Dark matter in QCD-like theories with a θ vacuum

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Based on 2405.10367 and work in progress, in collaboration with: C. Garcia-Cely & G. Landini.

SIMPs (Hochberg/Kuflik/Volansky/Wacker 2014)

✔ χ is the lightest particle in a nearly secluded dark sector.

✔ Ω_χ is set by NCP in the dark sector (negligible $\sigma_{\chi+\chi\to \mathrm{SM+SM}}$):

$\chi+\chi+\chi\to\chi+\chi$,

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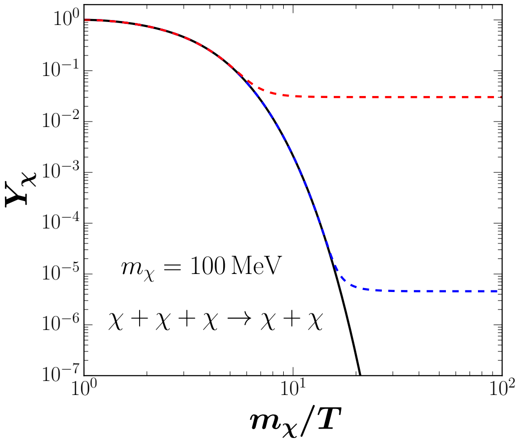
✔ $m_\chi \sim \alpha_{\mathrm{eff}}(T_{\mathrm{eq}}^2 M_{\mathrm{Pl}})^{1/3}$
$\sim$ 10 MeV - 1 GeV.

☞ $\chi+\chi\to\chi+\chi$ scatterings are constrained by cluster obs.

☞ Thermal equilibrium with SM via $\chi+\mathrm{SM}\to\chi+\mathrm{SM}$.

$$\frac{dn_\chi}{dt}+3Hn_\chi=-\langle\sigma_\chi v^2\rangle_{3\to2}\left[n_\chi^3-n_\chi^2 n_{\chi,\mathrm{eq}}\right].$$

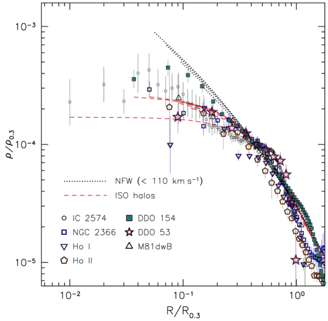


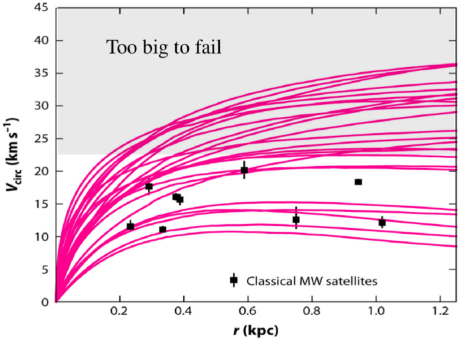
Challenges on the collisionless CDM

N-body simulations (at small scales $\lesssim \mathcal{O}(100 \text{ kpc})$ predict:

✔ universal halo profiles with a large central density, in contranst to certain observations (→ shallower density).
Core vs cusp: $\rho \sim r^{\alpha}$.(Moore94; Burkert95)

✔ very massive satellite halos in the Milky Way, which have not been obverved.(Boylan-Kolchin+,11)





✐ These issues may be mitigated or resolved by incorporating dissipative baryonic processes (gas cooling, star formation, supernova feedback) into simulations. e.g. Madau, Shen, Governato 2014, ...]

Self-interacting DM (SIDM) (Spergel/Steinhardt 99)

➡ It is a promising alternative to collisionless CDM.

✔ Allows DM particles elastic scatterings, $\chi + \chi \to \chi + \chi$, in the inner regions of astrophysical objects, thus decreasing the central density.

✔ Strong χ self-scatterings in dwarfs:

$$\sigma_{\chi\chi}/m_\chi \sim 1 - 10\,\mathrm{cm}^2/\mathrm{g}, \ \ \text{for} \ \ v_\chi \sim 10 - 200\,\mathrm{km/s}.$$

✔ Caution: weaker χ self-scatterings in clusters:

$$\sigma_{\chi\chi}/m_\chi \lesssim 0.5\,\mathrm{cm}^2/\mathrm{g}, \ \ \text{for} \ \ v_\chi \sim \mathcal{O}(1000)\,\mathrm{km/s}.$$

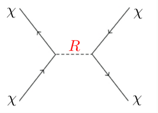
☞ A self-scattering cross section that decreases with the DM velocity can lead to a better fit.

Resonant SIDM (Chu/Garcia-Cely/Murayama 2018)

Resonant scattering in present-epoch halos.

$$\sigma(v_\chi) = \sigma_0 + \frac{128\pi\,\Gamma^2 (m_\chi^2 v_R^2)^{-1}}{m_\chi^2\left(v_\chi^2 - v_R^2\right)^2 + 4\Gamma^2 v_\chi^2/v_R^2}, \quad v_R = 2\sqrt{\frac{m_R - 2m_\chi}{m_\chi}}.$$

$\chi + \chi \to R \to \chi + \chi$



✔ Resonant enhancement for DM relative velocities $v_\chi \approx v_R$.

✔ $\sigma(v_\chi)$ can successfully account for the required interactions.

☞ It requires a near-threshold resonance with $v_R \sim (10^{-7}, 10^{-2})$.

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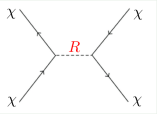

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QCD-like theories

✔ Based on a dark $SU(N_c)$ gauge group ($SO(N_c)$ or $Sp(N_c)$ also applies).
✔ N_f flavors of Dirac quarks in $d = N_c$ fund. rep. of $SU(N_c)$.

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{q}i\not{D}q - (\bar{q}_L M q_R + \text{h.c.}) , \quad M = \text{diag}(m_1, \cdots , m_{N_f}).$$

☞ Massless quarks implies the global flavor sym:

$$G_F = SU(N_f)_L \times SU(N_f)_R \Rightarrow \quad q_{L(R)} \to \exp^{i\alpha^a_{L(R)}\lambda^a} q_{L(R)}.$$

✔ Dark strong interactions confine at some energy scale Λ (in analogy with ordinary QCD).

➢ Giving rise to a fermion condensate $\langle \bar{q}q \rangle \sim \Lambda^3$

➢ This induces a ChPT for $m_q \ll \Lambda$: spontaneously breaks G_F:

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V .$$

☞ The unbroken vectorial subgroup is defined via $\alpha^a_L = \alpha^a_R$.

Dark pions

For $m_q \ll \Lambda$, M softly breaks G_F (still a good symmetry).

☞ This leads to $N_f^2 - 1$ pseudo-Goldstone bosons, π^a, described by chiral perturbation theory: $U = e^{i\Pi/f_\pi}$, $\Pi = \pi^a \lambda^a$.

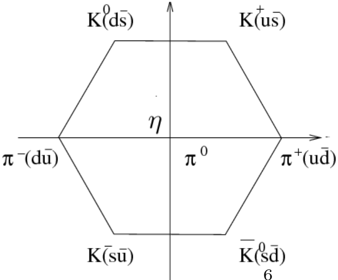
$$\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4} \mathrm{Tr}[|\partial_\mu U|^2] + \frac{f_\pi^2 B_0}{2} \mathrm{Tr}[MU + \mathrm{h.c.}] - \frac{N_c \epsilon^{\mu\nu\rho\sigma}}{240\pi^2 f_\pi^5} \mathrm{Tr}\left[\Pi \partial_\mu \Pi \partial_\nu \Pi \partial_\rho \Pi \partial_\sigma \Pi\right]$$

☞ f_π: dark meson constant such that $\Lambda \sim 4\pi f_\pi/\sqrt{N_c}$.

☞ Degenerate quark masses $\to m_\pi^2 = 2B_0 m_q$, $\quad \langle \bar{q} q \rangle \equiv -B_0 f_\pi^2$.

For $N_c = N_f = 3$:

$$\frac{\Pi}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\pi_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\pi_8 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\pi_8 \end{pmatrix}.$$



Dark pions as SIMPs (Hochberg/Kuflik/Murayama/Volansky/Wacker 2014)

✔ DM is a thermal relic in the form of π.

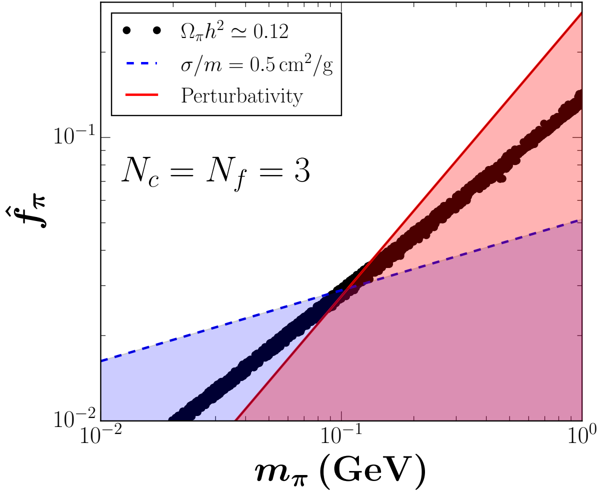
✔ Ω_π is set $3 \to 2$.

✔ π's can also scatter:

$$\mathcal{L}_{2\to 2} = \frac{m_\pi^2}{48 f_\pi^2}\,\mathrm{Tr}\left[\Pi^4\right] .$$

$$\sigma_{2\to 2} \propto m_\pi^2/f_\pi^4 .$$

☞ v-indep: no SIDM.



Ω_π in tension with the bound from clusters of galaxies: $\sigma_0/m_{\rm DM} \gtrsim 1\ \mathrm{cm}^2/\mathrm{g}$ in the perturbativity region.

✔ $N_c \geq 3$, N_f flavors of light Dirac quarks in the fund. rep.:

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{g^2\theta}{32\pi^2}F\widetilde{F} + \bar{q}i\not{D}q - (\bar{q}_L M q_R + \text{h.c.}) , \qquad \theta \ll 1.$$

$M = \text{diag}(m_1, \cdots, m_{N_f})$, with $m_i \neq 0$ for all flavors.

✔ Due to the chiral anomaly, under $q_{L,R} \to e^{\mp i\theta Q/2} q_{L,R}$:

$$\theta \to \theta(1 - \text{Tr}\, Q),$$

$$M \to M_\theta = e^{i\theta Q/2} M e^{i\theta Q/2}.$$

✔ Taking a transformation with $\text{Tr}\, Q = 1$, we move the θ parameter from the $F\widetilde{F}$ term to the quark mass matrix .

θ-vacuum in QCD-like: new interactions

The Pion interactions get modified to (Kamada/Kim/Sekiguchi 2017)

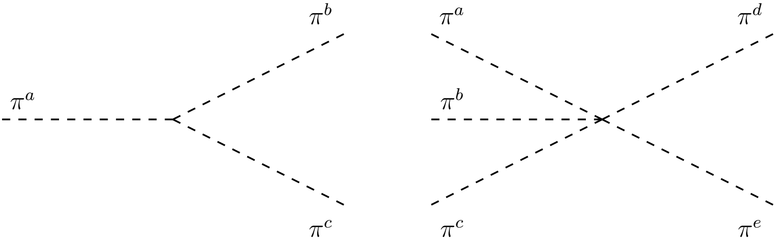
$$\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4}{\rm Tr}[\partial_\mu U^\dagger \partial^\mu U] + \frac{f_\pi^2}{2} B_0 {\rm Tr}[M_\theta U + U^\dagger M_\theta^\dagger] + \mathcal{L}_{\rm WZW}\,.$$

It gives rise to interactions with an odd number of mesons:

☞ $$\mathcal{L}_\theta = \frac{B_0\theta}{3 f_\pi {\rm Tr} M^{-1}}\left(d_{abc}\pi_a\pi_b\pi_c - \frac{c_{abcde}}{10 f_\pi^2}\pi_a\pi_b\pi_c\pi_d\pi_e \right),$$

$$f_{abc} = -i{\rm Tr}([\lambda_a,\lambda_b]\lambda_c)/4 \qquad d_{abc} = {\rm Tr}(\{\lambda_a,\lambda_b\}\lambda_c)/4,$$

$$c_{abcde} = (\delta_{ab}d_{cde} + \delta_{cd}d_{abe})/N_f + \frac{1}{2}d_{abm}d_{cdn}d_{mne}.$$



New cubic interaction — Extra quintic interaction

BM1: $N_c = N_f = 3, \quad m_u \le m_d \le m_s$

☞ Same notation of ordinary QCD.

✔ Non-relativistic resonant scattering: $v_R \lesssim 0.1$.

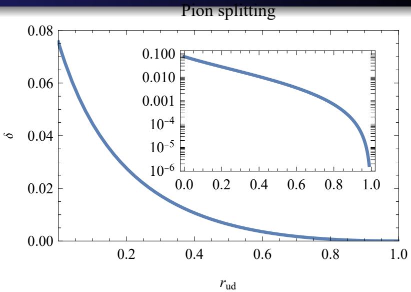
☞ m_u/m_s is a function of v_R and $r_{ud} = m_u/m_d$.

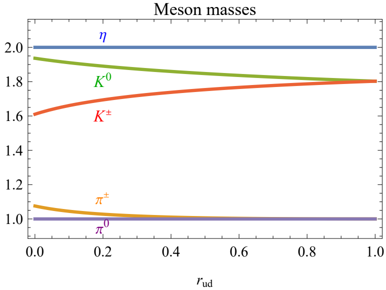
☞ Ratios of meson masses are fixed in terms of r_{ud}.

✔ η acts as unstable resonance:

$$\mathcal{L}^{(\mathrm{BM1})}_{\eta\pi\pi} = \frac{B_0\theta\,\cos(3\theta_{\eta\pi})}{\sqrt{3}f_\pi \mathrm{Tr}M^{-1}}\eta\pi^0\pi^0.$$

$$\begin{cases} m_{\eta^0} = \sqrt{(2+v_R^2/4)}m_{\pi^0} \\ m_K \\ m_{\pi^\pm} = (1+\delta)m_{\pi^0} \\ m_{\pi^0} \end{cases} .$$





☞ Neglecting $\mathcal{O}(\theta^2)$ corrections:

$$m^2_{\pi^\pm} = B_0(m_u+m_d), \;\; m^2_{K^\pm} = B_0(m_u+m_s), \;\; m^2_{K,\bar{K}^0} = B_0(m_d+m_s).$$

Cosmological implications: resonant $3 \to 2$ processes

✔ η behaves as a catalyzer, inducing NCP such as

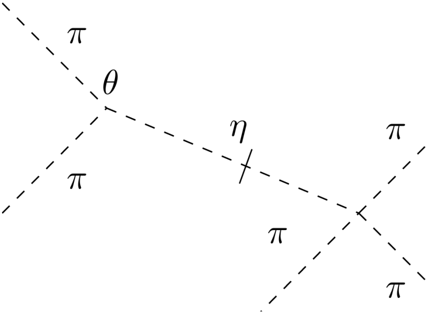
$$\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM}, \quad \eta \to \pi_{\rm DM}\pi_{\rm DM}, \quad \pi_{\rm DM}\pi_{\rm DM} \to \eta.$$

✔ For $v_R \lesssim 0.1$ and $\theta \gtrsim 10^{-4}$: $\eta \leftrightarrow \pi_{\rm DM}\pi_{\rm DM}$ are both active even after all other NCP have frozen-out.

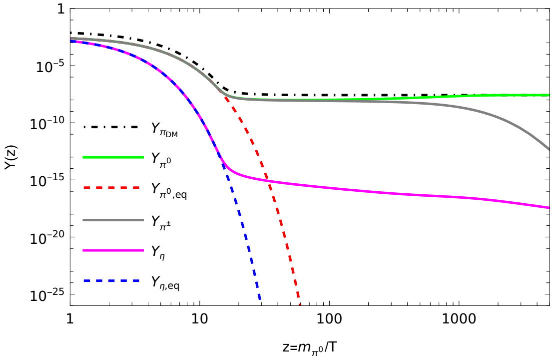
✔ $\pi_{\rm DM}\pi_{\rm DM}\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM}$ is dominated by the exchange of an on-shell η resonance.

$$sHz\frac{dY_{\pi_{\rm DM}}}{dz} = +\,2\gamma_D(\eta \to \pi_{\rm DM}\pi_{\rm DM})\left(\frac{Y_\eta}{Y_{\eta,{\rm eq}}} - \frac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},{\rm eq}}}\right) + \gamma_2(\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM})\left(\frac{Y_\eta}{Y_{\eta,{\rm eq}}}\frac{Y_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},{\rm eq}}} - \frac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},{\rm eq}}}\right)$$

$$sHz\frac{dY_\eta}{dz} = -\,\gamma_D(\eta \to \pi_{\rm DM}\pi_{\rm DM})\left(\frac{Y_\eta}{Y_{\eta,{\rm eq}}} - \frac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},{\rm eq}}}\right) - \gamma_2(\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM})\left(\frac{Y_\eta}{Y_{\eta,{\rm eq}}}\frac{Y_{\pi_{\rm DM}}}{Y_{\pi_{\rm DM},{\rm eq}}} - \frac{Y^2_{\pi_{\rm DM}}}{Y^2_{\pi_{\rm DM},{\rm eq}}}\right)$$



Resonant $3 \to 2$ processes



- ☞ $\eta \leftrightarrow \pi_{\rm DM}\pi_{\rm DM}$ establish chemical eq.: $n_\eta/n_{\pi_{\rm DM}}^2 = (n_\eta/n_{\pi_{\rm DM}}^2)_{\rm eq}$.
- ☞ As $Y_\eta \ll Y_{\pi_{\rm DM}} \Rightarrow Y_{\pi_{\rm DM}} + 2Y_\eta \simeq Y_{\pi_{\rm DM}}$.

$$\frac{dY_{\pi_{\rm DM}}}{dz} \simeq -\langle \sigma_{\eta\pi} v \rangle \frac{sY_{\eta,\rm eq}}{zH} \left(\frac{Y_{\pi_{\rm DM}}^3}{Y_{\pi_{\rm DM},\rm eq}^2} - \frac{Y_{\pi_{\rm DM}}^2}{Y_{\pi_{\rm DM},\rm eq}} \right).$$

$\sigma_{\eta\pi} \equiv \sigma(\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM})$: independent of θ.

✔ $\Omega_{\rm DM}$ is determined by s-wave 2-to-2.

BM1: parameter space

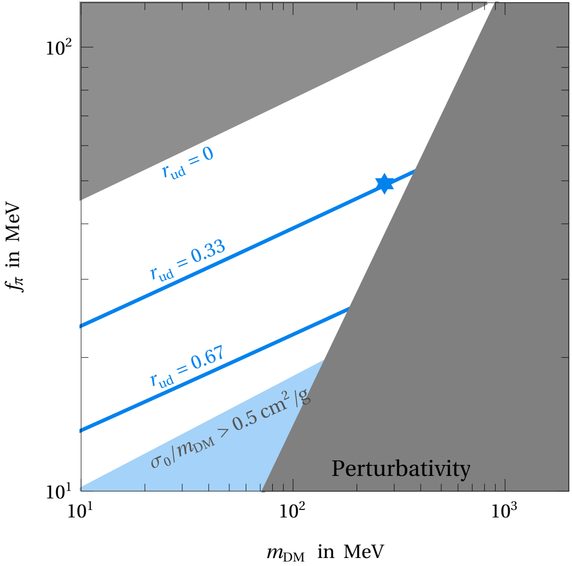
✔ $\Omega_{\text{Planck}} h^2$ is reproduced along the plane for different values of r_{ud}.

☞ Upper gray region DM is overabundant for any physical value of r_{ud}.

☞ Blue region is excluded by obs. of galaxy clusters: $\sigma_0/m_{\text{DM}} > 0.5\ \text{cm}^2/\text{g}$.

☞ In the dark-gray region ChPT breaks down: $m_{\text{DM}}/f_\pi > 4\pi/\sqrt{N_c}$.

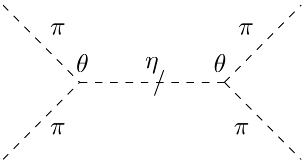
✔ Results are independent of v_R (if $v_R \lesssim 0.1$) but depend on δ (or r_{ud}), as long as the small dependence of the masses on v_R and θ is neglected.



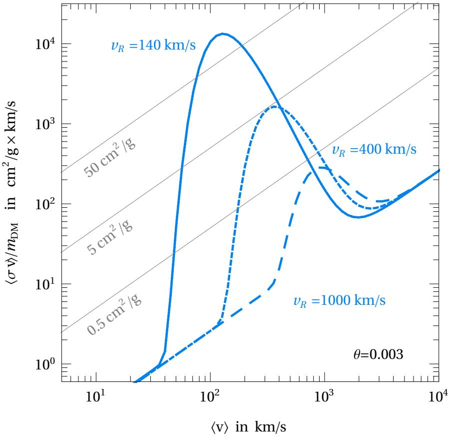
Astrophysical implications

$\sigma(v)$ can successfully account for the required interactions in small-scale halos while evading the cluster constraints.

- The θ angle could also induce resonant scattering among DM particles in present-epoch halos.



☞ $\sigma(v)/m_{\rm DM} \lesssim 100\,{\rm cm}^2/{\rm g}$ for $v \lesssim 100\,{\rm km/s} = 0.0003c$.



Velocity-average cross section at different scales for three different values of v_R.

Summary

✔ We pointed out a production mechanism of DM in QCD-like theories where $\theta \neq 0$.

✔ The θ-induced scatterings might address the apparent small-scale anomalies of the collisionless cold DM paradigm.

✔ The θ-dependent effects discussed here are generic features of QCD-like theories.

Outlook

- Beyond $SU(N_c)$ gauge groups: qualitatively similar effects arise for $SO(N_c)$ or $Sp(N_c)$.
  In particular, with $N_c \geq 4$, gauge confinement breaks flavor symmetry as $SU(N_f) \to SO(N_f)$ for $SO(N_c)$ or $SU(N_f) \to Sp(N_f)$ for $Sp(N_c)$.

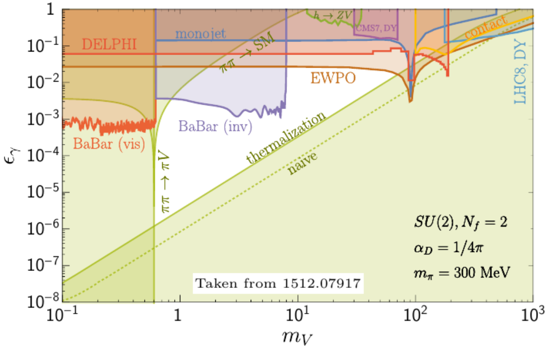
- SM portal: hidden dark photons; axion-like particles; vector mesons.

- Emission of GWs if the ChPT is first order.
  For $\theta = 0$, there are studies of QCD-like benchmarks for critical temperatures of order $T_* \sim 100$ MeV with a GW spectrum peaking at frequencies in the sub-mHz band (PTA signal).

- Additional source of CP violation in QCD-like sectors. Implications for the matter-antimatter asymmetry?.

Thermalization via a dark photon

$$\mathcal{L} \supset -\frac{1}{4}\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} - \frac{1}{2}\sin\chi\mathcal{A}_{\mu\nu}\mathcal{B}_Y^{\mu\nu} + \frac{1}{2}m_V^2\mathcal{A}_\mu\mathcal{A}^\mu .$$



Constraints on the DP for $SU(2) = Sp(2)$:

✔ kinetic equilibrium $\pi\ell \to \pi\ell$ during freeze-out;

✔ dominance of the $\pi\pi\pi \to \pi\pi$ over $\pi\pi \to V^* \to \ell^+\ell^-$;

✔ subdominance of $\pi\pi \to \pi V$.

BM2: mass spectrum

☞ Fix $N_f = n + 1$ with $n \geq 3$ and choose the quark mass matrix as $M = \mathrm{diag}(m, ..m, \mu)$ with $0 < m < \mu$.

☞ Under the remnant $SU(n)$, the $N_f^2 - 1$ mesons organize as:

- ⋆ $n^2 - 1$ pions π (adjoint of $SU(n)$): $m_\pi^2 = 2B_0 m$.
- ⋆ $2n$ kaons K ((anti)fund. reps.): $m_K^2 = B_0(m + \mu)$.
- ⋆ one singlet: the η resonance; $m_\eta^2 = 2B_0\left(m + n\mu\right)/(n+1)$.

➙ $m_\eta = (2 + v_R^2/4)m_\pi \Rightarrow \mu = m[(n+1)(2 + v_R^2/4)^2 - 1]/n$.

➙ For $v_R \lesssim 0.1 \Rightarrow m_K^2 = B_0 m(5 + 3/n) + \mathcal{O}(v_R^2)$.

Example: $N_f = 4$

$m_\pi^2 = 2B_0 m(1 - 25\theta^2/512)$, $m_\eta^2 = 8B_0 m(1 - 5\theta^2/1024)$:
➙ $v_R \approx 0.4\theta$, $\mu \simeq 5m$, $m_\eta \simeq 2m_\pi$, $m_K \simeq \sqrt{3}m_\pi$.

$$\mathcal{L}_{\eta\pi\pi}^{(\mathrm{BM2})} = \frac{B_0\theta}{\sqrt{n(n+1)/2}f_\pi \mathrm{Tr}M^{-1}}\,\eta\,\pi\cdot\pi.$$

BM2: cosmological implications

- The total DM relic density: $\Omega_{\rm DM} = Y_K + Y_\pi$.
- K and π are maintained in chemical equilibrium through 2-to-2 scatterings, which freeze out after $z_{\rm fo}$.
- But kaons are heavier: $Y_K/Y_\pi \sim \exp\left[-(m_K - m_{\rm DM})z/m_{\rm DM}\right]$.
- For $m_K \simeq \sqrt{3}m_\pi$: $\lesssim 10^{-3}$ for $z > 10$.
- Kaons can thus be safely neglected and $Y_{\pi_{\rm DM}} \simeq Y_\pi$.
- $\sigma(\eta\pi \to \pi\pi) \equiv \sigma_{\eta\pi} \Rightarrow \langle\sigma_{\eta\pi} v\rangle = \frac{\sqrt{5}m_{\rm DM}^2(n^2-4)}{192\pi f_\pi^4 n^2(n+1))}$.

✖ Due to the large number of degenerate π, $\Omega_{\rm DM}$ is reproduced in parameter regions in tension with the cluster bound.

☞ This suggests that the DM must lie in a small representation, which can be done breaking the mass degeneracy as in BM1. This is a generic feature of other color and flavor groups.