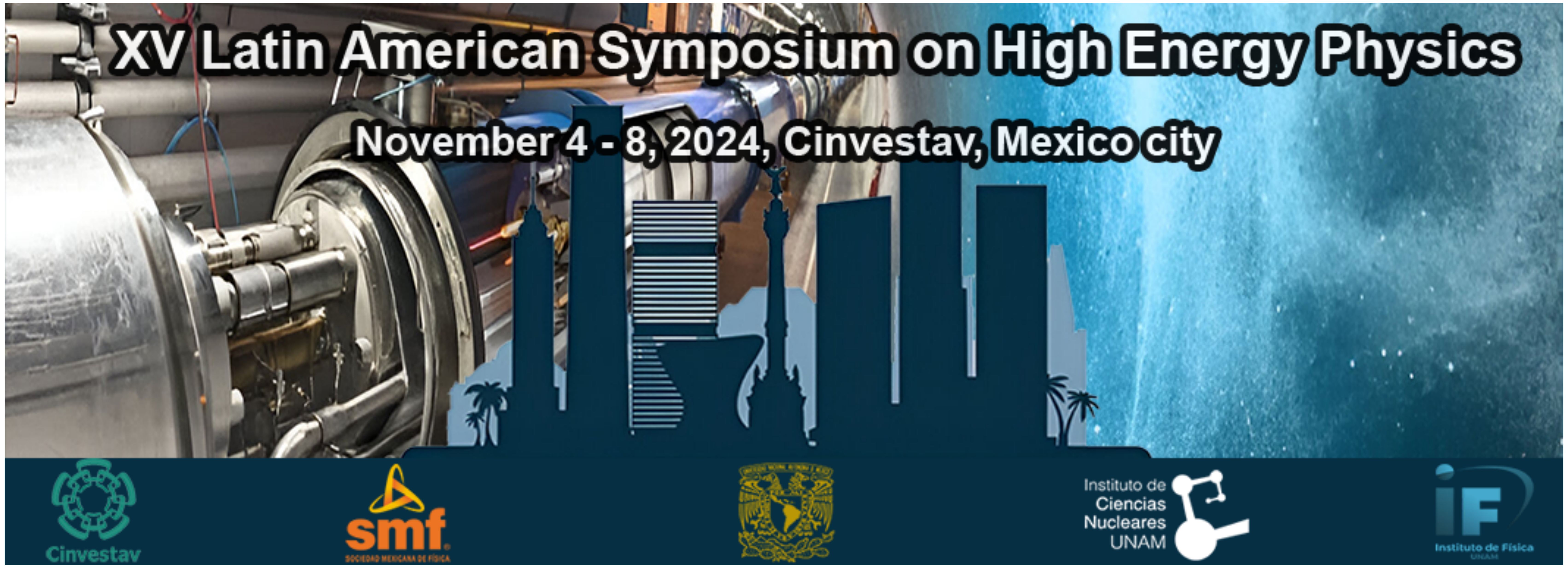
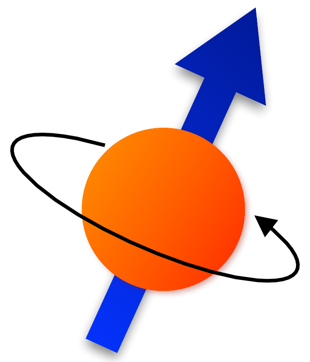
Muon g-2 Theory

Aida X. El-Khadra
University of Illinois



Outline

- Introduction
- Muon g-2 Theory Initiative
- data-driven HVP
- lattice HVP
- HLbL
- Summary and Outlook
- Appendix



- ``The anomalous magnetic moment of the muon in the SM": 1st White Paper published in 2020 (132 authors, 82 institutions) [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
- ``Prospects for precise predictions of a_μ in the SM": 2022 Snowmass Summer Study, arXiv:2203.15810
- Summary statement on the status of Muon g-2 Theory in the SM: https://muon-gm2-theory.illinois.edu

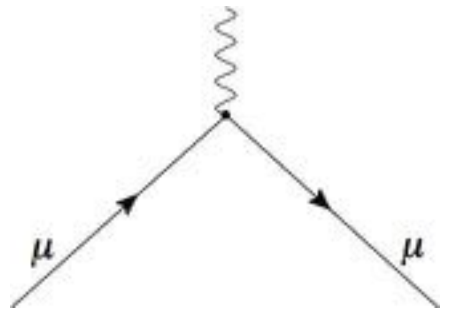
➠ Lattice 2024 conference

➠ Muon g-2 TI workshop @ KEK (9-13 Sep 2024)

Anomalous magnetic moment

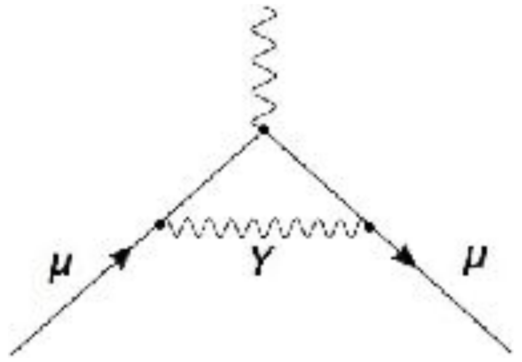
The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g\frac{e}{2m}\vec{S}$

Dirac (leading order): $g = 2$



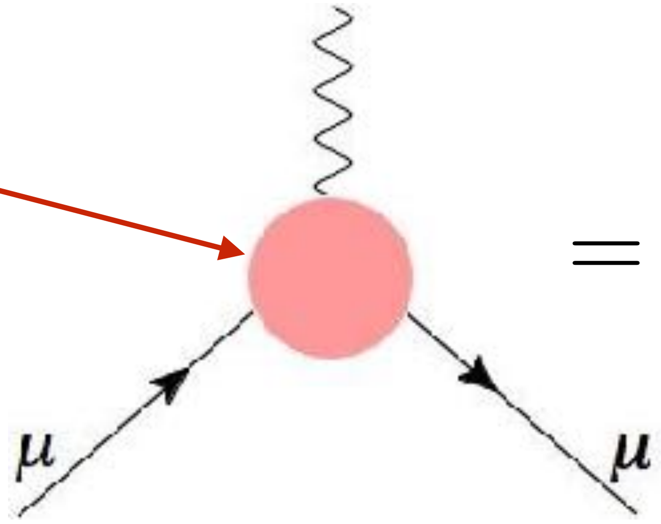
$= (-ie)\,\bar{u}(p')\gamma^{\mu}u(p)$

Quantum effects (loops):



$\Rightarrow g = 2\left(1 + \frac{\alpha}{2\pi}\right)$

All SM particles contribute



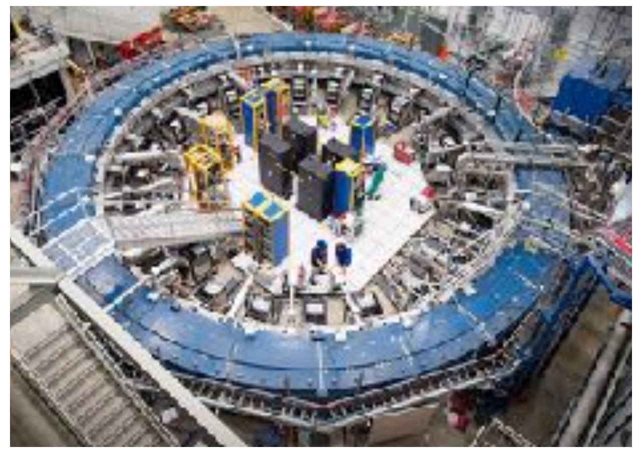
$$= (-i\,e)\,\bar{u}(p')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right]u(p)$$

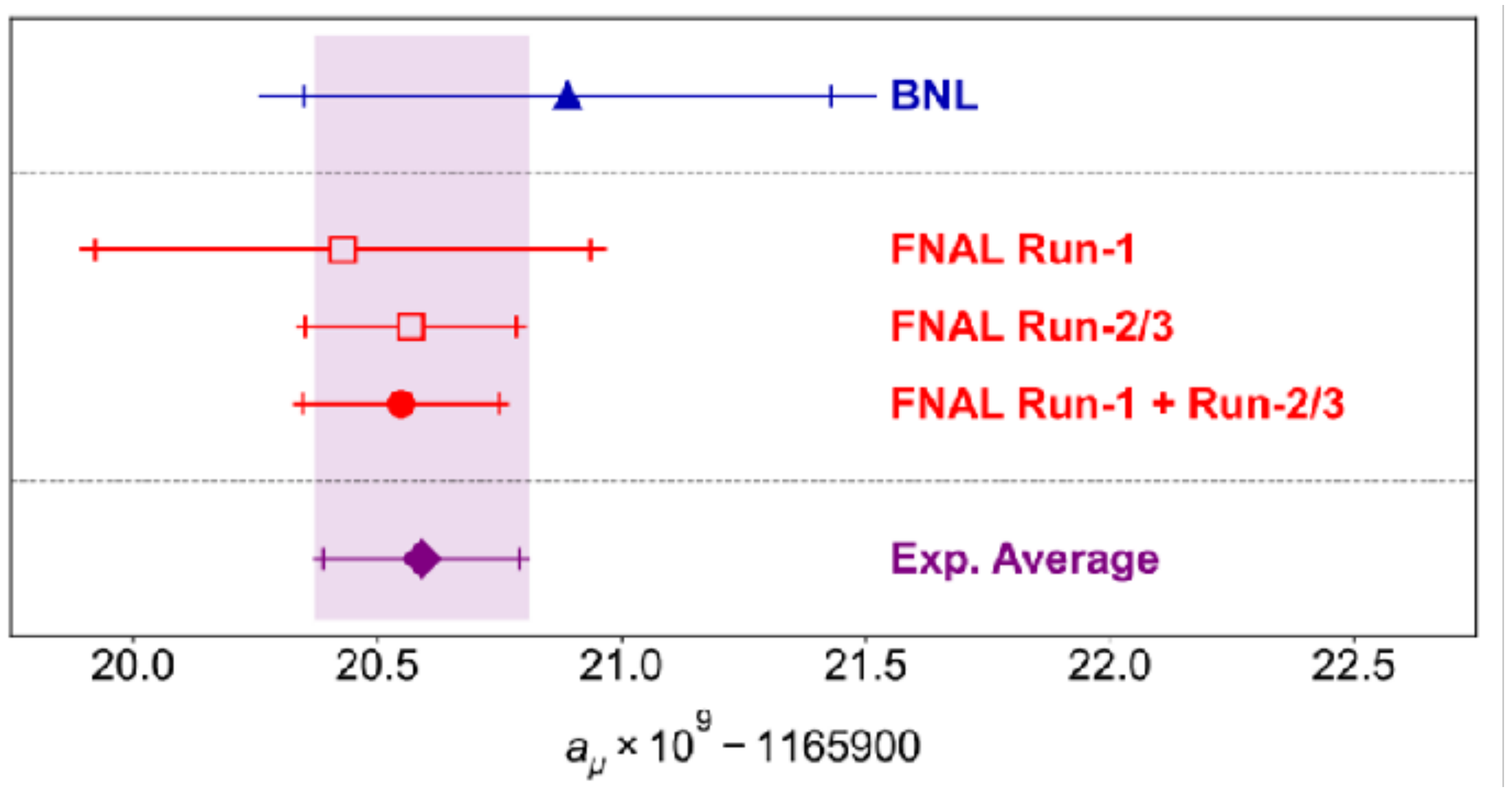
Note: $F_1(0) = 1$ and $g = 2 + 2\,F_2(0)$

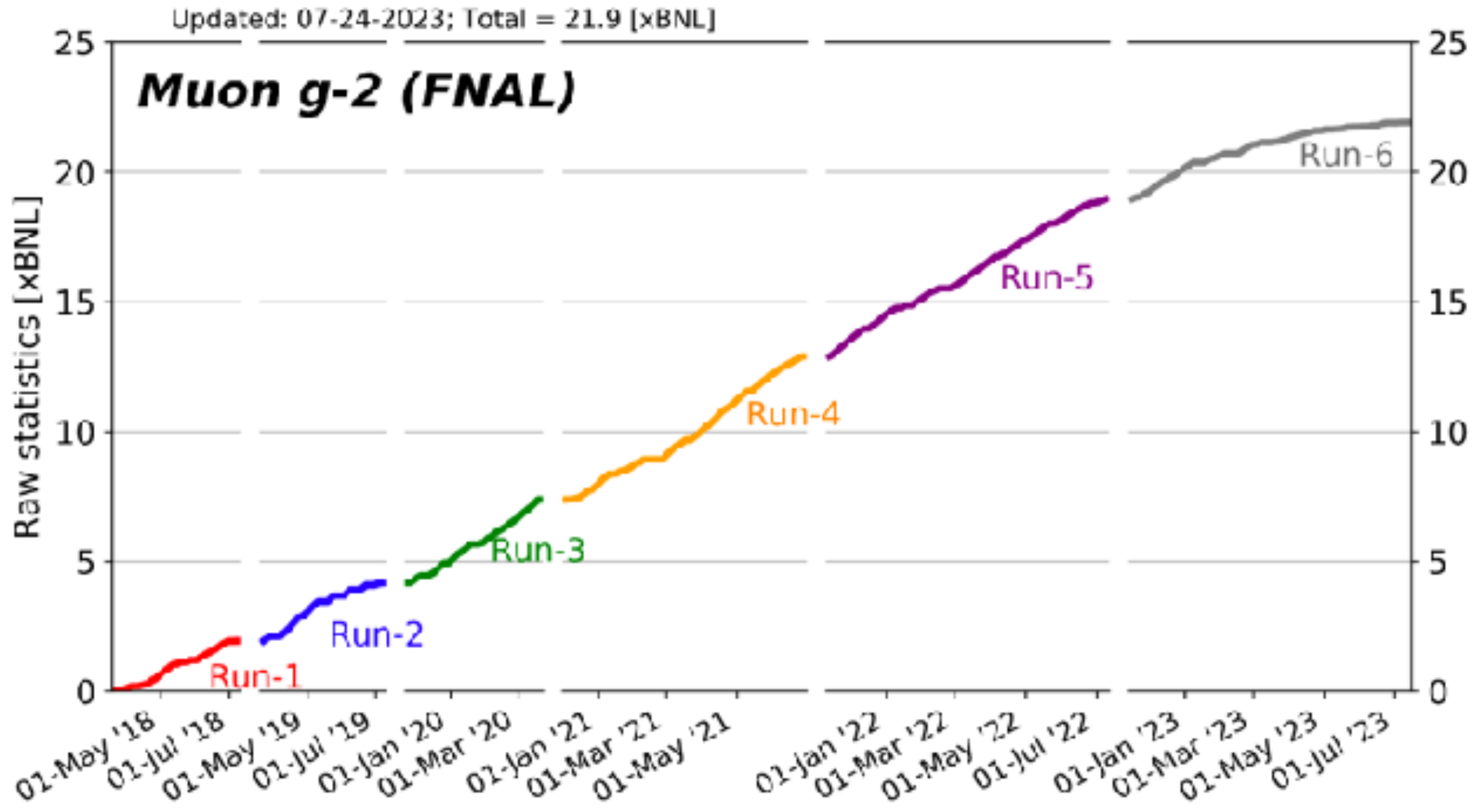
Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0)$ $= \frac{\alpha}{2\pi} + O(\alpha^2) + \ldots = 0.00116\ldots$

Muon g-2 experiment

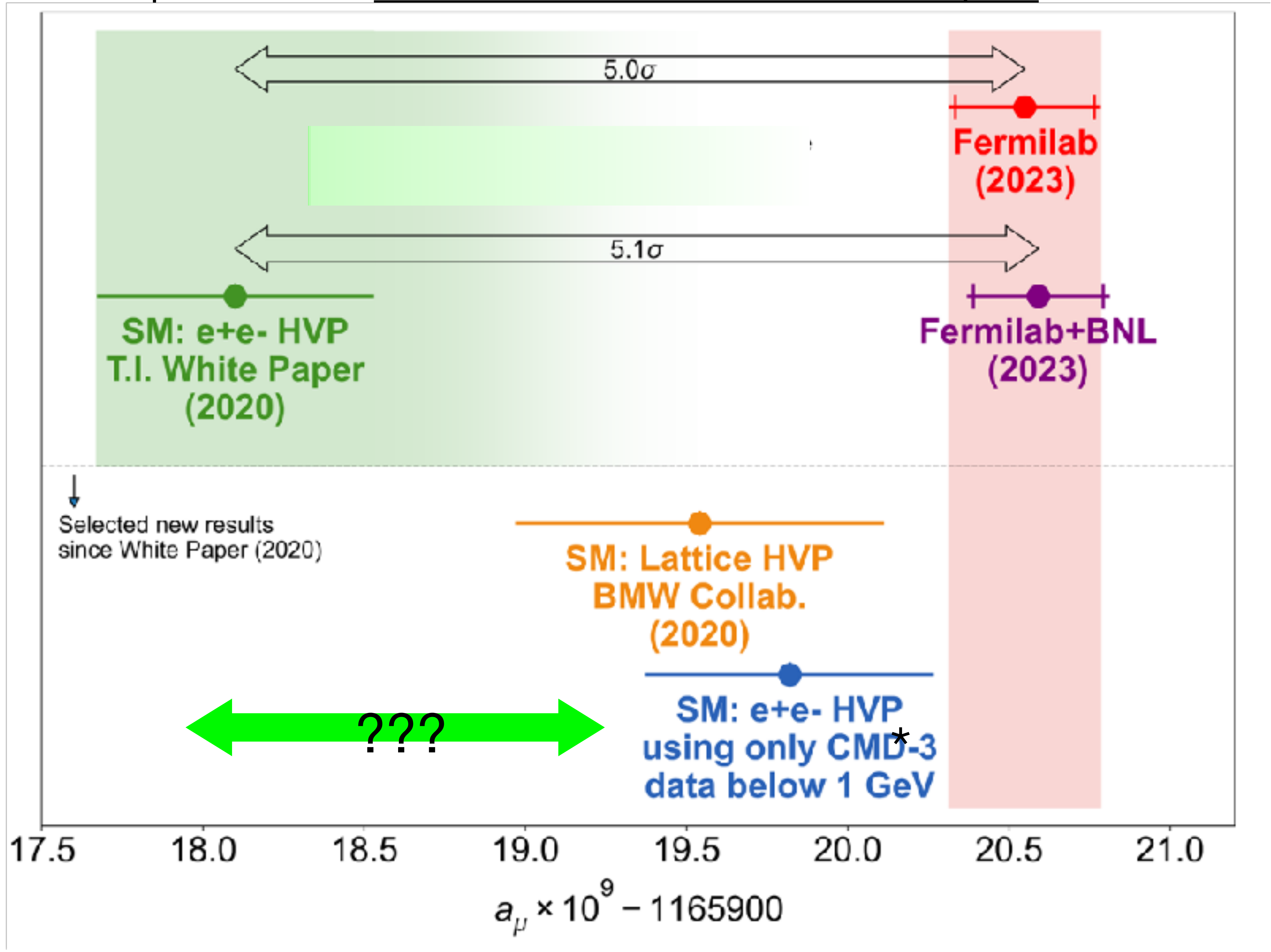
- The Fermilab experiment released the measurement result from their run 2&3 data on 10 Aug 2023. [D. Aguillard et al, 2308.06230]
- Run 6 completed summer 2023.
- Release of final measurement result expected in 2025







adapted from J. Mott @ Scientific Seminar, 10



Muon g-2: SM contributions

$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

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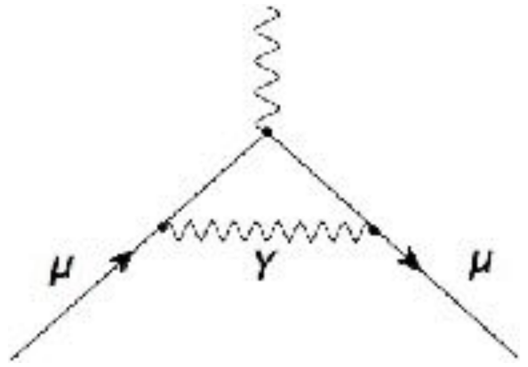
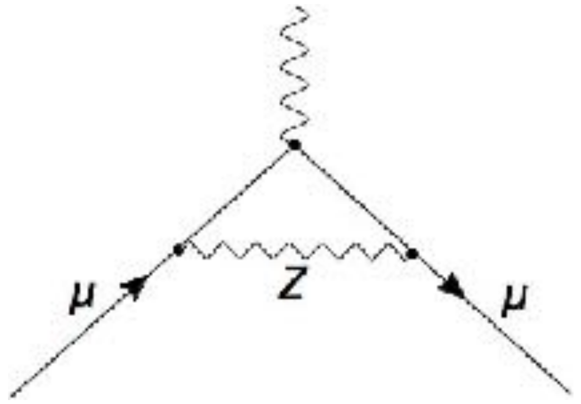
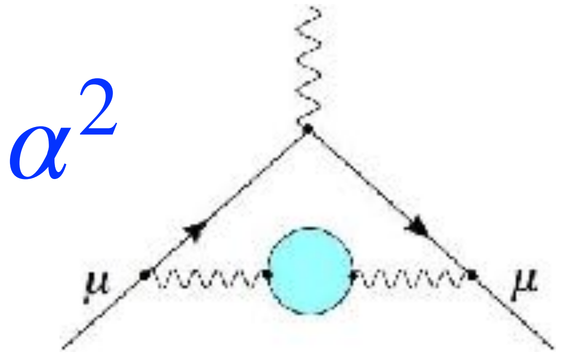
QED 

+… (5 loops) $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

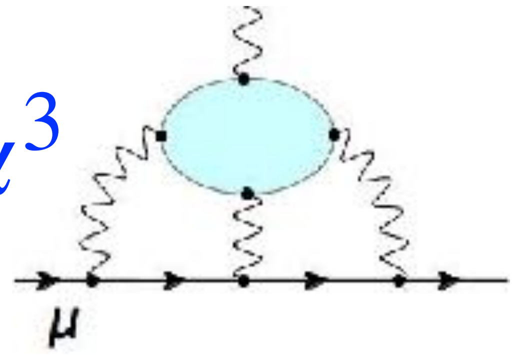
EW 

+… (2 loops) $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic corrections

HVP α^2 

+… (NNLO) $6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

HLbL α^3 

+… (NLO) $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

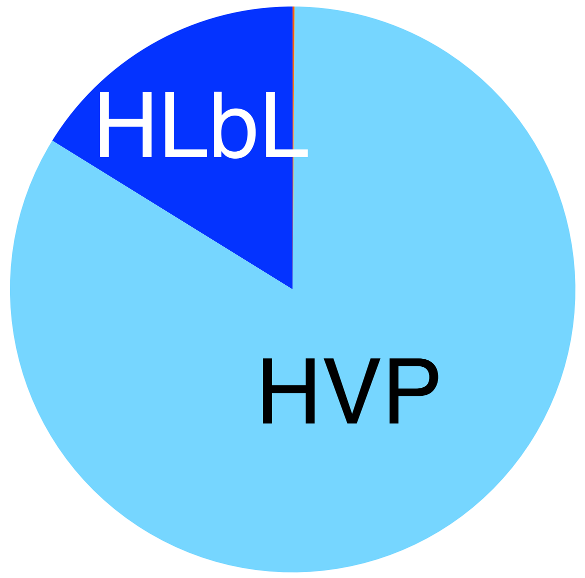
Muon g-2: SM contributions

$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

contribution

QED
EW
HVP
HLbL

error^2



$116\,584\,718.9\,(1) \times 10^{-11}$ — 0.001 ppm

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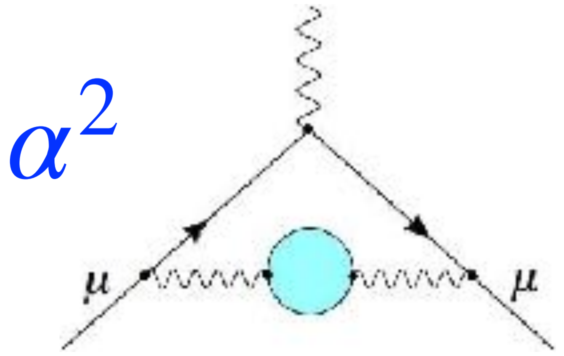
Hadronic corrections

HVP α^2



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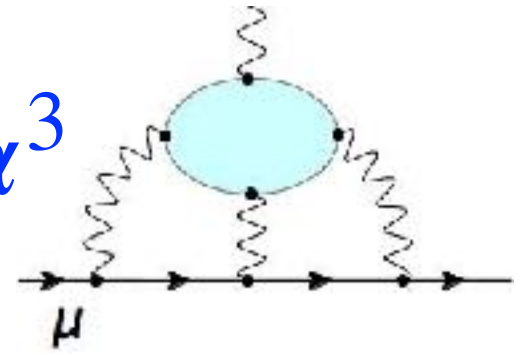
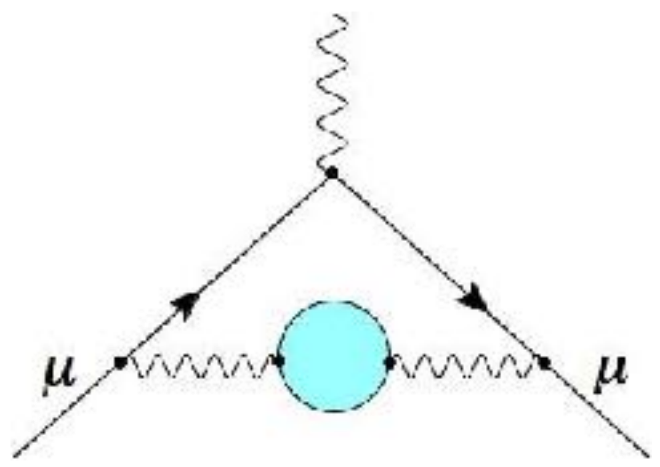
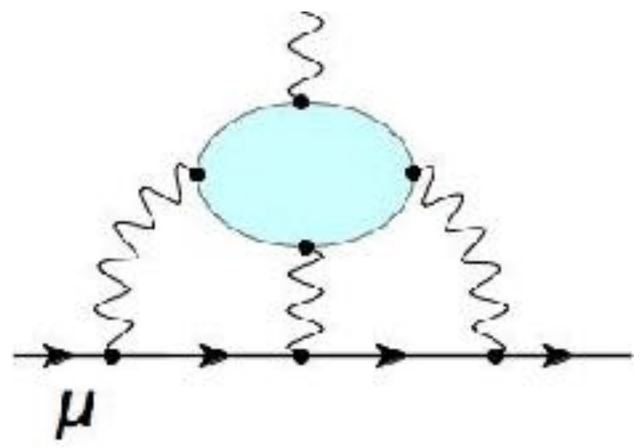


+… (NLO)

$92\,(18) \times 10^{-11}$ [20%] — 0.15 ppm

Muon g-2: hadronic corrections



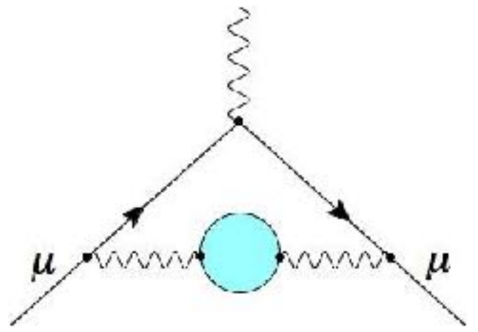


- Hadronic contributions are obtained by integrating over all possible virtual photon momenta, integral is weighted towards low q^2.
- Cannot use perturbation theory to reliably compute the hadronic bubbles
- Two-point & four-point functions:

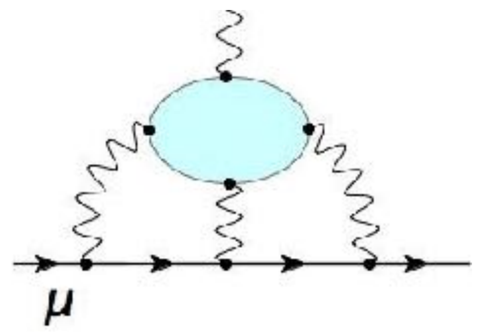
  HVP: $\langle 0 | T\{j_\mu j_\nu\} | 0 \rangle$ HLbL: $\langle 0 | T\{j_\mu j_\nu j_\rho j_\sigma\} | 0 \rangle$

Two independent approaches

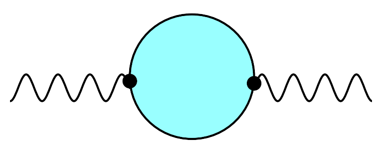
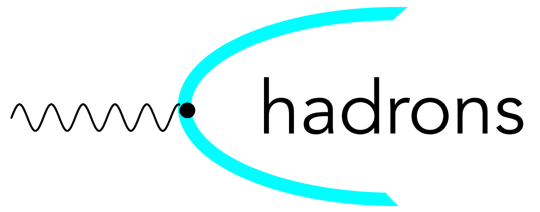
1. Dispersive, data-driven
2. Lattice QCD



Hadronic Corrections



- Dispersive, data-driven:
  HVP: integrate hadronic cross section over CM energy:

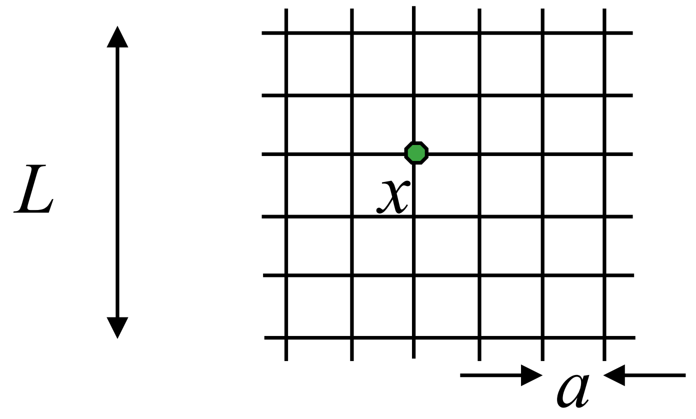
$Im[$  $] \sim |$ 

$|^2$ ➠ $$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2) = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\mathrm{exp}}(s)$$

Many experiments (over 20+ years) have measured the e^+e^-cross sections for (almost) all channels over the needed energy range with increasing precision.

For HLbL: new dispersive approach

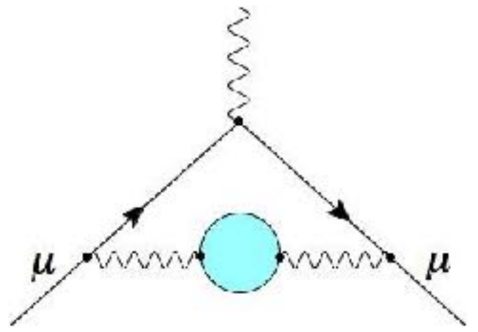
- Direct calculation using Euclidean Lattice QCD ➠ $$a_\mu^{\mathrm{HVP,LO}} = 4\,\alpha^2 \int_0^\infty dt\, C(t)\, \tilde{w}(t)$$



Approximations:
discrete space-time (spacing a)
finite spatial volume (L), and time extent (T)
…

- *ab-initio* method to quantify QCD effects
- already used for simple hadronic quantities with high precision
- requires large-scale computational resources
- allows for entirely SM theory based evaluations

Integrals are evaluated numerically using Monte Carlo methods.



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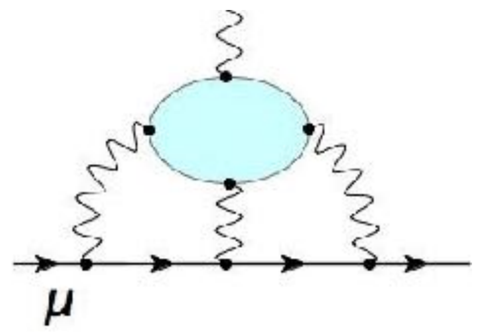

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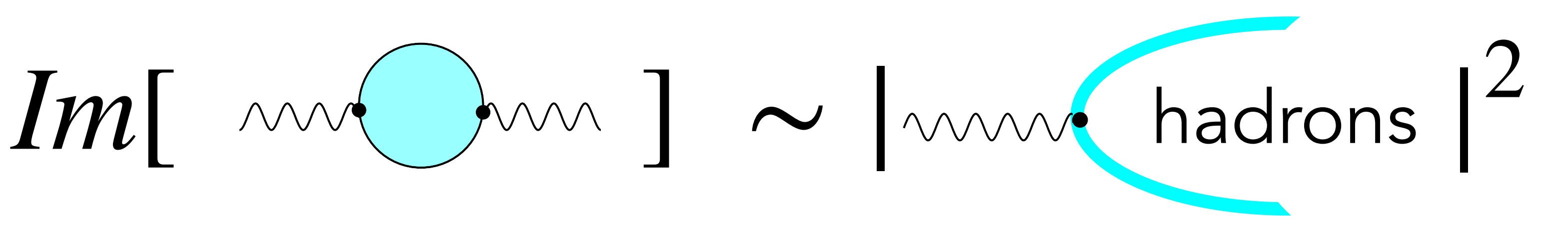

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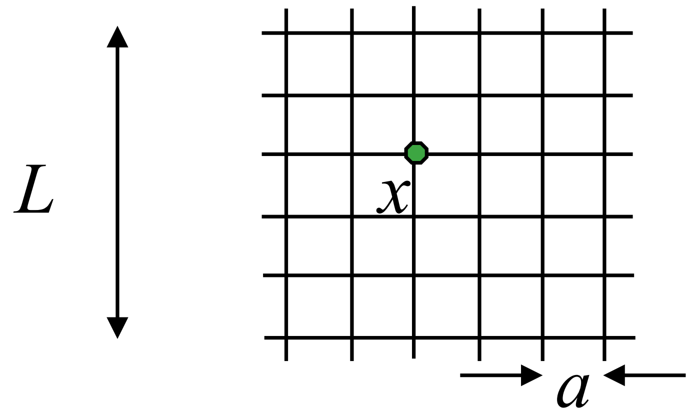

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Muon g-2 Theory Initiative

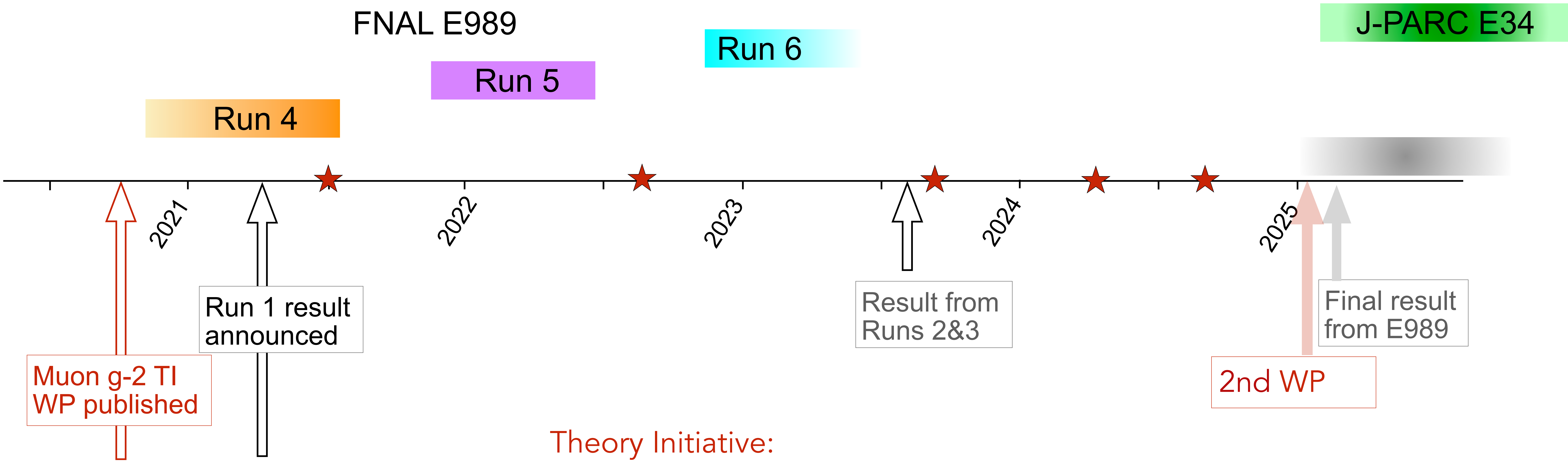
Steering Committee

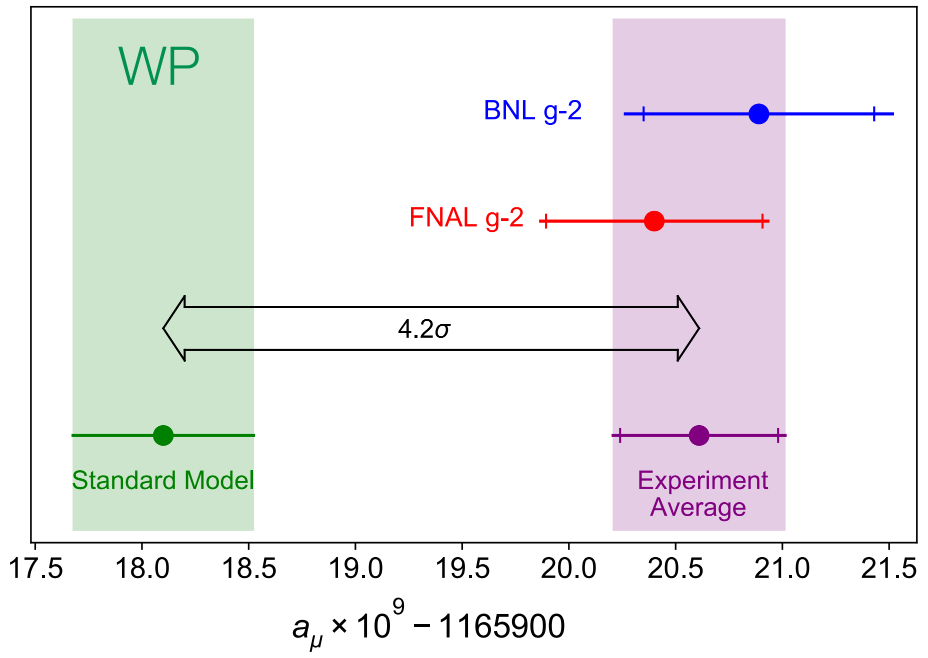
- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
  J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston)
  Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

https://muon-gm2-theory.illinois.edu

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the SM prediction
- assess reliability of uncertainty estimates
- summarize the theory status: White Papers
- organize workshops to bring the different communities together:
  - First plenary workshop near Fermilab: 3-6 June 2017
  - …
  - Virtual Spring 2024 TI workshop hosted by UIUC: 15-17, 23-24 Apr 2024
  - Seventh plenary workshop hosted by KEK/KMI (Japan): 9-13 Sep 2024
  - Eight plenary workshop: Orsay (France), 8-12 Sep 2025
  - Ninth and tenth plenary workshops: US, UK

Near-term timeline





Theory Initiative:

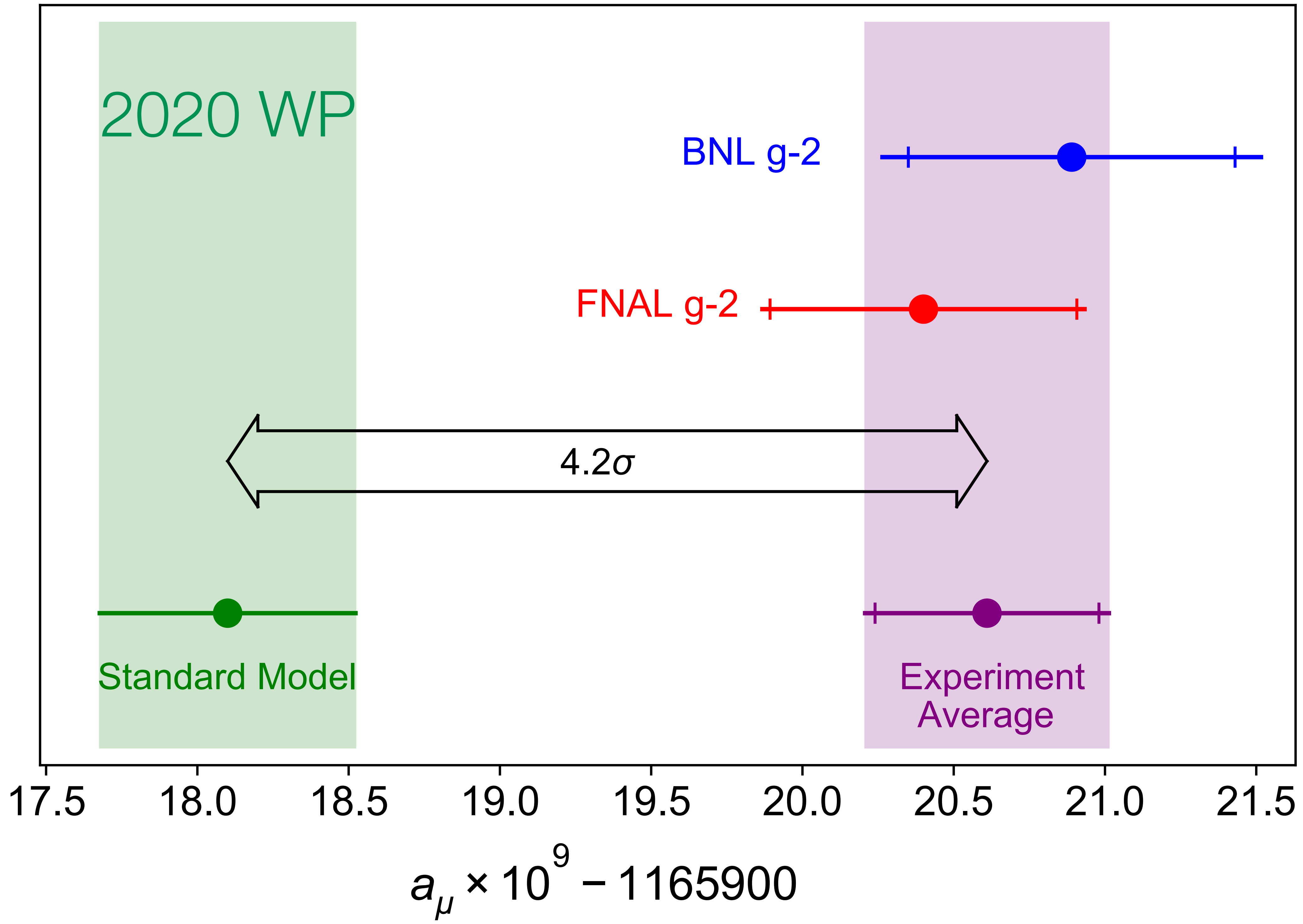
- ☆ CMD-3 seminar (virtual): 27 March 2023 at 8:00am US CDT
- ☆ 2nd CMD-3 discussion meeting
- ☆ 8/9/2023: Status of Muon g-2 Theory in SM

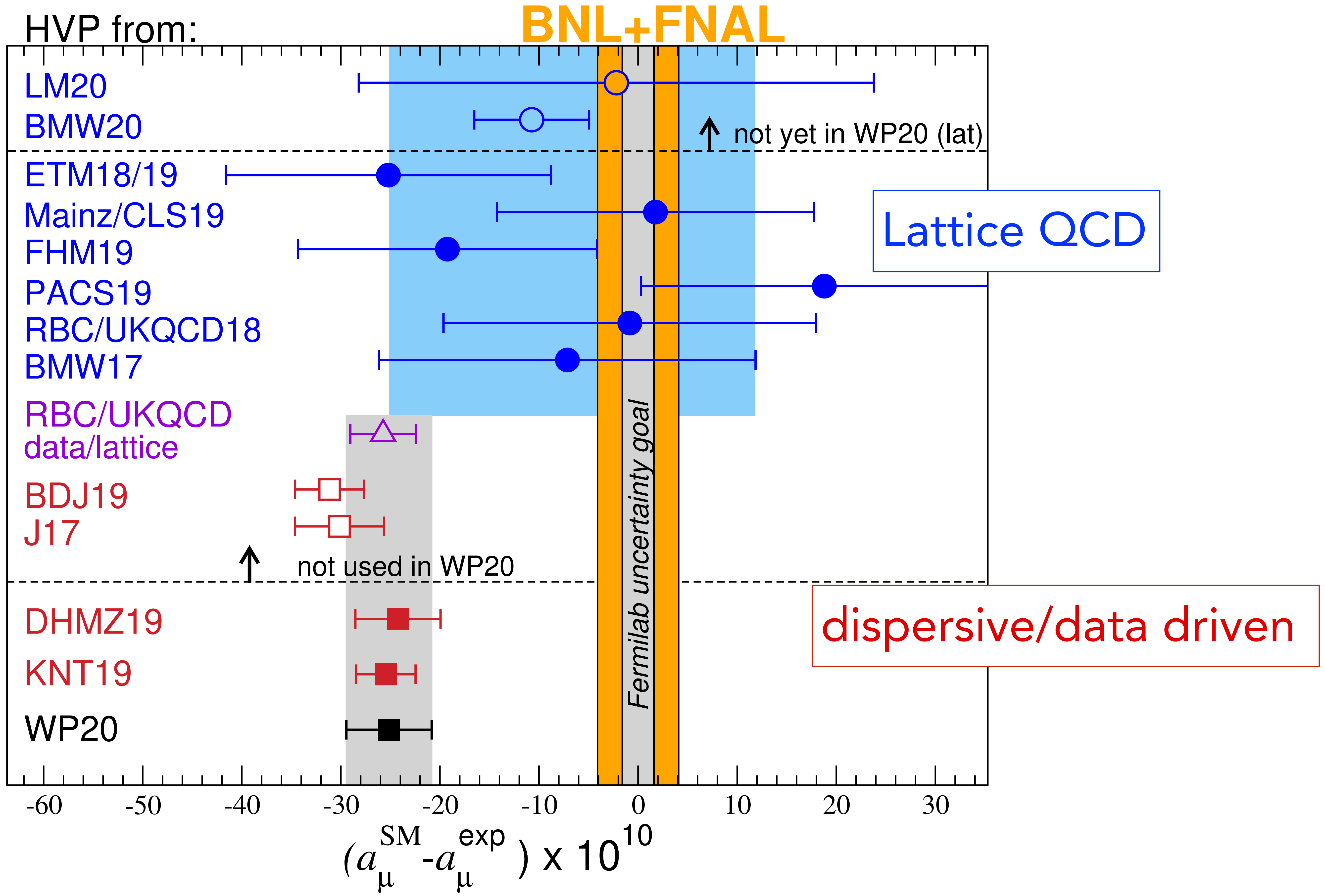
★ TI workshops:

Jun 2021 @ KEK (virtual) Sep 2023 @ Bern Sep 2024 @ KEK & KMI
Sep 2022 @ Higgscentre Apr 2024 (virtual)

Experiment vs SM theory

- 2021

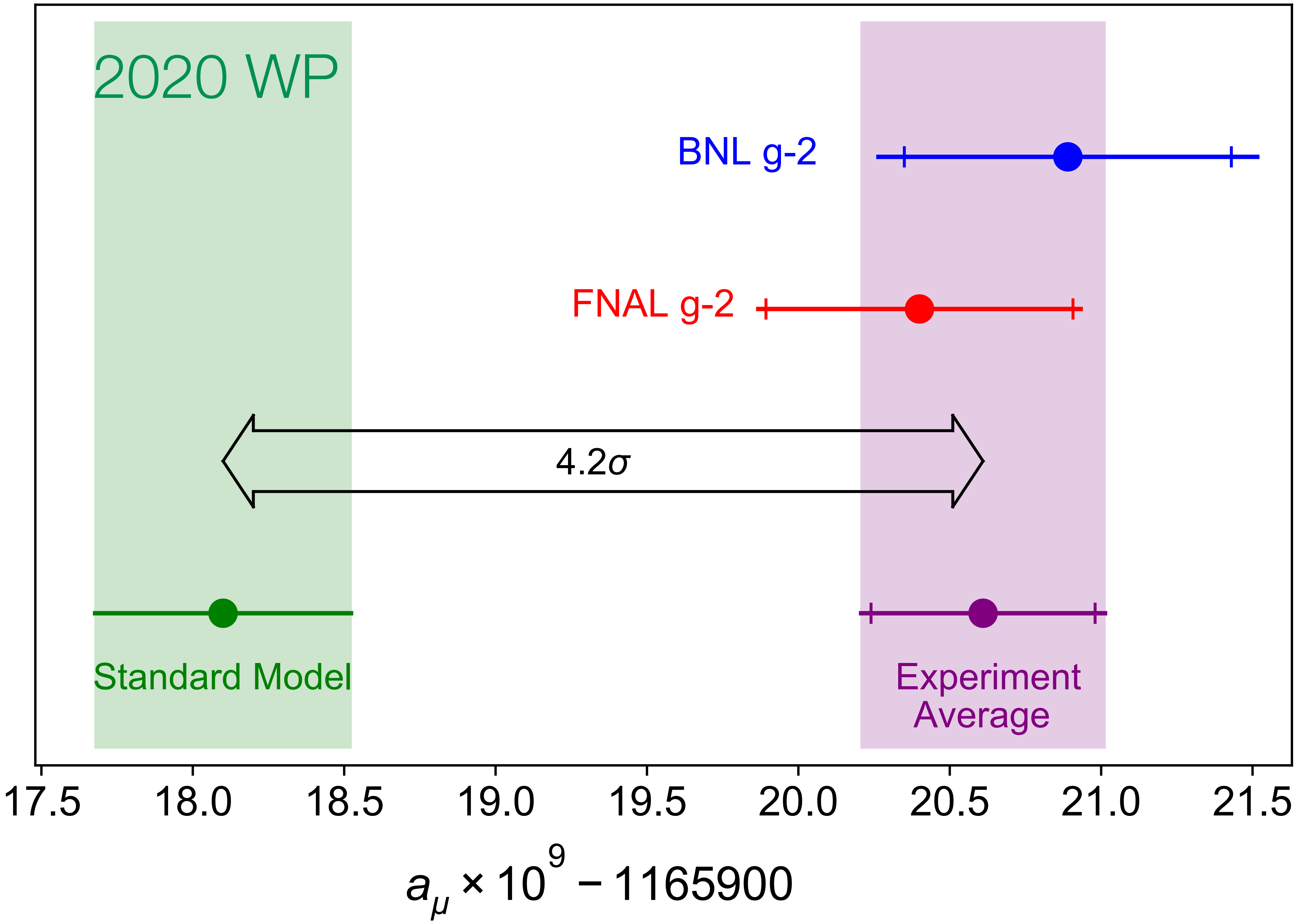




$$a_\mu^{\rm SM} \quad a_\mu^{\rm HVP} + [a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}]$$

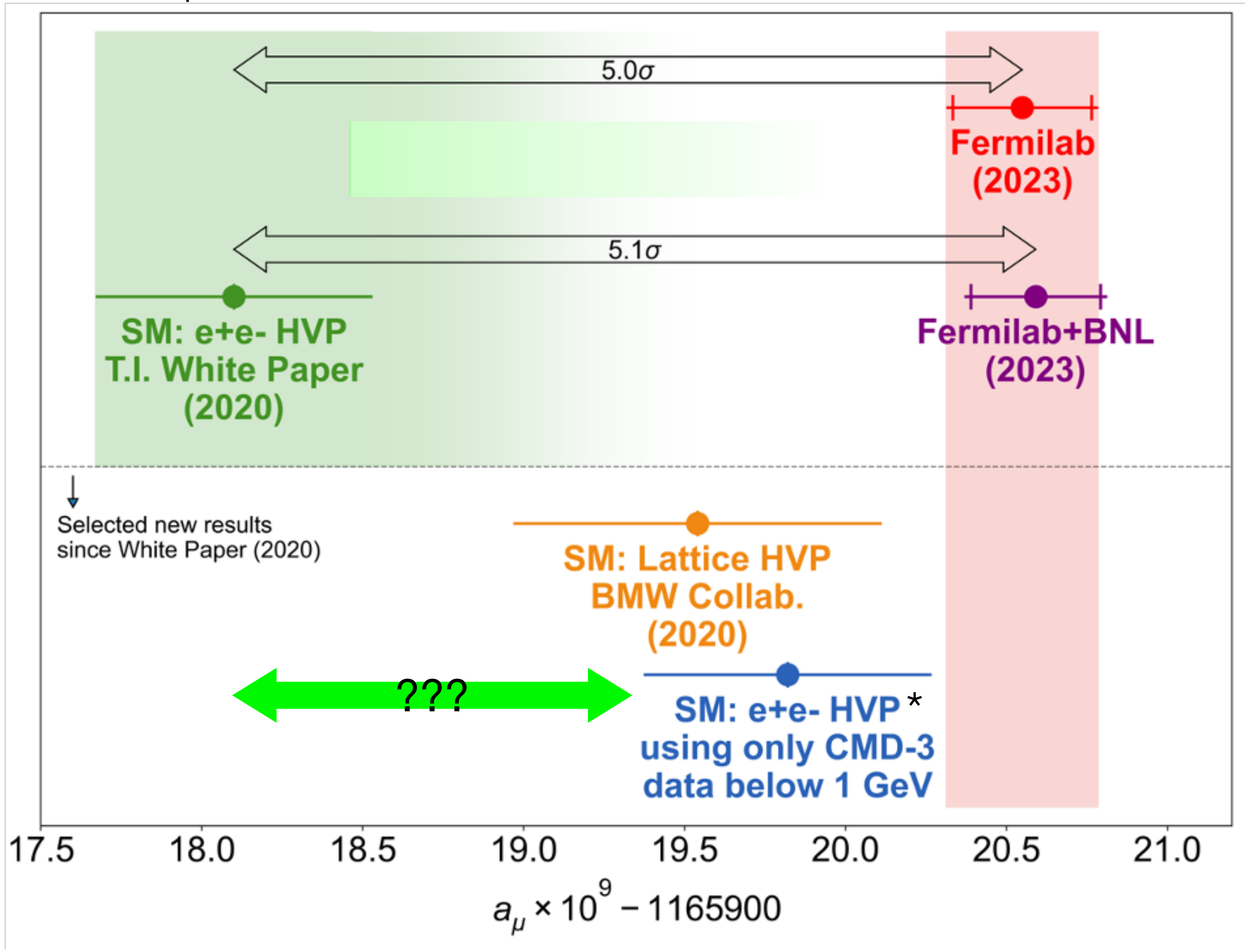
Experiment vs SM theory

- 2021



- 2023

adapted from J. Mott @ Scientific Seminar, 10 Aug 2023



*A. Keshavarzi @ Lattice 2023

Experiment vs SM theory

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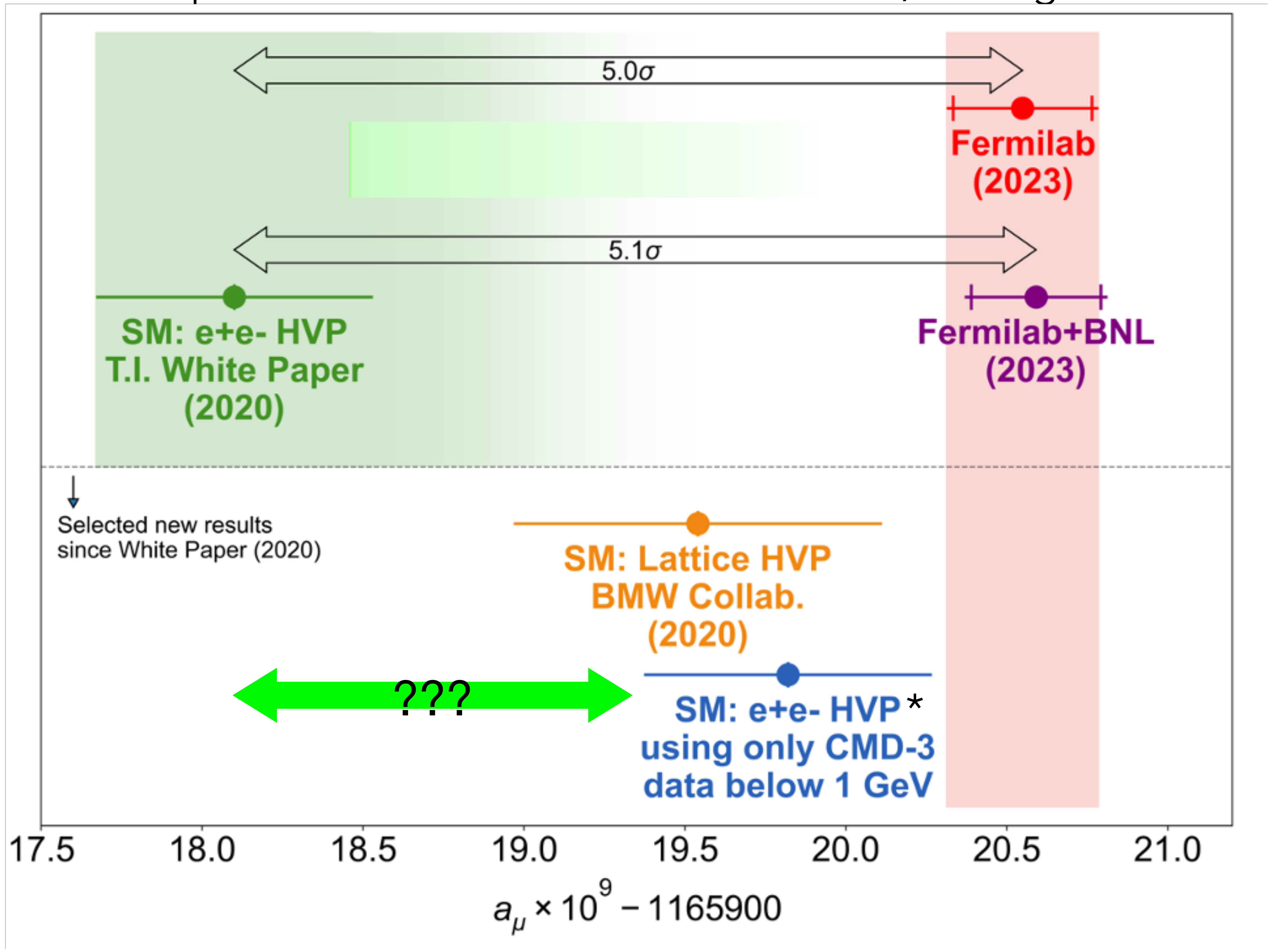
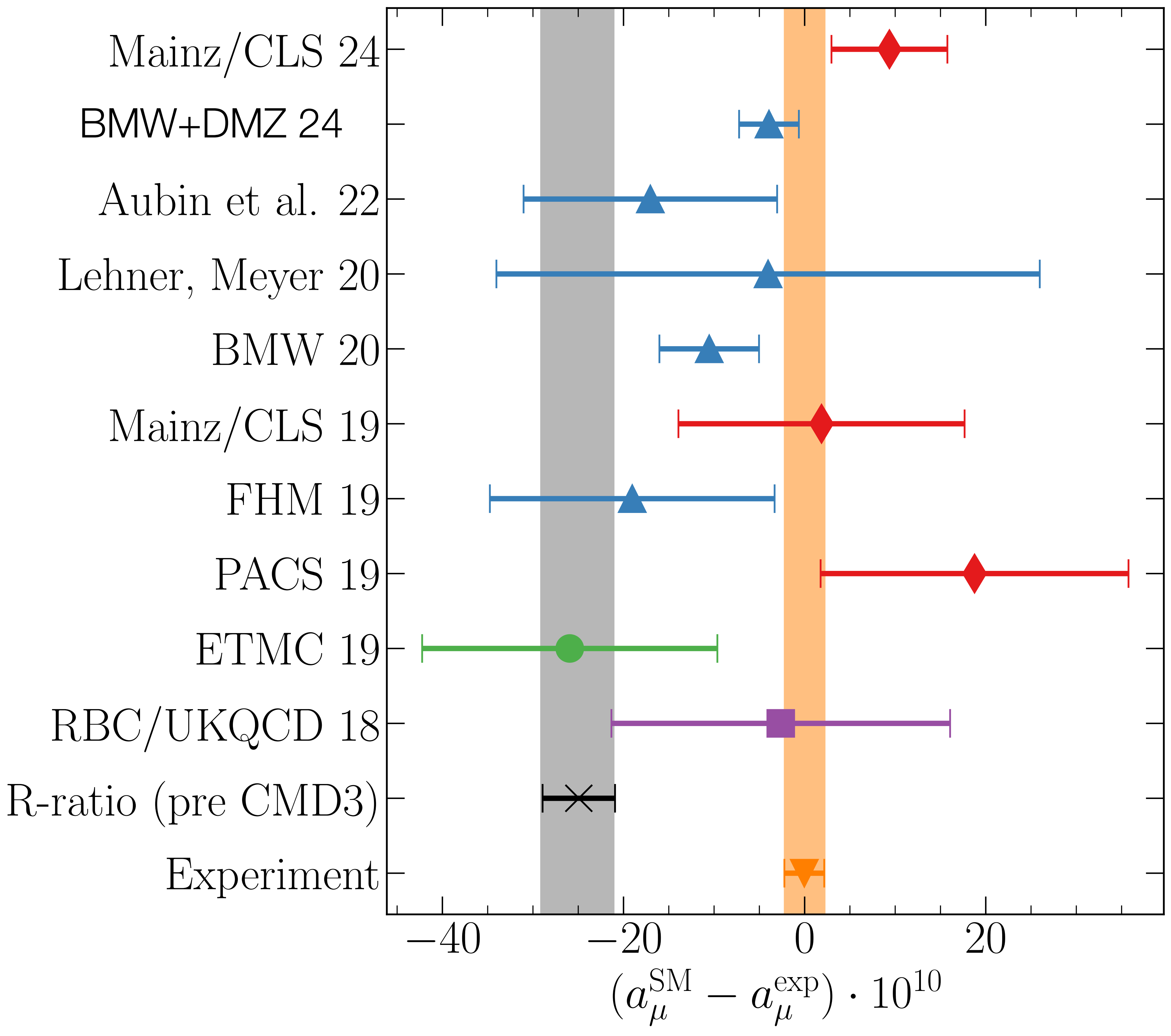


*A. Keshavarzi @ Lattice 2023

- 2024

New LQCD results (all using blind analyses):
BMW+DMZ 24 [arXiv:2407.10913]: LQCD+R-ratio (hybrid)
RBC/UKQCD: Lehner@Lattice 2024
Mainz: Kuberski @ KEK 2024
FNAL/HPQCD/MILC: exp. fall 2024

S. Kuberski @ KEK 2024



Overview of the experiments

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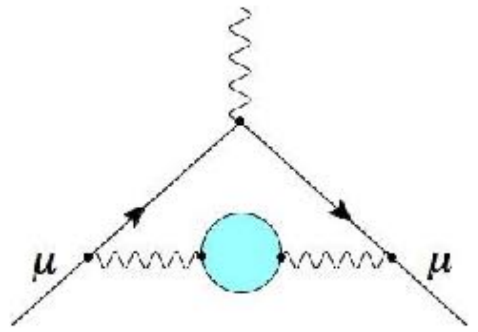
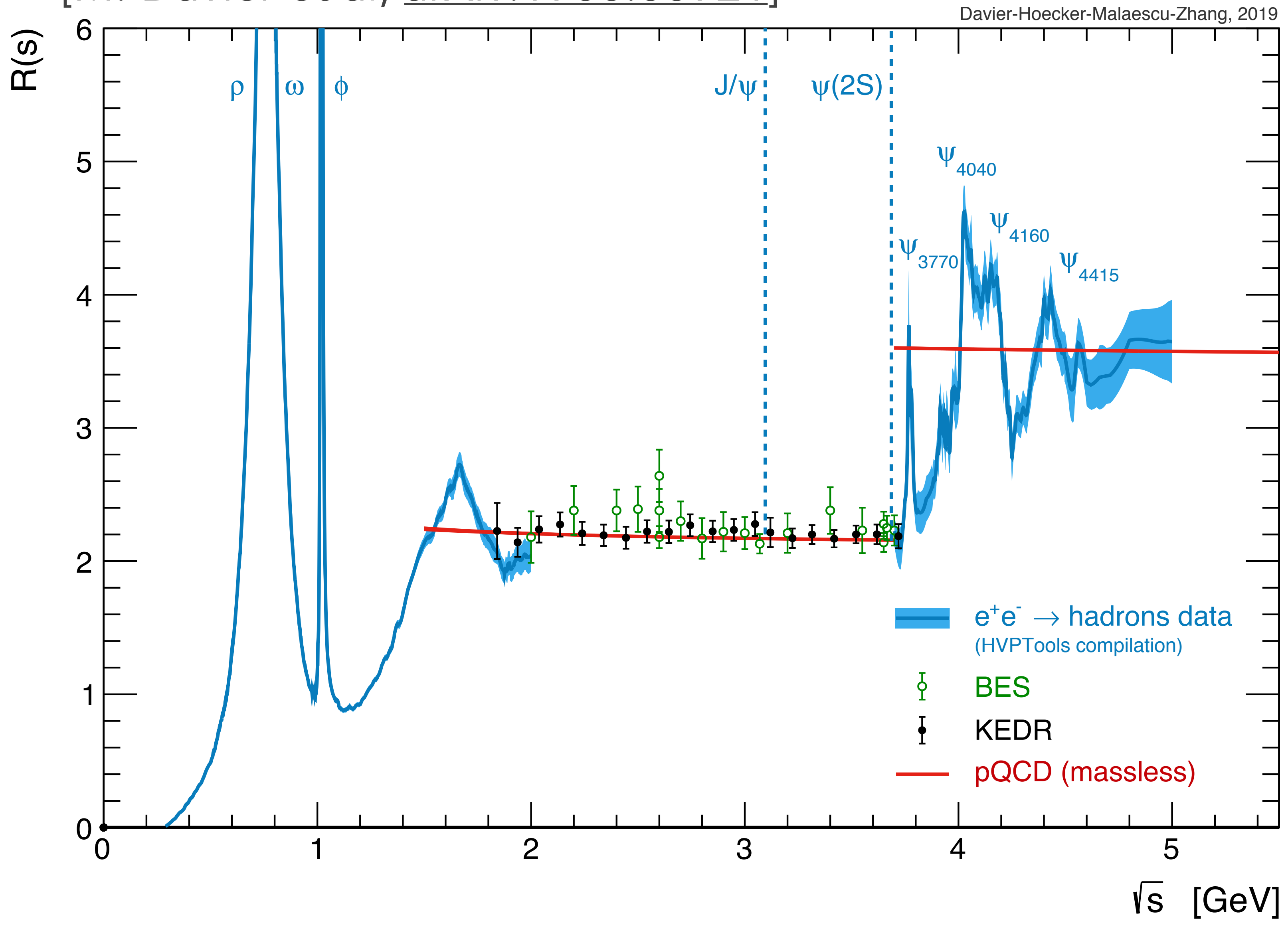


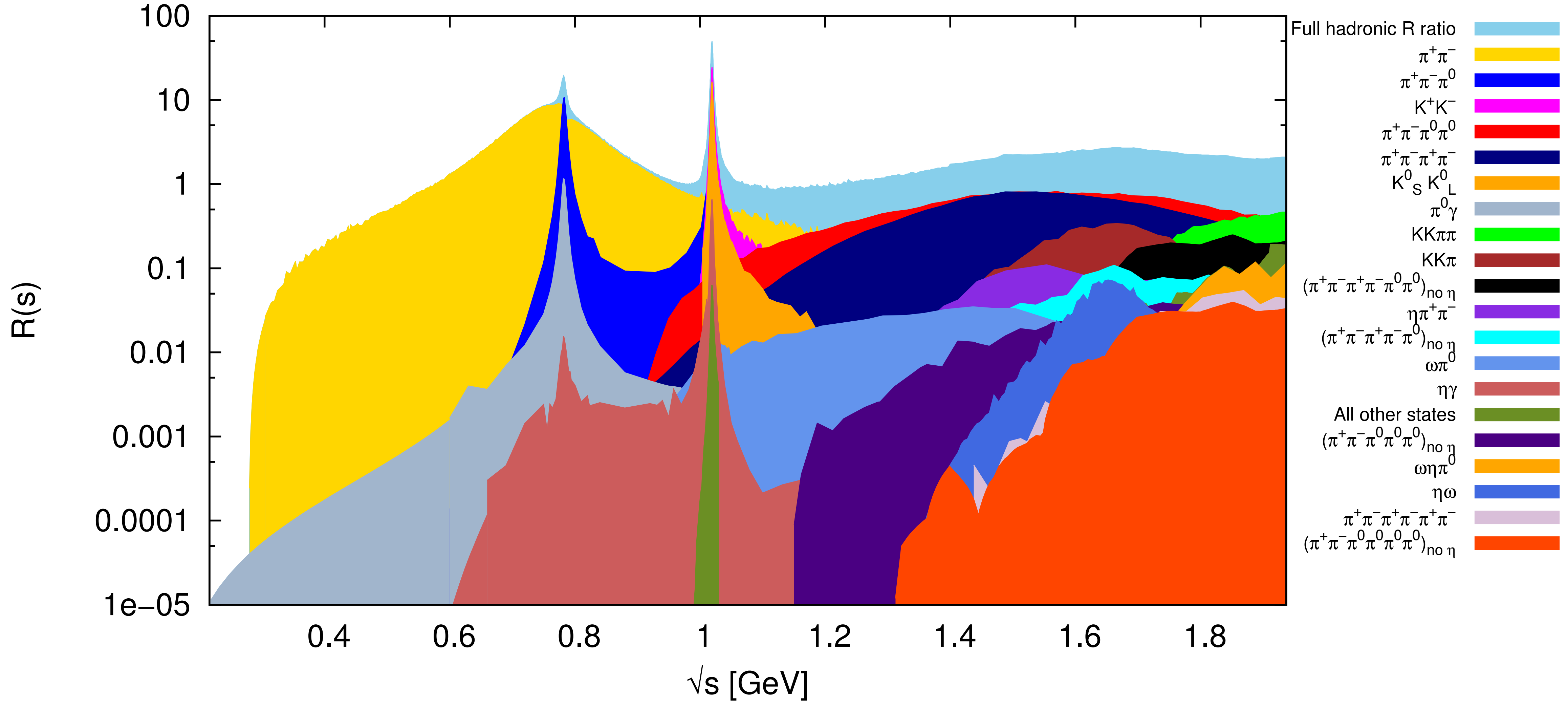
HVP: data-driven

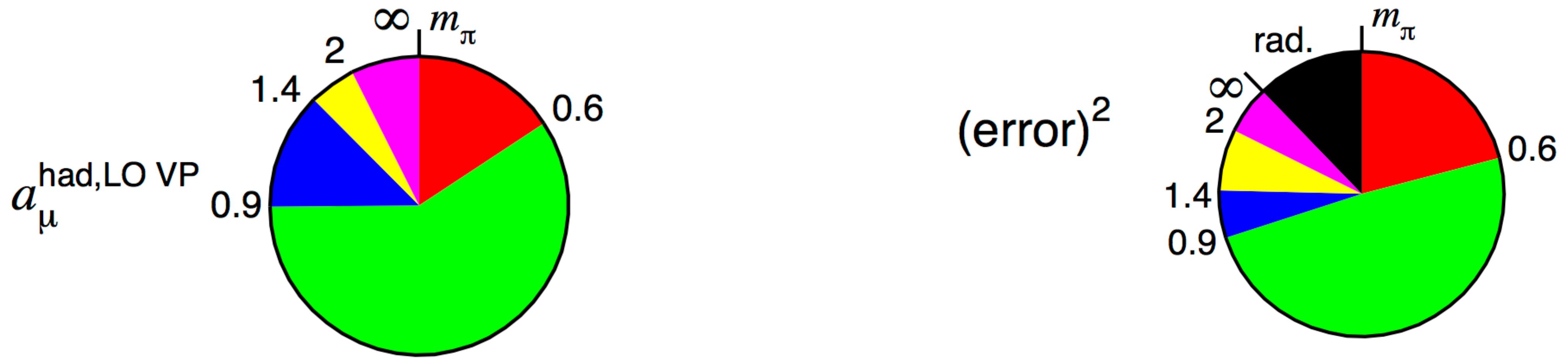
$$a_\mu^{\mathrm{HVP,LO}} = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\mathrm{exp}}(s)$$

[M. Davier et al, arXiv:1908.00921]



[A. Keshavarzi et al, arXiv:1802.02995]





- $\sigma_{\mathrm{had}}(s)$ defined to include real & virtual photons
- direct integration method: no modelling of $\sigma_{\mathrm{had}}(s)$, summing up contributions from all hadronic channels
- total hadronic cross section $\sigma_{\mathrm{had}}(s)$ from > 100 data sets in 35+ channels summed up to $\sqrt{s} \sim 2\,\mathrm{GeV}$
- $\sqrt{s} > 2$ GeV: inclusive data + pQCD + narrow resonances
- two independent compilations (DHMZ, KNT)

Tensions (of up to 3σ) between data sets:
➠ conservative procedure to include differences in error estimate

HVP: data-driven

Since 2020:

- 20+ papers with new experimental measurements for $\sigma_{\rm had}(s)$, for example: $\pi\pi, \pi\pi\pi, K\bar{K}\pi, \eta\pi\gamma, 4\pi, \pi\pi\pi\eta, \ldots$
- all ~ consistent with previous results

HVP: data-driven

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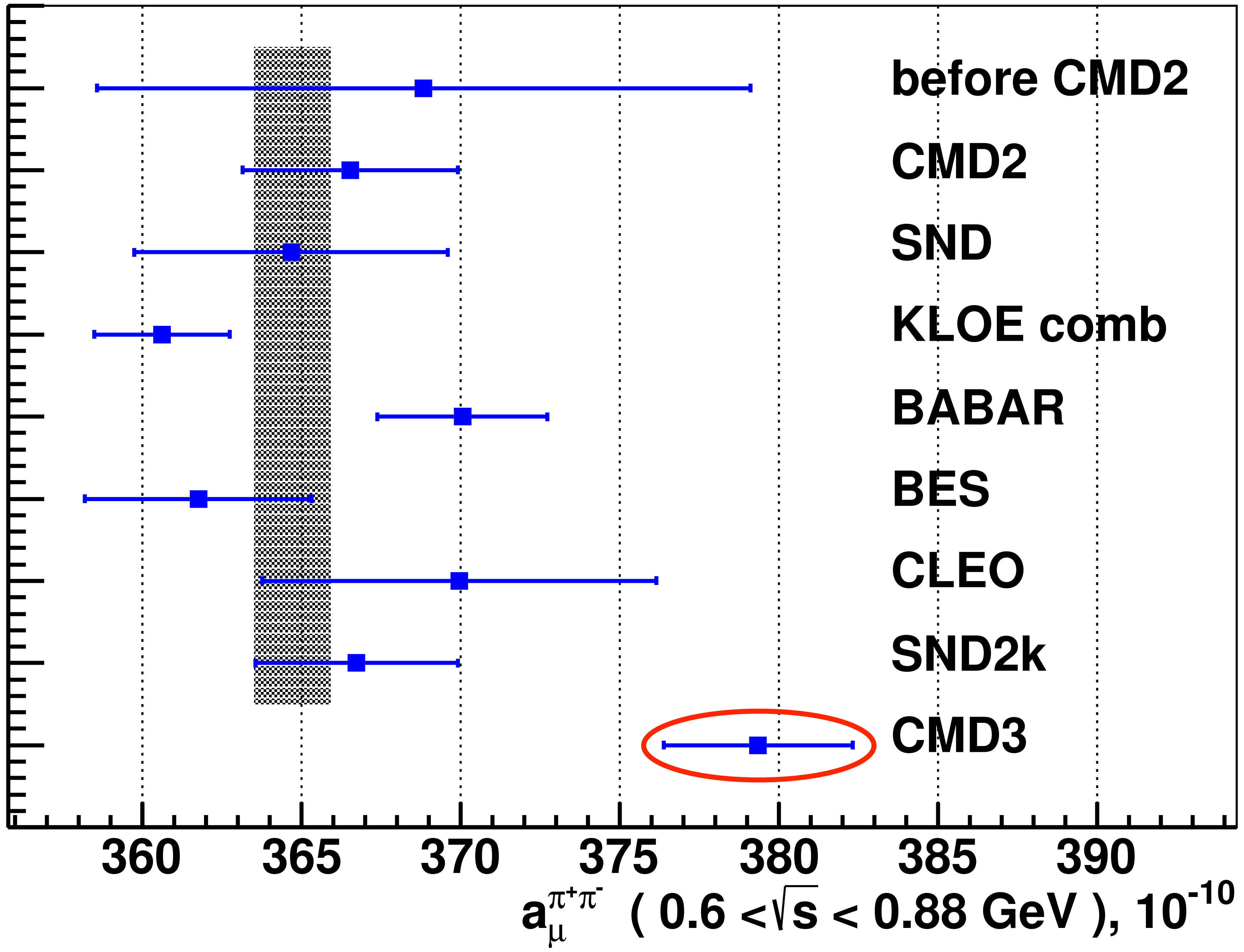
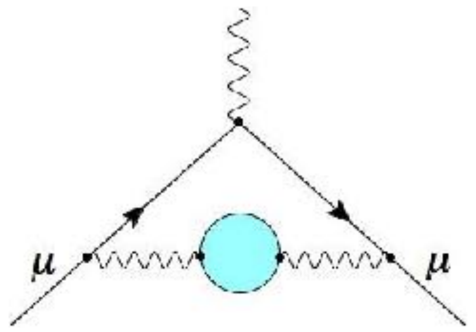
Feb 2023: from CMD-3 [F. Ignatov et al, arXiv:2302.08834, PRD 2024]



A new puzzle!
- discrepancies between experiments now $\gtrsim (3-5)\,\sigma$ need to be understood/resolved
- (virtual) scientific seminar + discussion panel on CMD-3 measurement March 27 (8:00 –11:00 am US CDT)
- 2nd CMD-3 discussion meeting (20 July 2023)
- Discussions are continuing….



HVP: data-driven

Updates presented at the 7th Muon g-2 TI workshop @ KEK

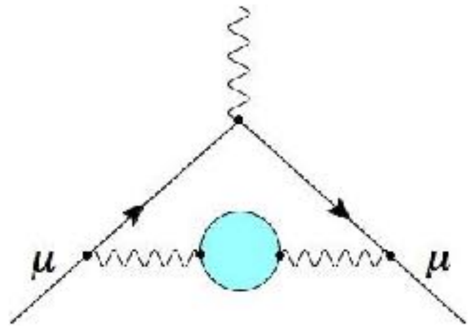
Ongoing work on experimental inputs:

- BaBar: ongoing analysis of large data set (not included before) in $\pi\pi$ channel, also $\pi\pi\pi$, other channels
- CMD3: ongoing analyses, comparisons with CMD-2 procedures, new data expected
- KLOE: ongoing analysis of large data set in $\pi\pi$ channel (not included before), other channels
- SND: new results for $\pi\pi$ channel, other channels in progress
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- Belle II: first results for $\pi\pi\pi$ in 2024, ramping up $\pi\pi$ analysis
  Better ultimate statistics than BaBar or KLOE; similar or better systematics for low-energy cross sections

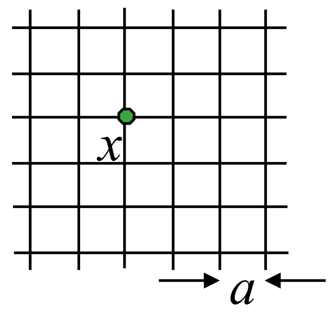
Ongoing work on theoretical aspects:

- better treatment of structure dependent radiative corrections (NLO) in $\pi\pi$ and $\pi\pi\pi$ channels
- new dispersive treatment [Colangelo at al, arXiv:2207.03495]
- Developing NNLO Monte Carlo generators (STRONG 2020 workshop https://agenda.infn.it/event/28089/)

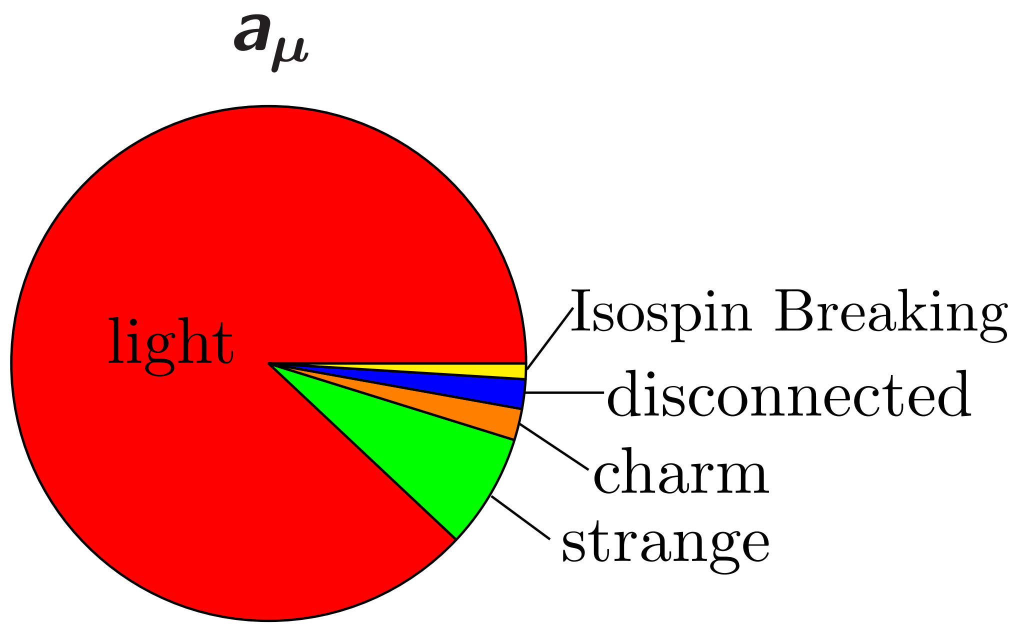
- including τ decay data: requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]



Lattice HVP: Introduction



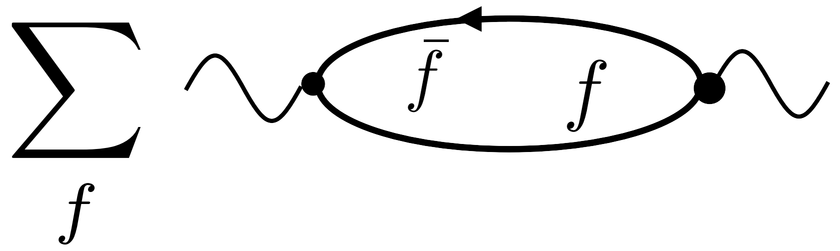
Calculate $a_\mu^{\rm HVP}$ in Lattice QCD: $$a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,\rm disc}^{\rm HVP,LO}$$



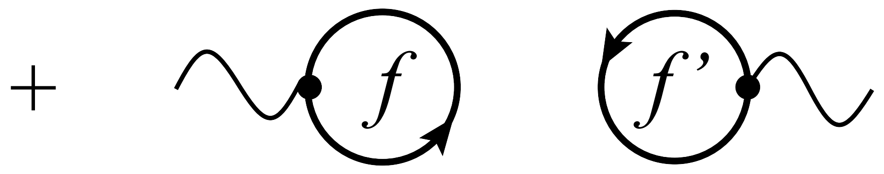
- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

  Note: almost always $m_u = m_d$

$f = ud, s, c, b$

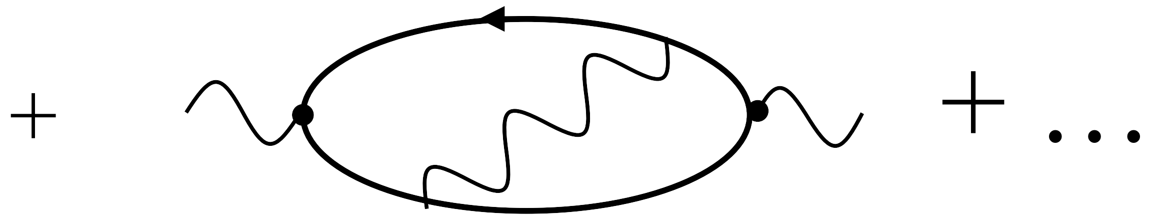


- light-quark connected contribution: $a_\mu^{\rm HVP,LO}(ud)$ ~90% of total
- s,c,b-quark contributions $a_\mu^{\rm HVP,LO}(s,c,b)$ ~8%, 2%, 0.05% of total



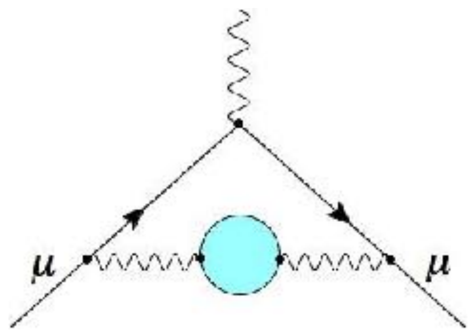
- disconnected contribution: $a_{\mu,\rm disc}^{\rm HVP,LO}$ ~2% of total

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:

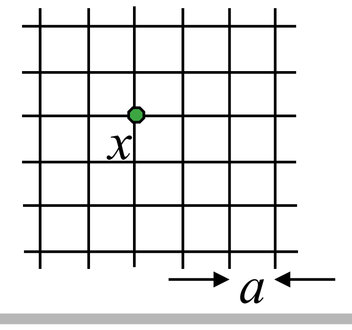


- Isospinbreaking (QED + $m_u \neq m_d$) corrections: $\delta a_\mu^{\rm HVP,LO}$ ~1% of total

$$a_\mu^{\rm HVP,LO} = a_\mu^{\rm HVP,LO}(ud) + a_\mu^{\rm HVP,LO}(s) + a_\mu^{\rm HVP,LO}(c) + a_{\mu,\rm disc}^{\rm HVP,LO} + \delta a_\mu^{\rm HVP,LO}$$



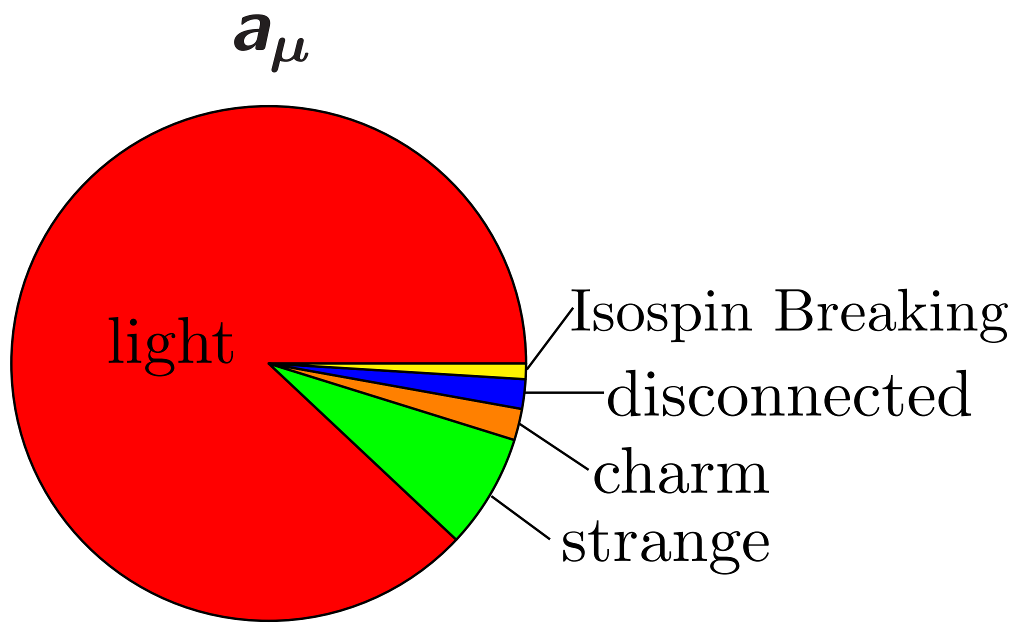
Lattice HVP: challenges



Calculate a_μ^{HVP} in Lattice QCD:

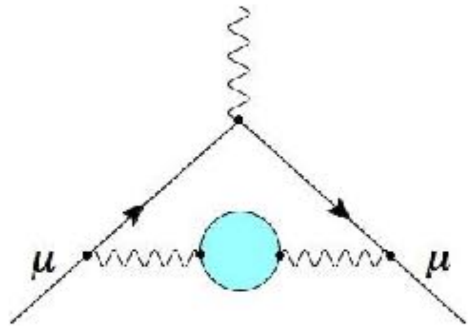
$$a_\mu^{\mathrm{HVP,LO}} = \sum_f a_{\mu,f}^{\mathrm{HVP,LO}} + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$$

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}}$$

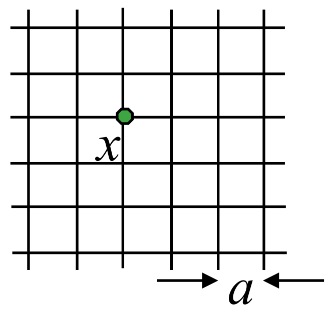


- $a_\mu^{\mathrm{HVP,LO}}$ needed with $< 0.5\,\%$ precision
- subpercent statistical precision:
  exponentially growing noise-to-signal in $C(t)$ as $t \to \infty$
  affects light-quark contributions
- sizable finite volume effects
- sensitivity to scale setting uncertainty
- control discretization effects
- quark-disconnected diagrams: control noise
- include isospin-breaking effects
  Separation of $a_\mu^{\mathrm{HVP,LO}}$ into $a_\mu^{\mathrm{HVP,LO}}(ud)$ and $\delta a_\mu^{\mathrm{HVP,LO}}$ is scheme dependent.

- light-quark connected contribution:
  $a_\mu^{\mathrm{HVP,LO}}(ud)$ ~90% of total
- *s*,*c*,*b*-quark contributions
  $a_\mu^{\mathrm{HVP,LO}}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution:
  $a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\mathrm{HVP,LO}}$ ~1% of total



Windows in Euclidean time



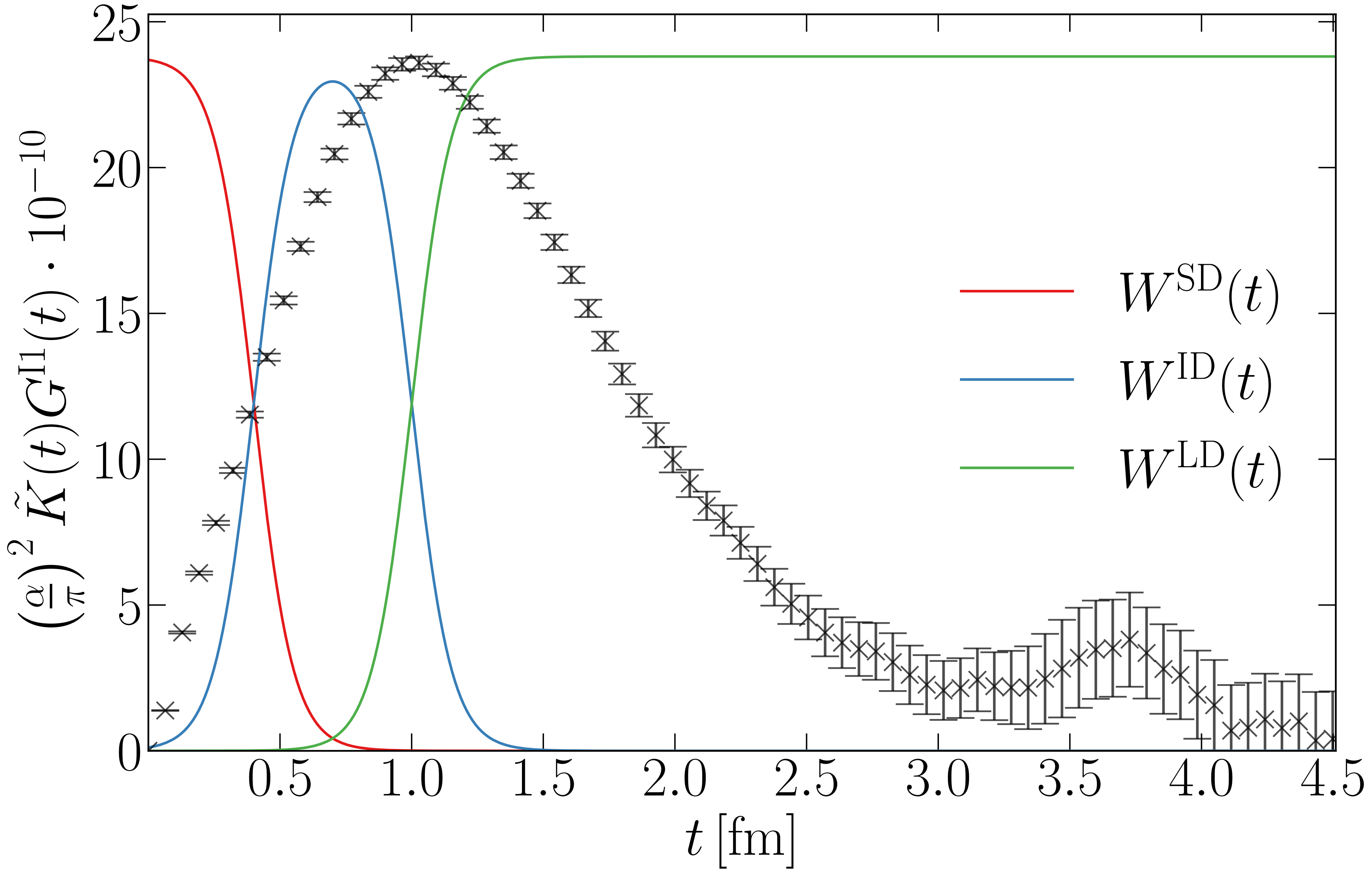
$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to consider the different time regions separately
  [T. Blum et al, arXiv:1801.07224, 2018 PRL]

$$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$$

Short Distance (SD)	$t : 0 \to t_0$
Intermediate (W)	$t : t_0 \to t_1$
Long Distance (LD)	$t : t_1 \to \infty$

S. Kuberski @ Lattice 2023



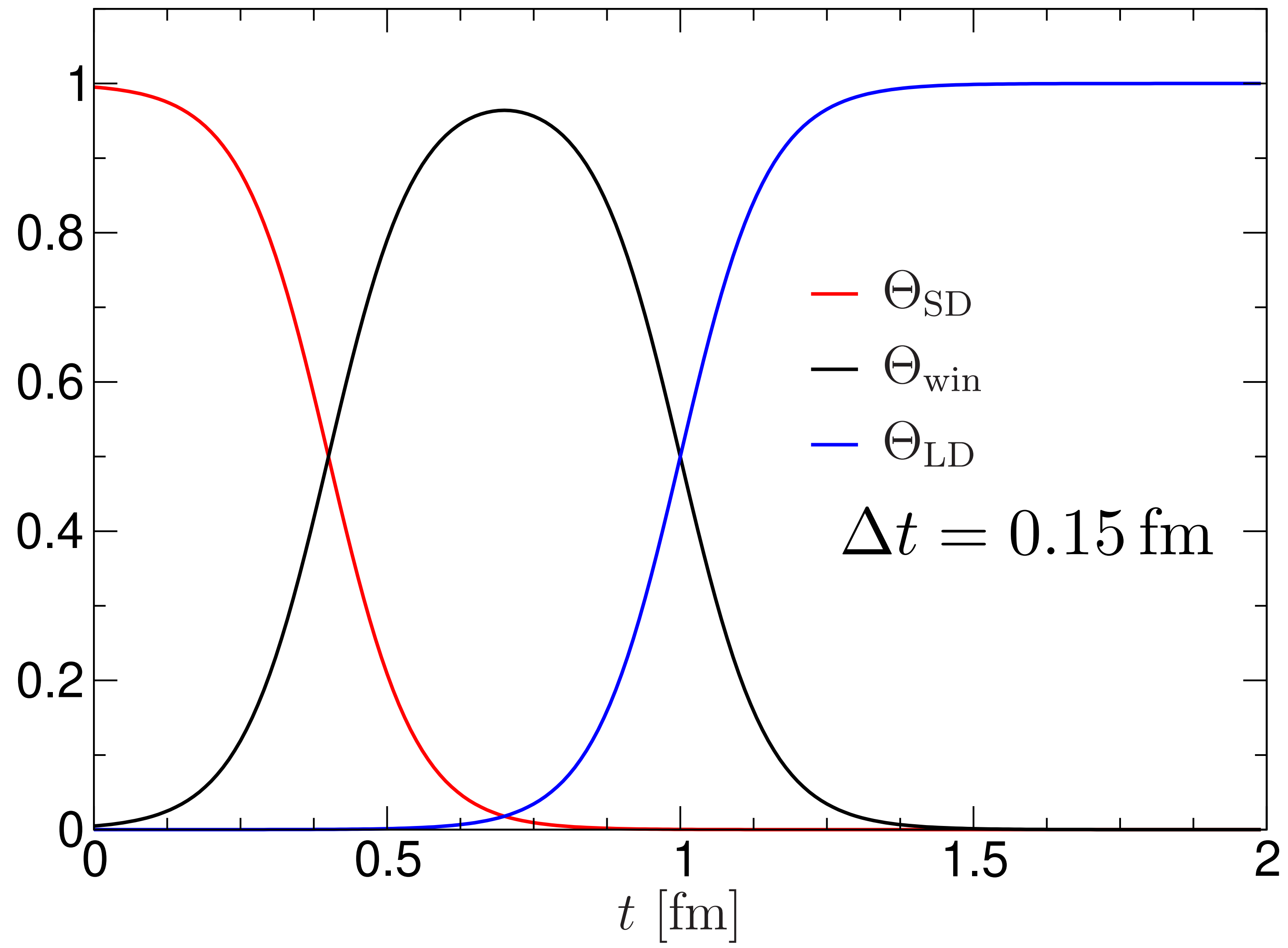
- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD; compare to disperse approach
- Internal cross check: compute each window separately (in continuum, infinite volume limits,…) and combine: $a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$

cross section inputs to windows observables

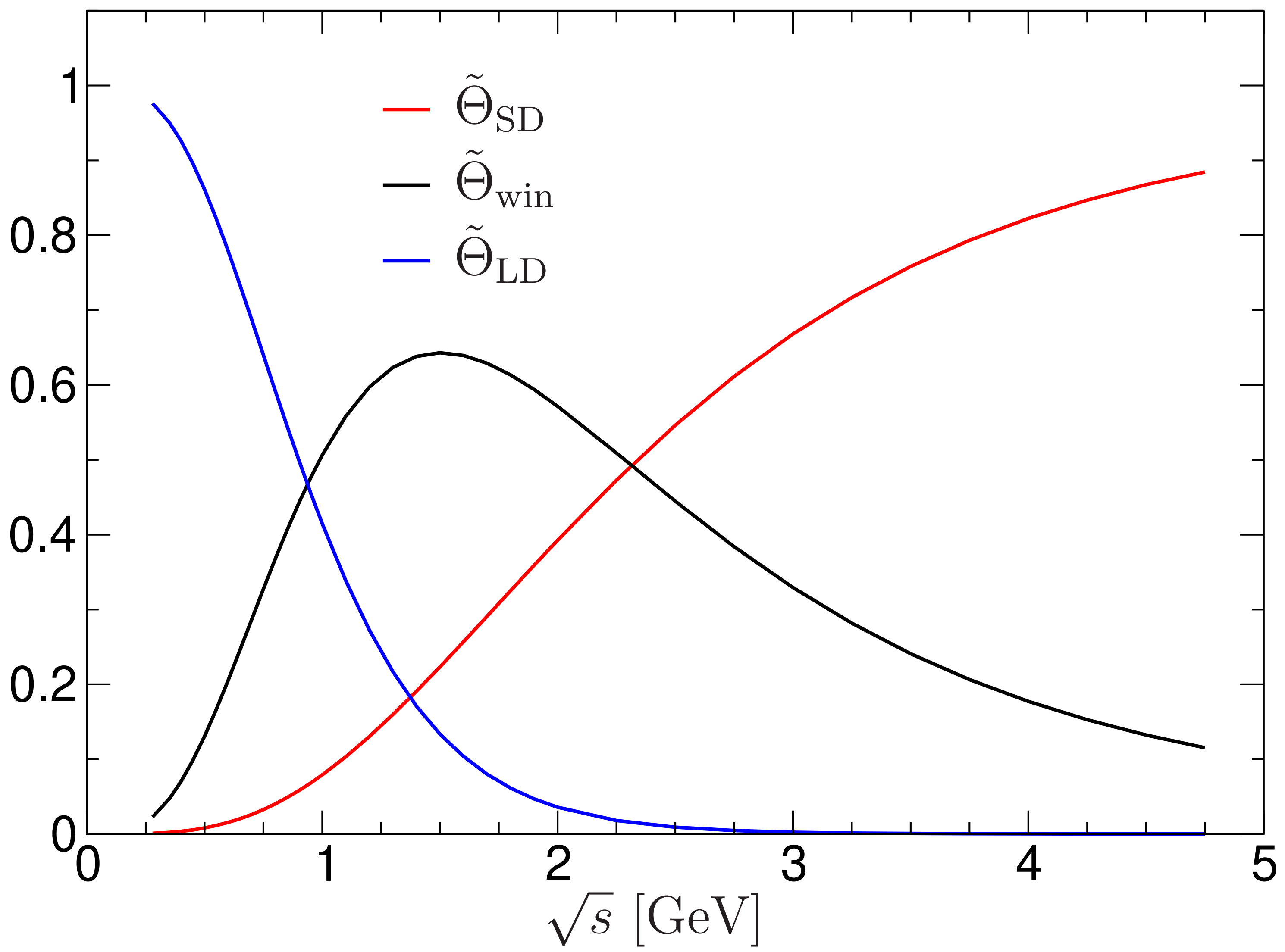
C. Davies @ Lattice 2024

Comparing data-driven and lattice HVP results

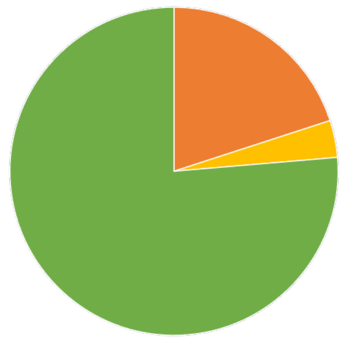
Colangelo et al 2205.12963



Mapping of window effects

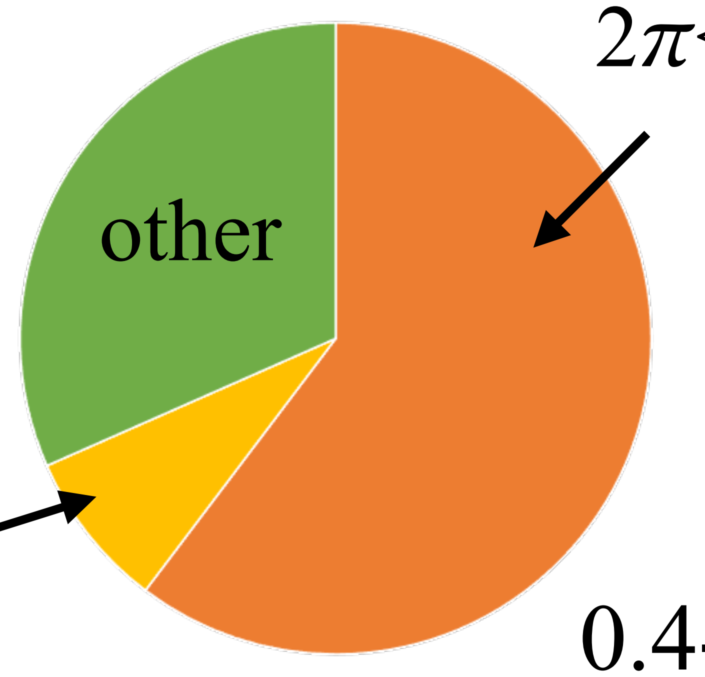


short-distance (SD)



0-0.4 fm

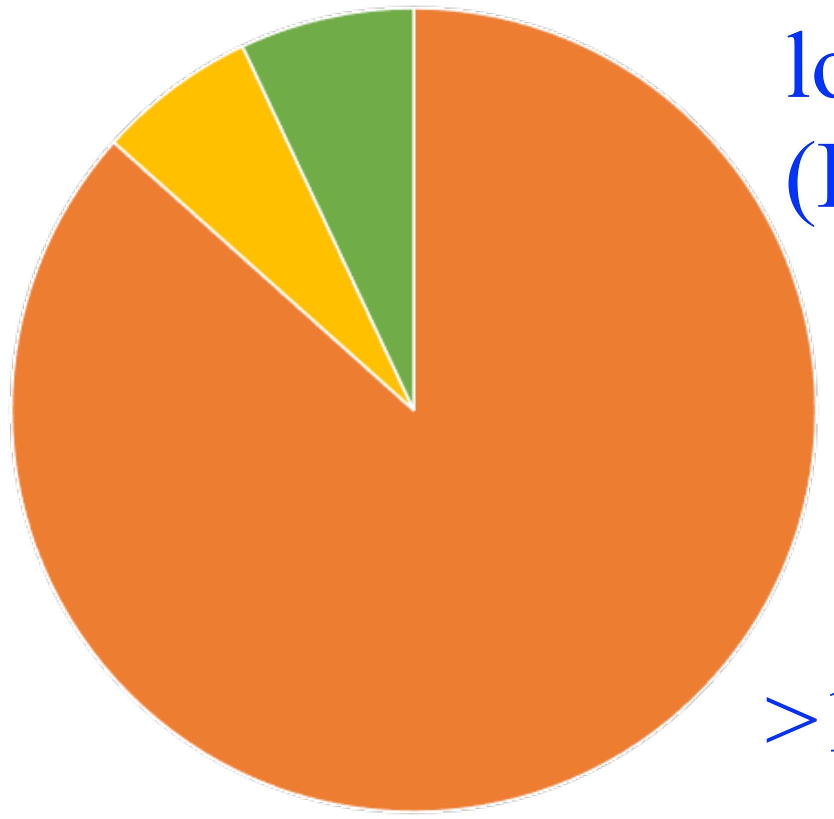
intermediate-distance



2π< 1 GeV

3π< 1.8 GeV

0.4-1.0 fm

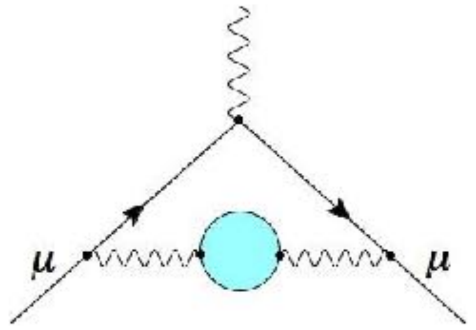


long-distance (LD)

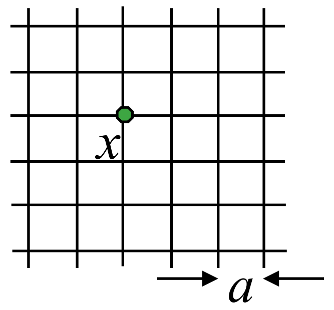
Data-driven contributions

>1.0 fm

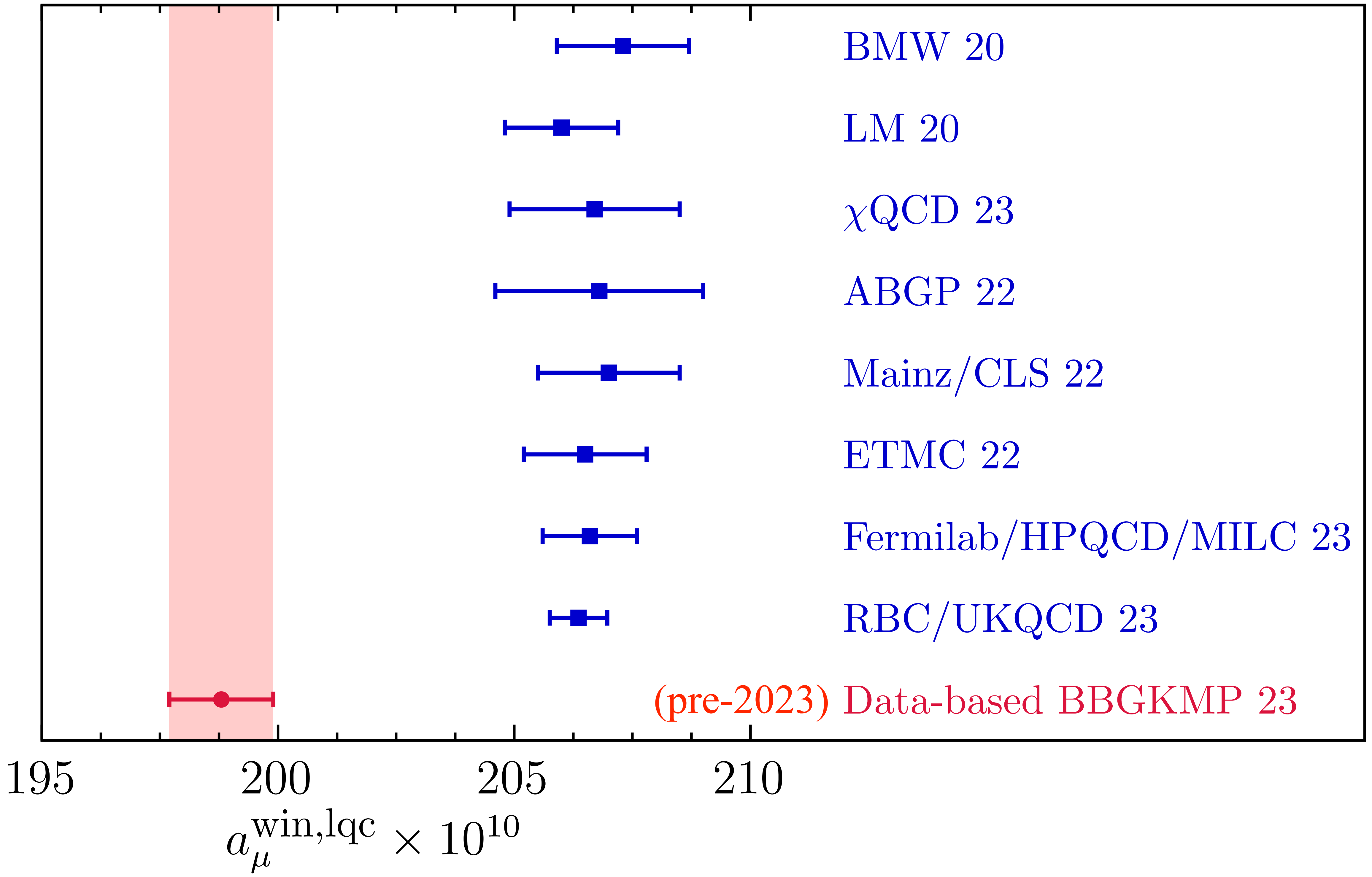
18



Lattice HVP: results for W



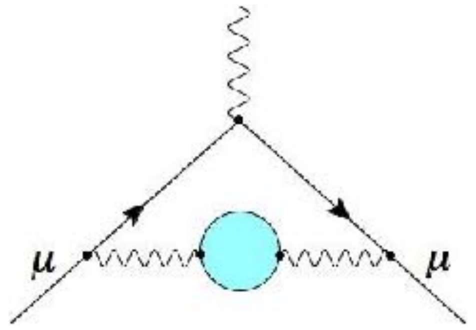
- new results in 2022-2024 for intermediate and short-distance windows.
- blind analyses by all lattice groups for results from 2023+
- focus on light-quark connected contribution to intermediate window:



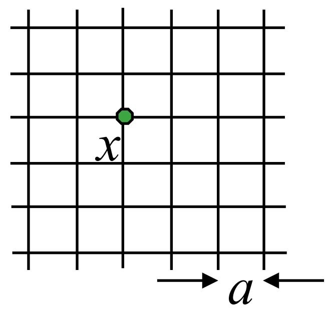
$\sim 5\sigma$ tension between LQCD and (pre-2023) data-driven evaluations for intermediate window

- dispersive evaluation of light-quark connected contribution [G. Benton, et al, arXiv:2306.16808]*

*based on disp. results for IB [Hoferichter et al, arXiv:2208.08993]



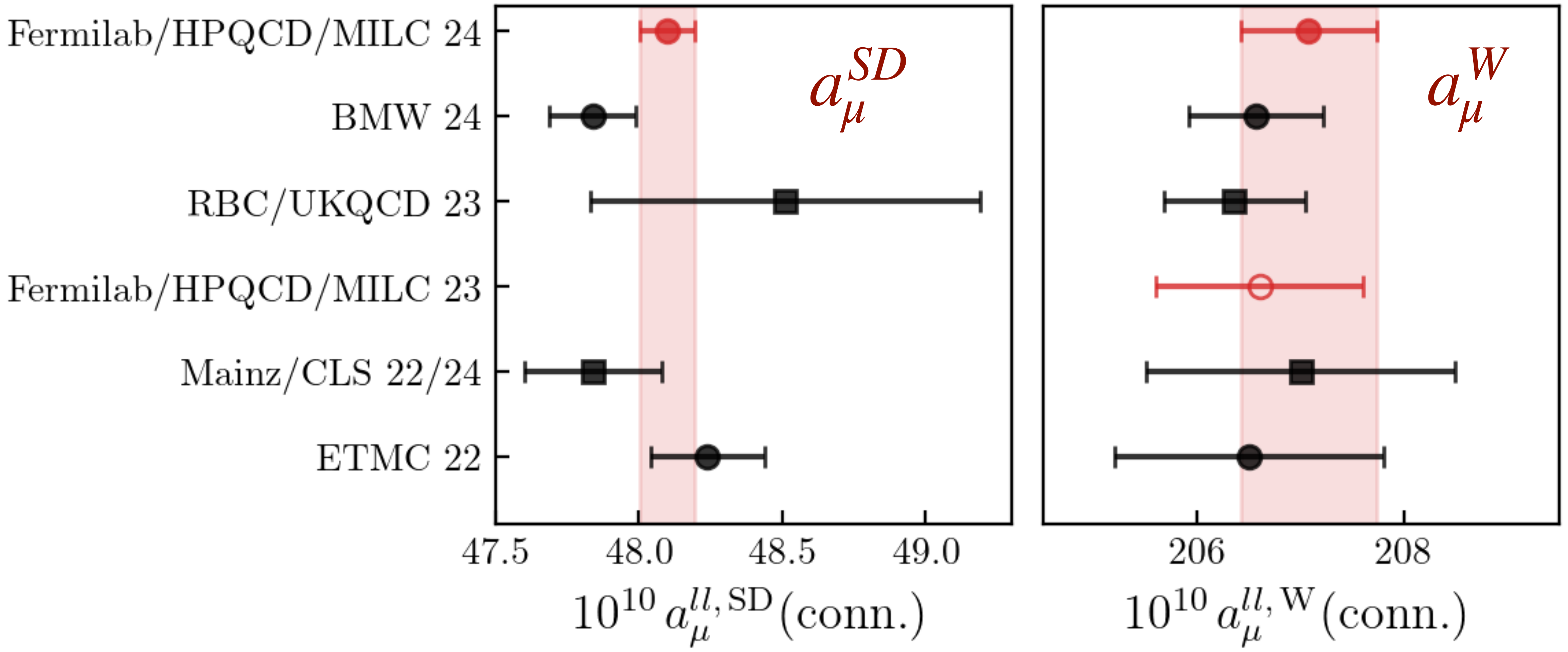
Lattice HVP: windows



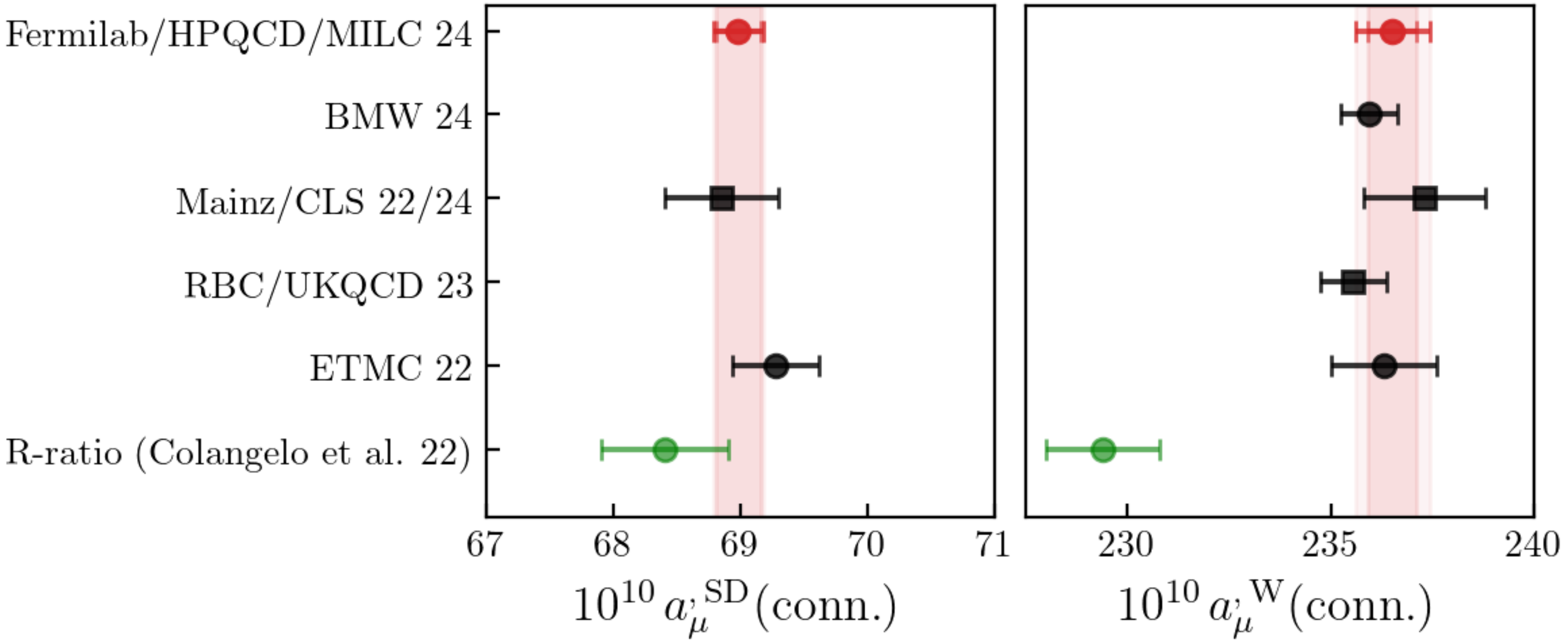
update: Fermilab/HPQCD/MILC 2024

S. Lahert talk @ KEK workshop:
Unblinded results for (all) contributions to a_μ^{SD} and a_μ^{W} (including correlations).

- light-quark connected

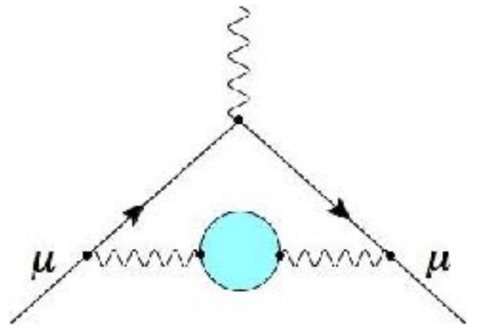


- total (u, d, s, c) + disc + IB

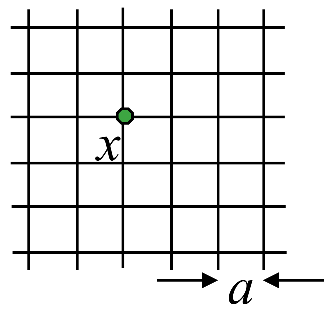


- W:
  - Consistent w/ all previous determinations
  - Leading uncertainty: scale setting (w_0 fm).
- SD: Good agreement with other groups.

➠ appendix for updates from BMW 2024 for W, SD windows



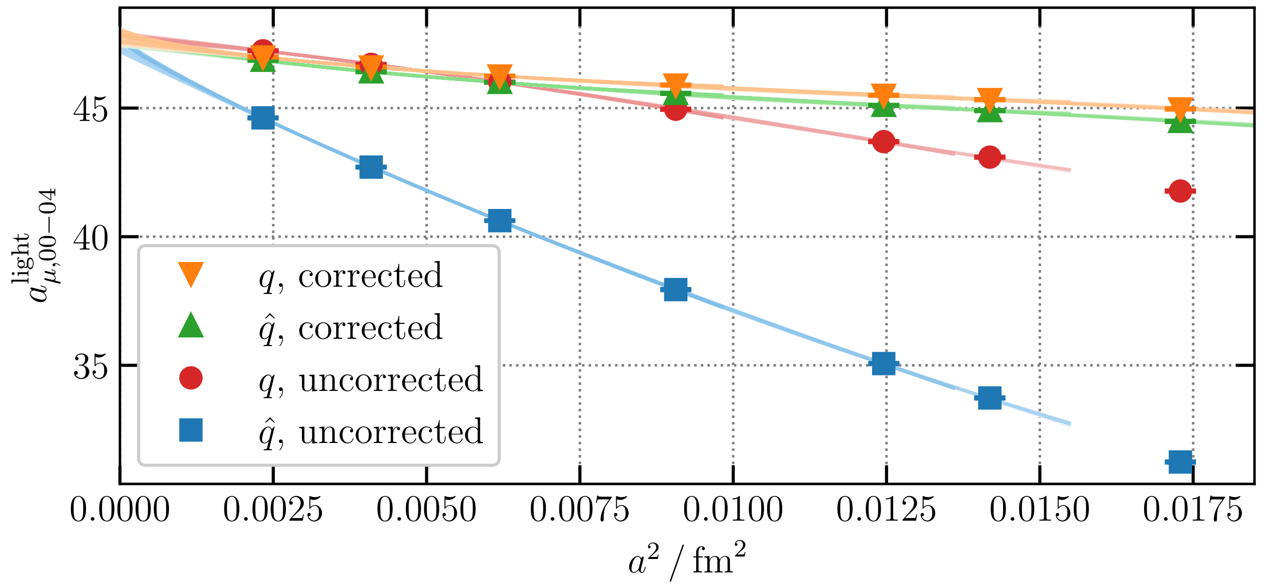
Lattice HVP: long-distance window



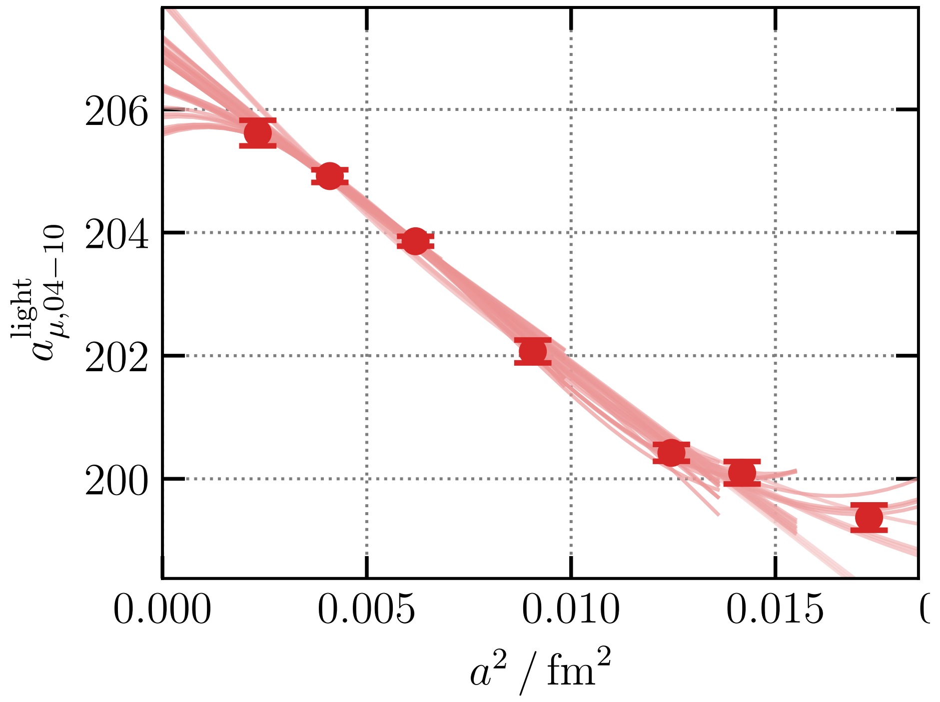
update: BMW+DMZ 2024

- BMW+DMZ 24 [A. Boccaletti et al, arXiv:2407.10913]

short-distance: 0-0.4 fm



intermediate-distance: 0.4-1.0 fm

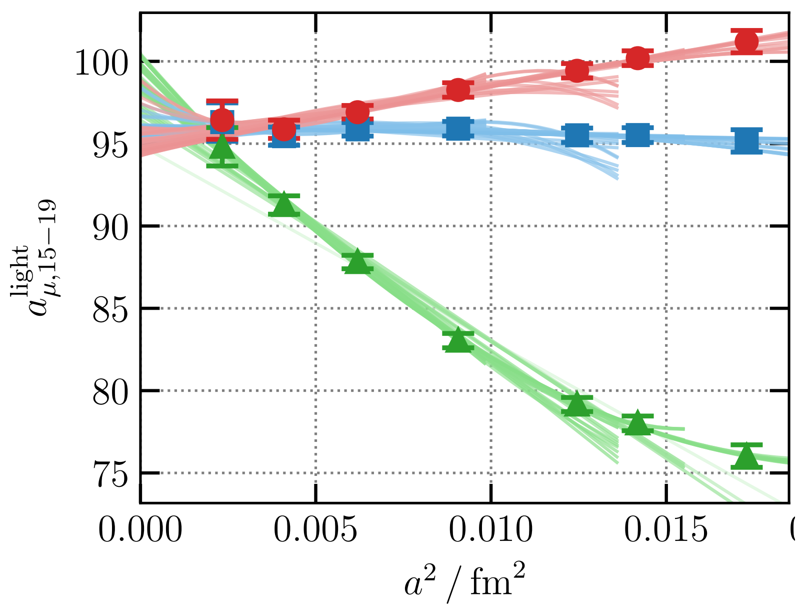


long-distance: 1.0 - ∞ fm or 1.0 - 2.8 fm

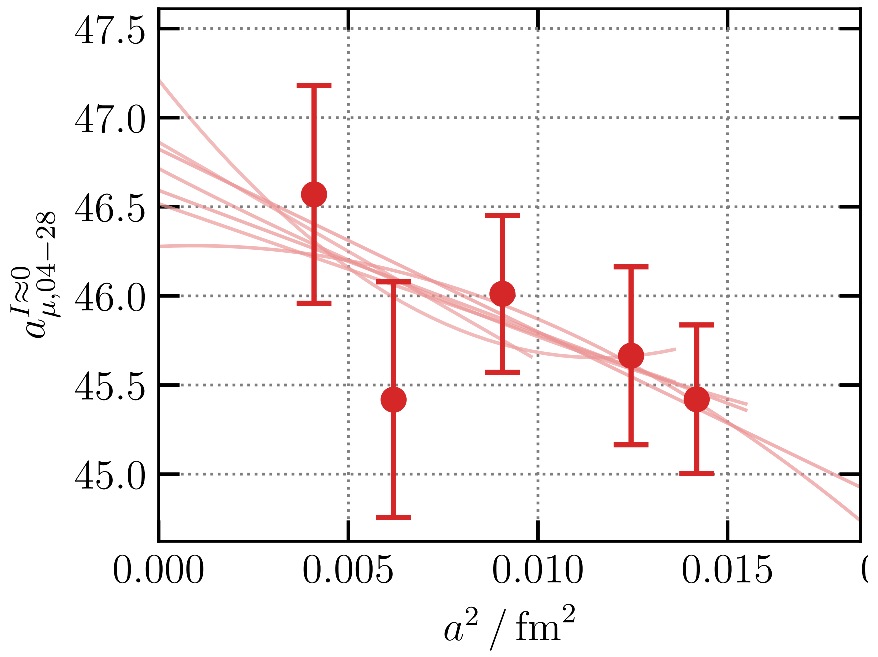
$\gtrsim 50\,\%$ of $a_\mu^{\rm HVP,LO}$

?

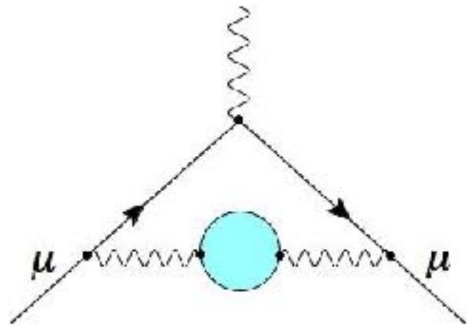
1.5-1.9 fm



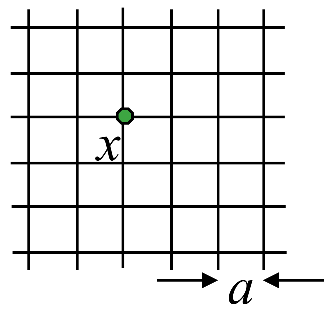
$I = 0$



No error budgets,
No plots of continuum extrapolations
No fit functions



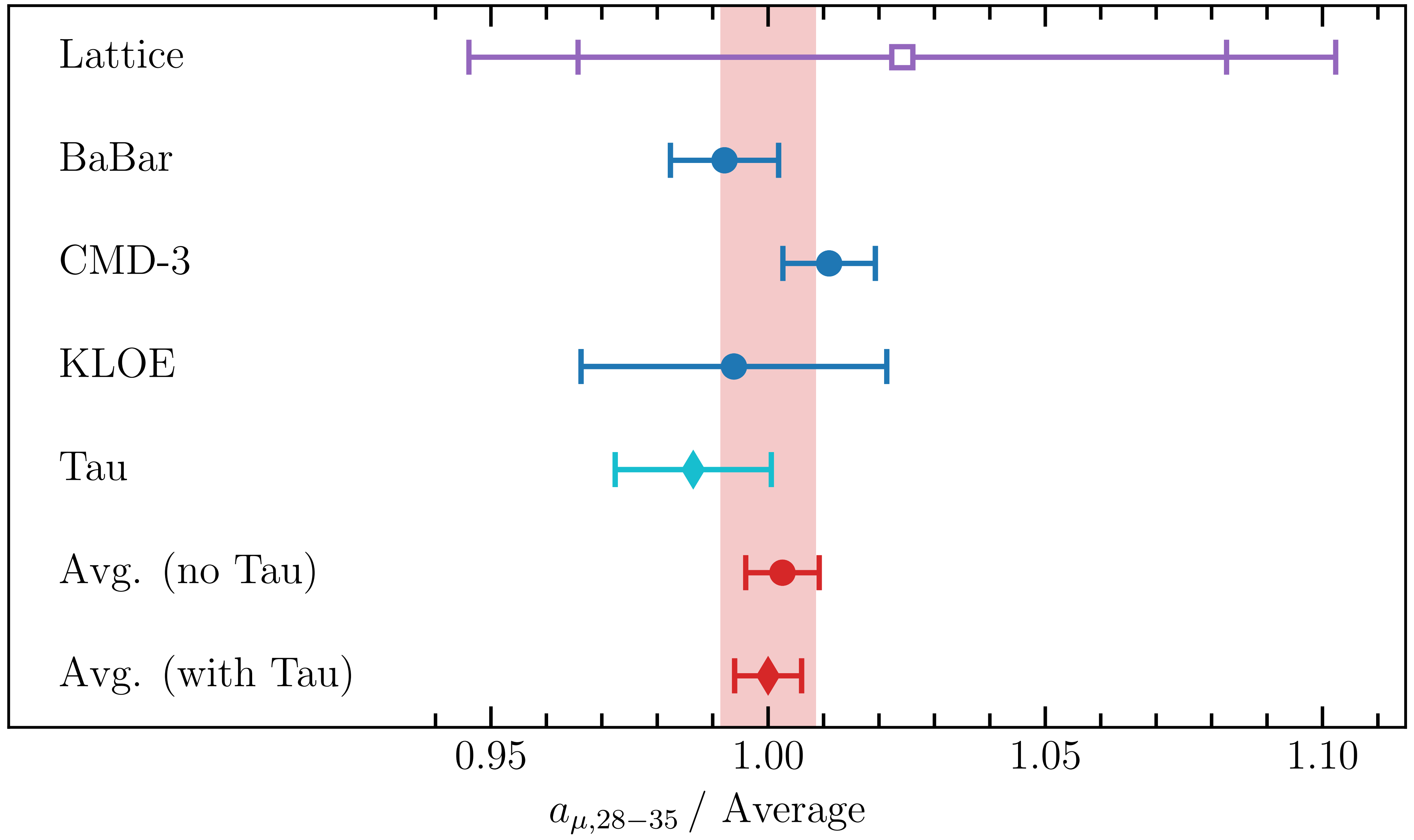
Lattice HVP: long-distance window



update: BMW+DMZ 2024

- BMW+DMZ 24 [A. Boccaletti et al, arXiv:2407.10913]

- statistical/systematic errors at long distances, $t \gtrsim 2.5$ fm still large:

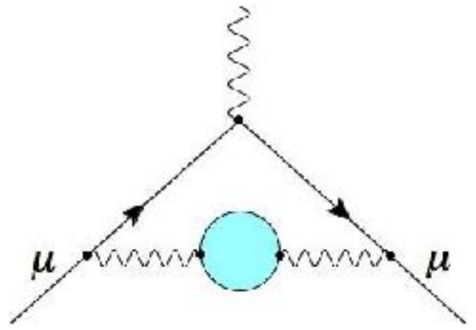


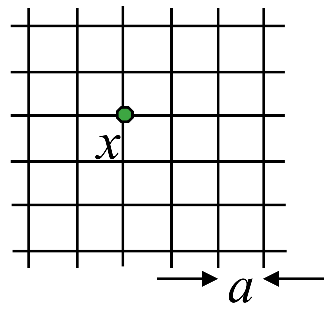
Laurent Lellouch @ Lattice @ CERN:

- Partial tail $a_{\mu,28\text{-}35}^{\text{LO-HVP}}$ for comparison with lattice dominated by cross section below ρ peak: $\sim$ 70% for $\sqrt{s} \leq 0.63$ GeV

- cross section measurements compatible for $\sqrt{s} < 0.5$ GeV
- not all measurements included in BMW/DMZ analysis
- constraints from analyticity, unitarity not included

Lattice HVP: long-distance window



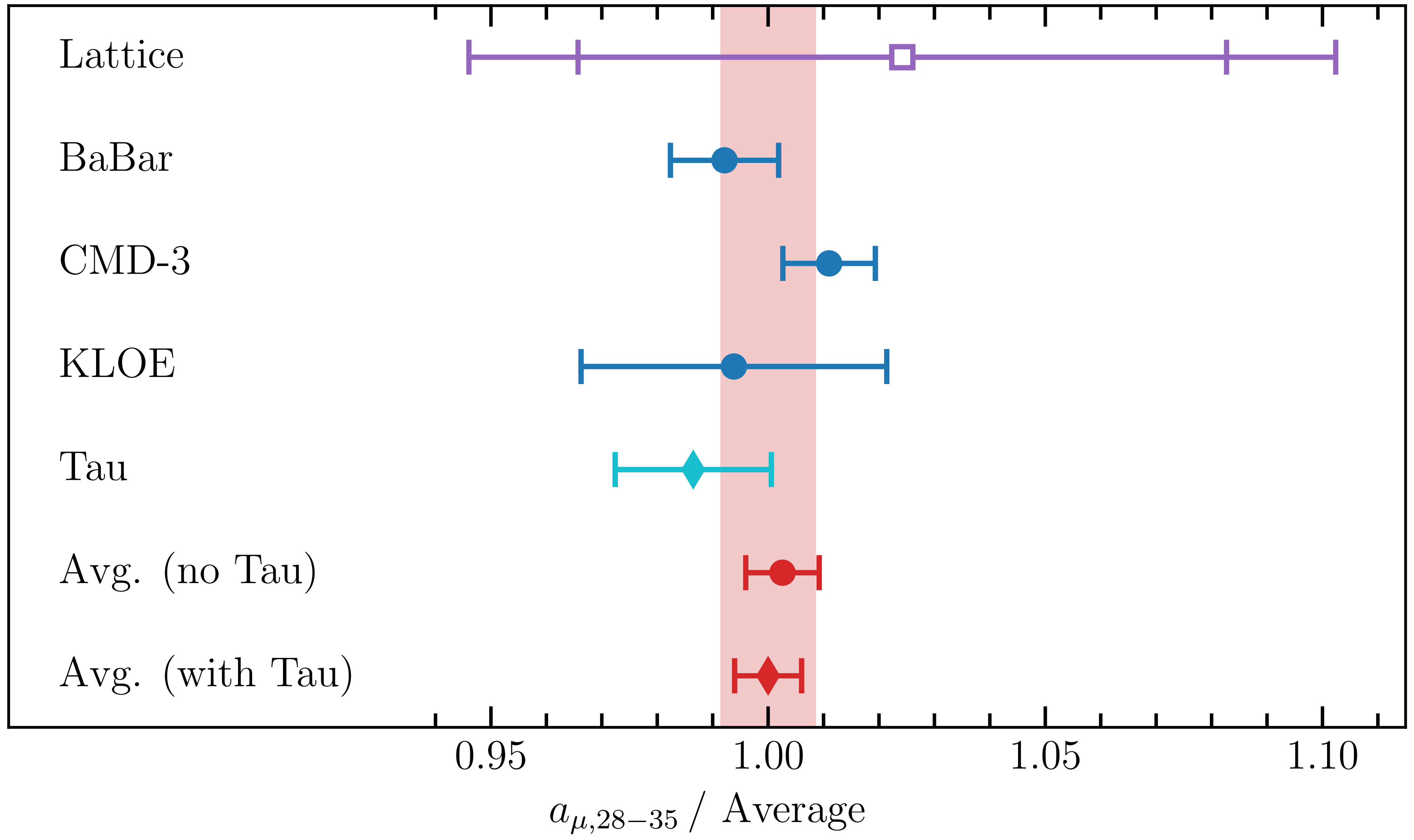


update: BMW+DMZ 2024

- BMW+DMZ 24 [A. Boccaletti et al, arXiv:2407.10913]

- statistical/systematic errors at long distances, $t \gtrsim 2.5$ fm still large:

[Laurent Lellouch Lattice @ CERN]



- All data-driven result agree very well
- Weighted average taken w/ and w/out τ: χ^2dof $= 1.1$ for both
- Final number: average w/ τ, PDG factor, and systematic = full difference τ/no-τ added linearly

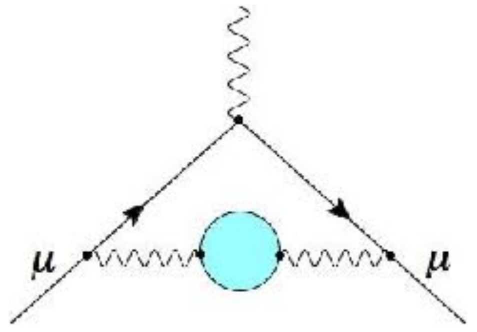
$$a^{\text{LO-HVP}}_{\mu,28\text{-}35} = 18.12(11)(5)[16]$$

- Excellent agreement w/ lattice, but uncertainty reduced by factor ~ 15
- Final number: average w/ τ, and systematic = full difference τ/no-τ added linearly

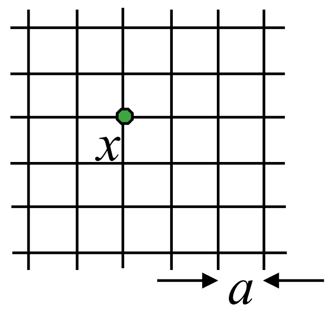
$$a^{\text{LO-HVP}}_{\mu,28\text{-}\infty} = 27.59(17)(9)[26]$$

- Only $\lesssim 5\%$ of final result for a_μ
- Contributes $\sim 65\%$ to total squared uncertainty uncertainty improvement: $5.5 \to 3.3$

This is a **hybrid evaluation:** combine lattice QCD calculation of one-sided window [Davies at at, arXiv:2207.04765] $a_\mu(t_1 = 2.8 \text{ fm})$ with data-driven evaluation of long tail, $t > 2.8$ fm.



Lattice HVP: long-distance window

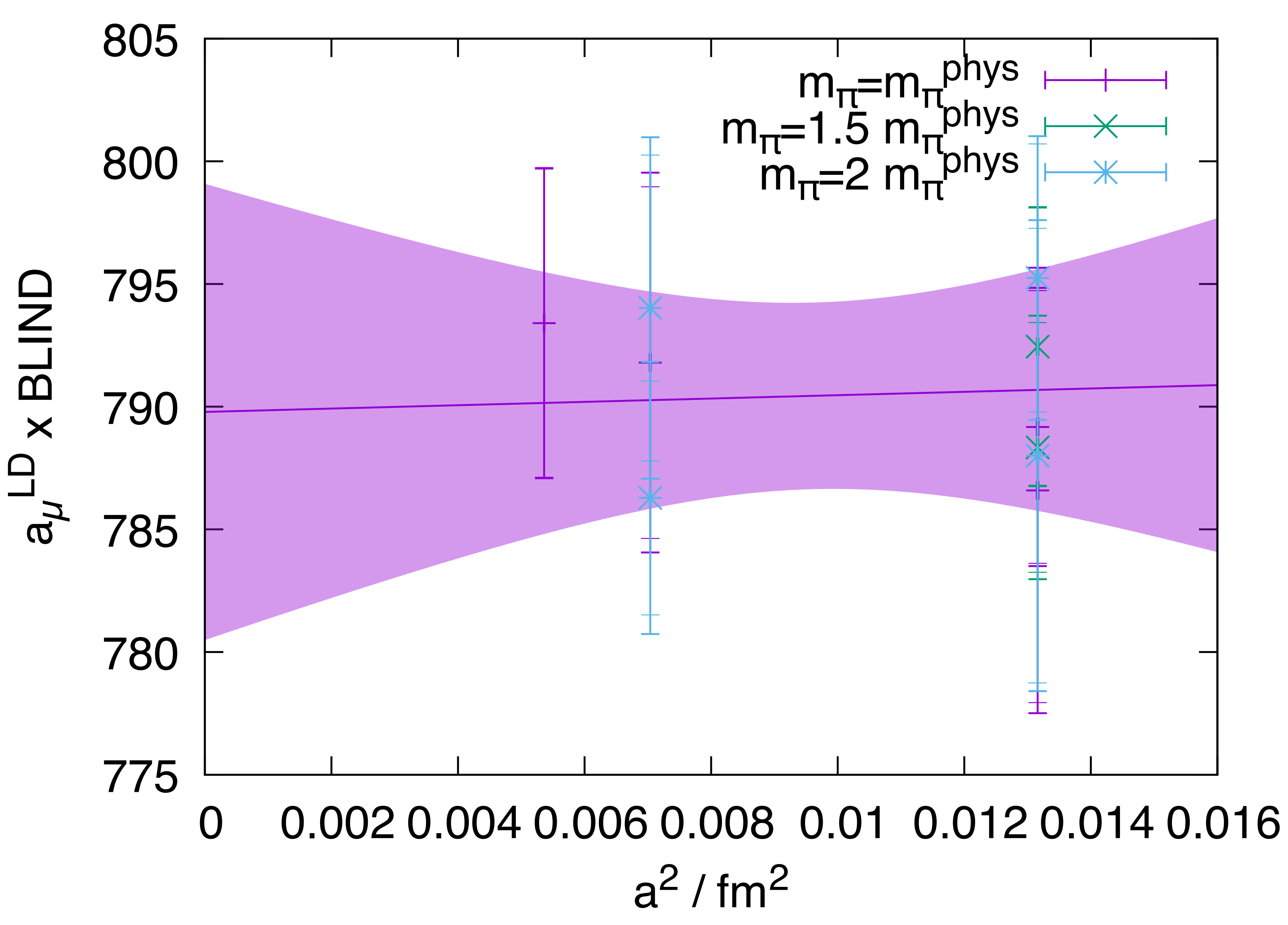


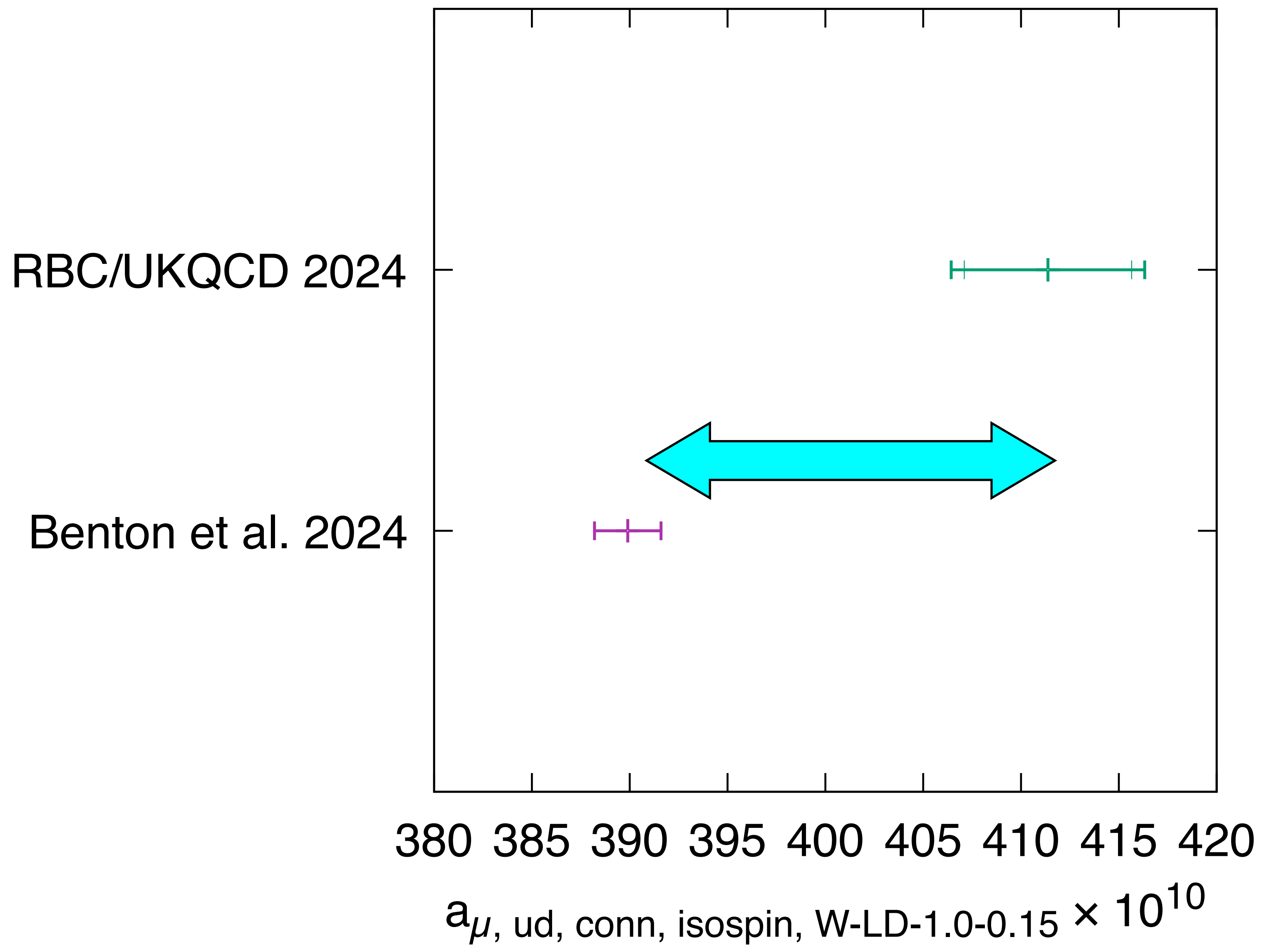
update: RBC/UKQCD 2024

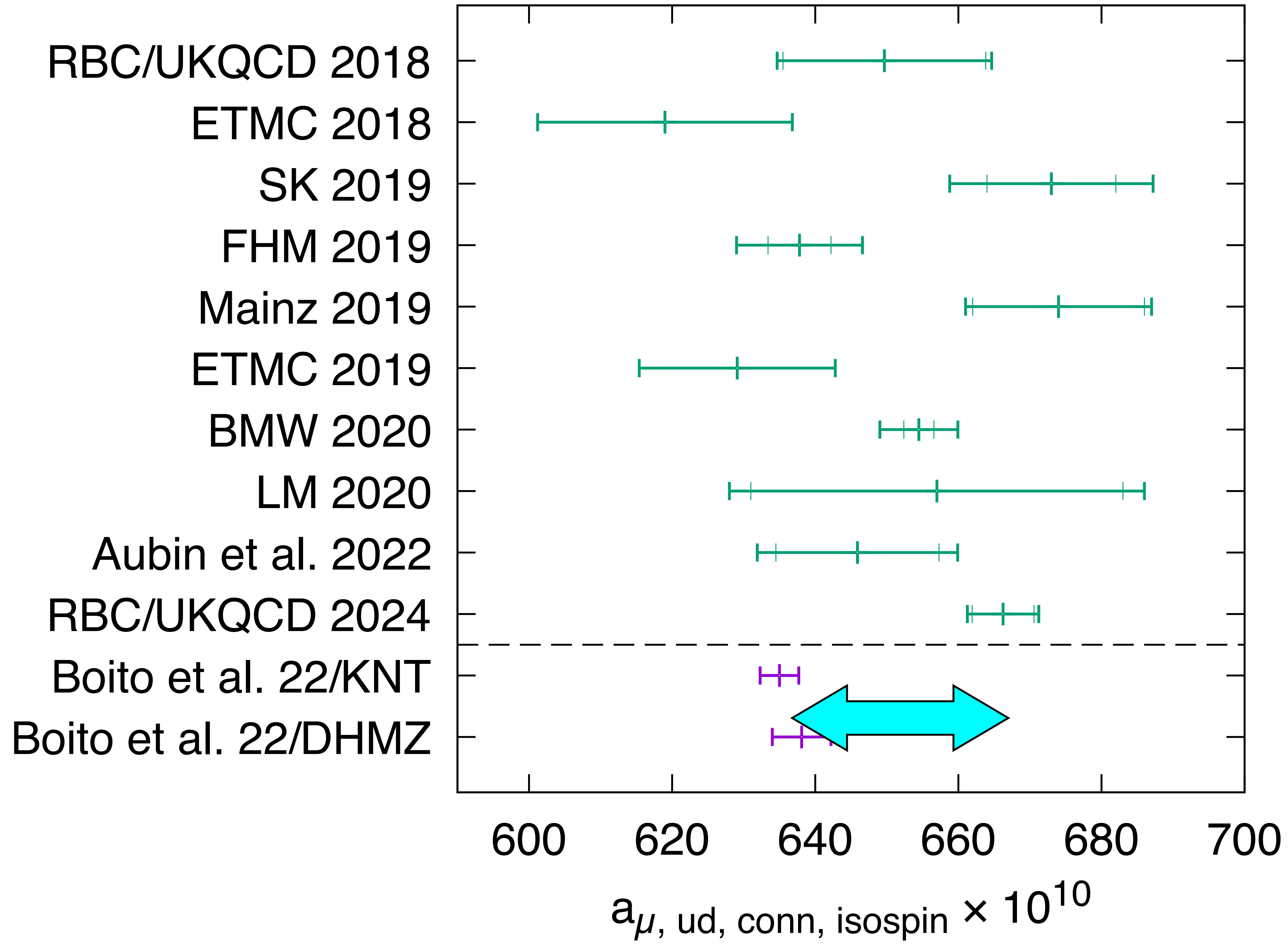
- long-distance window a_μ^{LD} and full a_μ^{ll}(conn.)

Unblinded results in BMW20 isospin-symmetric world

C. Lehner @ Lattice 2024





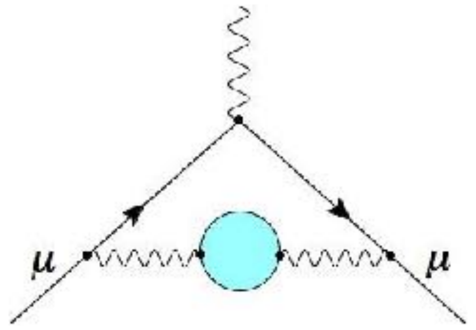


- ``BMW20 world'': fixed $w_0 = 0.1716$ fm
  scale uncertainty not included
- paper in progress

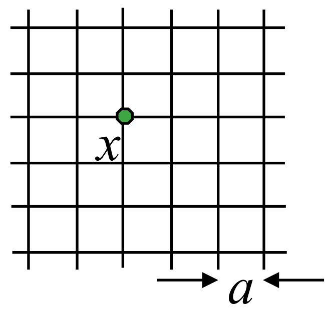
Result for $a_\mu^{\rm iso\ lqc}$ with 7.5/1000 precision.

$$a_\mu^{\rm LD\ iso\ lqc} = 411.4(4.3)_{\rm stat.}(2.3)_{\rm syst.} \times 10^{-10}\,,$$

$$a_\mu^{\rm iso\ lqc} = 666.2(4.3)_{\rm stat.}(2.5)_{\rm syst.} \times 10^{-10}\,.$$



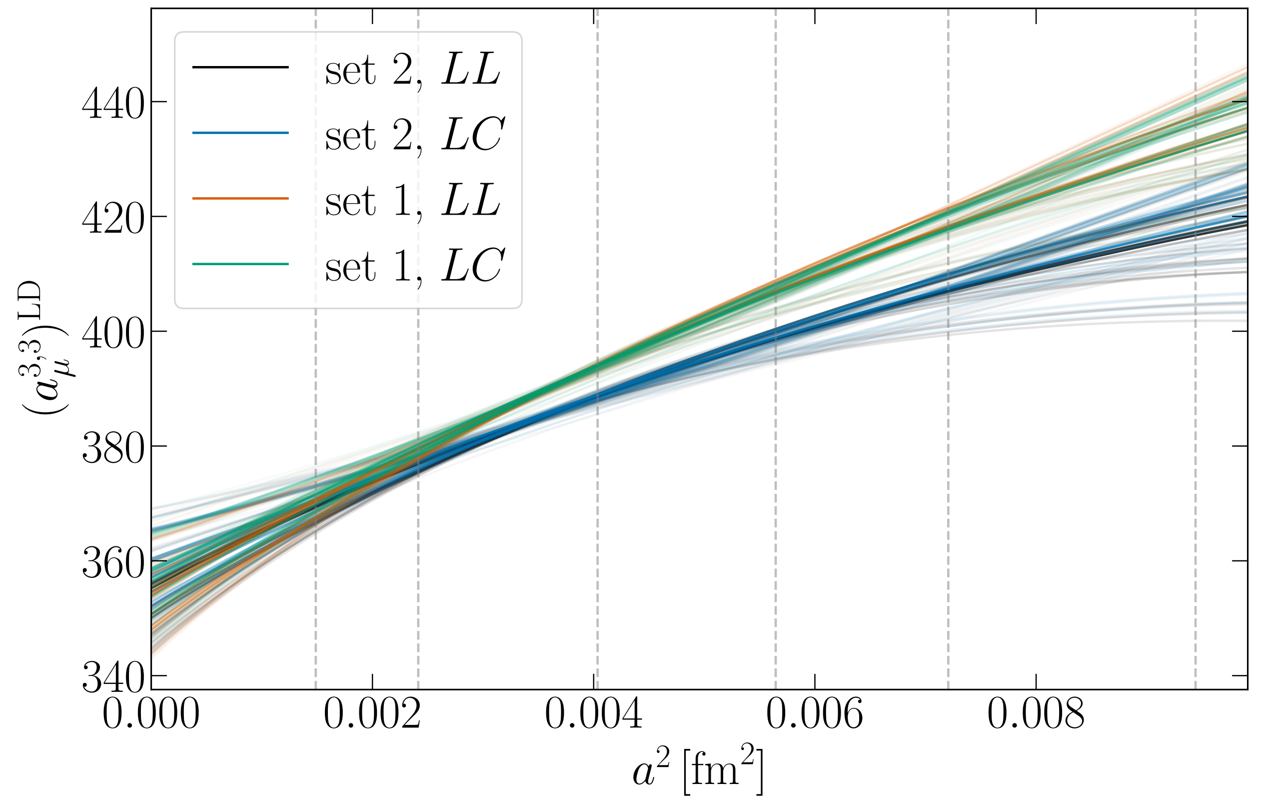
Lattice HVP: long-distance window



update: Mainz 2024

S. Kuberski @ KEK 2024

$(a_\mu^{\text{hvp}})^{\text{LD}}$ in the isovector channel: cutoff dependence



- Dependence of $(a_\mu^{3,3})^{\text{LD}}$ on a^2 at physical quark masses.
- Four sets of data (colors) differ by $\mathrm{O}(a^2)$.
- Each line represents a fit in the model average.
- Include terms à la $[\alpha_s(1/a)]^{0.395}\, a^2$ [Husung, 2401.04303].

$(a_\mu^{\text{hvp}})^{\text{LD}}$: Status and outlook

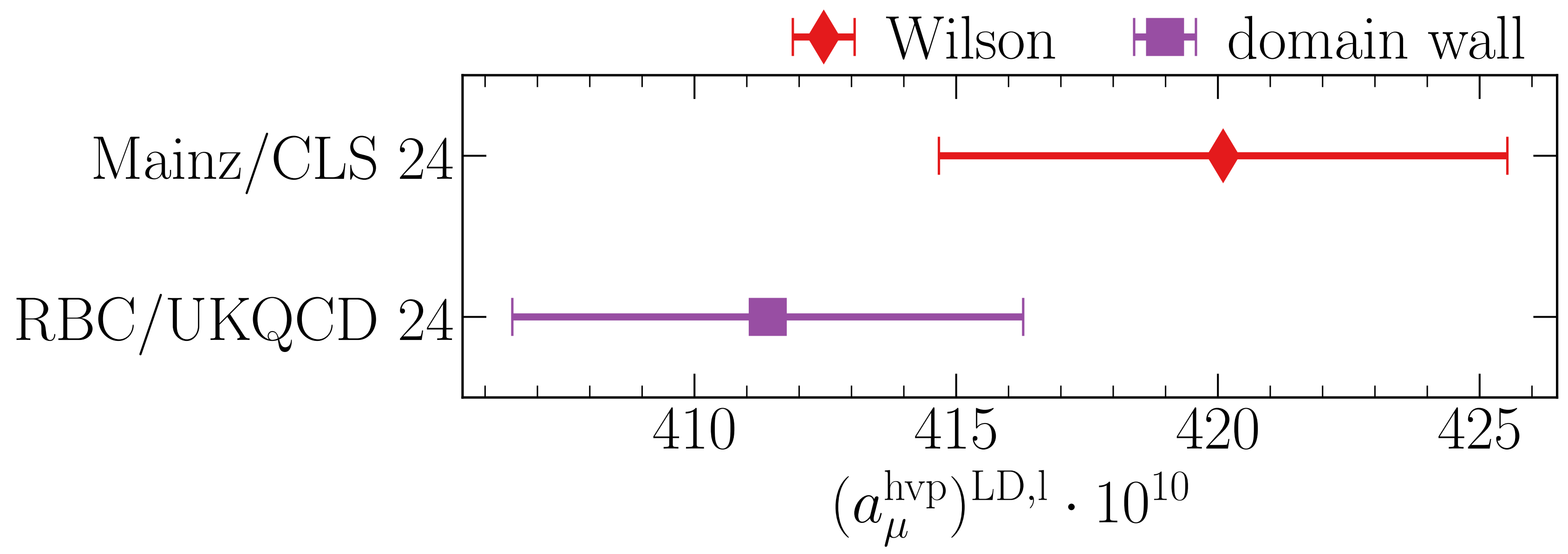
Achievements

- High statistical precision at m_π^{phys} and excellent control of the m_π dependence.
- Large span of lattice spacings to control the continuum extrapolation.
- Compute full isoQCD $(a_\mu^{\text{hvp}})^{\text{LD}}$ to 1.3% precision (statistics dominated).

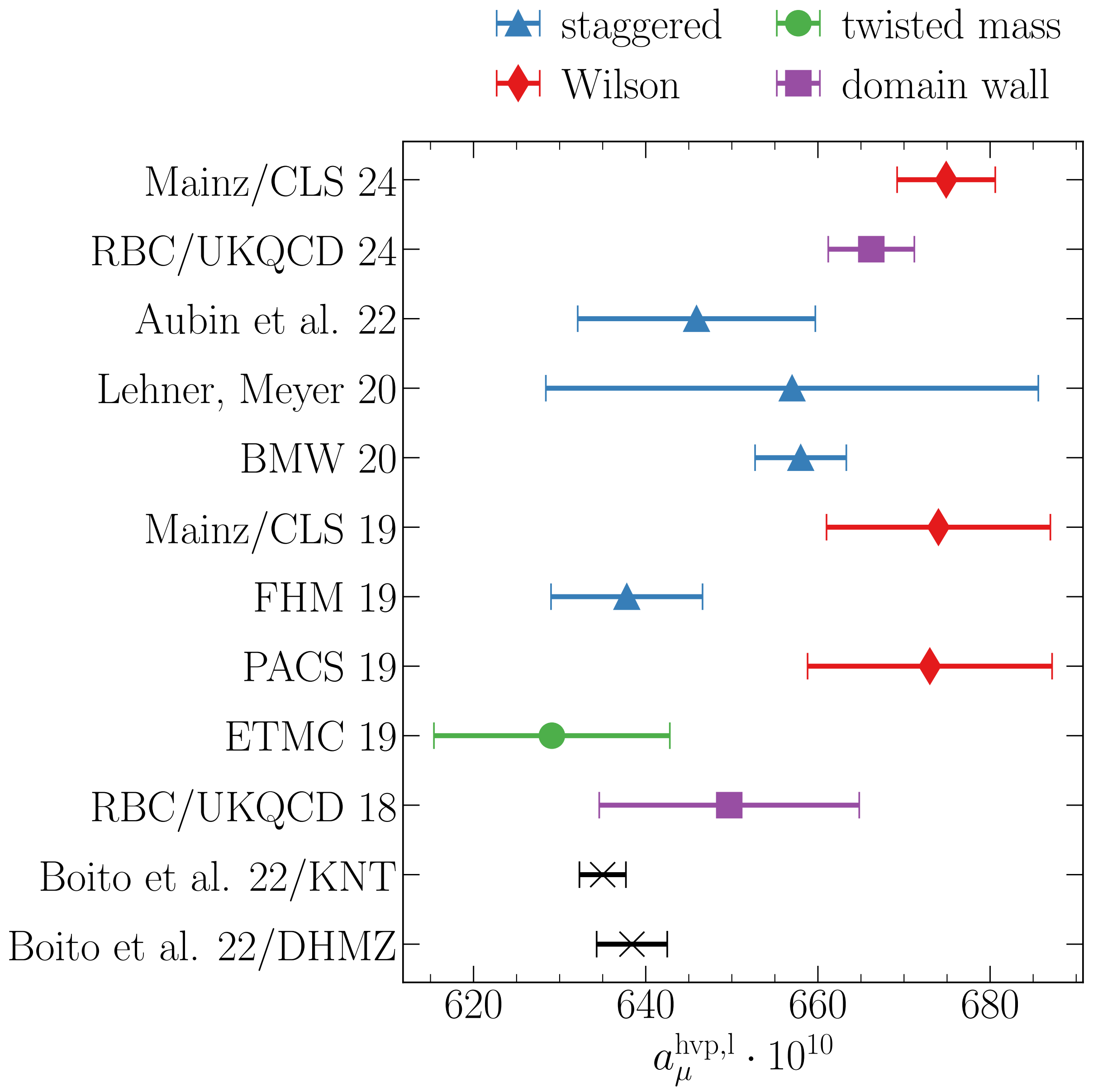
Outlook

- More data at fine lattice spacing and m_π^{phys} is being computed.
- Strong scale dependence in the long-distance regime:
  - We observe a strong scheme dependence: due to differences in the scale setting?
  - The global status of gradient flow scales is unsatisfactory [FLAG23].

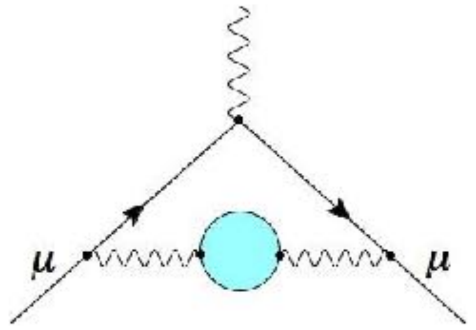
Comparison LD window (light):



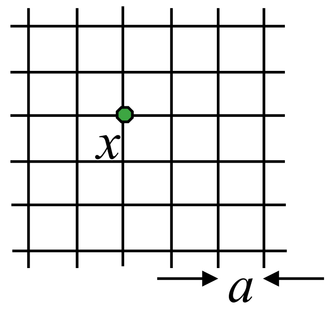
Comparison for full light-quark connected:



- Compute contributions to a_μ^{hvp} in isoQCD (Mainz world) by combinations with $(a_\mu^{\text{hvp}})^{\text{SD}}$ and $(a_\mu^{\text{hvp}})^{\text{ID}}$.
- We (will) publish the derivatives w.r.t. the input that defines our scheme. See [Portelli] for a comparison of schemes.
- $a_\mu^{\text{hvp,l}}$ determined to 0.8% precision
- Excellent compatibility of Mainz/CLS 19 with Mainz/CLS 24.

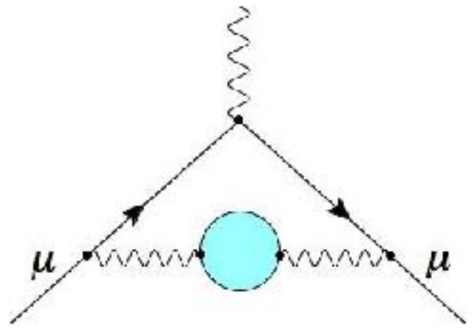


Lattice HVP: outlook

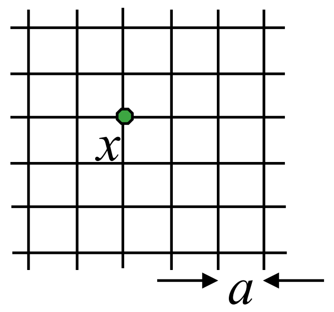


For total HVP and long-distance window:

- expect unblinded lattice results from FNAL/HPQCD/MILC (fall 2024) and ETM (~2025)
  - ➠ consolidated (?) lattice HVP for White Paper 2



Lattice HVP: outlook

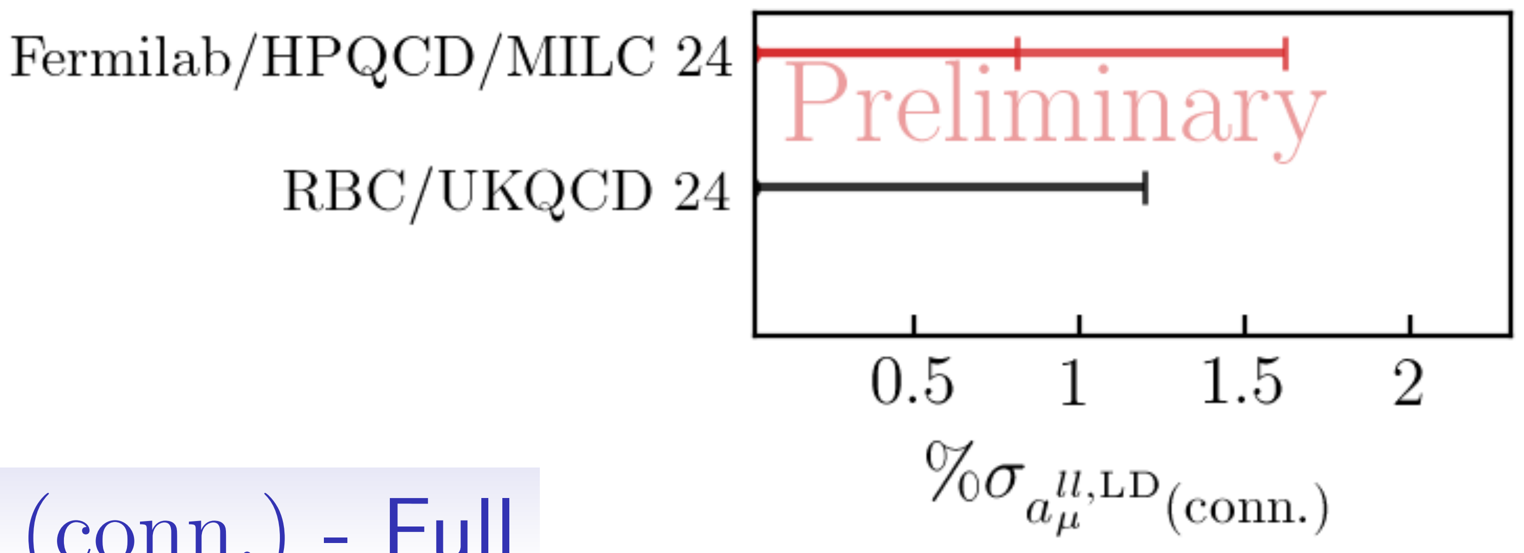


update: Fermilab/HPQCD/MILC 2024

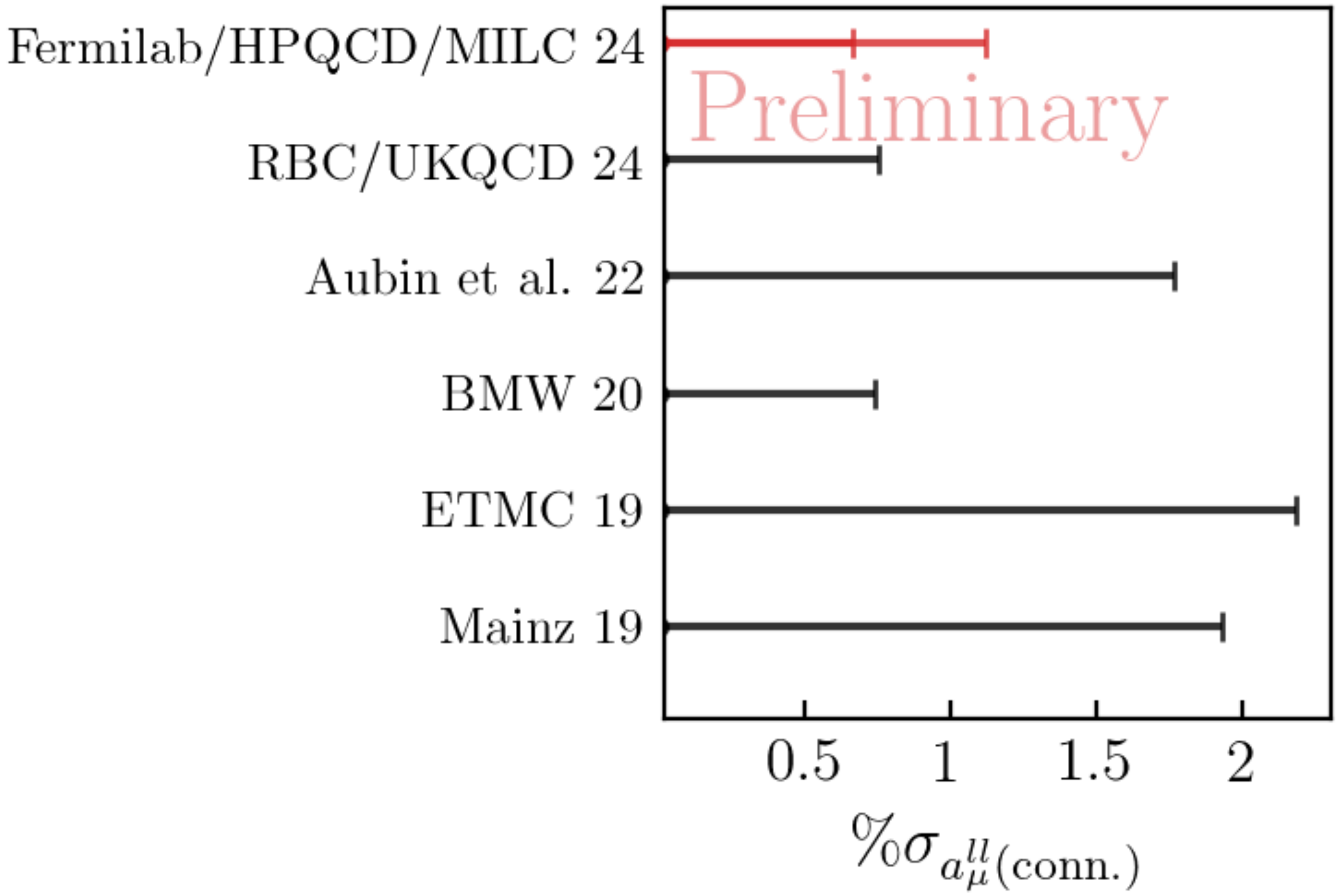
M. Lynch @ Lattice 2024
S. Lahert @ KEK 2024

long-distance window a_μ^{LD} and full $a_\mu^{ll}(\mathrm{conn.})$

$a_\mu^{ll}\,(\mathrm{conn.})$ - LD window



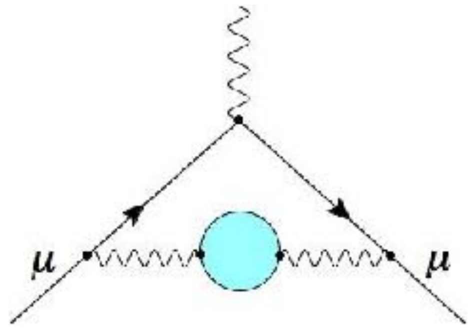
$a_\mu^{ll}\,(\mathrm{conn.})$ - Full



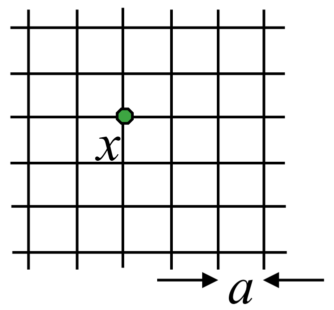
- Inner error w/o scale setting (w_0 fm) uncertainty
- Scale setting is now dominant error contributor.

We expect further improvements in stat., sys. uncertainties from...

- Generation of correlator data at a lattice spacing of 0.04 fm is in progress.
- Improved scale setting via M_Ω.
- Joint fit analysis with multiple vector current discretizations
- Direct finite volume study: $L \sim 5.5$ fm $\to L \sim 11$ fm (at $a = 0.09$ fm) to replace EFT-based FV error estimates.
- Calculation of two-pion contributions to vector-current correlation functions at finer lattice spacings.



Lattice HVP: full window

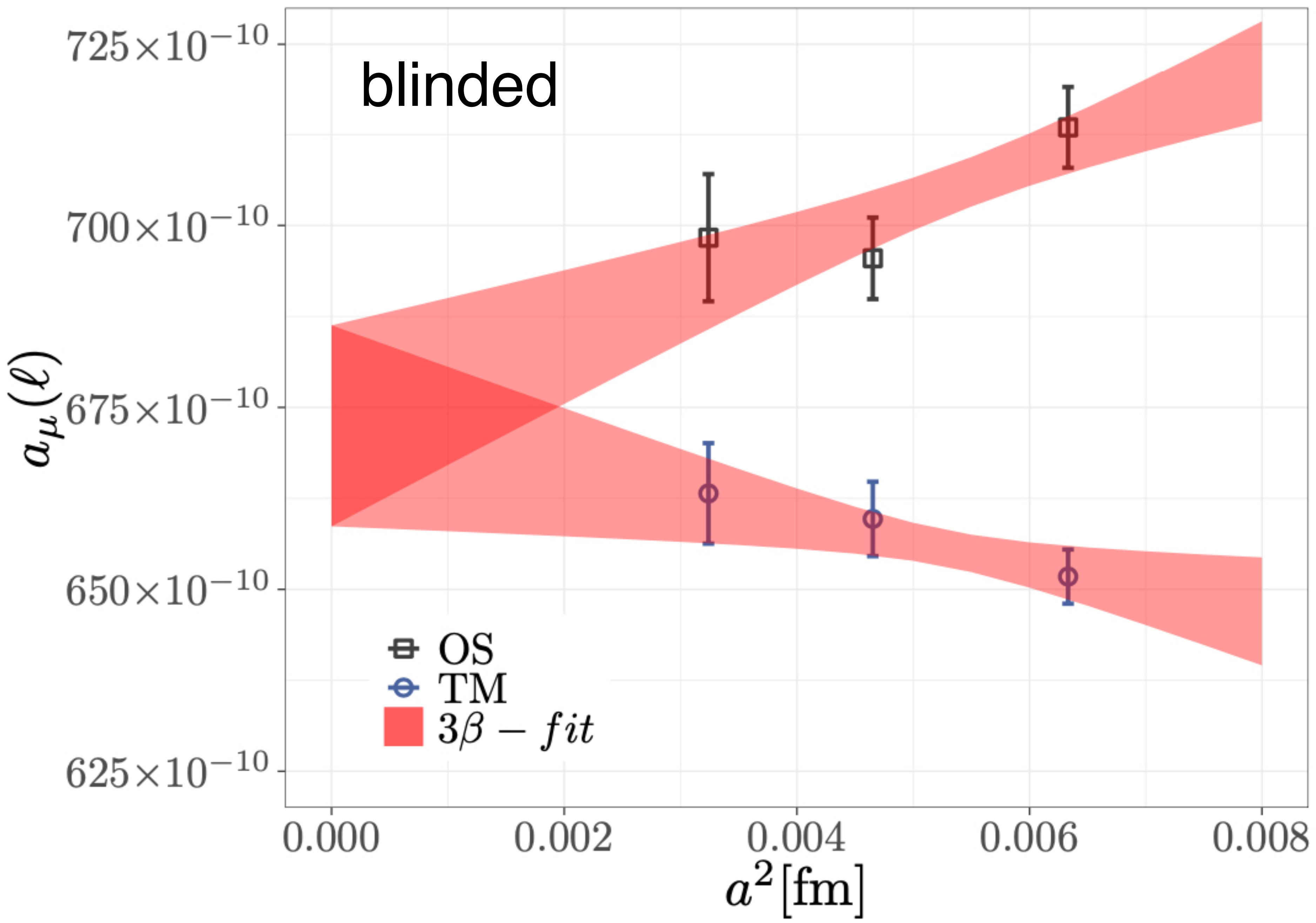


update: ETMC 2024

M. Garofalo @ Lattice 2024

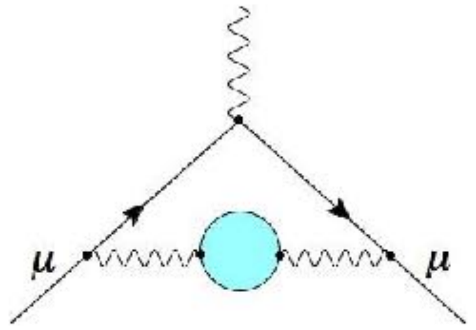
U. Wenger @ KEK 2024

ℓ-quark connected [Preliminary]

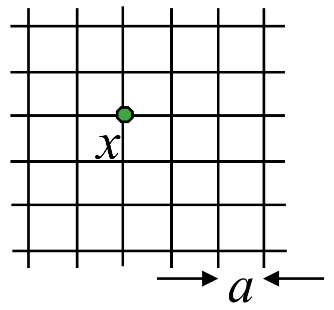


- Data interpolated to $L = 5.46$ fm:
  - ⇒ using GS/MLLGS approach
- Correction to isoQCD point:
  - ⇒ calculation in progress
- Significant reduction in uncertainty possible:
  - ⇒ additional final lattice spacing
  - ⇒ higher statistics in progress
  - ⇒ EFT model for TM lattice artefacts ($\rho\pi\gamma$-model)

also prelim. results for strange & charm



Lattice HVP: outlook



For total HVP and long-distance window:

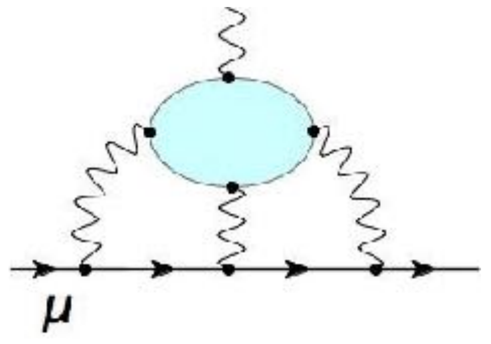
- expect unblinded lattice results from FNAL/HPQCD/MILC (fall 2024) and ETM (~2025)
  ➠ consolidated (?) lattice HVP for White Paper 2
- Including $\pi\pi$ states for refined long-distance computation (Mainz, RBC/UKQCD, FNAL/MILC)
- smaller lattice spacings to test continuum extrapolations crucial

➠ Slides from Lattice 2024 conference

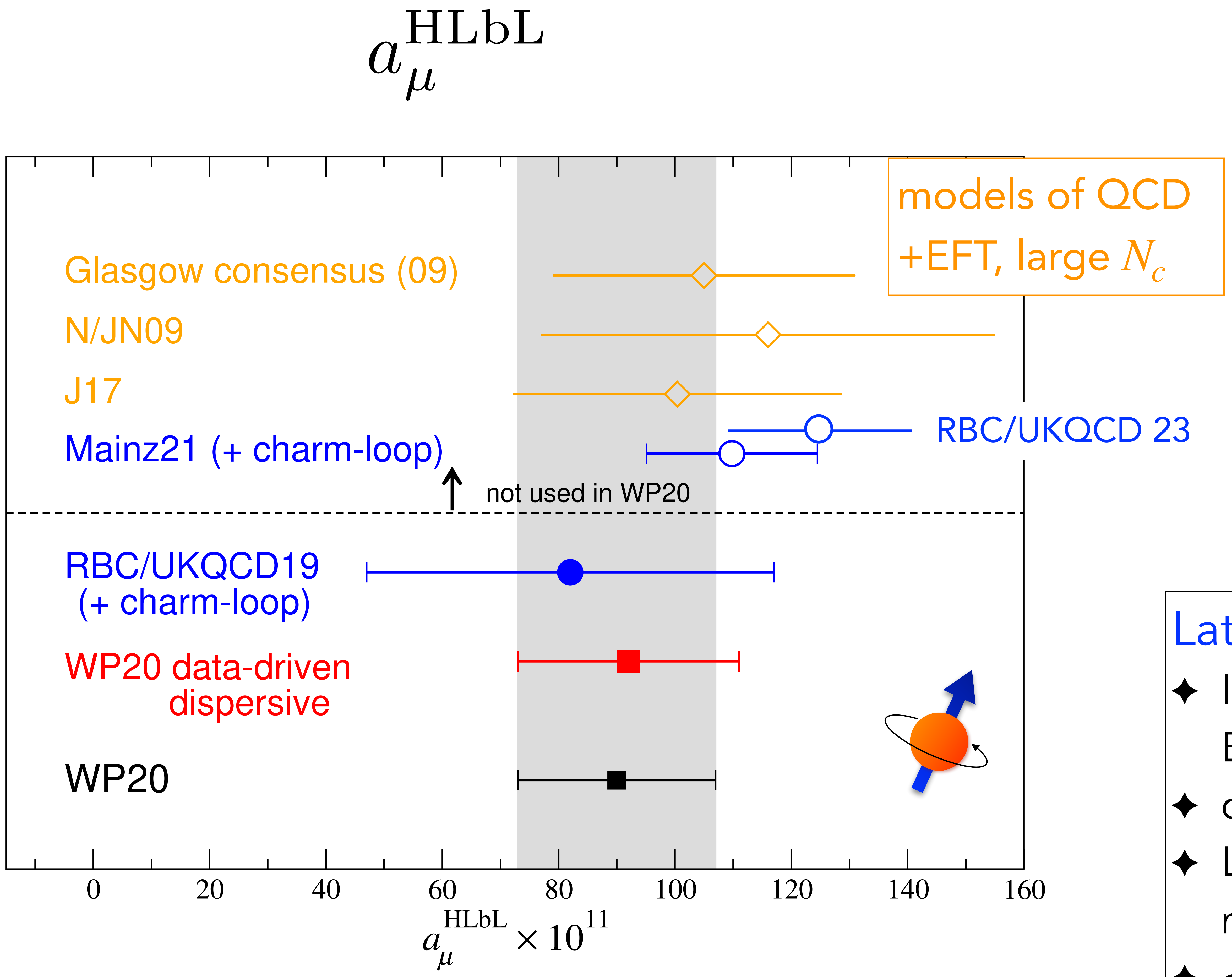
➠ Slides from Muon g-2 TI workshop @ KEK

More windows:

- Use linear combinations of finer windows to locate the tension (if it persists) in $\sqrt{s}$ [Colangelo et al, arXiv:12963]
- One-sided windows (excluding the long-distance region $t \gtrsim 2.5\,\mathrm{fm}$) to test data-driven evaluations [Davies at at, arXiv:2207.04765]



Hadronic Light-by-Light: Summary



Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ✦ can also use lattice results as inputs
- ✦ current 20% uncertainty ⇨ ~10% feasible

Lattice QCD+QED:

- ✦ Independent calculations by four groups (Mainz, RBC/UKQCD, ETM, BMW)
- ✦ consistent with each other and with previous calculations
- ✦ Lattice groups are continuing to improve their calculations, adding more statistics, lattice spacings, physical mass ensembles
- ✦ ongoing LQCD calculations of π, η, η' transition form factors to determine pseudo scalar pole contributions
  [Mainz, ETMC, BMW, RBC/UKQCD]

Summary

- consistent results from independent, precise LQCD calculations for light-quark connected contribution to **intermediate window** $a_\mu^{\rm W}$ (~ 1/3 of $a_\mu^{\rm HVP,LO}$) ➠ $\sim 5\,\sigma$ tension with (pre-2023) data-driven results
- 2 new LQCD results for long-distance contribution with $\sim 0.8\,\%$ precision, more coming soon!
  ➠ check consolidation, develop lattice HVP average for White Paper 2
- Programs and plans in place for:
  - data-driven HVP:
    new analyses from BaBar, KLOE, CMD3, SND, Belle II,…. will shed light on current discrepancies
    improved treatment of structure dependent radiative corrections (NLO) in $\pi\pi$ and $\pi\pi\pi$ channels
  - lattice HVP: **if no tensions** between independent lattice results, $\sim 0.5\,\%$ possible
  - dispersive HLbL and lattice HLbL: no puzzles, steady progress, $\sim 10\,\%$
- Need to understand tensions between data-driven and lattice HVP if they persist
- including τ decay data in data-driven approach:
  requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]

➠ continued coordination by Theory Initiative: 2nd WP in progress

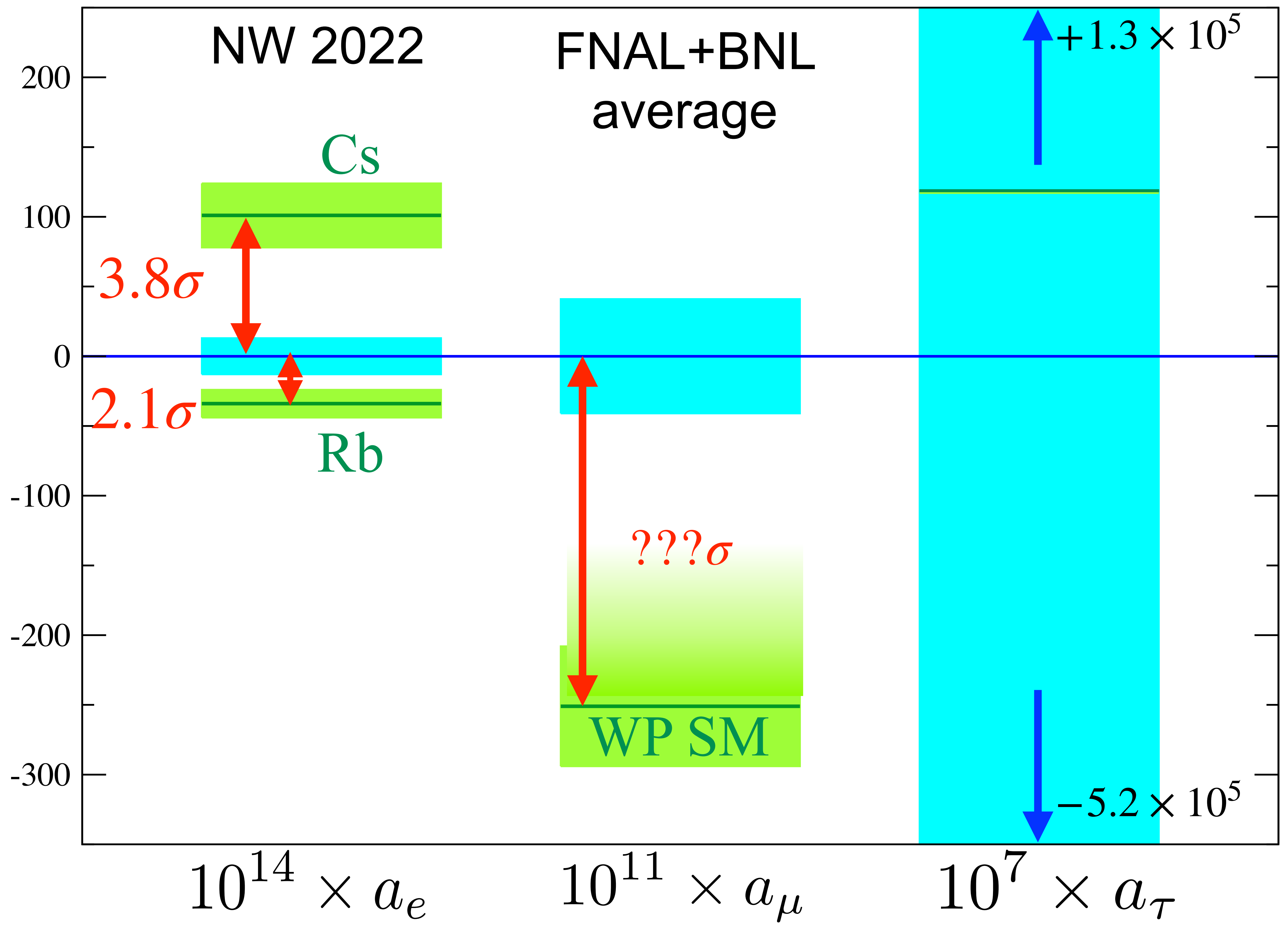
Beyond the SM possibilities

- a_μ is loop-induced, conserves CP & flavor, flips chirality.
- Generically expect: $a_\mu^{\rm NP} \sim a_\mu^{\rm EW} \times \frac{M_W^2}{\Lambda^2} \times \text{couplings}$
  - ➠ the difference between exp-WP20 is large: $\Delta a_\mu = 249\,(48) \times 10^{-11} \;>\; a_\mu(\rm EW)$
- Will likely be different if using (consolidated) lattice HVP average.
- Tensions between data-driven and lattice HVP results:

  - Can new physics hide in the low-energy $\sigma(e^+e^- \to \pi\pi)$ cross section?
    ➠ No [Luzio, et al, arXiv:2112.08312]
  - New boson at ~ 1GeV decays into $\mu^+\mu^-, e^+e^-$, affects $\sigma(e^+e^- \to \pi\pi)$ indirectly [L. Darmé et al, arXiv:2112.09139]
  - Neutral, long-lived hadrons, heretofore undetected? [Farrar, arXiv:2206.13460]
  - Z′ at < 1 GeV, coupling to 1st gen matter particles [Coyle, Wagner, arXiv:2305.02354]

Lepton moments summary

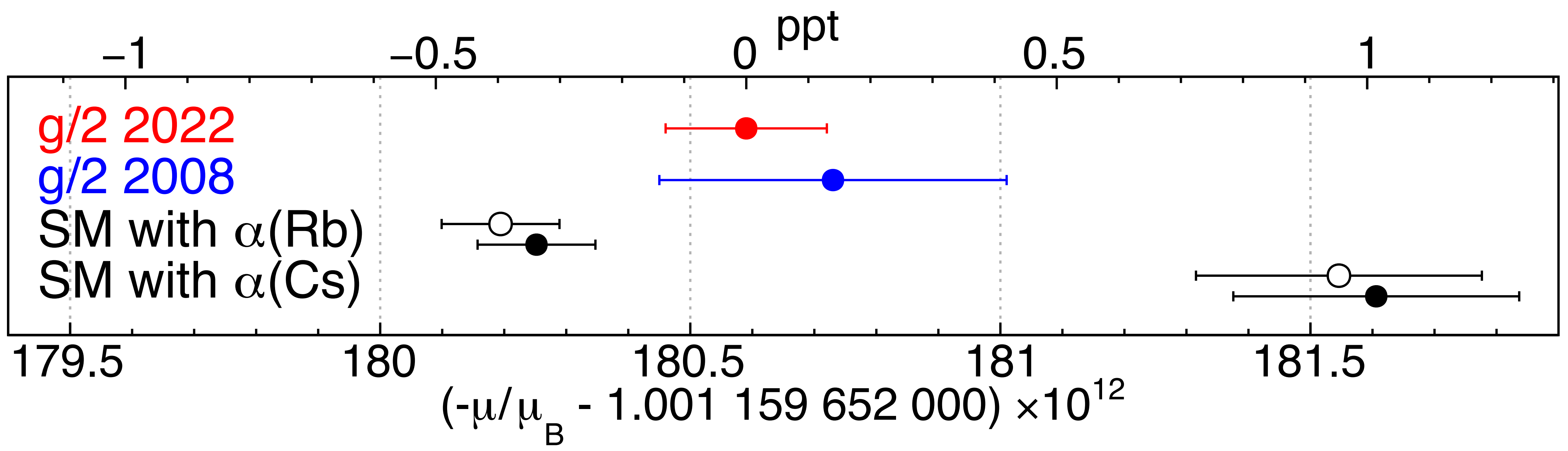
$$a_\ell^{\rm SM} - a_\ell^{\rm Exp}$$



Sensitivity to heavy new physics: $a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$

$(m_\mu/m_e)^2 \sim 4 \times 10^4$

[X. Fan et al (NW group), arXiv:2209.13084]



Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

Outlook

- Experimental program beyond 2025:
  - J-PARC: Muon g-2/EDM
  - CERN: MUonE
  - Fermilab: future muon campus experiments?
  - Belle II, BESIII, Novosibirsk,…
  - Chiral Belle (?)
- Data-driven/dispersive program beyond 2025:
  - development of NNLO MC generators
  - for HLbL, improved experimental/lattice inputs together with further development of dispersive approach
- MUonE will provide a space-like determination of HVP
- Lattice QCD beyond 2025:
  - access to future computational resources (coming Exascale) will enable improvements of all errors (statistical and systematic)
  - concurrent development of better methods and algorithms (gauge-field sampling, noise reduction) will accelerate progress
  - **beyond g-2: a rich program relevant for all areas of HEP**

Outlook

$m_{ud},\, m_u/m_d,\, m_s$ ⟸ $a_\mu^{\text{HVP LO}}$ a_μ^{HLbL} d_n MEs for light nuclei

$m_c,\, m_b$

$\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$ PDFs, GPDs, TMDs,.. $\langle NN | O_i | NN \rangle$

$\alpha_{\overline{MS}}(m_z)$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$ $\Lambda_b \to (p, \Lambda_c, \Lambda)\, \ell\nu$ $D_s \to \ell\nu\gamma$ $B \to \pi\pi\, \ell\nu\, \ldots$

$\hat{B}_K, \ldots$ nucleon form factors, .. $K^+ \to \ell^+\nu\,(\gamma)$ $B \to X_c \ell\nu$, other inclusive decay rates, …

$f_{B_{(s)}}, \ldots$ $g_A,\, g_T,\, g_S$ $\pi\pi \to \rho$ $\pi \to \ell^+\ell^-,\, K \to \gamma\gamma$

$f^{K\to\pi}(q^2)$ $f_{+,0,T}^{B\to D}(q^2), \ldots$ $\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $K^+ \to \pi^+\ell^+\ell^-$ …

$f_{K^\pm}$ $f_{+,0,T}^{B\to\pi}(q^2), \ldots$ $\langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $\Delta M_K,\, \epsilon_K$ $K^+ \to \pi^+\nu\bar{\nu}$

Complexity →

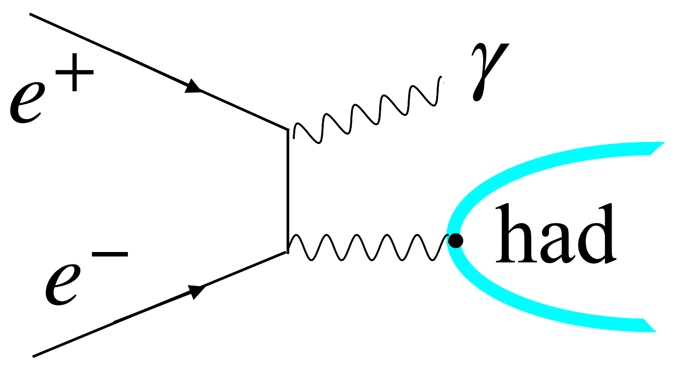
✓ LQCD flagship results	First complete LQCD results, large(ish) errors	First results, physical params, incomplete systematics	new methods, pilot projects, unphysical kinematics	new ideas, first studies


Thank you!
NVIDIA
NEC
THE UNIVERSITY of EDINBURGH
STORJ
Atos
UNIVERSITY of WASHINGTON
U.S. DEPARTMENT OF ENERGY
Office of Science
INSTITUTE for NUCLEAR THEORY
CENTRE FOR THEORETICAL PHYSICS Higgs
UK LFT
Fermilab
KEK
ILLINOIS
URA UNIVERSITIES RESEARCH ASSOCIATION
RIKEN BNL Research Center
Fermi Research Alliance LLC
epcc
HIM
HELMHOLTZ
Helmholtz-Institut Mainz
SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL
PRISMA
UNIVERSITÄT BERN
AEC
ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS
STRONG 2020
IP3
UCONN
DEPARTMENT OF PHYSICS
JGU
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Appendix

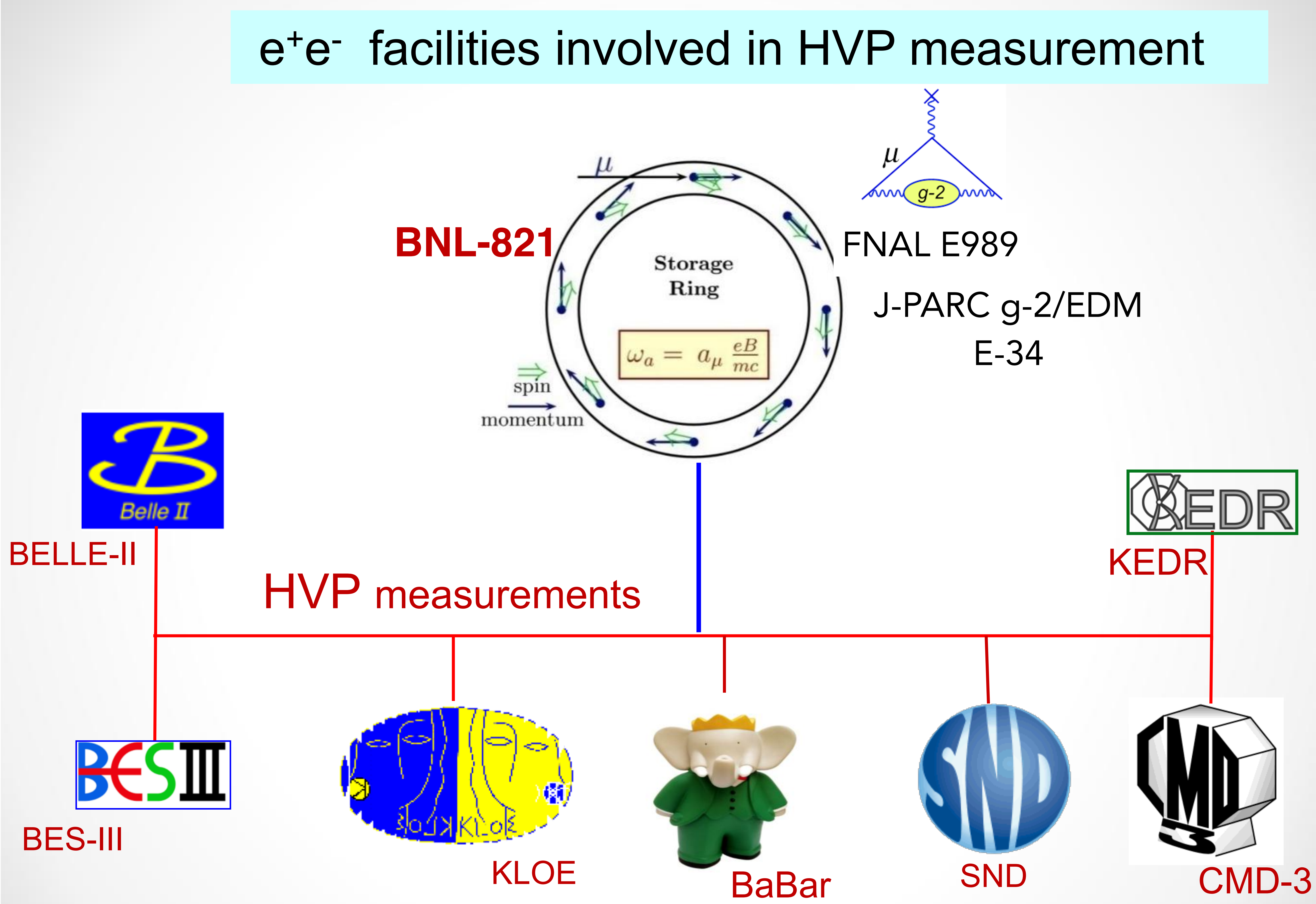
Experimental Inputs to HVP

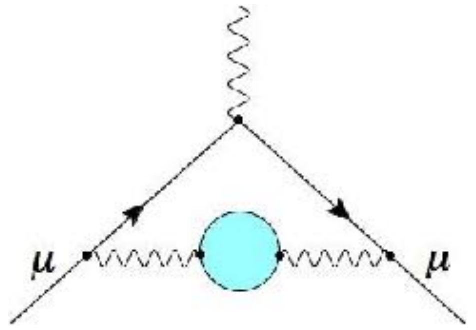
- two exp. approaches
  - ``Direct scan'': change CM energy of e^+e^- beams
  - ``Radiative Return'': with fixed e^+e^- CM energy, select events with initial state radiation (ISR)



- complemented by:
  - MC generators for $\sigma_{\rm had}(s)$ (e.g. PHOKARA)
  - detailed studies of radiative corrections (now known through NLO)

S. Serednyakov (for SND) @ HVP KEK workshop





HVP: data-driven

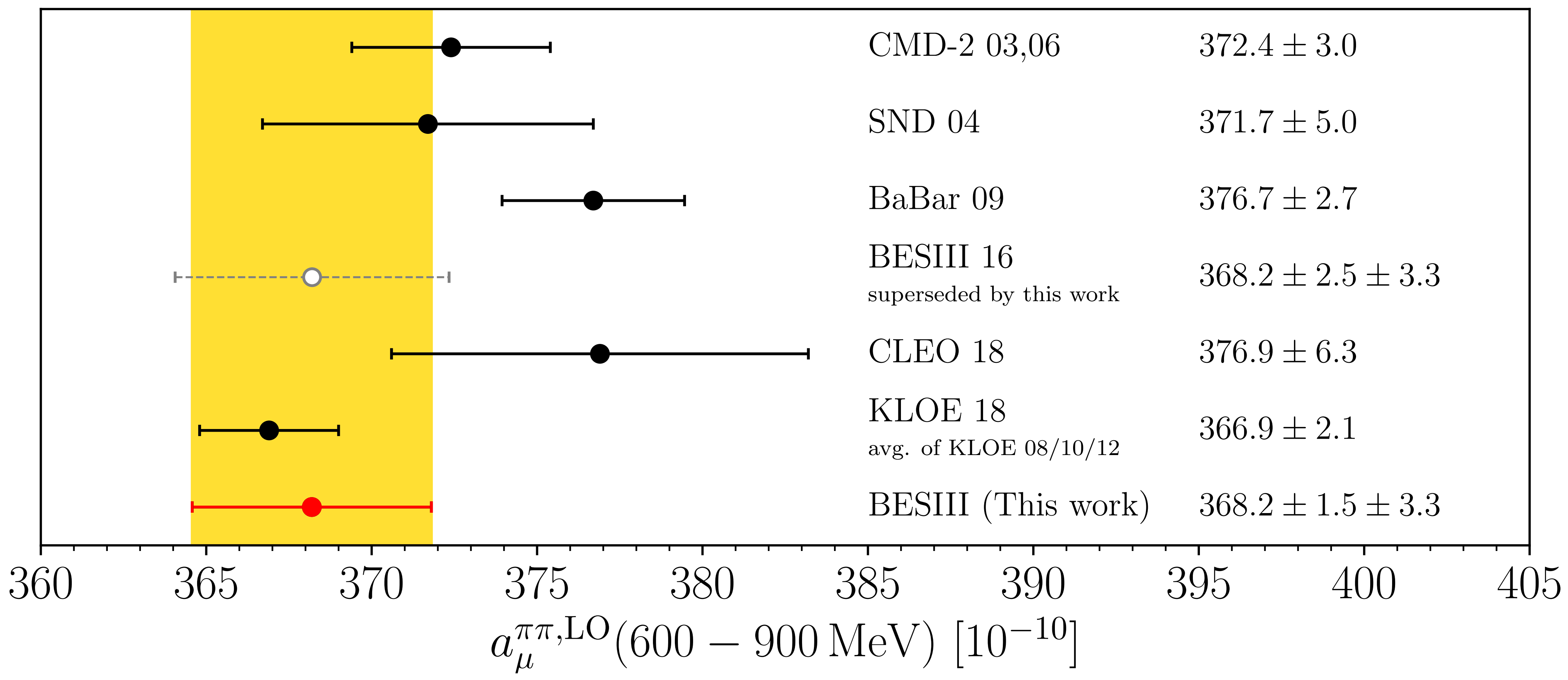
2020 White Paper [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]

Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs and treatment of correlations between systematic errors
- include results using constraints from unitarity & analyticity in $\pi\pi$ and $\pi\pi\pi$ channels
  [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019]

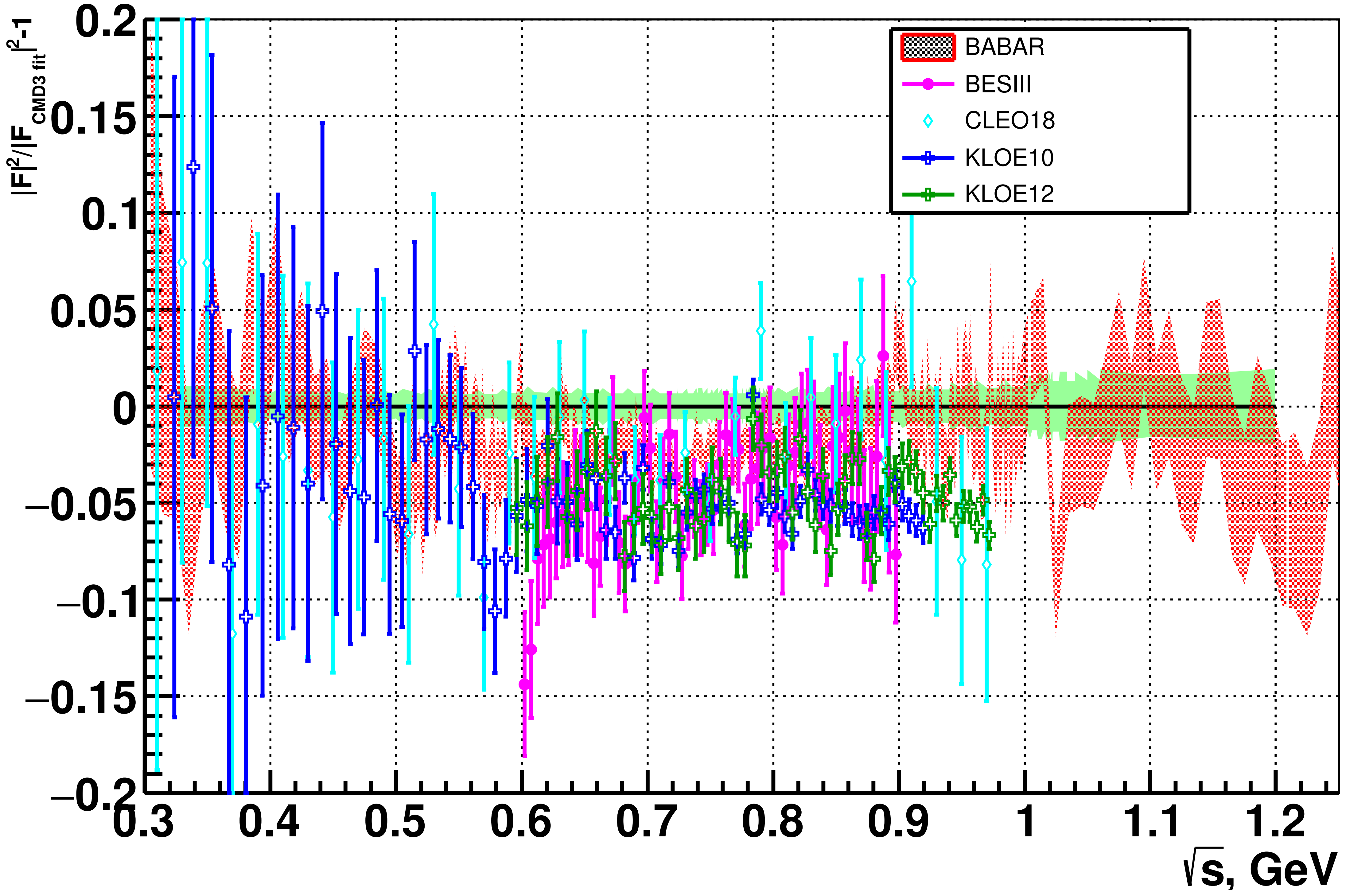
$$a_\mu^{\rm HVP,LO} = 693.1\,(2.8)_{\rm exp}\,(0.7)_{\rm DV+pQCD}\,(2.8)_{\rm BaBar-KLOE} \times 10^{-10}$$
$$= 693.1\,(4.0) \times 10^{-10}$$

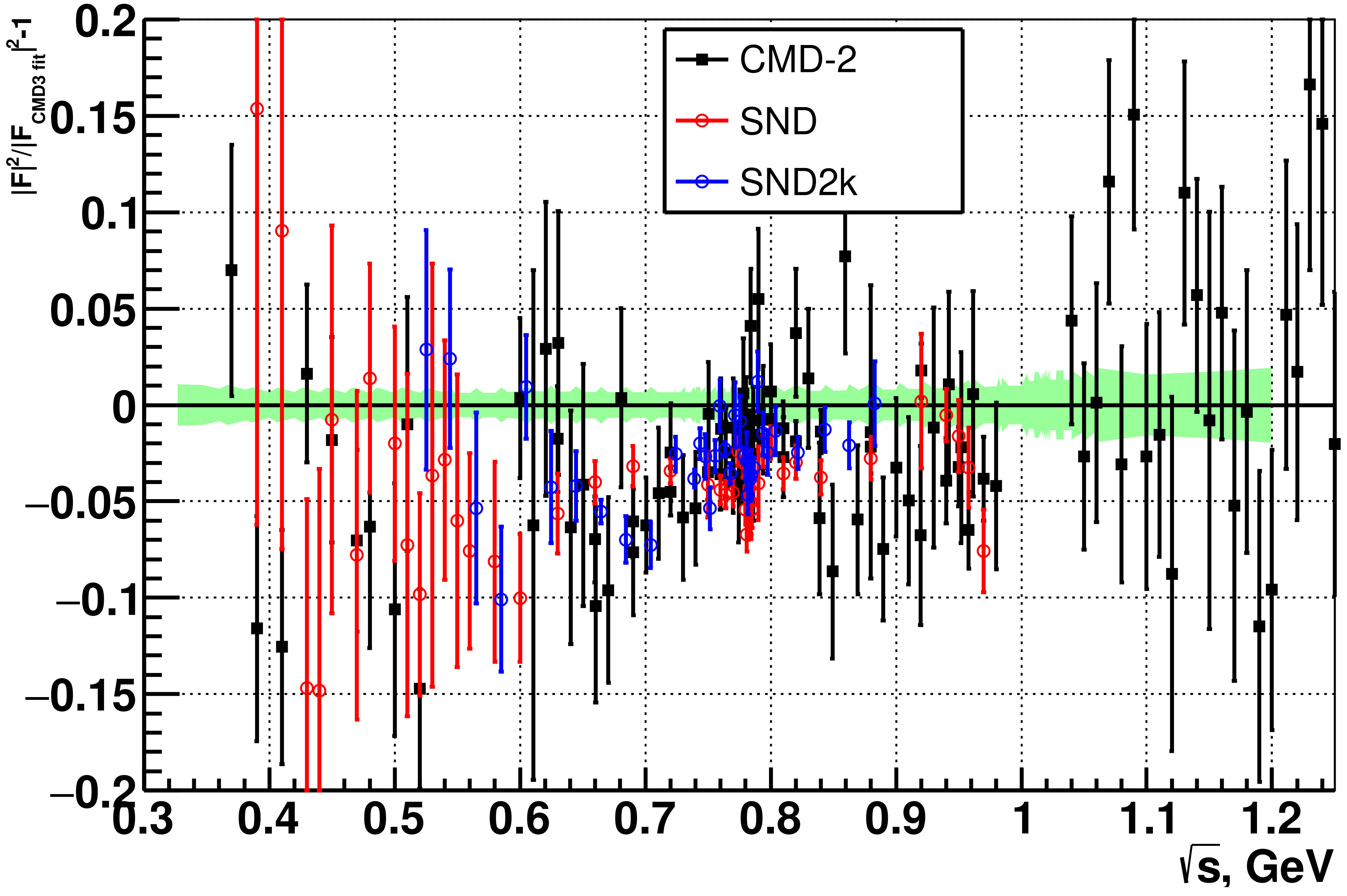
[M. Ablikim et al (BES III), arXiv:2009.05011]



cross section comparisons

[CMD-3, F. Ignatov et al, arXiv:2302.08834, PRD2024]





- For $\sqrt{s} \lesssim 0.6$ GeV: good consistency between cross section measurements
- For 0.6 GeV $\lesssim \sqrt{s} \lesssim 1$ GeV: significant differences between measurements

analyticity and unitarity constraints for $\sigma(e^+e^- \to \pi\pi)$

M. Hoferichter

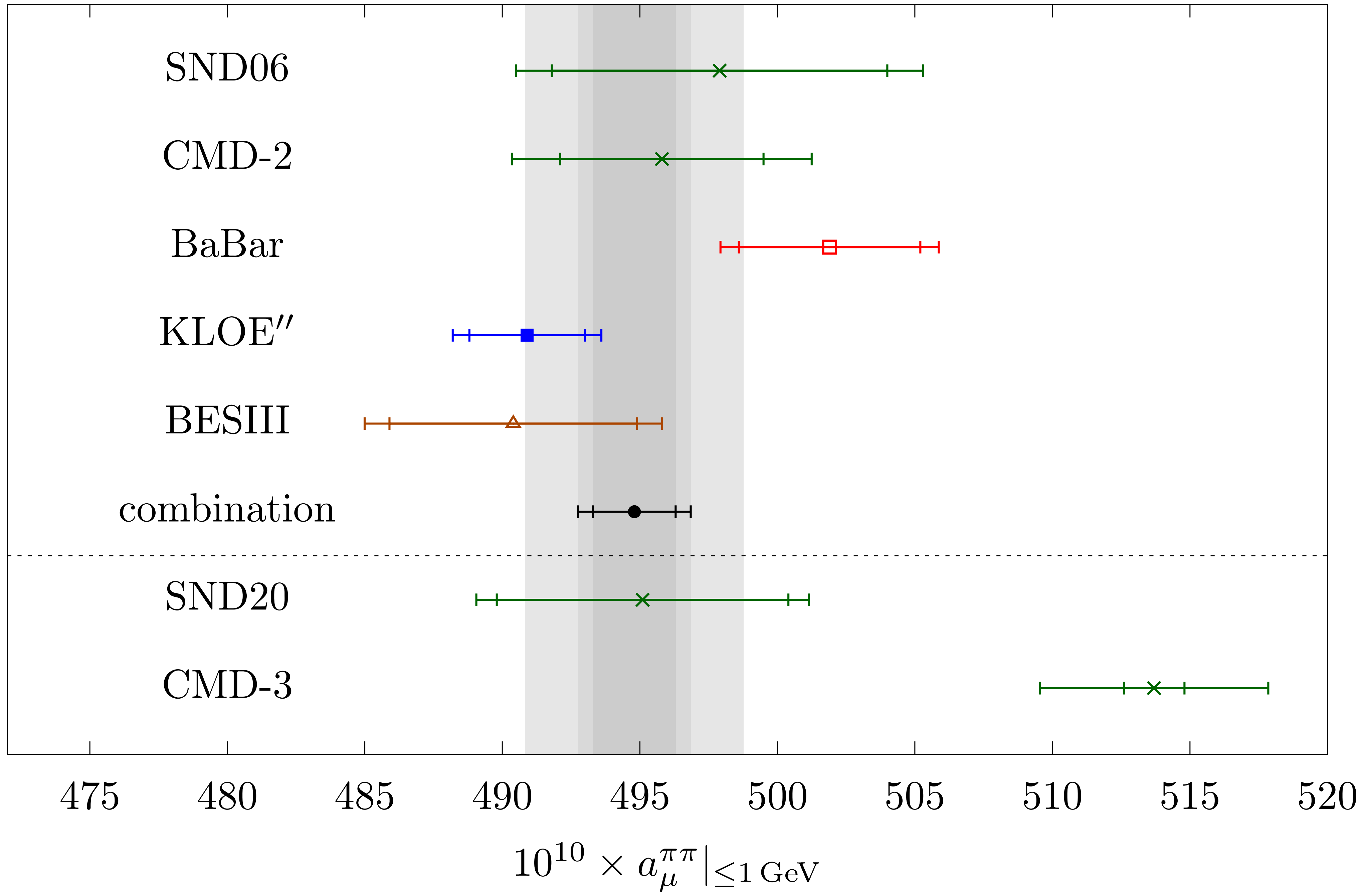
The pion form factor from dispersion relations

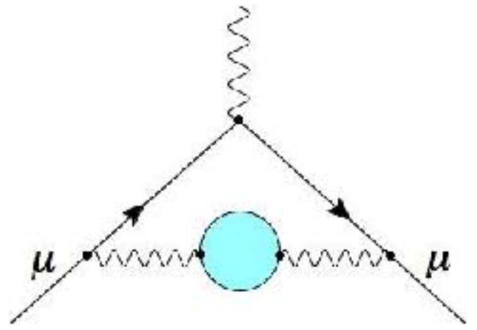
$$F_\pi^V(s) = \underbrace{\Omega_1^1(s)}_{\text{elastic } \pi\pi \text{ scattering}} \times \underbrace{G_\omega(s)}_{\text{isospin-breaking } 3\pi \text{ cut}} \times \underbrace{G_{\text{in}}(s)}_{\text{inelastic effects: } 4\pi, \ldots}$$

$$\Omega_1^1(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{s'(s'-s)}\right\} \qquad G_\omega(s) \simeq 1 + \frac{s\epsilon_\omega}{M_\omega^2 - s - iM_\omega\Gamma_\omega}$$

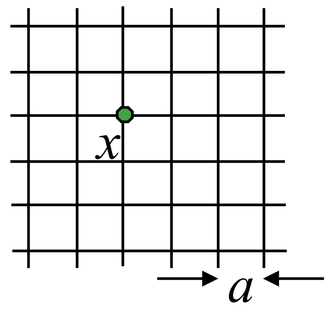
- $e^+e^- \to \pi^+\pi^-$ cross section subject to strong constraints from **analyticity**, **unitarity**, **crossing symmetry**, leading to dispersive representation with few parameters Colangelo, MH, Stoffer, 2018, 2021, 2022, work in progress
  - **Elastic $\pi\pi$ scattering**: two values of phase shifts
  - **ρ–ω mixing**: ω pole parameters and residue
  - **Inelastic states**: conformal polynomial

$\hookrightarrow$ correlations with $\pi\pi$ phase shifts, pion charge radius, ...





Lattice HVP: Introduction



Leading order HVP contribution: $$a_\mu^{\rm HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972; E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]

- Calculate $a_\mu^{\rm HVP,LO}$ in Lattice QCD

  Start with correlation function of EM currents: $C(t) = \frac{1}{3}\sum_{i,x}\langle j_i^{\rm EM}(x,t) j_i^{\rm EM}(0,0)\rangle$ $\quad j_\mu^{\rm EM} = \sum_f q_f\, \bar\psi_f(x,t)\,\gamma_\mu\,\psi_f(x,t)$ $\quad f = u, d, s, c, \ldots$

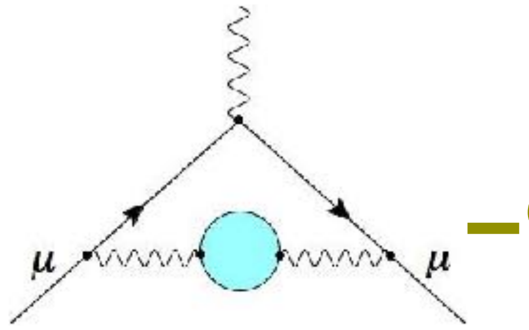
  Fourier transform yields $\hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t) \left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)\right]$

  [D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

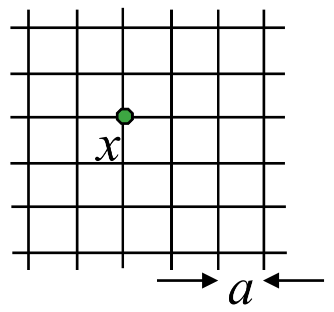
  so that $a_\mu^{\rm HVP,LO}$ can be obtained as an integral over Euclidean time, aka time momentum representation (TMR):

$$a_\mu^{\rm HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, w(Q^2)\, \hat{\Pi}(Q^2) = 4\,\alpha^2 \int_0^\infty dt\, C(t) \int_0^\infty dQ^2 w(Q^2) \left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)\right]$$

$$\Rightarrow \quad a_\mu^{\rm HVP,LO} = 4\,\alpha^2 \int_0^\infty dt\, C(t)\, \tilde{w}(t)$$

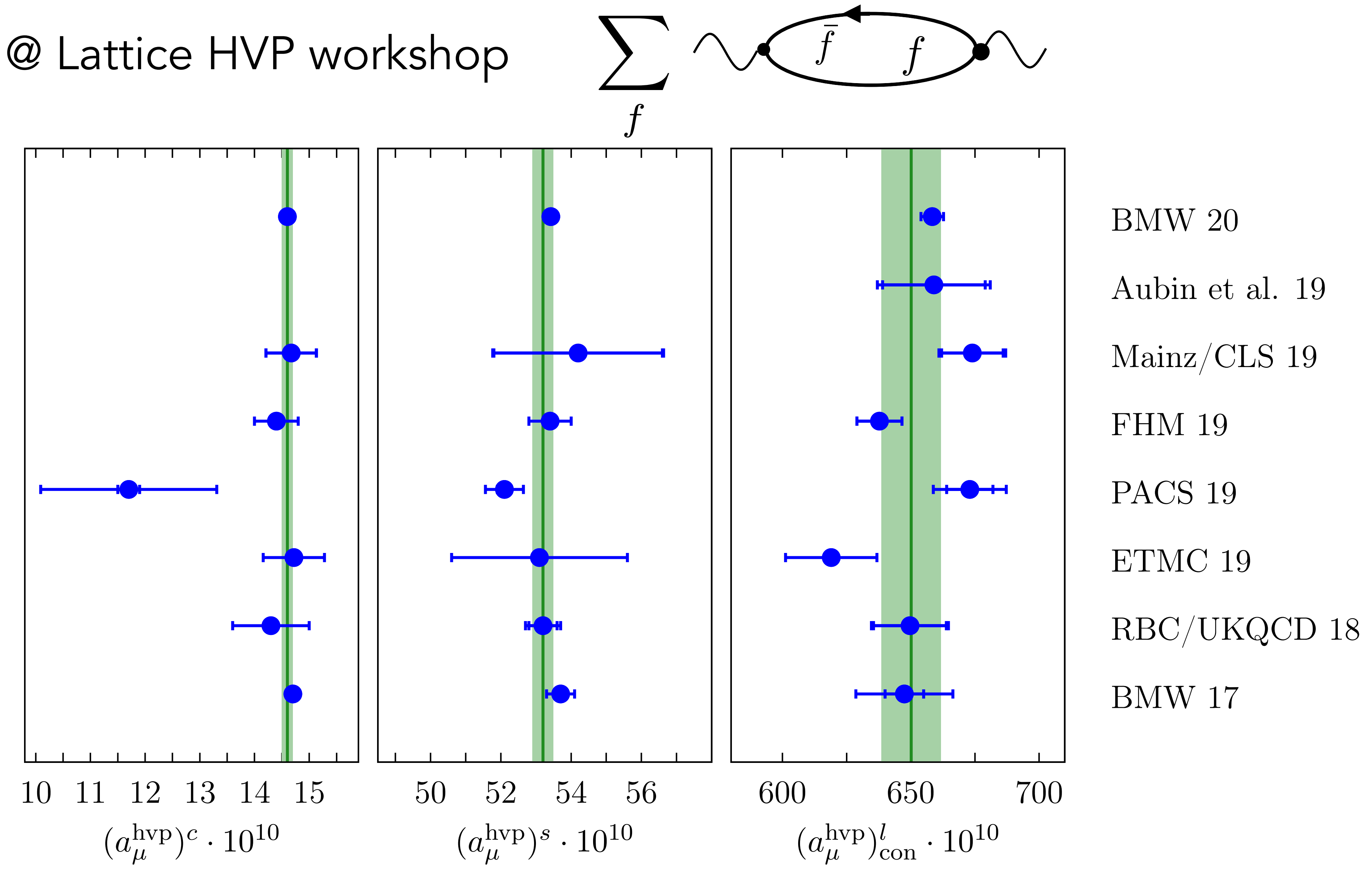


Lattice HVP: summary of contributions

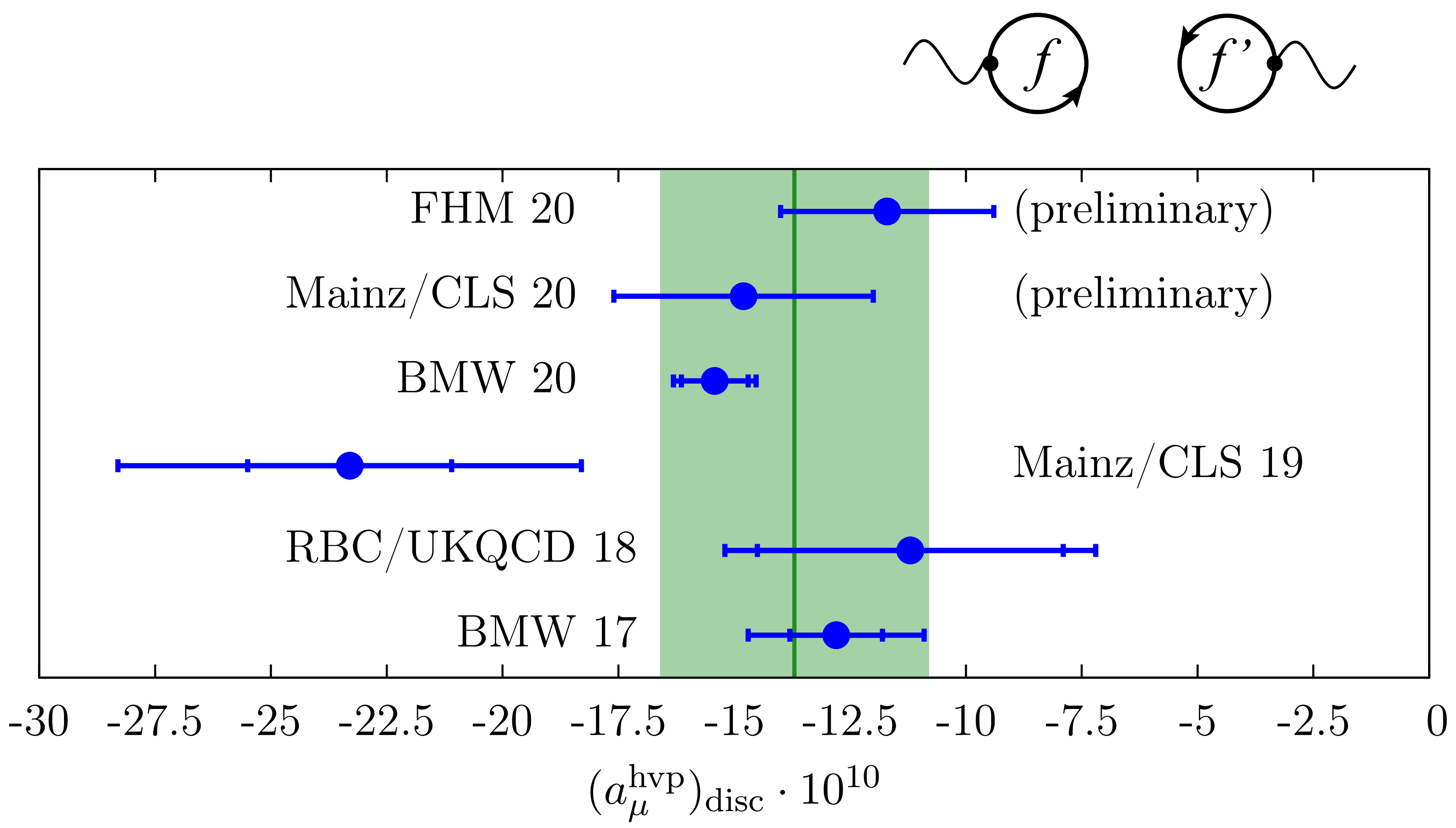


H. Wittig @ Lattice HVP workshop

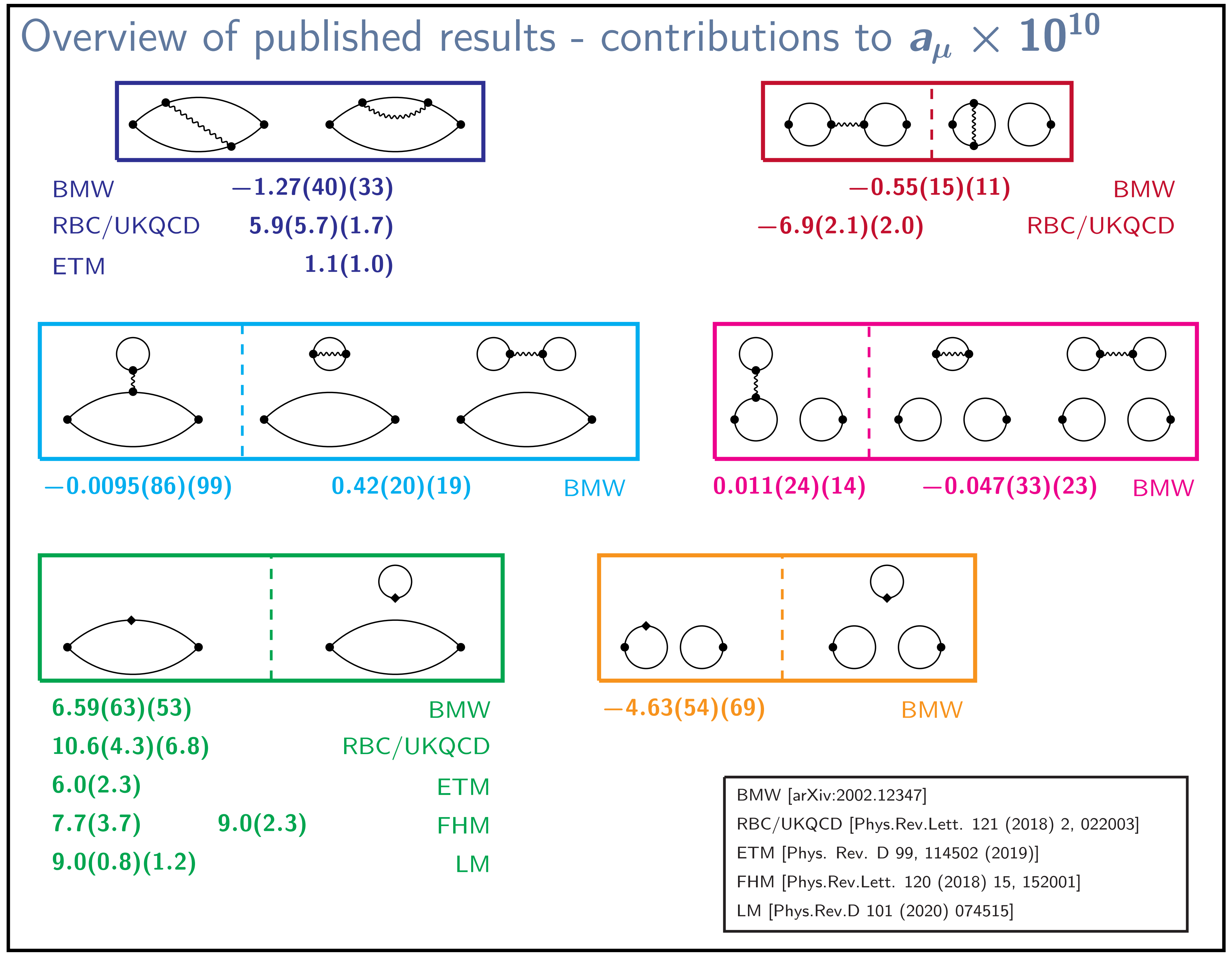
- Charm, strange contributions already well determined.
- Mild tensions for light contribution



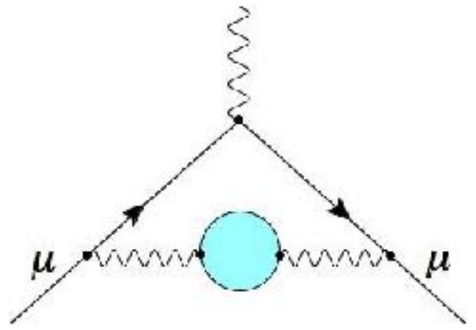
Consistent results with increasing precision



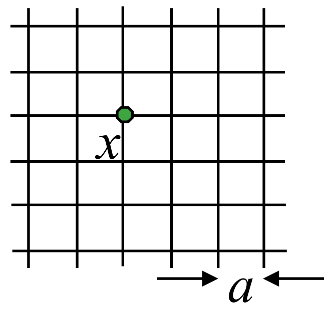
V. Gülpers @ Lattice HVP workshop



- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions: $\delta a_\mu^{\rm IB} \lesssim 1\,\%$



Lattice HVP: long-distance tail



$$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i^{\rm EM}(x,t)\, j_i^{\rm EM}(0,0)\rangle$$

- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$
  - ✦ bounding method:

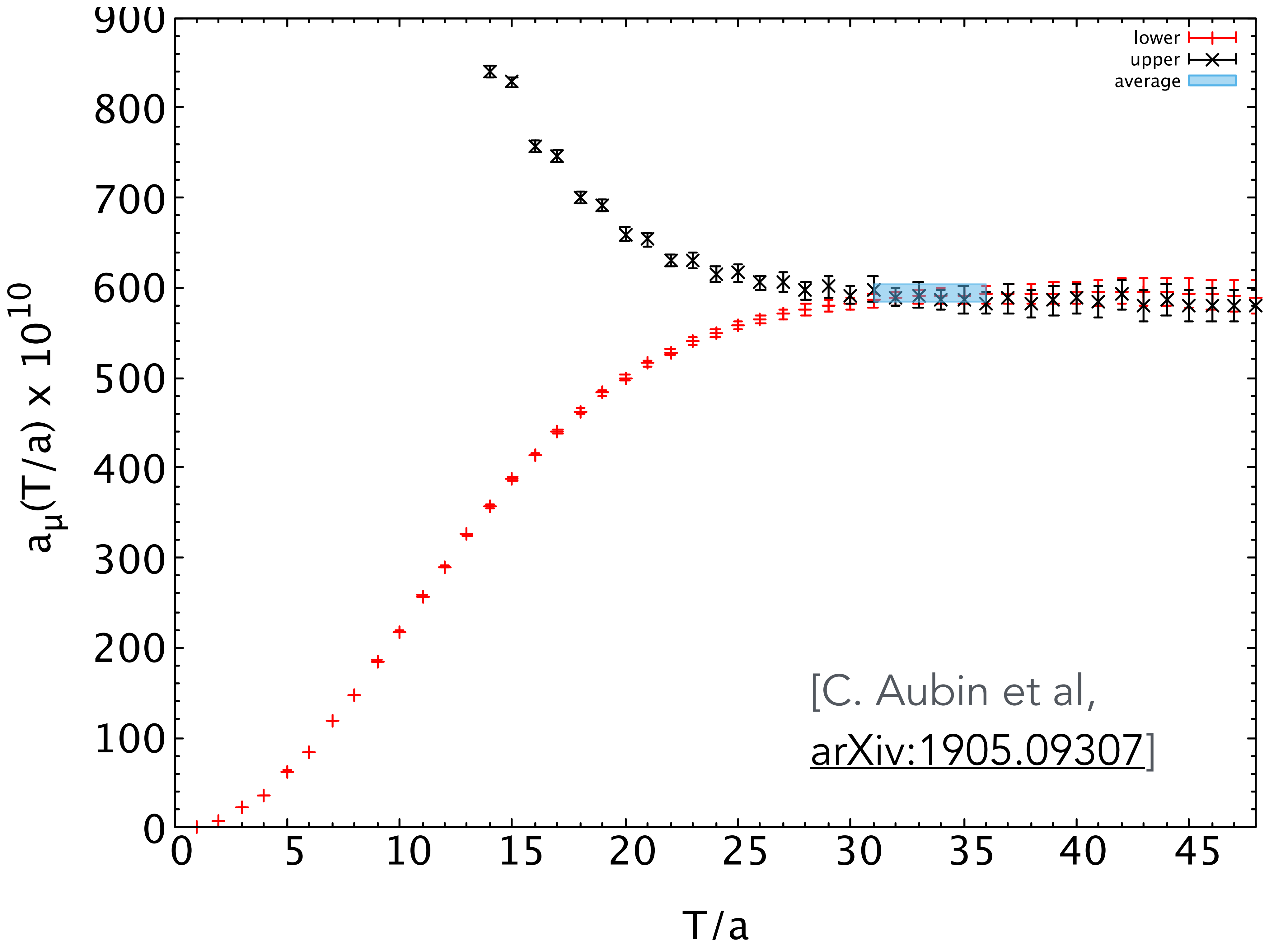
    [Borsanyi et al, PRL 2018, Blum et al, PRL 2018]

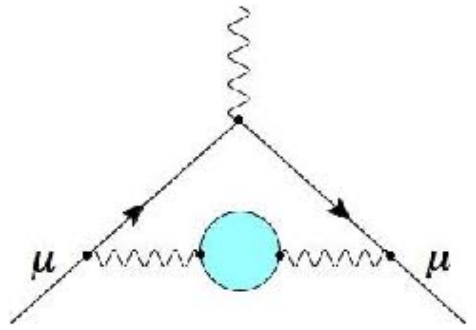
    for $t > t_c$: $0 \le C(t_c)\, e^{-\bar{E}_{t_c}(t-t_c)} \le C(t) \le C(t_c)\, e^{-E_0(t-t_c)}$

    $\bar{E}_{t_c}$: effective mass of $C(t)$ at t_c

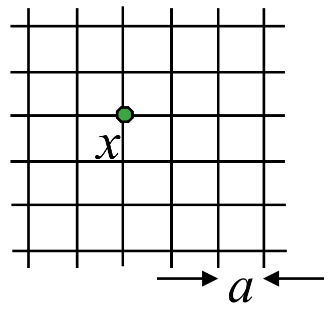
    E_0: ground state energy

    replace $G(t > t_c)$ with upper and lower bound, vary t_c





Lattice HVP: long-distance tail (again)

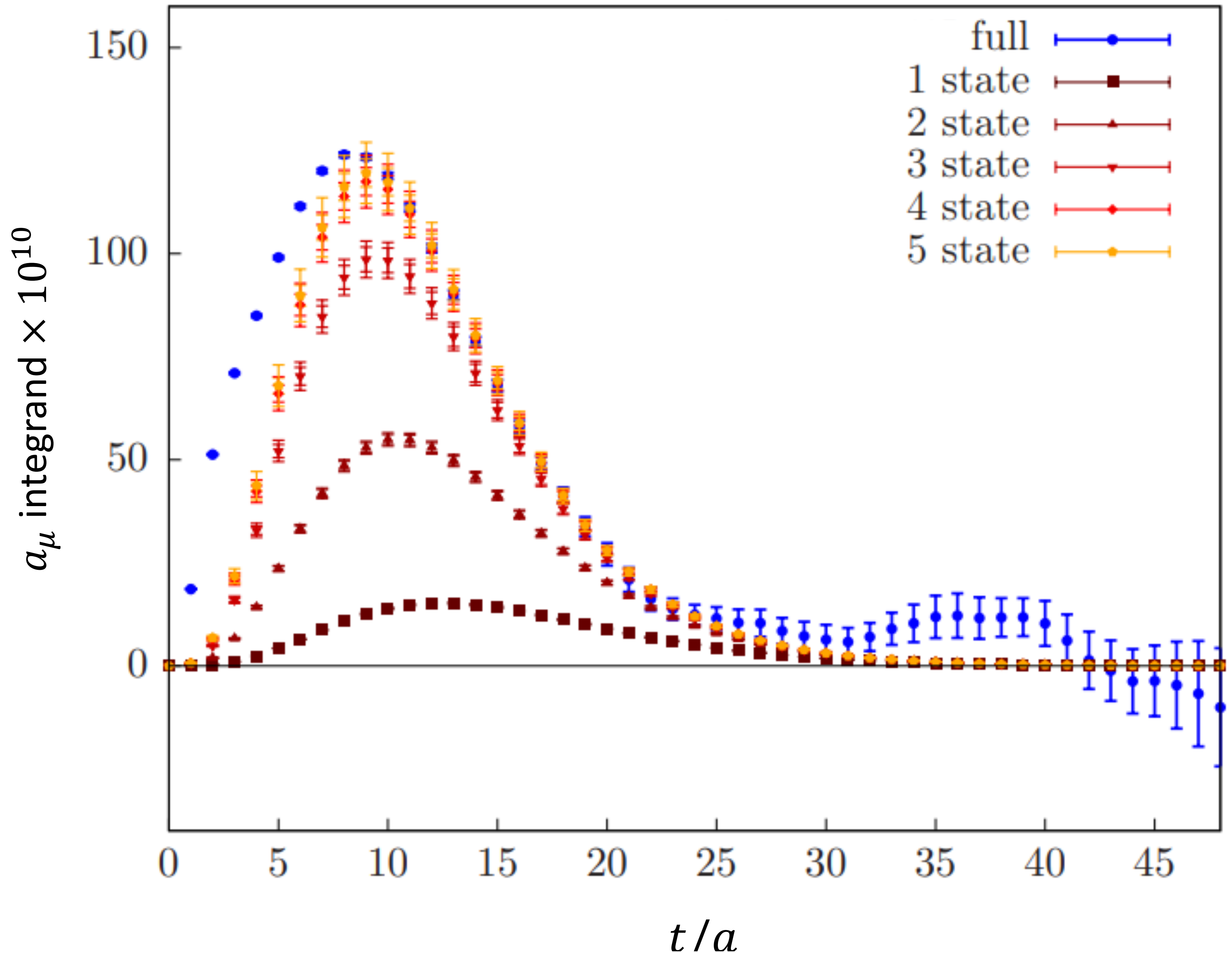


$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\mathrm{EM}}(x,t)\, j_i^{\mathrm{EM}}(0,0) \rangle$$

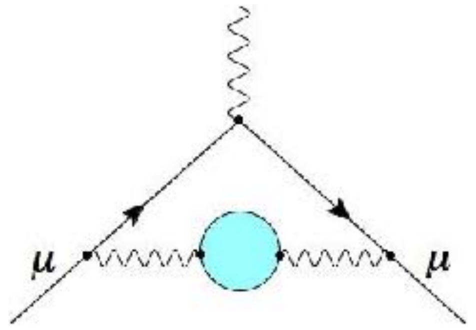
- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $C(t > t_c)$
  - ✦ can be used to improve bounding method:

    $$C(t) \rightarrow C(t) - \sum_{n=0}^{N} A_n^2 e^{-E_n t}$$

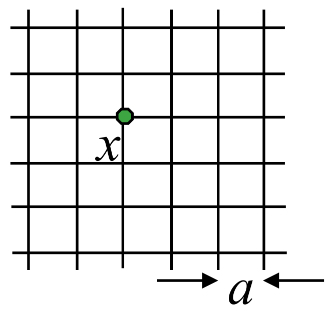
    use E_{N+1} in upper bound
  - ✦ yields big reduction in stat. errors (compared with bounding method)



J. Mckeon @ Lattice 2024 (RBC/UKQCD)
See also:
[Bruno et al, RBC/UKQCD, arXiv:1910.11745]



Lattice HVP: long-distance tail

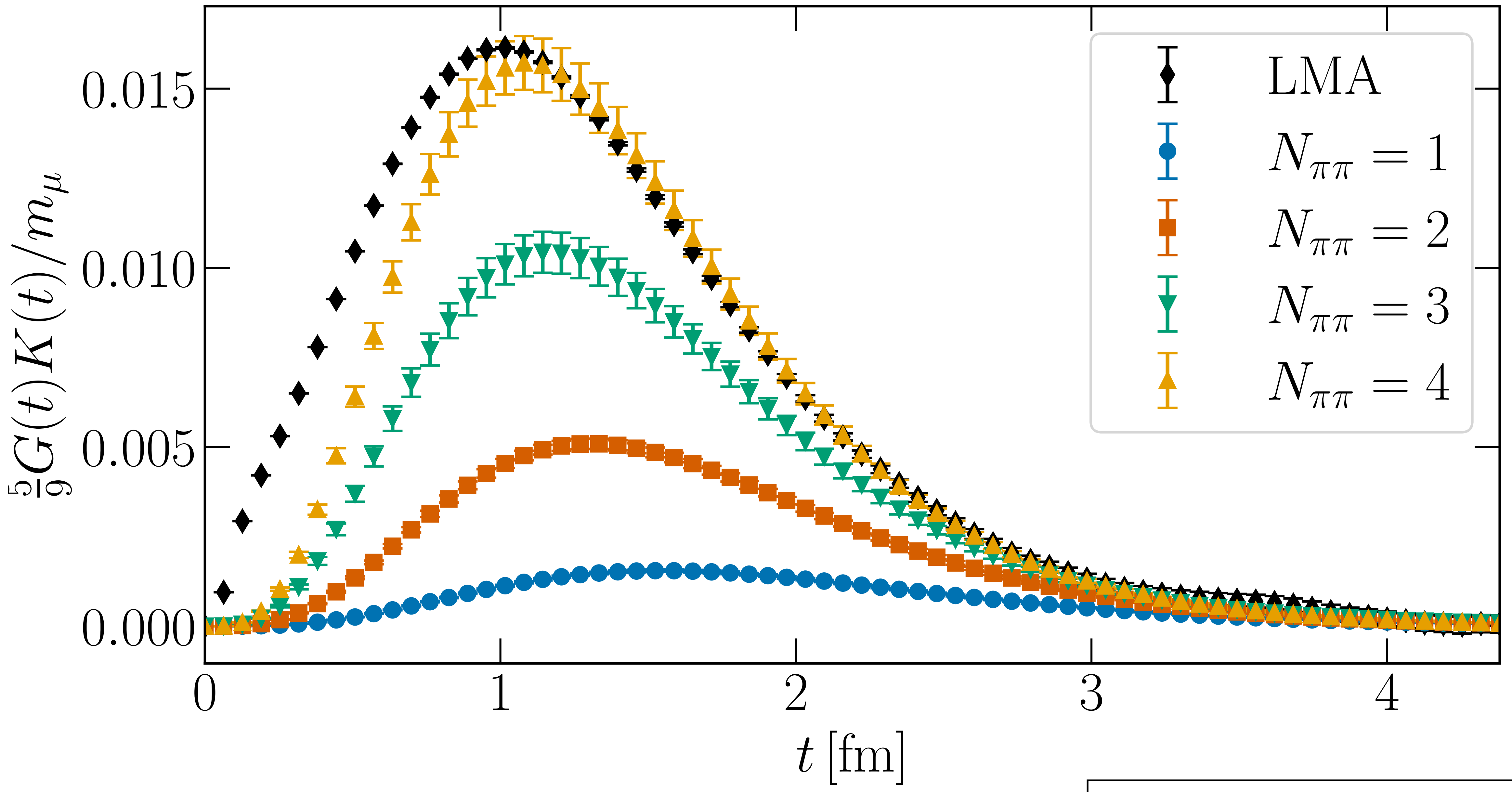


$$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i^{\rm EM}(x,t)\, j_i^{\rm EM}(0,0)\rangle$$

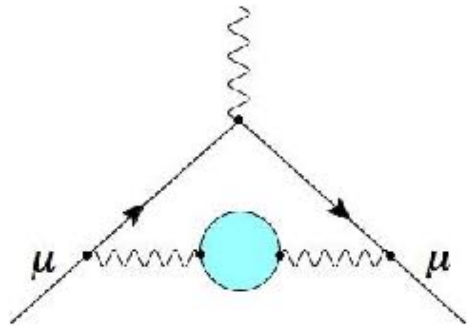
- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $C(t > t_c)$
  - ✦ can be used to improve bounding method:

    $$C(t) \to C(t) - \sum_{n=0}^{N} A_n^2\, e^{-E_n t}$$

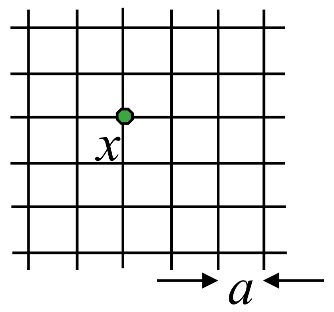
    use E_{N+1} in upper bound
  - ✦ yields $\sim \times 2$ reduction in stat. errors



S. Kuberski & N. Miller @ Lattice 2024 (Mainz)
See also: A. Gerardin et al, PRD 2019



Lattice HVP: long-distance tail

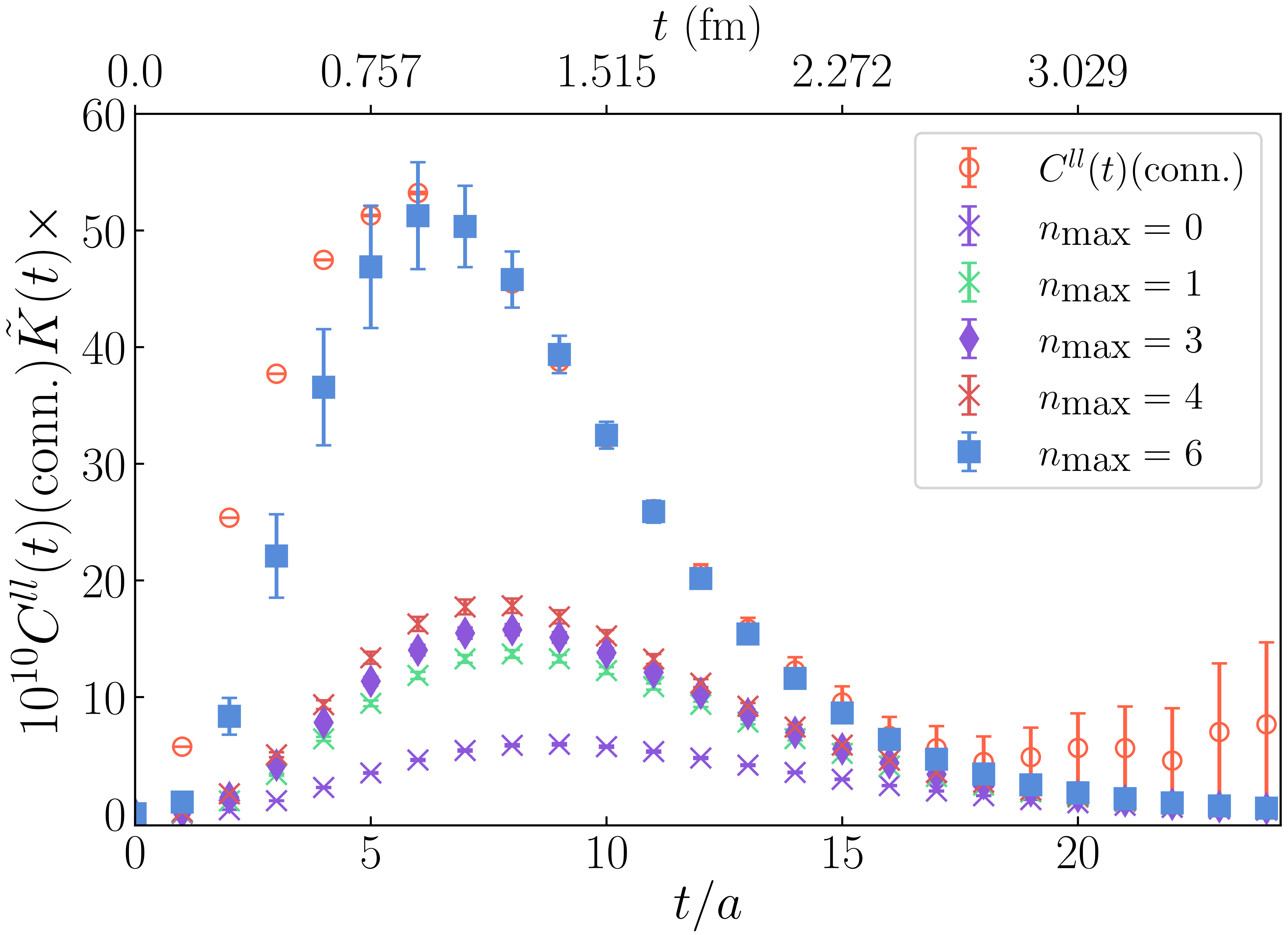


$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\mathrm{EM}}(x,t)\, j_i^{\mathrm{EM}}(0,0) \rangle$$

- Start with spectral decomposition: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $C(t > t_c)$
  - ✦ can be used to improve bounding method:

    $$C(t) \to C(t) - \sum_{n=0}^{N} A_n^2 e^{-E_n t}$$

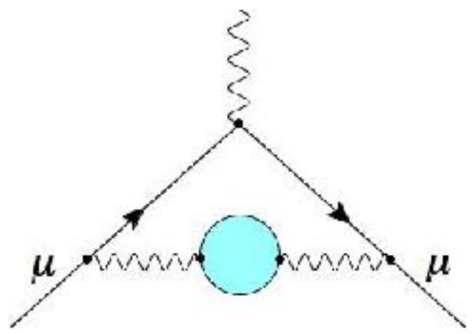
    use E_{N+1} in upper bound
  - ✦ yields $\sim \times 2.5$ reduction in stat. errors (compared with bounding method)



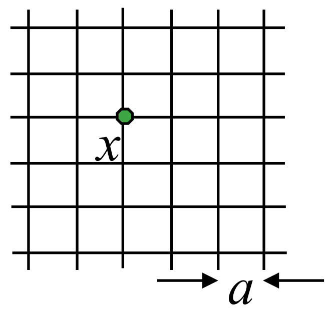
[Lahert, et al, arXiv:2409.00756]
see also:
[Lahert et al, arXiv:2112.11647]

First LQCD calculation with staggered multi-pion operators

see also:
[Frech for BMW @ Lattice 2024]



Long-distance tail (ud)

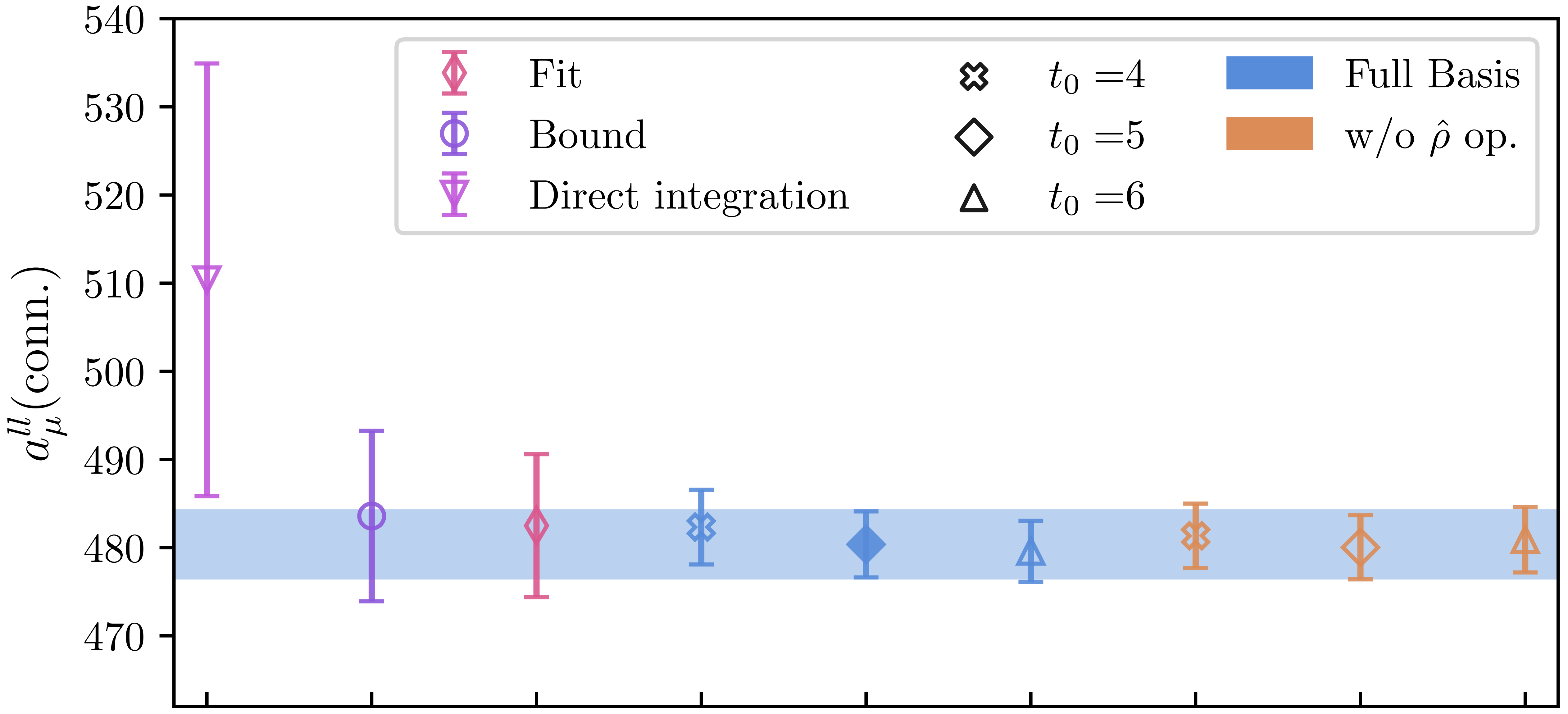


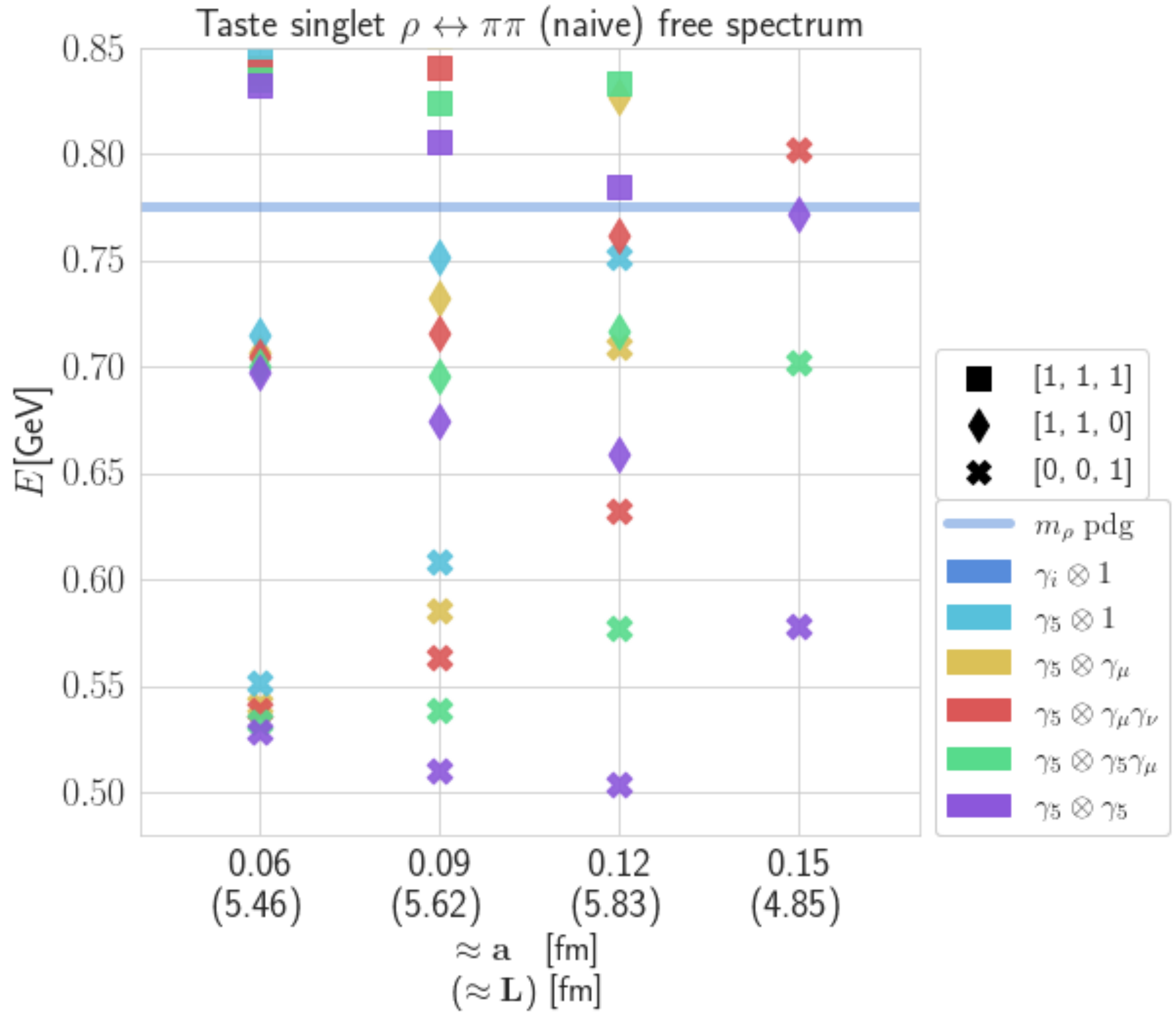
[Shaun Lahert et al, arXiv: 2409.00756]

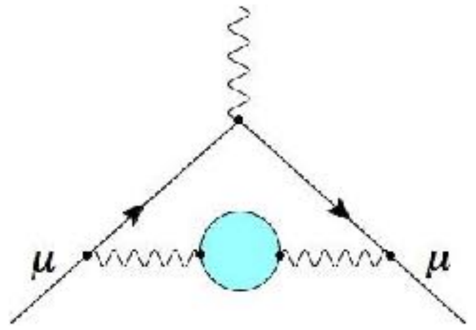
$$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i^{\rm EM}(x,t)\, j_i^{\rm EM}(0,0)\rangle$$

- Spectral reconstruction: $C(t) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t}$
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ First LQCD calculation with staggered multi-pion operators
  - ✦ Construct matrix of correlators (2,3,4-point functions)
  - ✦ Use GEVP to obtain energies and amplitudes for $\pi\pi$ states
  - ✦ Reconstruct vector-current correlator

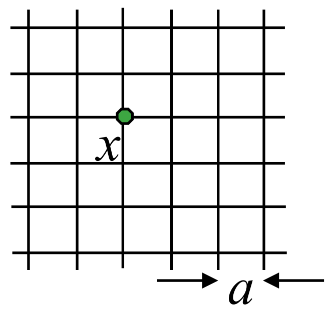
$$\mathbf{C}(t) = \begin{pmatrix} C(t)_{J,\tilde{J}\to J,\tilde{J}} & C(t)_{J,\tilde{J}\to\pi\pi} \\ C(t)_{\pi\pi\to J,\tilde{J}} & C(t)_{\pi\pi\to\pi\pi} \end{pmatrix}$$







Lattice HVP: results

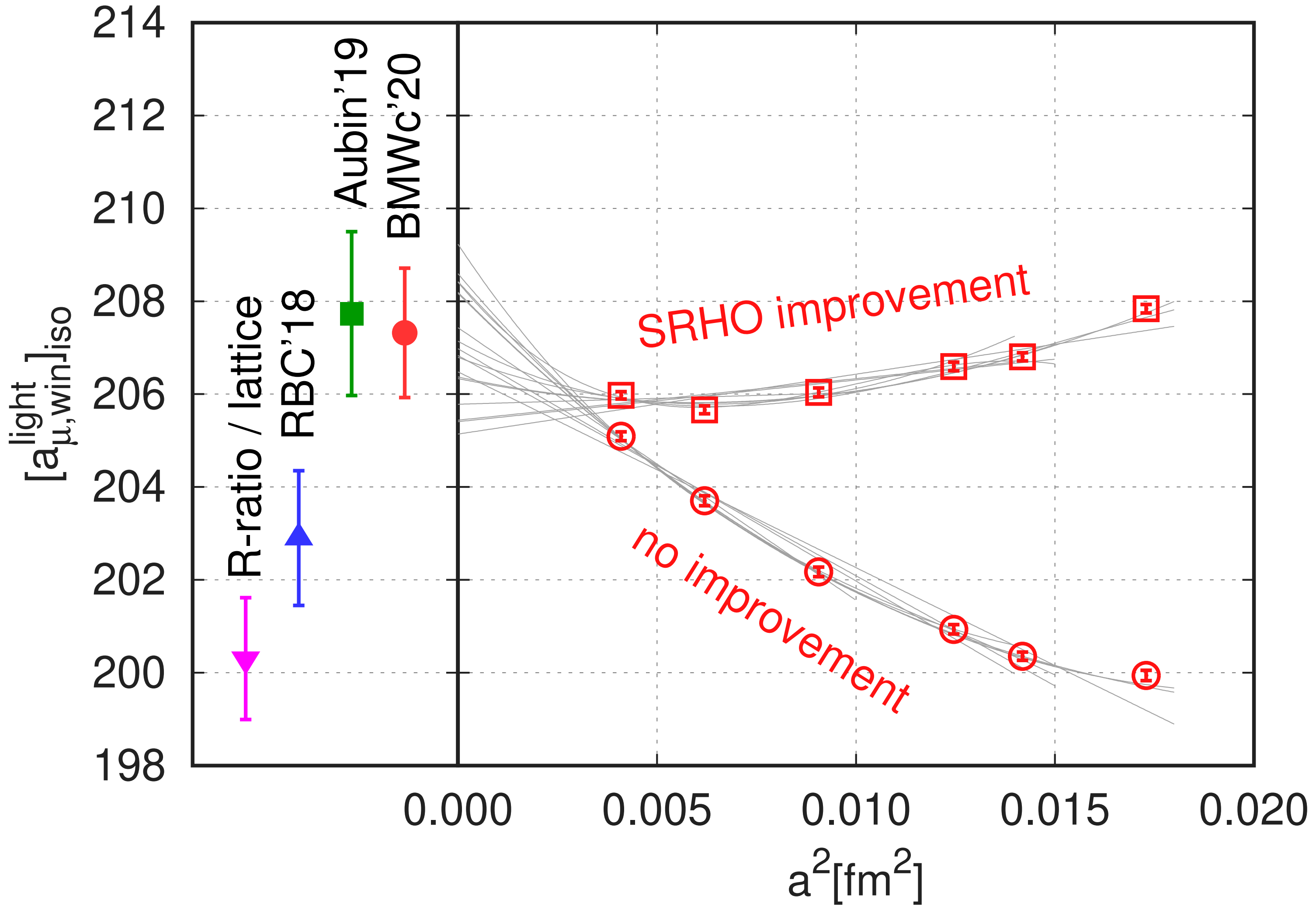


In 2020 WP:

- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\rm HVP,LO} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 [Sz. Borsanyi et al, arXiv:2002.12347, 2021 Nature] first LQCD calculation with sub-percent $(0.8\,\%)$ error in tension with data-driven HVP (2.1σ)
- Further tensions for intermediate window:

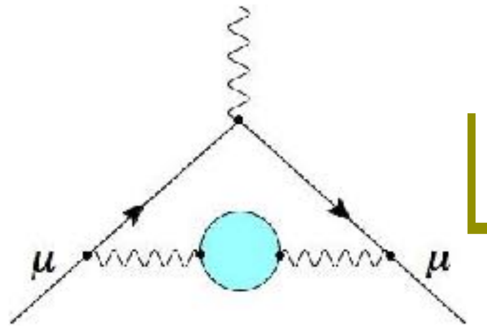
  -3.7σ tension with data-driven evaluation

  -2.2σ tension with RBC/UKQCD18

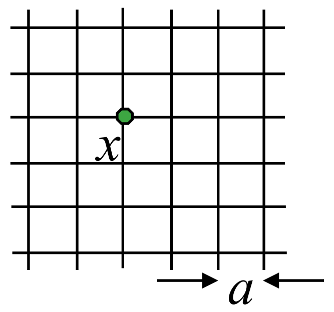


Staggered fermions:

- taste-breaking effects (which yield taste splittings) are significant (sometimes dominant) source of discretization errors
- possible to use EFT schemes (ChPT, Chiral Model, MLLGS) to correct for taste-splitting effects before taking continuum extrapolation: continuum limit should not be affected



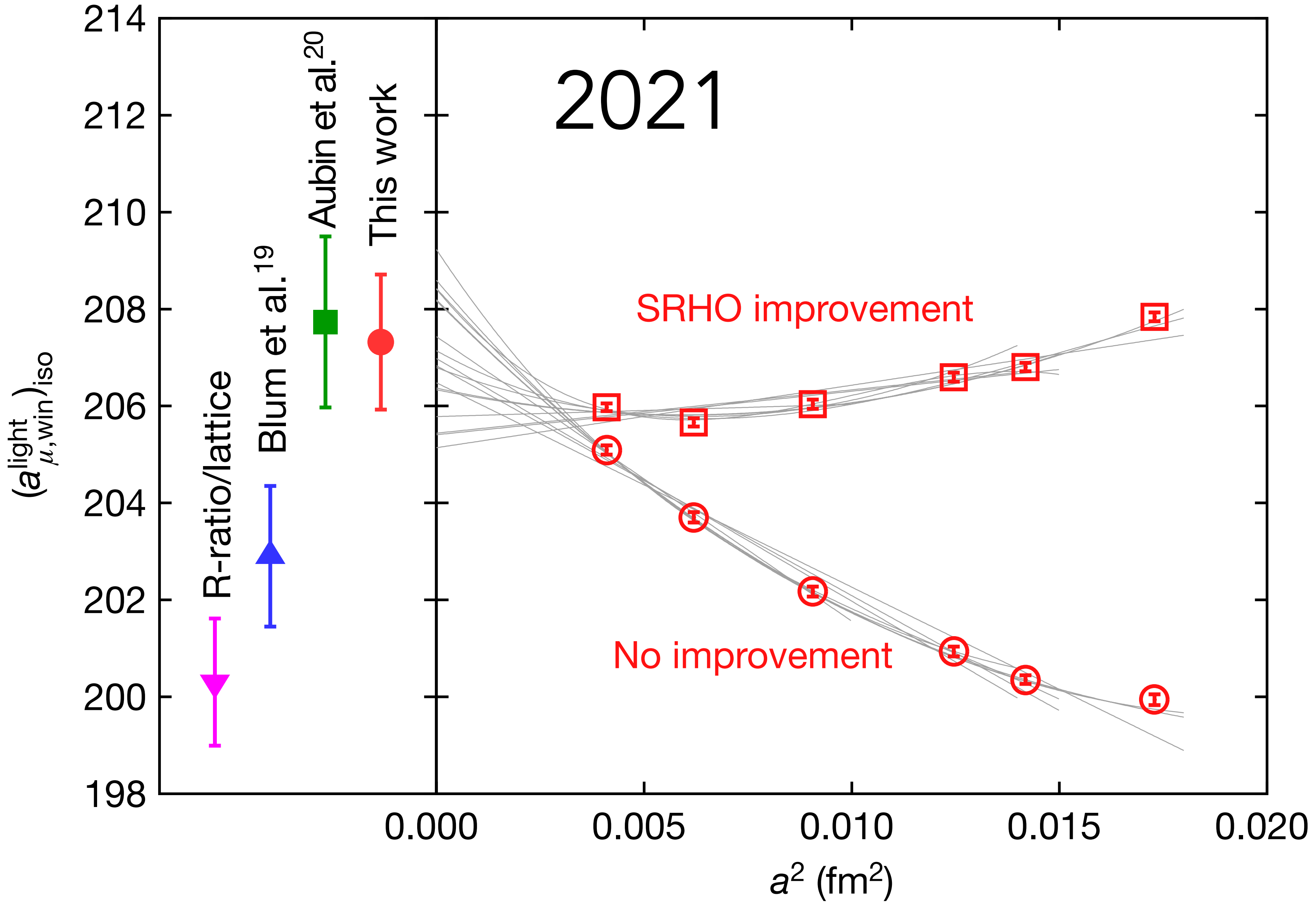
Lattice HVP: intermediate window (W)



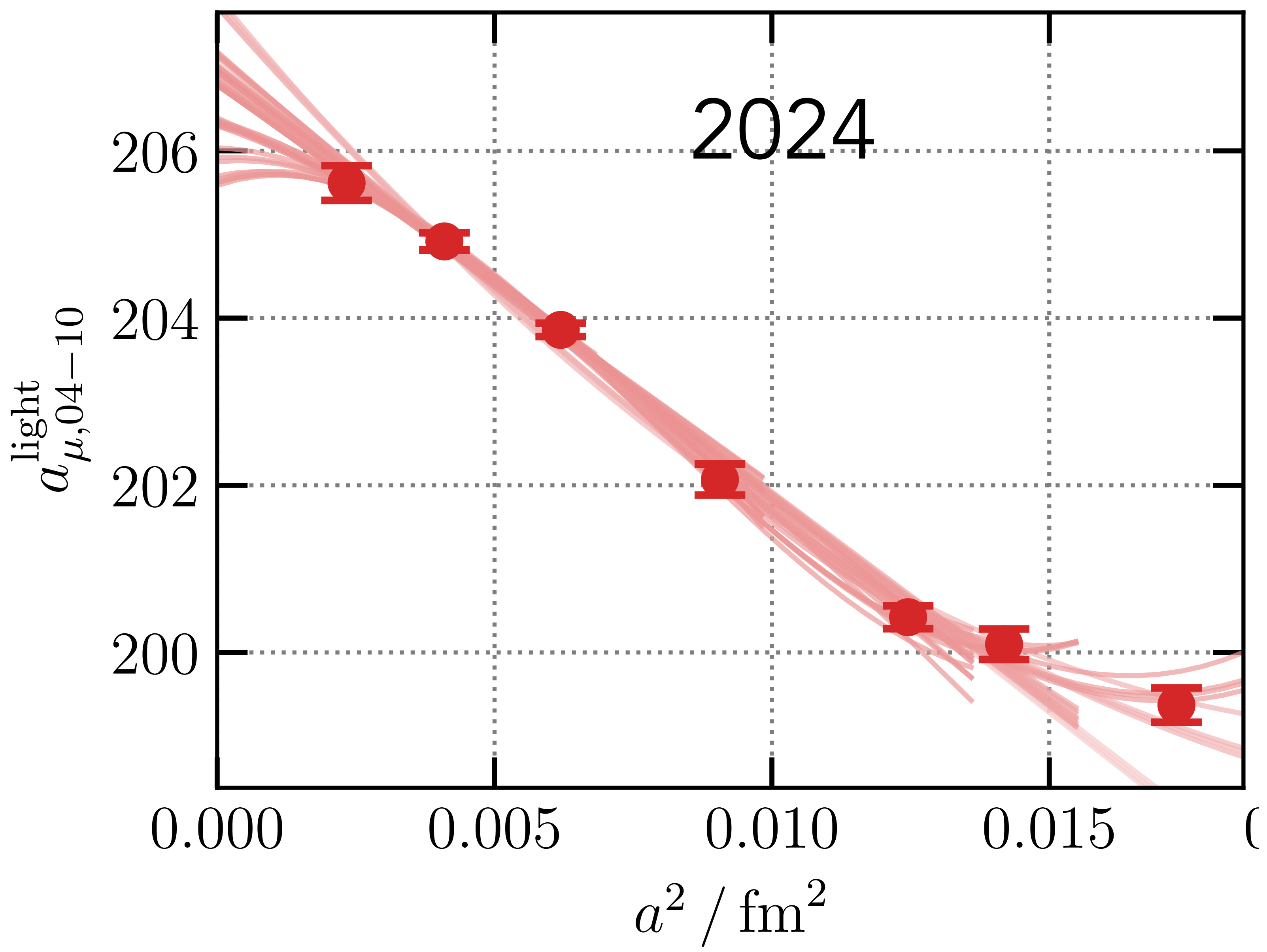
update: BMW+DMZ 2024

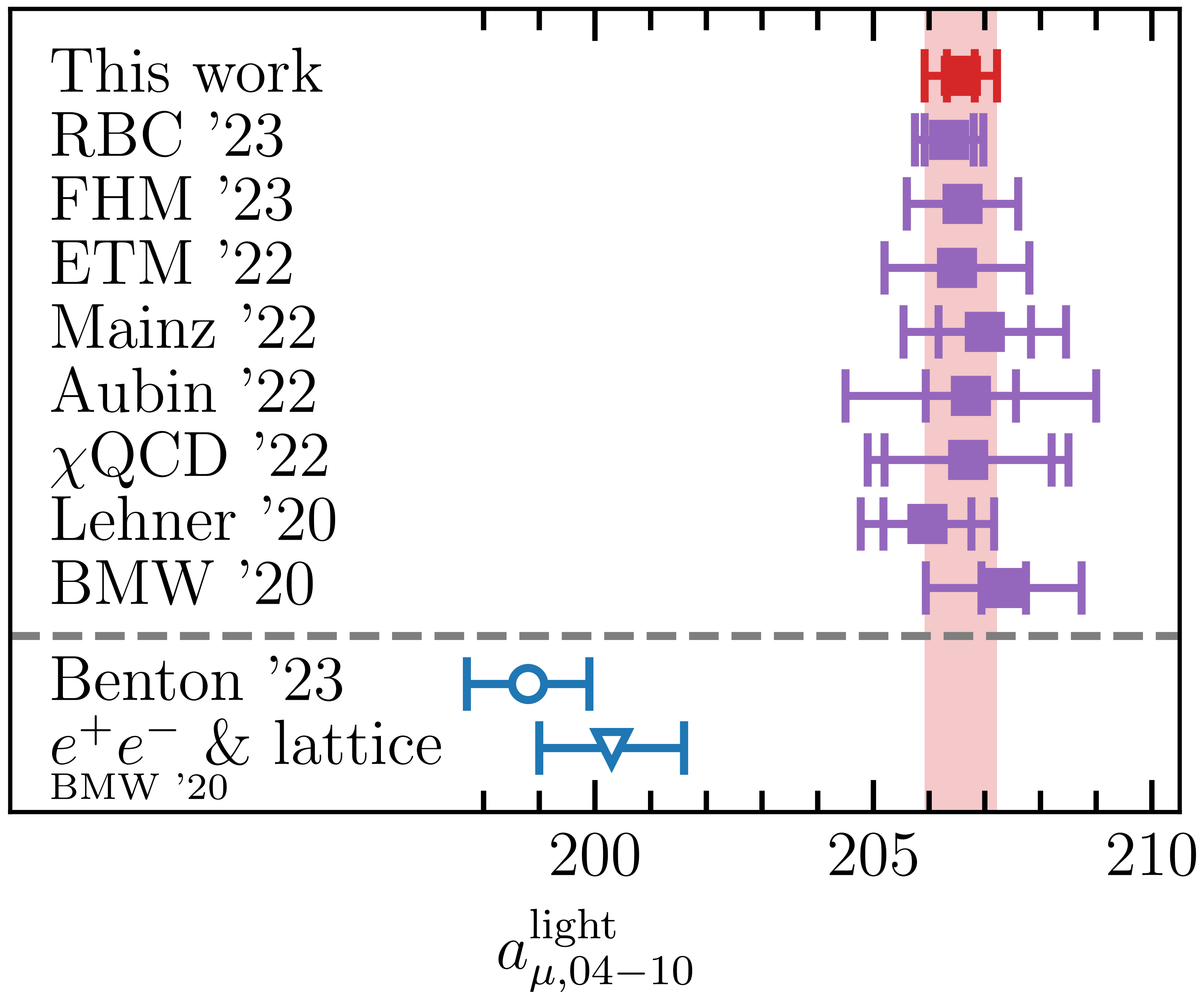
- intermediate window, a_μ^W

- BMW 20 [Sz. Borsanyi et al, arXiv:2002.12347, 2021 Nature]



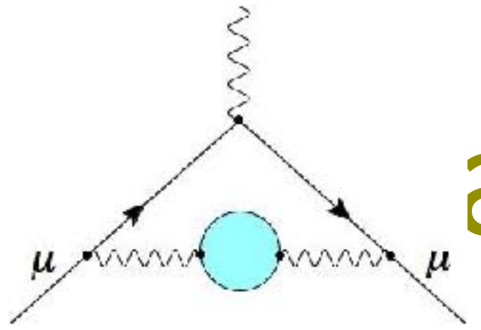
- BMW-DMZ 24 [A. Boccaletti et al, arXiv:2407.10913]



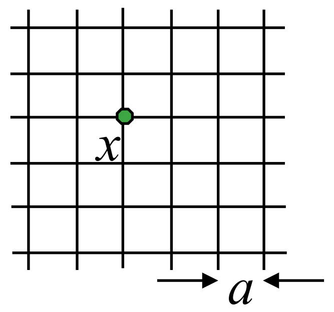


- Improvement:
  correct lattice data for discretization effects due to taste-splittings before taking continuum limit.

Continuum extrapolations obtained only from data not corrected for taste-splittings.

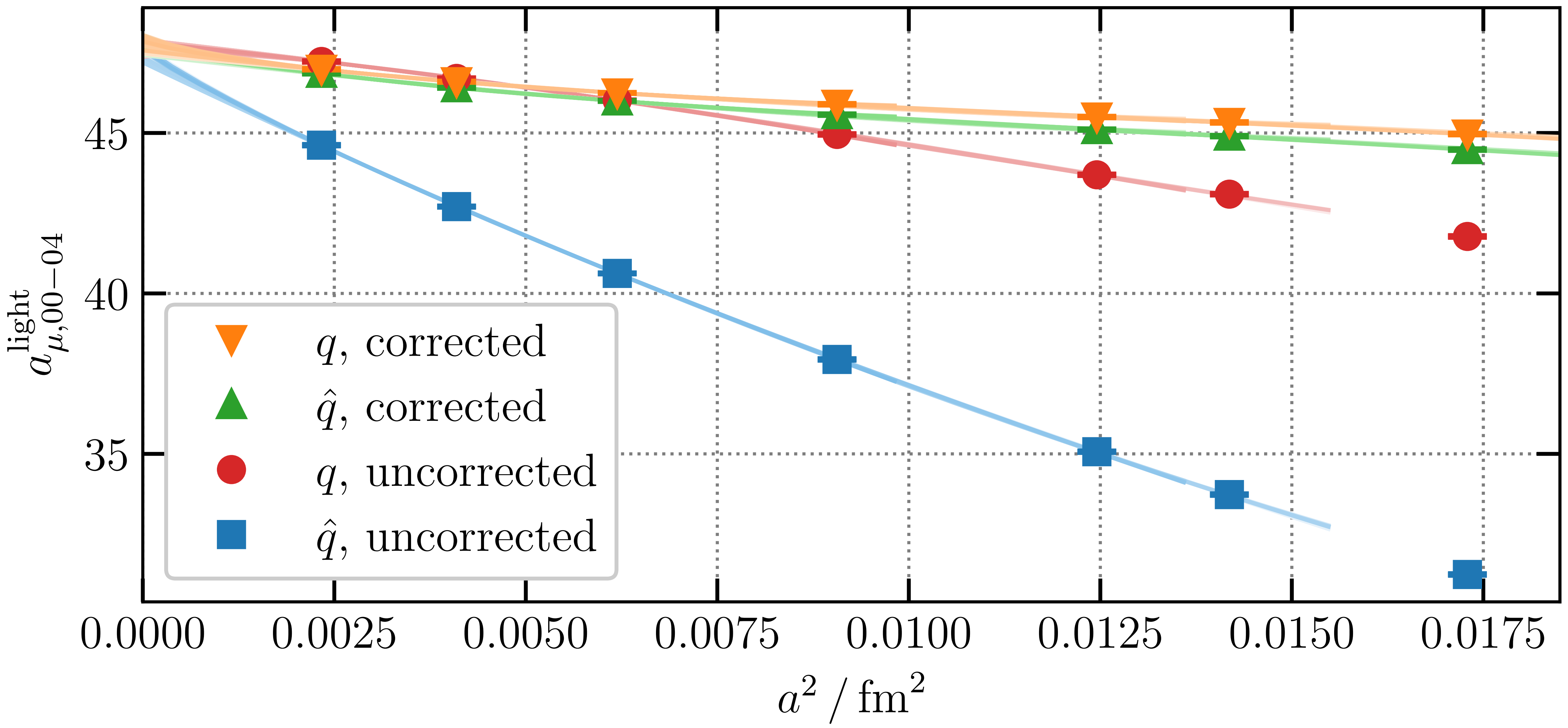


attice HVP: short-distance window (SD)



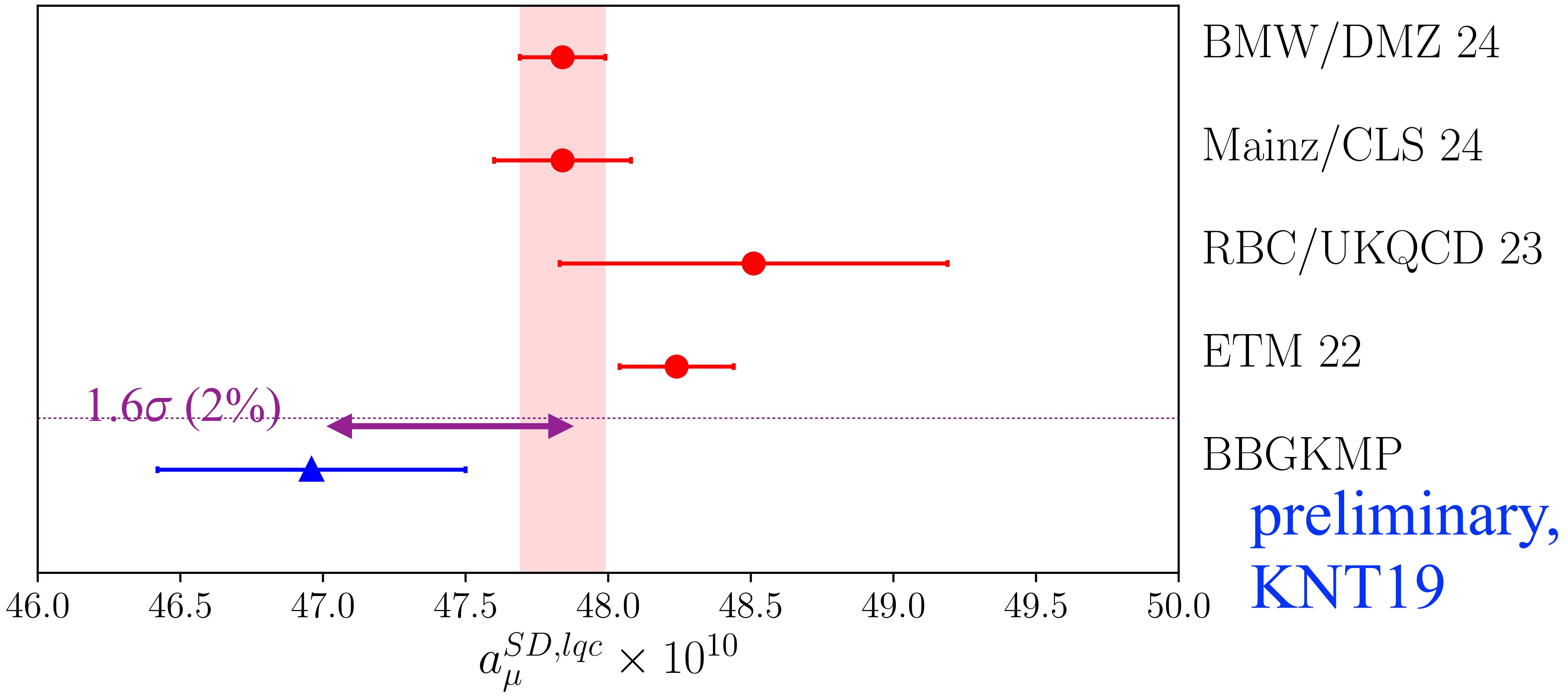
update: BMW+DMZ 2024

- BMW 24 [A. Boccaletti et al, arXiv:2407.10913]



- short-distance window, a_μ^{SD}

C. Davies @ Lattice 2024



- corrected: remove log-enhanced discretization effects at tree-level

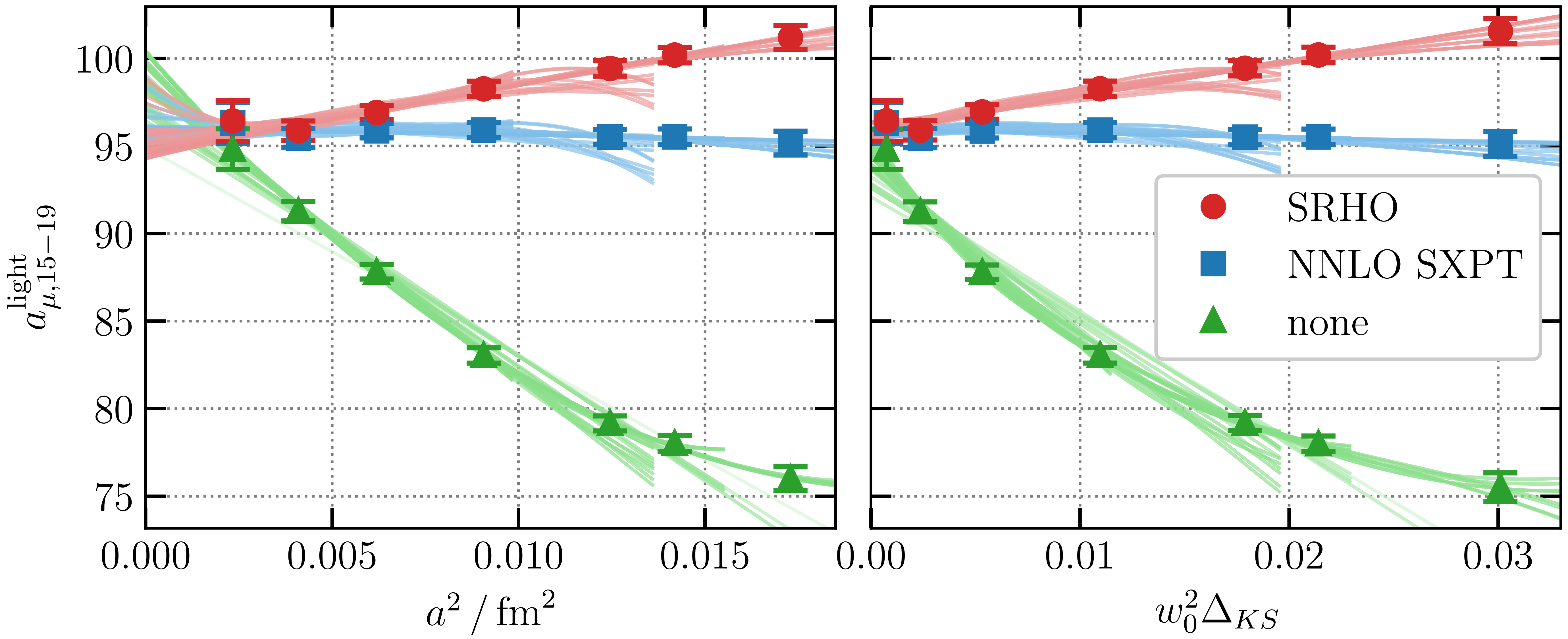
- $\hat{q}$: $\left[t^2 - \frac{4}{(aQ)^2}\sin^2\left(\frac{aQt}{2}\right)\right] \to \left[t^2 - \frac{4}{(a\hat{Q})^2}\sin^2\left(\frac{aQt}{2}\right)\right]$

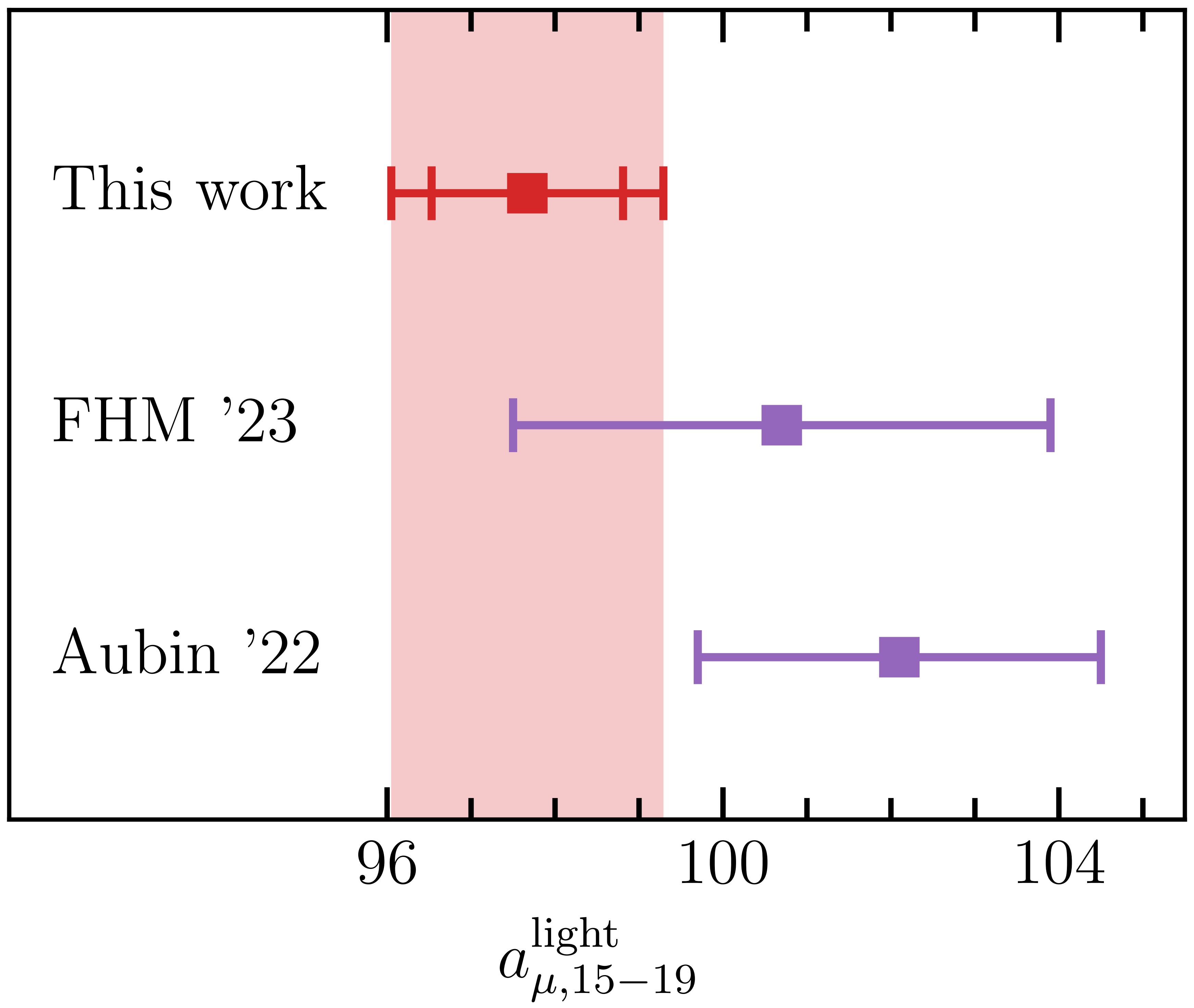
small tension in SD with pre-2023 data-driven evaluation

Lattice HVP: 2nd window (W2)

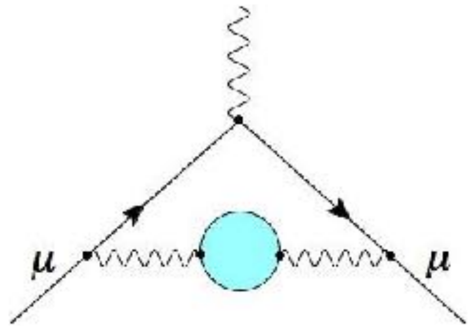
update: BMW+DMZ 2024

- [A. Boccaletti et al, arXiv:2407.10913]

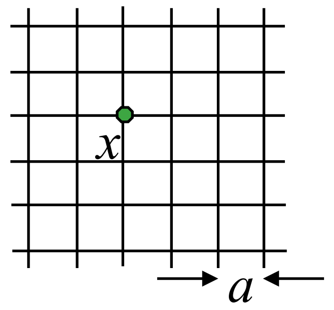




Continuum extrapolations of data with ``No improvement" (green) excluded from model average.



Lattice HVP: windows



update: Fermilab/HPQCD/MILC 2024

FNAL/HPQCD/MILC @ Lattice 2024:

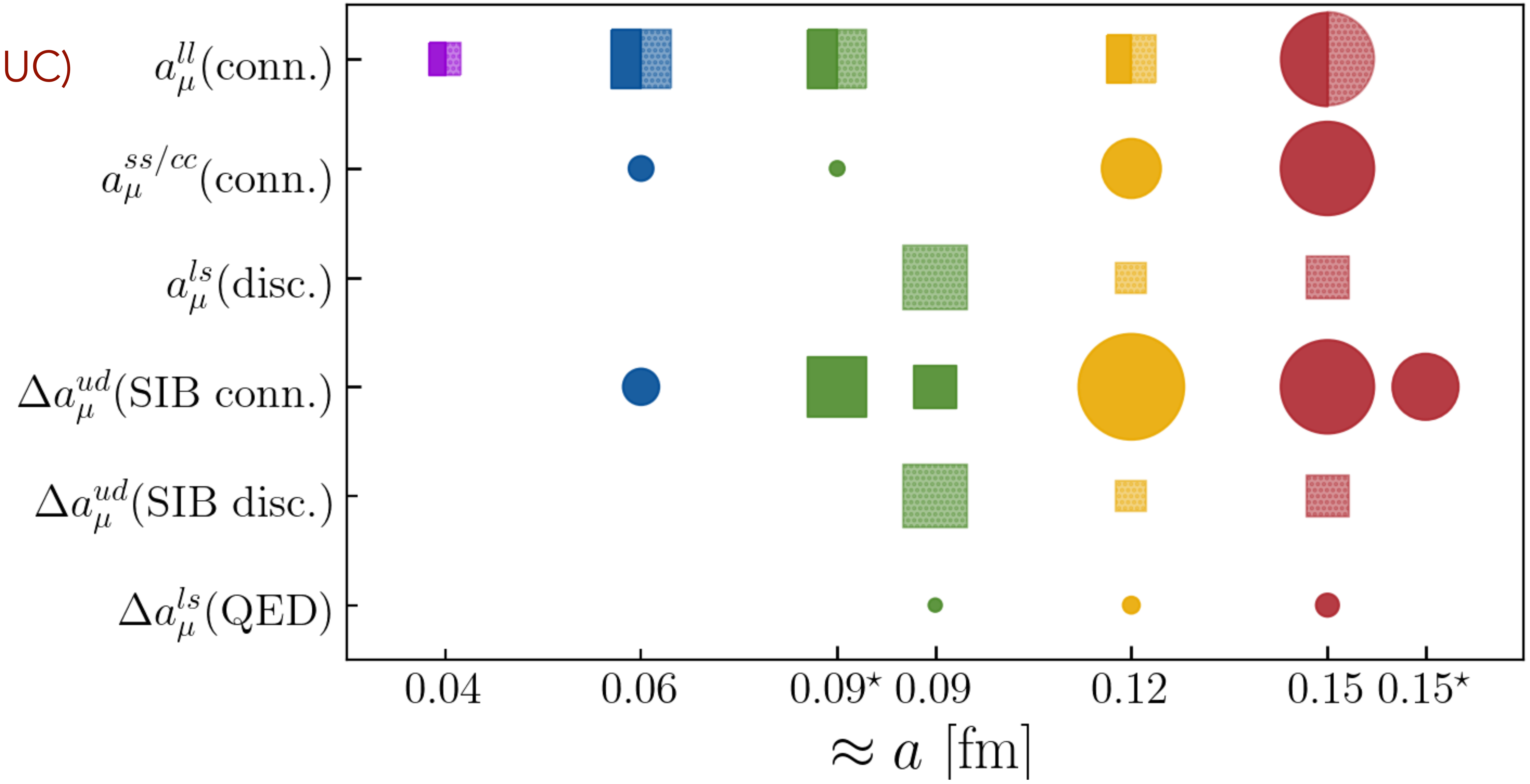
Shaun Lahert (Utah) & Michael Lynch (UIUC)

Shaun Lahert (Utah)

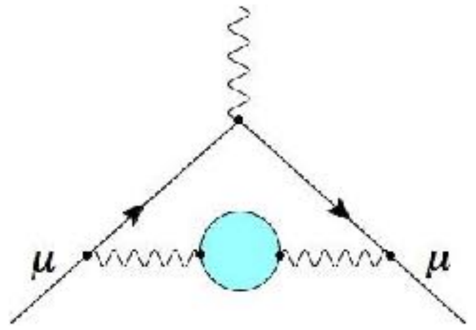
David Clarke (Utah)

Jake Sitison (U Colorado)

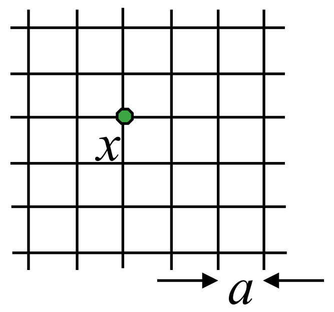
Craig McNeile (Plymouth)



- Solid color (local current) hatched (one-link)
- Squares: low-mode improved.
- Size $\sim$ statistics



Lattice HVP: long-distance window

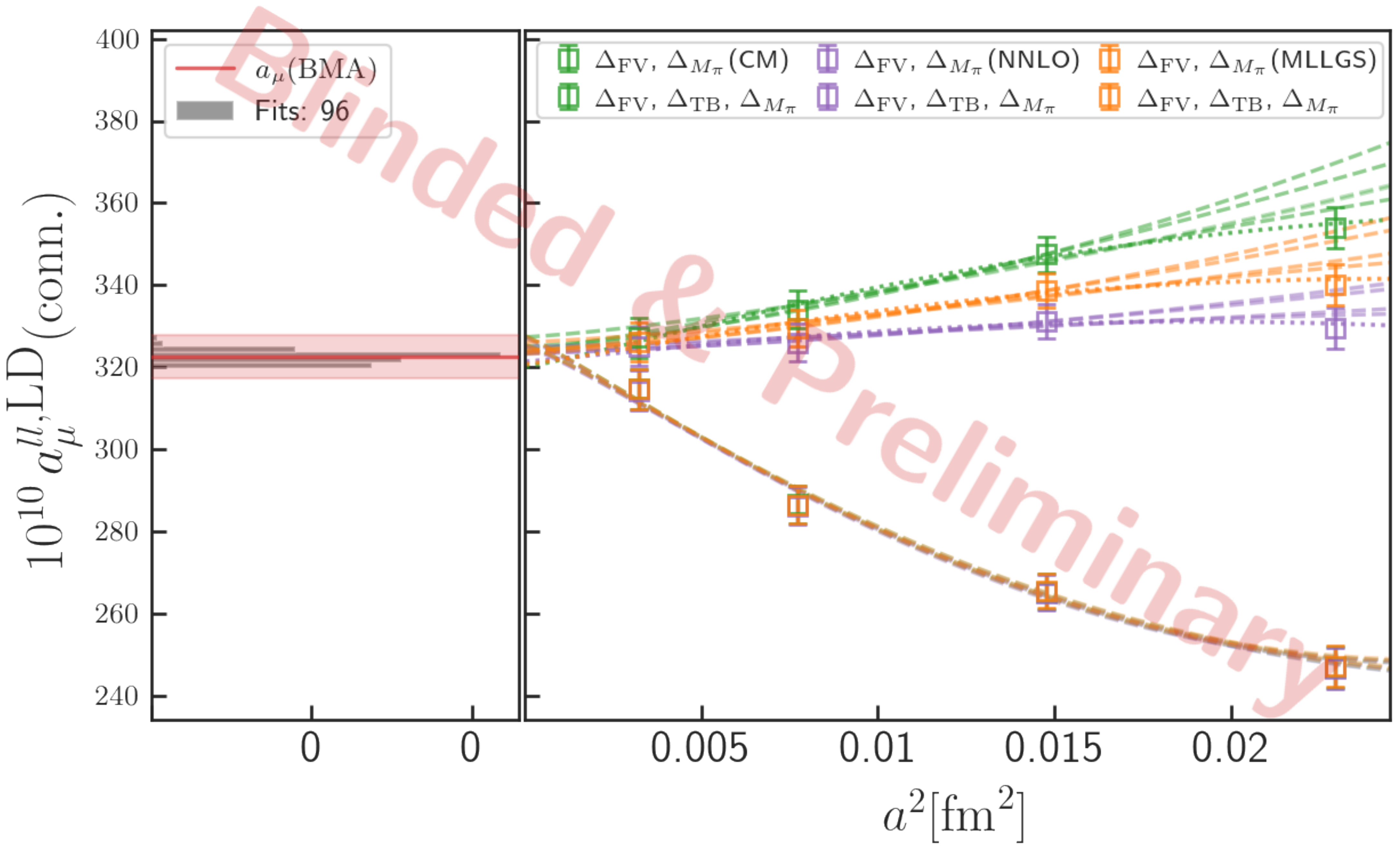


update: Fermilab/HPQCD/MILC 2024

Michael Lynch @ Lattice 2024

- long-distance window a_μ^{LD} and full a_μ^{ll}(conn.)

a_μ^{ll} (conn.) - LD window



a_μ^{ll} (conn.) - Full