1. **Title Slide**

Good afternoon, everyone. My name is Luis Carlos Parra Lara, and I'm excited to present my work titled "The Role of Goldstone Modes to Determine the Sound Velocity in Isospin-Imbalanced strongly interacting Matter."

—-------------------------------Introduction—-------------------------------

2. **Motivation Slide**

Let's dive into the motivation behind this study. As many of you are aware, Lattice Quantum Chromodynamics, or LQCD, has emerged as a powerful tool for exploring the properties of strongly interacting matter. It provides high-precision results, particularly for isospin-imbalanced systems at zero baryonic density, where the sign problem is not present.

Recent and not too recent LQCD calculations reveal a notable peak in the speed of sound within isospin-imbalanced matter. This finding suggests the presence of critical physical behaviors and a potential phase transition, making it a fascinating phenomenon in the context of Quantum Chromodynamics. Understanding this peak is not merely an academic exercise; it has implications for various fields in physics, from the behavior of nuclear matter in neutron stars, passing to the conditions in the early universe to particle collisions where the pion condensation happens.

However, to fully grasp the LQCD results, especially regarding the sound velocity peak, we require a complementary analytical theory. This theory can help us interpret the numerical data and uncover the underlying physics.

3. **Sound Velocity Peaks Slide**

Now, let's take a closer look at the observed sound velocity peaks in isospin-imbalanced matter.

[**Pause to show the slide**]

In this figure, we can see the squared speed of sound, plotted against the isospin chemical potential. The data from Abbott's group, shown in green, reveals a peak around 2.5 times the pion mass, followed by a smooth decline. In contrast, Brandt's group, depicted in blue, exhibits a sharper peak at a lower value of 1.6 times the pion mass.

These results highlight the intriguing behavior of sound velocity in different calculations.

4. **Approach and the Role of the Sigma Model Slide**

Moving on, let's discuss our approach and the role of the sigma model in our analysis.

First, while LQCD provides reliable data in specific regimes, its computational limitations, particularly in high-density environments, demand additional analytical frameworks.

This is where the linear sigma model with quarks comes in. The LSMq approximates QCD physics with about 20% accuracy for two flavors. Its simplicity allows us to explore the underlying physics in greater depth, making it an invaluable tool in our investigation.

Despite its approximations, the LSMq captures essential features of QCD and provides qualitative insights that are particularly useful for studying phase transitions and sound velocity in isospin-imbalanced matter.

The ultimate goal of our work is to develop a theoretical framework that aligns with LQCD data while revealing the physics behind the sound velocity peak. This understanding is crucial, as it can shed light on the implications of these phenomena across various areas of physics.

—-------------------------------LSMq—-------------------------------

5. Theoretical Framework Slide

Now, let’s delve into the theoretical framework underlying our study.

We base our model on the linear sigma model with two-flavor quarks, which incorporates mesonic fields, denoted by σ and π, alongside fermionic fields represented by ψ. The Lagrangian density for our model can be expressed as follows:

[**Pause to show the Lagrangian**]

In this Lagrangian, the first part comprises the kinetic terms for σ and π, which describe the dynamics of the mesonic degrees of freedom. Next, the potential terms, (...), lead to spontaneous symmetry breaking, resulting in a shift of the field σ to a vacuum expectation value, $\sigma \to \sigma + v$.

Furthermore, we have interaction terms, including (...), which capture the interactions between mesons and fermions.

In this framework, we will introduce the isospin chemical potential to study the system's behavior under isospin imbalance, which leads to the intriguing phenomenon of pion condensation.

6. Isospin Chemical Potential Slide

Now, let’s discuss the introduction of the isospin chemical potential.

We modify the covariant derivative for the charged pions to account for the isospin chemical potential:

[**Pause to show the covariant derivative equation**]

This adjustment affects the energies of the charged pion states. We apply a similar modification for the fermionic fields, which allows us to explore the dynamics under isospin imbalance.

When the chemical potential exceeds the pion mass in vacuum, the plus pion becomes a Goldstone boson. This leads to the possibility of charged pion condensation, represented by:

[**Pause to show the pion condensation equation**]

Here, Δ represents the magnitude of the condensation, and θ is a free parameter associated with the symmetry that remains after the spontaneous symmetry breaking of the U(1) group.

7. Key Elements from the Lagrangian Slide

Now let's extract some key elements from the resulting Lagrangian that are crucial for our analysis.

[**Pause to show the key elements list**]

First, we have the tree-level potential, which determines the ground state configuration and drives spontaneous symmetry breaking. It is given by:

[**Pause to show the tree-level potential equation**]

This potential plays a vital role in characterizing the ground state of our system.

Next, we consider the fermionic propagator, denoted S^{-1}, which encapsulates the dynamics of fermions influenced by the pion condensate and isospin chemical potential. This propagator is essential for understanding how fermionic excitations behave in this framework.

Lastly, we have the mesonic propagator,D^{-1}, which describes the behavior of mesons, particularly how their dispersion relations are modified due to the presence of the isospin chemical potential.

These components provide the necessary groundwork for calculating interactions, scattering amplitudes, and effective masses within the system, allowing us to probe deeper into the nature of isospin-imbalanced matter.

8. Inverse Propagator of Mesons Slide

Now, we turn our attention to the inverse propagator of the mesons.

For simplicity, we set θ=0. The inverse propagator is represented by the following matrix:

[**Pause to show the inverse propagator equation**]

This expression captures the essential mixing of the mesons, a crucial factor in our analysis. The diagonal terms correspond to the dynamics of the scalar and pseudoscalar mesons, while the off-diagonal elements illustrate how the charged mesons couple with the sigma meson. This coupling significantly influences the dynamics of the system, as it can lead to modified dispersion relations and mixing behaviors that we will explore further.

The mass terms in the Lagrangian are defined as:

[**Pause to show the mass definitions**]

Here, mσ2m_\sigma^2mσ2, mπ02m_{\pi_0}^2mπ02, and mch2m_{ch}^2mch2 denote effective mass squares for the sigma and charged mesons, derived from the parameters of our model. These relations show how the vacuum expectation values and interaction strengths contribute to the meson masses.

One-Loop Corrections, Fermions Slide

Next, we consider the one-loop corrections from fermions.

The contribution to the one-loop effective potential from fermions can be expressed as:

[**Pause to show the fermionic potential equation**]

Here, Sf−1S_f^{-1}Sf−1 is a non-diagonal 2×22 \times 22×2 matrix in flavor space. At zero temperature, we find that the one-loop effective potential simplifies to:

[**Pause to show the simplified potential equation**]

In this equation, Nc=3N_c = 3Nc=3 represents the number of colors, and EΔuE_\Delta^{u}EΔu and EΔdE_\Delta^{d}EΔd describe the energy contributions from the up and down quarks, respectively.

We have chosen the chemical potentials for the quarks as μd=−μI/2\mu_d = -\mu_I/2μd=−μI/2 and μu=μI/2\mu_u = \mu_I/2μu=μI/2 to ensure consistency with our isospin chemical potential framework. These energy terms highlight how fermionic excitations interact with the condensate and contribute to the effective potential.

One-Loop Corrections, Bosons Slide

Finally, we will discuss the one-loop corrections from bosons.

The contribution to the one-loop effective potential from bosons is given by:

[**Pause to show the bosonic potential equation**]

In this expression, Db−1D_b^{-1}Db−1 denotes the non-diagonal boson inverse propagator.

It's essential to note the structure of the boson inverse propagator: only the neutral pion does not mix with the other mesons. The charged pions and the sigma meson exhibit mixing. In this context, mπ02m_{\pi_0}^2mπ02 corresponds to the square of the neutral pion mass, while mσ2m_\sigma^2mσ2 and mch2m_{ch}^2mch2 are not the actual squares of the sigma and charged pion masses. Instead, they represent useful combinations of parameters that will be critical in our further analysis of the system's behavior.

Goldstone Modes and Symmetry Breaking

In this section, we address the implications of breaking the U(1)IU(1)_{I}U(1)I global symmetry, which results in the emergence of a Goldstone boson. To explore the conditions imposed on the vacuum expectation value v and the parameter Δ by the presence of a massless mode, we analyze the limit where the components of K^μ approach zero. **[Point to the equation: determinant of the inverse boson propagator]** Setting the determinant of the inverse boson propagator to zero gives us the following equation:

[Point to the conditions derived from the equation]

Goldstone Modes and Symmetry Breaking

From the analysis of these equations, we identify three potential solutions. However, within the region defined by $\mu_I \geq m_\pi$, only two of these solutions are real, specifically given by:

[Point to the equations for Δ_1 and Δ_2]

It is noteworthy that Δ_2 corresponds to the condition required for the existence of a massless mode at tree-level, while Δ_1 is derived from loop corrections. This illustrates a non-trivial relationship between v, μ_I, and Δ.

[Emphasize the statement about physical states]

Relationship Between Couplings

Next, we explore the crucial relationship between the couplings in our model, established through the renormalization procedure. The Ward-Takahashi identity provides the following important relation:

[Point to the relationship between couplings]

Moreover, at tree level, these couplings can be expressed in terms of physical masses:

[Point to the equations for λ and g]

For $N_c = 3$, we derive the mass relation:

[Point to the mass relation]

This relationship is critical for ensuring the internal consistency of the model and for connecting the low-energy effective theory with the underlying microscopic parameters.

Squared Speed of Sound vs. Isospin Chemical Potential

In this slide, we present the relationship between the squared speed of sound and the isospin chemical potential. **[Point to the graph]** At tree level, we can already observe a peak, indicating that the theory predicts a non-trivial behavior even before considering any further corrections. When we analyze the case with only fermions included, **[point to the smooth peak]** the peak becomes very smooth and barely noticeable. However, as we incorporate mesons into the model, **[point to the pronounced peak]** the peak re-emerges prominently.

This observation strongly suggests that the origin of the peak is fundamentally linked to the intrinsic dynamics of the mesons. **[Emphasize the importance of proper treatment]** It is only through their proper and self-consistent treatment that we can truly reveal their nature and the underlying physics contributing to this behavior.

Robustness of the Model

Moving on to this slide, we discuss the robustness of our model. **[Point to the second graph]** After applying the physical restrictions we previously discussed, the system reduces to a model with only two free parameters: the pion mass and the quark mass. We observe that fixing the pion mass while varying mfm_fmf does not lead to the disappearance of the peak. **[Point to the significance of the peak's persistence]**

In fact, the peak persists across a wide range of physical quark masses, indicating a remarkable stability in the behavior of the system. **[Emphasize the robustness of mesonic dynamics]** This robustness further reinforces the critical role of mesonic dynamics in determining the properties of the system.

Conclusions

As we conclude, let's summarize the key findings of our analysis. **[Point to the first bullet]** Within the linear sigma model, we have demonstrated that the sound velocity peak in isospin-imbalanced matter is closely linked to the emergence of Goldstone modes, which arise from spontaneous symmetry breaking. This connection is critical in understanding the dynamics of the system.

[Point to the second bullet] Our theoretical framework aligns well with results from Lattice QCD, providing a comprehensive understanding of the peak's behavior and its implications for strongly interacting matter. This agreement highlights the robustness of our model.

[Point to the third bullet] Furthermore, we note that the interplay between the isospin chemical potential and the pion condensation mechanism plays a crucial role in shaping the dynamics of the system, impacting various physical observables. This insight is vital for future explorations in the field.

[Point to the fourth bullet] Looking ahead, our future work will focus on refining our calculations by increasing the number of free parameters to include the ρ\rhoρ mesons. We also aim to extend our analysis to finite temperature, allowing us to explore more interesting phenomena such as the BEC-BCS transition, chiral restoration, the triple point, and how the peak behaves as a function of temperature and isospin chemical potential.

Acknowledgments

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Finally, this presentation is based on our collaborative work titled "On the origin of the peak of the sound velocity for isospin imbalanced strongly interacting matter," authored by Ayala, Lopes, Farias, and myself, which is available on arXiv.

Questions

In closing, I want to thank you all for your attention! **[Point to the audience]** I now welcome any questions you may have. **[Pause for audience interaction]**