On magnetized Bose-Einstein charged scalar condensate stars.

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Motivation

Collapse of massive stars

↓ Neutron stars ↓ Black holes

Neutron stars → Inner composition models

Inner composition models → Fermions: quarks, neutrons, ets.

Inner composition models ↓

Condensate of pions, superfluid states of protons, neutrons or pions NOT EXCLUDED

Bose-Einstein condensate stars (BECs)

- CO → interacting gas of bosons.
  ↓
  sustains them against gravitational collapse
- BECs can be considered as an alternative model of neutron star nuclei if we assume that this core is composed exclusively of bosonized nuclear matter.

Chavanis, P.H., Harko, Tiberiu. 2012,Phys.Rev.D,86(6), 064011

Objectives

- Revisited the condensation of a magnetized scalar charged boson gas for low temperature regime and an arbitrary value of magnetic field.

- Reproduce the diffuse condensation.

- Reproduce the two step condensation previously obtained for non-relativistic bosons.

Bose-Einstein condensation

$B = 0$

Critical temperature T_c (different from zero) from wich condensation will start such that $\mu(T_c) = \varepsilon$. → Increase of particles in the state with zero energy

Strong criterion

$B \neq 0$

Finite fraction of the total particle density in the ground and in states in the neighborhood at some temperature $T > 0$ → Weak criterion

→ No critical temperature defined → Diffuse phase transition

- *On Bose-Einstein condensation in any dimensión, H. Perez Rojas Physics Letters A 234 (1997) 13-19.*
- *Condensation may occur in a constant magnetic field, H. Perez Rojas Physics Letters B 379 (1996) 148-152.*
- *Quintero Angulo, A. Pérez Martínez and H. Pérez Rojas, Phys . Rev. C 96 (2017)045810.*

Thermodynamic potential

Constant and uniform magnetic field in the abscissa direction x

$$\Omega(T,\mu,B) = -\frac{eB}{4\pi^2\beta}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty} dp_3 \ln\left|\left(1-e^{-\beta(\varepsilon-\mu)}\right)\left(1-e^{-\beta(\varepsilon+\mu)}\right)\right| - \frac{eB}{4\pi^2}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty} dp_3 \varepsilon$$

Statistical (first term) — Vacuum (second term)

- $m_B = \sqrt{m^2 + eB}$
- $\beta = \frac{1}{T}$
- $\varepsilon_n(p_{||}) = \sqrt{m^2 + p_{||}^2 + 2eB(n+1/2)}$
- $b = \frac{B}{B_c} = \frac{qB}{m^2}$

m_B: ground state energy

m: boson mass (pion)$\approx 140MeV$

q: boson charge

β: inverse temperature

ε: particle energy spectrum

μ: chemical potential

B: magnetic field

B_c: critical magnetic field

$$B_c = 2.08 \times 10^{18} G$$

Vacuum energy

$$\Omega_{vac}(B) = -\frac{1}{4\pi^2}\int_0^\infty \frac{ds}{s^3}\left((eBs)\,coth(eBs) - 1 - \frac{(eBs)^2}{3}\right) e^{-m^2 s}$$

Vacuum energy

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At low temperatures regime $T \ll m_B$

$$\frac{\Omega_{st}(t,\mu,b)}{m^4} = -\frac{b(1+b)^{1/4} t^{3/2}}{2\pi^2}\left[Li_{3/2}(z) + \sum_{n=1}^{\infty} \frac{z^n}{n^{5/2}} \frac{n}{e^{n\gamma} - 1}\right]$$

- $t = \frac{T}{m}$
- $x = \mu/m$

- $Li_s(z) \equiv \sum_{n=1}^{\infty} \frac{z^n}{n^s}$
- $z \equiv e^{(\mu - m_B)/T}$
- $\gamma = \frac{b}{t\sqrt{1+b}}$

$Li_s(z)$: Polylogarithmic function

z: Fugacity

γ: scaled magnetic field

A. Ayala, M. Loewe, J. C. Rojas and C. Villavicencio, arXiv : hep ph], Oct2012.

Thermodynamic properties

Particle density

$$N = N_0 + N_n = -\left(\frac{\partial\Omega}{\partial\mu}\right)_{T,B}$$

Entropy density

$$S = -\left(\frac{\partial\Omega}{\partial T}\right)_{B,\mu}$$

$$\frac{N(t,z,b)}{m^3} = -\frac{b(1+b)^{1/4}t^{1/2}}{2\pi^2}\left[Li_{1/2}(z) + \sum_{n=1}^{\infty}\frac{z^n}{n^{3/2}}\frac{n}{e^{\frac{nb\sqrt{1+b}}{t}}-1}\right]$$

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Energy density $E = \Omega + \mu N + TS$

$$\frac{E(t,z,b)}{m^4} = \frac{b\sqrt{m_B t}}{(2\pi)^{\frac{3}{2}}}\left[m_B Li_{1/2}(z) + \frac{t}{2}Li_{1/2}(z)\right] + \sum_{n=1}^{\infty}\frac{\sqrt{t}bz^n}{(2\pi n)^{\frac{3}{2}}\sqrt{m_B}(e^{n\gamma} - 1)^2}$$

Thermodynamic properties

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Specific heat per volume

$$C_v = \left(\frac{\partial E}{\partial T}\right)_{B,N} = \left(\frac{\partial E}{\partial T}\right)_{B,\mu} + \left(\frac{\partial E}{\partial \mu}\right)_{T,B}\left(\frac{\partial \mu}{\partial T}\right)_{B,N}$$

$$\left(\frac{\partial \mu}{\partial T}\right)_{B,N} = \frac{\partial y}{\partial x}$$

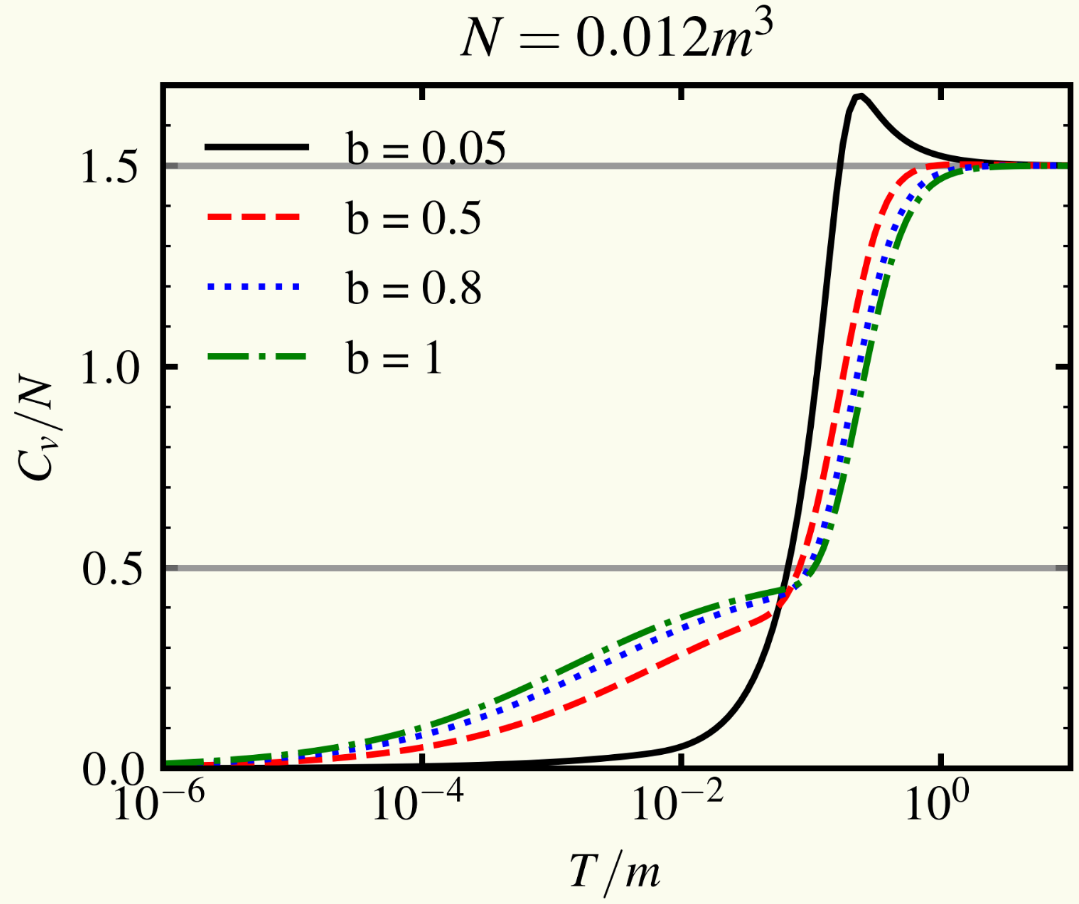
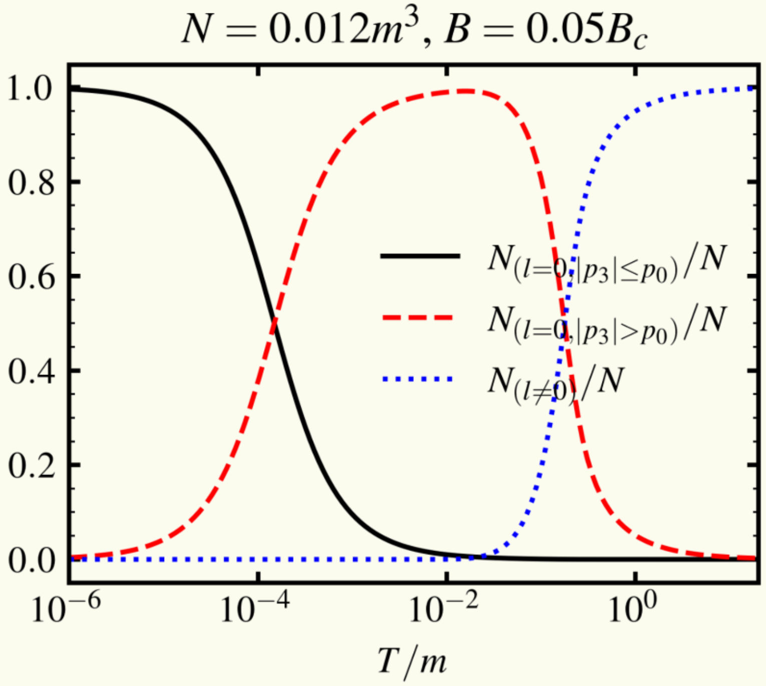


Figure 1: Specific heat $C_v\ /N$ as a function of the temperature in the low temperature regime, at a constant density.



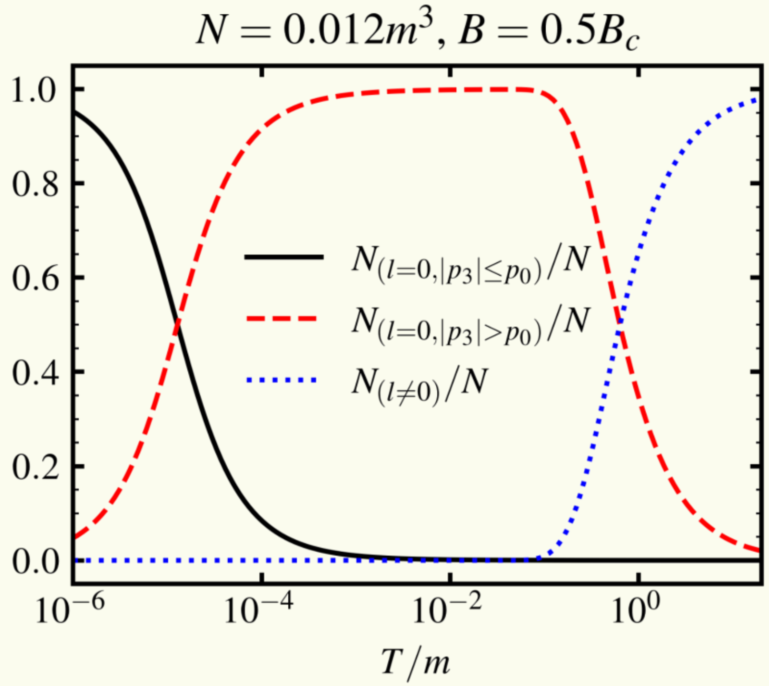


Figure 2,3: The figure shows the particle density as a function of temperature for a fixed density $N\ =\ 0.012$ and magnetic field, $b\ =\ 0.05$ and $b\ =\ 0.5$.

**R. L. Delgado, P. Bargueño, and F. Sols , DOI: 10.1103/PhysRevE.86.031102, September 2012.*

Magnetic properties of a charged boson gas

Magnetization

$$M = -\left(\frac{\partial \Omega}{\partial B}\right)_{T,\mu}$$

Magnetic susceptibility

$$\chi = \left(\frac{\partial M}{\partial B}\right)_{T,\mu}$$

$$M_0(t,z,b) = \frac{\sqrt{t}}{8\sqrt{2}(1+b)^{3/4}\pi^{3/2}}\left(-2b\sqrt{1+b}Li_{1/2}(z) + (4+5b)tLi_{3/2}(z)\right)$$

$T \rightarrow 0$

$$M_0(t,z,b) = -\frac{N_{LLL}}{2}\sqrt{\frac{\pi}{2(1+b)}}$$

$$M_n(t,z,b) = -\frac{t^{3/2}}{8\sqrt{2}(1+b)^{3/4}\pi^{3/2}}\left(2\sqrt{1+b}Li_{3/2}(z) - tLi_{5/2}(z)\right)$$

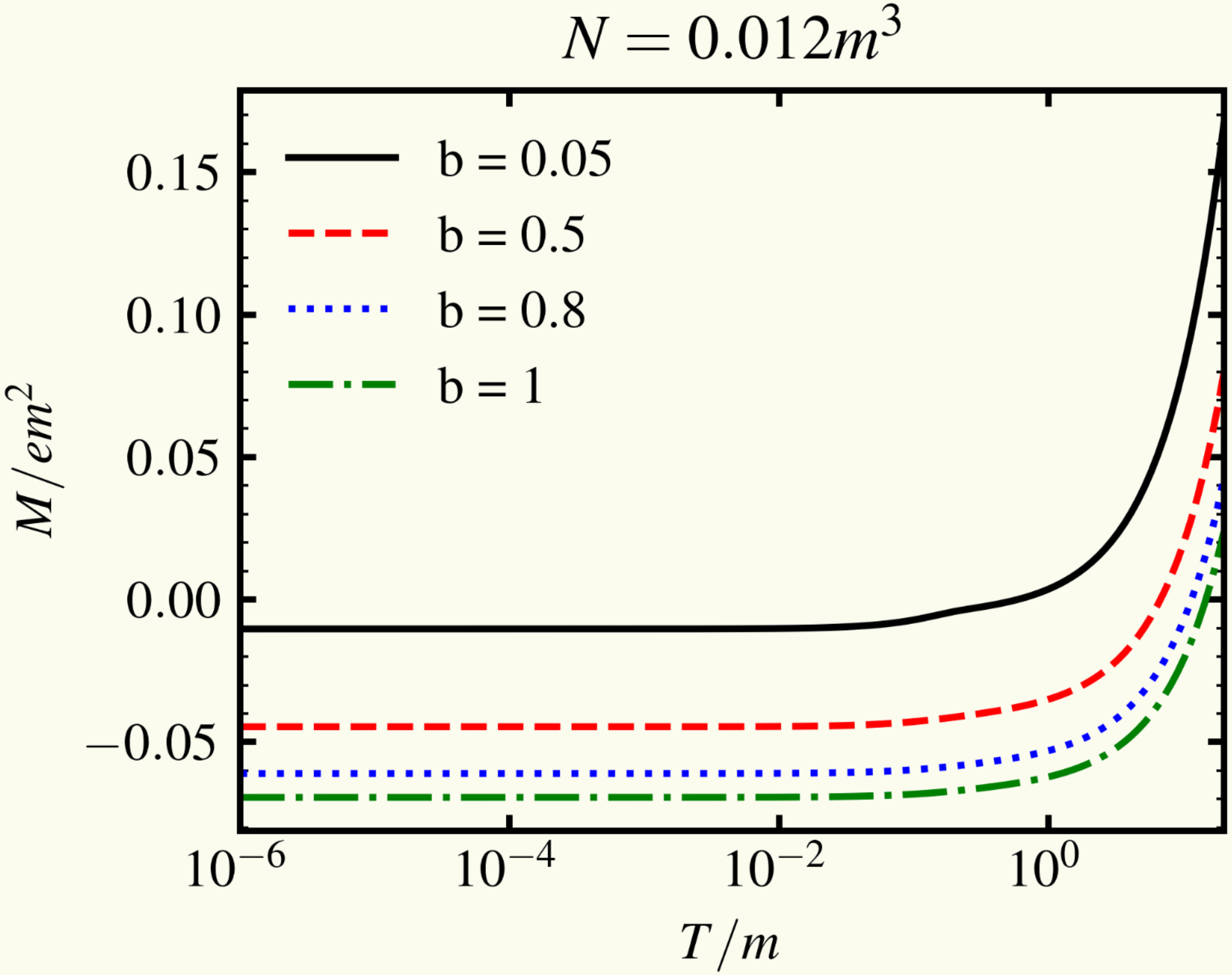
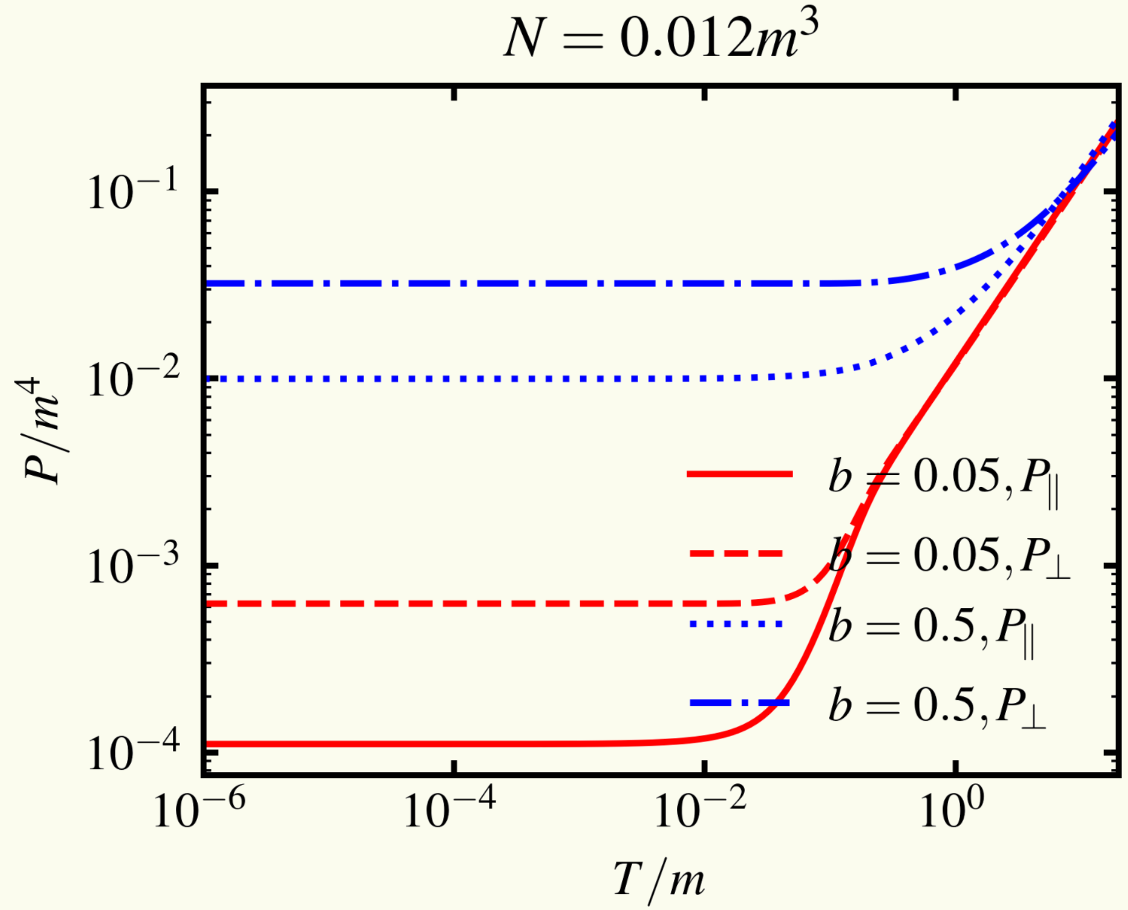


Figure 3: The charged boson magnetization as a function of temperature. The LLL is the term that contributes most to the magnetization. Magnetization is negative at low temperatures and increases to positive values as temperature rises.

Equation of State



$$P_{||} = -\Omega$$
$$P_{\perp} = -\Omega - MB$$

Figure 3: Charged scalar boson gas pressure as a function of temperature. At low temperature regime we can observe an anisotropy in the temperatures. The difference between the pressures is negligible at higher values of T.

Summary

- We reproduce with a relativistic spectrum (in case of low temperature regime) the results already obtained from Bargueño* (non-relativistic spectrum).
- We confirm the two step condensation through the analisis of the population of levels when temperature is dimished with a fixed number of particles. The first step is the accumulation of the particles in the zeroth Landau level ($n = 0, p_3 \neq 0$) and the 2nd step is the accumulation of a macroscopic number of particles around the ground state ($n = 0, p_3 = 0$).
- We observe the condensation steps in the specific heat with the emergence of two plateaus, one for $C_v = 0.5$ (1st step) and the other one for $C_v = 0$ (2nd step) that shows the diffuse transition to the condensation for a charged scalar boson gas in presence of a magnetic field.

Perspective

- Study the macrophysics by using the EoS taking into account the interaction between bosons and the General Relativity to get the observables, mass and radius.