Electroweak precision tests and global fits

A. Freitas

University of Pittsburgh

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1. Electroweak precision tests
2. Global electroweak fit
3. Precision physics with future e^+e^- colliders
4. Theoretical calculations

Electroweak precision tests

Experimental data:

- LEP/SLC: Z(W)-boson properties
- LHC/TeVatron: M_{W}, M_{H}, m_{t}, $\sin^2\theta^{\ell}_{\mathrm{eff}}$
- Other experiments:
  a_μ, PVES, G_μ, α_{s}, ...

Fit theory model to data:

- **SM** parameters
  M_{Z}, M_{W}, M_{H}, m_{t}, α_{s}, α^*, $m^{**}_{f\neq t}$
  * fixed   ** mostly fixed/negligible
- **BSM** models:
  SM + new particle masses/couplings
- **SMEFT/HEFT**:
  SM + Wilson coeff. of higher-dim. ops.


LEP (CERN)


SLC (SLAC)


LHC (CERN)

Electroweak precision observables

Fermi constant (from μ decay):

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}(1-M_W^2/M_Z^2)M_W^2}(1+\Delta r)$$

$\rightarrow$ prediction of M_W

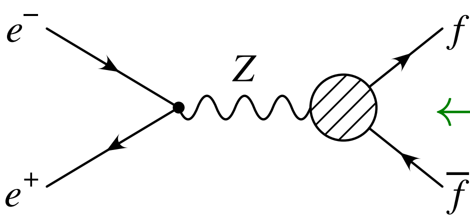
$e^+e^- \rightarrow f\bar{f}$ for $E_{CM} \sim M_Z$:

Width $\Gamma_Z = \sum_f \Gamma_{ff}$

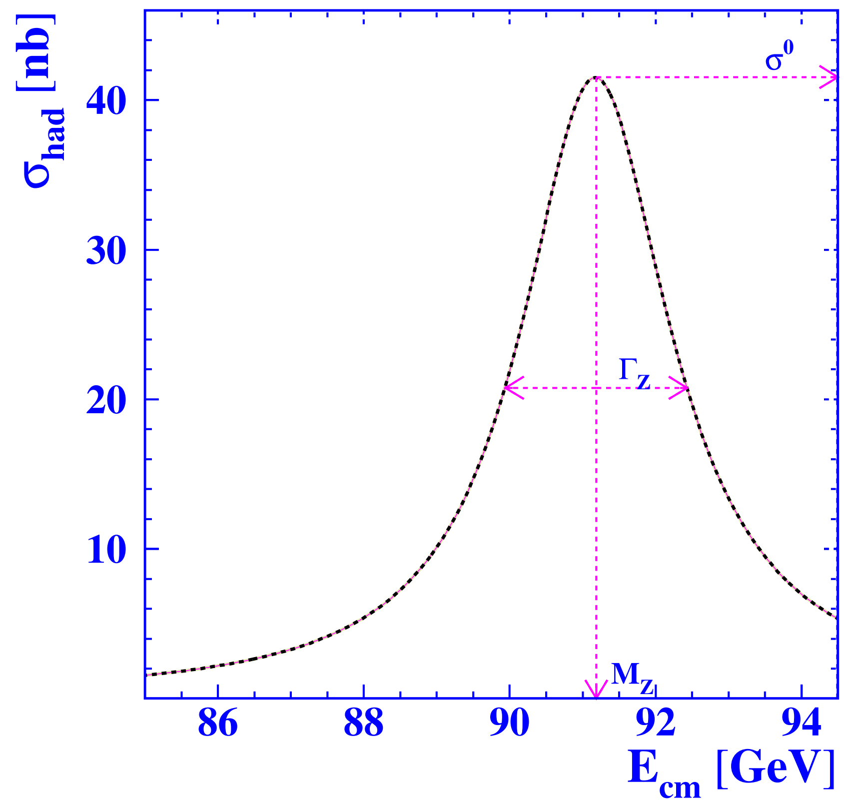
Braching ratios $R_f = \Gamma_{ff}/\Gamma_Z$

$$\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} = \frac{12\pi}{M_Z^2}R_e R_f$$

$$\Gamma_{ff} = C\left[(g_L^f)^2 + (g_R^f)^2\right]$$



$\leftarrow g_L^f, g_R^f$



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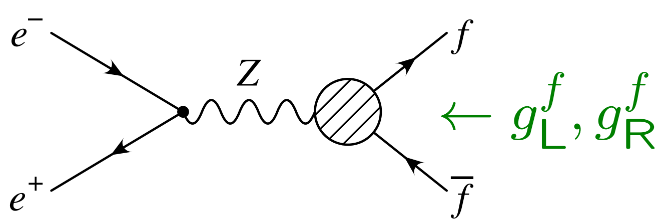
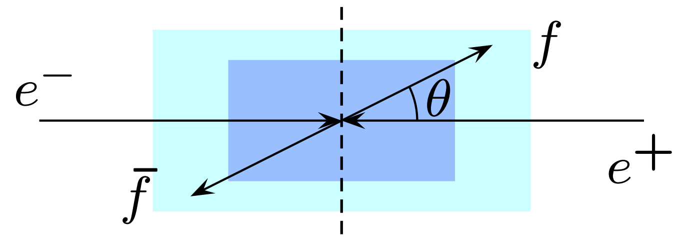


Asymmetries in $e^+e^- \rightarrow f\bar{f}$:

$$A_{FB} \equiv \frac{\int_{\theta>\frac{\pi}{2}} d\sigma - \int_{\theta<\frac{\pi}{2}} d\sigma}{\int_{\theta>\frac{\pi}{2}} d\sigma + \int_{\theta<\frac{\pi}{2}} d\sigma} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

$$A_{LR} \equiv \frac{\sigma_{e_L} - \sigma_{e_R}}{\sigma_{e_L} + \sigma_{e_R}} = \mathcal{A}_e$$

$$\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$$



$$\mathcal{A}_f = \frac{2(1-4\sin^2\theta_{\text{eff}}^f)}{1+(1-4\sin^2\theta_{\text{eff}}^f)^2}$$

$$\sin^2\theta_{\text{eff}}^f = \frac{g_R^f}{2|Q_f|(g_R^f - g_L^f)}$$

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

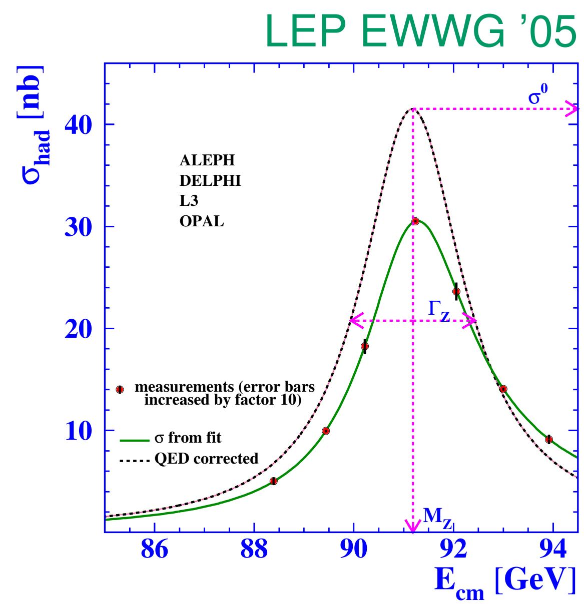
- Subtraction of γ-exchange, γ–Z interference, box contributions:

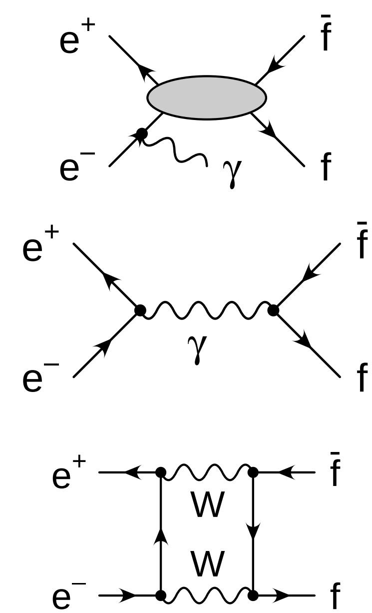
$$\sigma_{\text{hard}} = \sigma_{\text{Z}} + \sigma_{\gamma} + \sigma_{\gamma\text{Z}} + \sigma_{\text{box}}$$

- Z-pole contribution:

$$\sigma_{\text{Z}} = \frac{R}{(s - M_{\text{Z}}^2)^2 + M_{\text{Z}}^2 \Gamma_{\text{Z}}^2} + \sigma_{\text{non-res}}$$

- Final-state radiation, initial-final interference, etc.
  $\longrightarrow$ Monte-Carlo programs, consistently matched to fixed-order calculations





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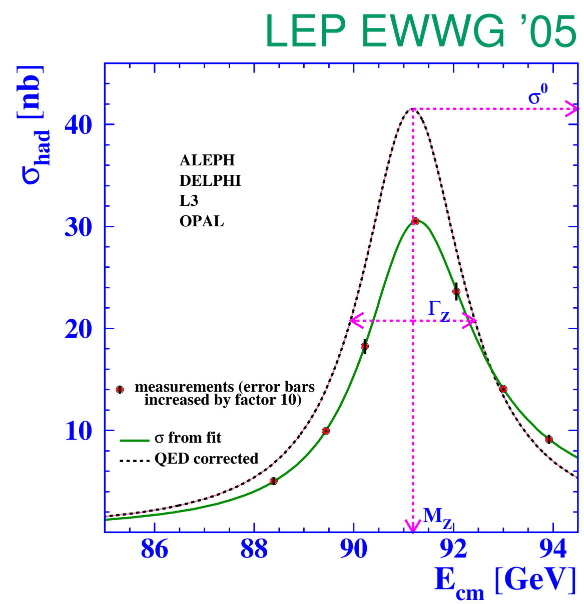
$$\sigma_{\text{hard}} = \sigma_{\text{Z}} + \underbrace{\sigma_\gamma + \sigma_{\gamma\text{Z}} + \sigma_{\text{box}}}$$

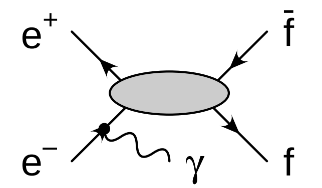
- Z-pole contribution: computed in SM (NLO)

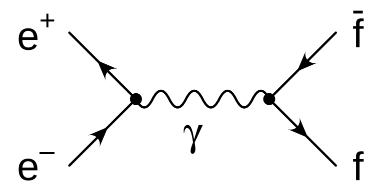
$$\sigma_{\text{Z}} = \frac{R}{(s - M_{\text{Z}}^2)^2 + M_{\text{Z}}^2\Gamma_{\text{Z}}^2} + \sigma_{\text{non-res}}$$

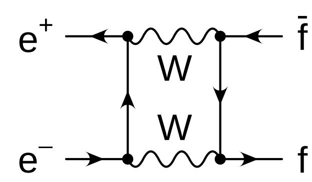
- Final-state radiation, initial-final interference, etc.
  $\longrightarrow$ Monte-Carlo programs, consistently matched to fixed-order calculations

- possible BSM physics?







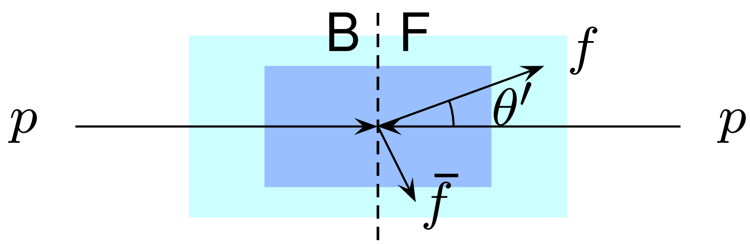


LHC measurements

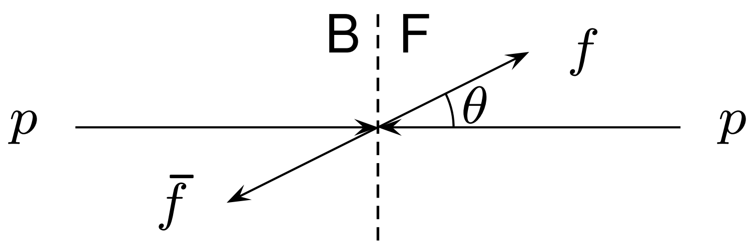
Forward-backward asymmetry:

"forward" defined through event boost

lab frame:

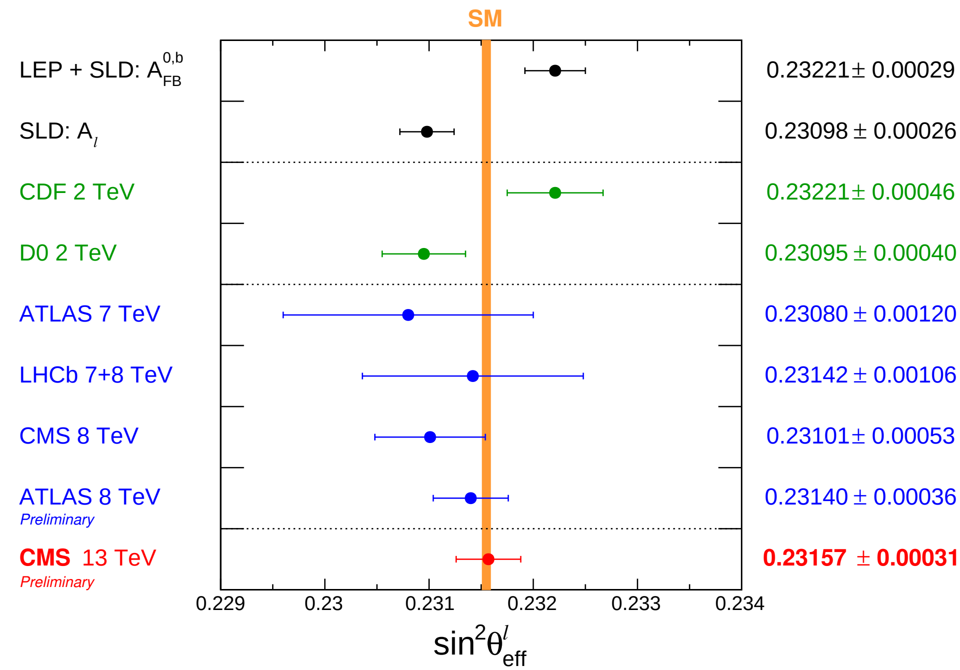


center-of-mass frame:



→ main systematics:
PDFs, QCD corrections

CMS '24



Averages:

PDG '24

$$\sin^2\theta^\ell_{\rm eff} = 0.23151 \pm 0.00016 \text{ (LEP+SLC)}$$

$$\sin^2\theta^\ell_{\rm eff} = 0.23146 \pm 0.00021 \text{ (LHC+TeV)}$$

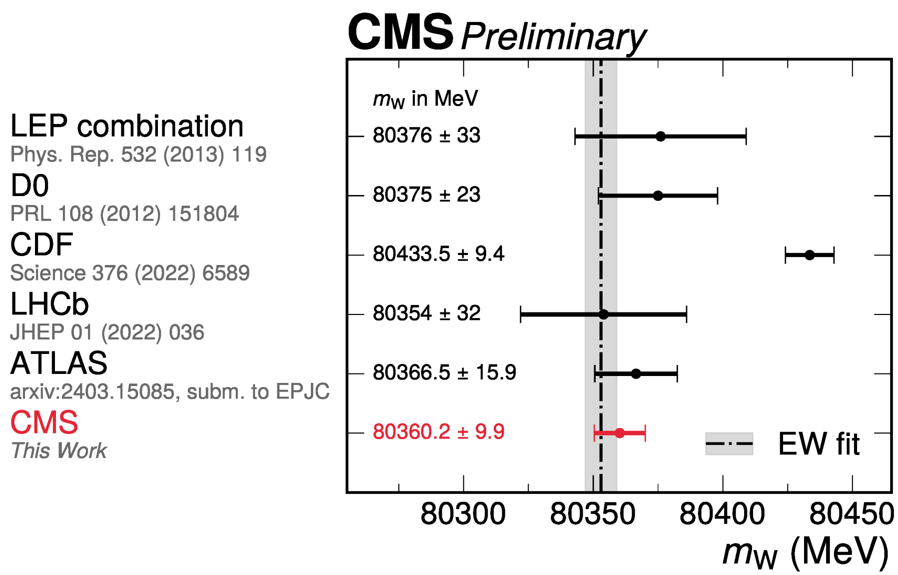
$$\sin^2\theta^\ell_{\rm eff} = 0.23149 \pm 0.00013 \text{ (colliders)}$$

LHC measurements

W mass:

from $pp \to W^{\pm} \to \ell^{\pm}\nu$,

using m_T and $p_{\ell,\perp}$ distributions



Averages

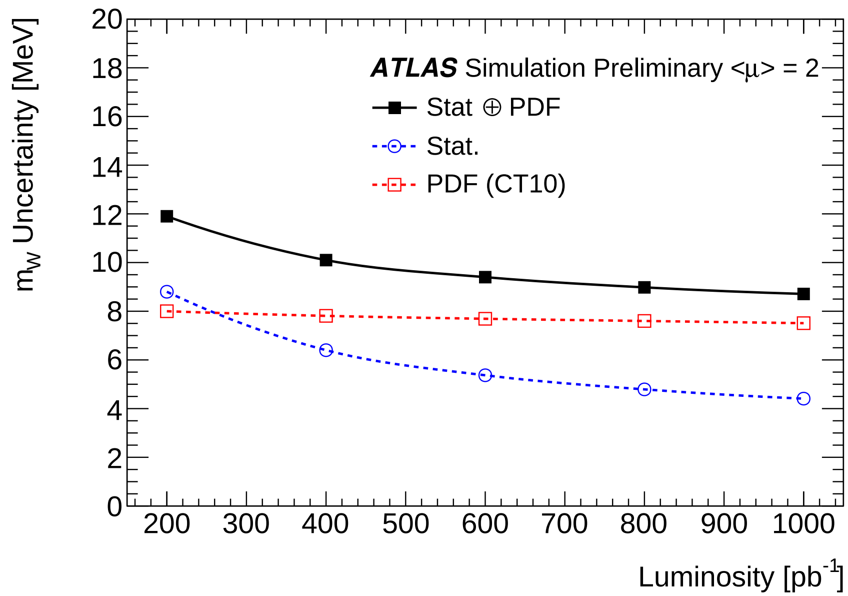
(approximate)

$$M_W = 80.360 \pm 0.008 \text{ GeV (w/o CDF II)}$$

$$M_W = 80.373 \pm 0.007 \text{ GeV (with CDF II)}$$

Ultimate precision at HL-LHC:

$\delta M_W \sim$ 5–10 MeV



$\rightarrow$ talk by M. Velasco

Other masses

- m_{t}: Most precise measurement at LHC: $\delta m_{\mathrm{t}} \sim 0.3$ GeV PDG '24

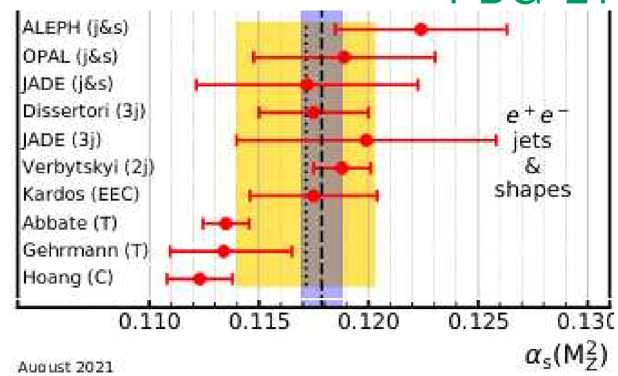
  Theoretical ambiguity in mass def.: Hoang, Plätzer, Samitz '18

$$m_{\mathrm{t}}^{\mathrm{CB}}(Q_0) - m_{\mathrm{t}}^{\mathrm{pole}} = -\tfrac{2}{3}\alpha_{\mathrm{s}}(Q_0)\, Q_0 + \mathcal{O}(\alpha_s^2 Q_0)$$
$$\approx 0.5 \pm 0.2_{\mathrm{pert.}} \pm 0.2_{\mathrm{np.}}\mathrm{GeV}$$

- $m_{\mathrm{b,c}}$: $\delta m_{b,c} \sim 8$ MeV (QCD sum rules) Erler, Masjuan, Spiesberger '16,22
- M_{H}: $M_{\mathrm{H}} = 125.10 \pm 0.09$ GeV (LHC)

- α_s: PDG '24
  - Most precise determination using Lattice QCD:
    $\alpha_s = 0.1184 \pm 0.0008$ FLAG '21

PDG '21



  - e^+e^- event shapes: $\alpha_s \sim 0.113...0.119$

    $\longrightarrow$ Large non-pertubative power corrections

    $\longrightarrow$ Systematic uncertainties?

  - Hadronic τ decays: $\alpha_s = 0.1173 \pm 0.0017$

    $\longrightarrow$ Non-perturbative uncertainties in OPE and from duality violation

    Pich '14; Boito et al. '15,18

  - Hadron colliders: jj, W/Z, $t\bar{t}$, DIS

    $\longrightarrow$ Most precise determination from $pp \rightarrow Z + X$ at 8 TeV:

    $\alpha_s = 0.1183 \pm 0.0009$ ATLAS '23

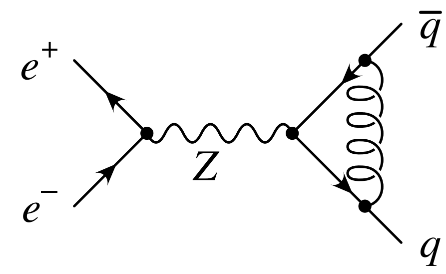
Strong coupling

- α_s:
  - Electroweak precision ($R_\ell = \Gamma_Z^{\text{had}}/\Gamma_Z^{\ell}$):
    $\alpha_s = 0.122 \pm 0.003$ PDG '24

    $\rightarrow$ Negligible non-perturbative QCD effects

    Theory input: N^{4}LO QCD corr. + NNLO EW

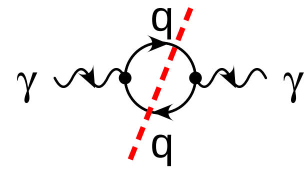
    Caviat: R_ℓ could be affected by new physics



Electromagnetic coupling

- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$

a) $\Delta\alpha_{\text{had}}$ from $e^+e^- \to$ had. using dispersion relation

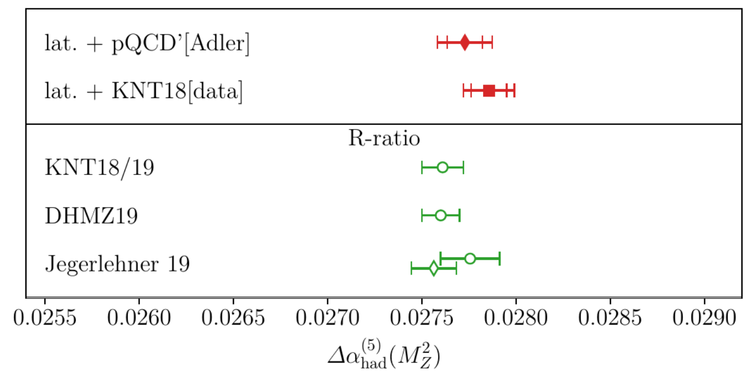


$\rightarrow$ Current precision $\sim 10^{-4}$

Davier et al. '19; Jegerlehner '19; Keshavarzi, Nomura, Teubner '19

b) $\Delta\alpha_{\text{had}}$ from Lattice QCD
(challenging but much progress)

Burger et al. '15
Cè et al. '22



Future improvements for methods (a) and (b):

- More precise exp./lattice data
- Full 4-loop pQCD for R-ratio / Adler function (for $|Q^2| \gg \Lambda_{\text{QCD}}$)
- More precise inputs for m_b, m_c, α_s

$\rightarrow \delta(\Delta\alpha_{\text{had}}) \lesssim 5 \times 10^{-5}$ likely achievable — Jegerlehner '19

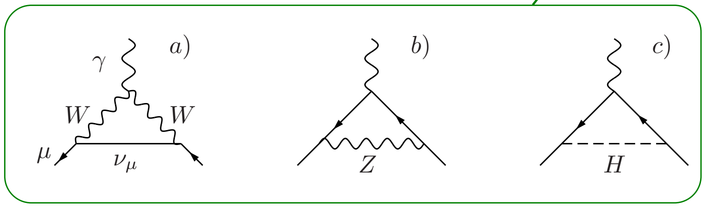
Muon magnetic moment

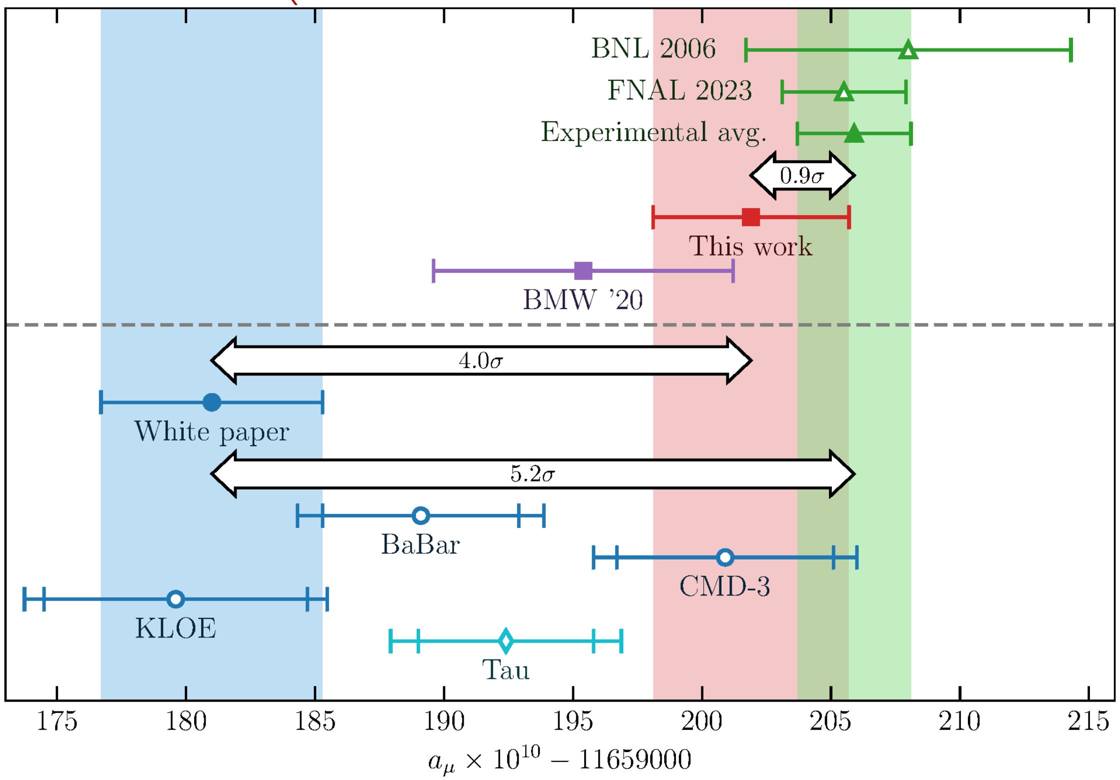
$a_\mu = \frac{g_\mu - 2}{2}$: minor correction from EW physics, but high precision

$a_\mu^{\text{exp}} = (1165920.59 \pm 0.22) \times 10^{-9}$ Muon g-2 '23

$a_\mu^{\text{th}} = (\underbrace{1165847.19}_{\text{QED}} + \underbrace{1.54}_{\text{EW}} + \underbrace{[65.58 \pm 0.25]}_{\text{HVP}} + \underbrace{[1.11 \pm 0.10]}_{\text{LBL}}) \times 10^{-9}$

→ talk by A. El Khadra

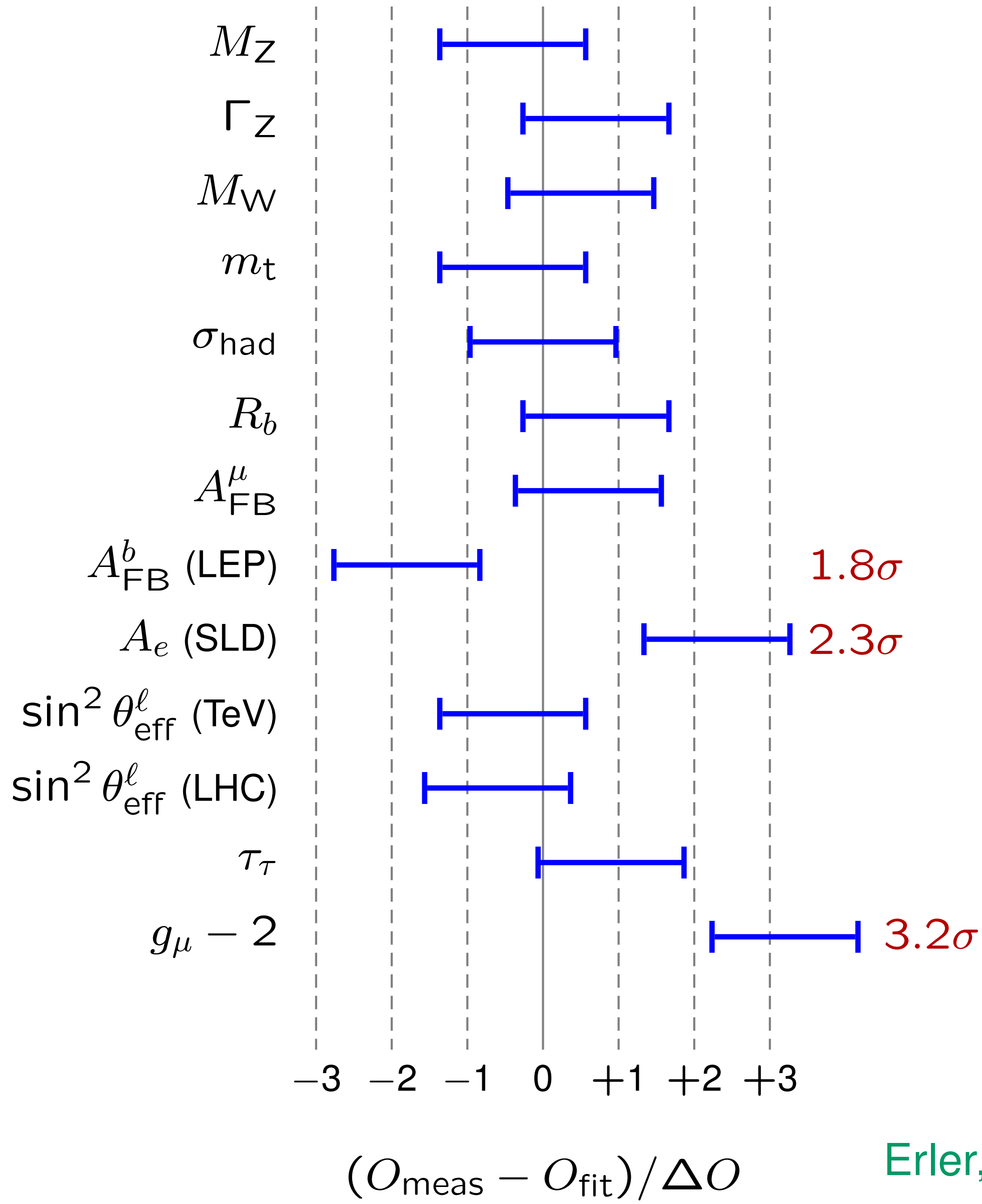




Discrepancies in evaluation of HVP contribution

BMW '24

Global electroweak fit



(data driven HVP incl. CMD-3 + BMW)

Erler, Freitas (RPP) '24

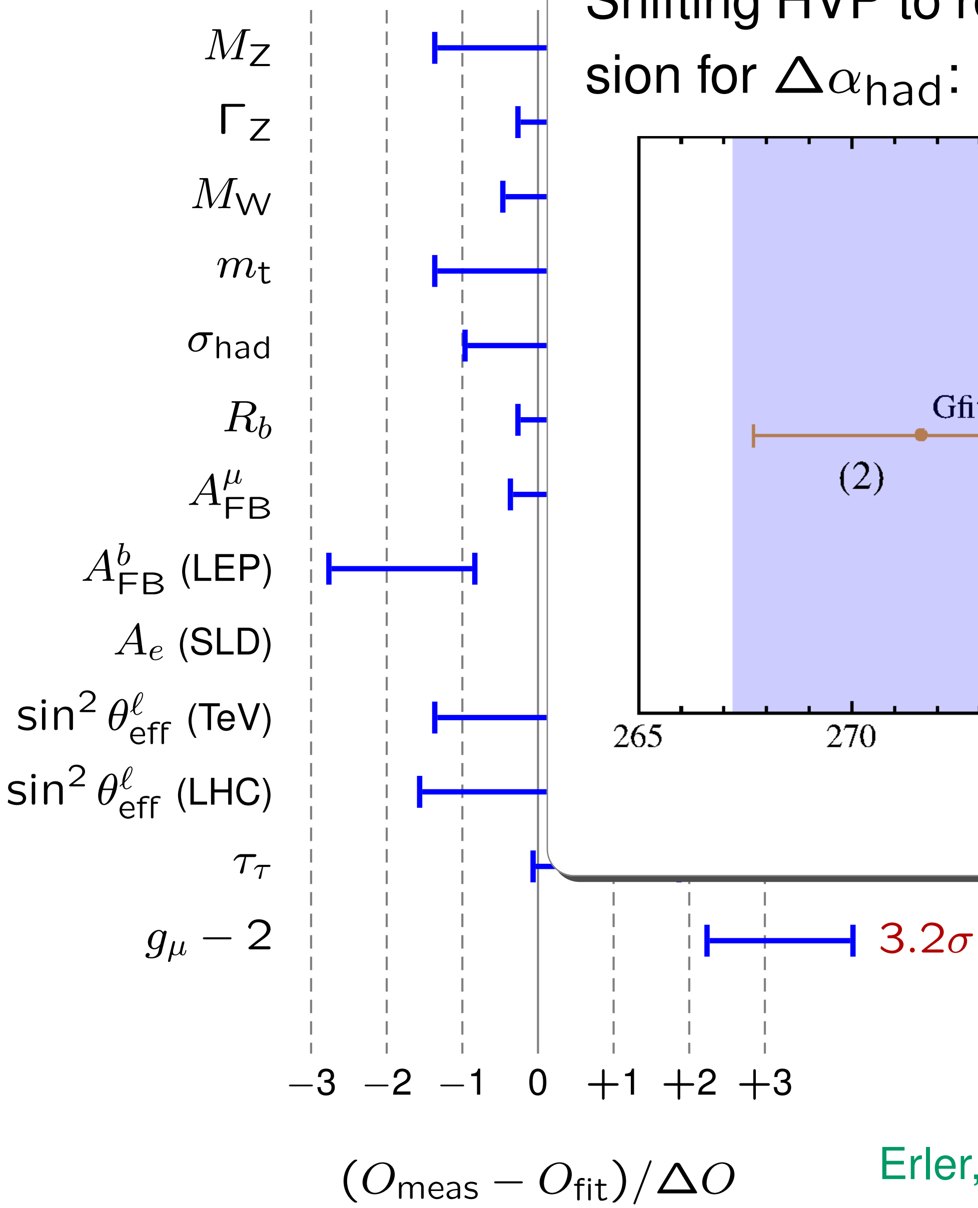
Very good agreement:

χ^2/d.o.f. $= 49.5/47$ ($p = 37\%$)

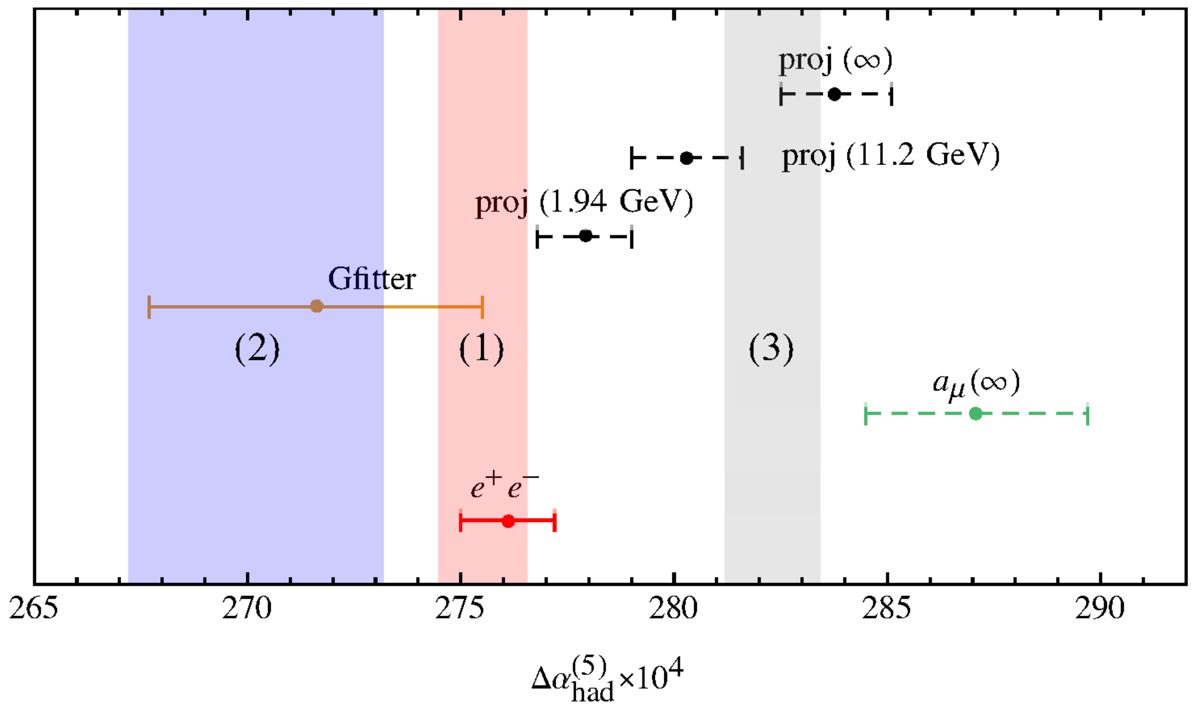
[without M_W from CDF II]

Most quantities measured with 1%–0.1% precision

Global electroweak fit



Shifting HVP to reconcile $g_\mu - 2$ slightly increases tension for $\Delta\alpha_{had}$: Crivellin, Hoferichter, Manzari, Montull '20



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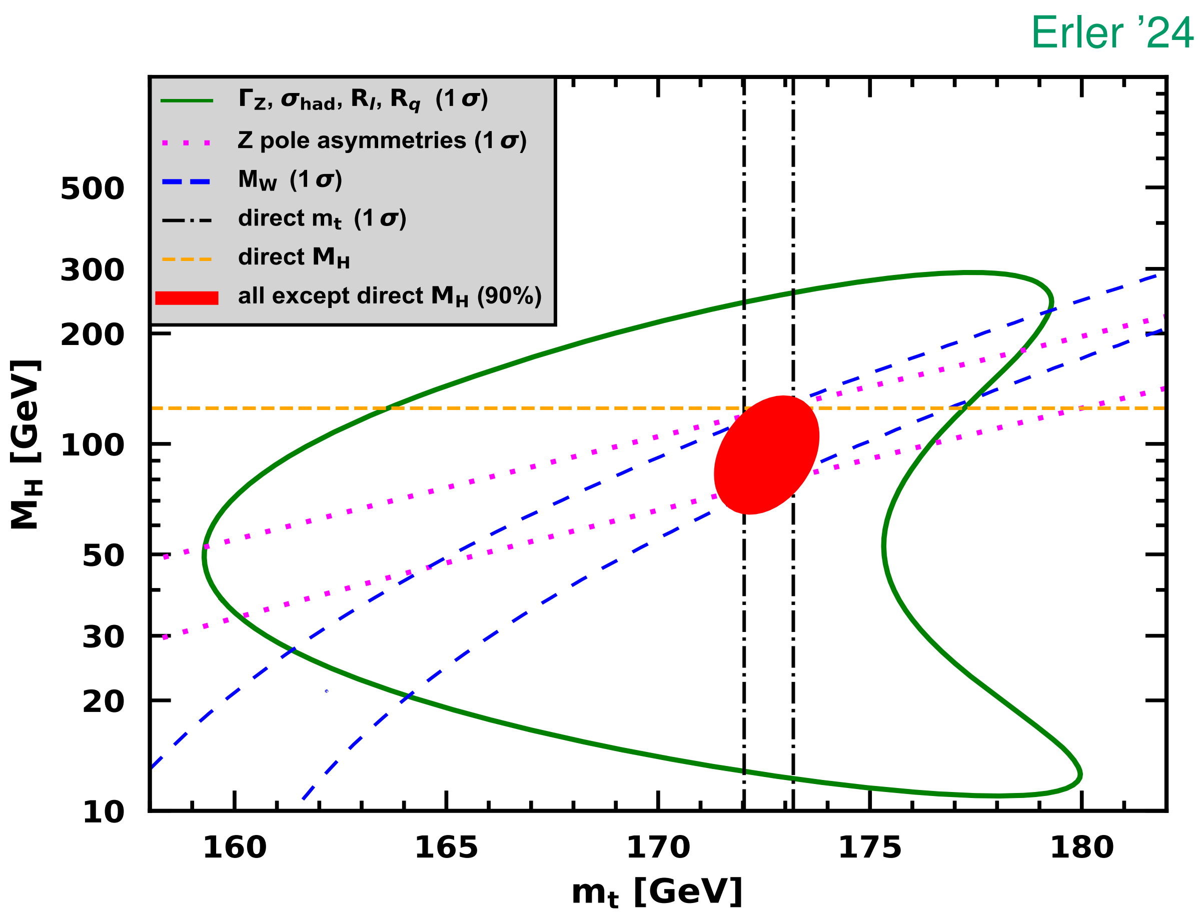
Global electroweak fit



Direct measurements:

$M_H = 125.10 \pm 0.09$ GeV

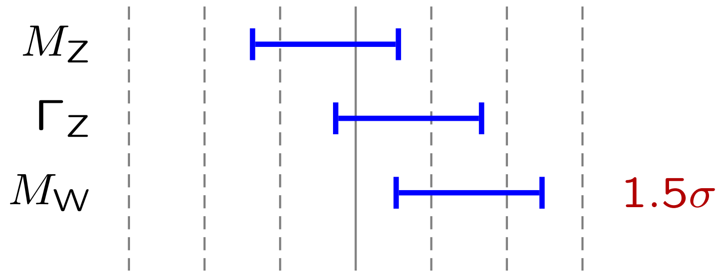
$m_t = 172.61 \pm 0.58$ GeV

Indirect prediction:

$M_H = 97^{+18}_{-16}$ GeV

$m_t = 175.2 \pm 1.8$ GeV

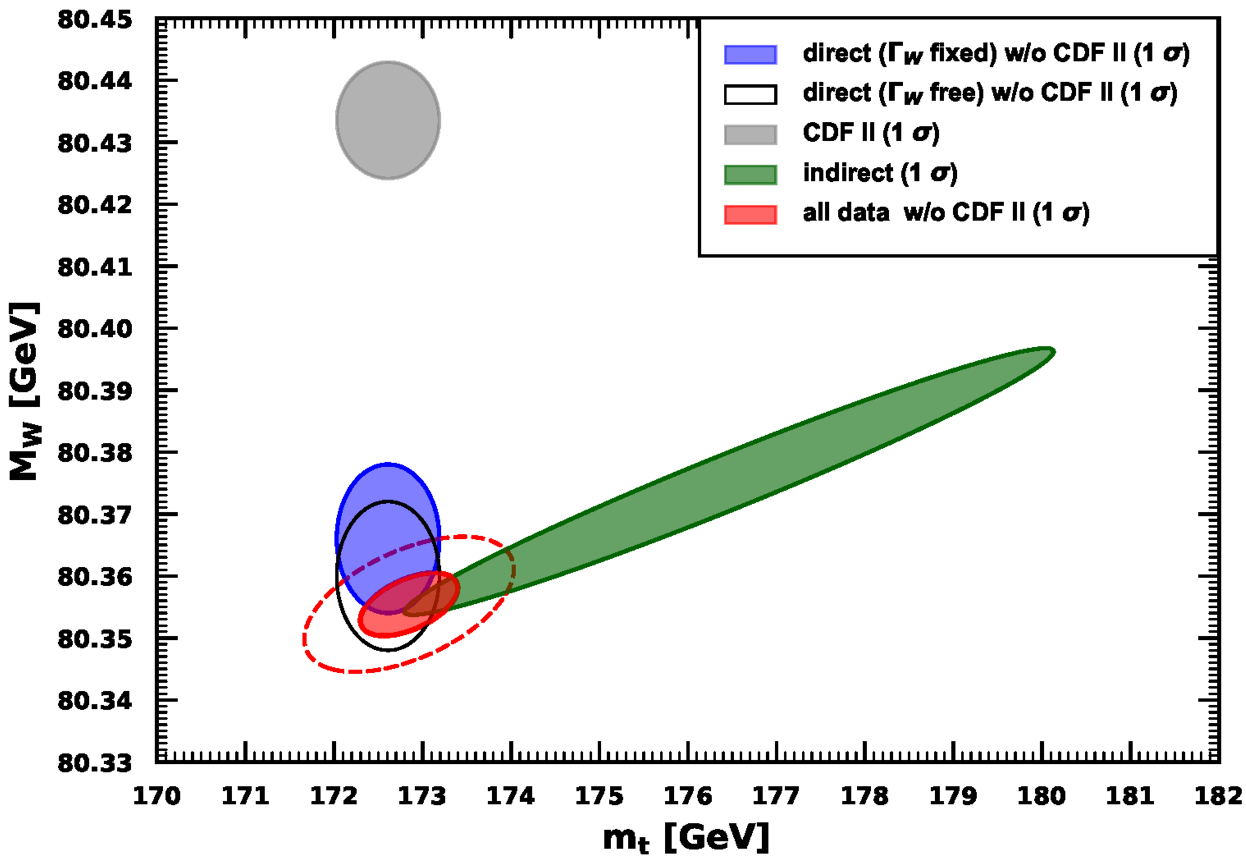
Global electroweak fit



Including M_W from CDF II:

Marginal agreement:

χ^2/d.o.f. $= 70.1/48$ ($p = 2\%$)



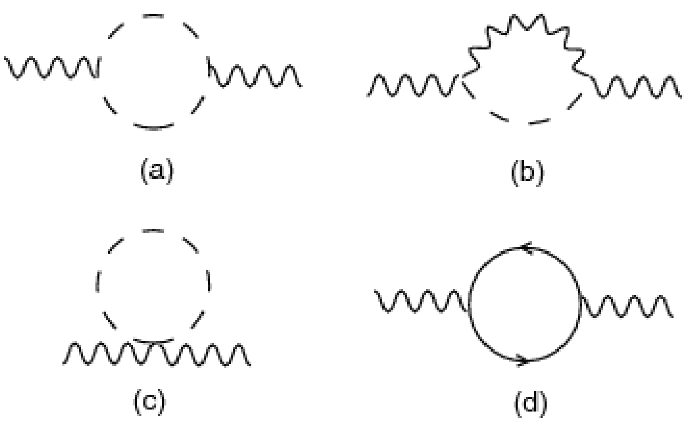
(plot without new CMS result)

Erler '24

Adding BSM oblique parameters:

$$\alpha T = \frac{\Sigma_{WW}(0)}{M_W} - \frac{\Sigma_{ZZ}(0)}{M_Z}$$

$\rightarrow$ $T = 0.08 \pm 0.02$



Including M_W from CDF II:

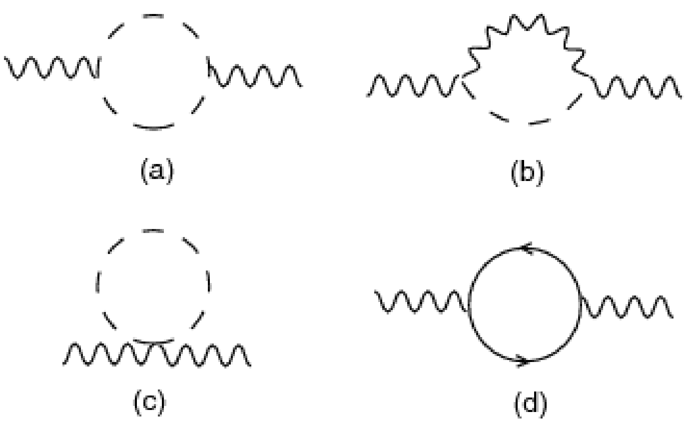
Good agreement:

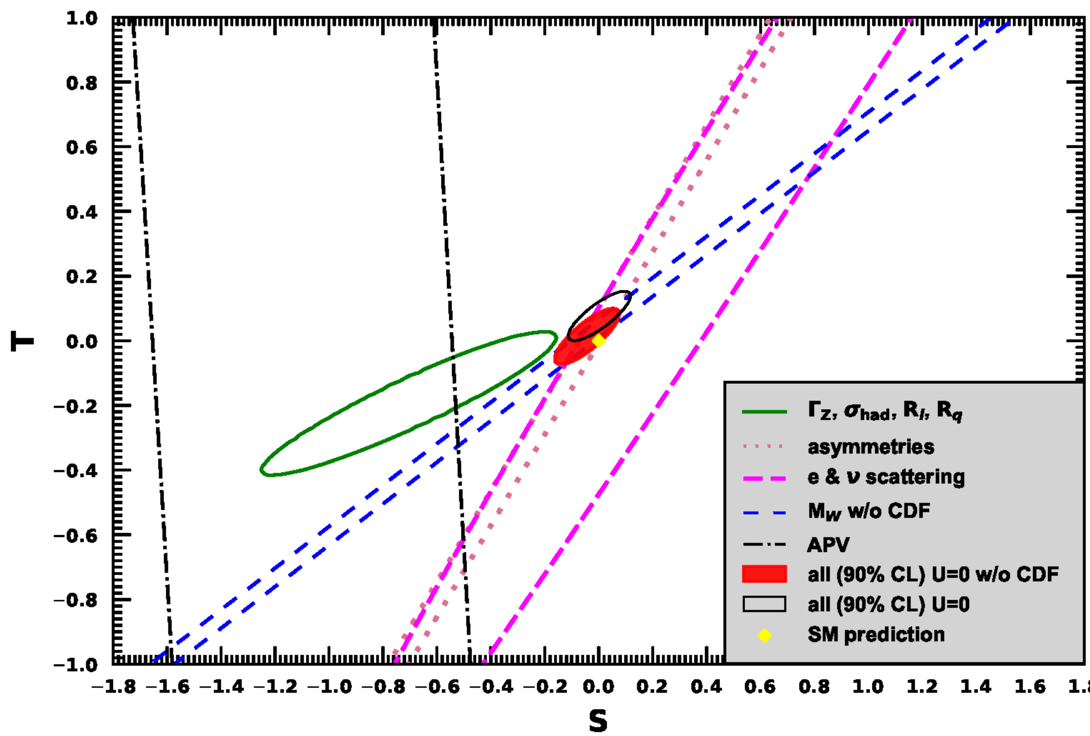
$\chi^2/\text{d.o.f.} = 58.5/47$ ($p = 12\%$)

Adding BSM oblique parameters:

$$\alpha T = \frac{\Sigma_{WW}(0)}{M_W} - \frac{\Sigma_{ZZ}(0)}{M_Z}$$

$$\frac{\alpha}{4s^2c^2} S = \frac{\Sigma_{ZZ}(M_Z^2) - \Sigma_{ZZ}(0)}{M_Z} + \frac{s^2 - c^2}{sc} \frac{\Sigma_{Z\gamma}(M_Z^2)}{M_Z} - \frac{\Sigma_{\gamma\gamma}(M_Z^2)}{M_Z}$$





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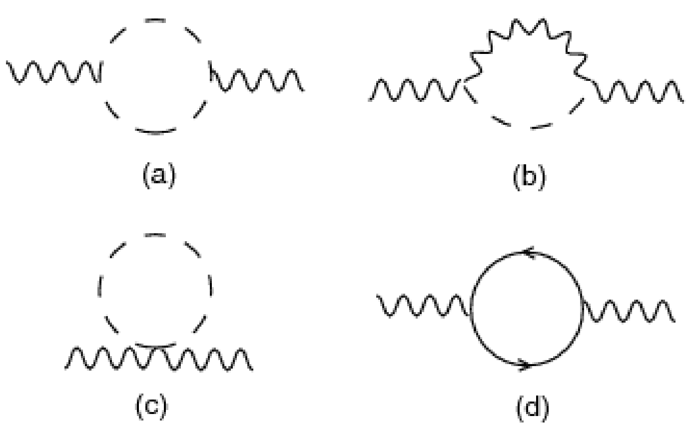

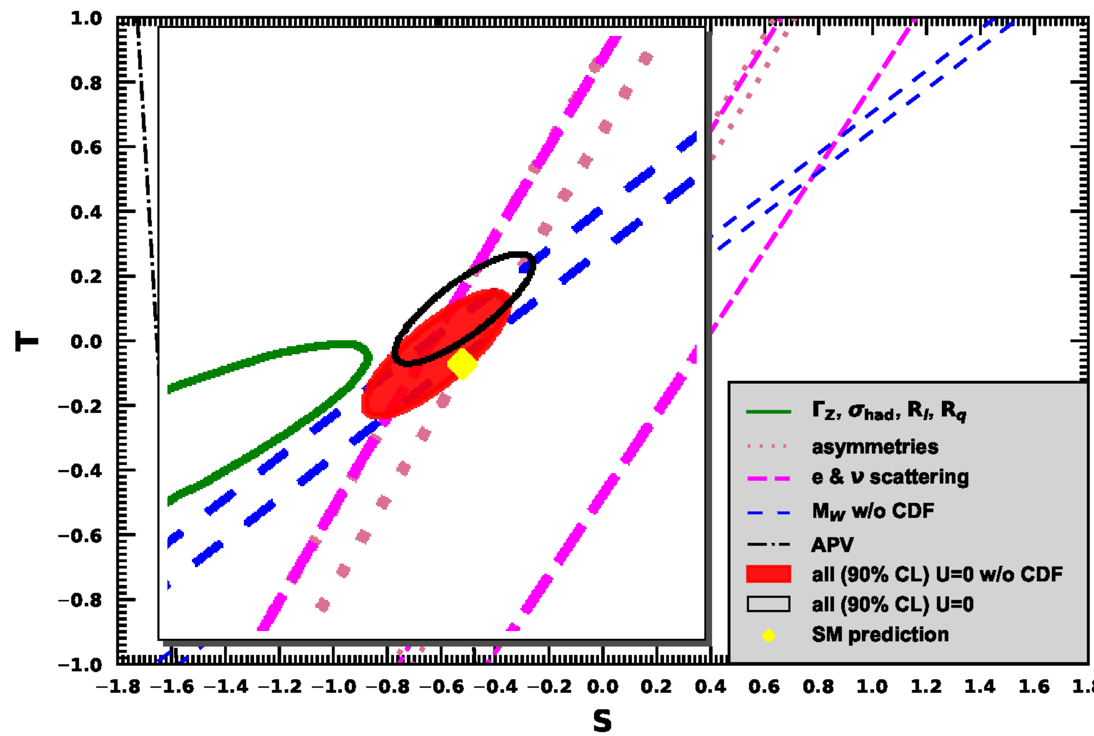

Global electroweak fit

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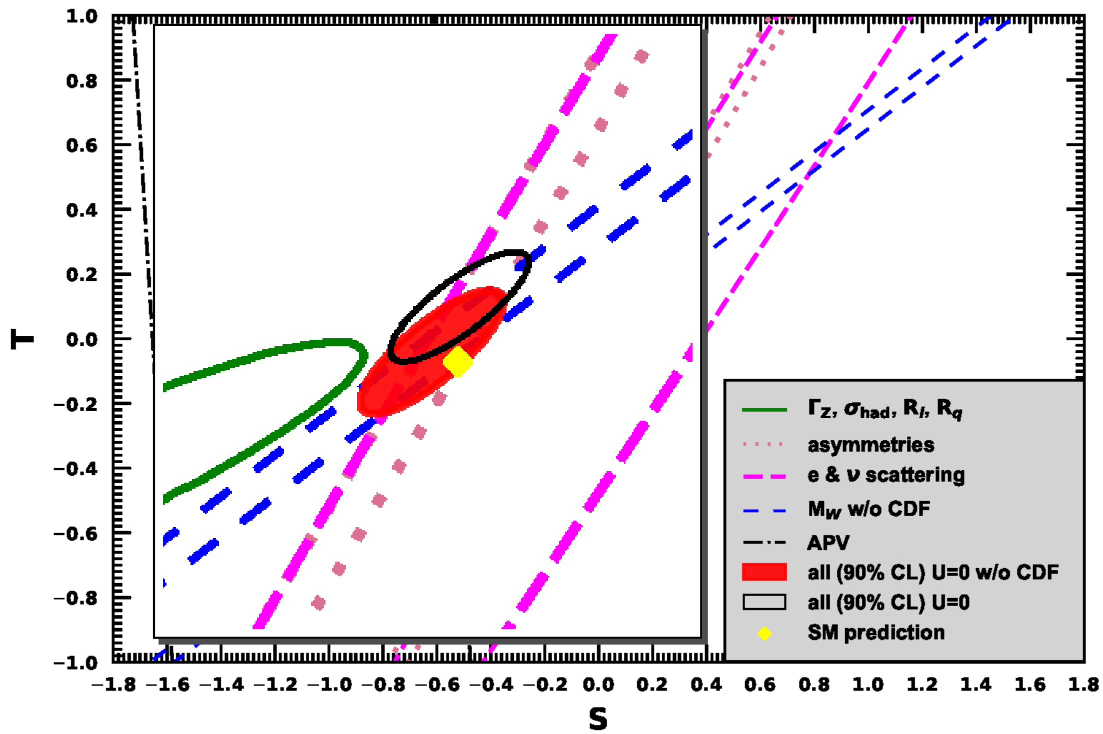
Dim-6 SMEFT studies confirm that T parameter effectively absorbs M_W shift

de Blas, Pierini, Reina, Silvestrini '22
Bagnaschi et al. '22; Balkin et al. '22



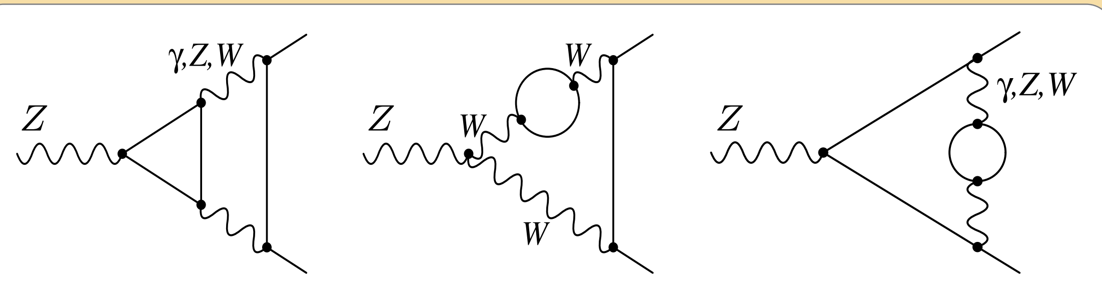
Theoretical predictions in the SM

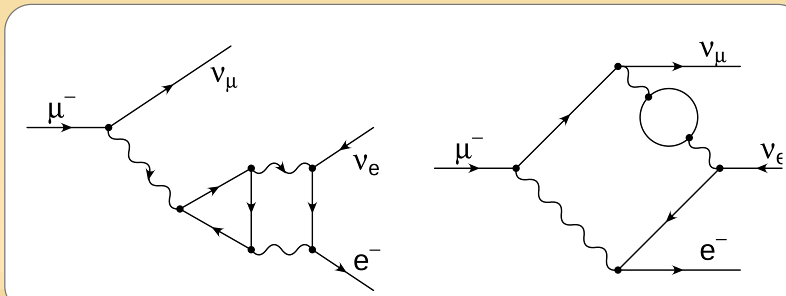
Objective: Comparison of measurements for pseudo-obs. (M_W, $\sin^2\theta^\ell_{eff}, \ldots$) with SM theory predictions

	δM_W [MeV]	$\delta \sin^2\theta^\ell_{eff}$ [10^{-5}]	
now	$\pm$ 8	$\pm$ 13	
1-loop	$\pm$ 450	$\pm$ 1000	Marciano, Sirlin '80
2-/3-loop QCD	$\pm$ 70	$\pm$ 45	Djouadi et al. '88 Chetyrkin, Kühn, Steinhauser '95
ferm. 2-loop EW	$\pm$ 50	$\pm$ 90	Freitas et al. '00 Awramik, Czakon '03 Awramik, Czakon, Freitas, Weiglein '04
bos. 2-loop EW	$\pm$ 2	$\pm$ 1	Awramik, Czakon, Freitas '06
leading 3-loop	$\pm$ 5	$\pm$ 25	Faisst, Kühn, Seidensticker, Veretin '03

computed from G_μ

Experimental precision sensitive to 2-/3-loop effects





Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

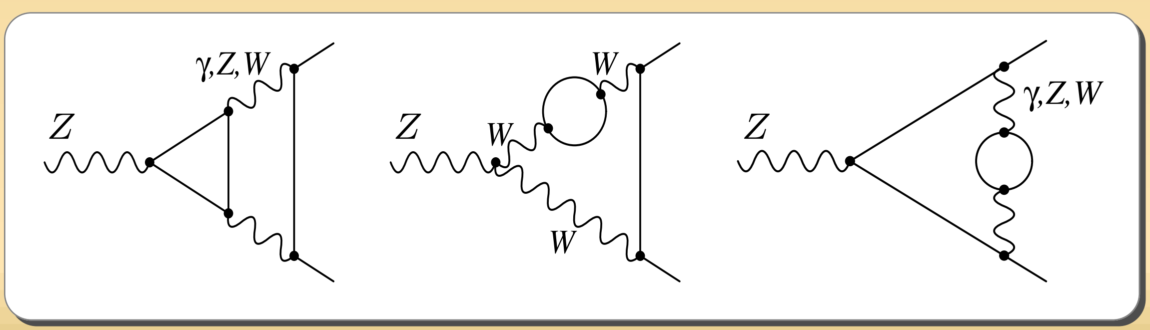
  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

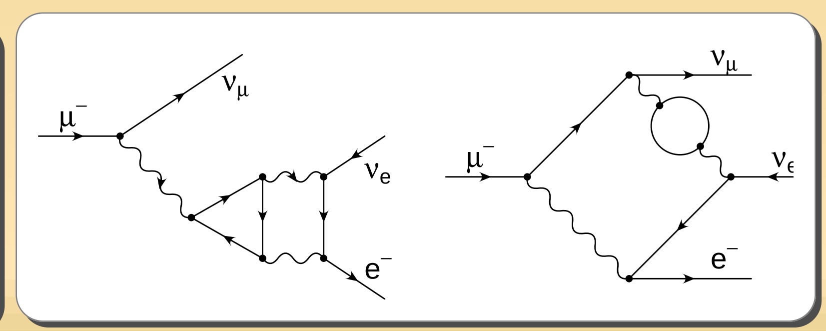
- Full 2-loop results for M_W, Z-pole observables

  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07
  Awramik, Czakon, Freitas, Kniehl '08
  Freitas '14
  Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

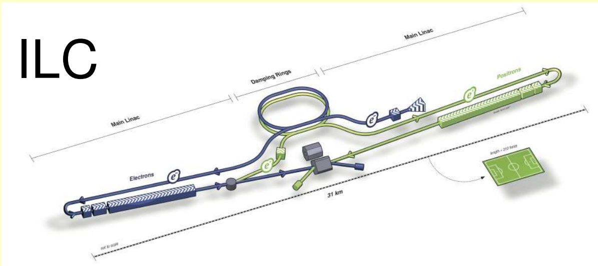
- Approximate 3- and 4-loop results (enhanced by Y_t and/or N_f)

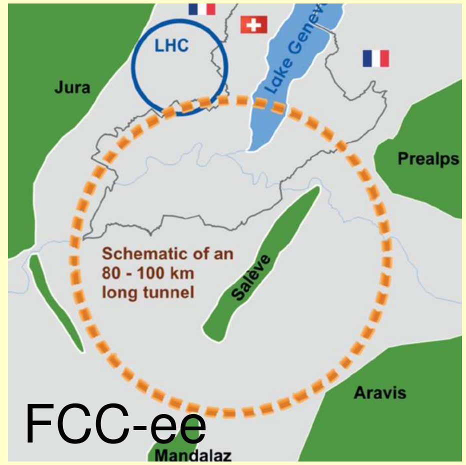
  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06
  Chen, Freitas '20

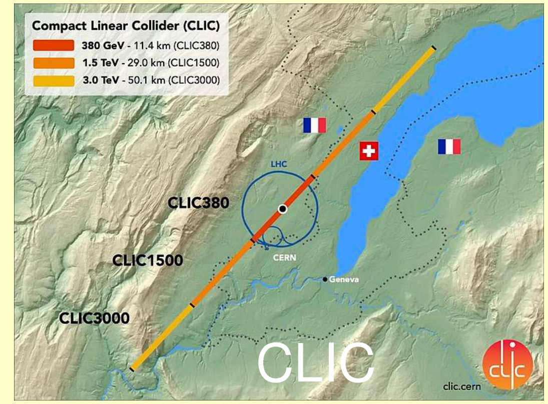


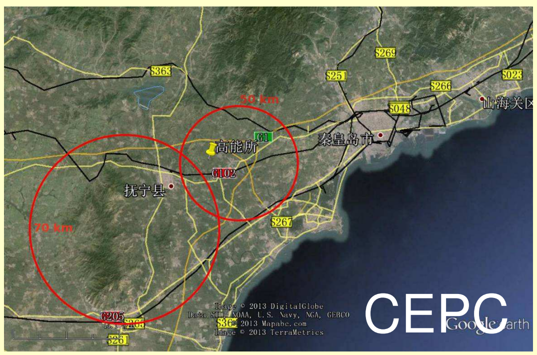


Precision physics with future e^+e^- colliders









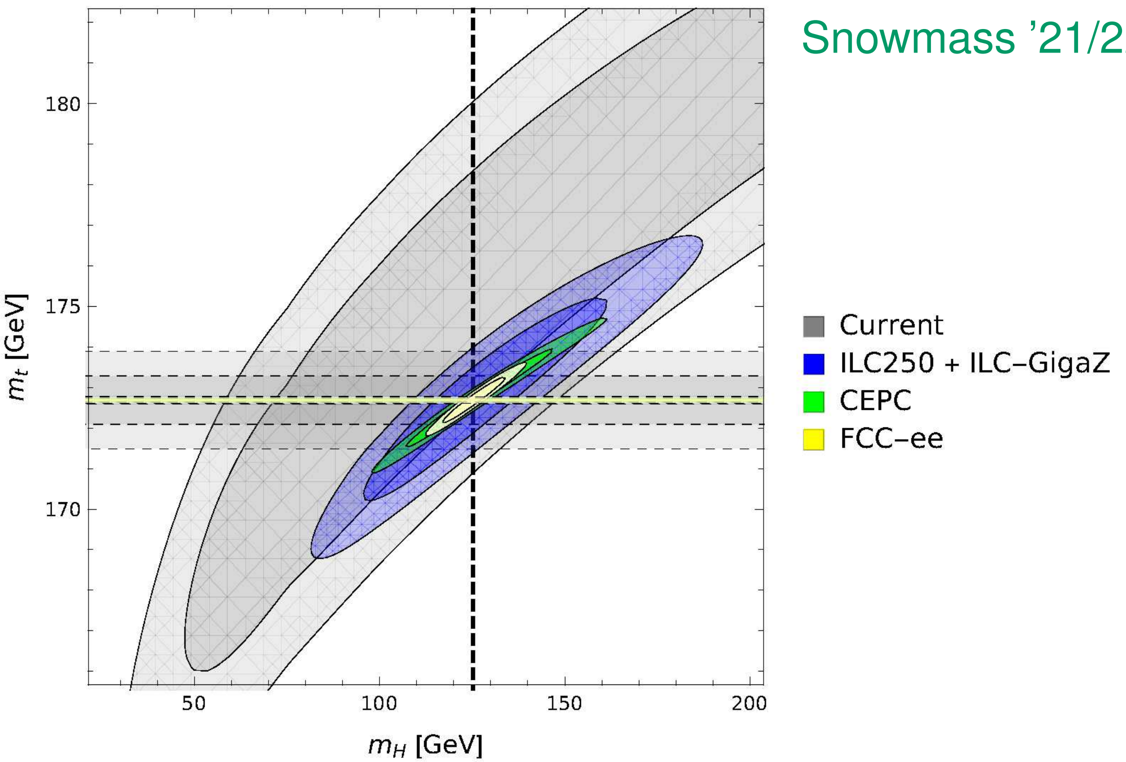
- circular colliders: high-lumi run at $\sqrt{s} \sim M_Z$
- linear colliders: radiative return $e^+e^- \to \gamma Z$

$\sqrt{s}$	M_Z	$2M_W$	240–250 GeV	350–380 GeV
ILC	*100 fb^{-1}*	*500 fb^{-1}*	2 ab^{-1}	200 fb^{-1} (10 pts.)
CLIC	–	–	–	1 ab^{-1}
FCC-ee	150 ab^{-1}	10 ab^{-1} (2 pts.)	5 ab^{-1}	1 ab^{-1} (8 pts.)
CEPC	100 ab^{-1}	6 ab^{-1} (3 pts.)	20 ab^{-1}	1 ab^{-1} ?

$\to$ talks by J. Guimaraes da Costa, G. Bernardi

Precision physics with future e^+e^- colliders

	Current exp.	ILC250	CEPC	FCC-ee
M_W [MeV]	8	2.4	0.5	0.4
Γ_Z [MeV]	2.3	1.5	0.025	0.025
$R_\ell = \Gamma_Z^{had}/\Gamma_Z^\ell$ $[10^{-3}]$	25	20	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{had}$ $[10^{-5}]$	66	23	4.3	6
$\sin^2\theta_{eff}^\ell$ $[10^{-5}]$	13	2	0.3	0.4



Snowmass '21/22

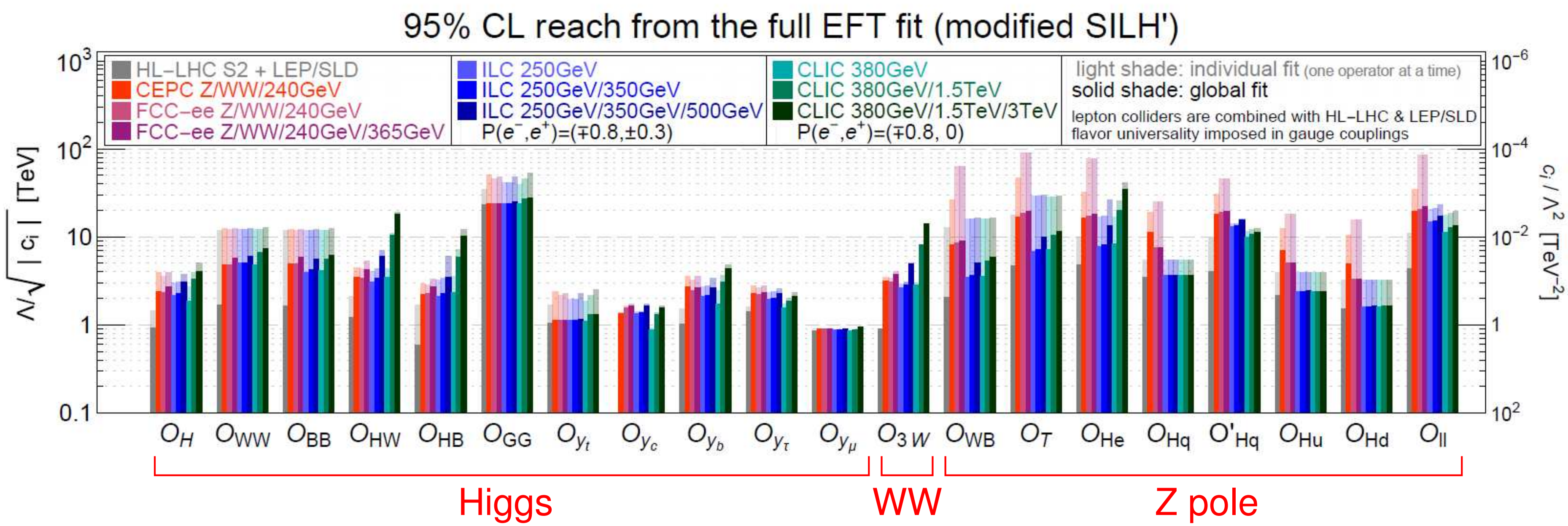
Precision physics with future e^+e^- colliders (SMEFT)

- Extension of SM by **higher-dimensional operators**:

Wilson '69
Weinberg '79

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\ldots + \frac{1}{\Lambda^2}\sum_i c_i\,\mathcal{O}_i^{(6)} + \ldots$$

- SMEFT dim-6 operators provide framework for comparing experiments



de Blas, Durieux, Grojean, Gu, Paul '19

Theory calculations: Uncertainties

- To probe new physics, compare EWPOs with SM theory predictions

- Need to take theory error into account:

	Current exp.	Current th.	CEPC	FCC-ee
$M_{\rm W}$ [MeV]	11–12	4	0.5	0.4
Γ_Z [MeV]	2.3	0.4	0.025	0.025
$R_\ell = \Gamma_Z^{\rm had}/\Gamma_Z^\ell$ [10^{-3}]	25	5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{\rm had}$ [10^{-5}]	66	10	4.3	6
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	13	4.5	0.3	0.4

- Theory error estimate is not well defined, ideally $\Delta_{\rm th} \ll \Delta_{\rm exp}$

- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Theory calculations: Uncertainty projections

- Estimated impact of future higher-order calculations — Freitas et al. ’19

	Current th.	Projected th.†	CEPC	FCC-ee
M_W [MeV]	4	1	0.5	0.4
Γ_Z [MeV]	0.4	0.15	0.025	0.025
$R_\ell = \Gamma_Z^{had}/\Gamma_Z^\ell$ [10^{-3}]	5	1.5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{had}$ [10^{-5}]	10	5	4.3	6
$\sin^2\theta_{eff}^\ell$ [10^{-5}]	4.5	1.5	0.3	0.4

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^2\alpha_s)$, leading 4-loop
(N_f^n = at least n closed fermion loops)

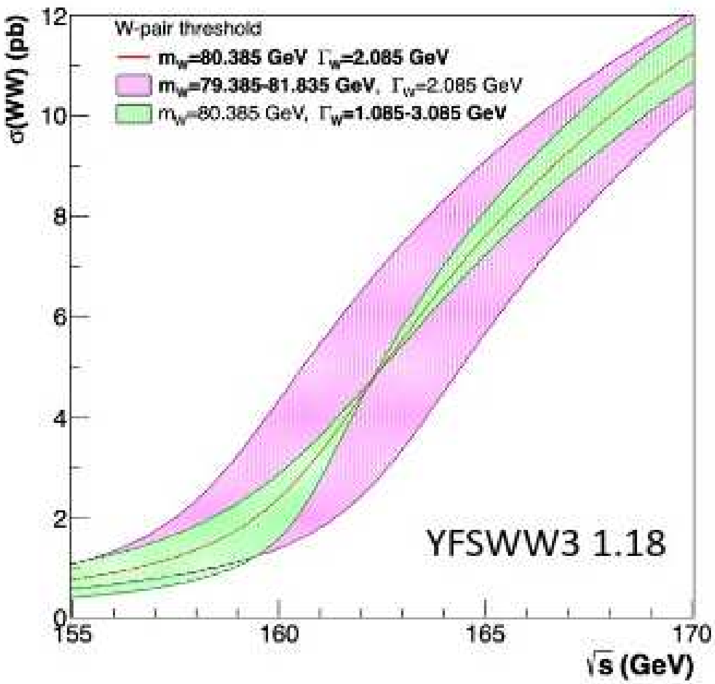
Note: Estimates (based on extrapolation of perturb. series and prefactors) are unreliable and only provide a rough guess

- Also need NNLO corrections for subtracted “backgrounds”
- More precise MC tools for multi-photon emission, hadronization, etc.

Jadach, Skrzypek ’19

WW threshold : W mass and width

Scans of possible E_1 E_2 data taking energies and luminosity fractions f (at the E_2 point)



A -minimum of $\Delta\Gamma_W$=0.91 MeV with Δm_W=0.55 MeV
taking data at E_1=**156.6** GeV E_2=**162.4** GeV f=**0.25**
yields **Δm_W=0.47 MeV** (as single par)

B- minimum of **Δm_W=0.28 MeV** **$\Delta\Gamma_W$=3.3 MeV** with
E_1=**155.5** GeV E_2=**162.4** GeV f=**0.95**
yields **Δm_W=0.28 MeV** (as single par)

C- minimum of **$\Delta\Gamma_W$=0.96 MeV +Δm_W=0.41 MeV** with
E_1=**157.5** GeV E_2=**162.4** GeV f=**0.45**
yields and **Δm_W=0.37 MeV** (as single par)

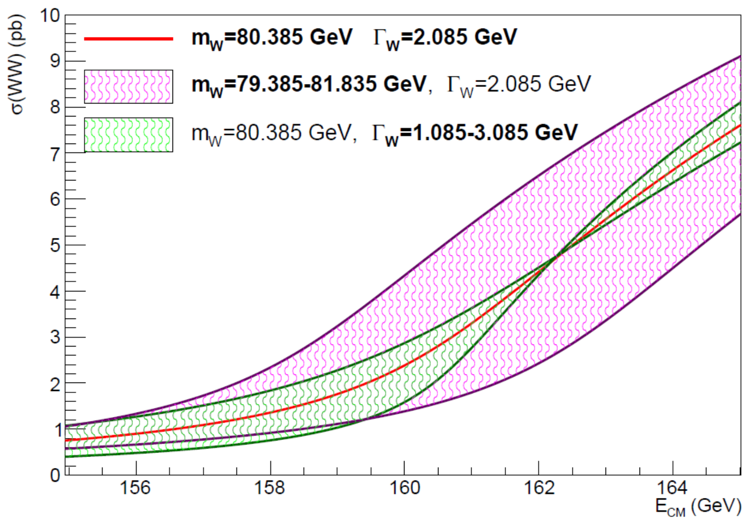
Δm_W, $\Delta\Gamma_W$: error on W mass and width from fitting both
Δm_W : error on W mass from fitting only m_W

WW threshold

a) Corrections near threshold enhanced by $1/\beta$ and $\ln\beta$

$$\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$

b) Non-resonant contributions are important



- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$

  Denner, Dittmaier, Roth, Wieders '05

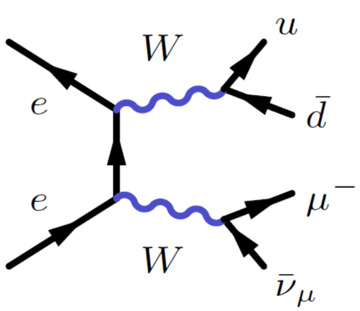
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$

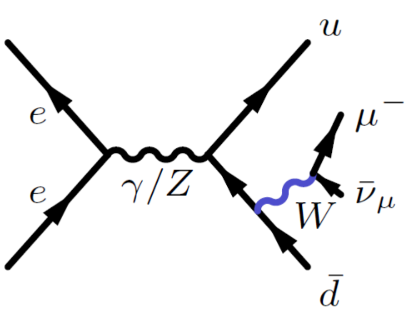
  Beneke, Falgari, Schwinn, Signer, Zanderighi '07

  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{th} M_W \sim 3$ MeV

    Actis, Beneke, Falgari, Schwinn '08

  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{th} M_W \lesssim 0.6$ MeV

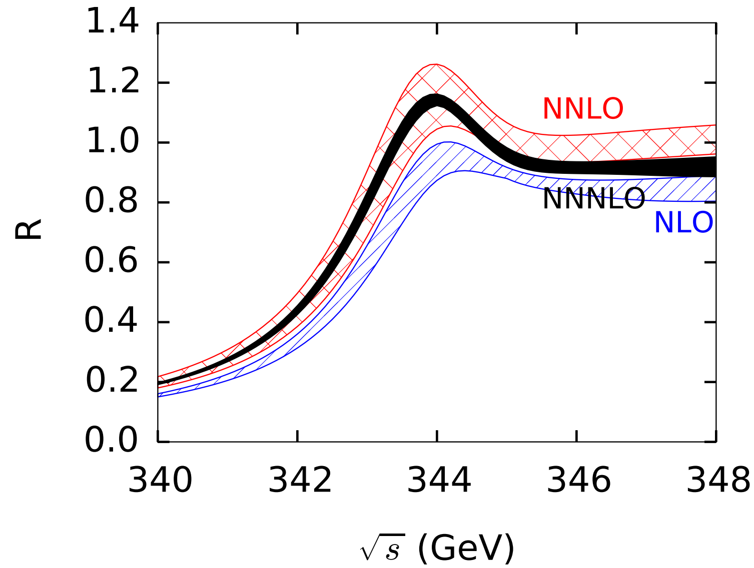




From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV:

Impact of theory modelling:

$$\delta m_{\mathrm{t}}^{\overline{\mathrm{MS}}} = [\quad]_{\mathrm{exp}} \oplus [50\ \mathrm{MeV}]_{\mathrm{QCD}} \oplus [10\ \mathrm{MeV}]_{\mathrm{mass\ def.}} \oplus [70\ \mathrm{MeV}]_{\alpha_s} > 100\ \mathrm{MeV}$$



Beneke et al. ’15

Top-quark mass

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV:

Impact of theory modelling:

$$\delta m_t^{\overline{\text{MS}}} = [\quad]_{\text{exp}} \oplus [50 \text{ MeV}]_{\text{QCD}} \oplus [10 \text{ MeV}]_{\text{mass def.}} \oplus [70 \text{ MeV}]_{\alpha_s} > 100 \text{ MeV}$$

future improvements:

$[20 \text{ MeV}]_{\text{exp}}$ (FCC-ee, CEPC)

$\oplus [30 \text{ MeV}]_{\text{QCD}}$ (h.o. resumm., N^{4}LO?)

$\oplus [10 \text{ MeV}]_{\text{mass def.}}$

$\oplus [15 \text{ MeV}]_{\alpha_s}$ ($\delta\alpha_s \lesssim 0.0002$)

$\lesssim 50$ MeV

Strong coupling

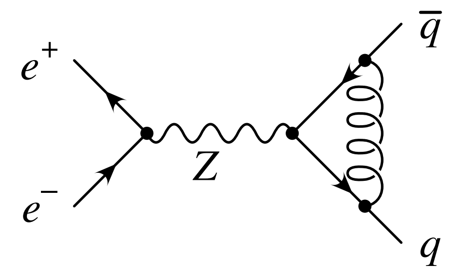
- α_s:
  - Electroweak precision ($R_\ell = \Gamma_Z^{had}/\Gamma_Z^\ell$):
    $\alpha_s = 0.122 \pm 0.003$ PDG '18

    $\rightarrow$ Negligible non-perturbative QCD effects

    FCC-ee: $\delta R_\ell \sim 0.001$
    $\Rightarrow \quad \delta\alpha_s < 0.0001$

    Theory input: N^{3}LO EW corr. + leading N^{4}LO
    to keep $\delta_{th} R_\ell \lesssim \delta_{exp} R_\ell$

    Caviat: R_ℓ could be affected by new physics



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    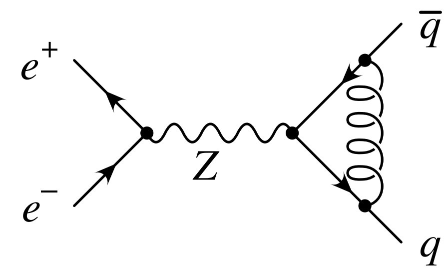

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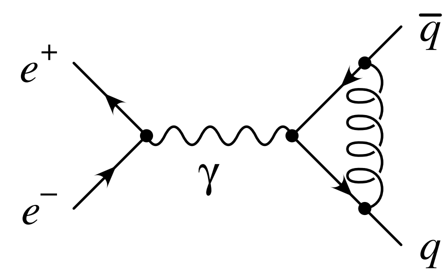
    FCC-ee: $\delta R_\ell \sim 0.001$
    $\Rightarrow \quad \delta\alpha_s < 0.0001$

    Caviat: R_ℓ could be affected by new physics

  - $R = \frac{\sigma[ee\to\text{had.}]}{\sigma[ee\to\mu\mu]}$ at lower $\sqrt{s}$

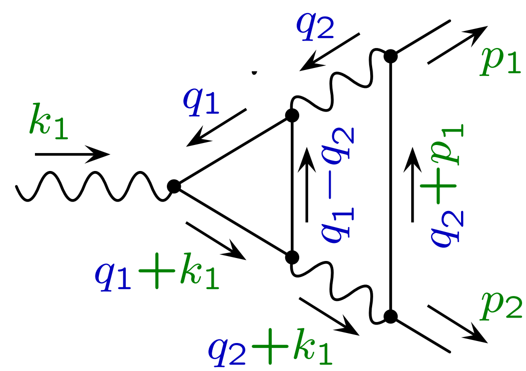
    e.g. CLEO ($\sqrt{s} \sim 9$ GeV): $\alpha_s = 0.110 \pm 0.015$
    Kühn, Steinhauser, Teubner '07

    $\rightarrow$ dominated by s-channel photon, less room for new physics
    $\rightarrow$ QCD still perturbative

    naive scaling to 50 ab^{-1} (BELLE-II): $\delta\alpha_s \sim 0.0001$

Theoretical calculations

Experimental precision requires inclusion
of **multi-loop corrections** in theory



Integrals over loop momenta:

$$\int d^4q_1 d^4q_2 \; f(q_1, q_2, p_1, k_1, ..., m_1, m_2, ...)$$

Challenges:

1. $\mathcal{O}(1000) - \mathcal{O}(10000)$ integrals

2. Individual integrals can be divergent (drop out for physical results)
$\rightarrow$ Regularization, renormalization

3. Multi-dimensional integrations, depending on multiple mass/momentum scales

General approaches:

- Analytical
- Numerical
- Approximations (expansions), specialized techniques, ...

Calculational techniques

Analytical techniques:

- Reduction to master integrals (MIs), reduced number of ints. by 10-100
  $\rightarrow$ computationally intensive
  $\rightarrow$ public programs `Reduze, FIRE, LiteRed, KIRA, FiniteFlow,` ...
  von Manteuffel, Studerus '12; Smirnov '13,14; Lee '13; Maierhoefer, Usovitsch, Uwer '17; Peraro '19
- Not fully understood function space of MIs
  (Goncharov polylogs, iterated elliptic integrals, hypergeometric functions, ...)
- Work best for problems with few (no) masses

Numerical techniques:

- Multi-dim. numerical integrations:
  - in momentum space: $4L$ dimensions ($L =$ # of loops)
  - in Feynman par. space: $P - 1$ dimensions ($P =$ # of propagators)

  $\rightarrow$ slowly converging, limited precision
- Numerical instabilites, in particule for diagrams with physical cuts
- Works best for problems with many masses

Calculational techniques

New techniques, e.g.:

- Numerical reduction to MIs, numerical MIs via differential equations (DEs)
  Mandal, Zhao '18, Czakon, Niggetiedt '20
- DEs with respect to auxialiary parameter, $\frac{1}{k_i^2 - m_i^2 + i\epsilon}$
  Liu, Ma, Wang '17
  Liu, Ma '18,21,22
- Series solutions of DEs — Moriello '19, Hidding '20
- Dispersion relations + Feynman parameters — Song, Freitas '21, 22

Summary

- **Electroweak precision tests** have played an important role in testing the Standard Model
- Today they probe physics beyond the Standard Model at **TeV scale**
- More data from **LHC** and **future** e^+e^- **colliders** will push the reach into the **multi-TeV** regime
- **Electroweak fits** rely on detailed theory calculations for QED effects, backgrounds, SM predictions, etc.
- Future precision data requires progress in **fixed-order loop calculations**, **MC tools** and **non-perturbative QCD effects**

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