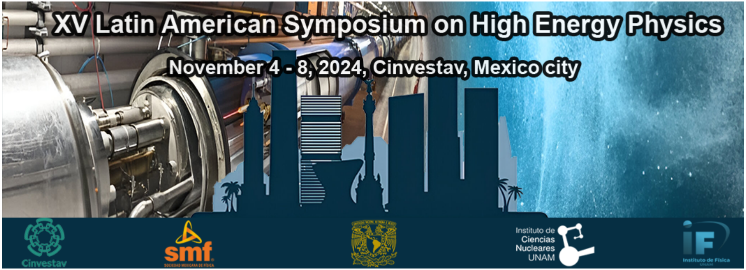
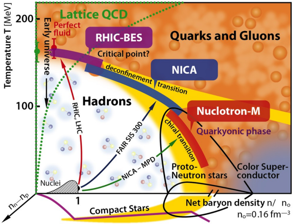


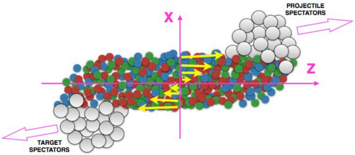
Thermal and baryon density modifications of the σ-boson propagator in an HTL-like approximation

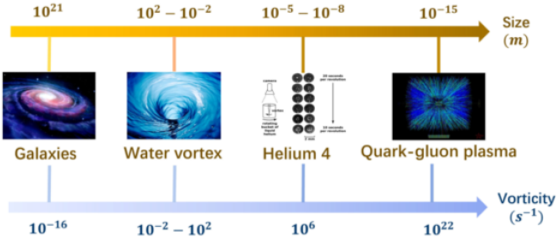
M.C. José Jorge Medina Serna

November 4^{th} of 2024

Heavy-ion collisions





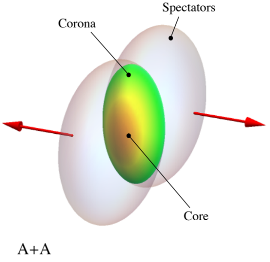


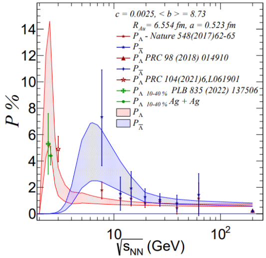
N. physics at JINR (official Web-Page).

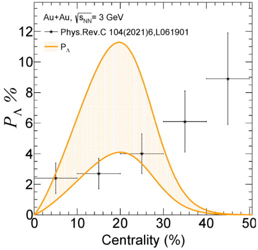
Lecture Notes in Physics, vol 987. Springer, Cham.

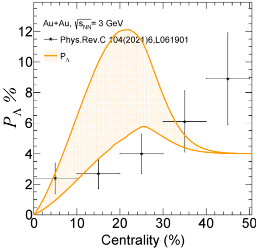
Nuclear Physics A, vol. 1005, p. 121752, 2021.

Core-Corona model for polarization of Λ







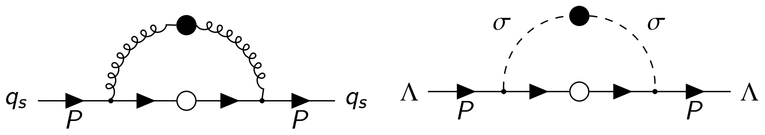


Particles 2023, 6, 405-415.

Λ interactions in the corona

The core, where the energy density is high, can be modeled as a QCD plasma.

The corona, where the energy density is low, can be modeled as a hadronic gas.



Then, we have to obtain the thermodynamic correction to the σ propagator.

Phys.Rev.D 109 (2024) 7, 074018

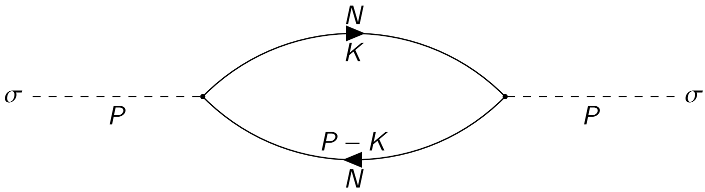
Physical Review C, 63(6), 065203.

One-loop thermodynamic correction to the scalar sigma meson

Considering the corona region as an baryonic gas, the one-loop correction to the σ propagator Δ^* at finite temperature T and baryonic chemical potential μ can be written in the imaginary time formalism as

$$\Delta^*(i\omega, p) = \frac{1}{\omega^2 + p^2 + m_\sigma^2 + \Pi}, \tag{1}$$

where Π is the σ self energy at one loop as is depicted in the Feynman diagram showed below.



Explicity, the σ self energy is

$$\begin{aligned}\Pi &= T\sum_n \int \frac{d^3k}{(2\pi)^3} g_\sigma^2 \, \mathrm{Tr}\left[(M_N - \not{K})(M_N - (\not{K} - \not{P}))\right] \\ &\times \tilde{\Delta}(K)\tilde{\Delta}(K-P). \end{aligned} \tag{2}$$

Where

$$\begin{aligned} P &= (i\omega, \vec{p}), \\ K &= (i\omega_n, \vec{k}). \end{aligned} \tag{3}$$

Tho compute expresion (2) we use the standar methods of sumation of frequencies which involve distributions functions and the energy of the particles.

$$\begin{aligned} E_1 &= \sqrt{M_N^2 + k^2} \\ E_2 &= \sqrt{M_N^2 + k^2 + p^2 - 2pk\cos\theta} \\ \tilde{n}_\pm(E) &= \frac{1}{e^{\frac{E\mp\mu}{T}} + 1} \end{aligned} \tag{4}$$

Approximation for $M_N > T$

We can make the change of variable $\xi^2 = M_N^2 + k^2$, $\xi d\xi = kdk$. This new variable has the information of the nucleon mass and allow us to make the following approximations,

$$
\begin{aligned}
E_1 &\approx \sqrt{\xi^2} = \xi \\
E_2 &\approx \sqrt{\xi^2 - 2\sqrt{\xi^2 - M_N^2}\,p\cos\theta + p^2} \\
&\approx \xi - p\cos\theta + \frac{p^2 - p^2x^2}{2\xi} \\
\tilde{n}_\pm(E_1) &\approx e^{-\frac{\xi}{T} \pm \frac{\mu}{T}} \equiv n_\pm^1 \\
\tilde{n}_\pm(E_2) &\approx n_\pm^1 - p\cos\theta \frac{dn_\pm^1}{d\xi} \equiv n_\pm^2
\end{aligned}
\tag{5}
$$

Leading order contribution to self energy

The leading order contribution term over M can be written as

$$\Pi = M_T p_0 \left((3x^3 - x) \log \left(\frac{x+1}{x-1} \right) + 6x^2 + \frac{2}{3} \right) - \frac{5 M_T p_0 x^4 \log \left(\frac{x^2 + \sqrt{3x^2+1} + 1}{x^2 - \sqrt{3x^2+1} + 1} \right)}{\sqrt{3x^2+1}} \tag{6}$$

Where we have defined

$$M_T = \frac{2g^2 M_N e^{-M/T} \sinh\left(\frac{\mu}{T}\right)}{\pi^2}$$
$$x = \frac{p0}{p} \tag{7}$$

Propagator with thermodynamic correction

If we notice that $\frac{x+1}{x-1} < 0$ and $\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1} < 0$ when $-1 < x < 1$, we can express the propagator as

$$\Delta^*(q_0) = \frac{1}{p_0^2 - p^2 - m_\sigma^2 - F(p_0, p) + i\pi A(p_0, p)\theta(p^2 - p_0^2)}, \quad (8)$$

Where we have defined

$$F(p_0,p) = M_T p_0 \left((3x^3 - x) \log \left(\left| \frac{x+1}{x-1} \right| \right) + 6x^2 + \frac{2}{3} \right) - \frac{5 M_T p_0 x^4 \log \left(\left| \frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1} \right| \right)}{\sqrt{3x^2+1}}, \quad (9)$$

and

$$A(p_0,p) = M_T p_0 \left((3x^3 - x) - \frac{5x^4}{\sqrt{3x^2+1}} \right) \Bigg]. \quad (10)$$

Spectral density

We can obtain the spectral density making the analytic continuation and taking the imaginary part of the propagator,

$$\rho_\sigma(p_0, p) = 2\mathrm{Im}\Delta^*(q_0 + i\eta, q). \tag{11}$$

Then, it is easy to show that

$$\begin{aligned}\rho_\sigma(p_0, p) &= 2\pi Z(\omega(p))\left[\delta(p_0 - \omega_\sigma(p)) - \delta(p_0 + \omega_\sigma(p))\right] \\ &+ \beta(p_0, p),\end{aligned} \tag{12}$$

with

$$\beta(p0, p) = \frac{A(p_0, p)\theta(p^2 - p_0^2)}{(p_0^2 - p^2 - m_\sigma^2 - F(p_0, p))^2 - (\frac{A(p_0,p)\pi}{2})^2}. \tag{13}$$

Poles of the propagator

Now, $\omega_\sigma(p)$ is the pole of

$$\frac{1}{p_0^2 - p^2 - m_\sigma^2 - F(p_0, p)}, \tag{14}$$

and $Z(\omega_\sigma(p))$ the residue. Then

$$\omega_\sigma(p)^2 - p^2 - m_\sigma^2 - F(\omega_\sigma(p), p) = 0. \tag{15}$$

Explicity, this is

$$\begin{aligned} \omega_\sigma(p)^2 &= p^2 + m_\sigma^2 + M_T p_0 \\ &\times \quad (\left((3x^3 - x) \log\left(\left| \frac{x+1}{x-1} \right| \right) + 6x^2 + \frac{2}{3} \right) \\ &- \frac{5x^4 \log\left(\left| \frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1} \right| \right)}{\sqrt{3x^2+1}} \;) \end{aligned} \tag{16}$$

Summary

The discussion and results obtained can be summary as follows:

- The RMF model can be used to calculated the σ self energy.
- We aproximate the propagator with HTL-like model wich has the information of the large mass of the nucleons.
- The spectral desity of the propagator has a similar behavior as the HTL model.
- The thermic σ-boson propagator can be used to estimated the Λ polarization contribution from the corona region.

BACKUP

Different sets of couplings in the RMF model

For the interactions of nucleons and Λ in the RMF model, some couplings are

	NL3	BigApple	TM1	IUFSU
M_σ [MeV]	508.194	492.730	511.198	491.5
g_σ	10.217	9.6699	10.0289	9.9713
$g_{\sigma\Lambda}/g_\sigma$	0.618896	0.616322	0.621052	0.616218

Traces of the Feynmann diagram

The trace give us

$$\Pi = 4T\sum_n \int \frac{d^3k}{(2\pi)^3} g_\sigma^2 \left[M_N^2 + \omega_n^2 + k^2 - \omega_n\omega - \vec{k}\vec{p}\right] \times \tilde{\Delta}(K)\tilde{\Delta}(K-P). \quad (17)$$

It is convenient to separate the last expression into three integrals,

$$\Pi_1 = 4T\sum_n \int \frac{d^3k}{(2\pi)^3} g_\sigma^2 \left[M_N^2 + \omega_n^2 + k^2\right] \tilde{\Delta}(K)\tilde{\Delta}(K-P),$$
$$\Pi_2 = -4T\sum_n \int \frac{d^3k}{(2\pi)^3} g_\sigma^2 \left[\vec{k}\vec{p}\right] \tilde{\Delta}(K)\tilde{\Delta}(K-P), \quad (18)$$
$$\Pi_3 = -4T\sum_n \int \frac{d^3k}{(2\pi)^3} g_\sigma^2 \left[\omega_n\omega\right] \tilde{\Delta}(K)\tilde{\Delta}(K-P).$$

Computation of Π_1

The term $\left[M_N^2 + \omega_n^2 + k^2\right]$ is the inverse of $\tilde{\Delta}(K)$, then

$$\begin{aligned}
\Pi_1 &= 4T \sum_n \int \frac{d^3k}{(2\pi)^3} g_\sigma^2 \, \tilde{\Delta}(K-P) \\
&= 4T \sum_n \int \frac{d^3k}{(2\pi)^3} g_\sigma^2 \, \tilde{\Delta}(K) \\
&= \frac{2g_\sigma^2}{E_1} \int \frac{d^3k}{(2\pi)^3} \left(1 - \tilde{n}_+(E_1) - \tilde{n}_-(E_1)\right). \qquad (19)
\end{aligned}$$

Now we apply the approximations and omit the temperature independent term to obtain

$$\begin{aligned}
\Pi_1 &= -2g^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{\xi} \left(e^{\frac{-\mu-\xi}{T}} + e^{\frac{\mu-\xi}{T}}\right) \\
&= -2g^2 \int \frac{d\xi d(\cos\theta)}{(2\pi)^2} \sqrt{\xi^2 - M^2} \cosh\left(\frac{\mu}{T}\right) \qquad (20)
\end{aligned}$$

Permormig the remaining integrals,

$$\Pi_1 = -\frac{2g^2}{\pi^2} MT\ K_1\left(\frac{M}{T}\right)\cosh\left(\frac{\mu}{T}\right), \quad (21)$$

where $K_1(x)$ is the modified Bessel function of second kind. Now, if we consider that $M >> T$, then de Bessel function behaves as

$$K_1(\frac{M}{T}) \approx \sqrt{\frac{\pi}{2}} e^{-\frac{M}{T}} \sqrt{\frac{T}{M}}. \quad (22)$$

Therefore

$$\Pi_1 \approx -\frac{\sqrt{2}g^2}{\pi^{\frac{3}{2}}} M^{\frac{1}{2}} T^{3/2} \cosh\left(\frac{\mu}{T}\right). \quad (23)$$

Computation of Π_2

The expresion for Π_2 is

$$\Pi_2 = -4T \sum_n \int \frac{d^3k}{(2\pi)^3} g_\sigma^2 \left[\vec{k}\vec{p}\right] \tilde{\Delta}(K)\tilde{\Delta}(K-P). \tag{24}$$

We use the following identity,

$$T \sum_n \tilde{\Delta}(i\omega_n - \mu, \vec{k})\tilde{\Delta}(i(\omega_n - \omega) + \mu, \vec{k} - \vec{p}) =$$

$$\frac{1}{E_1 E_2}\left[\frac{1 - \tilde{n}_+(E1) - \tilde{n}_-(E2)}{i\omega - E_1 - E_2} - \frac{1 - \tilde{n}_-(E1) - \tilde{n}_+(E2)}{i\omega + E_1 + E_2}\right] - \tag{25}$$

$$\frac{1}{E_1 E_2}\left[\frac{\tilde{n}_-(E1) - \tilde{n}_-(E2)}{i\omega + E_1 - E_2} - \frac{\tilde{n}_+(E1) - \tilde{n}_+(E2)}{i\omega - E_1 + E_2}\right].$$

Now let us define

$$
\begin{aligned}
I_1 &= \int d\xi \; (\xi^2 - M_N^2)\xi \; \frac{(n_+^1 + n_-^2)}{E_1 \; E_2(p_0 - \Delta E^+)}, \\
I_2 &= \int d\xi \; (\xi^2 - M_N^2)\xi \; \frac{(n_-^1 + n_+^2)}{E_1 \; E_2(p_0 + \Delta E^+)}, \\
I_3 &= \int d\xi \; (\xi^2 - M_N^2)\xi \; \frac{(n_-^1 - n_-^2)}{E_1 \; E_2(p_0 + \Delta E^-)}, \\
I_4 &= \int d\xi \; (\xi^2 - M_N^2)\xi \; \frac{(n_+^1 - n_+^2)}{E_1 \; E_2(p_0 - \Delta E^-)}.
\end{aligned} \tag{26}
$$

With

$$
\begin{aligned}
\Delta E^+ &= E_1 + E_2, \\
\Delta E^- &= E_1 - E_2.
\end{aligned} \tag{27}
$$

Then we can write

$$
\Pi_2 = -\frac{4 g_\sigma^2 p}{(2\pi)^2} \int d(\cos\theta) \cos\theta \left[-(I_1 - I_2) - (I_3 - I_4) \right] \tag{28}
$$

At leading order, we can write

$$\begin{aligned}\Pi_2 &= \frac{2g^2 p_0 e^{-\frac{M}{T}}}{\pi^2}\Big(\frac{\left(48x^4+26x^2+1\right)}{3x^2\left(x^2+1\right)} \\ &+ \frac{\left(22x^4+13x^2+1\right)\log\left(\frac{x+1}{x-1}\right)}{2\left(x^3+x\right)} \\ &+ \frac{\left(2x^2+1\right)\left(19x^2+5\right)\log\left(\frac{\sqrt{1+3x^2}+x-1}{\sqrt{1+3x^2}-x+1}\right)}{\left(x^2+1\right)\sqrt{\left(1+3x^2\right)}}\Big) \\ &\times \left(p_0\cosh\left(\frac{\mu}{T}\right) - T\sinh\left(\frac{\mu}{T}\right)\right)\end{aligned}$$

(29)

Computation of Π_3

The expresion for Π_3 is

$$\Pi_3 = -4T \sum_n \int \frac{d^3k}{(2\pi)^3} g_\sigma^2 \; [\omega_n \omega] \, \tilde{\Delta}(K) \tilde{\Delta}(K-P). \tag{30}$$

We use the following identity,

$$\begin{aligned} & T \sum_n \omega_n \tilde{\Delta}(i\omega_n - \mu, \vec{k}) \tilde{\Delta}(i(\omega_n - \omega) + \mu, \vec{k} - \vec{p}) = \\ & \frac{i}{E_2} \left[\frac{1 - \tilde{n}_+(E1) - \tilde{n}_-(E2)}{i\omega - E_1 - E_2} + \frac{1 - \tilde{n}_-(E1) - \tilde{n}_+(E2)}{i\omega + E_1 + E_2} \right] - \\ & \frac{i}{E_2} \left[\frac{\tilde{n}_-(E1) - \tilde{n}_-(E2)}{i\omega + E_1 - E_2} + \frac{\tilde{n}_+(E1) - \tilde{n}_+(E2)}{i\omega - E_1 + E_2} \right] \end{aligned} \tag{31}$$

Now let us define

$$\begin{aligned}
J_1 &= \int d\xi \, \sqrt{\xi^2 - M_N^2} \, \xi \, \frac{(n_+^1 + n_-^2)}{E_2(p_0 - \Delta E^+)}, \\
J_2 &= \int d\xi \, \sqrt{\xi^2 - M_N^2} \, \xi \, \frac{(n_-^1 + n_+^2)}{E_2(p_0 + \Delta E^+)}, \\
J_3 &= \int d\xi \, \sqrt{\xi^2 - M_N^2} \, \xi \, \frac{(n_-^1 - n_-^2)}{E_2(p_0 + \Delta E^-)}, \\
J_4 &= \int d\xi \, \sqrt{\xi^2 - M_N^2} \, \xi \, \frac{(n_+^1 - n_+^2)}{E_2(p_0 - \Delta E^-)}.
\end{aligned} \tag{32}$$

With

$$\begin{aligned}
\Delta E^+ &= E_1 + E_2, \\
\Delta E^- &= E_1 - E_2.
\end{aligned} \tag{33}$$

Then we can write

$$\Pi_3 = -\frac{4 g_\sigma^2 p_0}{(2\pi)^2} \int d(\cos\theta) \left[-(J_1 + J_2) + (J_3 + J_4)\right] \tag{34}$$

At leading order, we can write

$$\Pi_3 = M_T p_0 \left((3x^3 - 1x) \log\left(\frac{x+1}{x-1}\right) + 6x^2 + \frac{2}{3} \right) - \frac{5 M_T p_0 x^4 \log\left(\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1}\right)}{\sqrt{3x^2+1}}. \tag{35}$$