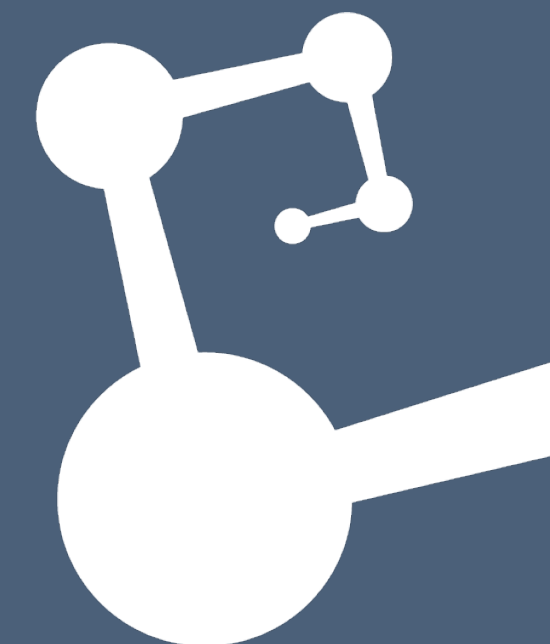




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Two-gluon one-photon vertex in the presence of a magnetic field:

General structure and one-loop approximation in the intermediate field regime

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Two-gluon one-photon vertex in a magnetic field and its explicit one-loop approximation in the intermediate field strength regime

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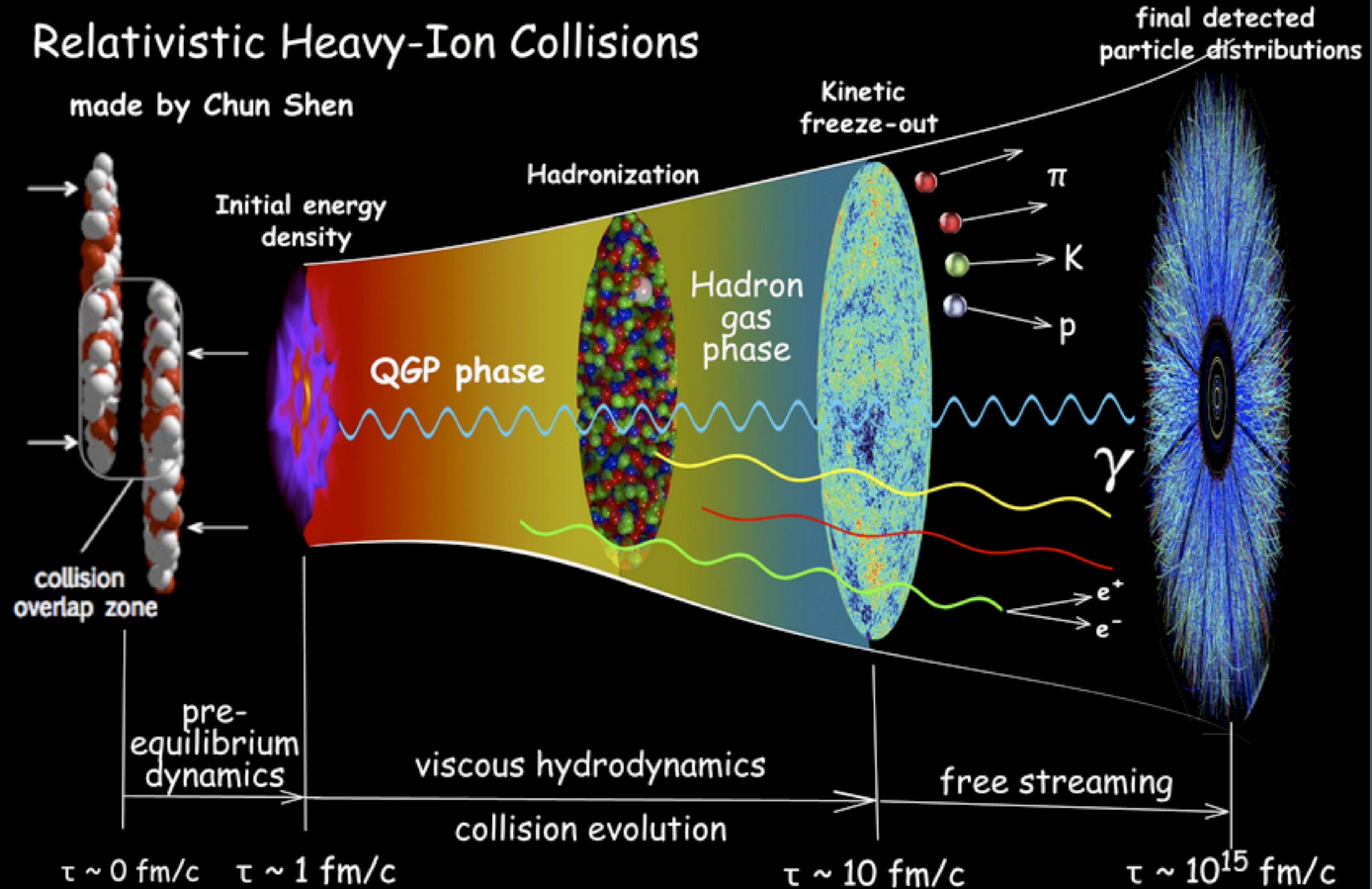
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We find the general structure for the two-gluon one-photon vertex in the presence of a constant magnetic field. We show that, when accounting for the symmetries satisfied by the strong and electromagnetic interactions under parity, charge conjugation and gluon interchange, and for gluons and photons on mass-shell, there exist only three possible tensor structures that span the vertex. These correspond to external products of the polarization vectors for each of the particles in the vertex. We also explicitly compute the one-loop approximation to this vertex in the intermediate field strength regime, which is the most appropriate one to describe possible effects of the presence of a magnetic field to enhance photon emission during preequilibrium in peripheral relativistic heavy-ion collisions. We show that the most favored direction for the photon to propagate is in the plane transverse to the field, which is consistent with a positive contribution to ν_2 and may help to understand the larger than expected elliptic flow coefficient measured in this kind of reaction.

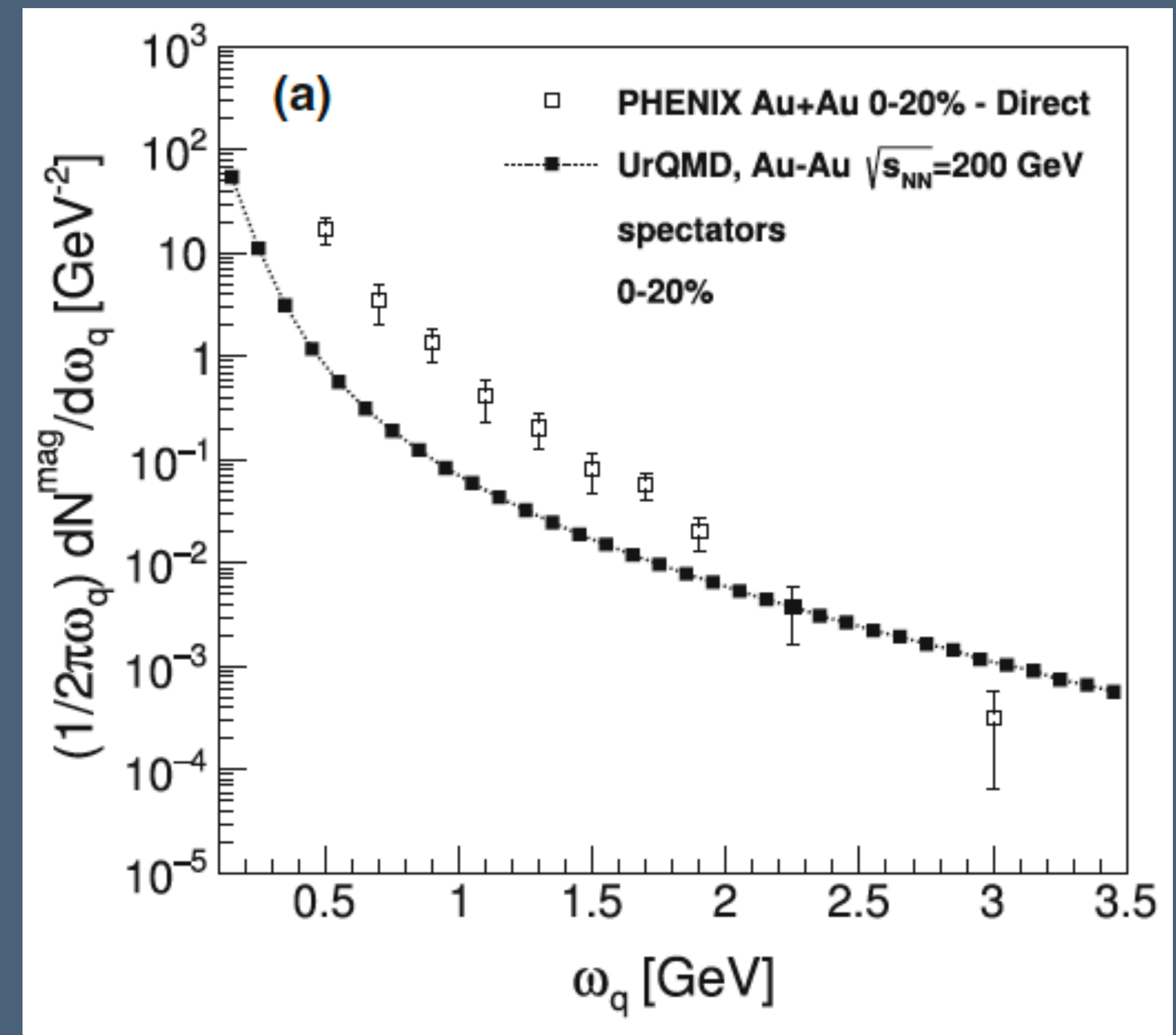
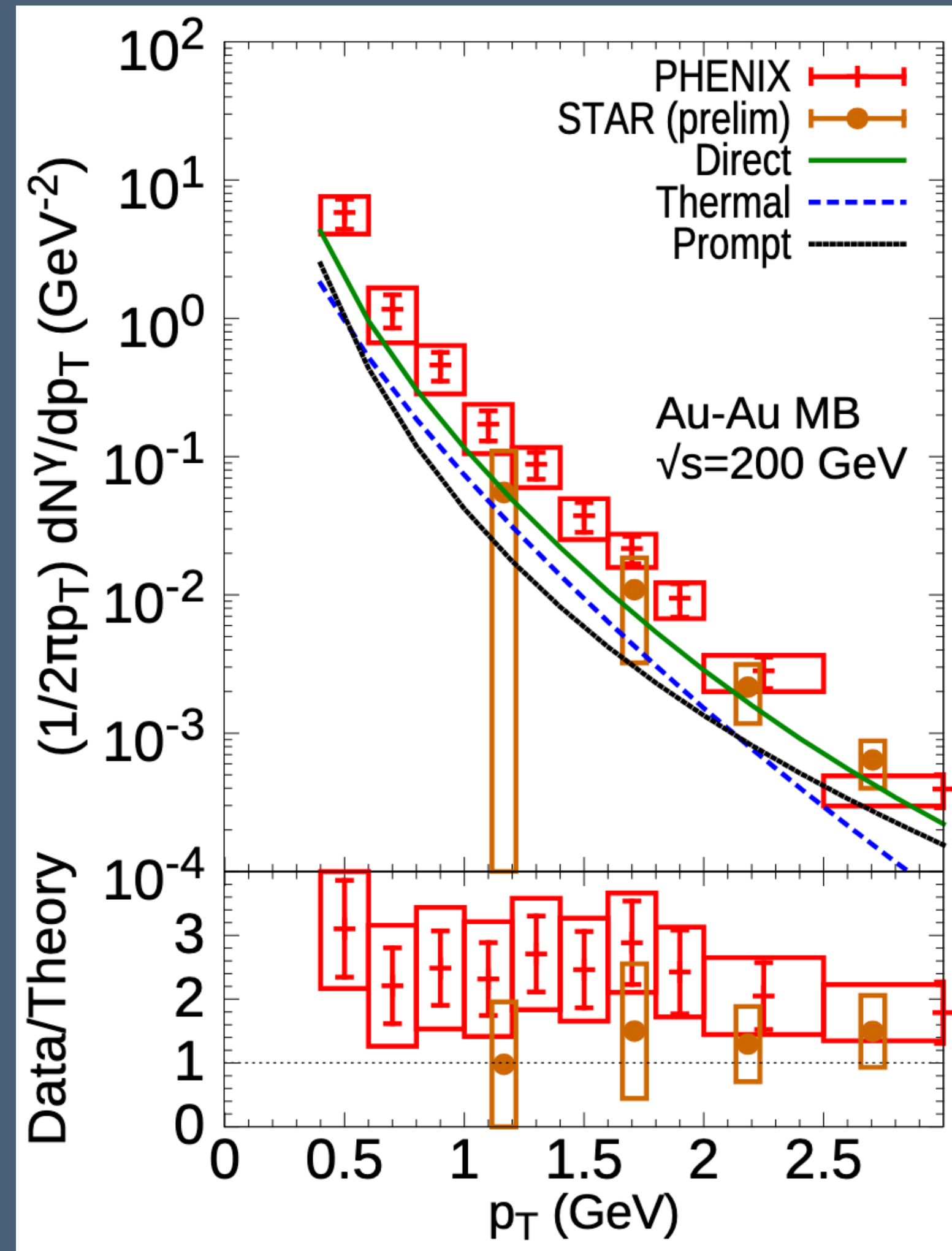
DOI: [10.1103/PhysRevD.110.076021](https://doi.org/10.1103/PhysRevD.110.076021)

Relativistic Heavy-Ion Collisions

made by Chun Shen



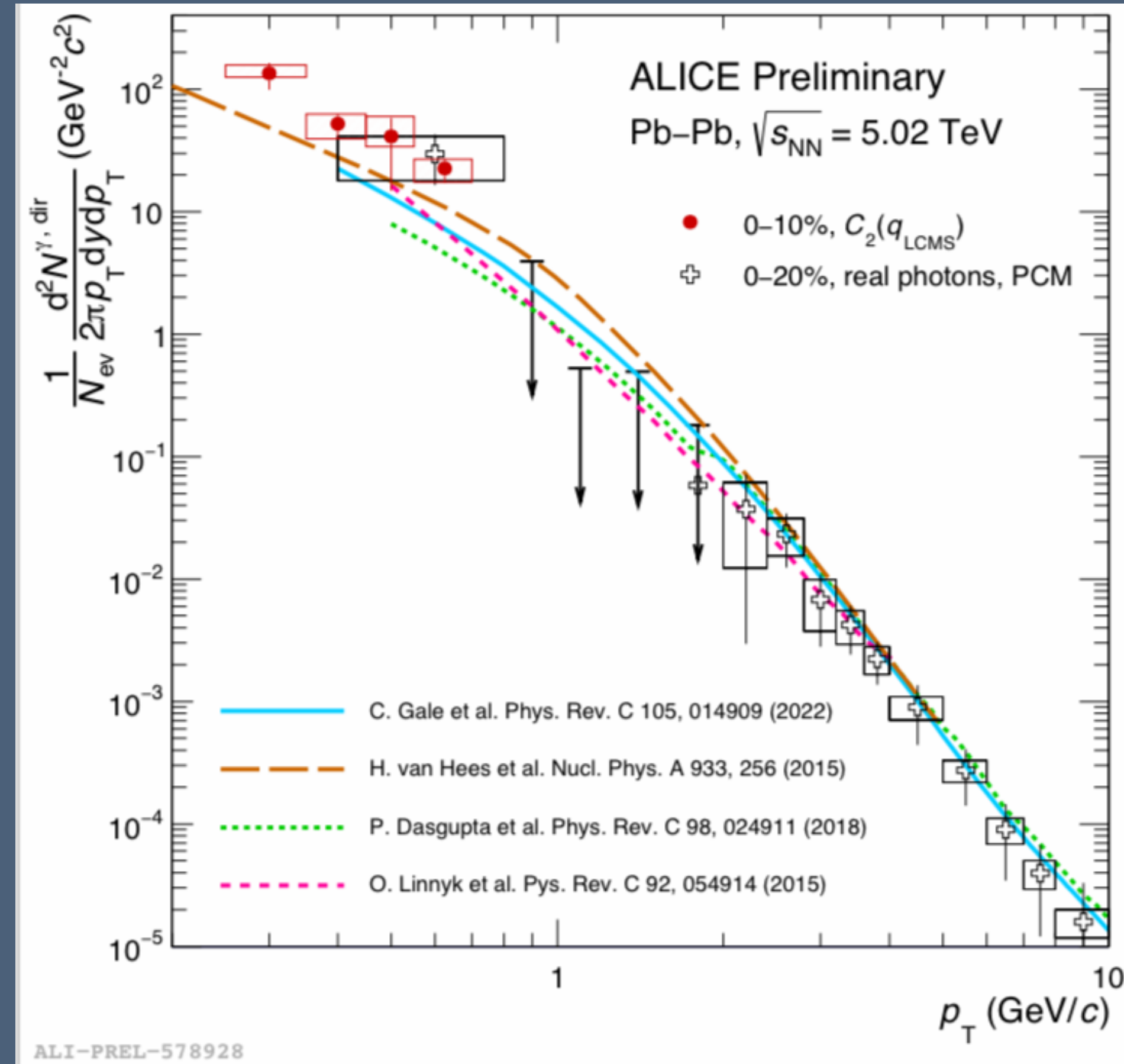
Photon puzzle



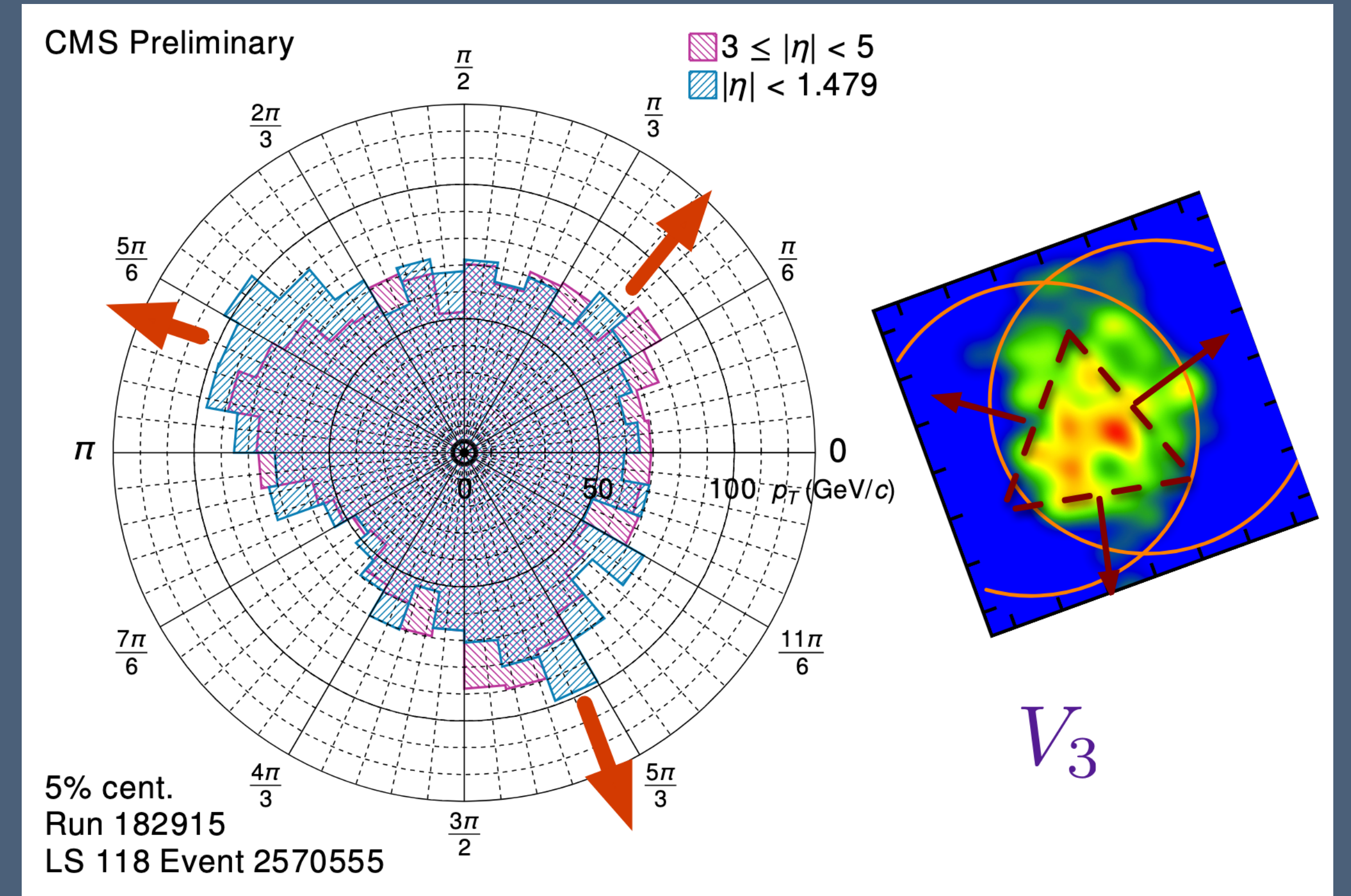
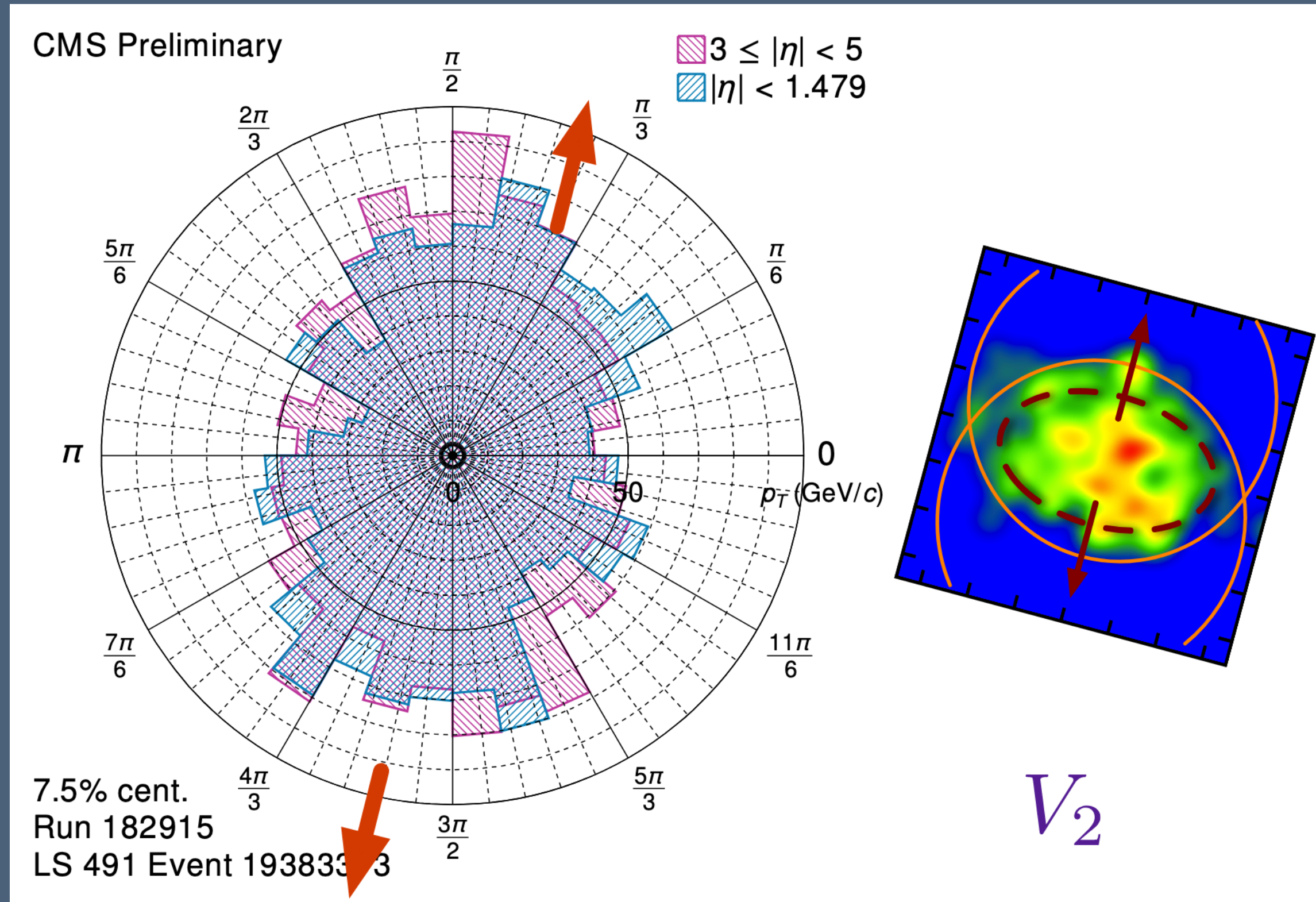
EPJ A **56**, 53 (2020), arXiv:1904.02938

PRC **93**, 044906 (2016), arXiv:1509.06738

Photon puzzle



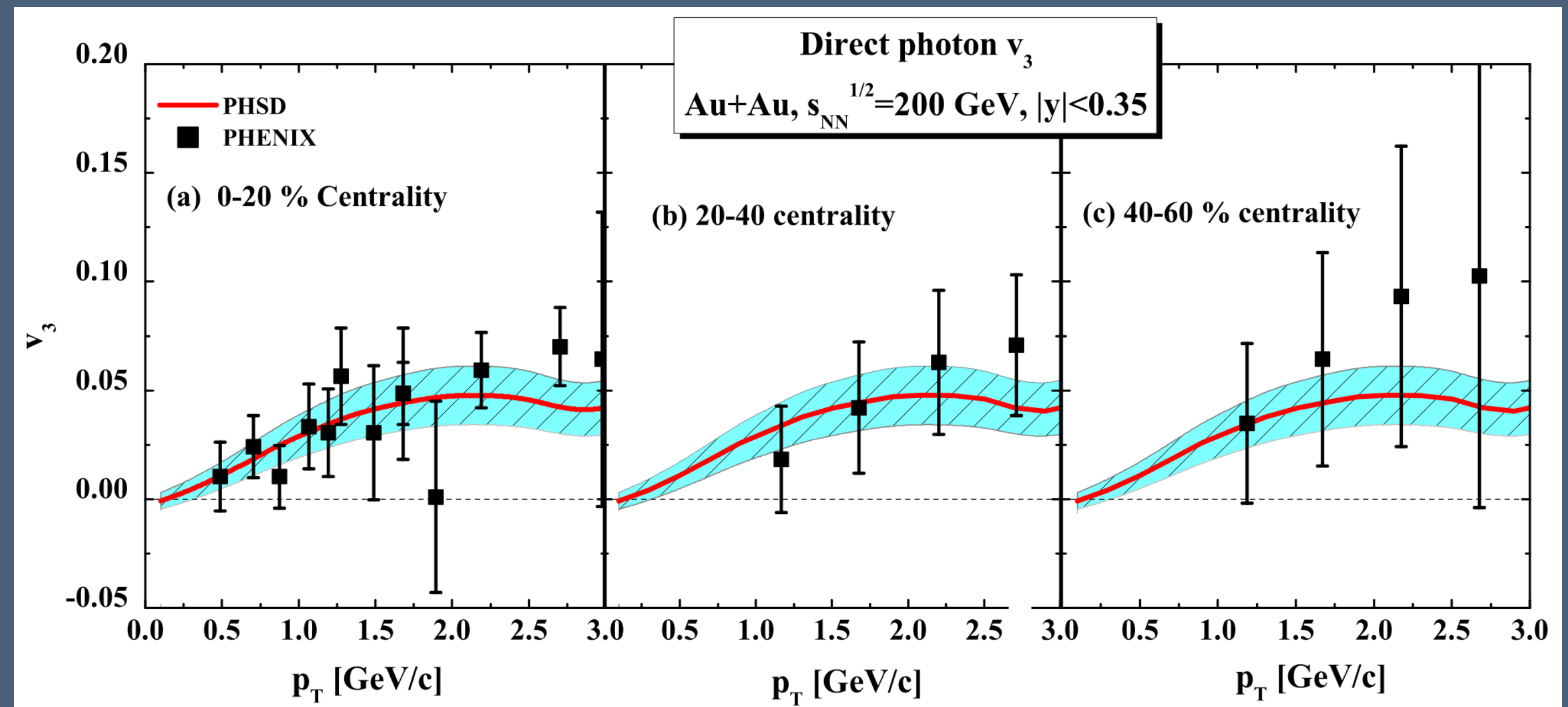
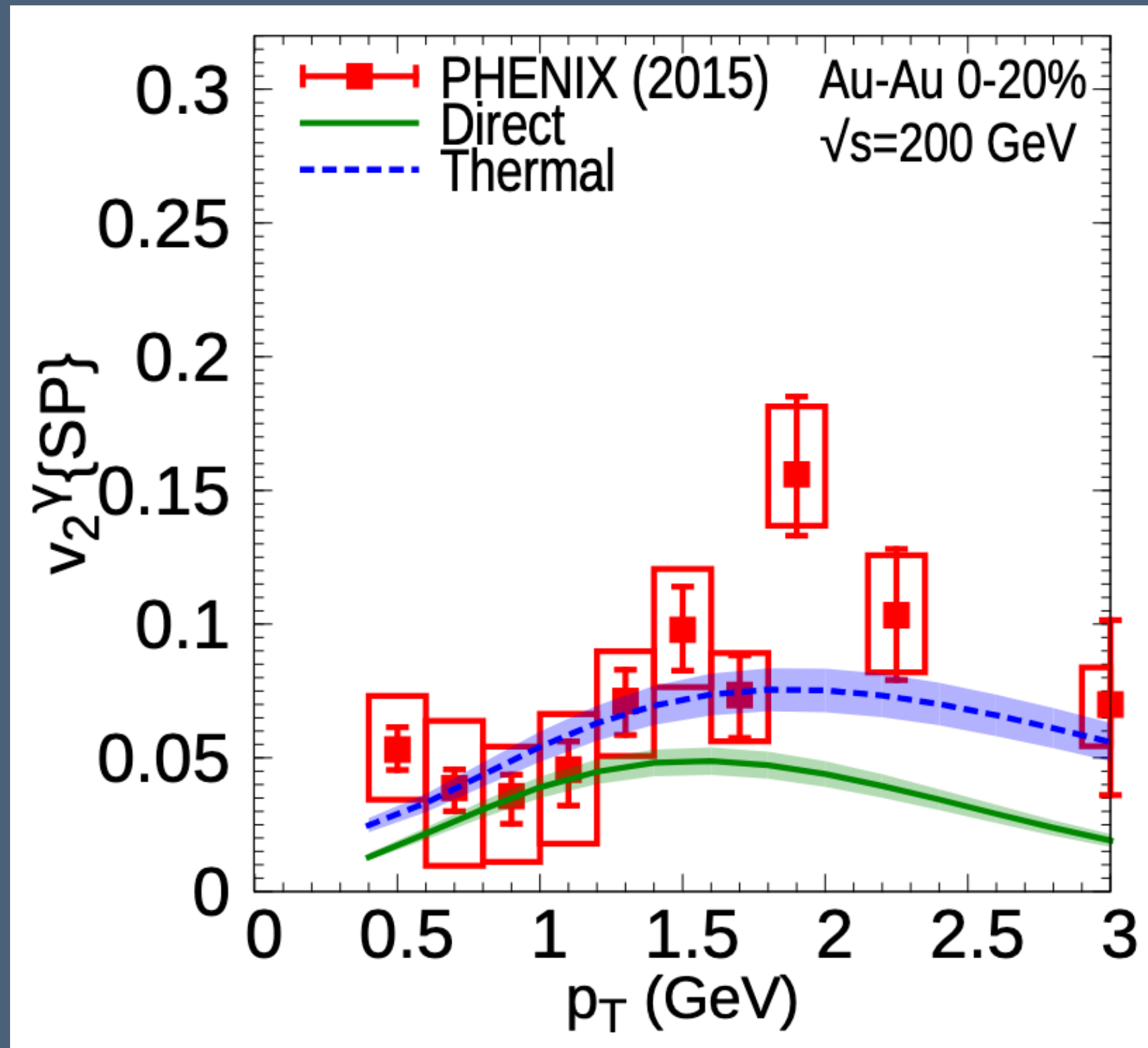
Elliptic and triangular flow



D. Teaney, ASU Colloquium

https://www.public.asu.edu/~ishovkov/colloquium/slides/Colloquium_Slides_Teaney.pdf

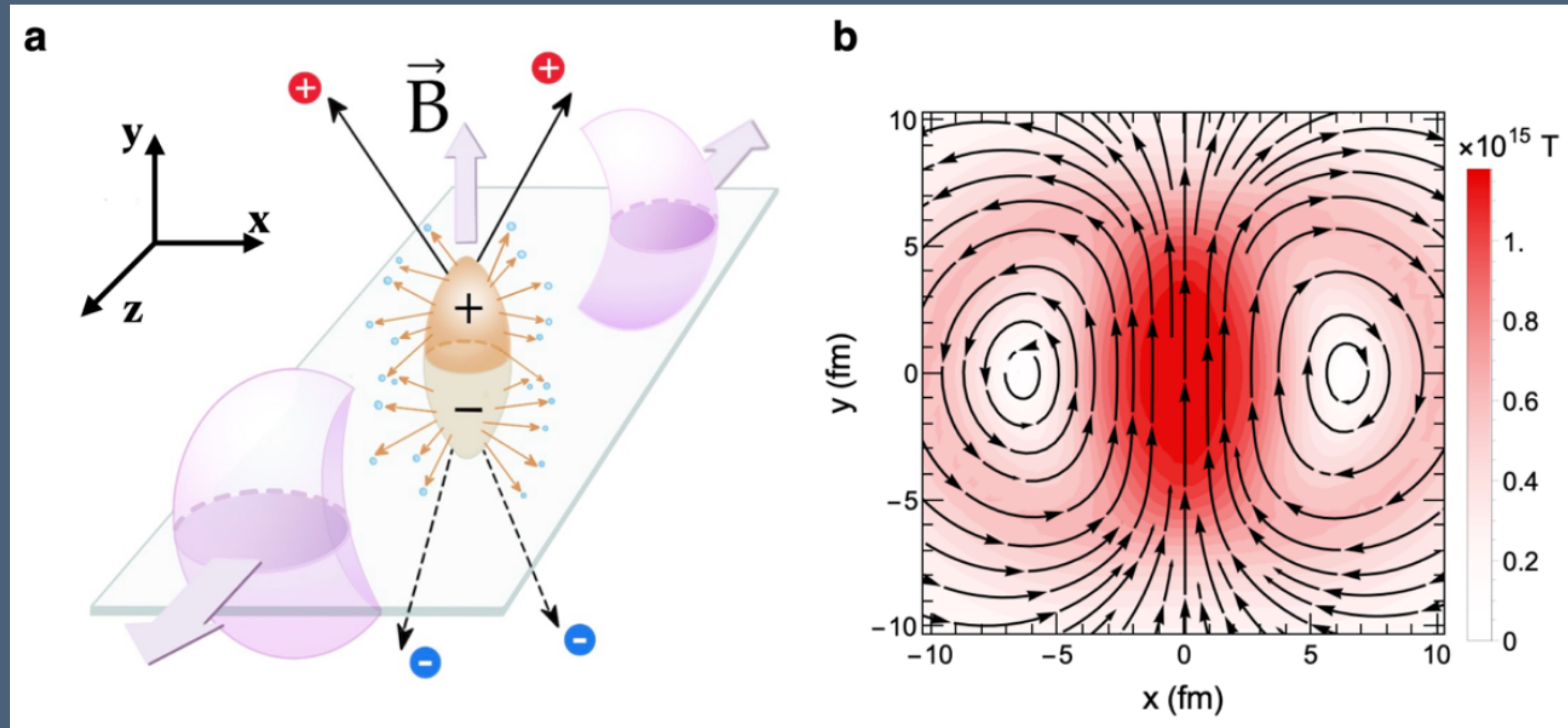
Photon puzzle



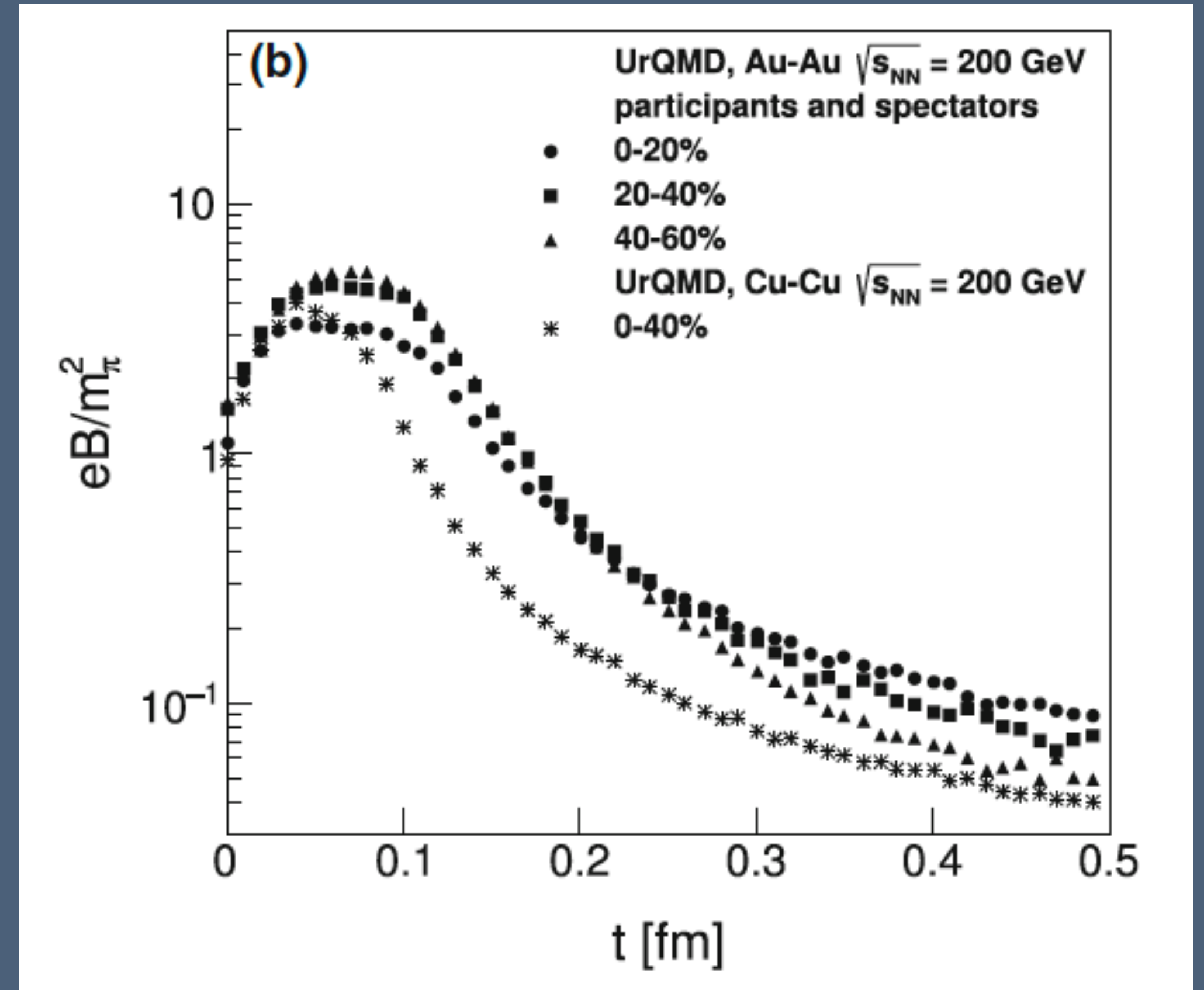
Prog. Part. Nucl. Phys. **87**, 50 (2016), arXiv:1512.08126

PRC **93**, 044906 (2016), arXiv:1509.06738

Magnetic fields



Nature Rev. Phys. **3**, 55 (2021), arXiv:2102.06623



EPJ A **56**, 53 (2020), arXiv:1904.02938

Two-gluon one-photon vertex

General structure

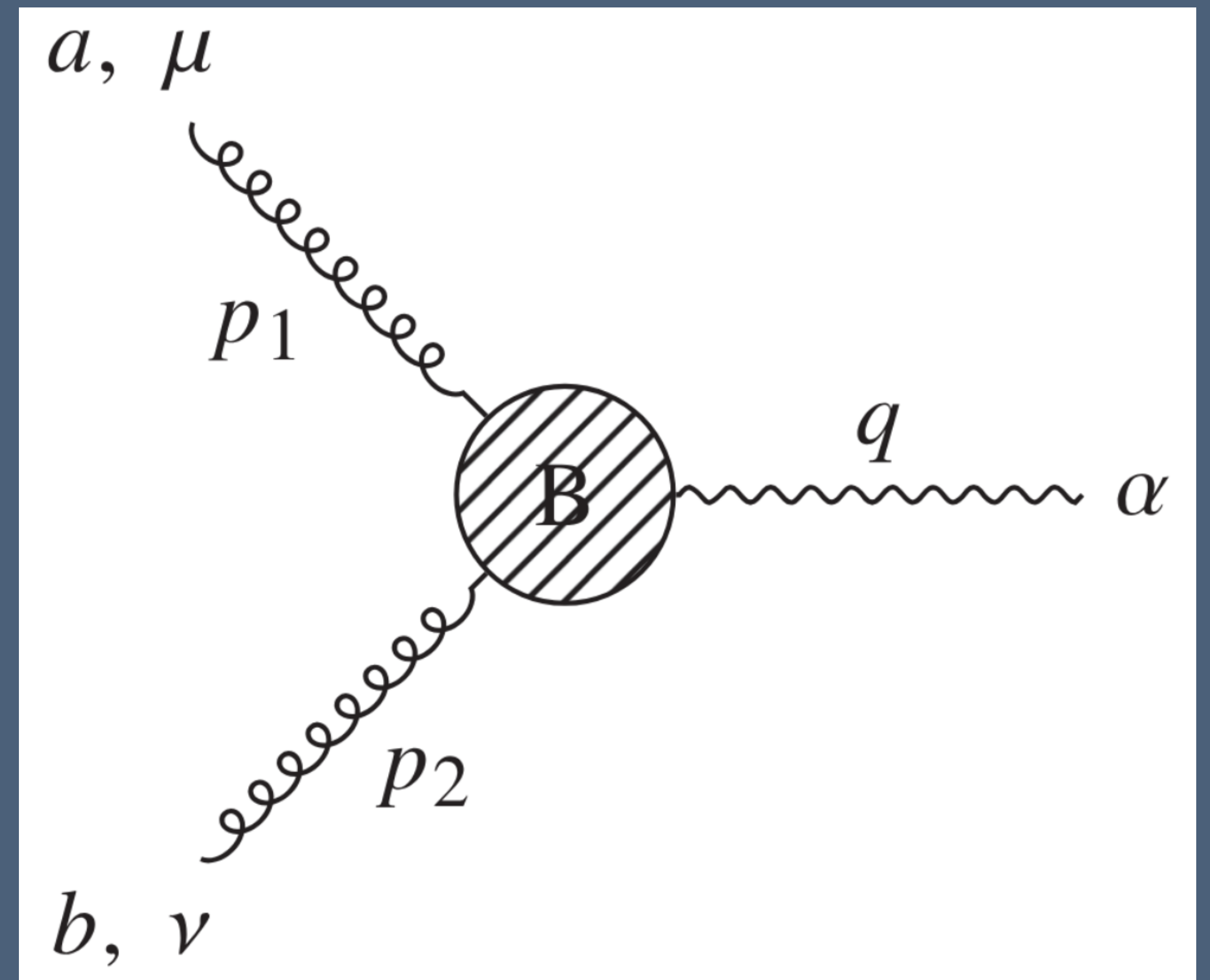
- From gauge invariance, the vertex must be transverse when contracted with the gluons and photons momenta
- The vertex must be symmetric under gluon exchange
- The vertex is invariant under CP
- The basis will be expressed as a set of polarization vectors and the photon's momentum:

- q^μ

- $l_q^\mu \equiv \hat{F}^{\mu\beta} q_\beta$

- $l_q^{*\mu} \equiv \hat{F}^{*\mu\beta} q_\beta$

- $k_q^\mu \equiv \frac{q^2}{l_q^2} \hat{F}^{\mu\beta} \hat{F}_{\beta\sigma} q^\sigma + q^\mu$



Two-gluon one-photon vertex

General structure

- Assuming a constant magnetic field in the $\hat{z}$ direction, the polarizations can be explicitly written
- In principle, there are 27 possible tensor structures that can be constructed with q^μ , $\hat{l}_q^{*,\mu}$, $\hat{l}_q^{*,\mu}$ and k_q^μ
- By considering the gluon exchange symmetry, that number is reduced to 18
- Assuming that the gauge bosons are on-shell, then $k_q^\mu \rightarrow q^\mu$
- Imposing the conservation of energy-momentum, which implies that the gauge bosons are collinear

Two-gluon one-photon vertex

General structure

- If we write the transformation properties of each coefficient as a_i^{CP} and consider all the possible Lorentz scalars that can be constructed, it is possible to see that odd structures with respect to $\hat{C}$ and $\hat{P}$, are not available to express the on-shell vertex
- Hence we arrive at

$$\bullet \Gamma_{ab}^{\mu\nu\alpha}(p_1, p_2, q)_{\text{on-shell}} = a_1^{++} \hat{l}_{p_1 a}^{\mu} \hat{l}_{p_2 b}^{\nu} \hat{l}_q^{\alpha} + a_2^{++} \hat{l}_{p_1 a}^{*\mu} \hat{l}_{p_2 b}^{*\nu} \hat{l}_q^{\alpha} + \frac{a_{10}^{++}}{2} \hat{l}_{p_1 a}^{\mu} \hat{l}_{p_2 b}^{*\nu} + \hat{l}_{p_1 a}^{*\mu} \hat{l}_{p_2 b}^{\nu} \hat{l}_q^{*\alpha}$$

Two-gluon one-photon vertex

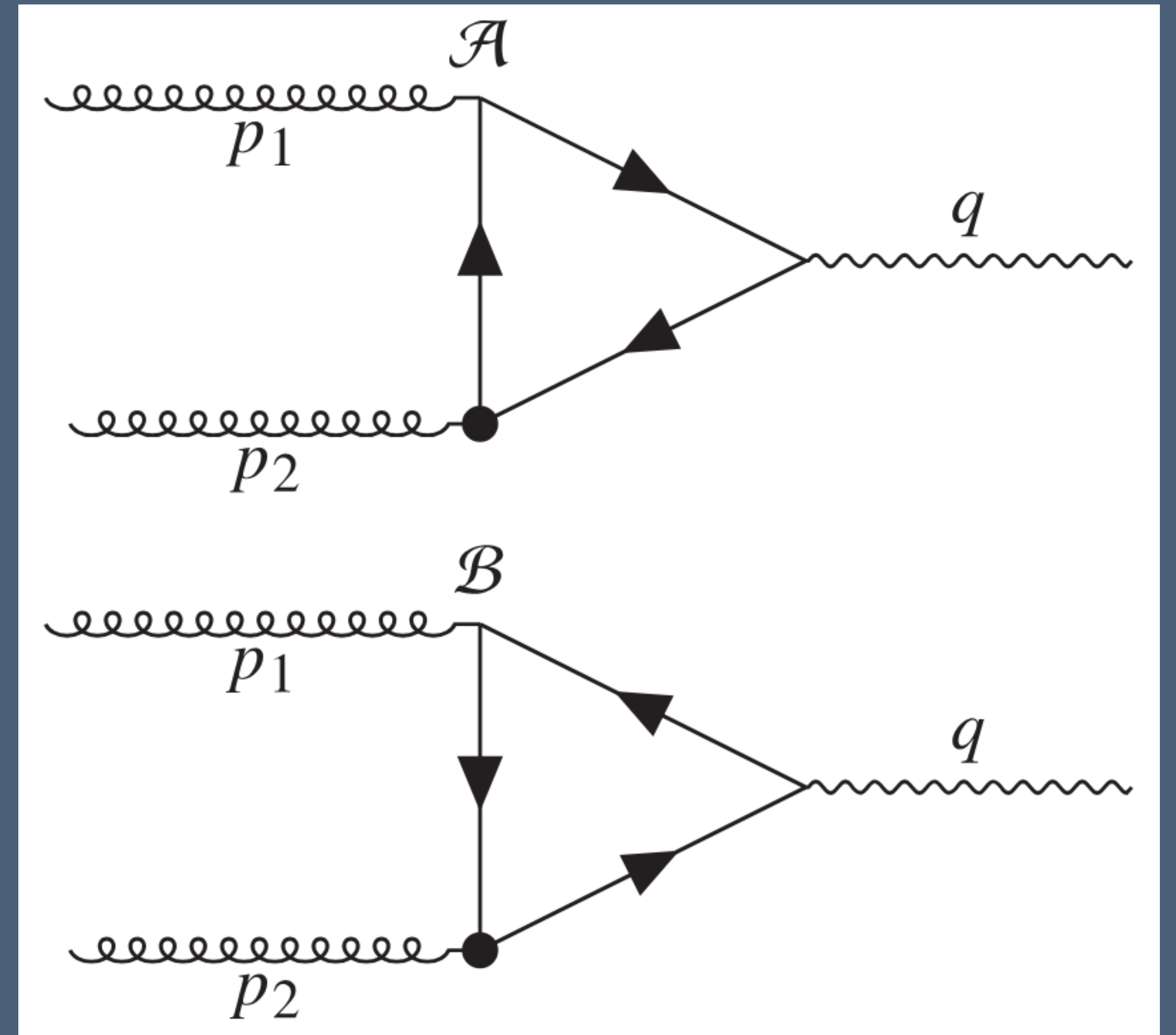
One-loop approximation

- At the leading order in α_s and α_{em} , the vertex is

$$\begin{aligned} \Gamma_{ab}^{\mu\nu\alpha} = & -ig^2 q_f \int d^4x d^4y d^4z \int \frac{d^4r_1}{(2\pi)^4} \frac{d^4r_2}{(2\pi)^4} \frac{d^4r_3}{(2\pi)^4} \\ & \times e^{-ir_3 \cdot (y-x)} e^{-ir_2 \cdot (x-z)} e^{-ir_1 \cdot (z-y)} e^{-ip_1 \cdot z} e^{-ip_1 \cdot z} e^{-ip_2 \cdot y} e^{-ip_3 \cdot x} \\ & \times \text{Tr} \gamma_\alpha S(r_2) \gamma_\mu t_a S(r_1) \gamma_\nu t_b S(r_3) \Phi(x, y, z, x) \\ & \times \text{Tr} \gamma_\alpha S(r_3) \gamma_\nu t_a S(r_1) \gamma_\mu t_b S(r_2) \Phi(x, z, y, x) \end{aligned}$$

- Where $\Phi(x, y, z, x)$ is the product of Schwinger phases and

$$S(p) = \int_0^\infty \frac{ds}{\cos(q_f B s)} e^{i s p_\parallel^2 + p_\perp^2 \frac{\tan(q_f B s)}{q_f B s} - m_f^2 + i\epsilon} e^{i q_f B s \Sigma_3 (m_f + |p_\parallel|) + \frac{p_\perp}{\cos(q_f B s)}}$$



Two-gluon one-photon vertex

One-loop approximation

- In the on-shell limit, the vertex is

$$\begin{aligned}
 \Gamma_{ab}^{\mu\nu\alpha} = & -i \frac{g^2 q_f^2 B}{2\pi^2} \text{Tr}[t_a t_b] \delta^4(p_1 + p_2 - q) \\
 & \times \int_0^\infty \frac{ds_1 ds_2 ds_3}{c_1^2 c_2^2 c_3^2} \frac{1}{t_1 t_2 t_3 - t_1 - t_2 - t_3} \frac{e^{-ism_f^2}}{s} \\
 & \times e^{-\frac{i}{s}(s_1 s_3 \omega_{p_1}^2 + s_2 s_3 \omega_{p_2}^2 + s_1 s_2 \omega_q^2) \frac{q_\perp^2}{\omega_q^2}} \\
 & \times e^{-\frac{i}{\omega_q^2} \frac{q_\perp^2}{|q_f B|} \frac{1}{t_1 t_2 t_3 - t_1 - t_2 - t_3} (t_1 t_3 \omega_{p_1}^2 + t_2 t_3 \omega_{p_2}^2 + t_1 t_2 \omega_q^2)} \\
 & \times \sum_{j=1}^{19} T_{\mathcal{A}_j}^{\mu\nu\alpha} + T_{\mathcal{B}_j}^{\mu\nu\alpha}
 \end{aligned}$$

$$\begin{aligned}
 T_{\mathcal{A}1}^{\mu\nu\alpha} + T_{\mathcal{B}1}^{\mu\nu\alpha} &= \text{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha \mathcal{A}_b \gamma^\nu \mathcal{A}_c] + \text{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu \mathcal{B}_b \gamma^\alpha \mathcal{B}_a], \\
 T_{\mathcal{A}2}^{\mu\nu\alpha} + T_{\mathcal{B}2}^{\mu\nu\alpha} &= m_f^2 \{ \text{Tr}[\gamma^\mu e_1 \gamma^\alpha e_2 \gamma^\nu \mathcal{A}_c] + \text{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu e_2 \gamma^\alpha e_1] \}, \\
 T_{\mathcal{A}3}^{\mu\nu\alpha} + T_{\mathcal{B}3}^{\mu\nu\alpha} &= m_f^2 \{ \text{Tr}[\gamma^\mu e_1 \gamma^\alpha \mathcal{A}_b \gamma^\nu e_3] + \text{Tr}[\gamma^\mu e_3 \gamma^\nu \mathcal{B}_b \gamma^\alpha e_1] \}, \\
 T_{\mathcal{A}4}^{\mu\nu\alpha} + T_{\mathcal{B}4}^{\mu\nu\alpha} &= m_f^2 \{ \text{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha e_2 \gamma^\nu e_3] + \text{Tr}[\gamma^\mu e_3 \gamma^\nu e_2 \gamma^\alpha \mathcal{B}_a] \}, \\
 T_{\mathcal{A}5}^{\mu\nu\alpha} + T_{\mathcal{B}5}^{\mu\nu\alpha} &= \frac{i}{s} \{ \text{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha e_2 \gamma_\parallel^\nu e_3] + \text{Tr}[\gamma^\mu e_3 \gamma_\parallel^\nu e_2 \gamma^\alpha \mathcal{B}_a] \}, \\
 T_{\mathcal{A}6}^{\mu\nu\alpha} + T_{\mathcal{B}6}^{\mu\nu\alpha} &= \frac{i}{s} \{ \text{Tr}[\gamma_\parallel^\mu e_1 \gamma^\alpha \mathcal{A}_b \gamma^\nu e_3] + \text{Tr}[\gamma_\parallel^\mu e_3 \gamma^\nu \mathcal{B}_b \gamma^\alpha e_1] \}, \\
 T_{\mathcal{A}7}^{\mu\nu\alpha} + T_{\mathcal{B}7}^{\mu\nu\alpha} &= \frac{i}{s} \{ \text{Tr}[\gamma^\mu e_1 \gamma_\parallel^\alpha e_2 \gamma^\nu \mathcal{A}_c] + \text{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu e_2 \gamma_\parallel^\alpha e_1] \}, \\
 T_{\mathcal{A}8}^{\mu\nu\alpha} + T_{\mathcal{B}8}^{\mu\nu\alpha} &= -\frac{i}{s} \{ \text{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha e_2 \gamma^\nu e_3] + \text{Tr}[\gamma^\mu e_3 \gamma^\nu e_2 \gamma^\alpha \mathcal{B}_a] \}, \\
 T_{\mathcal{A}9}^{\mu\nu\alpha} + T_{\mathcal{B}9}^{\mu\nu\alpha} &= -\frac{i}{s} \{ \text{Tr}[\gamma^\mu e_1 \gamma^\alpha \mathcal{A}_b \gamma^\nu e_3] + \text{Tr}[\gamma^\mu e_3 \gamma^\nu \mathcal{B}_b \gamma^\alpha e_1] \}, \\
 T_{\mathcal{A}10}^{\mu\nu\alpha} + T_{\mathcal{B}10}^{\mu\nu\alpha} &= -\frac{i}{s} \{ \text{Tr}[\gamma^\mu e_1 \gamma^\alpha e_2 \gamma^\nu \mathcal{A}_c] + \text{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu e_2 \gamma^\alpha e_1] \}, \\
 T_{\mathcal{A}11}^{\mu\nu\alpha} + T_{\mathcal{B}11}^{\mu\nu\alpha} &= \frac{iq_f B}{(t_1 t_2 t_3 - t_1 - t_2 - t_3)} \{ \text{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha \gamma^\nu] + \text{Tr}[\gamma^\mu \gamma^\nu \gamma^\alpha \mathcal{B}_a] \}, \\
 T_{\mathcal{A}12}^{\mu\nu\alpha} + T_{\mathcal{B}12}^{\mu\nu\alpha} &= \frac{iq_f B}{(t_1 t_2 t_3 - t_1 - t_2 - t_3)} \{ \text{Tr}[\gamma^\mu \gamma^\alpha \mathcal{A}_b \gamma^\nu] + \text{Tr}[\gamma^\mu \gamma^\nu \mathcal{B}_b \gamma^\alpha] \}, \\
 T_{\mathcal{A}13}^{\mu\nu\alpha} + T_{\mathcal{B}13}^{\mu\nu\alpha} &= \frac{iq_f B}{(t_1 t_2 t_3 - t_1 - t_2 - t_3)} \{ \text{Tr}[\gamma^\mu \gamma^\alpha \gamma^\nu \mathcal{A}_c] + \text{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu \gamma^\alpha] \}, \\
 T_{\mathcal{A}14}^{\mu\nu\alpha} + T_{\mathcal{B}14}^{\mu\nu\alpha} &= -\frac{iq_f B}{(t_1 t_2 t_3 - t_1 - t_2 - t_3)} \{ \text{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha \gamma_\perp^\nu] + \text{Tr}[\gamma^\mu \gamma_\perp^\nu \gamma^\alpha \mathcal{B}_a] \}, \\
 T_{\mathcal{A}15}^{\mu\nu\alpha} + T_{\mathcal{B}15}^{\mu\nu\alpha} &= -\frac{iq_f B}{(t_1 t_2 t_3 - t_1 - t_2 - t_3)} \{ \text{Tr}[\gamma_\perp^\mu \gamma^\alpha \mathcal{A}_b \gamma^\nu] + \text{Tr}[\gamma_\perp^\mu \gamma^\nu \mathcal{B}_b \gamma^\alpha] \}, \\
 T_{\mathcal{A}16}^{\mu\nu\alpha} + T_{\mathcal{B}16}^{\mu\nu\alpha} &= -\frac{iq_f B}{(t_1 t_2 t_3 - t_1 - t_2 - t_3)} \{ \text{Tr}[\gamma^\mu \gamma_\perp^\alpha \gamma^\nu \mathcal{A}_c] + \text{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu \gamma_\perp^\alpha] \}, \\
 T_{\mathcal{A}17}^{\mu\nu\alpha} + T_{\mathcal{B}17}^{\mu\nu\alpha} &= \frac{iq_f B t_1}{2(t_1 t_2 t_3 - t_1 - t_2 - t_3)} \{ \text{Tr}[\gamma^\mu \mathcal{A}_a \gamma^\alpha \gamma_\perp^\nu \gamma_\perp^\sigma] \hat{F}_{\beta\sigma} + \text{Tr}[\gamma^\mu \gamma_\perp^\sigma \gamma^\nu \gamma_\perp^\beta \gamma^\alpha \mathcal{B}_a] \hat{F}_{\sigma\beta} \}, \\
 T_{\mathcal{A}18}^{\mu\nu\alpha} + T_{\mathcal{B}18}^{\mu\nu\alpha} &= -\frac{iq_f B t_2}{2(t_1 t_2 t_3 - t_1 - t_2 - t_3)} \{ \text{Tr}[\gamma^\mu \gamma_\perp^\beta \gamma^\alpha \mathcal{A}_b \gamma^\nu \gamma_\perp^\sigma] \hat{F}_{\beta\sigma} + \text{Tr}[\gamma^\mu \gamma_\perp^\sigma \gamma^\nu \mathcal{B}_b \gamma^\alpha \gamma_\perp^\beta] \hat{F}_{\sigma\beta} \}, \\
 T_{\mathcal{A}19}^{\mu\nu\alpha} + T_{\mathcal{B}19}^{\mu\nu\alpha} &= \frac{iq_f B t_3}{2(t_1 t_2 t_3 - t_1 - t_2 - t_3)} \{ \text{Tr}[\gamma^\mu \gamma_\perp^\beta \gamma^\alpha \gamma_\perp^\sigma \mathcal{A}_c] \hat{F}_{\beta\sigma} + \text{Tr}[\gamma^\mu \mathcal{B}_c \gamma^\nu \gamma_\perp^\sigma \gamma^\alpha \gamma_\perp^\beta] \hat{F}_{\sigma\beta} \}, \\
 \mathcal{A}_a &= -\left(\frac{s_3 \omega_{p_1} + s_2 \omega_q}{s \omega_q} \right) \not{e}_1 + \frac{(t_3 \omega_{p_1} + t_2 \omega_q) \not{q}_\perp - t_2 t_3 \omega_{p_2} \gamma^\sigma \hat{F}_{\sigma\beta} q_\perp^\beta}{(t_1 t_2 t_3 - t_1 - t_2 - t_3) \omega_q} \\
 \mathcal{A}_b &= \left(\frac{s_1 \omega_q + s_3 \omega_{p_2}}{s \omega_q} \right) \not{e}_2 - \frac{(t_3 \omega_{p_2} + t_1 \omega_q) \not{q}_\perp + t_1 t_3 \omega_{p_1} \gamma^\sigma \hat{F}_{\sigma\beta} q_\perp^\beta}{(t_1 t_2 t_3 - t_1 - t_2 - t_3) \omega_q} \\
 \mathcal{A}_c &= \left(\frac{s_1 \omega_{p_1} - s_2 \omega_{p_2}}{s \omega_q} \right) \not{e}_3 + \frac{(-t_1 \omega_{p_1} + t_2 \omega_{p_2}) \not{q}_\perp + t_1 t_3 \omega_q \gamma^\sigma \hat{F}_{\sigma\beta} q_\perp^\beta}{(t_1 t_2 t_3 - t_1 - t_2 - t_3) \omega_q} \\
 \mathcal{B}_a &= \left(\frac{s_3 \omega_{p_1} + s_2 \omega_q}{s \omega_q} \right) \not{e}_1 - \frac{(t_3 \omega_{p_1} + t_2 \omega_q) \not{q}_\perp + t_2 t_3 \omega_{p_2} \gamma^\sigma \hat{F}_{\sigma\beta} q_\perp^\beta}{(t_1 t_2 t_3 - t_1 - t_2 - t_3) \omega_q}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{B}_b &= -\left(\frac{s_1 \omega_q + s_3 \omega_{p_2}}{s \omega_q} \right) \not{e}_2 + \frac{(t_3 \omega_{p_2} + t_1 \omega_q) \not{q}_\perp - t_1 t_3 \omega_{p_1} \gamma^\sigma \hat{F}_{\sigma\beta} q_\perp^\beta}{(t_1 t_2 t_3 - t_1 - t_2 - t_3) \omega_q} \\
 \mathcal{B}_c &= -\left(\frac{s_1 \omega_{p_1} - s_2 \omega_{p_2}}{s \omega_q} \right) \not{e}_3 + \frac{(t_1 \omega_{p_1} - t_2 \omega_{p_2}) \not{q}_\perp + t_1 t_3 \omega_q \gamma^\sigma \hat{F}_{\sigma\beta} q_\perp^\beta}{(t_1 t_2 t_3 - t_1 - t_2 - t_3) \omega_q}.
 \end{aligned}$$

Two-gluon one-photon vertex

One-loop approximation at the intermediate field regime

- Since we are interested in the regime where $q_f B \sim m_\pi^2$ and $q_\perp \gtrsim 500$ MeV, the hierarchy of scales is $m_f^2 < |q_f B| < q_\perp^2$
- If we perform the change of variable $s_i = s v_i$ with $v_3 = 1 - v_1 - v_2$, $v_1 \leq 1$ and $v_2 \leq 1 - v_1$, in general we have integrals of the form

$$\cdot \sum_{j=1}^{19} \int ds dv_1 dv_2 \csc^{i+1}(x) e^{i \text{Arg}} G_j^{\mu\nu\alpha}, \text{ where } i = 1, 2, 3, \text{ Arg} = \frac{q_\perp^2}{|q_f B|} x^3 F(v_1, v_2) \text{ and}$$

$$G_j^{\mu\nu\alpha} = i \frac{g^2 q_f}{16\pi^2} \text{Tr}[t_a t_b] x \sec(v_1 x) \sec(v_2 x) \sec(x(1 - v_1 - v_2)) T_{\mathcal{A}_j}^{\mu\nu\alpha} + T_{\mathcal{B}_j}^{\mu\nu\alpha}, \text{ with } s = x / q_f B$$

- Where the region of the integrand that provides the largest contribution to the integral comes from $q_f B s \ll 1$, thus, we have neglected terms proportional to m_f^2

Two-gluon one-photon vertex

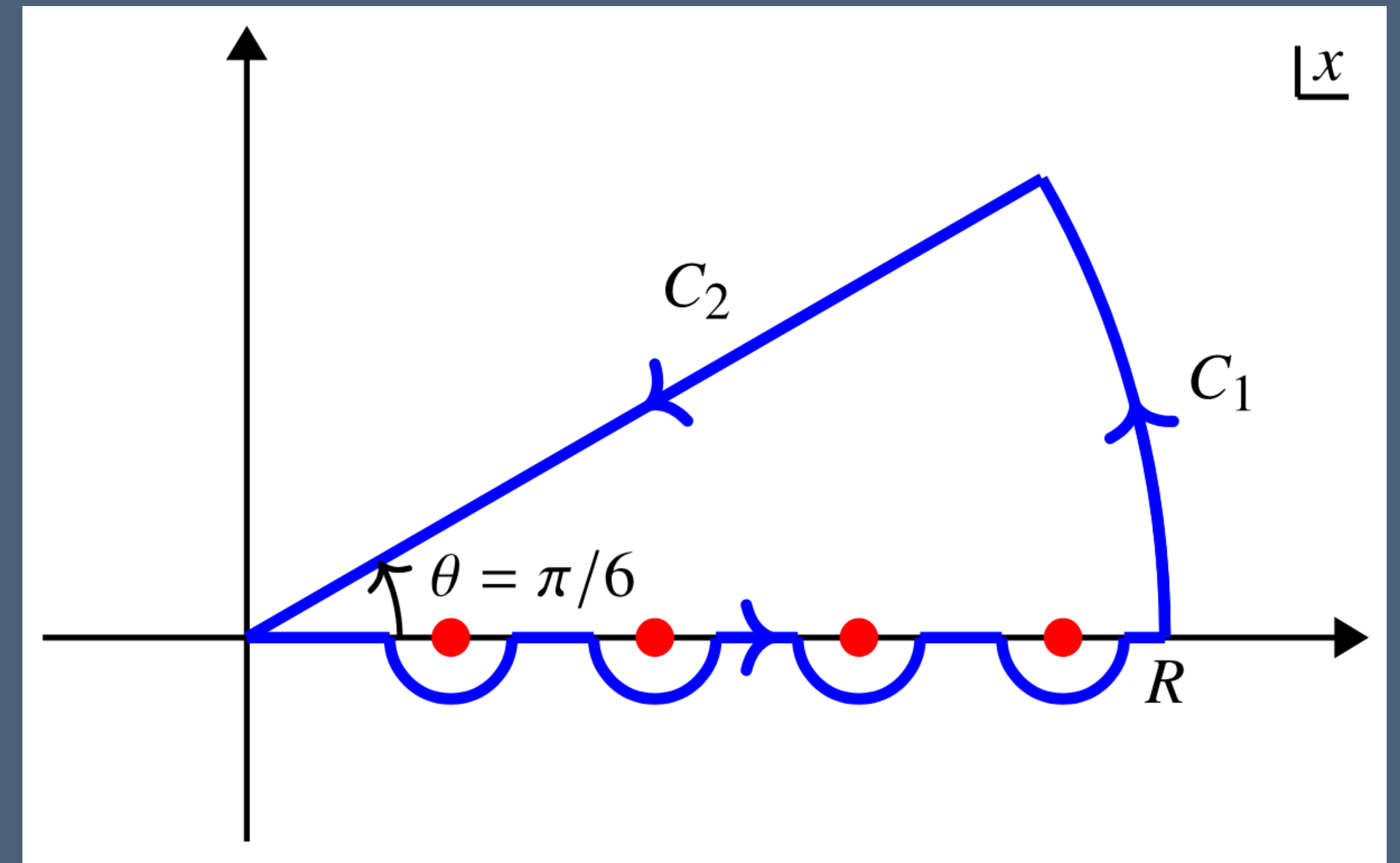
One-loop approximation at the intermediate field regime

- There are 3 kinds of integrands, all proportional to $\csc^i(x)$, with $i = 2, 3, 4$ and the principal value can be obtained as

$$\begin{aligned} \int_0^\infty dx \csc^{j+1}(x) e^{i\text{Arg} G_j(x, v_1, v_2)} &= \sum \text{Res}(\csc^{j+1}(x) e^{i\text{Arg} G_j(x, v_1, v_2)}, n\pi) \\ &\quad - \int_0^\infty d\tau \csc^{j+1}(\tau e^{i\pi/6}) e^{-\frac{q_1^2}{q_f B} \tau^3 F(v_1, v_2)} G_j(\tau e^{i\pi/6}, v_1, v_2) \end{aligned}$$

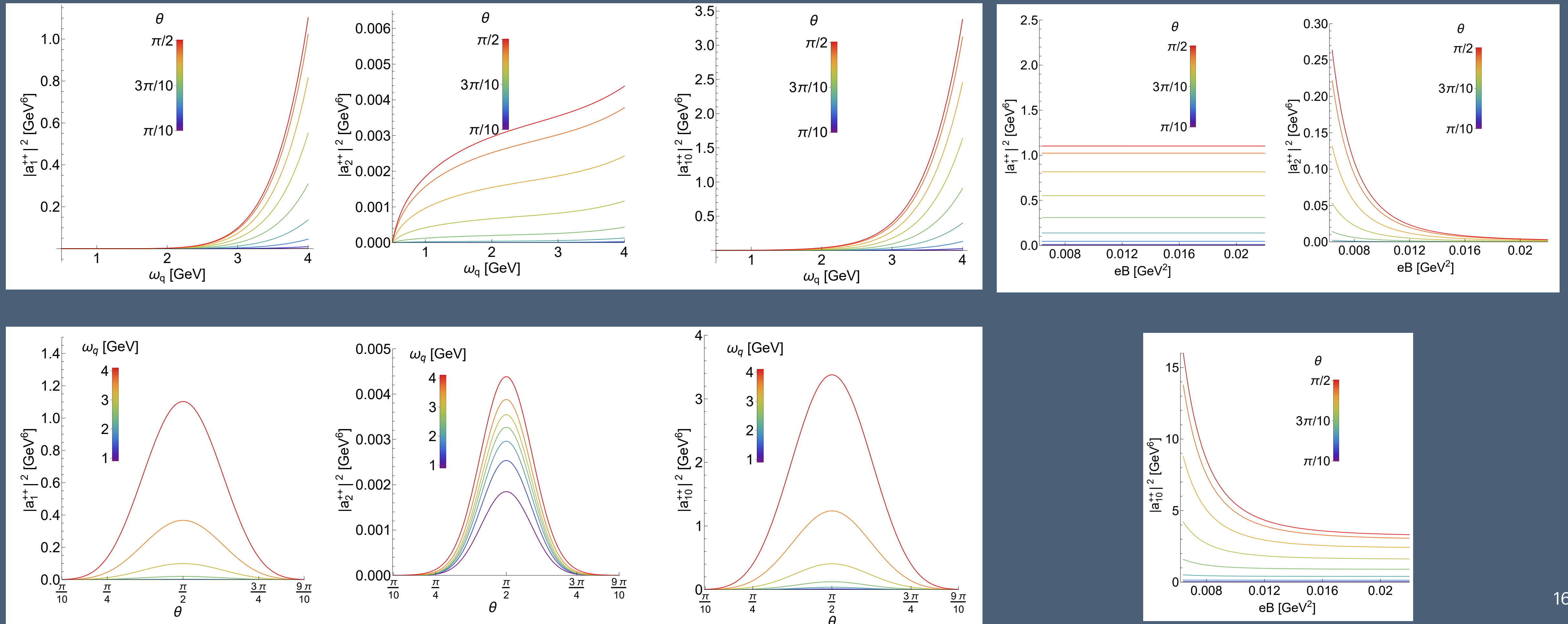
- To perform the integrals over v_1 and v_2 we resort to the stationary phase approximation

$$\begin{aligned} \int dv_1 dv_2 \mathcal{F}(v_1, v_2) e^{i\chi\psi(v_1, v_2)} &\approx \sum_{\vec{v}^0 \in \Upsilon} e^{i\chi\psi(\vec{v}^0)} |\det(\text{Hess}(\Psi(\vec{v}^0)))|^{1/2} \\ &\quad \times e^{i\frac{\pi}{4} \text{sign}(\text{Hess}(\Psi(\vec{v}^0)))} \frac{2\pi^{2/2}}{\chi} \mathcal{F}(\vec{v}^0) + \mathcal{O}(\chi^{-1/2}) \end{aligned}$$



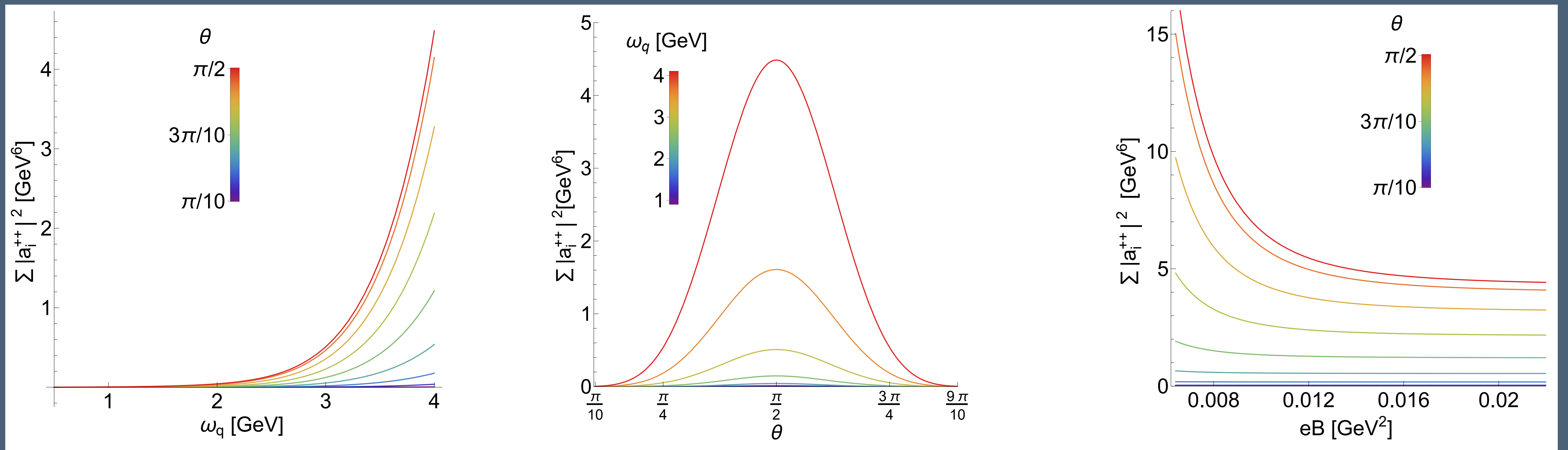
Two-gluon one-photon vertex

One-loop approximation at the intermediate field regime



Two-gluon one-photon vertex

One-loop approximation at the intermediate field regime



Summary and conclusions

- We found the general structure of the two-gluon one-photon vertex in the presence of a constant magnetic field
- We have obtained it at the one-loop level in the intermediate field regime, which is the most appropriate one to describe the possible effects of the presence of a magnetic field as a catalyzer of photon emission during preequilibrium
- The most favored direction of photon propagation is transverse to the magnetic field (contribution to positive elliptic flow)
- These findings will be used for the computation of the photon yield and elliptic flow coefficient

Thank you !