Heavy-quark mass relation from standard-model boson operator representation in terms of fermions

Jaime Besprosvany,

in collaboration with Rebeca Sánchez

Instituto de Física
Universidad Nacional Autónoma de México, Mexico City

XV Latin American Symposium on High Energy Physics,
Nov. 4-8, 2024, Mexico City

J. Besprosvany and R. Romero, “Heavy quarks within the electroweak multiplet", *Phys. Rev. D* 99, 073001 (2019). arXiv:1701.01191[hep-ph],

Jaime Besprosvany and Rebeca Sánchez, “Heavy-quark mass relation from standard-model boson operator expansion in terms of fermions", *submitted*

Plan

- Motivation: multiplet structure. Standard-model puzzle: independent scalar-vector, Yukawa sectors.
- 1) Beyond the standard model (SM)

Spin-extended model, (7+1)-dimensional chiral states and operators.

- 2) SM projection

Top-quark mass from SM. Scalar-field uniqueness: electroweak scalar-vector and Yukawa scalar-fermion term comparison. Quark-mass relation.

- 3) Collider physics

Chiral-scalar signature: type II two-Higgs models.

Motivation: multiplet structure

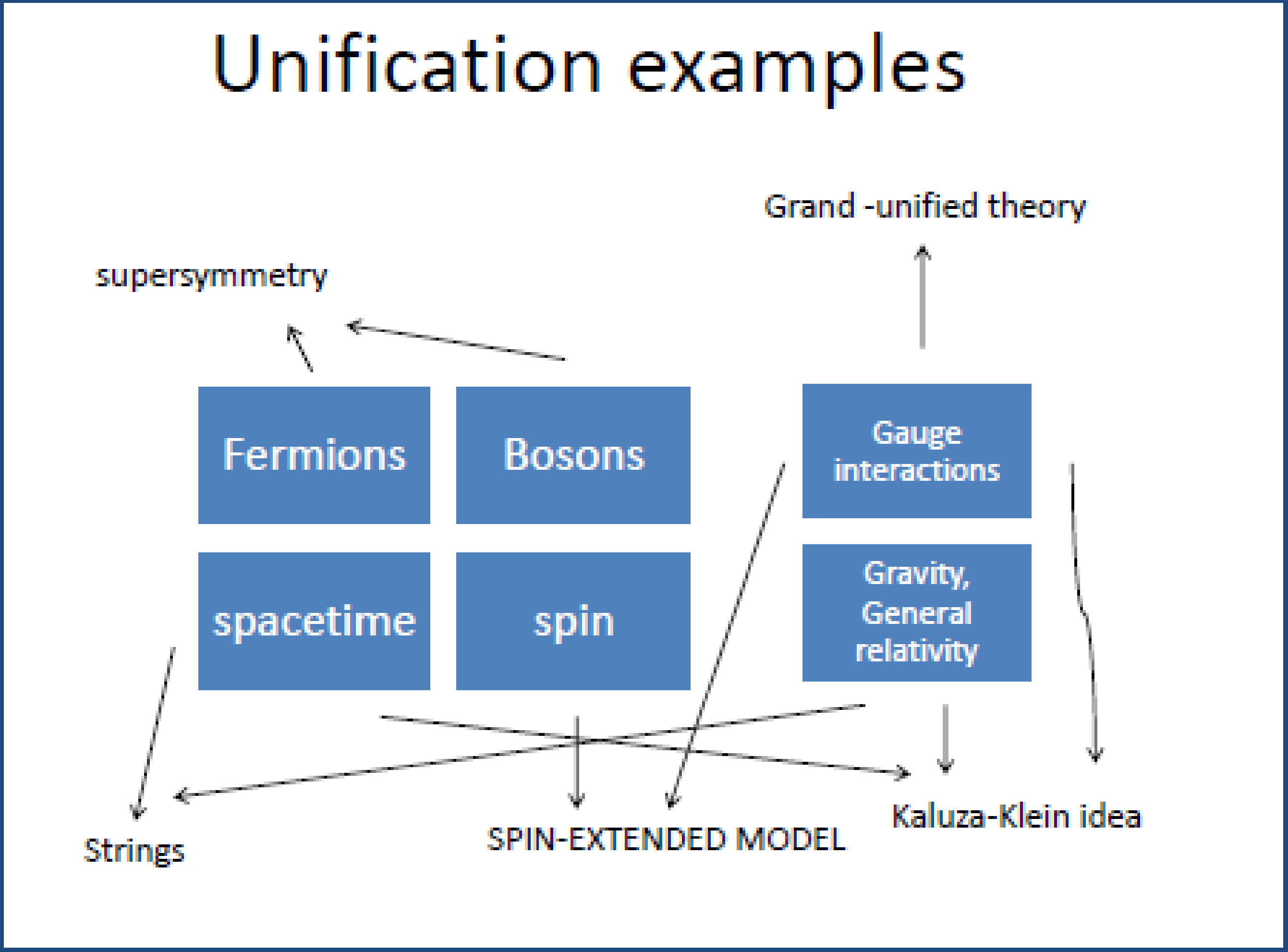
Electroweak-related puzzles in the standard model:

•Fermion-mass parameters; Yukawa sector independent of scalar-vector.
•Origin of electroweak symmetry breaking (Higgs mechanism).

	Masses (GeV)	Spin	Weak I^2	Hypercharge Y
• $W^{+/-}$	80.4	1	1	0
• Z	91.2	1	0	0
• H	126	0	½	1
• t	173	½	½ ,0	1/3, 4/3
• b	4	½	½ ,0	1/3, -2/3

Composite-multiplet structure suggested

Spin-extended model within standard-model extensions



Standard-model mass generation

Scalar-vector (SV) Lagrangian

$$\mathcal{L}_{SV} = \mathbf{H}^\dagger(x)\left[\frac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{W}^\mu(x) + \frac{1}{2}g'B^\mu(x)\right]\left[\frac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{W}_\mu(x) + \frac{1}{2}g'B_\mu(x)\right]\mathbf{H}(x)$$

Higgs doublet

$$\mathbf{H}(x) = \exp[i\boldsymbol{\theta}(x)\cdot\boldsymbol{\tau}/(2v)]\left[\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h(x) \end{pmatrix} + \mathbf{v}\right]$$

Mass-generation component

$$\mathcal{L}_{MV} = \frac{1}{4}\mathbf{v}^\dagger\left[g\boldsymbol{\tau}\cdot\mathbf{W}^\mu(x) + g'B^\mu(x)\right]\left[g\boldsymbol{\tau}\cdot\mathbf{W}_\mu(x) + g'B_\mu(x)\right]\mathbf{v}$$

Vacuum expectation value $\mathbf{v} = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\mathcal{L}_{MV} = \frac{1}{4}\mathbf{v}^\dagger\left[g\boldsymbol{\tau}\cdot\mathbf{W}^\mu(x) + g'B^\mu(x)\right]\left[g\boldsymbol{\tau}\cdot\mathbf{W}_\mu(x) + g'B_\mu(x)\right]\mathbf{v}$$

Vector boson masses, from Higgs mechanism

Mass-generating Lagrangian

$$\mathcal{L}_{MV} = \frac{1}{g^2+g'^2}\mathrm{v}^\dagger\left[(g^2 I_3 - \frac{1}{2}g'^2 Y)Z^\mu(x)\right]\left[(g^2 I_3 - \frac{1}{2}g'^2 Y)Z_\mu(x)\right]\mathrm{v}+$$
$$\mathrm{v}^\dagger\left[\frac{1}{\sqrt{2}}gI^- W^{-\mu}(x)\right]\left[\frac{1}{\sqrt{2}}gI^+ W^+{}_\mu(x)\right]\mathrm{v} =$$
$$\frac{1}{2}Z_\mu(x)Z^\mu(x)M_Z^2 + W^-{}_\mu(x)W^{+\mu}(x)M_W^2$$

Quadratic-form fields

$$W^\pm{}_\mu(x) = \frac{1}{\sqrt{2}}[W_\mu^1(x) \mp iW_\mu^2(x)]$$

$$Z_\mu(x) = \frac{1}{\sqrt{g^2+g'^2}}[-gW_\mu^3(x) + g'B_\mu(x)]$$

$$A_\mu(x) = \frac{1}{\sqrt{g^2+g'^2}}[g'W_\mu^3(x) + gB_\mu(x)]$$

$$gg'A_\mu(x)(I_3 + \tfrac{1}{2}Y)\mathrm{v} = 0$$ Photon mass 0

Vector masses

$$M_Z^2 = \frac{1}{g^2+g'^2}\mathrm{v}^\dagger(g^2 I_3 - \frac{1}{2}g'^2 Y)(g^2 I_3 - \frac{1}{2}g'^2 Y)\mathrm{v} = (g^2+g'^2)v^2/4$$

$$M_W^2 = \frac{g^2}{2}\mathrm{v}^\dagger I^- I^+ \mathrm{v} = g^2 v^2/4.$$

Spin-Isospin degree-of-fredom basis

$$\mathcal{L}_{SV} = \mathbf{H}^\dagger(x)[\frac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{W}^\mu(x) + \frac{1}{2}g' B^\mu(x)][\frac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{W}_\mu(x) + \frac{1}{2}g' B_\mu(x)]\mathbf{H}(x) =$$

$$\frac{1}{2}\mathrm{tr}\mathbf{H}^\dagger(x)\gamma_0[\frac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{W}^\nu(x) + \frac{1}{2}g' B^\nu(x)]\gamma_\nu[\frac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{W}^\mu(x) + \frac{1}{2}g' B^\mu(x)]\gamma_\mu\gamma_0\mathbf{H}(x)$$

$$A_\mu = g_{\mu\nu}A^\nu = \frac{1}{4}\mathrm{tr}\gamma_\mu\gamma_\nu A^\nu$$

Fermion creation-annihilation operator $q_{ks}=a_q a_k a_s$

Isospin $q=t, b,$ four-momentum $k,$ spin s=↑,↓

Anticommutation relations $\{\bar{q}_i^\dagger, q'_j\} = \delta_{ij}\delta_{qq'}$ $\{q_i, q'_j\} = \{q_i^\dagger, q'^\dagger_j\} = 0$

Normalization conditions $\langle 0|q_i q_i^\dagger|0\rangle = \langle 0|\bar{q}_i\bar{q}_i^\dagger|0\rangle = 1$ $q_i|0\rangle = 0$ $\bar{q}_i|0\rangle = 0$

Chiral basis

Right-handed, left-handed components

Common quantum numbers for particle-creation and antiparticle-annihilation operators

$$q_i^\dagger = \frac{1}{\sqrt{2}}(q_{Li}^\dagger + q_{Ri}^\dagger)$$

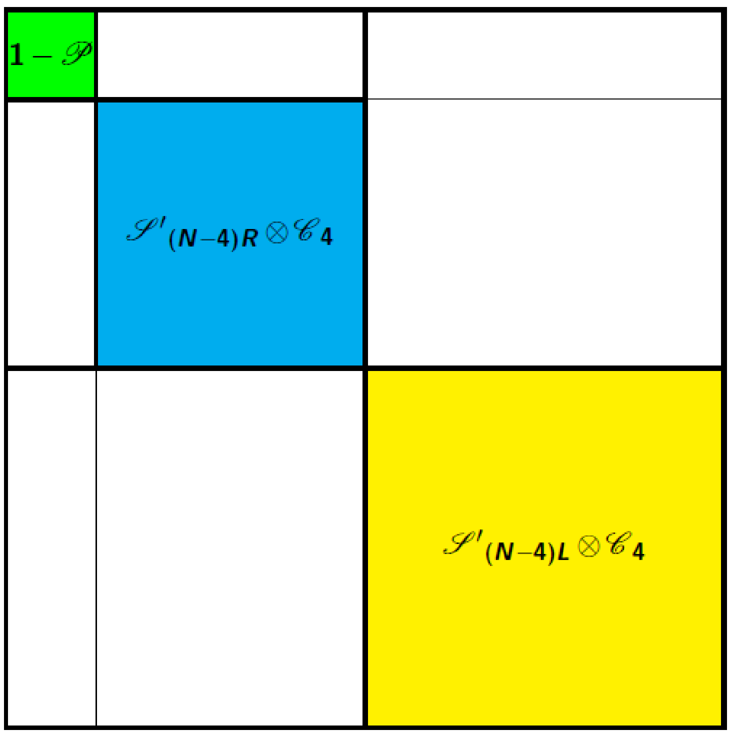
$$\bar{q}_i = \frac{1}{\sqrt{2}}(q_{Li}^\dagger - q_{Ri}^\dagger)$$

$$\{q_{Qi}^\dagger, q'_{Q'j}\} = \delta_{ij}\delta_{QQ'}\delta_{qq'} \qquad Q, Q' = R, L$$

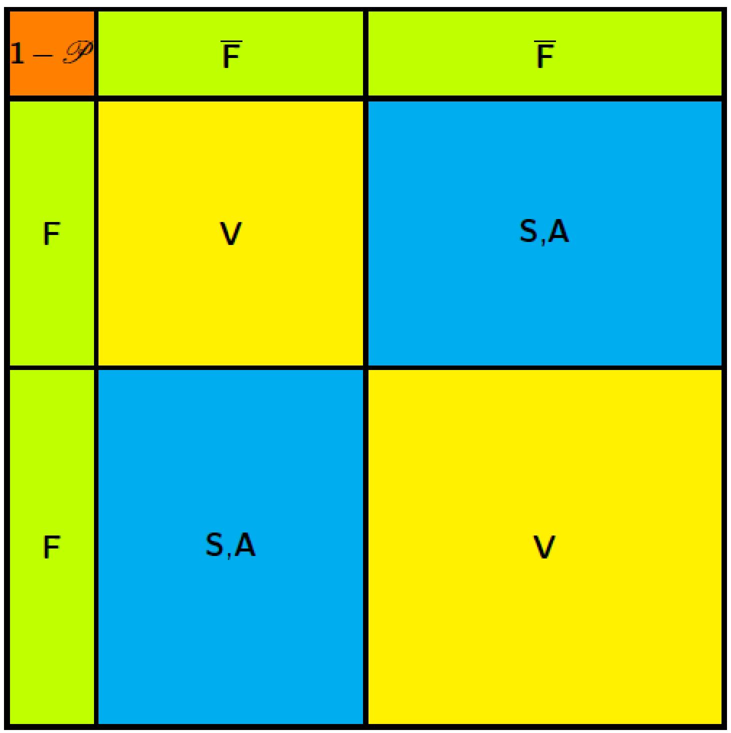
Spin-space structure, at each dimension

- Finite number of partitions at each *d*, consistent with Lorentz symmetry
- Operators: gauge and flavor (only act on fermions)
- States: fermions and bosons
- Chiral components

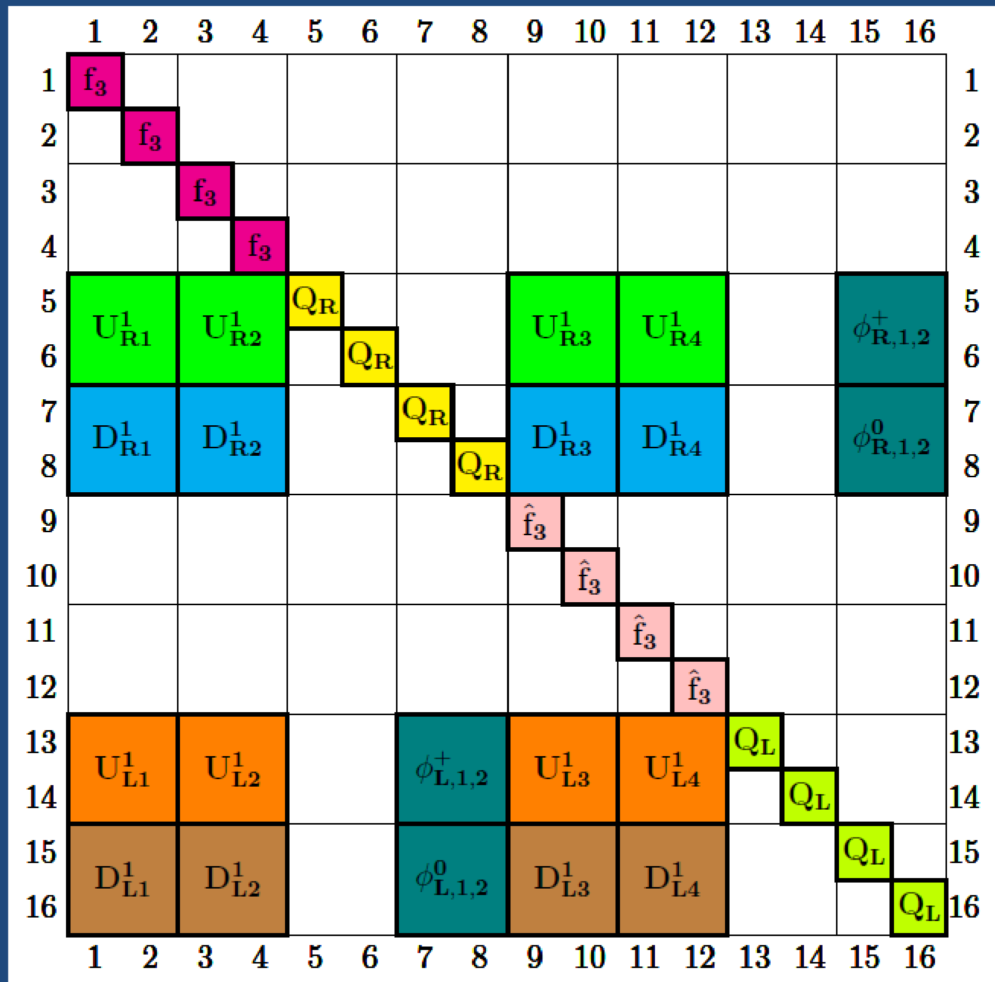
Operators



States



States in (7+1)-dimensional space



Use of conventional and spin bases

spin basis → conventional basis

- Finite number of possible partitions, consistent with 4-d Lorentz symmetry.
- Constrain representations and interactions at given dimension.

spin basis ← conventional basis

Reinterpretation of fields:

- Standard-model projection.
- SV: scalar operator acting over vectors
- SF: scalar operator acting over fermions

Conventional and spin-extended bases, Lagrangian equivalence: fermion-vector

conventional basis

spin-extended basis

Field formulation: $A_\mu(x) = g_\mu{}^\nu A_\nu(x)$

$A_\mu(x)\gamma_0\gamma^\mu$

$$\mathcal{L}_{FV} = \bar{\mathrm{q}}_L(x)[i\partial_\mu + \frac{1}{2}g\tau^a W^a_\mu(x) + \frac{1}{6}g' B_\mu(x)]\gamma^\mu \mathrm{q}_L(x) +$$

$$\bar{t}_R(x)[i\partial_\mu + \frac{2}{3}g' B_\mu(x)]\gamma^\mu t_R(x) + \bar{b}_R(x)[i\partial_\mu - \frac{1}{3}g' B_\mu(x)]\gamma^\mu b_R(x)$$

$$\mathrm{q}_L(x) = \begin{pmatrix} t_L(x) \\ b_L(x) \end{pmatrix}$$

$$t_L(x) = \begin{pmatrix} \psi^1_{tL}(x) \\ \psi^2_{tL}(x) \end{pmatrix}$$

$$\mathcal{L}_{FV} = \mathrm{tr}\{\mathbf{\Psi}^\dagger_{qL}(x)[i\partial_\mu + gI^a W^a_\mu(x) + \frac{1}{2}g' Y_o B_\mu(x)]\gamma^0\gamma^\mu \mathbf{\Psi}_{qL}(x) +$$

$$\Psi^\dagger_{tR}(x)[i\partial_\mu + \frac{1}{2}g' Y_o B_\mu(x)]\gamma^0\gamma^\mu \Psi_{tR}(x) + \Psi^\dagger_{bR}(x)[i\partial_\mu + \frac{1}{2}g' Y_o B_\mu(x)]\gamma^0\gamma^\mu \Psi_{bR}(x)\} P_f$$

$$\mathbf{\Psi}_{qL}(x) = \sum_\alpha \psi^\alpha_{tL}(x) T^\alpha_L + \psi^\alpha_{bL}(x) B^\alpha_L$$

SV Lagrangian and scalar t-b spin representation

Scalar correspondence

$$\mathbf{H}(\mathbf{x}) \to \phi_1(x) - \phi_2(x)$$

$$\tilde{\mathbf{H}}^\dagger(x) \to \phi_1(x) + \phi_2(x).$$

$$\mathbf{H}_t(x) = \phi_1(x) + \phi_2(x),\ \mathbf{H}_b(x) = \phi_1(x) - \phi_2(x)$$

$$R_5 = \frac{1}{2}(1+\tilde{\gamma}_5),\ \text{e. g., } R_5\mathbf{H}_t(x)L_5 = \mathbf{H}_t(x)$$

$$L_5\mathbf{H}_t(x)R_5 = 0,\ R_5\mathbf{H}_b(x)L_5 = 0$$

$$\mathbf{H}_{af}(x) = a\phi_1(x) + f\phi_2(x)$$

$$\mathbf{H}_{af}(x) = \frac{1}{\sqrt{2}}(\chi_t\mathbf{H}_t(x) + \chi_b\mathbf{H}_b(x))$$

$$\chi_t = \frac{1}{\sqrt{2}}(a+f),\ \chi_b = \frac{1}{\sqrt{2}}(a-f)$$

SV spin representation

$$\mathbf{F}''(x) = [i\partial_\mu + gW^i_\mu(x)I^i + \frac{1}{2}g'B_\mu(x)Y_o]\gamma_0\gamma^\mu$$

$$\mathcal{L}_{SV} = \mathrm{tr}\{[\mathbf{F}''(x), \mathbf{H}_{af}(x)]^\dagger_\pm[\mathbf{F}''(x), \mathbf{H}_{af}(x)]_\pm\}_{\mathrm{sym}}$$

Scalar-vector scalar-fermion comparison

$$\langle \eta_3(x) \rangle = v, \ \langle \mathbf{H}(x) \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

scalar-vector symmetry

Z-vector mass

Higgs mechanism

$$\langle \mathbf{H}_{af}(x) \rangle = H_n = \frac{v}{2}(\chi_t H_t^0 + \chi_b H_b^0),$$

$$\begin{aligned} \mathcal{L}_{SZm0} &= \mathrm{tr}[H_n, W_0^3(x) g I^3 + B_0(x) \frac{1}{2} g' Y_o]^\dagger [H_n, W_0^3(x) g I^3 + B_0(x) \frac{1}{2} g' Y_o] \qquad (11) \\ &= Z_0^2(x) \frac{1}{g^2 + g'^2} \mathrm{tr}[H_n, g^2 I^3 - \frac{1}{2} g'^2 Y_o]^\dagger [H_n, g^2 I^3 - \frac{1}{2} g'^2 Y_o] = \frac{1}{2} Z_0^2(x) m_Z^2, \end{aligned}$$

Top-quark mass

Higgs mechanism

$$H_m = \langle \mathbf{H}_m(x) \rangle = m_t H_t^0 + m_b H_b^0$$

$$H_m^h T_M^1 = m_t T_M^1, \quad H_m^h T_M^{c1} = -m_t T_M^{c1},$$

$$H_m^h B_M^1 = m_b B_M^1, \quad H_m^h B_M^{c1} = -m_b B_M^{c1}, \qquad (13)$$

where $H_m^h = H_m + H_m^\dagger$, and T_M^{c1}, B_M^{c1} correspond to negative-energy solution states

Spin-space connection: vector and fermion masses

vector

$$m_Z = v\sqrt{g^2 + g'^2}/2$$

fermion

massive quarks	H_m^h	Q	$\frac{3i}{2}B\gamma^1\gamma^2$
$T_M^1 = \frac{1}{\sqrt{2}}(T_L^1 + T_R^1)$	m_t	$2/3$	$1/2$
$B_M^1 = \frac{1}{\sqrt{2}}(B_L^1 - B_R^1)$	m_b	$-1/3$	$1/2$
$T_M^{c1} = \frac{1}{\sqrt{2}}(T_L^1 - T_R^1)$	$-m_t$	$2/3$	$1/2$
$B_M^{c1} = \frac{1}{\sqrt{2}}(B_L^1 + B_R^1)$	$-m_b$	$-1/3$	$1/2$

Table 3: Massive quark eigenstates of H_m^h

$$\Rightarrow \quad \sqrt{2}H_n = H_m$$

Correspondence's physical interpretation

- Dynamical: action of scalar on fermion and vector share the same effect: common Hamiltonian H.

$$[H+H^{\dagger},F] \qquad \text{vs} \qquad [H,V]^{\dagger}\,[H,V]$$

- Symmetry: e. g., $SU(2)_L xU(1)$ fundamental-adjoint representation connection.

- Compositeness: Standard-model gauge structure. No information on whether this a formal or physical feature.

from gauge invariance. Formally, a boson field $B_o(x)$ expansion may be obtained using $B_o(x) = \sum_{lk} F_l(x)F_k(x) + [B_o(x) - \sum_{lk} F_l(x)F_k(x)]$, where $F_l(x)$, $F_k(x)$ are fermion fields reproducing $B_o(x)$'s quantum numbers, and the last two terms give corrections.

Quark-mass relation

The "punchline:" $|\langle Z|\sqrt{2}H_n|Z\rangle|^2 = m_Z^2$ and $\langle t|H_m + H_m^\dagger|t\rangle = m_t$,

$$\sqrt{2}H_n = H_m$$

Higgs mechanism

$$\langle \mathbf{H}^\dagger_{af}(x)\mathbf{H}_{af}(x)\rangle = (|a|^2 + |f|^2)v^2/2 = (|\chi_t|^2 + |\chi_b|^2)v^2/2 = v^2/2$$

$$(|a|^2 + |f|^2)v^2/2 = |m_t|^2 + |m_b|^2 = v^2/2$$

$$\Rightarrow$$

$$m_t = \sqrt{v^2/2 - m_b^2} \simeq 173.90 \text{ GeV}$$

$$v = 246 \text{ GeV}$$

$$m_b = 4 \text{ GeV}$$

For colliders: 2 chiral Higgs doublet

General two-Higgs doublet produces Yukawa term with flavor changing neutral currents.

Chiral Higgs

$$\begin{aligned}
\phi_1 &\to \frac{1}{2}(1+\tilde{\gamma}_5)\gamma^0\phi_1\frac{1}{2}(1-\tilde{\gamma}_5),\\
\phi_2 &\to \frac{1}{2}(1-\tilde{\gamma}_5)\gamma^0\phi_2\frac{1}{2}(1+\tilde{\gamma}_5),\\
\tilde{\phi}_1 &\to \frac{1}{2}(1+\tilde{\gamma}_5)\gamma^0\tilde{\phi}_1\frac{1}{2}(1-\tilde{\gamma}_5),\\
\tilde{\phi}_2 &\to \frac{1}{2}(1-\tilde{\gamma}_5)\gamma^0\tilde{\phi}_2\frac{1}{2}(1+\tilde{\gamma}_5),
\end{aligned}$$

leads to feasible Type II Yukawa:

Type II 2HDM

$$\begin{aligned}
\mathcal{L}^{\text{Chiral 2HDM}}_{Yukawa} \supset & tr\left[\sum_{iq} y^{U(2)}_{iq}\Psi^{\dagger}_{iR}(x)\tilde{\phi}_1\mathbf{\Psi}_{qL}(x)Y^{F(1)}_{iq} + \sum_{jq} y^{D(1)}_{qj}\mathbf{\Psi}^{\dagger}_{qL}(x)\phi_1\Psi_{jR}(x)Y^{F(1)}_{qj}\right] + hc\\
& + tr\left[\sum_{iq} y^{U(2)}_{iq}\Psi^{\dagger}_{iR}(x)\tilde{\phi}_2\mathbf{\Psi}_{qL}(x)Y^{F(2)}_{iq} + \sum_{jq} y^{D(1)}_{qj}\mathbf{\Psi}^{\dagger}_{qL}(x)\phi_2\Psi_{jR}(x)Y^{F(2)}_{qj}\right] + hc
\end{aligned}$$

Relevant points

- A spin BSM extension constrains SM fields, reproduces the electroweak interaction and its chiral property, generating also chiral scalars.
- In a SM projection, the *same* scalar field within SV and SF terms connects such sectors, and determines the quark mass.
- Chiral 2 Higgs-doublet models lack FCNC, and are therefore still feasible candidates to be detected at colliders.