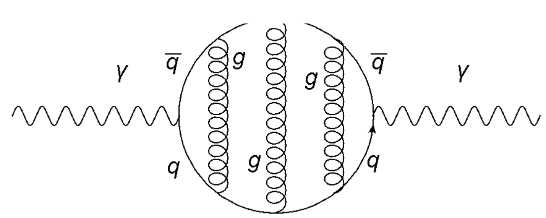
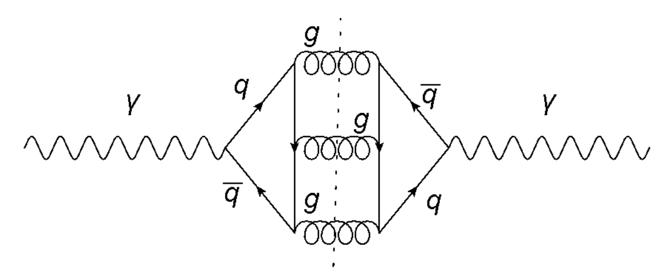


Disconnected contributions

σ - h.o. coefficients



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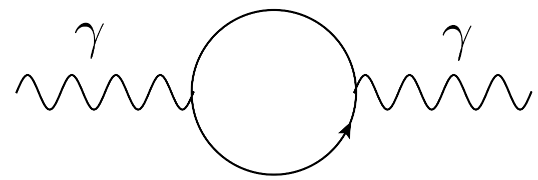

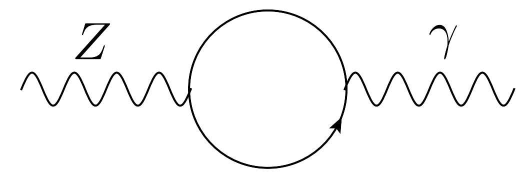

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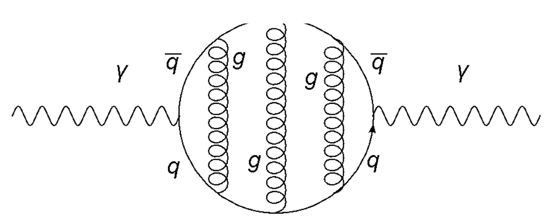

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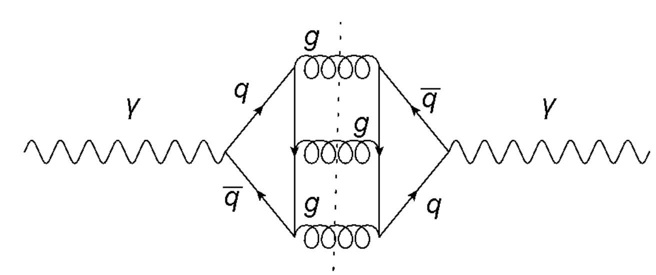
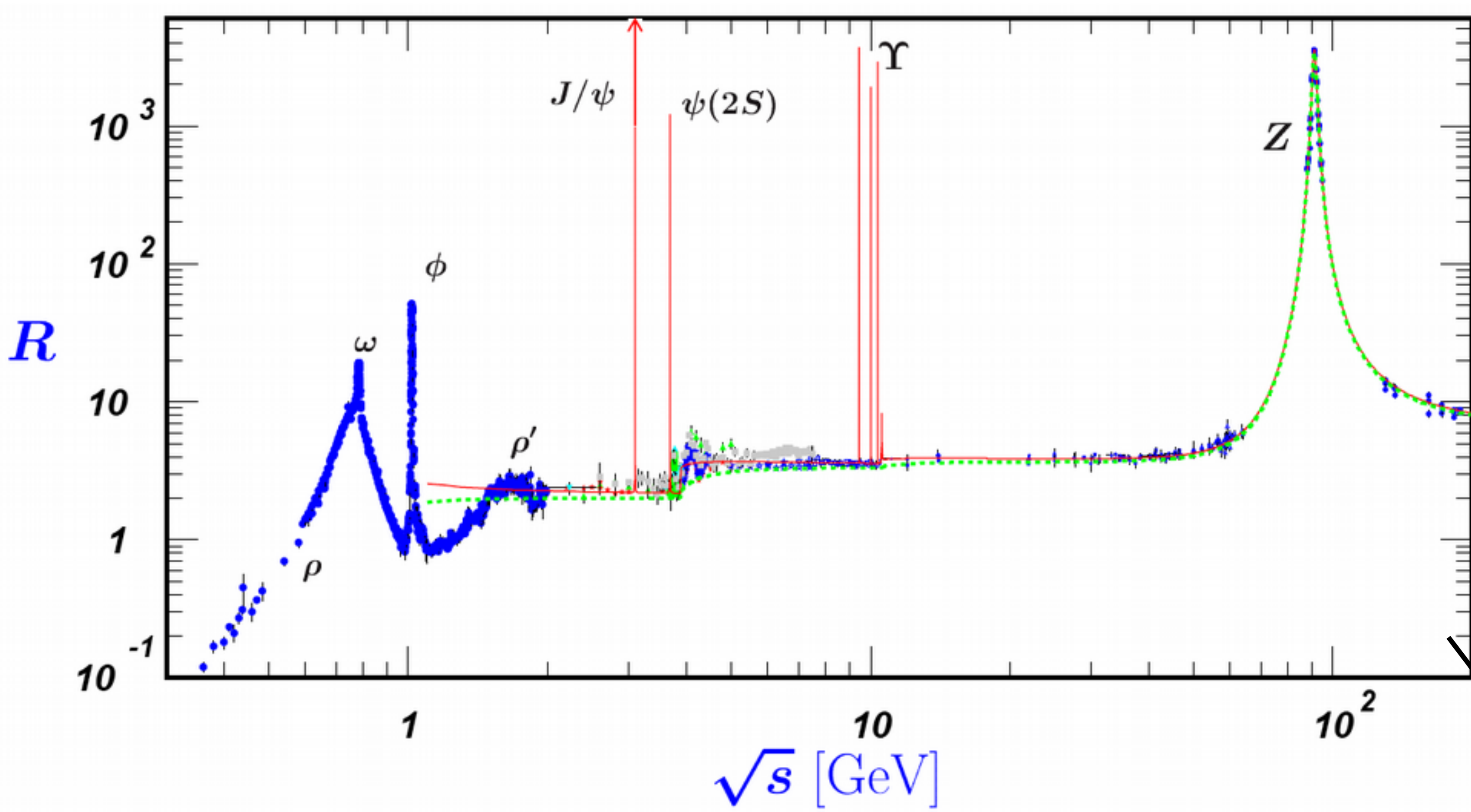


Run from Z-pole down: integrate heavy d.o.f. step by step, match at threshold

Running $\sin^2\theta$

By the time one gets down to low scale QCD is non-perturbative - use experimental input + dispersion relation

Use exp. known R(s)= $\sigma(e^+e^- \to \text{hadrons}) / \sigma(e^+e^- \to \mu^+\mu^-)$



Talk by Aida this morning in the context of muon g-2

At scale μ_0 = 2GeV: use pQCD input

$$\Delta\hat{\alpha}^{(3)}(\mu_0) = \frac{\alpha}{3\pi}\int_{4m_\pi^2}^{\mu_0^2} ds \frac{R(s)}{s - i\epsilon} + 4\pi I^{(3)}$$

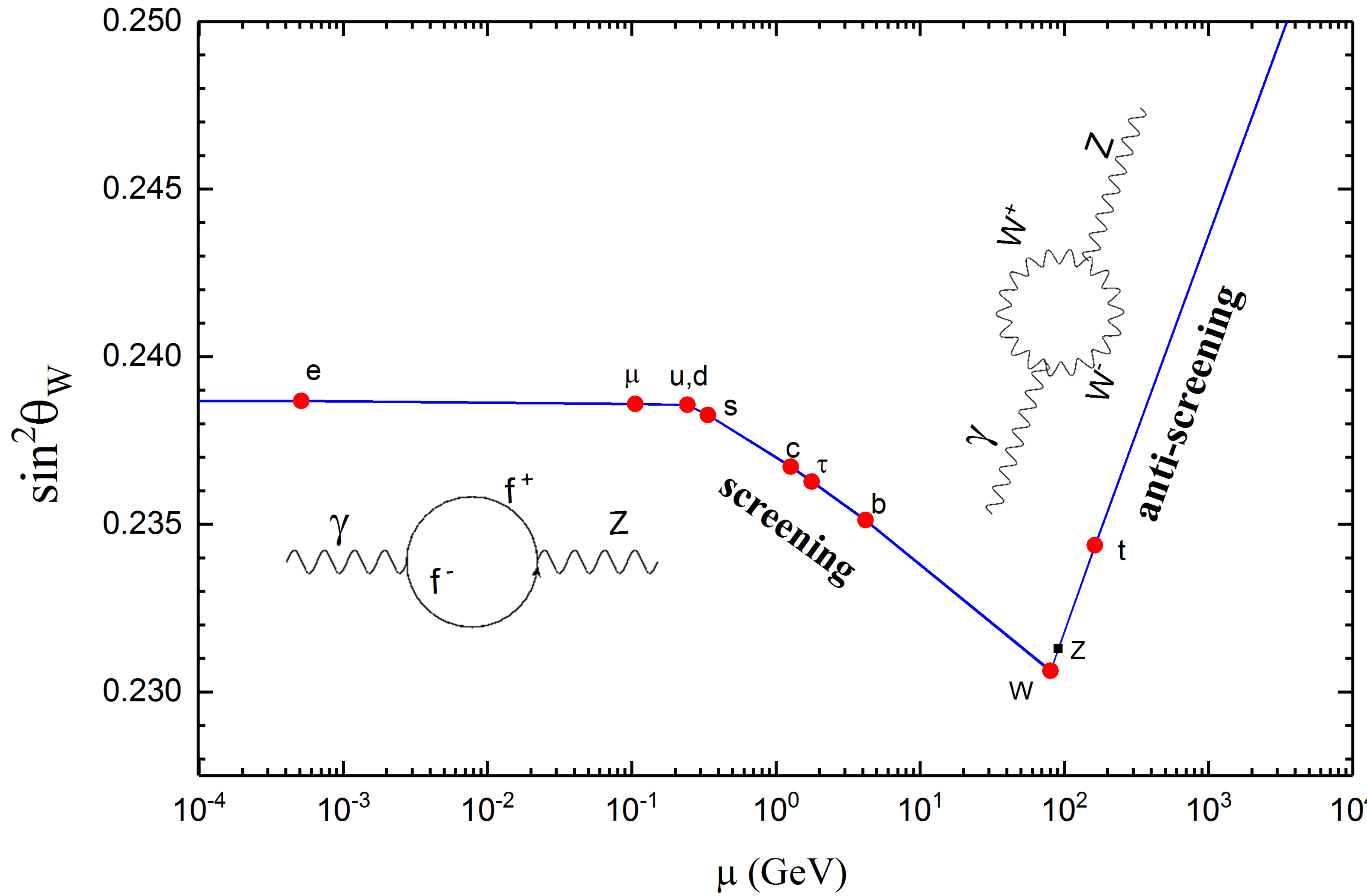
Final step - flavor rotate R to get Z coupling from e.-m. coupling

Running $\sin^2\theta$

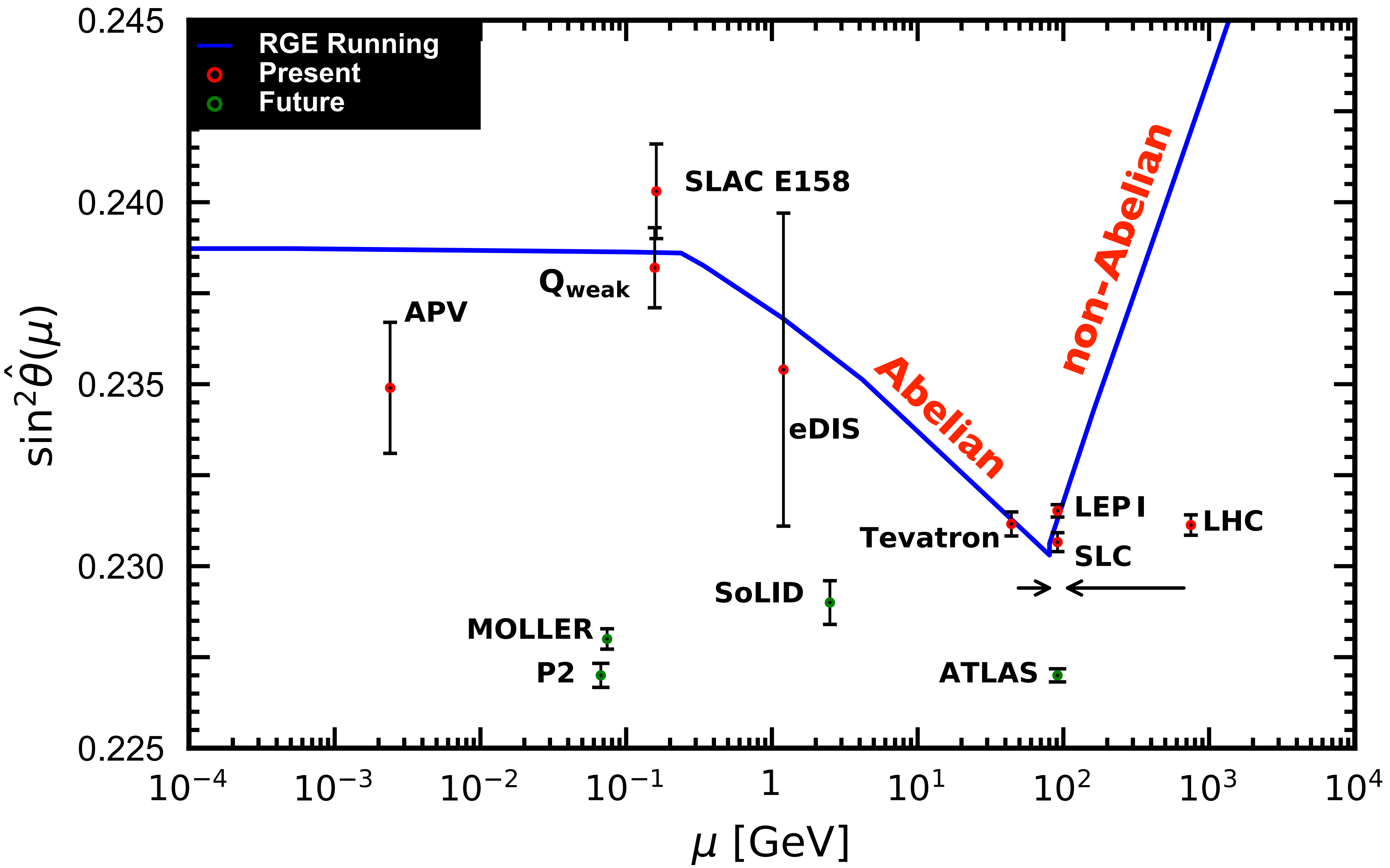
SM prediction for low energy:

$$\sin^2\hat{\theta}(0) = 0.23868(5)(2)$$

Erler, Ferro Hernandez, arXiv:1712.09146



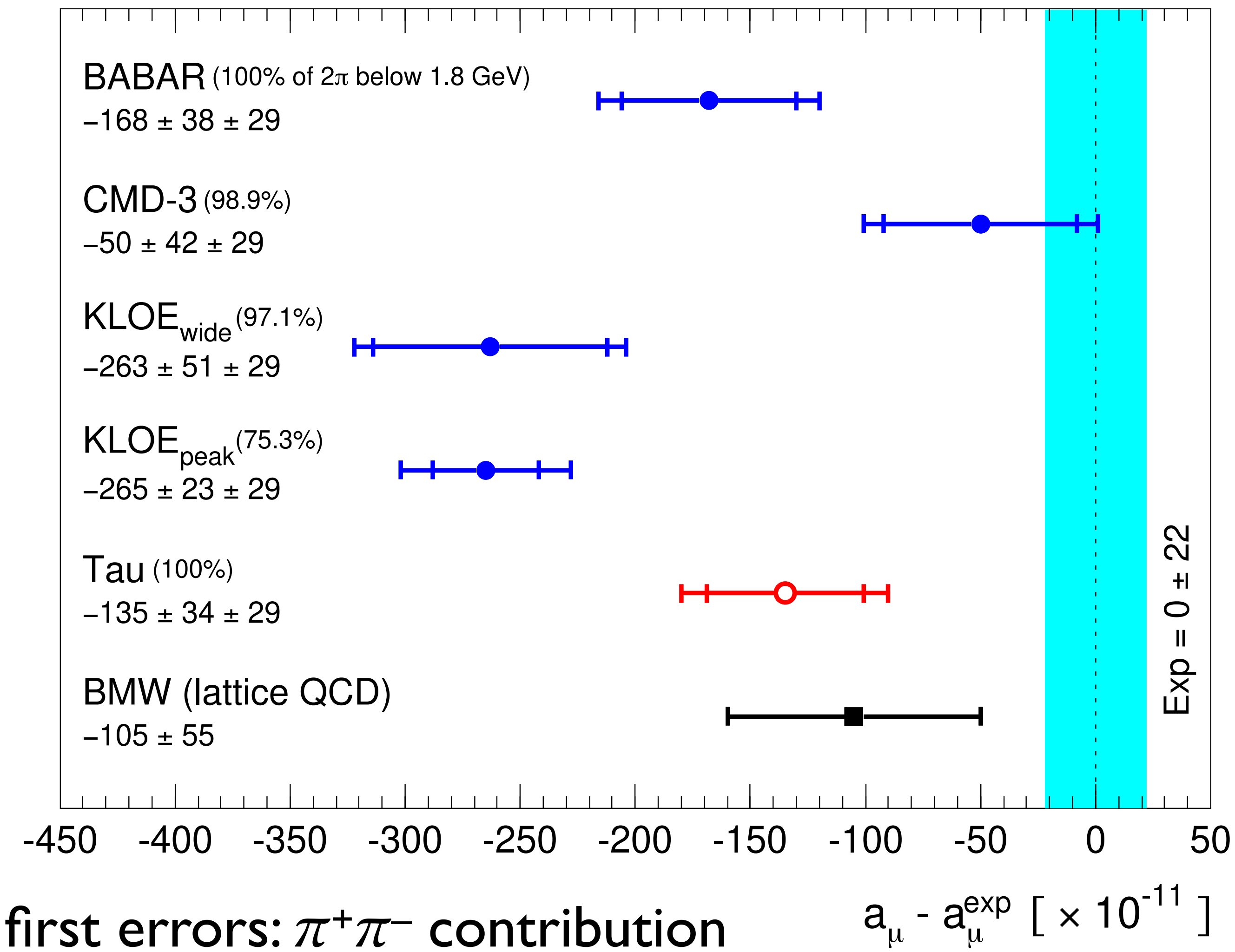
Experimental tests of running $\sin^2\hat{\theta}(\mu)$



updated from
Ferro-Hernández & JE
arXiv:1712.09146

HVP: LQCD vs Data for a_μ

Talk by Aida this morning in the context of muon g-2: data-driven vs. LQCD HVP



first errors: $\pi^+\pi^-$ contribution

BaBar and earlier data based on
Davier et al. arXiv:1908.00921

CMD–3 and figure from
Davier et al., arXiv:2312.02053

KLOE based on
Davier et al. arXiv:1908.00921

after isospin rotation according to
Davier et al., arXiv:2312.02053

Borsanyi et al., arXiv:2002.12347

Δα from Cè et al., arXiv:2203.08676
also enters through correlations

HVP: LQCD vs Data for a_μ and $\sin^2\hat\theta(0)$

LQCD - Data discrepancy also seen for WMA

Lattice HVP with flavor separation

M. Ce et al, arXiv 2203.08676

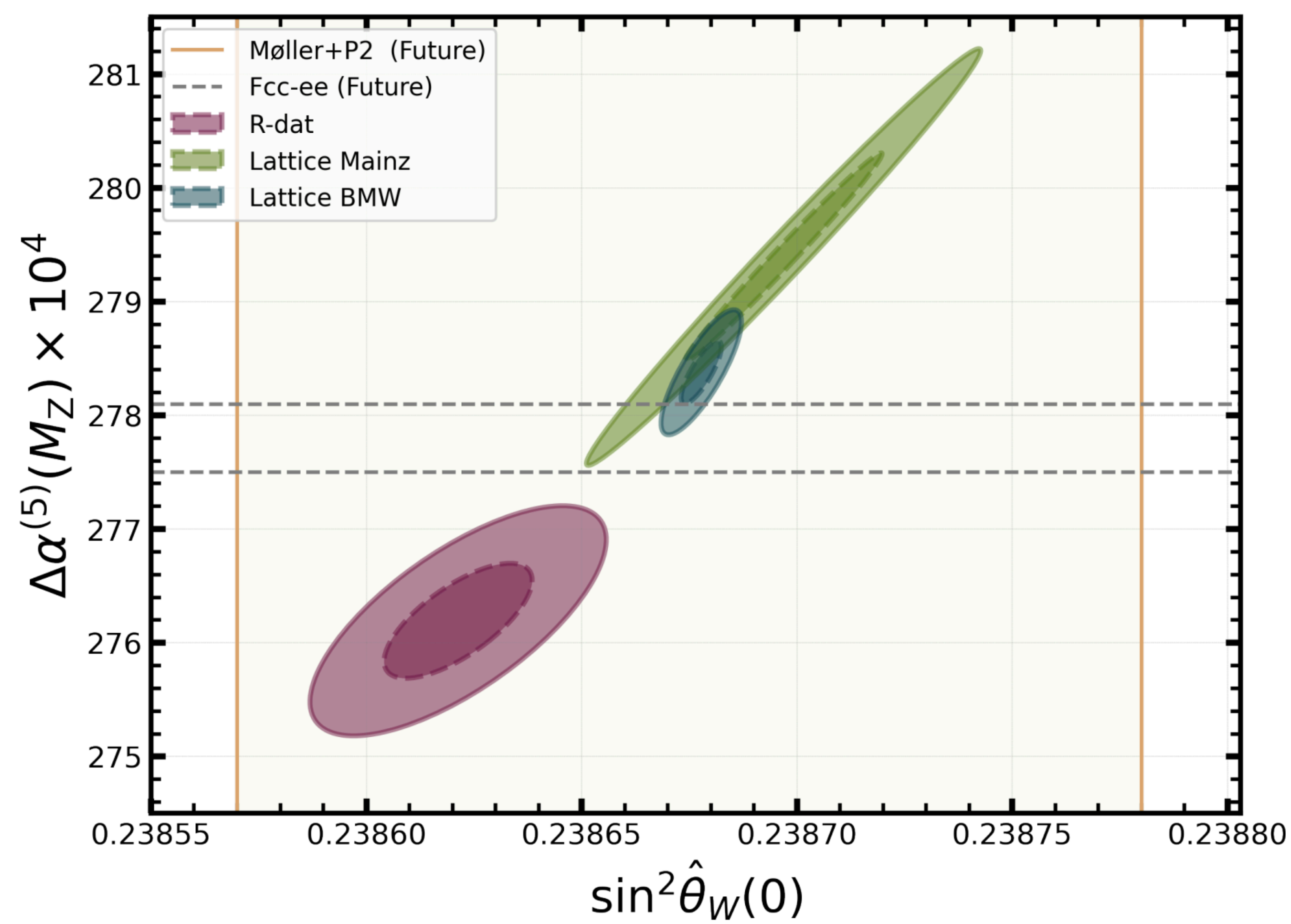
parameter	result $\times 10^4$	correlations		
Π_{disc}	-3.8 ± 0.2	1.0	0.8	0.8
Π_s	83.0 ± 1.4	0.8	1.0	0.96
Π_{ud}	587.8 ± 8.3	0.8	0.96	1.0

Defining $\hat{s}^2(0) = \hat\kappa(0)\ \sin^2\hat\theta(M_Z)$

$$\hat\kappa(0)_{\text{lat}} - \hat\kappa(0)_{e^+e^-} = (3.3 \pm 1.3) \times 10^{-4}$$

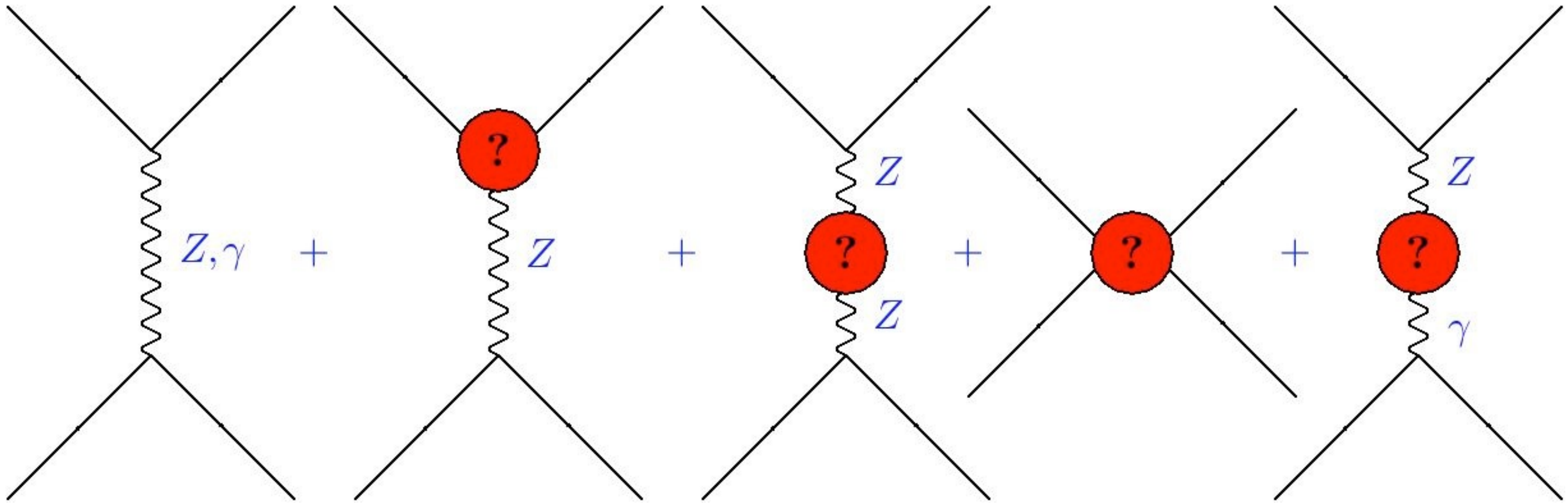
Erler, Ferro Hernandez, Kuberski, arXiv 2406.16691

Precision of future PVES experiments not enough to resolve this discrepancy



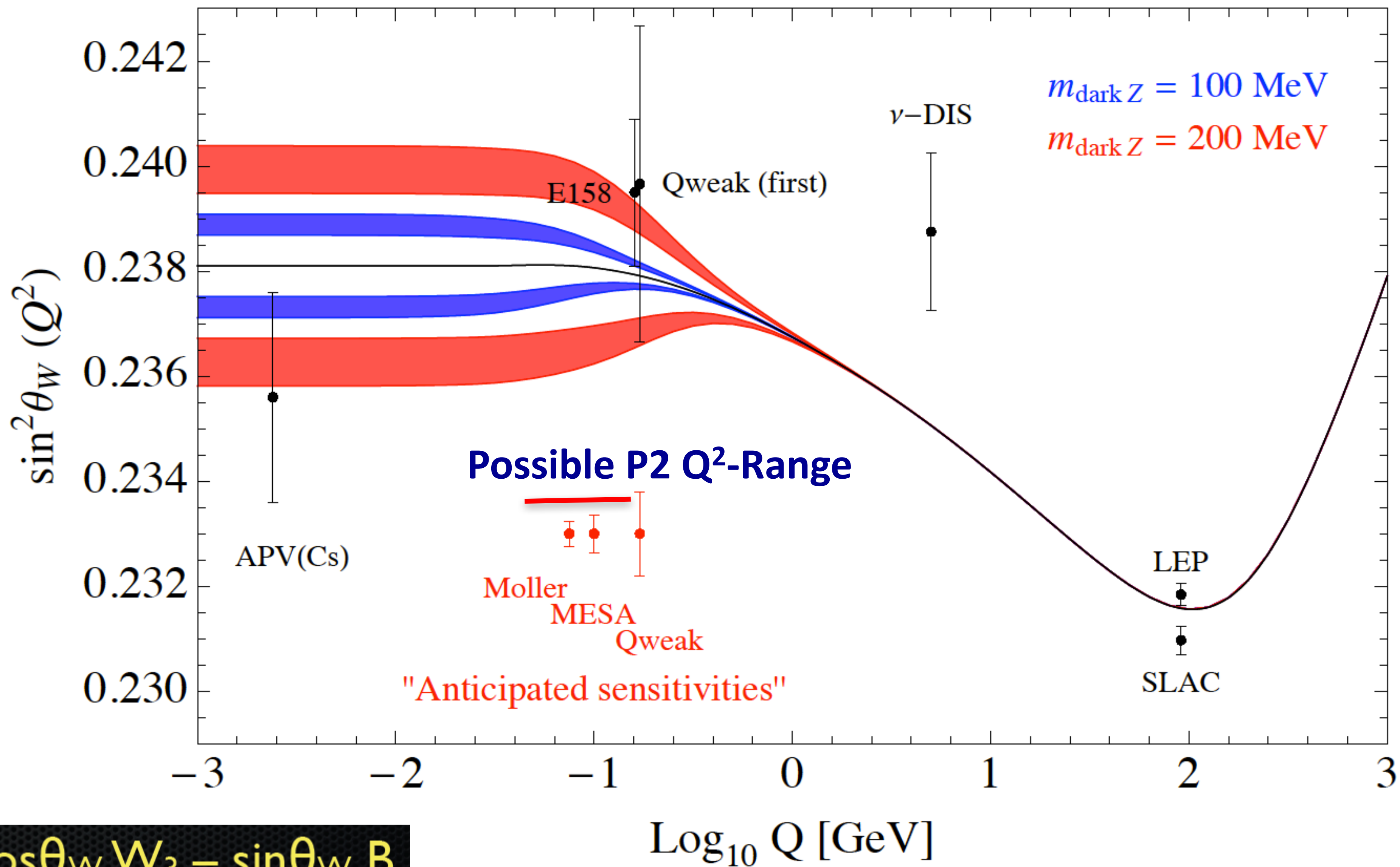
Sensitivity to New Physics

Running $\sin^2 \hat{\theta}(\mu)$ and New Physics



* Z-Z′ mixing: modification of Z vector coupling
* oblique parameters: STU (also need M_W and Γ_Z)
* new amplitudes: off- versus on-Z pole measurements (e.g. heavy Z′)
* dark Z: renormalization group evolution (low versus very low energy measurements)

Running $\sin^2\hat{\theta}(\mu)$ and Dark Parity Violation



$Z = \cos\theta_W W_3 - \sin\theta_W B$

$A = \sin\theta_W W_3 + \cos\theta_W B$

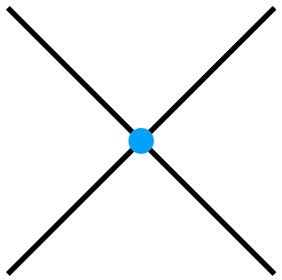
Bill Marciano

Heavy BSM reach of modern low-energy experiments: up to 49 TeV

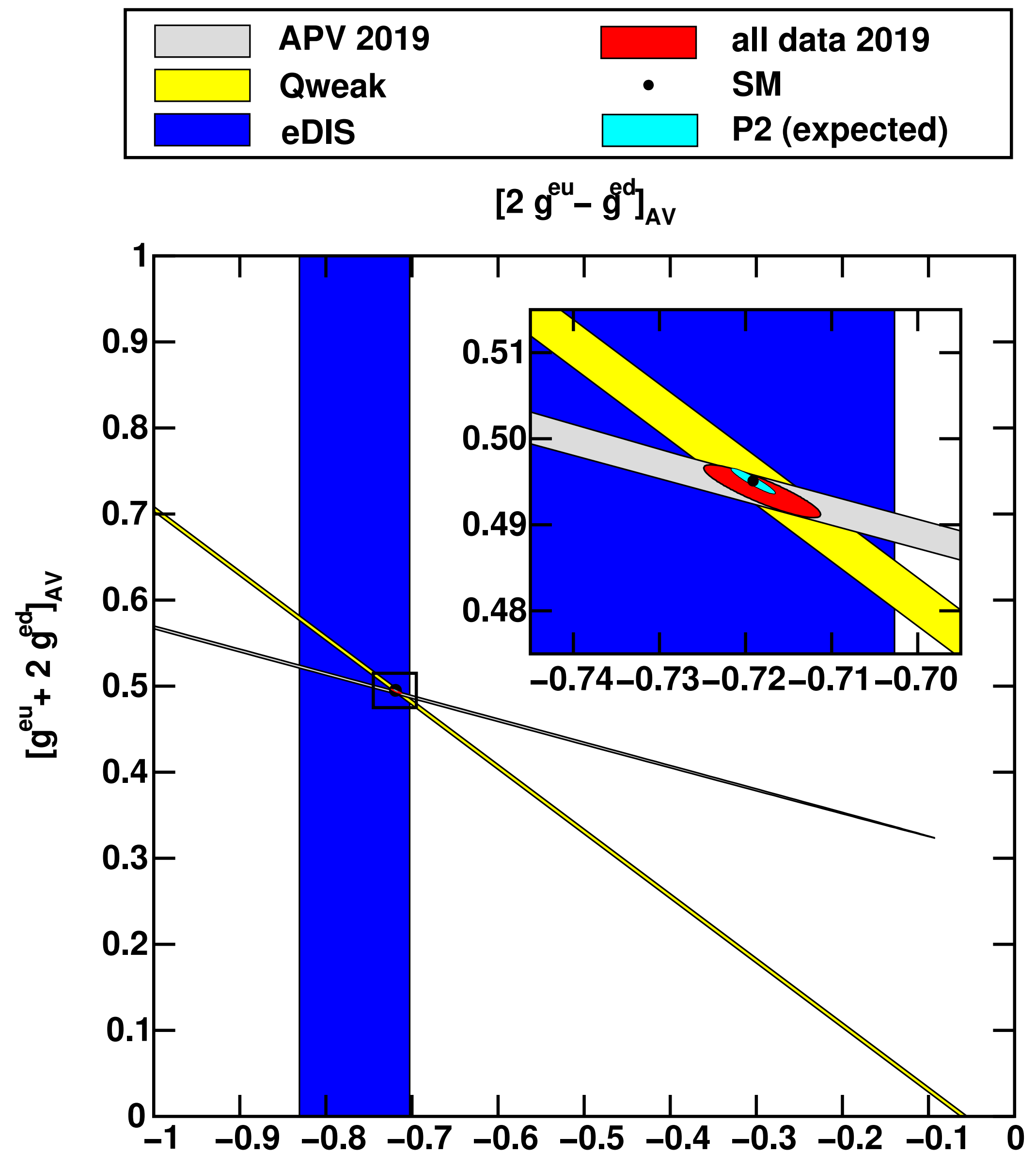
Sensitivity to light dark gauge sector - complementary to colliders

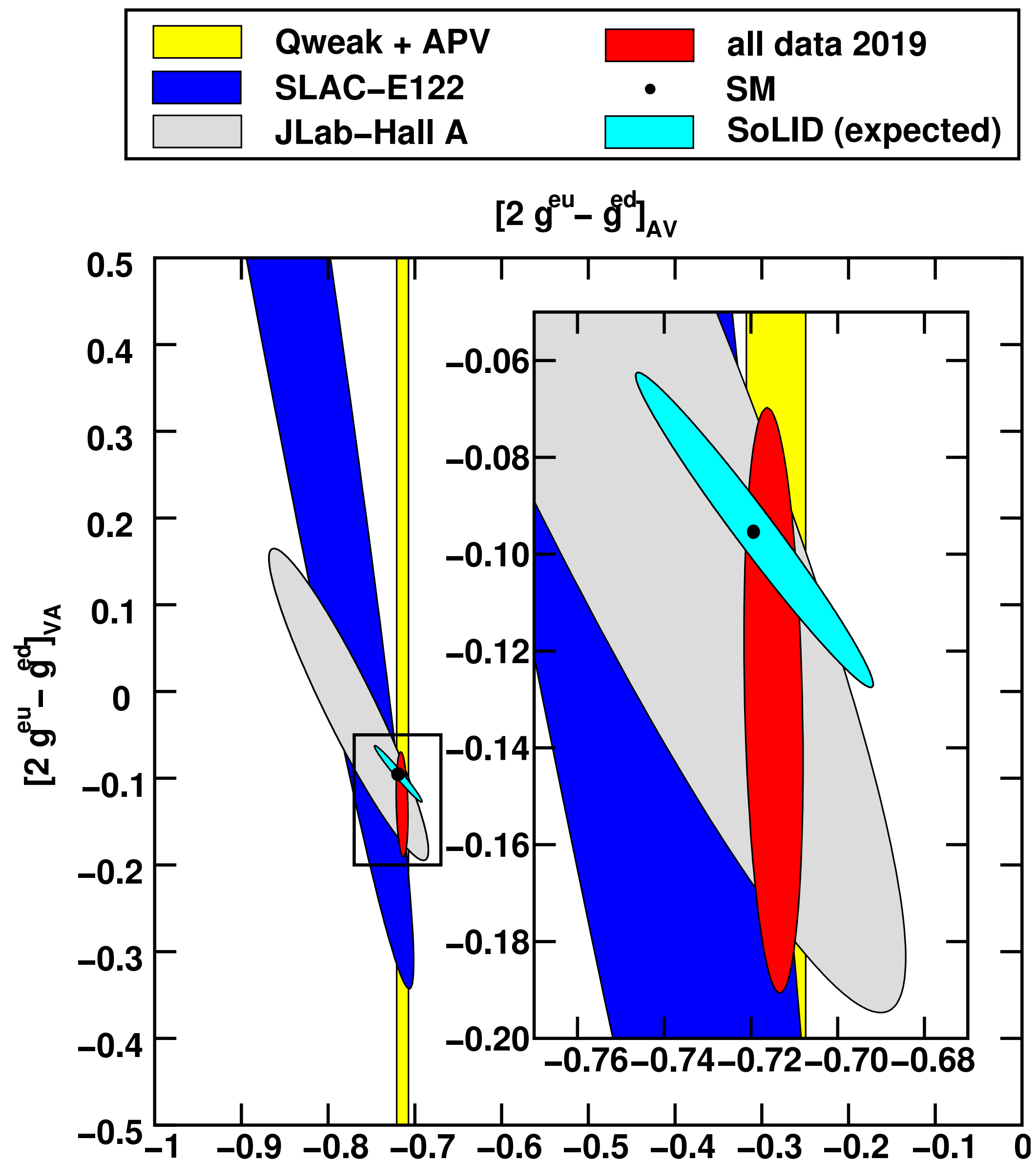
Talk by Mayda Velasco on Wed

Parity-Violating 4-fermion e-q couplings

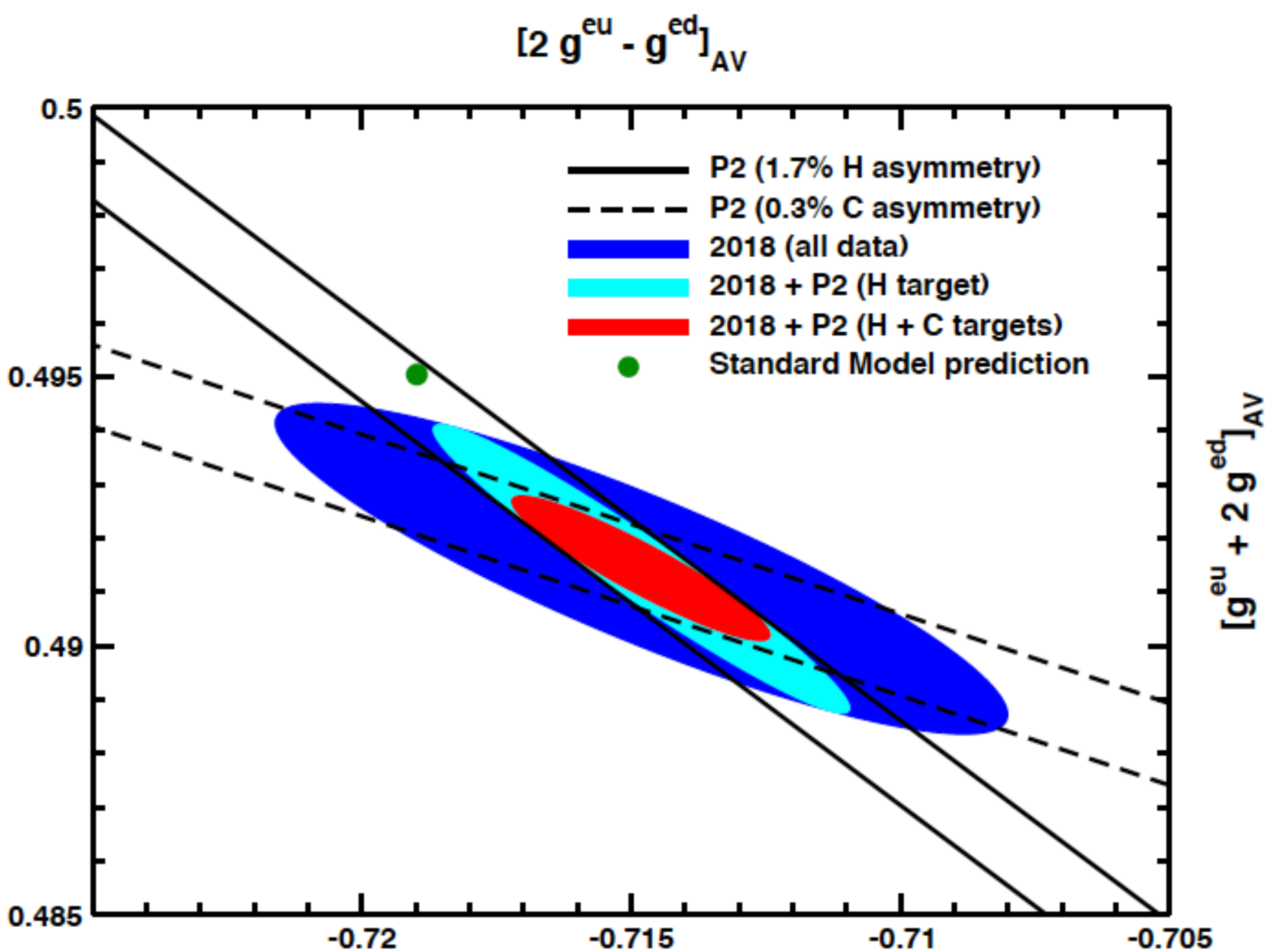


$$\propto \frac{G_F}{\sqrt{2}} \left[g_{AV}^{eq}\, \bar{e}\gamma_\mu\gamma_5 e\, \bar{q}\gamma^\mu q + g_{VA}^{eq}\, \bar{e}\gamma_\mu e\, \bar{q}\gamma^\mu\gamma_5 q \right]$$





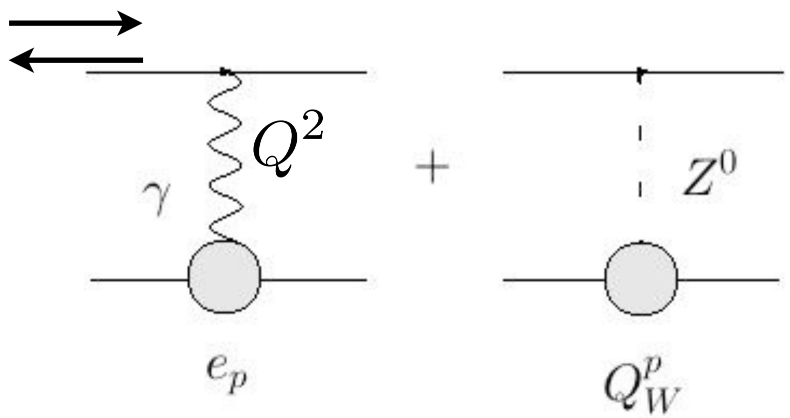
PV 4-fermion e-q couplings (zoom to Mainz)



- Quark-vector-electron-axial vector couplings
- Sensitivity down to masses of 70 MeV and up to masses of 50 TeV

Weak Charges and Weak Mixing Angle from PVES

Parity-Violating Electron Scattering (PVES)



$$\sigma_{\rm TOT} \propto |\mathcal{A}_{\rm EM} + \mathcal{A}_{\rm wk}|^2 \propto |\mathcal{A}_{\rm EM}|^2 + \underbrace{|\mathcal{A}_{\rm wk}|^2}_{\text{negligible}} + 2|\mathcal{A}_{\rm EM}|\cdot|\mathcal{A}_{\rm wk}|$$

$$A^{\rm PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[Q_W^p + Q^2 B(Q^2)\right]$$

Weak charge from PVES on proton:

$$Q_W^p = \lim_{Q^2\to 0}\left[-\frac{4\sqrt{2}\pi\alpha}{G_F Q^2}A^{exp}\right]$$

In SM at tree-level: accidentally suppressed

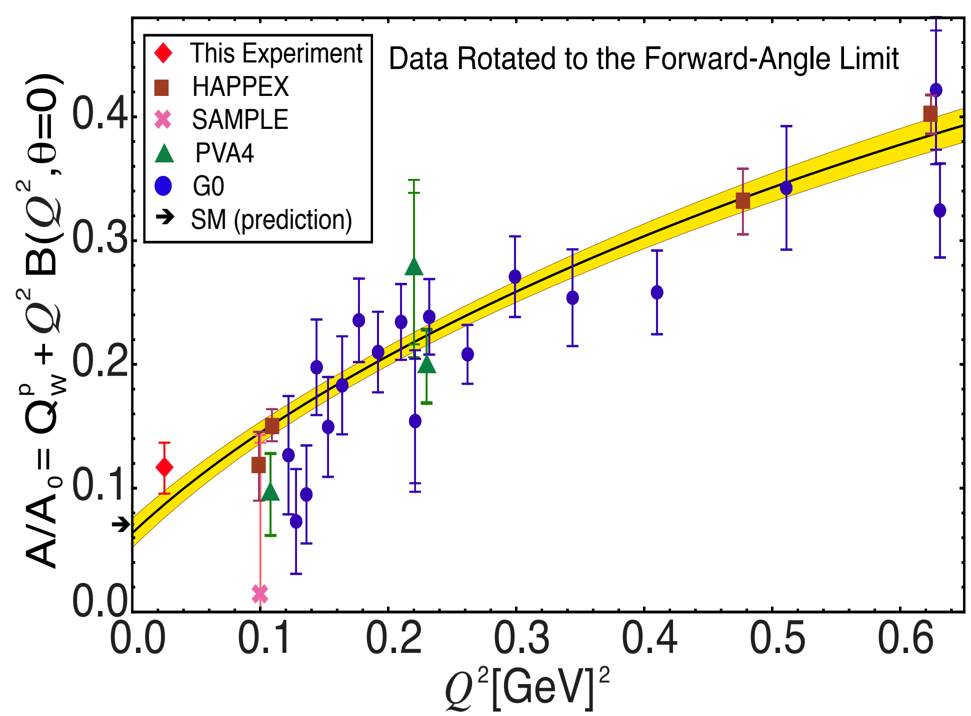
$$Q_W^{p,\,\rm tree} = 1 - 4\sin^2\theta_W \approx 0.07$$

A sensitive test of running of θ_W at low energy:

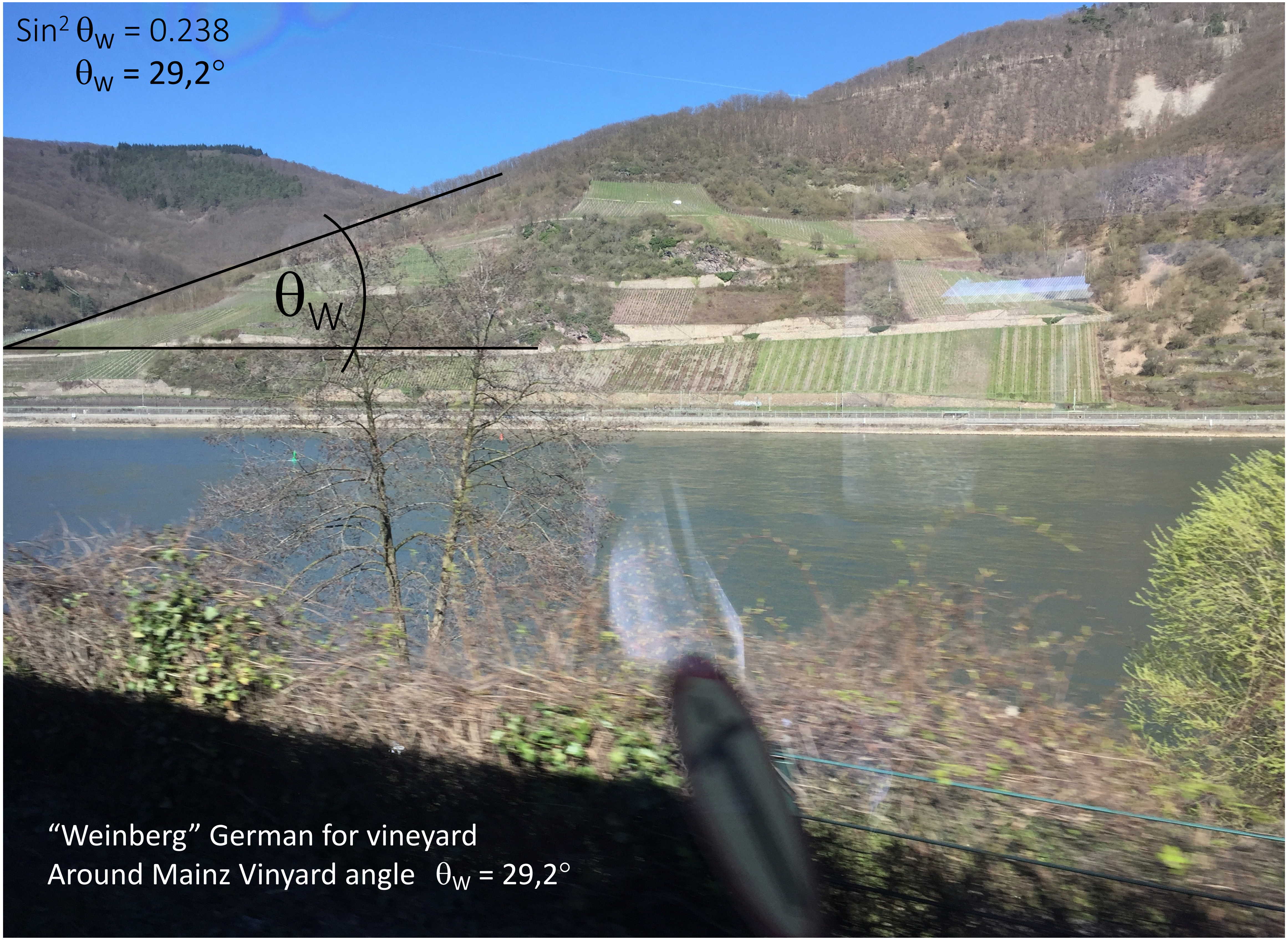
2% measurement of Q_W -> 0.14% on $\sin^2\theta_W$

B(Q^2) - from non-forward PVES data

Young et al. '07;
Androic et al. [Qweak Coll.], '13



Weinberg angle near Mainz

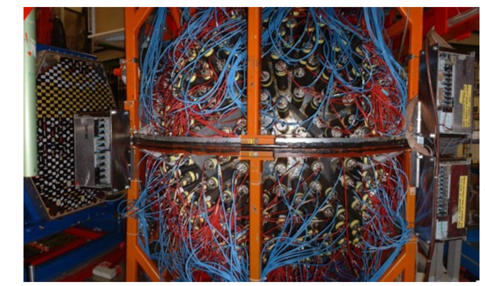


MAMI and MESA

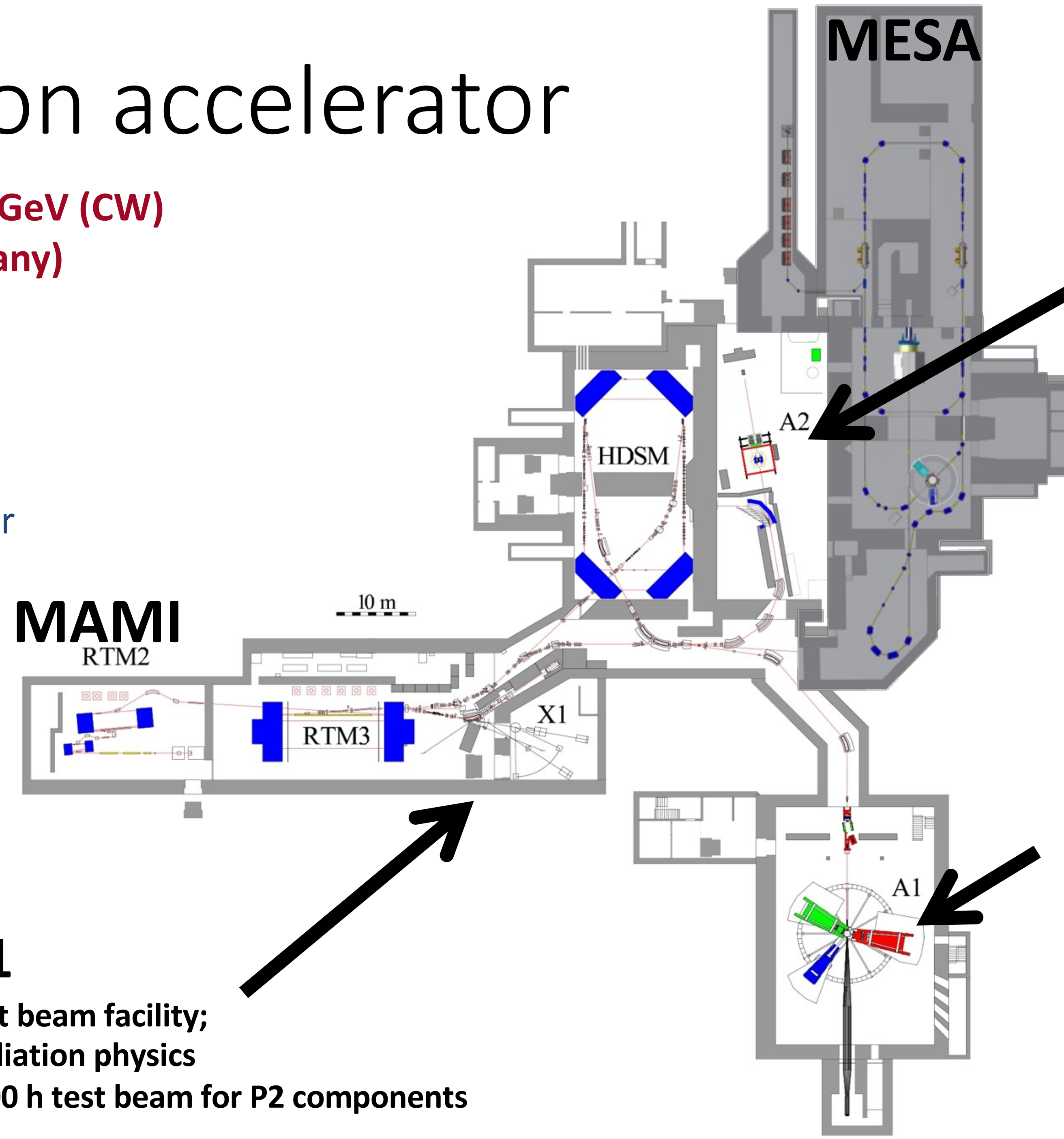
MAMI Electron accelerator

Electron Accelerator E_{max} =1.6 GeV (CW)
operated at JGU Mainz (Germany)
Hallmarks

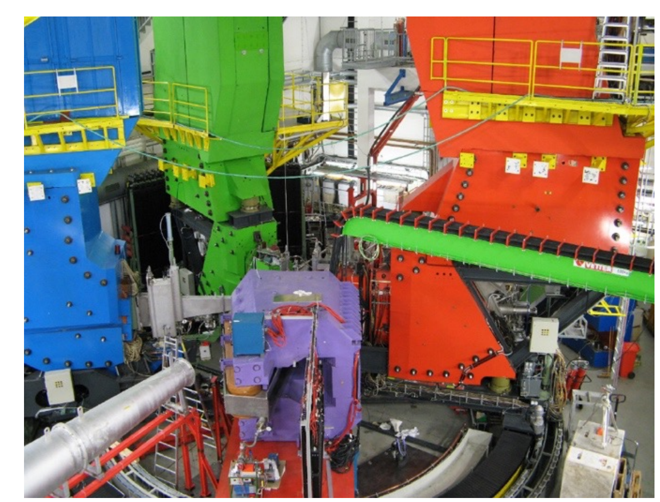
- Intensity max. 100 μA
- Resolution σ_E < 0.100 MeV
- Polarization 85%
- Reliability: up to 7000 h / year



Tagged Photon Scattering (A2 hall)
Crystal Ball / TAPS calorimeters;
Polarized frozen-spin target



Electron scattering
(A1 hall)
High resolution
Magnetic spectrometers

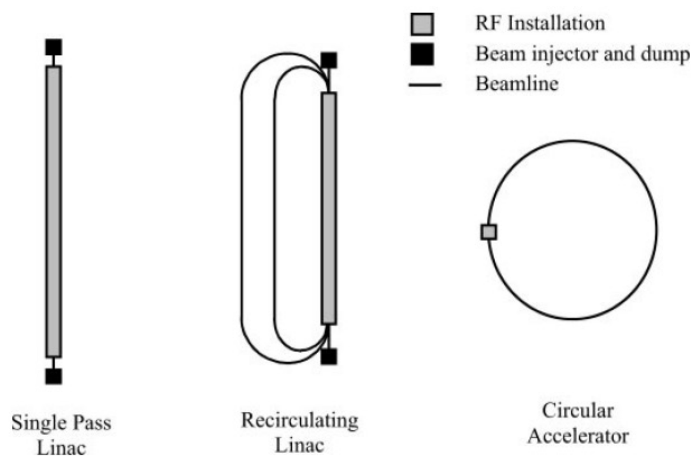


X1

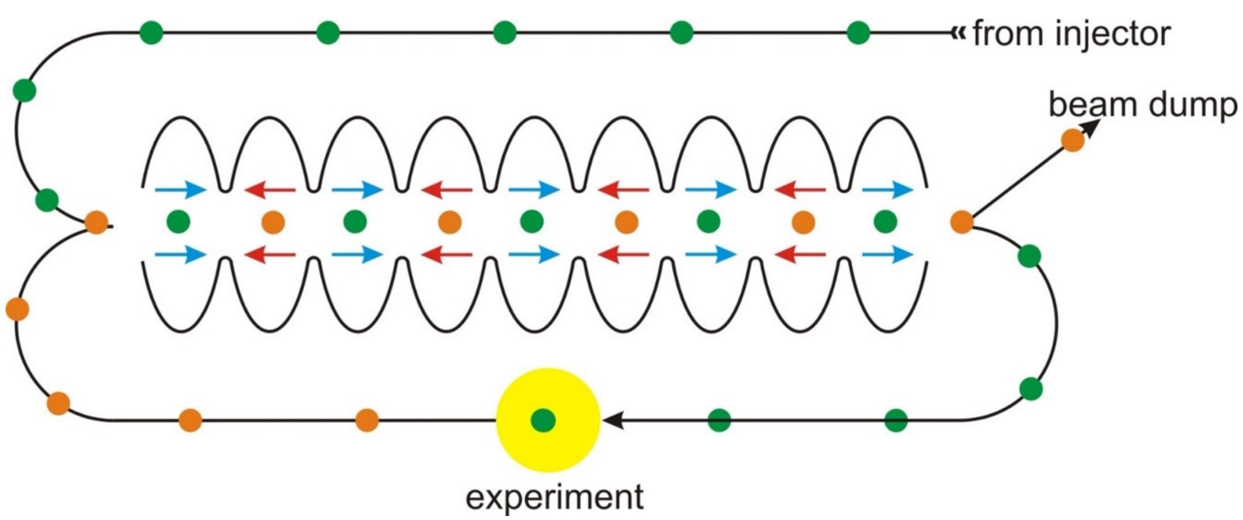
Test beam facility;
Radiation physics
2000 h test beam for P2 components

MESA:
Mainz Energy-recovering Superconducting Accelerator

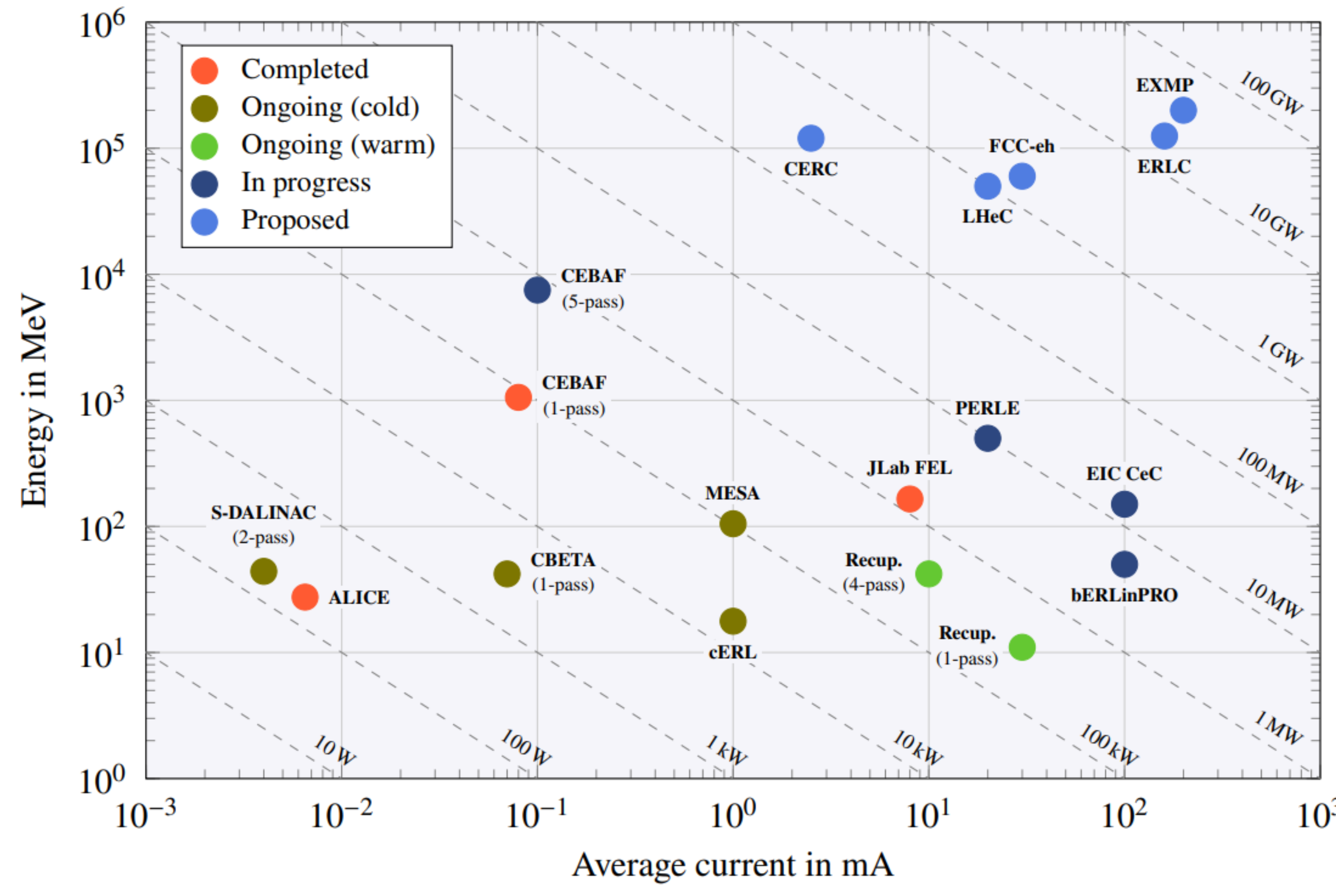
MESA Accelerator



Energy Recovering LINAC



ERLs world-wide (status fall 2022)



→ MESA is one of few ongoing ERL activities

→ The **first** ERL facility with a target in the beam for physics experiments

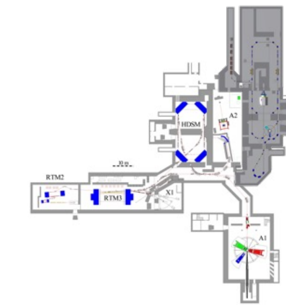
MESA: Parameters & Progress

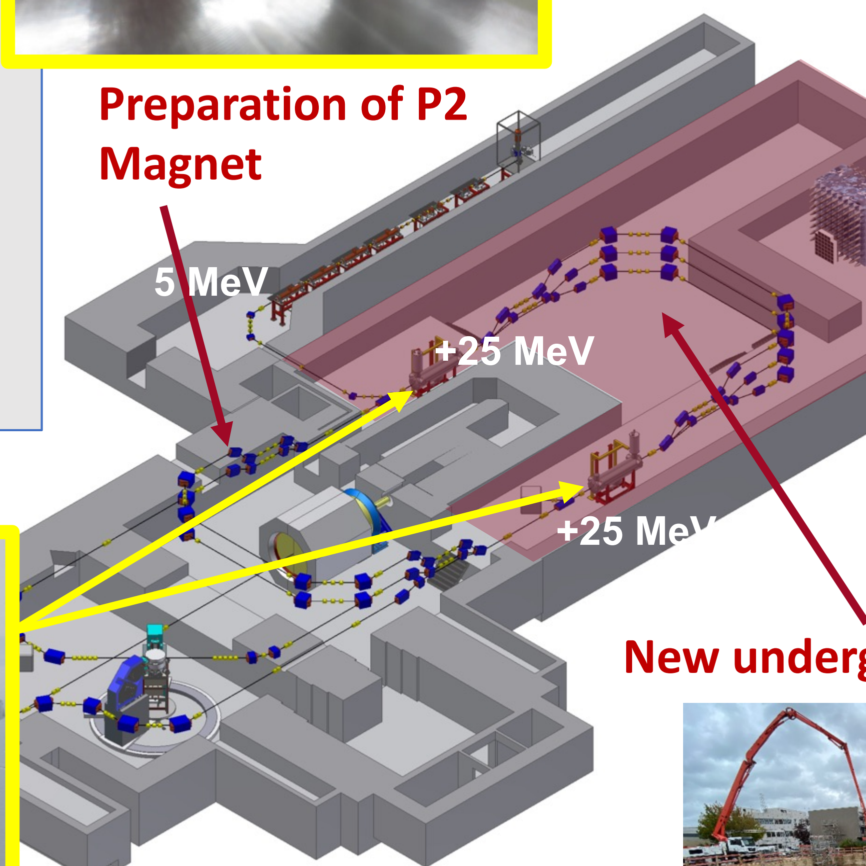
Mesa accelerator

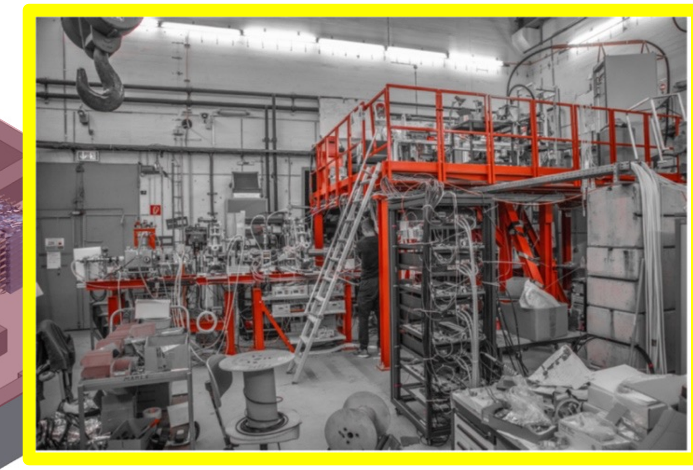
Key parameters MESA:

- Two operation modes: extracted beam (EB) or energy recovering (ERL)
- Max. beam energy 155 MeV (EB), 105 MeV (ERL)
- Beam current 150 µA (EB), 1 mA (ERL)
- Superconducting cavities
- Start commissioning 2024
- New research building (par. 91b GG)
- Can run in parallel to MAMI

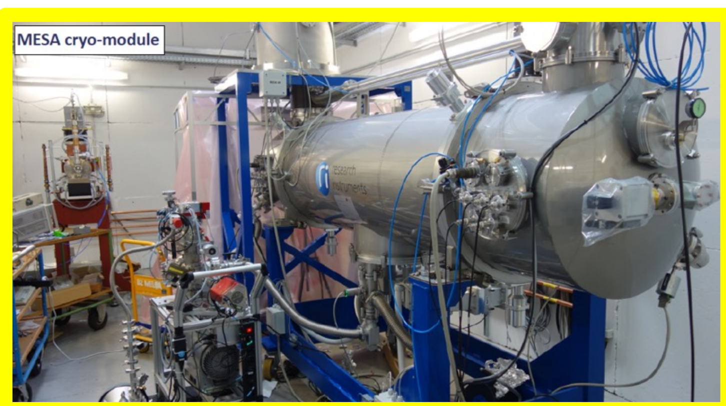
Preparation of P2 Magnet







Polarized Source Test Setup

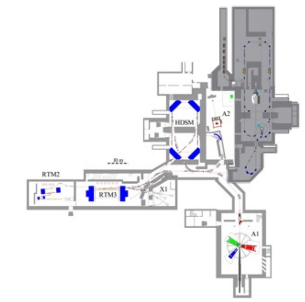


Cryomodules successfully tested

New underground experimental hall (par. 91b GG)

MESA physics

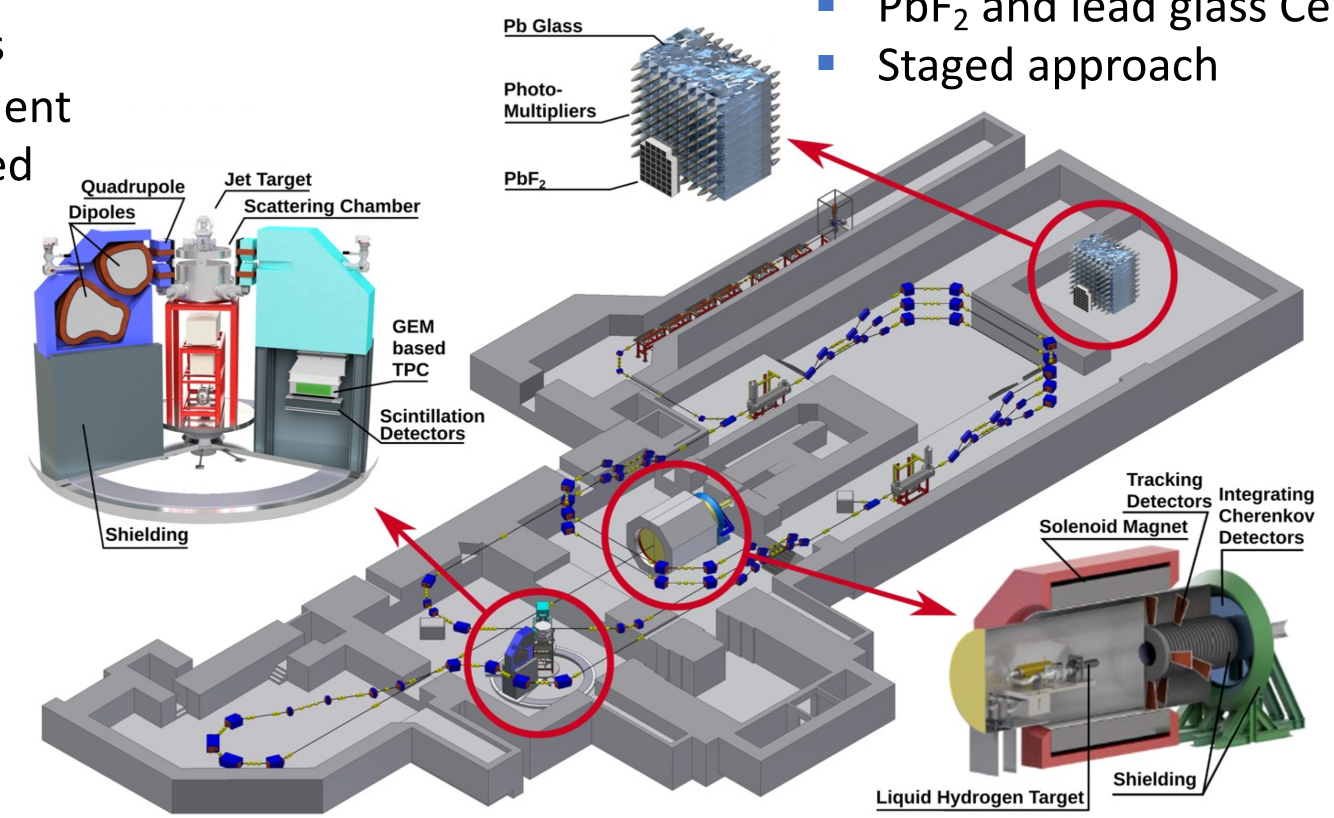
MESA experiments



DarkMESA
- Beam dump experiment
- Direct detection of light dark matter
- PbF_2 and lead glass Cerenkov calorimeter
- Staged approach

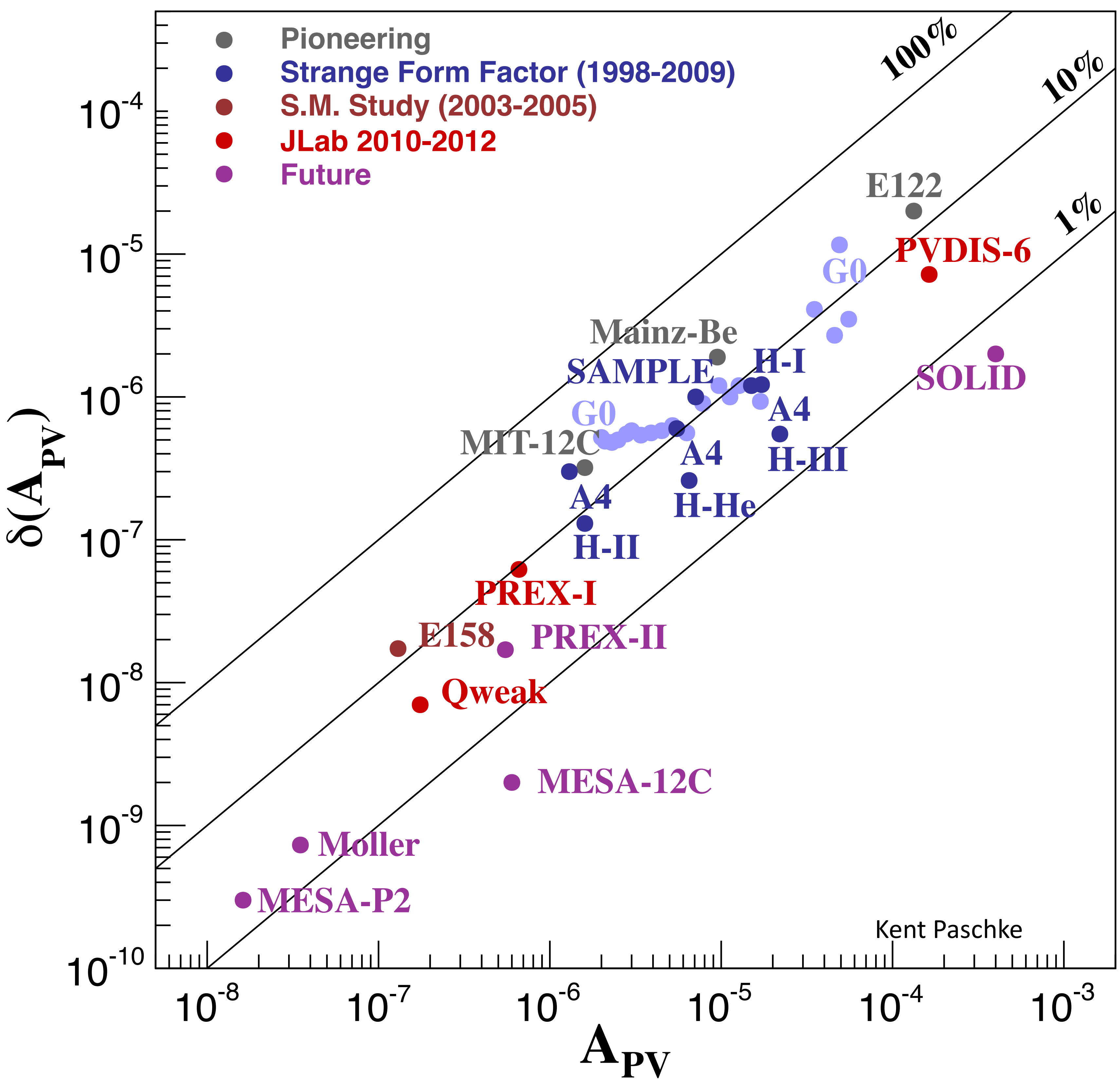
MAGIX experiment
- Operated in ERL mode of MESA
- Double-arm spectrometers
- Internal gas target experiment
- Gas jet target commissioned at A1/MAMI already

Main components of MAGIX and P2 presently constructed in industry and assembled in house (funding via major research instrumentation program of federal government)



P2
- Extracted beam mode
- Parity violation experiment
- 10^{22} Electrons / a
- $\sin^2\theta_W$, neutron skin, etc.

PVES Experiments Summary



Future PVES Programs vs. Qweak

P2 @ MESA
E=155 MeV,
Forward: $\theta = (25 - 45)^\circ$
Backward: $\theta \sim 145^\circ$ (?)
Commissioning 2025 - Physics 2026-

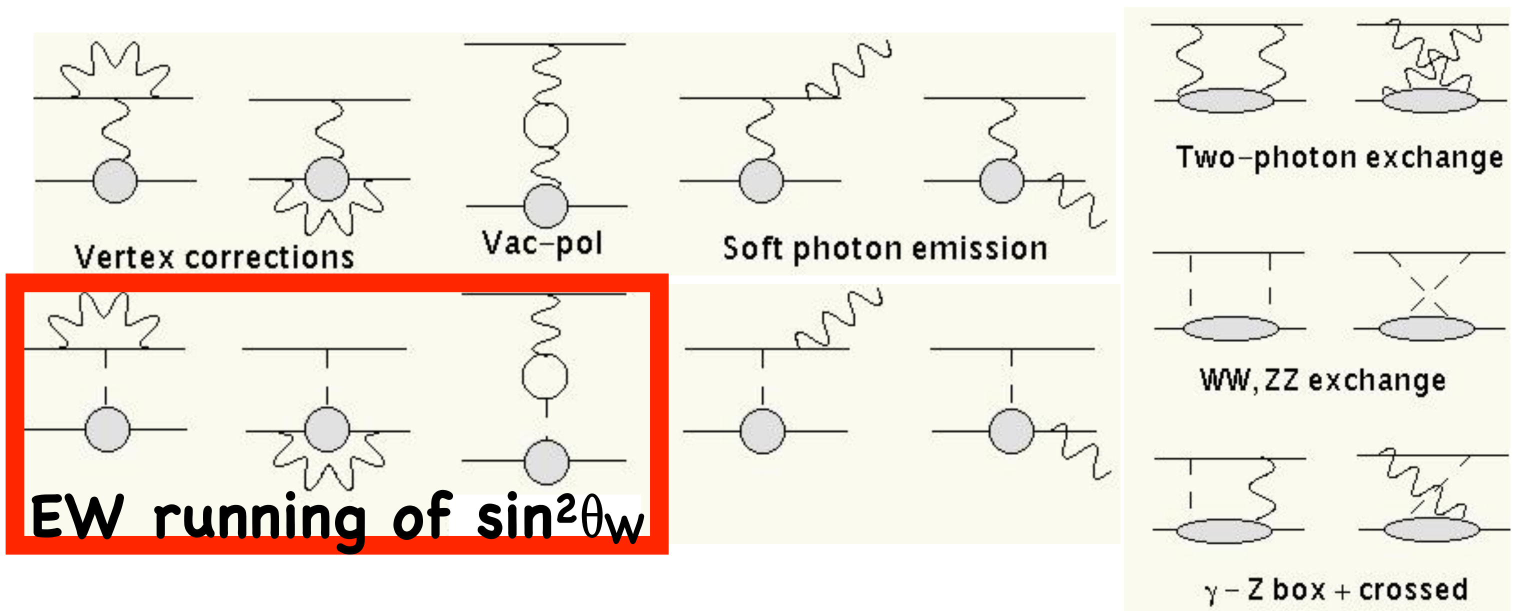
Qweak@Jlab	P2@MESA Hydrogen	P2@MESA Carbon
A_{ep}=-226.5 ppb	A_{ep}=-28 ppb	A_{eC}= 416.3 ppb
ΔA_{ep}= 9.3 ppb	ΔA_{ep}= 0.5 ppb ppb=1/√N Factor 19 After 11,000 h	ΔA_{ep}^{stat}= 2.7 ppb after 300 h ΔA_{ep}^{stat}= 0.9 ppb after 2500 h
$\Delta A_{ep}/A_{ep}$= 4.2 %	$\Delta A_{ep}/A_{ep}$= 1.8 %	$\Delta A_{ep}/A_{ep}^{stat}$= 0.6 % (0.2 %) Polarimetry!
$\Delta \sin^2\theta_W/\sin^2\theta_W$= 0.46 %	$\Delta \sin^2\theta_W/\sin^2\theta_W$= 0.15 %	$\Delta \sin^2\theta_W/\sin^2\theta_W$= 0.6 % (0.3%)
	Aux. measurem. backward angle	Aux. measurem. backward angle

MOLLER @ JLab: PV ee scattering
e-scattering off atomic electrons in LH2 target
E=11 GeV, $\theta = 5$ mrad
Commissioning 2026 - Physics 2026-8

$$A^{PV} \approx -32\,\text{ppm}$$
$$\delta A^{PV} = -0.8\,\text{ppm}(2.5\%)$$
$$\delta A^{PV}/A^{PV} = 2.5\,\%$$
$$\delta \sin^2\theta_W/\sin^2\theta_W = 0.11\,\%$$

Weak Charges in Presence of Radiative Corrections

Proton’s weak charge with radiative corrections: EW boxes



Hadronic effects under control

$$Q_W^{p,\,1-\text{loop}} = (1+\Delta_\rho+\Delta_e)(1-4\sin^2\hat{\theta}_W+\Delta_e') + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}$$

Marciano, Sirlin ’83,84; Erler, Musolf ’05

Non-universal correction - depends on kinematics and hadronic structure

Marciano and Sirlin ’84:
γZ-box mainly universal (large log)
same for PV in atoms and e-scattering
Residual dependence on hadronic scale Λ
No energy dependence assumed

$$\Box_{\gamma Z} = \frac{5\hat{\alpha}}{2\pi}(1-4\hat{s}^2)\left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right)+C_{\gamma Z}(\Lambda)\right]$$

0.0052±0.0005 (7.3±0.7% of Q_W)

γZ-Box from Dispersion Relations

γZ-box from forward dispersion relation

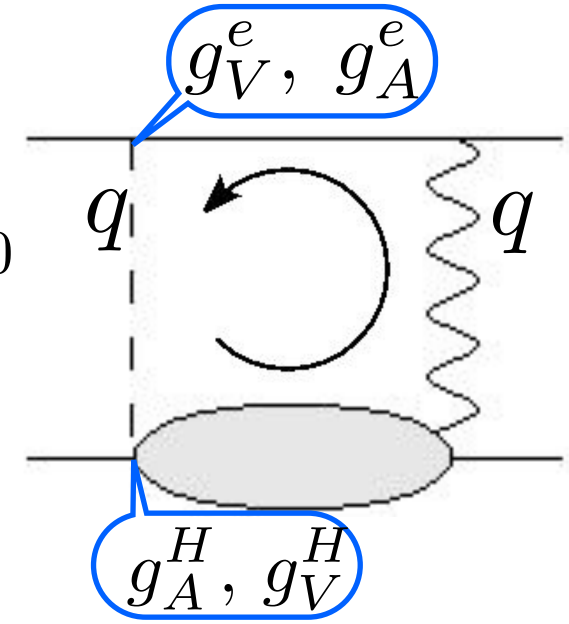
Imaginary part = on-shell states = data
Real part: from unitarity + analyticity + symmetries

MG, Horowitz '09; MG, Horowitz, Ramsey-Musolf '11

$$k^\mu = (E, \vec{k})$$

$$Q^2 = -q_\mu q^\mu \geq 0$$
$$W^2 = (p+q)^2$$

$$p^\mu = (M, \vec{0})$$



Lower blob: forward interference Compton tensor

$$\mathrm{Im}W^{\mu\nu} = -\hat{g}^{\mu\nu} F_1^{\gamma Z} + \frac{\hat{p}^\mu \hat{p}^\nu}{(p\cdot q)} F_2^{\gamma Z} + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p\cdot q)} F_3^{\gamma Z}$$

Forward dispersion relations

$$\mathrm{Re}\Box_{\gamma Z_V}(E) = \frac{2E}{\pi}\int_0^\infty dQ^2 \int_{W_\pi^2}^\infty dW^2 \left[A F_1^{\gamma Z}(W^2, Q^2) + B F_2^{\gamma Z}(W^2, Q^2) \right]$$

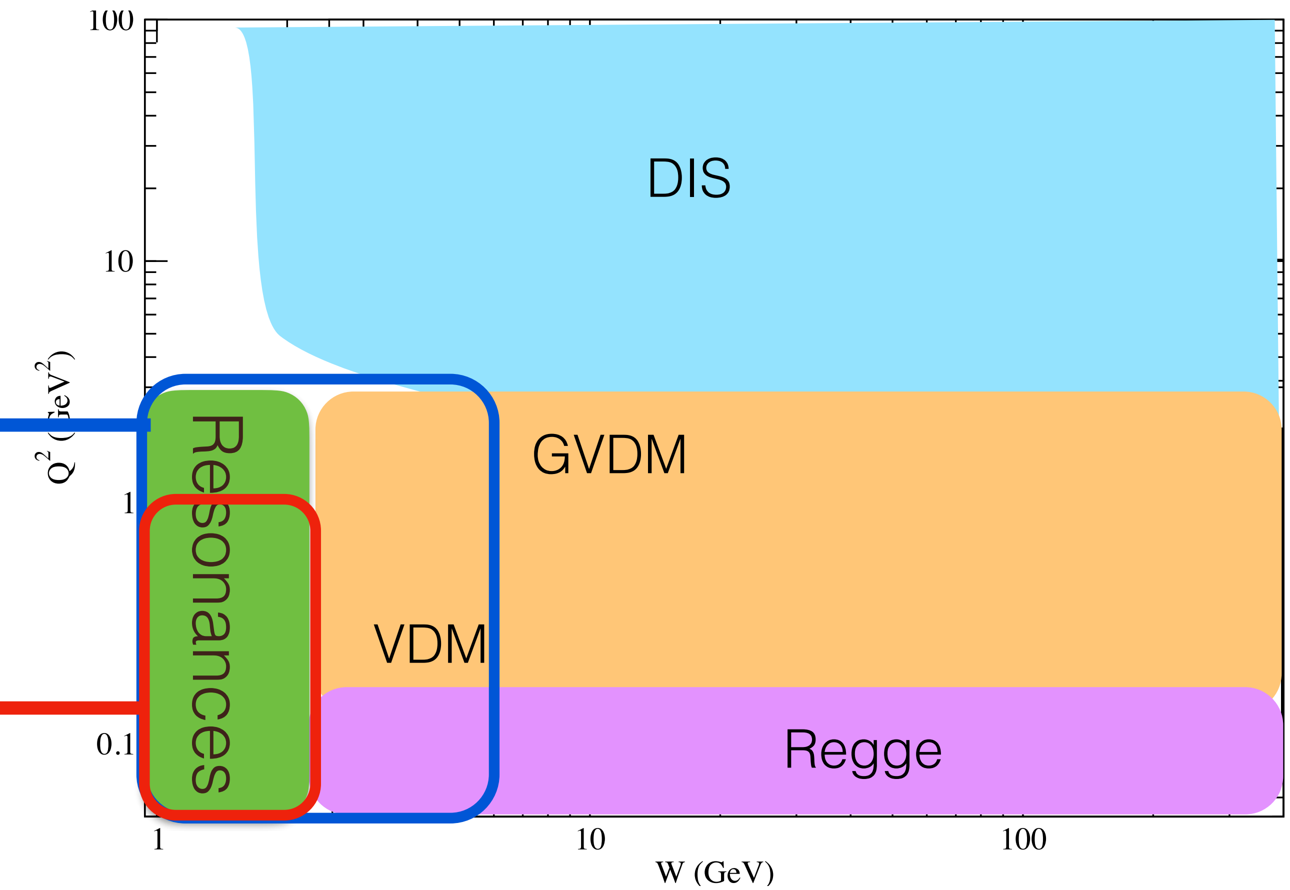
$$\mathrm{Re}\Box_{\gamma Z_A}(E) = \frac{2}{\pi}\int_0^\infty dQ^2 \int_{(M+m_\pi)^2}^\infty dW^2\ C F_3^{\gamma Z}(W^2, Q^2)$$

Inclusive PV data
- little available

Energy dependence of γZ-Box from Dispersion Relations

Model dependence stems from that in the flavor separation/rotation

Importance of the input for the integral:



For QWeak energy E = 1.165 GeV
Main contribution: W < 5 GeV, Q^2 < 2 GeV^2

For P2 energy E = 155 MeV
Main contribution: W < 2.5 GeV, Q^2 < 1 GeV^2

Energy dependence required
a formal redefinition of the weak charge

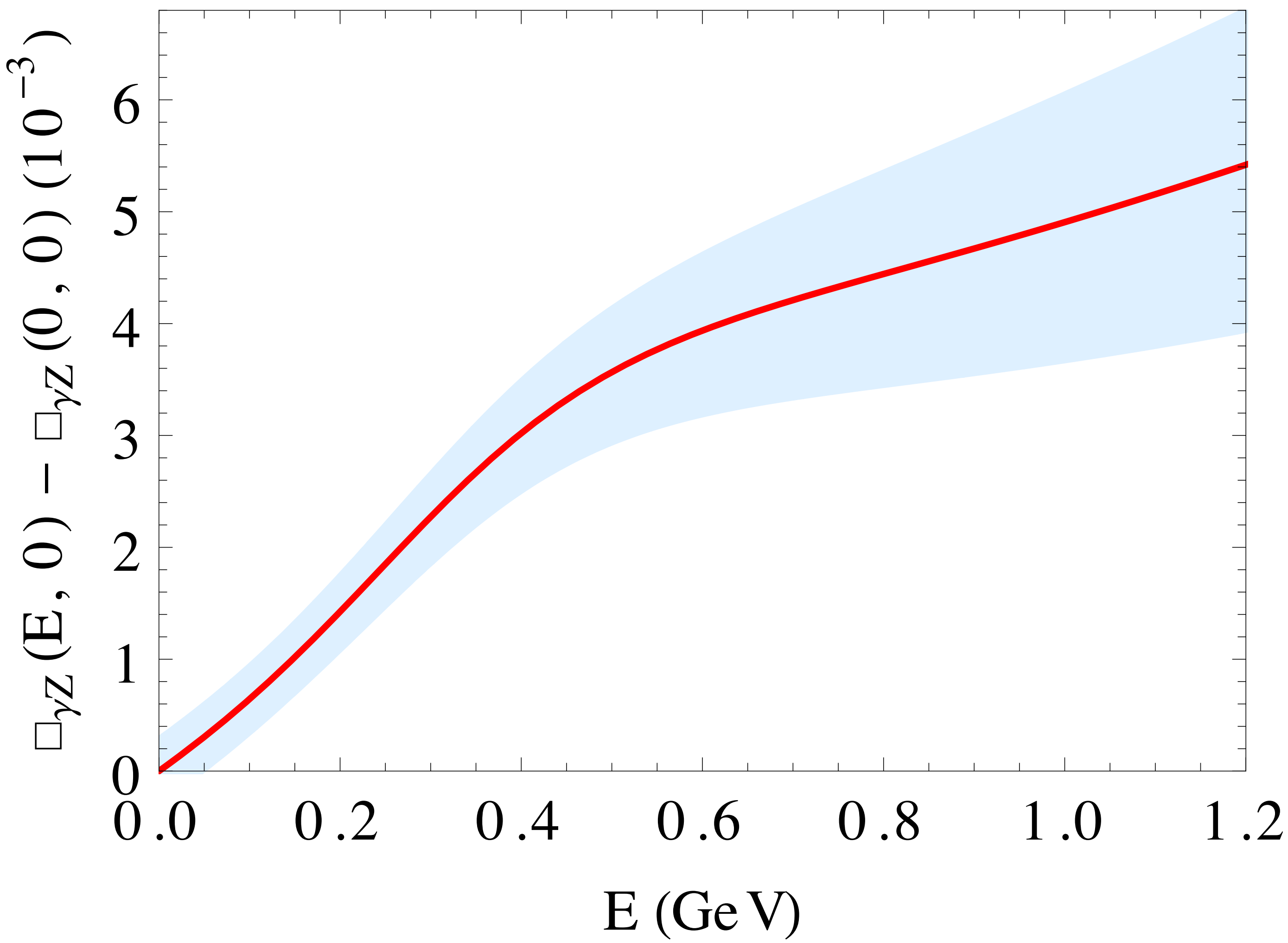
$$Q_W^p = \lim_{E,\, Q^2 \to 0} \left[-\frac{4\sqrt{2}\pi\alpha}{G_F Q^2} A^{exp} \right]$$

Steep energy dependence of $\Box_{\gamma Z}^{V}$ (much smaller for smaller energy)

- Furnished a strong motivation for the P2 experiment in Mainz at E=155 MeV

$$\Box_{\gamma Z}^{V}(E = 1.165\,\mathrm{GeV}) = 0.0054(20) \to \Box_{\gamma Z}^{V}(E = 155\,\mathrm{MeV}) = 0.0011(2)$$

Status of the energy-dependent γZ-Box



MG, Horowitz, PRL 102 (2009) 091806;
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Hall et al., PRD 88 (2013) 013011;
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Hall et al., PLB 731 (2014) 287;
MG, Zhang, PLB 747 (2015) 305;
Hall et al., PLB 753 (2016) 221;
MG, Spiesberger, Zhang, PLB 752 (2016) 135;

QWEAK energy: $\mathrm{Re}\,\Box^{A+V}_{\gamma Z}(E = 1.165\,\mathrm{GeV}) = (9.3 \pm 1.5) \times 10^{-3}$ (mostly vector box)

QWEAK final result: Q^p_W = 0.0719 ± 0.0045 (error mostly experimental)

P2 energy: $\mathrm{Re}\,\Box^{A+V}_{\gamma Z}(E = 155\,\mathrm{MeV}) = (5.4 \pm 0.4) \times 10^{-3}$ (mostly axial box)

P2 expectation: Q^p_W = 0.0713 ± 0.0013

Summary and Outlook

Running of WMA: precise prediction of the Standard Model

Hadronic uncertainties under control

Future experiments (P2, Moller) will provide stringent tests of (B)SM
Will probe (semi)leptonic operators (heavy BSM) and light Dark Z

Sensitive to New Physics down to 70 MeV and up to 50 TeV:
complementary to colliders

Not in this presentation:
Atomic PV, Neutron skin program; PV DIS @ SOLID