Neutral pion screening mass in a magnetized medium

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Overview

- Why is strong-field physics important?
- Interplay between strong magnetic fields and QCD.
- Linear sigma model with quarks (LSMq)
- Analysis of the neutral pion screening mass
- Results
- Summary and perspectives

Strong-(Electromagnetic)Field Physics

- High energy physics (heavy ion collisions)
- Astrophysics (neutron stars)

Strong-Field Physics

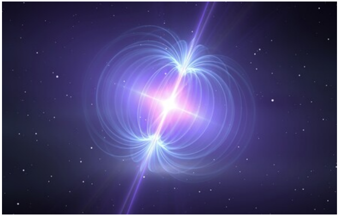


Figure: magnetar ($10^{13} - 10^{15}$ G.)

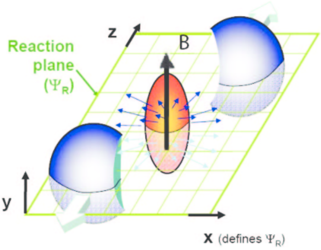


Figure: Heavy ion collisions ($10^{18} - 10^{19}$ G.)

Comparing magnetic field strengths

- Earth's magnetic field: 0.6 G
- Common comercial magnet: 100 G
- Strongest magnetic field produced in labs: 4.5×10^5 G
- Magnetars: $(10^{13} - 10^{15})G$
- Heavy ion collisions: $(10^{18} - 10^{19})$ G

Interplay between strong magnetic fields and QCD

- Magnetic catalysis at zero temperature.
- Inverse magnetic catalysis around T_C.
- Chiral magnetic effect.
- Electromagnetic fields provide a powerful probe to explore the properties of the QCD vacuum.

Why screening masses of neutral pions?

Since the dynamics of chiral symmetry breaking is dominated by pions, the lightest of all quark-antiquark bound states, it then becomes important to explore how the properties of the pion are affected by the presence of magnetic fields.

In this work we will study the effect of a constant magnetic field B in the screening mass of neutral pions.

Screening Mass

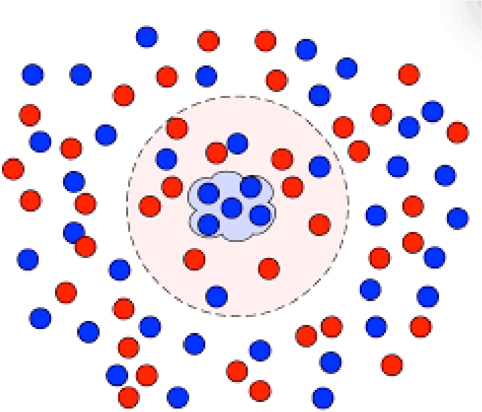


Figure: Debye mass (inverse of Debye length)

Pole and screening masses ($B \neq 0$, $T = 0$)

- Pole mass

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_3^2 = p_\perp^2 = 0} = 0$$

- Screening mass B breaks Lorentz invariance and defines $\parallel$ and $\perp$
  - Longitudinal

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_\perp^2 = 0} = 0$$

  - Transverse

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_3^2 = 0} = 0$$

Pole and screening masses (definitions)

$$E^2 = u_\perp^2 \mathbf{q}_\perp^2 + u_\parallel^2 q_3^2 + m_{\pi^0,pole}^2$$

$$m_{\pi^0,scr.\perp} = \frac{m_{\pi^0,pole}}{u_\perp}$$

$$m_{\pi^0,scr.\parallel} = \frac{m_{\pi^0,pole}}{u_\parallel}$$

$$u_\perp \equiv u_\perp(B,T) \ , \ \ u_\parallel \equiv u_\parallel(T)$$

Pole and screening masses (special cases)

- **(i)T=0, B=0**

$$u_\perp = u_\parallel = 1$$

$$m_{\pi^0,pole} = m_{\pi^0,scr,\parallel} = m_{\pi^0,scr,\perp}$$

- **(ii)T≠0, B=0**

$$u_\perp = u_\parallel = u \neq 1$$

$$m_{\pi^0,pole} \neq m_{\pi^0,scr,\perp} = m_{\pi^0,scr,\parallel}$$

$u < 1$ in order to satisfy causality

Pole and screening masses (special cases)

- **(iii)B$\neq$0, T=0**

$$u_{\perp} \neq u_{\parallel} \ \ but \ \ u_{\parallel} = 1$$

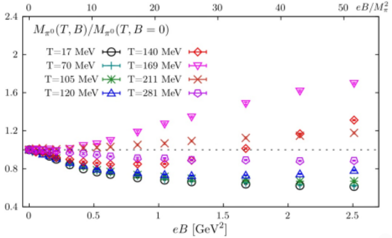
$$m_{\pi^0,pole} = m_{\pi^0,scr,\parallel} < m_{\pi^0,scr,\perp}$$

- **(iv)B$\neq$0, T$\neq$0**

$$u_{\perp} < u_{\parallel} < 1$$

$$m_{\pi^0,pole} < m_{\pi^0,scr,\parallel} < m_{\pi^0,scr,\perp}$$

Comparison with LQCD and the NJL model



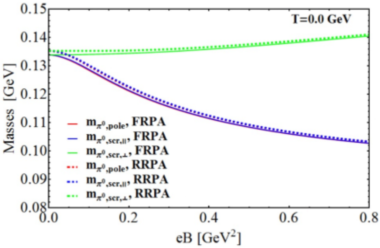


Figure: H. T. Ding, S. T. Li, J. H. Liu, and X. D. Wang, Phys. Rev D105, 034514 (2022), and B. Sheng, Y. Wang, X. Wang, and L. Yu, Phys. Rev. D103 (2021) 9, 094001.

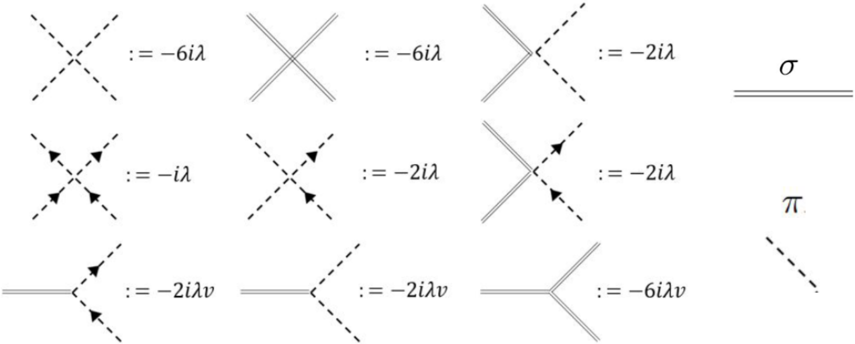
LSMq Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu \partial_\mu \psi \\ - ig\bar{\psi}\gamma^5 \vec{\tau} \cdot \vec{\pi}\psi - g\bar{\psi}\psi\sigma \, .$$

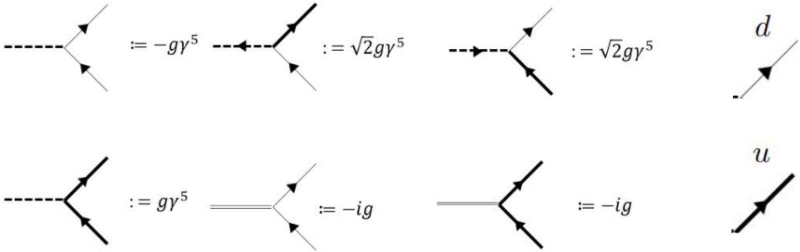
- it implements the SSB of: $SU(2)_L \times SU(2)_R \to SU(2)_V$.
- $m_\pi(v) = \sqrt{\lambda v^2 - a^2} = 0$ at VEV.
- $m_f(v) = gv$.
- $m_\sigma(v) = \sqrt{3\lambda v^2 - a^2}$

$\mathcal{L} \to \mathcal{L} + h(\sigma + v)$ in order to give the correct vacuum pion mass.

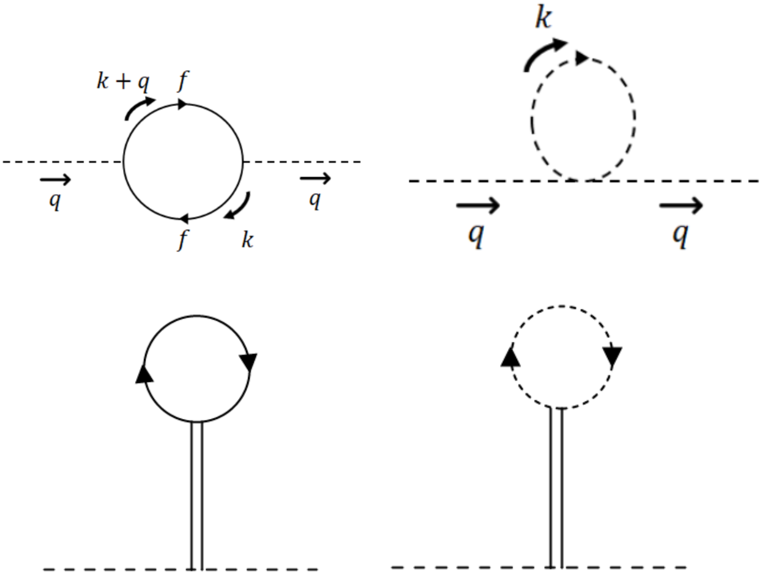
Feynman rules for boson-boson interactions



Feynman rules for boson-fermion interactions



Relevant Feynman diagrams



Feynman rules for the fermionic contribution to $\pi_{f\bar{f}}$ (vertices and propagator)

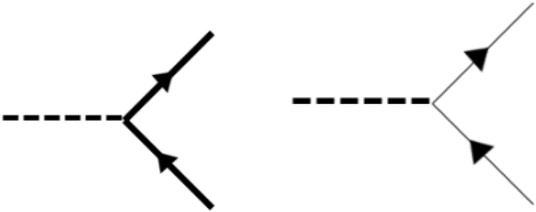


Figure: π_0-u quark vertex=gγ^5; π_0-d quark vertex=-gγ^5

$$iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp\left[is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon\right)\right]$$

$$\times\left\{(m_f + \not{p}_\parallel)\Big(\cos(qBs) + \gamma^1\gamma^2 \sin(qBs)\Big) - \frac{\not{p}_\perp}{\cos(qBs)}\right\}$$

$$iS(p) \to i\frac{(m_f + \not{p})}{p^2 - m_f^2 + i\epsilon}$$

Neutral pion self-energy (fermion contribution)

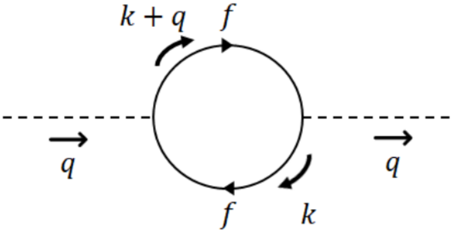


Figure: Neutral pion self-energy

$$-i\pi_{f\bar{f}} = -g^2 \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma^5 iS(k)\gamma^5 iS(k+q)\right] + c.c..$$

Small magnetic field approximation for the propagator

$$iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp\left[is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon\right)\right]$$
$$\times \left\{(m_f + \not{p}_\parallel)\Big(\cos(qBs) - \gamma^1\gamma^2 \sin(qBs)\Big) + \frac{\not{p}_\perp}{\cos(qBs)}\right\}$$

$$iS(p) \to iS^0(p) + iS^1(p) + iS^2(p)$$

,

$$iS^0(p) = i\frac{m_f + \not{p}}{p^2 - m^2}$$

$$iS^1(p) = |q_f B|\gamma^1\gamma^2 \frac{m_f + \not{p}_\parallel}{(p^2 - m^2)^2} sign(q_f B)$$

$$iS^2(p) = -2i|q_f B|^2 \frac{p_\perp^2(\not{p}_\parallel + m_f) - (m_f^2 - p_\parallel^2)\not{p}_\perp}{(p^2 - m_f^2)^4}$$

Small magnetic field approximation for the self-energy

$$-i\pi_{f\bar{f}} = -g^2 \int \frac{d^4k}{(2\pi)^4} Tr[\gamma^5 (iS^0(q) + iS^1(q) + iS^2(q))\gamma^5$$
$$\times (iS^0(q-p) + iS^1(q-p) + iS^2(q-p))] \, .$$

Traces

$$Tr[\gamma^5 iS^0(q)\gamma^5 iS^1_{(q-p)}] = 0$$

$$Tr[\gamma^5 iS^1(q)\gamma^5 iS^0(q-p)] = 0$$

$$Tr[\gamma^5 iS^0(q)\gamma^5 iS^2(q-p)] = \frac{2(q_f B)^2}{(q^2 - m_f^2 + i\epsilon)((q-p)^2 - m_f^2 + i\epsilon)^4} \Big[4q_\perp \cdot (q-p)_\perp (m_f^2 - (q-p)_\parallel^2) + 4m_f^2 (q-p)_\perp^2 - 4q_\parallel \cdot (q-p)_\parallel (q-p)_\perp^2\Big]$$

Traces

$$Tr[\gamma^5 iS^0(q)\gamma^5 iS^2(q-p)] = \frac{2(q_f B)^2}{(q^2-m_f^2+i\epsilon)^4((q-p)^2-m_f^2+i\epsilon)} \Big[4q_\perp\cdot(q-p)_\perp(m_f^2-q_\parallel^2) + 4m_f^2(q-p)_\perp^2 - 4q_\parallel\cdot(q-p)_\parallel(q-p)_\perp^2\Big]$$

$$Tr[\gamma^5 iS^1(q)\gamma^5 iS^1(q-p)] = \frac{(q_f B)^2[4m_f^2-4q_\parallel\cdot(q-p)_\parallel]}{(q^2-m_f^2+i\epsilon)^2((q-p)^2-m_f^2+i\epsilon)^2}$$

Self Energy

$$-i\pi_{f\bar{f}} = -g^2 \int \frac{d^4k}{(2\pi)^4} Tr[\gamma^5(iS^0(q) + iS^1(q) + iS^2(q))\gamma^5$$
$$\times(iS^0(q-p) + iS^1(q-p) + iS^2(q-p))]$$
$$= -i\pi^{00}_{f\bar{f}} - i\pi^{11}_{f\bar{f}} - i\pi^{02}_{f\bar{f}} - i\pi^{20}_{f\bar{f}}\,.$$

There are no odd-powers in the magnetic field, there are just even powers

Feynman parameters

$$\frac{1}{A_1^{m1} A_2^{m2} \cdots A_n^{mn}} = \int_0^1 dx_1 dx_2 ... dx_n \delta(\sum x_i - 1) \frac{\prod x_i^{mi-1}}{(\sum x_i A_i)^{\sum mi}} \times \frac{\Gamma(m_1 + ... + m_n))}{\Gamma(m_1) \cdots \Gamma(m_n)} .$$

Momentum integrals

$$\int \frac{d^4\ell}{(2\pi)^4}\,[\ldots] \to \int \frac{d^2\ell_\perp}{(2\pi)^2} \int \frac{d^2\ell_\parallel}{(2\pi)^2}\,[\ldots]\,.$$

$$\int \frac{d^d\ell}{(2\pi)^d}\frac{1}{(\ell^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}}\frac{\Gamma(n-d/2)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-d/2}$$

$$\int \frac{d^d\ell}{(2\pi)^d}\frac{\ell^2}{(\ell^2-\Delta)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}}\frac{d}{2}\frac{\Gamma(n-d/2-1)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-d/2-1}$$

$$\int \frac{d^d\ell_E}{(2\pi)^d}\frac{1}{(\ell_E^2+\Delta)^n} = \frac{1}{(4\pi)^{d/2}}\frac{\Gamma(n-d/2)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-d/2}$$

$$\int \frac{d^d\ell_E}{(2\pi)^d}\frac{\ell_E^2}{(\ell_E^2+\Delta)^n} = \frac{1}{(4\pi)^{d/2}}\frac{d}{2}\frac{\Gamma(n-d/2-1)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-d/2-1}$$

Final expressions

$$\left(\pi^{11}_{f\bar{f}}\right)_\perp = \frac{g^2(q_f B)^2}{4\pi^2}\int_0^1 dxx(1-x)\left[\frac{2m_f^2 + x(1-x)p_\perp^2}{(x(1-x))p_\perp^2 + m_f^2)^2}\right]$$

$$\left(\pi^{02}_{f\bar{f}} + \pi^{20}_{f\bar{f}}\right)_\perp = -\frac{g^2(q_f B)^2}{6\pi^2}\int_0^1 dxx(1-x)^3\left[\frac{3m_f^2 p_\perp^2 + x(1-x)p_\perp^4}{(x(1-x))p_\perp^2 + m_f^2)^3}\right]$$

$$\begin{aligned} f(p_0, p_\perp, p_\parallel, B) =& \pi_{f\bar{f}} - \lim_{B\to 0}\pi_{f\bar{f}} \\ =& \pi^{11}_{f\bar{f}} + \pi^{02}_{f\bar{f}} + \pi^{20}_{f\bar{f}} \end{aligned}$$

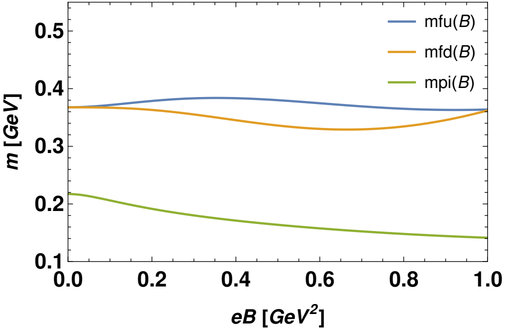
$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_3^2 = 0} = 0$$

Parameters

Parameters at zero magnetic field

$$g = \frac{m_f}{v} = 2.75; \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2 f_\pi^2} = 15$$

Magnetic field dependence of masses



1

[1] Masses taken from S. S. Avancini, M. Coppola, N. N. Scoccola and J. C. Sodré, Phys. Rev. D **104**, no.9, 094040 (2021), and S. S. Avancini, R. Farias, M. B. Pinto, W. R. Tavares, Phys. Letters B 767 (2017) 247-252 .

Transverse Mass

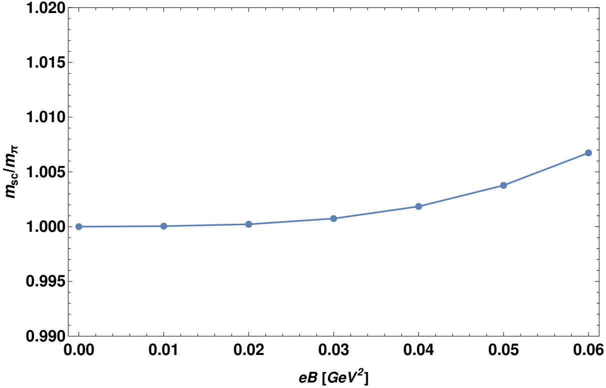


Figure: 'Transverse' Screening mass as a function of B, $g = 2.75$.

2

[2]Masses taken from S. S. Avancini, M. Coppola, N. N. Scoccola and J. C. Sodré, Phys. Rev. D **104**, no.9, 094040 (2021), and S. S. Avancini, R. Farias, M. B. Pinto, W. R. Tavares, Phys. Letters B 767 (2017) 247-252 .

Summary and perspectives

Summary:

- We have calculated the neutral pion self-energy in the LSMq.
- We have obtained the 'transverse' screening mass as a function of B for small B.
- We have compared our results with LQCD and NJL, and we have found a nice agreement that shows the slow increasing behavior of the transverse screening mass as a function of B.

Perspectives:

- We will complete the study of the 'transverse' screening mass as a function of B beyond small B.
- We will study the case where $T \neq 0$.
- We will repeat the calculation for the longitudinal screening mass using the same techniques.

Thank You

Neutral pion self-energy (fermion contribution)

$$-i\pi_{f\bar{f}} = -4g^2 \int_0^\infty \int_0^\infty \frac{dsds'}{\cos(qBs)\cos(qBs')}$$

$$\times \int \frac{d^4k}{(2\pi)^4} e^{is\left(k_\parallel^2 - k_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon\right)} e^{is'\left((k+p)_\parallel^2 - (k+p)_\perp^2 \frac{\tan(qBs')}{qBs'} - m_f^2 + i\epsilon\right)}$$

$$\left\{ \cos[qB(s+s')][m_f^2 - k_\parallel \cdot (k+p)_\parallel] + \frac{k_\perp \cdot (k_\perp + p_\perp)}{\cos(qBs)\cos qBs'} \right\}$$

Important integrals and convenient change of variables

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$u = s + s'$$

$$s = u(1 - v)$$

$$s' = uv$$

$$\frac{\partial(s, s')}{\partial(u, v)} = u$$

Neutral pion self-energy (fermion contribution)

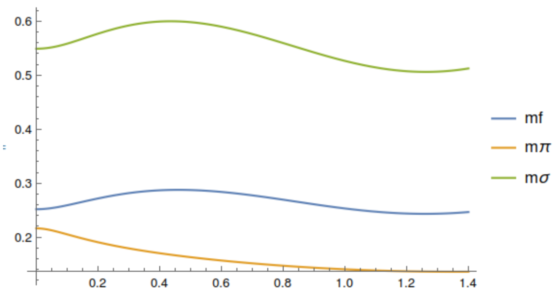
$$
\begin{aligned}
\pi_{f\bar{f}}(q) &= -4g^2\frac{|q_fB|}{(4\pi)^2}\int_0^1 dv\int_0^\infty du \\
&\times \exp[-i\frac{q_\perp^2}{|q_fB|}\frac{\sin(|q_fB|u(1-v))\sin(|q_fB|uv)}{\sin(|q_fB|u)}] \\
&\times e^{-iq_3^2uv(1-v)}e^{iq_0^2uv(1-v)}e^{-ium_f^2}e^{-u\epsilon} \\
&\times \left\{\frac{m_f^2}{\tan(|q_fB|u)}+\frac{|q_fB|}{\sin^2(|q_fB|u)}\right. \\
&\times \left(\frac{-q_\perp^2}{|q_fB|}\frac{\sin(|q_fB|u(1-v))\sin(|q_fB|uv)}{\sin(|q_fB|u)}-i\right) \\
&+ \left.\frac{1}{u\tan(|q_fB|u)}\left(\frac{1}{i}-uv(1-v)(q_3^2-q_0^2)\right)\right\}.
\end{aligned}
\tag{1}
$$

Parameters

Parameters at zero magnetic field

$$g = \frac{m_f}{v} = 2.75; \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2 f_\pi^2} = 15$$

Magnetic field dependence of masses



3

[3] S. S. Avancini, R. Farias, M. B. Pinto, W. R. Tavares, Phys. Letters B 767 (2017) 247-252.

Transverse Mass

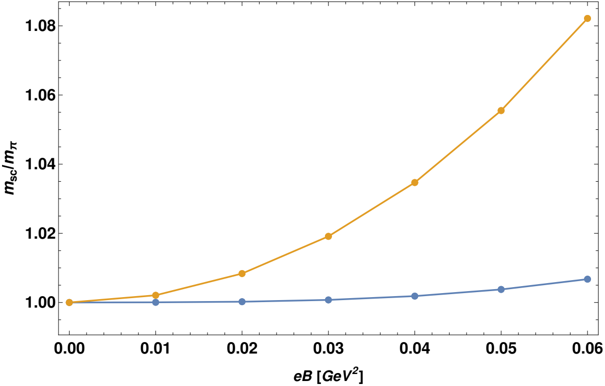


Figure: 'Transverse' Screening mass as a function of B,
$g = 0.33, \lambda = 2.5$.

4

[4]Masses taken from S. S. Avancini, M. Coppola, N. N. Scoccola and J. C. Sodré, Phys. Rev. D **104**, no.9, 094040 (2021), and S. S. Avancini, R. Farias, M. B. Pinto, W. R. Tavares, Phys. Letters B 767 (2017) 247-252 .

Longitudinal Mass

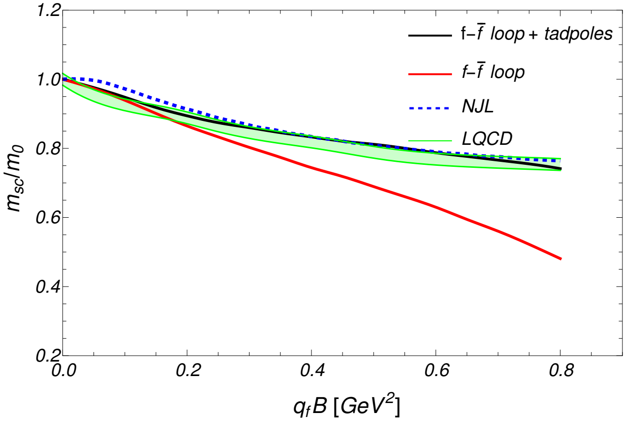


Figure: 'Longitudinal' Screening mass as a function of B,
$g = 0.33, \lambda = 2.5$.

5

[5] A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Neutral pion self-energy (fermion contribution)

$$\pi_{\bar{f}f} = \frac{-4g^2 qB}{(4\pi)^2} \int_0^1 dv \int_0^\infty du \exp\left(-ix\right) \exp\left(-u\epsilon\right) \left[\frac{m_f^2}{\tan\left(qBu\right)} \right.$$
$$- \frac{qB}{\sin^2\left(qBu\right)} \left(\frac{p_\perp^2}{|qB|} \frac{\sin\left(qBu(1-v)\right)\sin\left(qBuv\right)}{\sin\left(qBu\right)} \right) - \frac{v(1-v)(p_3^2 - p_0^2)}{\tan(qBu)}$$
$$\left. - \frac{iqB}{\sin\left(qBu\right)} - \frac{i}{u\tan\left(qBu\right)} \right]$$

where x is given by:

$$x = \frac{p_\perp^2}{qB} \frac{\sin(qBu(1-v))\sin(qBuv)}{\sin(qBu)} + p_3^2 uv(1-v) - p_0^2 uv(1-v) + m_f^2 u$$

$B \to 0$ limit of $\pi_{\bar{f}f}$

$$\lim_{B\to 0} \pi_{\bar{f}f} = -\frac{4g^2}{(4\pi)^2}\int_0^1 dv \int_0^\infty \frac{du}{u} e^{-ix_0} e^{-u\epsilon} \times \left\{ m_f^2 - v(1-v)(p_\perp^2 - p_3^2) + v(1-v)p_0^2 - \frac{2i}{u} \right\}$$

where

$$x_0 = uv(1-v)(p_\perp^2 + p_3^2) - p_0^2 uv(1-v) + m_f^2$$