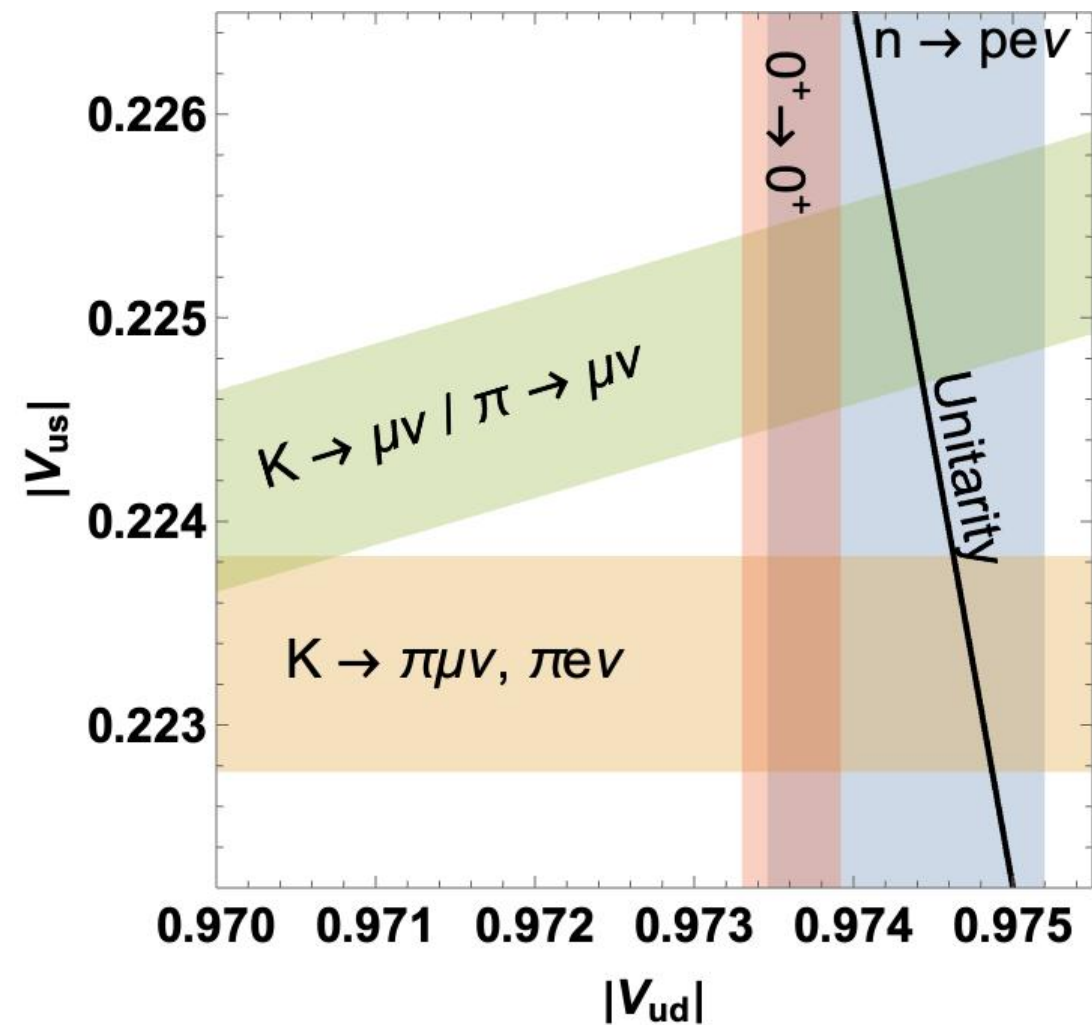




# Precision Tests of the Standard Model with Cabibbo Unitarity and Nuclear $\beta$ -Decays



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Neutron beta decay review: MG, Seng, Universe **2023**, 9(9), 422, arXiv:**2307.01145**

Nuclear beta decay review: MG, Seng, Ann.Rev.Part.Nucl.Sci. 74 (2024) 23-47, arXiv:**2311.00044**

# Outline

Beta decay, radiative corrections and the Standard Model

Cabibbo anomaly: have we found BSM?

Summary of experimental measurements

Radiative corrections to  $\beta$ -decays: overall setup

$\gamma W$ -box: Dispersion Theory, lattice QCD and EFT

Nuclear corrections

Summary & Outlook

What to work on to win a Nobel prize?

# What to work on to win a Nobel prize?

Beta decay has been an excellent choice for a century!

1896- Becquerel discovers spontaneous radioactivity of uranium, identified  $\beta$  with the electron

1898- Curie-Sklodowska, Curie discover polonium and radium

1899- Rutherford systematized  $\alpha$ ,  $\beta$ ,  $\gamma$  rays, identified  $\alpha$  with He-4

1934- F.&I. Joliot-Curie discovered  $\beta^+$  decay with  $\beta^+$  - positron

1956- Lee & Yang proposed parity non conservation in  $\beta$ -decay, confirmed by Wu experiment

1961- Glashow proposed electroweak unification

1967- Weinberg & Salam implemented Higgs mechanism

1973- Neutral weak current discovered at CERN

1973- Kobayashi, Maskawa: 3-flavor quark mixing matrix



**1903**



**1908**



**1935**



**1957**



**1979**



**2008**



# That was the bright side...

Niepce de Saint-Victor: observed radioactivity in 1857 cited in Becquerel-father's book



Cox, McIlwraith, Kurrelmeier (1928); Chase (1929-30)  
“Apparent evidence of polarization in a beam of beta rays”



1930: Pauli postulated existence of neutrinos

1934: Fermi formulated the contact theory of beta decay



1938: Klein predicted  $M_W \sim \sqrt{4\pi\alpha\sqrt{2}/G_F} \sim 100 \text{ GeV}$



1957: Wu's experiment was crucial to prove Lee-Yang's conjecture, but Chien-Shiung Wu was not awarded the NP



1963- Cabibbo: proposed 2-flavor quark mixing  
to reconcile  $\mu$ ,  $\beta$ , K decay rates



# Precision Era: V-A + Radiative Corrections

V - A theory (Sudarshan&Marshak and Gell-Mann&Feynman 1957); S-PS not excluded

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Radiative corrections to muon decay: important evidence for V-A theory

RC to muon decay - UV finite for V-A but divergent for S-PS

Muon lifetime  $\tau_\mu = 2196980.3(2.2)ps$   $\rightarrow$  Fermi constant  $G_\mu = 1.1663788(7) \times 10^{-5} GeV^{-2}$

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Kinoshita, Sirlin, Behrends, ...

1-loop RC to spectrum:

$$\Delta P^0 d^3p = \frac{\alpha}{2\pi} P^0 d^3p \left[ 6 \ln \frac{\Lambda}{M_p} + \text{finite} \right]$$

UV cut-off

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1967: Sirlin applied current algebra:

general UV behavior of  $\beta$  decay rate at 1-loop

$$\frac{\alpha}{2\pi} P^0 d^3p \ 3[1 + 2\bar{Q}] \ln(\Lambda/M)$$

$\bar{Q}$  : average charge of fields involved:  $1 + 2\bar{Q}_{\mu,\nu_\mu} = 0$  but  $1 + 2\bar{Q}_{n,p} = 2$

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Eventually, massive W-boson renders RC to beta decay UV-finite

# Precision, Universality and CKM unitarity

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Cabibbo: strength shared between 2 generations

$$|G_V^{\Delta S=0}| = \cos \theta_C G_\mu$$

Cabibbo unitarity:  $\cos^2 \theta_C + \sin^2 \theta_C = 1$

$$|G_V^{\Delta S=1}| = \sin \theta_C G_\mu$$

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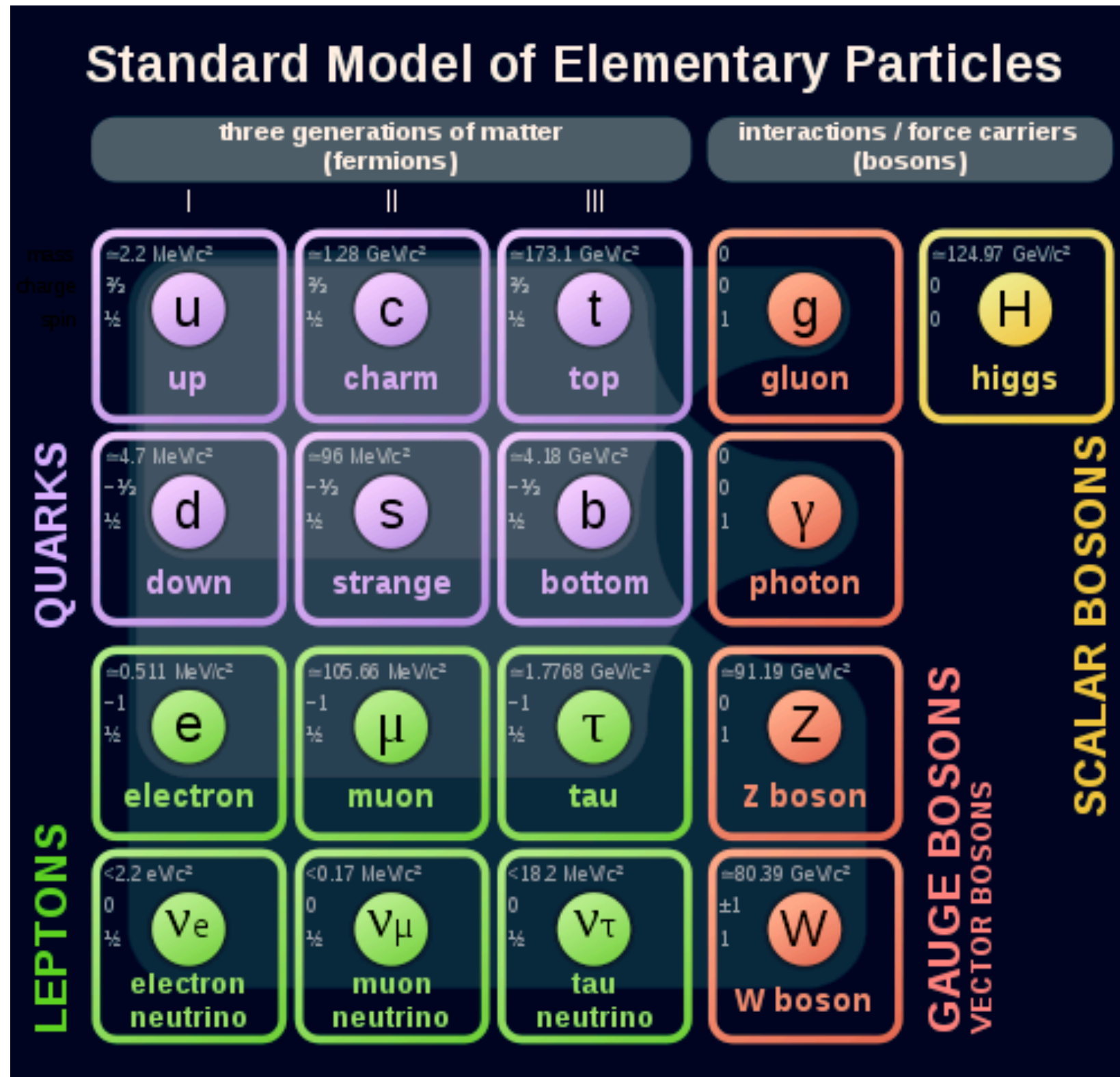
Kobayashi & Maskawa: 3 flavors + CP violation — CKM matrix V

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM unitarity - completeness of the SM:  $VV^\dagger = \mathbf{1}$

Top row unitarity constraint:  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

# Detailed understanding of $\beta$ decays largely shaped the Standard Model



# Cabibbo Angle Anomaly: Status and BSM interpretation

# Status of Cabibbo unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

$\sim 0.95$      $\sim 0.05$      $\sim 10^{-5}$

$V_{ud}$  and  $V_{us}$  determinations  
inconsistent with the SM

Superaligned nuclear  $\beta$ :  $|V_{ud}| = 0.9737(3)$

At variance with kaon decays + Cabibbo unitarity

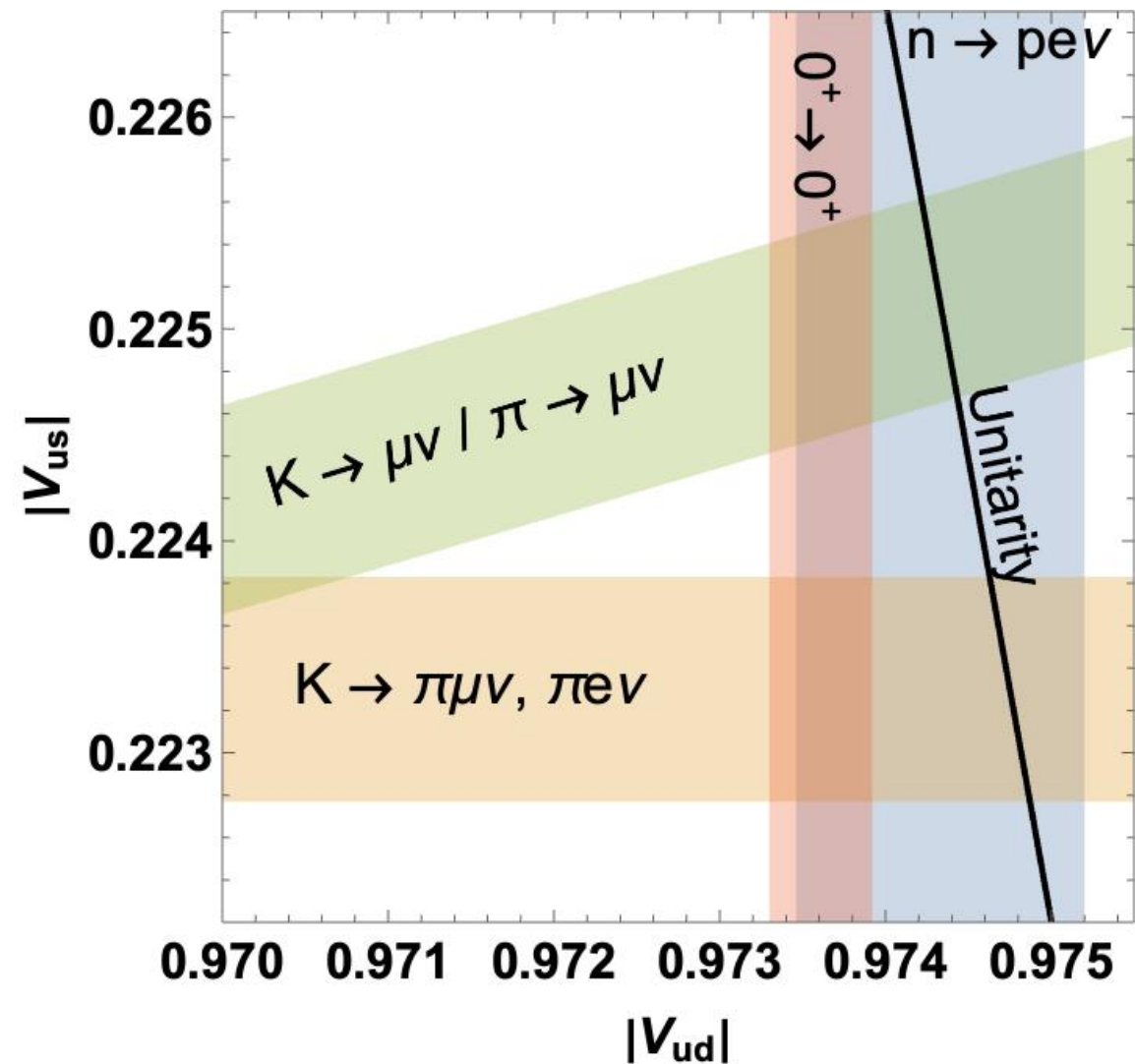
$K \rightarrow \pi \ell \nu$ :  $|V_{us}| = 0.2233(5)$

Unitarity  $\rightarrow |V_{ud}| = \sqrt{1 - |V_{us}|^2} = 0.9747(1)$

$\frac{K \rightarrow \mu \nu}{\pi \rightarrow \mu \nu}$ :  $|V_{us}/V_{ud}| = 0.2311(5)$

Unitarity  $\rightarrow |V_{ud}| = [1 + |V_{us}/V_{ud}|^2]^{-1/2} = 0.9743(1)$

} PDG [ $S = 2.5$ ]:  $|V_{us}| = 0.2243(8)$   
 } Unitarity  $\rightarrow |V_{ud}| = 0.9745(2)$



But consistent with the free neutron decay:  $|V_{ud}| = 0.9743(9)$

# CAA summary - 3 anomalies!

3 observables:  $|V_{us}|^{K\ell 3}$ ,  $|V_{us}/V_{ud}|^{K\mu 2}$ ,  $V_{ud}$   
2 quantities to determine:  $V_{us}$ ,  $V_{ud}$



**3 ways to test unitarity**

$$\Delta_{\text{CKM}}^{(1)} = |V_{ud}|^2 + |V_{us}^{K\ell 3}|^2 - 1 = -0.00176(56) \quad -3.1\sigma$$

$$\Delta_{\text{CKM}}^{(2)} = |V_{ud}|^2 \left[ 1 + \left( \left| \frac{V_{us}}{V_{ud}} \right|^{K\mu 2} \right)^2 \right] - 1 = -0.00098(58) \quad -1.7\sigma$$

$$\Delta_{\text{CKM}}^{(3)} = |V_{us}^{K\ell 3}|^2 \left[ \left( \frac{1}{|V_{us}/V_{ud}|^{K\mu 2}} \right)^2 + 1 \right] - 1 = -0.0164(63) \quad -2.6\sigma$$

Can it be a signal of BSM?

# CAA in presence of RH currents

- In SM,  $W$  couples only to LH chiral fermion states
- New physics with couplings to RH currents could explain both unitarity deficit and  $K_{\ell 3}$ - $K_{\mu 2}$  difference
- Define  $\epsilon_R$  = admixture of RH currents in non-strange sector  
 $\epsilon_R + \Delta\epsilon_R$  = admixture of RH currents in strange sector

**Cirigliano et al.**  
**PLB 838 (2023)**

$$\Delta_{\text{CKM}}^{(1)} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2$$

$$\Delta_{\text{CKM}}^{(2)} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2$$

$$\Delta_{\text{CKM}}^{(3)} = 2\epsilon_R + 2\Delta\epsilon_R(2 - V_{us}^2)$$

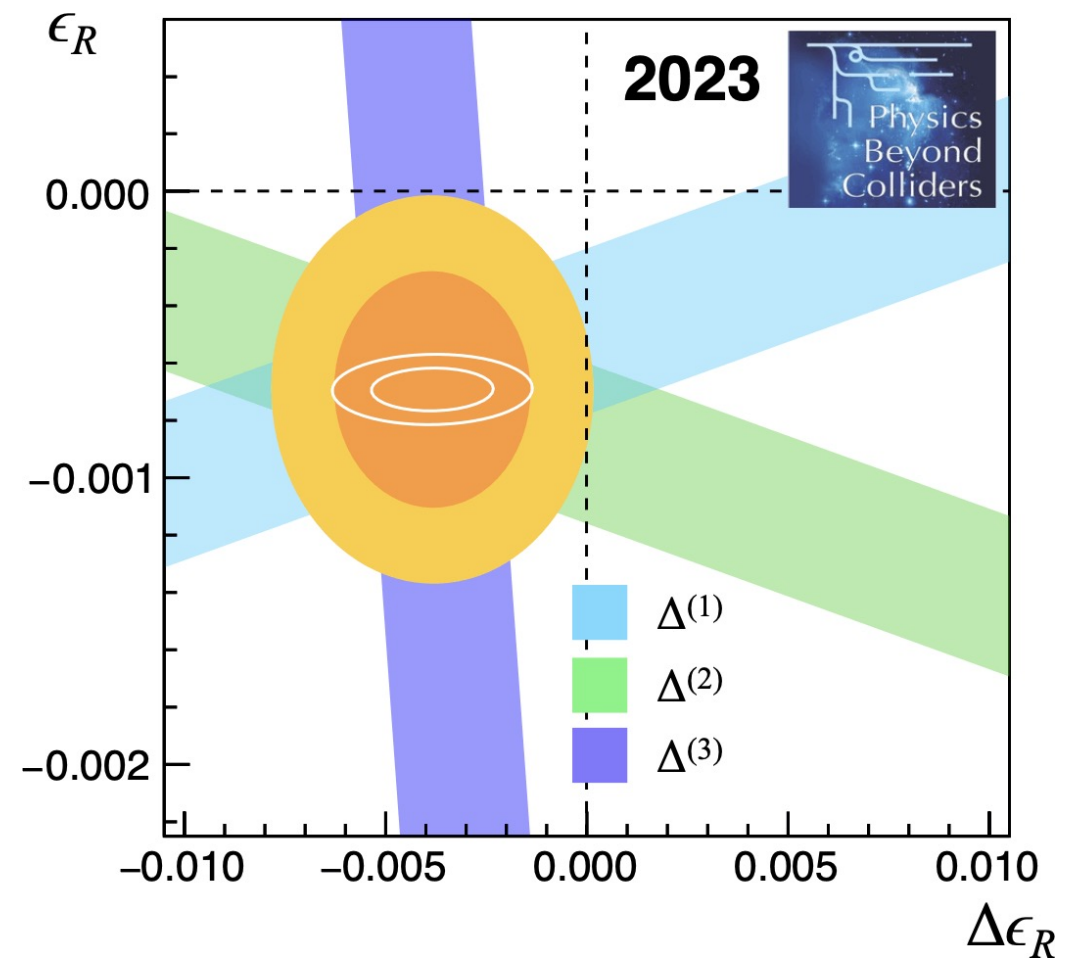
$$r \equiv \left( \frac{1 + \Delta_{\text{CKM}}^{(2)}}{1 + \Delta_{\text{CKM}}^{(3)}} \right)^{1/2} = \frac{V_{us}}{V_{ud}} \left| \frac{K_{\ell 2}/\pi_{\ell 2}}{V_{us}^{K_{\ell 3}}/V_{ud}^{\beta}} \right| = 1 - 2\Delta\epsilon_R$$

From current fit:

$$\epsilon_R = -0.69(27) \times 10^{-3} \quad (2.5\sigma)$$

$$\Delta\epsilon_R = -3.9(1.6) \times 10^{-3} \quad (2.4\sigma)$$

$$\epsilon_R = \Delta\epsilon_R = 0 \text{ excluded at } 3.1\sigma$$



Review the “ $\sigma$ ” that defines the significance of the Cabibbo angle anomaly!

$V_{us}$  from world data



$V_{us} / V_{ud}$  from  $K_{\mu 2} = K \rightarrow \mu\nu / \pi_{\mu 2} = \pi \rightarrow \mu\nu$

$$\frac{|V_{us}|}{|V_{ud}|} \frac{f_K}{f_\pi} = \left( \frac{\Gamma_{K_{\mu 2}(\gamma)} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2}(\gamma)} m_{K^\pm}} \right)^{1/2} \frac{1 - m_\mu^2 / m_{\pi^\pm}^2}{1 - m_\mu^2 / m_{K^\pm}^2} \left( 1 - \frac{1}{2} \delta_{EM} - \frac{1}{2} \delta_{SU(2)} \right)$$

## Inputs from experiment:

From  $K^\pm$  BR fit: [KLOE, CNTR]

$$\mathbf{BR}(K_{\mu 2}^\pm) = \mathbf{0.6358(11)}$$

$$\mathbf{\tau_{K^\pm} = 12.384(15) \text{ ns}}$$

From PDG:

$$\mathbf{BR}(\pi_{\mu 2}^\pm) = \mathbf{0.9999}$$

$$\mathbf{\tau_{\pi^\pm} = 26.033(5) \text{ ns}}$$

## Inputs from theory:

$\delta_{EM}$  Long-distance EM corrections

$\delta_{SU(2)}$  Strong isospin breaking

$$f_K / f_\pi \rightarrow f_{K^\pm} / f_{\pi^\pm}$$

$f_K / f_\pi$  Ratio of decay constants

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for  $SU(2)$ -breaking:  $f_{K^\pm} / f_{\pi^\pm}$

# New development: QCD + QED on the Lattice

**Giusti et al.**  
PRL 120 (2018)

## First lattice calculation of EM corrections to $P_{12}$ decays

- Ensembles from ETM
- $N_f = 2+1+1$  Twisted-mass Wilson fermions

$$\delta_{SU(2)} + \delta_{EM} = -0.0122(16)$$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld '11:

$$\delta_{SU(2)} + \delta_{EM} = -0.0112(21)$$

**Di Carlo et al.**  
PRD 100 (2019)

Update, extended description, and systematics of Giusti et al.

$$\delta_{SU(2)} + \delta_{EM} = -0.0126(14)$$

$$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(28)_{\text{BR}}(20)_{\text{corr}}$$

# $f_K/f_\pi$ on the lattice and result for $V_{us}/V_{ud}$

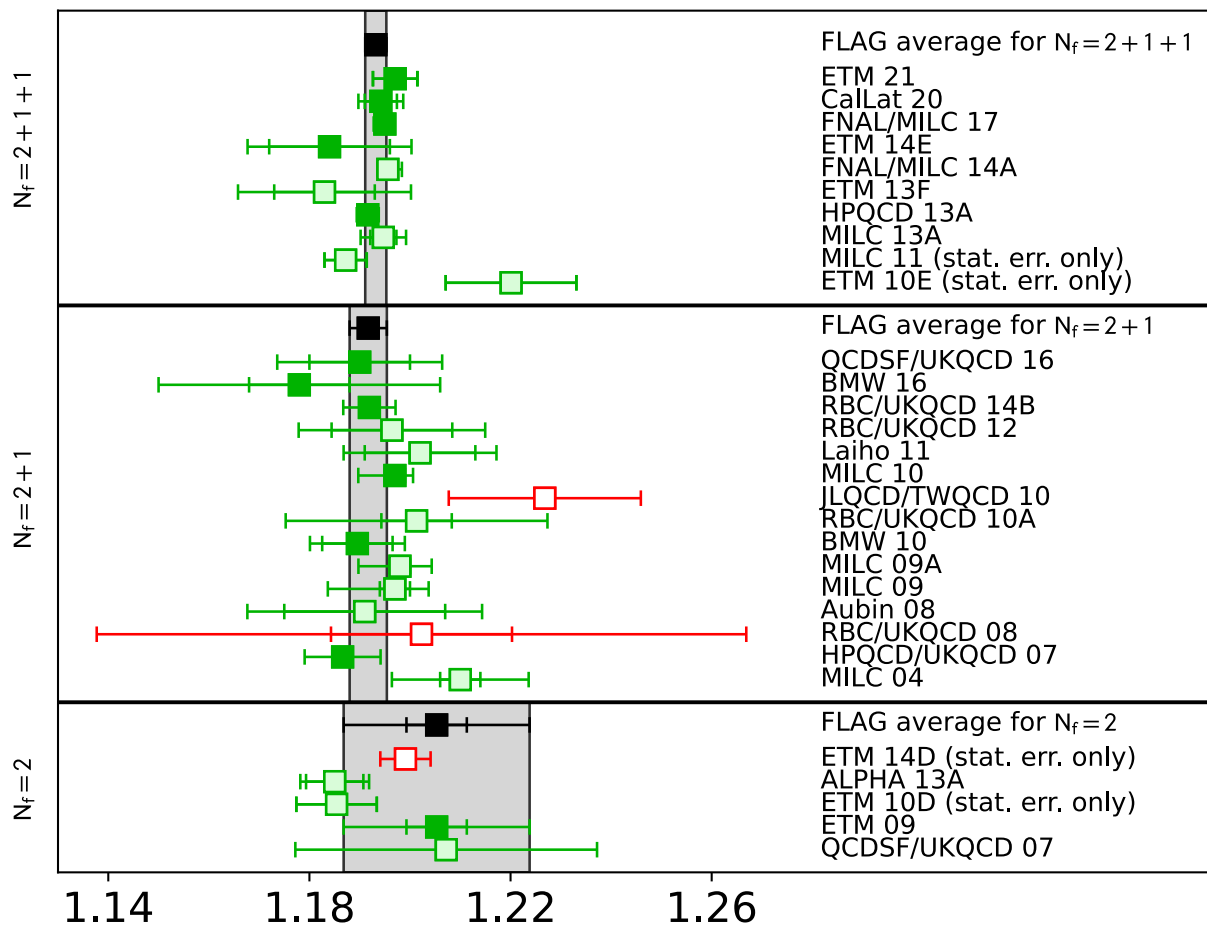
Some lattice groups include strong isospin breaking, others work in isospin limit

Isospin breaking  $\frac{f_{K^\pm}}{f_{\pi^\pm}} = \frac{f_K}{f_\pi} \sqrt{1 + \delta_{SU(2)}}$

ISB from  $\chi$ PT  $\delta_{SU(2)} \approx \sqrt{3} \epsilon_{SU(2)} \left[ -\frac{4}{3} (f_K/f_\pi - 1) + \frac{2}{3(4\pi)^2 f_0^2} \left( M_K^2 - M_\pi^2 - M_\pi^2 \ln \frac{M_K^2}{M_\pi^2} \right) \right]$

FLAG2023

$f_{K^\pm}/f_{\pi^\pm}$



LQCD ( $N_f = 2 + 1 + 1$ ):

$$f_{K^\pm}/f_{\pi^\pm} = 1.1934(19)$$

$$f_K/f_\pi = 1.1978(22)$$

LQCD ( $N_f = 2 + 1$ ):

$$f_{K^\pm}/f_{\pi^\pm} = 1.1917(37)$$

$$f_K/f_\pi = 1.1946(34)$$

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23108(23)_{\text{exp}}(42)_{\text{lat}}(16)_{\text{ISB}}$$

$$(51)_{\text{tot}} = 0.22\%$$

$V_{us}$  from  $K\ell 3 = K \rightarrow \pi e \nu, \pi \mu \nu$

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{K\ell}(\lambda_{K\ell}) \left( 1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM} \right)$$

with  $K \in \{K^+, K^0\}$ ;  $\ell \in \{e, \mu\}$ , and:

$C_K^2$  1/2 for  $K^+$ , 1 for  $K^0$

$S_{EW}$  Universal SD EW correction (1.0232)

## Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$  Rates with well-determined treatment of radiative decays:

- Branching ratios:  $K_S, K_L, K^\pm$
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$  Integral of form factor over phase space:  $\lambda$ s parameterize evolution in  $t$

- $K_{e3}$ : Only  $\lambda_+$  (or  $\lambda_+', \lambda_+''$ )
- $K_{\mu 3}$ : Need  $\lambda_+$  and  $\lambda_0$

## Inputs from theory:

$f_+^{K^0\pi^-}(0)$  Hadronic matrix element (form factor) at zero momentum transfer ( $t = 0$ )

$\Delta_K^{SU(2)}$  Form-factor correction for  $SU(2)$  breaking

$\Delta_{K\ell}^{EM}$  Form-factor correction for long-distance EM effects

# Fit to $K\ell 3$ branching ratios of $K_S, K_L, K^\pm$

## Fit to $K_S$ rate data (2022)

### 7 input measurements:

- KLOE '06** BR  $\pi^0\pi^0/\pi^+\pi^-$
- NA48**  $\Gamma(K_S \rightarrow \pi e \nu)/\Gamma(K_L \rightarrow \pi e \nu), \tau_S$
- KLOE '11**  $\tau_S$
- KTeV '11**  $\tau_S$
- KLOE-2 '22** BR  $\pi e \nu/\pi^+\pi^-$  **New!**
- KLOE-2 '20** BR  $\pi \mu \nu/\pi^+\pi^-$

### 2 possible constraints:

- $\Sigma \text{BR} = 1$
- **BR( $K_{e3}$ )/BR( $K_{\mu 3}$ ) = 0.6640(17)**  
From ratio of phase-space integrals from current fit to dispersive  $K_{\ell 3}$  form factor parameters

Only sum constraint used for fit

Parameter	Value
BR( $\pi^+\pi^-(\gamma)$ )	69.20(5)%
BR( $\pi^0\pi^0$ )	30.69(5)%
BR( $K_{e3}$ )	7.15(6) $\times 10^{-4}$
BR( $K_{\mu 3}$ )	4.56(20) $\times 10^{-4}$
$\tau_S$	89.58(4) ns

$\chi^2/\text{ndf} = 0.36/3$  (Prob = 95%)

Little correlation for  $K_{e3} K_{\mu 3}$  from fit

10-20% correlations with  $\pi^0\pi^0/\pi^+\pi^-$

Input measurements essentially unchanged

## Fit to $K_L$ rate data (2010)

### 21 input measurements:

- 5 KTeV** ratios
- NA48** BR( $K_{e3}/2$  track)
- 4 KLOE** BRs  
with dependence on  $\tau_L$
- KLOE, NA48** BR( $\pi^+\pi^-/K_{\ell 3}$ )
- KLOE, NA48** BR( $\gamma\gamma/3\pi^0$ )
- BR( $2\pi^0/\pi^+\pi^-$ ) from  $K_S$  fit, Re  $\varepsilon'/\varepsilon$
- KLOE**  $\tau_L$  from  $3\pi^0$
- Vosburgh '72**  $\tau_L$
- KTeV** BR( $\pi^+\pi^-\gamma/\pi^+\pi^-(\gamma)$ )
- E731, 2 KTeV** BR( $\pi^+\pi^-\gamma_{\text{DE}}/\pi^+\pi^-\gamma$ )

### 1 constraint: $\Sigma \text{BR} = 1$

Parameter	Value	$S$
BR( $K_{e3}$ )	0.4056(9)	1.3
BR( $K_{\mu 3}$ )	0.2704(10)	1.5
BR( $3\pi^0$ )	0.1952(9)	1.2
BR( $\pi^+\pi^-\pi^0$ )	0.1254(6)	1.3
BR( $\pi^+\pi^-(\gamma_{\text{IB}})$ )	1.967(7) $\times 10^{-3}$	1.1
BR( $\pi^+\pi^-\gamma$ )	4.15(9) $\times 10^{-5}$	1.6
BR( $\pi^+\pi^-\gamma_{\text{DE}}$ )	2.84(8) $\times 10^{-5}$	1.3
BR( $2\pi^0$ )	8.65(4) $\times 10^{-4}$	1.4
BR( $\gamma\gamma$ )	5.47(4) $\times 10^{-4}$	1.1
$\tau_L$	51.16(21) ns	1.1

$\chi^2/\text{ndf} = 19.8/12$  (Prob = 7.0%)

Essentially same result as 2010 fit  
Current PDG (since '09): 37.4/17 (0.30%)

## Fit to $K^\pm$ rate data (2014)

### 17 input measurements:

- 3 old**  $\tau$  values in PDG
- KLOE**  $\tau$
- KLOE** BR  $\mu\nu, \pi\pi^0$
- KLOE** BR  $K_{e3}, K_{\mu 3}$   
with dependence on  $\tau$
- NA48/2** BR  $K_{e3}/\pi\pi^0, K_{\mu 3}/\pi\pi^0$
- E865** BR  $K_{e3}/K_{\text{Dal}}$
- 3 old** BR  $\pi\pi^0/\mu\nu$
- KEK-246**  $K_{\mu 3}/K_{e3}$
- KLOE** BR  $\pi\pi\pi, \pi\pi^0\pi^0$   
(Bisi '65 BR  $\pi\pi^0\pi^0/\pi\pi\pi$  removed)

Parameter	Value	$S$
BR( $\mu\nu$ )	63.58(11)%	1.1
BR( $\pi\pi^0$ )	20.64(7)%	1.1
BR( $\pi\pi\pi$ )	5.56(4)%	1.0
BR( $K_{e3}$ )	5.088(27)%	1.2
BR( $K_{\mu 3}$ )	3.366(30)%	1.9
BR( $\pi\pi^0\pi^0$ )	1.764(25)%	1.0
$\tau_\pm$	12.384(15) ns	1.2

$\chi^2/\text{ndf} = 25.5/11$  (Prob = 0.78%)  
compare PDG '16: 53/28 (0.26%)

With **ISTRA+ '14** BR( $K_{e3}^-/\pi^-\pi^0$ )

- **BR( $K_{e3}$ ) = 5.083(27)%**
- Negligible changes in other parameters, fit quality

### 1 constraint: $\Sigma \text{BR} = 1$

Much more selective than PDG fit  
PDG '16: 35 inputs, 8 parameters

Compared to Kl2:

Many decay channels

multiple consistency checks possible

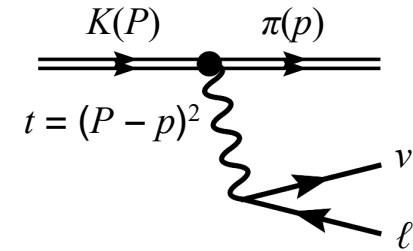
# $K - \pi$ transition form factors

**Hadronic matrix element:**

$$\langle \pi | J_\alpha | K \rangle = f(0) \times [\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha]$$

$K_{e3}$  decays: Only **vector form factor**:  $\tilde{f}_+(t)$

$K_{\mu 3}$  decays: Also need **scalar form factor**:  $\tilde{f}_0(t) = \tilde{f}_+ + \tilde{f}_- \frac{t}{m_K^2 - m_\pi^2}$



To extract  $V_{us}$ : need a phase-space integral over the form factor

Parametrize FFs  $\rightarrow$  best option: dispersion representation

Perform a fit to Dalitz plot

Dispersive + ChPT parametrization

Bernard et al. '09

$$\tilde{f}_+(t) = \exp \left[ \frac{t}{m_\pi^2} (\Lambda_+ - H(t)) \right]$$

$$\tilde{f}_0(t) = \exp \left[ \frac{t}{m_K^2 - m_\pi^2} (\ln C - G(t)) \right]$$

$H, G$ : obtained from  $K-\pi$  phase shifts

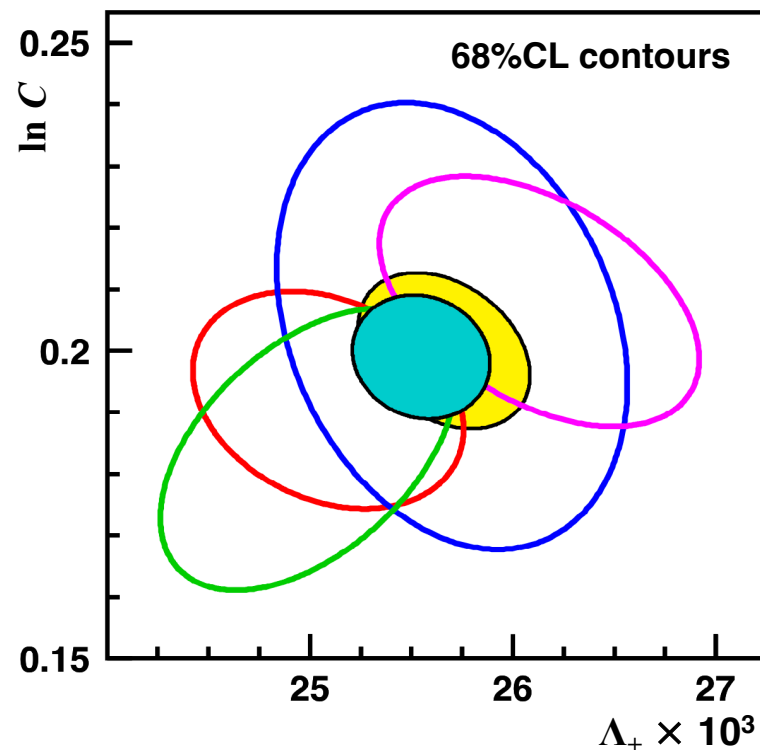
$\Lambda_+, \ln C$  - free parameters

$K_{\ell 3}$  avgs from

**KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48  $K_{e3}$  data included in fits but not shown

**2010 fit** **Current**



$\Lambda_+ \times 10^3$	$= 25.55 \pm 0.38$
$\ln C$	$= 0.1992(78)$
$\rho(\Lambda_+, \ln C)$	$= -0.110$
$\chi^2/\text{ndf}$	$= 7.5/7$ (38%)

**Integrals**

Mode	Update	2010
$K^0_{e3}$	<b>0.15470(15)</b>	0.15476(18)
$K^+_{e3}$	<b>0.15915(15)</b>	0.15922(18)
$K^0_{\mu 3}$	<b>0.10247(15)</b>	0.10253(16)
$K^+_{\mu 3}$	<b>0.10553(16)</b>	0.10559(17)

Only tiny changes in central values

# Long-distance EM correction

## Mode-dependent corrections $\Delta^{\text{EM}}_{K\ell}$ to phase-space integrals $I_{K\ell}$ from EM-induced Dalitz plot modifications

- Values depend on acceptance for events with additional real photon(s)
- All recent measurements assumed fully inclusive

## FlaviaNet analysis and updates used Cirigliano et al. '08

- Comprehensive analysis at fixed order  $e^2p^2$

**Seng et al.**  
**JHEP 07 (2022)**

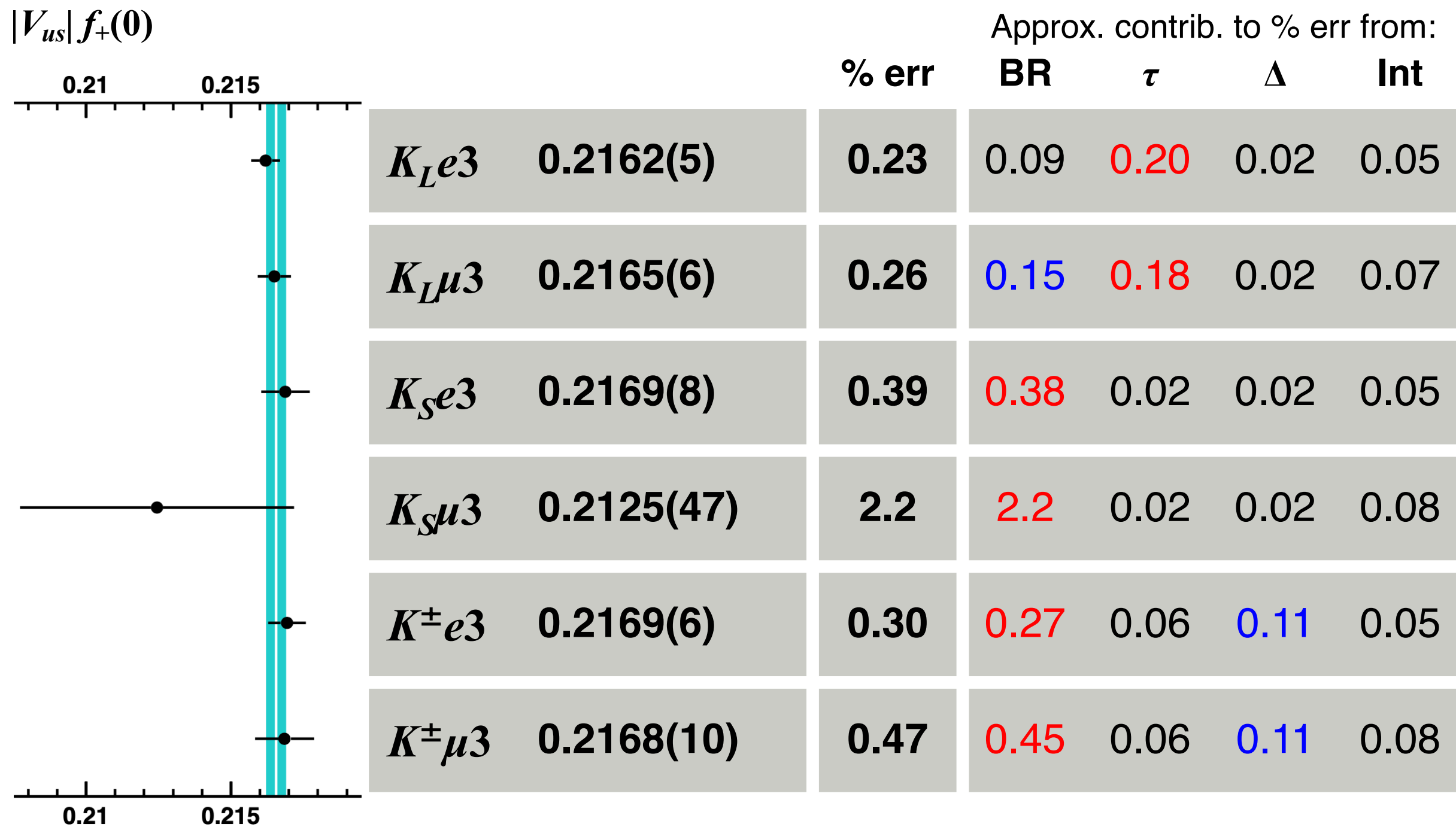
Calculation of complete EW RC using hybrid current algebra and ChPT with resummation of largest terms to all chiral orders

- Reduced uncertainties at  $O(e^2p^4)$
- Lattice evaluation of QCD contributions to  $\gamma W$  box diagrams
- Conventional value of  $S_{\text{EW}}$  subtracted from results for use with standard formula for  $V_{us}$

	Cirigliano et al. '08	Seng et al. '21
$\Delta^{\text{EM}}(K^0_{e3})$ [%]	$0.50 \pm 0.11$	<b><math>0.580 \pm 0.016</math></b>
$\Delta^{\text{EM}}(K^+_{e3})$ [%]	$0.05 \pm 0.12$	<b><math>0.105 \pm 0.023</math></b>
$\Delta^{\text{EM}}(K^+_{\mu3})$ [%]	$0.70 \pm 0.11$	<b><math>0.770 \pm 0.019</math></b>
$\Delta^{\text{EM}}(K^0_{\mu3})$ [%]	$0.01 \pm 0.12$	<b><math>0.025 \pm 0.027</math></b>

Seng, Galviz, Meißner 1910.13208; Seng, Galviz, MG, Meißner 2103.04843; Seng, Galviz, MG, Meißner 2203.05217; Feng, MG, Jin, Ma, Seng 2003.09798; Ma, Feng, MG, Jin, Seng 2102.12048

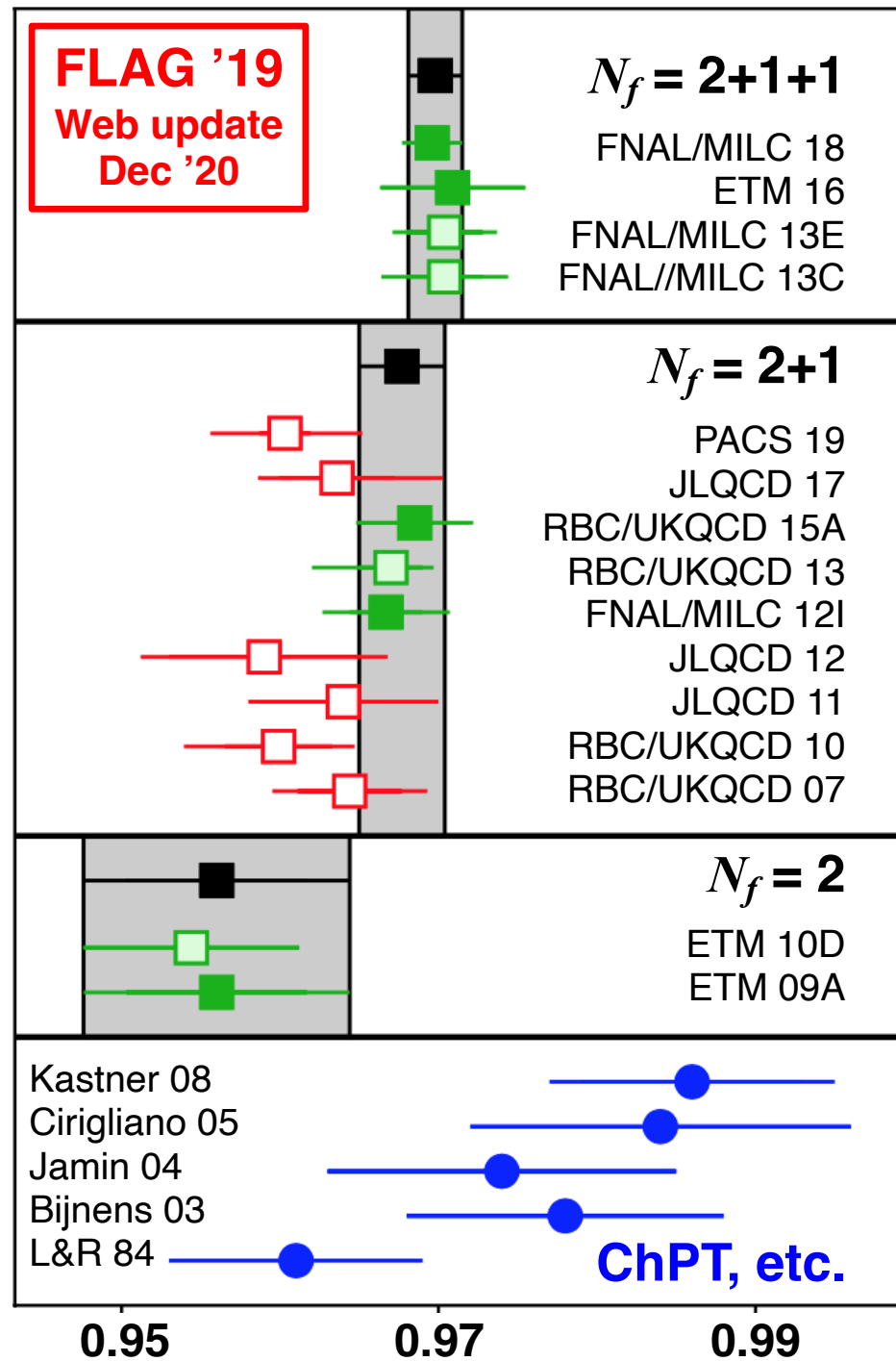
# $|V_{us}|f_+(0)$ from world data



**Average:  $|V_{us}|f_+(0) = 0.21656(35)$      $\chi^2/\text{ndf} = 1.89/5$  (86%)**



# $f_+(0)$ on the lattice and extraction of $|V_{us}|$



## FLAG '21 averages:

$$N_f = 2+1+1 \quad f_+(0) = 0.9698(17)$$

Uncorrelated average of:

**FNAL/MILC 18:** HISQ, 5sp,  $m_\pi \rightarrow 135$  MeV,  
new ensembles added to FNAL/MILC 13E

**ETM 16:** TwMW, 3sp,  $m_\pi \rightarrow 210$  MeV, full  $q^2$   
dependence of  $f_+, f_0$

$$N_f = 2+1 \quad f_+(0) = 0.9677(27)$$

Uncorrelated average of:

**FNAL/MILC 12I:** HISQ,  $m_\pi \sim 300$  MeV

**RBC/UKQCD 15A:** DWF,  $m_\pi \rightarrow 139$  MeV

**JLQCD 17** not included because only single  
lattice spacing used

$$\text{ChPT} \quad f_+(0) = 0.970(8)$$

**Ecker 15, Chiral Dynamics 15:**

Calculation from Bijmans 03,  
with new LECs from Bijmans, Ecker 14

$$|V_{us}| = 0.22330(35)_{\text{exp}}(39)_{\text{lat}}(8)_{\text{IB}} \\ (53)_{\text{tot}} = 0.24\%$$

# Summary on $|V_{us}|$

High-precision world data on leptonic and semileptonic decay channels

Theoretical evaluation of form factors, decay constants, radiative corrections as important as data

Past ~5 years: ChPT is officially superseded by lattice; LQCD results very consistent!

Great consistency among semileptonic channels

$K\mu 2$  channel: first ever QCD+QED calculation on the lattice

$$\Delta_{\text{CKM}}^{(3)} = |V_{us}^{K\ell 3}|^2 \left[ \left( \frac{1}{|V_{us}/V_{ud}|^{K\mu 2}} \right)^2 + 1 \right] - 1 = -0.0164(63) \quad -2.6\sigma$$

$K\mu 2$  -  $K\ell 3$  discrepancy confirmed

Cabibbo anomaly even w/o info on  $V_{ud}$ !

Upcoming exp. Belle II, NA62, KOTO

$$\text{Unitarity} \rightarrow |V_{ud}^{K\ell 3}| = \sqrt{1 - |V_{us}^{K\ell 3}|^2} = 0.9747(1)$$

$$\text{Unitarity} \rightarrow |V_{ud}^{K\mu 2}| = [1 + |(V_{us}/V_{ud})^{K\mu 2}|^2]^{-1/2} = 0.9743(1)$$

Hyperon and tau decays: not precise enough or inconsistent — Belle II, Super Tau-Charm Factory?

$$|V_{us}| = 0.2250(27) \quad (\text{Hyperon Decays})$$

$$|V_{us}| = 0.2207(14) \quad (\text{Tau Decays})$$

$$|V_{ud}^{\text{Hyp}}| = 0.9743(6)$$

$$|V_{ud}^{\tau}| = 0.9753(3)$$

$V_{ud}$  from world data

# $V_{ud}$ from neutron decay

Neutron decay: 2 measurements needed

$$|V_{ud}|^2 = \frac{5024.7 \text{ s}}{\tau_n(1 + 3g_A^2)(1 + \Delta_R^V)}$$

RC  $\Delta_R^V$ : bottleneck since 40 years

Pre-2018:  $\Delta_R^V = 0.02361(38)$  *Marciano, Sirlin PRL 2006*

Post-2018:  $\Delta_R^V = 0.02479(21)$  *MG, Seng Universe 2023*

Since 2018: DR+data+pQCD+EFT+LQCD

$\Delta_R^V$  uncertainty: factor 2 reduction

*C-Y Seng et al., PRL 2018; PRD 2019*

*A. Czarnecki, B. Marciano, A. Sirlin, PRD 2018*

*K. Shiells et al, PRD 2021; L. Hayen PRD 2021*

*P-X Ma, X. Feng, MG, L-C Jin, et al 2308.16755*

**Experiment:** factor 3-5 uncertainties improvement; discrepancies in  $\tau_n$  and  $g_A$

$3.4\sigma$   $\curvearrowright$

$$g_A = -1.27641(56)$$

$$g_A = -1.2677(28)$$

**PERKEO-III** *B. Märkisch et al, Phys.Rev.Lett. 122 (2019) 24, 242501*

**aSPECT** *M. Beck et al, Phys. Rev. C101 (2020) 5, 055506; 2308.16170*

$4\sigma$   $\curvearrowright$

$$\tau_n = 877.75(28)_{-12}^{+16}$$

$$\tau_n = 887.7(2.3)$$

**UCN $\tau$**  *F. M. Gonzalez et al. Phys. Rev. Lett. 127 (2021) 162501*

**BL1 (NIST)** *Yue et al, PRL 111 (2013) 222501*

PDG average

$$|V_{ud}^{\text{free n}}| = 0.9743 (3)_{\tau_n} (8)_{g_A} (1)_{RC} [9]_{total}$$

Single best measurements only

$$|V_{ud}^{\text{free n}}| = 0.9740 (2)_{\tau_n} (3)_{g_A} (1)_{RC} [4]_{total}$$

# $V_{ud}$ from semileptonic pion decay

Pion decay  $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ : theoretically cleanest, experimentally tough

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)} \frac{\Gamma_{\pi\ell^3}}{0.3988(23) \text{ s}^{-1}} \quad |V_{ud}^{\pi\ell^3}| = 0.9739 (27)_{exp} (1)_{RC}$$

RC to semileptonic pion decay

$\delta$  uncertainty: factor 3 reduction

ChPT:  $\delta = -0.0334(10)_{\text{LEC}}(3)_{\text{HO}}$  *Cirigliano et al, 2003; Passera et al, 2011*

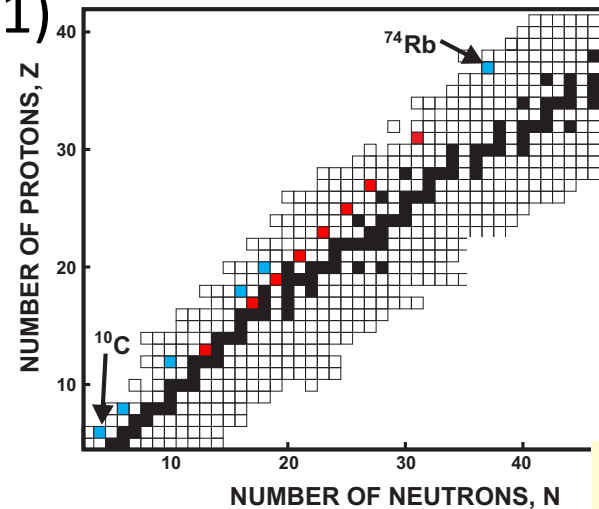
DR + LQCD + ChPT:  $\delta = 0.0332(1)_{\gamma W}(3)_{\text{HO}}$  *Feng et al, 2020; Yoo et al, 2023*

Future exp: 1 o.o.m. (PIONEER @ PSI)

Talk by Saul Cuen-Rochin on Monday

# $V_{ud}$ from superallowed $0^+ - 0^+$ nuclear decays

1. Transitions within  $J^P=0^+$  isotriplets ( $T=1$ )
2. Elementary process:  $p \rightarrow n e^+ \nu$
3. Only conserved vector current
4. 15 measured to better than 0.2%
5. Internal consistency as a check
6. SU(2) good  $\rightarrow$  corrections  $\sim$  small

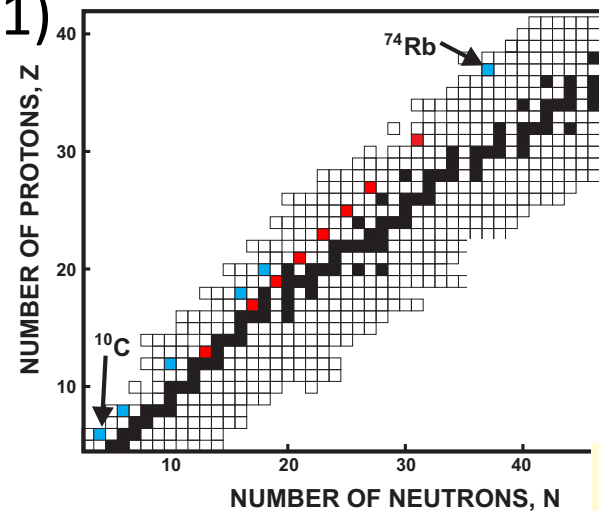


${}^6_6\text{C} \rightarrow {}^5_5\text{B}$
${}^8_8\text{O} \rightarrow {}^7_7\text{N}$
${}^{10}_{10}\text{Ne} \rightarrow {}^9_9\text{F}$
${}^{12}_{12}\text{Mg} \rightarrow {}^{11}_{11}\text{Na}$
${}^{14}_{14}\text{Si} \rightarrow {}^{13}_{13}\text{Al}$
${}^{16}_{16}\text{S} \rightarrow {}^{15}_{15}\text{P}$
${}^{18}_{18}\text{Ar} \rightarrow {}^{17}_{17}\text{Cl}$
${}^{20}_{20}\text{Ca} \rightarrow {}^{19}_{19}\text{K}$
${}^{22}_{22}\text{Ti} \rightarrow {}^{21}_{21}\text{Sc}$
${}^{24}_{24}\text{Cr} \rightarrow {}^{23}_{23}\text{V}$
${}^{26}_{26}\text{Fe} \rightarrow {}^{25}_{25}\text{Mn}$
${}^{28}_{28}\text{Ni} \rightarrow {}^{27}_{27}\text{Co}$

${}^{26m}_{13}\text{Al} \rightarrow {}^{26}_{12}\text{Mg}$
${}^{34}_{17}\text{Cl} \rightarrow {}^{34}_{16}\text{S}$
${}^{38m}_{19}\text{K} \rightarrow {}^{38}_{18}\text{Ar}$
${}^{42}_{21}\text{Sc} \rightarrow {}^{42}_{20}\text{Ca}$
${}^{46}_{23}\text{V} \rightarrow {}^{46}_{22}\text{Ti}$
${}^{50}_{25}\text{Mn} \rightarrow {}^{50}_{24}\text{Cr}$
${}^{54}_{27}\text{Co} \rightarrow {}^{54}_{26}\text{Fe}$
${}^{62}_{31}\text{Ga} \rightarrow {}^{62}_{30}\text{Zn}$
${}^{66}_{33}\text{As} \rightarrow {}^{66}_{32}\text{Ge}$
${}^{70}_{35}\text{Br} \rightarrow {}^{70}_{34}\text{Se}$
${}^{74}_{37}\text{Rb} \rightarrow {}^{74}_{36}\text{Kr}$

# $V_{ud}$ from superallowed $0^+ - 0^+$ nuclear decays

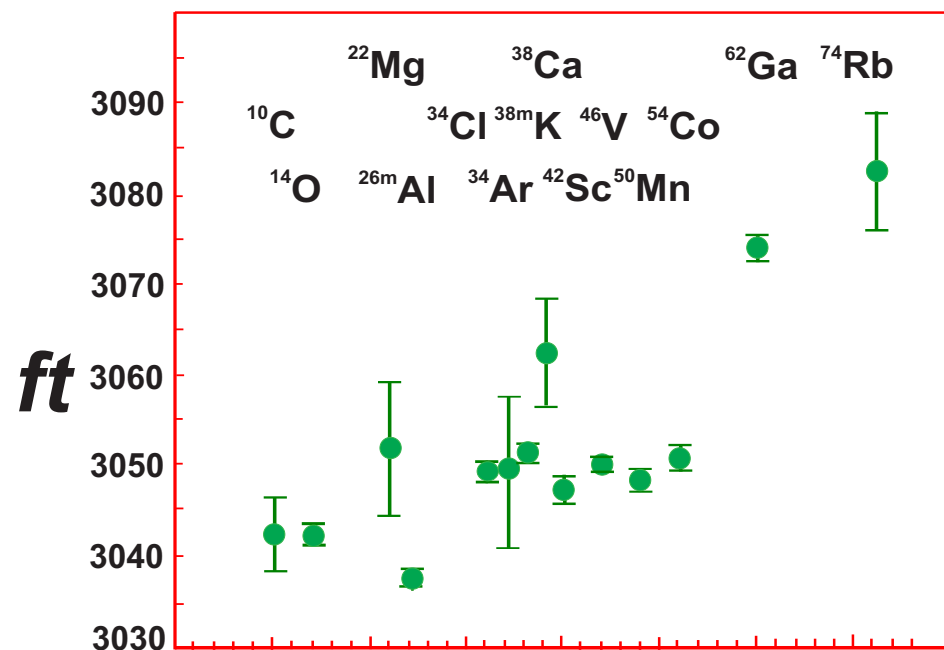
1. Transitions within  $J^P=0^+$  isotriplets ( $T=1$ )
2. Elementary process:  $p \rightarrow n e^+ \nu$
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${}_{6}^{10}\text{C} \rightarrow {}_{5}^{10}\text{B}$
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${}_{20}^{38}\text{Ca} \rightarrow {}_{19}^{38}\text{K}$
${}_{22}^{42}\text{Ti} \rightarrow {}_{21}^{42}\text{Sc}$
${}_{24}^{46}\text{Cr} \rightarrow {}_{23}^{46}\text{V}$
${}_{26}^{50}\text{Fe} \rightarrow {}_{25}^{50}\text{Mn}$
${}_{28}^{54}\text{Ni} \rightarrow {}_{27}^{54}\text{Co}$

${}_{13}^{26m}\text{Al} \rightarrow {}_{12}^{26}\text{Mg}$
${}_{17}^{34}\text{Cl} \rightarrow {}_{16}^{34}\text{S}$
${}_{19}^{38m}\text{K} \rightarrow {}_{18}^{38}\text{Ar}$
${}_{21}^{42}\text{Sc} \rightarrow {}_{20}^{42}\text{Ca}$
${}_{23}^{46}\text{V} \rightarrow {}_{22}^{46}\text{Ti}$
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${}_{33}^{66}\text{As} \rightarrow {}_{32}^{66}\text{Ge}$
${}_{35}^{70}\text{Br} \rightarrow {}_{34}^{70}\text{Se}$
${}_{37}^{74}\text{Rb} \rightarrow {}_{36}^{74}\text{Kr}$

Exp.: **f** - phase space (Q value)  
**t** - partial half-life ( $t_{1/2}$ , branching ratio)



ft values: same within  $\sim 2\%$  but not exactly!

Reason: SU(2) slightly broken

- a. RC (e.m. interaction does not conserve isospin)
- b. Nuclear WF are not SU(2) symmetric  
(proton and neutron distribution not the same)

# $V_{ud}$ extraction: Universal RC and Universal Ft

To obtain  $V_{ud}$   $\rightarrow$  absorb all decay-specific corrections into universal  $\mathbf{Ft}$

$$ft(1 + \text{RC} + \text{ISB}) = \mathcal{F}t(1 + \Delta_{\text{R}}^{\text{V}}) = ft(1 + \delta'_{\text{R}})(1 - \delta_{\text{C}} + \delta_{\text{NS}})(1 + \Delta_{\text{R}}^{\text{V}})$$

$\sim$  Measured  $\uparrow$  QED  $\rightarrow$  Isospin-breaking  $\rightarrow$  Nuclear structure  $\rightarrow$  Universal RC  $\uparrow$

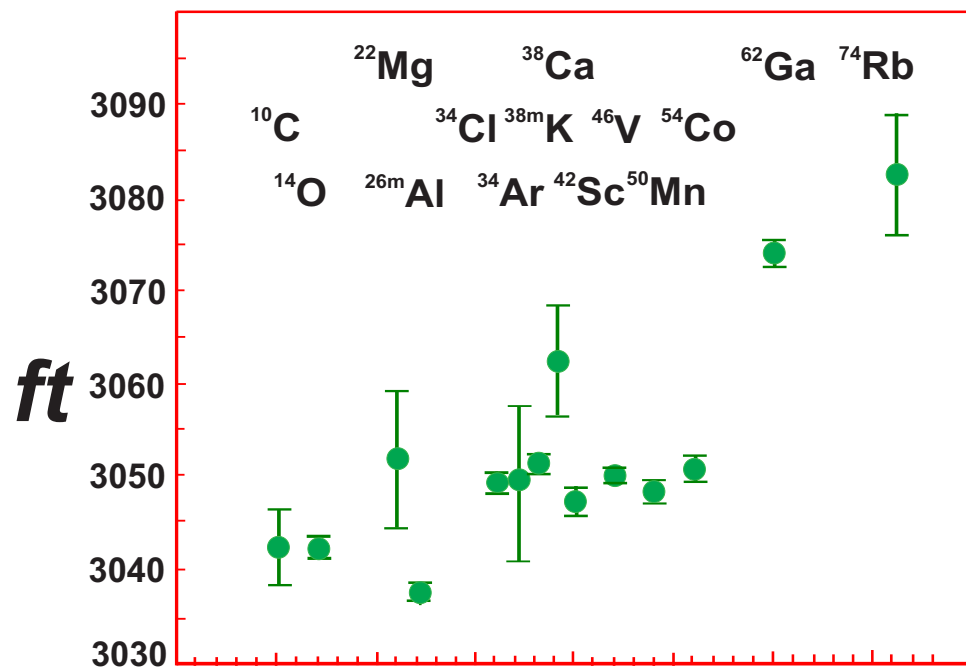


# $V_{ud}$ extraction: Universal RC and Universal Ft

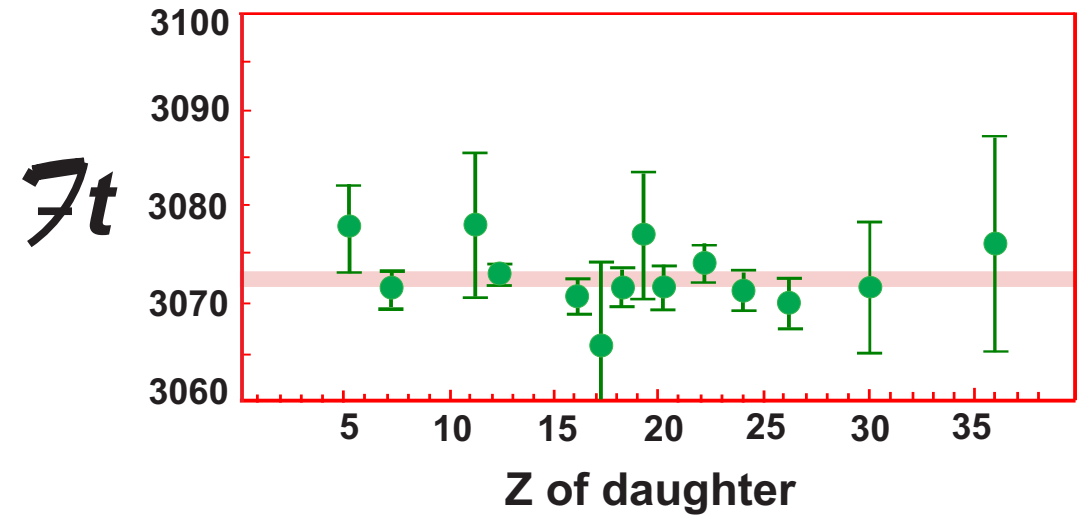
To obtain  $V_{ud}$  → absorb all decay-specific corrections into universal  $\overline{Ft}$

$$ft(1 + \text{RC} + \text{ISB}) = \mathcal{F}t(1 + \Delta_R^V) = ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS})(1 + \Delta_R^V)$$

$\uparrow$   $\sim$  Measured      QED      Isospin-breaking      Nuclear structure      Universal RC



→



Average of 14 decays

Hardy, Towner 1972 - 2020

Pre-2018:  $\overline{Ft} = 3072.1 \pm 0.7 s$

PDG 2024:  $\overline{Ft} = 3072 \pm 2 s$

$$|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1 + \Delta_R^V)}$$

$$|V_{ud}^{0^+-0^+}| = 0.9737 (1)_{exp,nucl} (3)_{NS} (1)_{RC} [3]_{total}$$

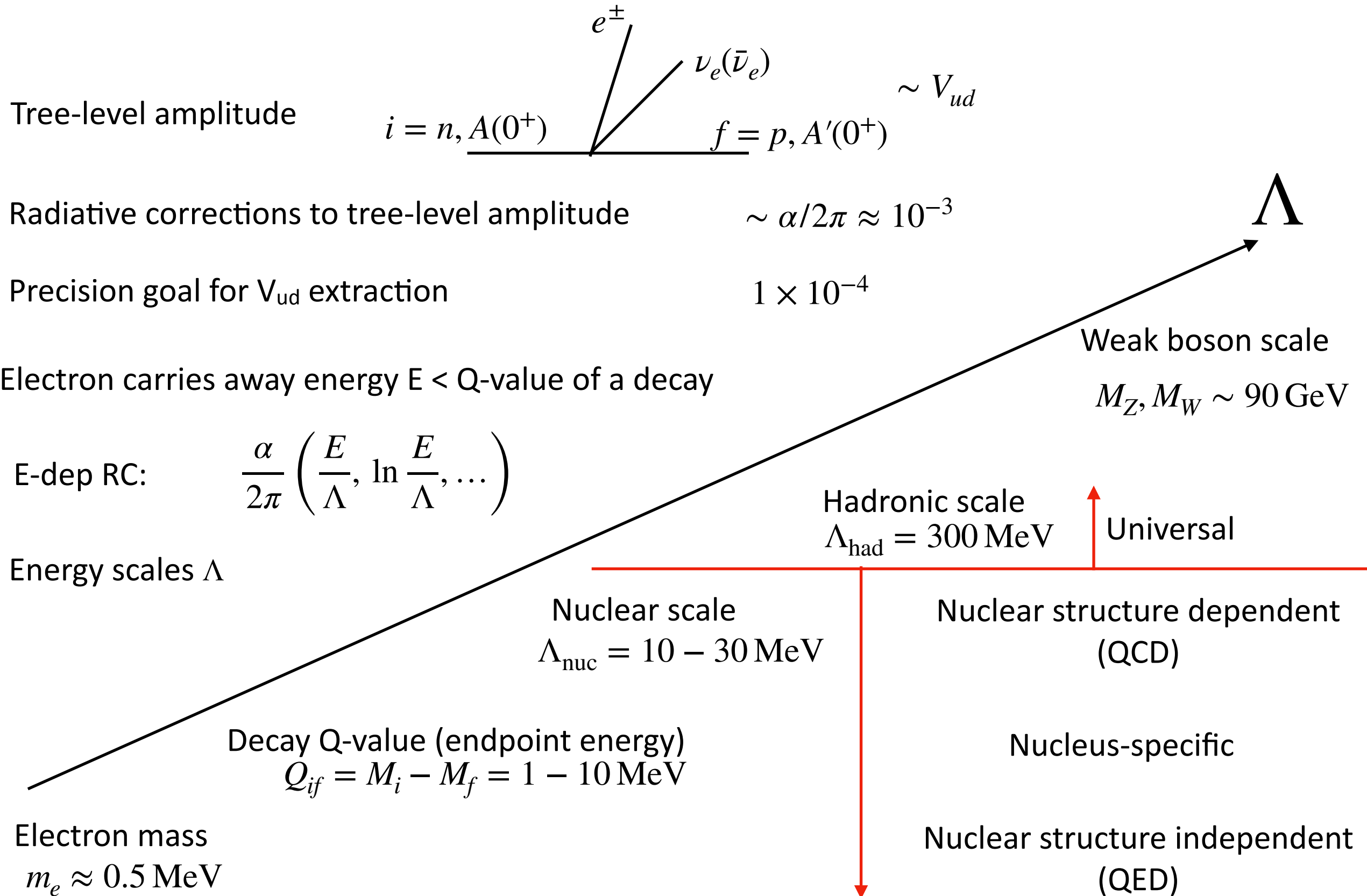
**Are all SM contributions under control?**



# Radiative Corrections to beta decay: Overall Setup



# RC to beta decay: overall setup



# RC to beta decay: separating scales

Generically: only IR and UV extremes feature large logarithms!

Works by Sirlin (1930-2022) and collaborators: all large logs under control

**IR: Fermi function (Dirac-Coulomb problem) + Sirlin function (soft Bremsstrahlung)**

**UV: large EW logs + pQCD corrections**

Inner RC:

energy- and model-independent

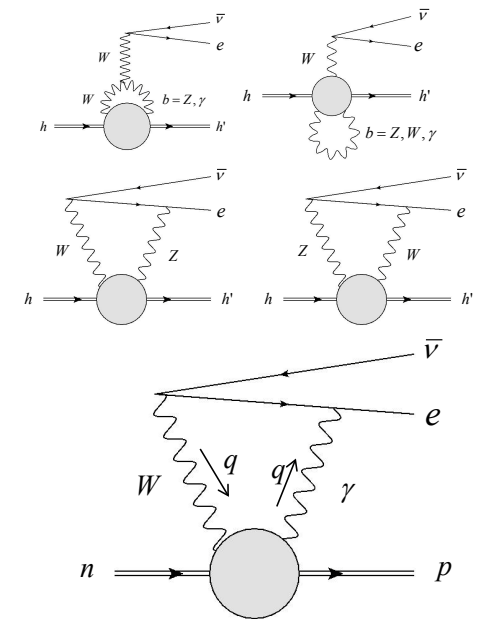
W,Z - loops

UV structure of SM

**$\gamma W$ -box: sensitive to all scales**

New method for computing EW boxes: dispersion theory

Combine exp. data with pQCD, lattice, EFT, ab-initio nuclear

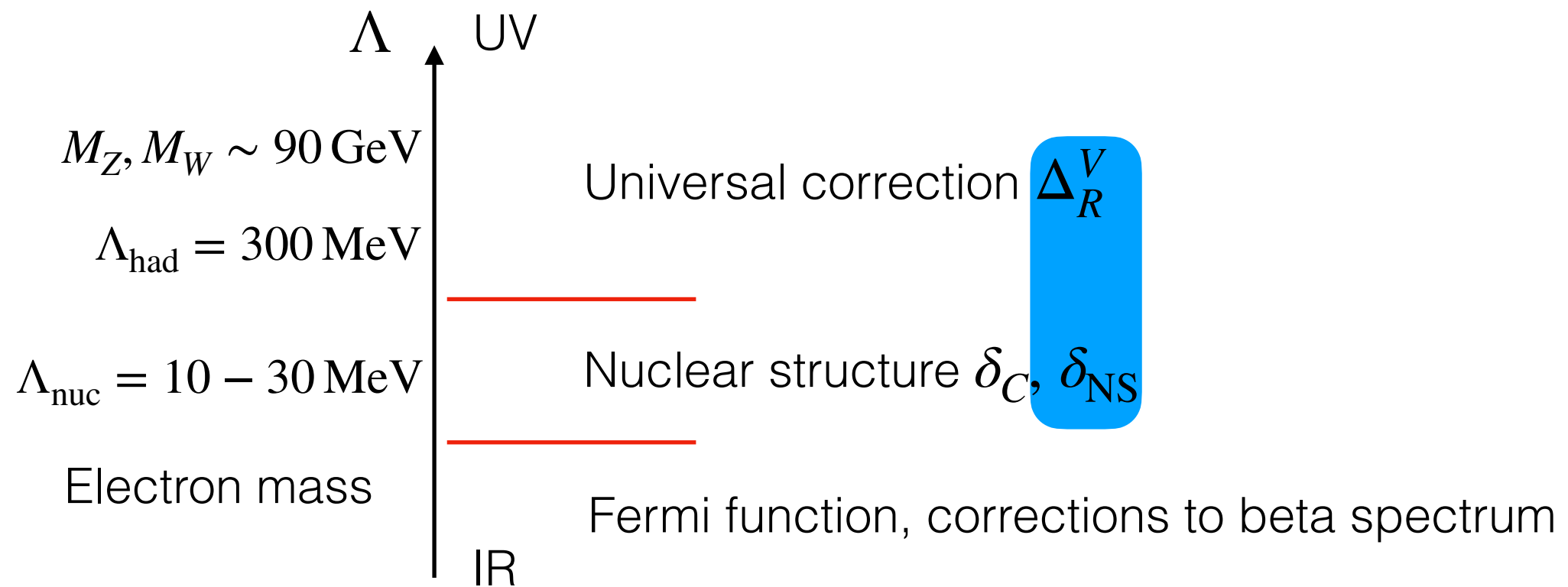


UV-sensitive  $\gamma W$ -box on free neutron  $\Delta_R^V$ : Sirlin, Marciano, Czarnecki 1967 - 2006

$$g_V^2 = |V_{ud}|^2 \left[ 1 + \frac{\alpha}{2\pi} \left\{ 3 \ln \frac{M_Z}{M_p} + \ln \frac{M_Z}{M_W} + \tilde{a}_g \right\} + \delta_{\text{QED}}^{\text{HO}} + 2 \square_{\gamma W} \right]$$

Nuclear structure:  $\delta_{\text{NS}} = 2(\square_{\gamma W}^{\text{Nucl}} - \square_{\gamma W}^{\text{free n}})$

All non-enhanced terms  $\sim \alpha/2\pi \sim 10^{-3}$  — only need to  $\sim 10\%$

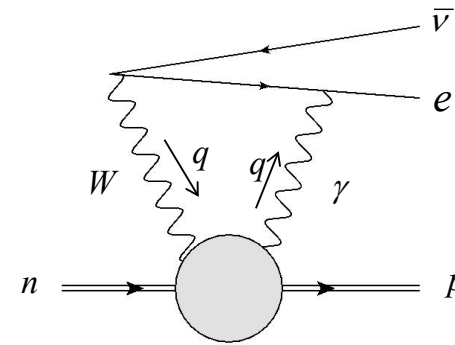


Unified Formalism for  $\Delta_R^V$  and  $\delta_{\text{NS}}$

Dispersion Theory of the  $\gamma W$ -box

# $\gamma W$ -box by Marciano & Sirlin

$$\square_{\gamma W}^{VA} = 4\pi\alpha \text{Re} \int \frac{d^4q}{(2\pi)^4} \frac{M_W^2}{M_W^2 + Q^2} \frac{Q^2 + \nu^2}{Q^4} \frac{T_3(\nu, Q^2)}{M\nu}$$



$$Q^2 = -q^2$$

$$\nu = (pq)/M$$

Marciano & Sirlin used loop techniques:

$$\square_{\gamma W}^{VA} = \frac{\alpha}{8\pi} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} F(Q^2)$$

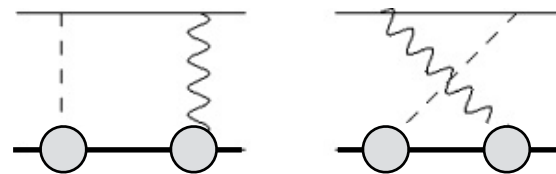
Short distance  $Q^2 \gg$

$$F^{\text{DIS}}(Q^2) = \frac{1}{Q^2} \quad \square_{\gamma W}^{\text{DIS}} = \frac{\alpha}{8\pi} \int_{\Lambda^2}^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} F^{\text{DIS}}(Q^2) = \frac{\alpha}{4\pi} \ln \frac{M_W}{\Lambda}$$

Finite  $Q^2$  -  
pQCD corrections:

$$F^{\text{DIS}} = \frac{1}{Q^2} \rightarrow \frac{1}{Q^2} \left[ 1 - \frac{\alpha_s^{\overline{MS}}}{\pi} \right]$$

Long distance  $Q^2 \ll$  - elastic box



MS 1987: asymptotic + pQCD + Born

$$\left[ \Delta_R^V \right]^{\gamma W} = \frac{\alpha}{2\pi} \left[ \ln \frac{M_W}{\Lambda} + A_g + 2C_B \right]$$

$\sim 4.1$ 
 $-0.24$ 
 $1.85$

Problem: connecting short and long distances

# $\gamma W$ -box by Marciano & Sirlin

MS 2006 update

Short distance:  
DIS to N<sup>3</sup>LO

$$F^{\text{DIS}} = \frac{1}{Q^2} \rightarrow \frac{1}{Q^2} \left[ 1 - \frac{\alpha_s^{\overline{MS}}}{\pi} - C_2 \left( \frac{\alpha_s^{\overline{MS}}}{\pi} \right)^2 - C_3 \left( \frac{\alpha_s^{\overline{MS}}}{\pi} \right)^3 \right]$$

GLS and Bjorken SR to N3LO  
Larin, Vermaseren 1997

$$Q^2 > Q_2^2$$

Interpolate between them

Vector Dominance Model Ansatz

$$F^{\text{INT}}(Q^2) = -\frac{1.490}{Q^2 + m_\rho^2} + \frac{6.855}{Q^2 + m_A^2} - \frac{4.414}{Q^2 + m_{\rho'}^2}$$

$$Q^2 < Q_1^2$$

Long distance: Born

$$F(Q^2) = F^B(Q^2)$$

$$\left[ \Delta_R^V \right]^{\gamma W} = \frac{\alpha}{2\pi} \left[ \underbrace{\ln \frac{M_W}{\Lambda}}_{\sim 3.86} + A_g + 2C_B \right]$$

1.78

$$\left[ \Delta_R^V \right]^{\gamma W} = \frac{\alpha}{2\pi} \left[ \underbrace{\ln \frac{M_W}{\Lambda}}_{\sim 3.77} + A_g + C^{\text{Int}} + 2C_B \right]$$

0.14(14) 1.66

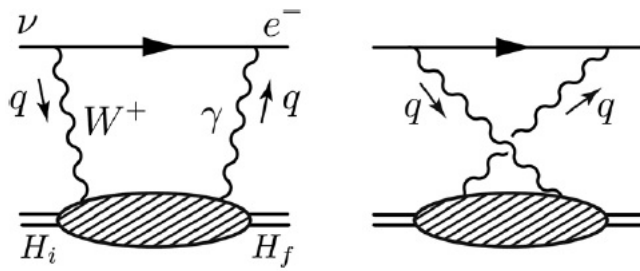
Uncertainty reduced by a factor  $\sim 2$



# $\gamma W$ -box from dispersion relations

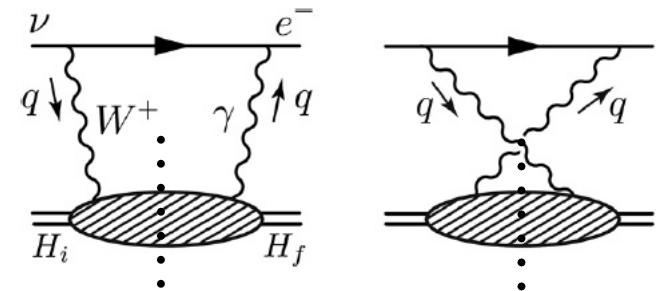
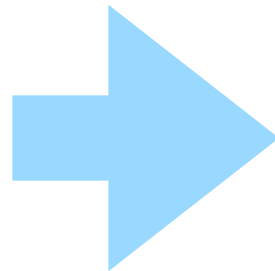
Model-dependent part or RC:  $\gamma W$ -box

$$\delta \mathfrak{M}_{\gamma W}^b = -i \frac{G_F V_{ud} e^2}{\sqrt{2}} L_\lambda \int \frac{d^4 k}{(2\pi)^4} \frac{m_W^2}{m_W^2 - k^2} \frac{\epsilon^{\mu\nu\alpha\lambda} k_\alpha}{[(p_e - k)^2 - m_e^2] k^2} T_{\mu\nu}$$



Generalized Compton tensor  
time-ordered product — complicated!

$$\int dx e^{iqx} \langle H_f(p) | T \{ J_{em}^\mu(x) J_W^{\nu,\pm}(0) \} | H_i(p) \rangle$$



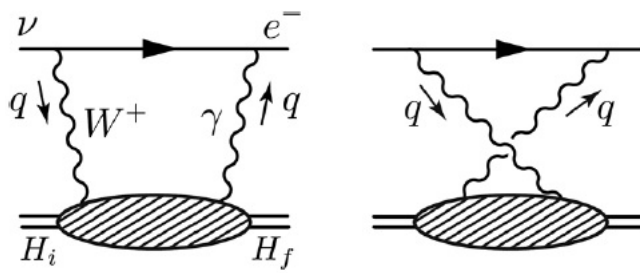
Commutator (Im part) - only on-shell  
hadronic states — related to data

$$\int dx e^{iqx} \langle H_f(p) | [J_{em}^\mu(x), J_W^{\nu,\pm}(0)] | H_i(p) \rangle$$

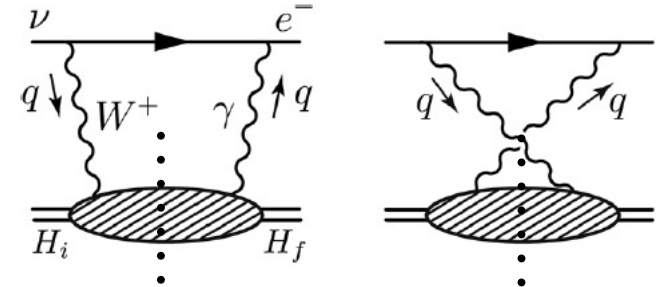
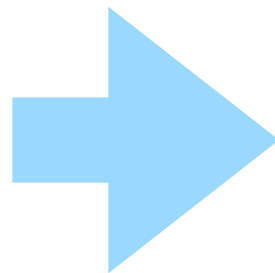
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Model-dependent part or RC:  $\gamma W$ -box

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Generalized Compton tensor  
time-ordered product — complicated!



Commutator (Im part) - only on-shell  
hadronic states — related to data

$$\int dx e^{iqx} \langle H_f(p) | T \{ J_{em}^\mu(x) J_W^{\nu,\pm}(0) \} | H_i(p) \rangle$$

Generalized (non-diagonal) Compton amplitudes

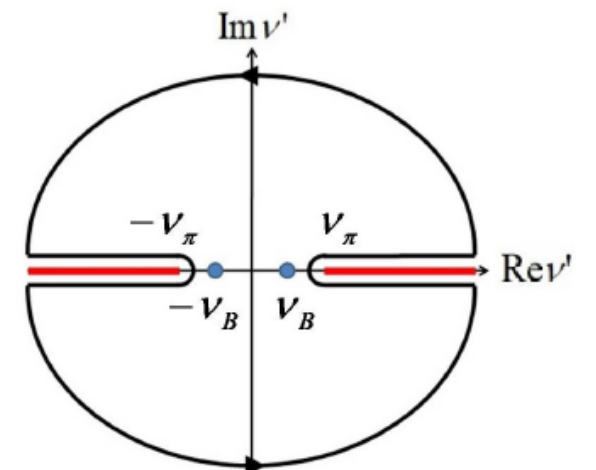
$$\int dx e^{iqx} \langle H_f(p) | [J_{em}^\mu(x), J_W^{\nu,\pm}(0)] | H_i(p) \rangle$$

Interference structure functions

Amplitudes = analytic functions inside the contour C in the complex  $\nu$ -plane with singularities on the real axis - poles + cuts

Discontinuity  $\sim$  structure functions

$$\text{Im } T_i^{\gamma W}(\nu, Q^2) = 2\pi F_i^{\gamma W}(\nu, Q^2)$$



# Universal RC from dispersion relations

Cauchy theorem (dispersion relation) + some algebra

$$\square_{\gamma W}^{b,e}(E_e) = \frac{\alpha}{\pi} \int_0^\infty dQ^2 \frac{M_W^2}{M_W^2 + Q^2} \int_{\nu_{\text{thr}}}^\infty \frac{d\nu'}{\nu'} \frac{\nu' + 2\sqrt{\nu'^2 + Q^2}}{(\nu' + \sqrt{\nu'^2 + Q^2})^2} \frac{F_{3,-}(\nu', Q^2)}{M f_+(0)} + \mathcal{O}(E_e^2)$$

$$\square_{\gamma W}^{b,o}(E_e) = \frac{2\alpha E_e}{3\pi} \int_0^\infty dQ^2 \int_{\nu_{\text{thr}}}^\infty \frac{d\nu'}{\nu'} \frac{\nu' + 3\sqrt{\nu'^2 + Q^2}}{(\nu' + \sqrt{\nu'^2 + Q^2})^3} \frac{F_{3,+}(\nu', Q^2)}{M f_+(0)} + \mathcal{O}(E_e^3)$$

Structure functions are measurable or may be related to data

# Universal RC from dispersion relations

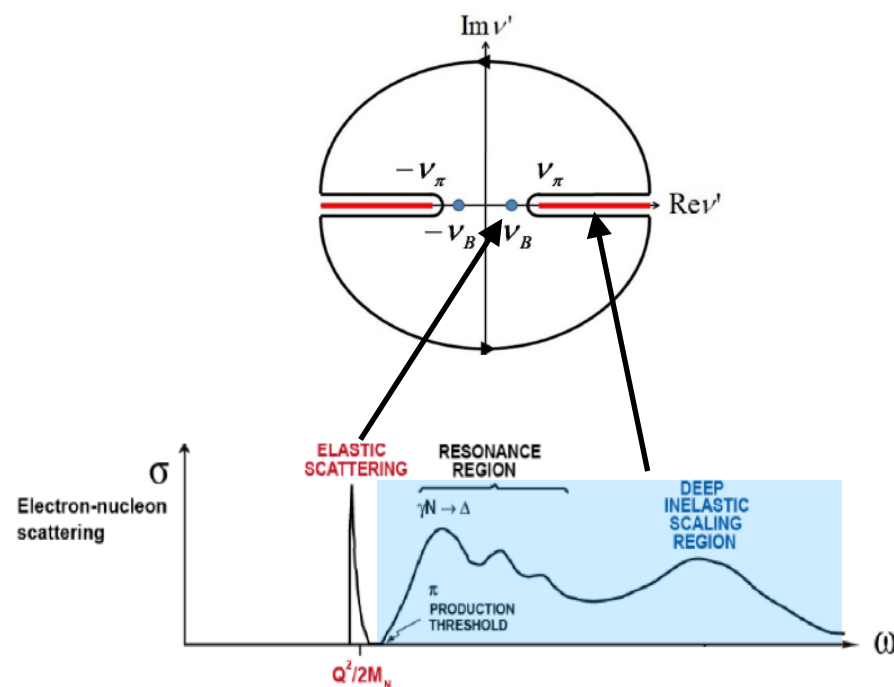
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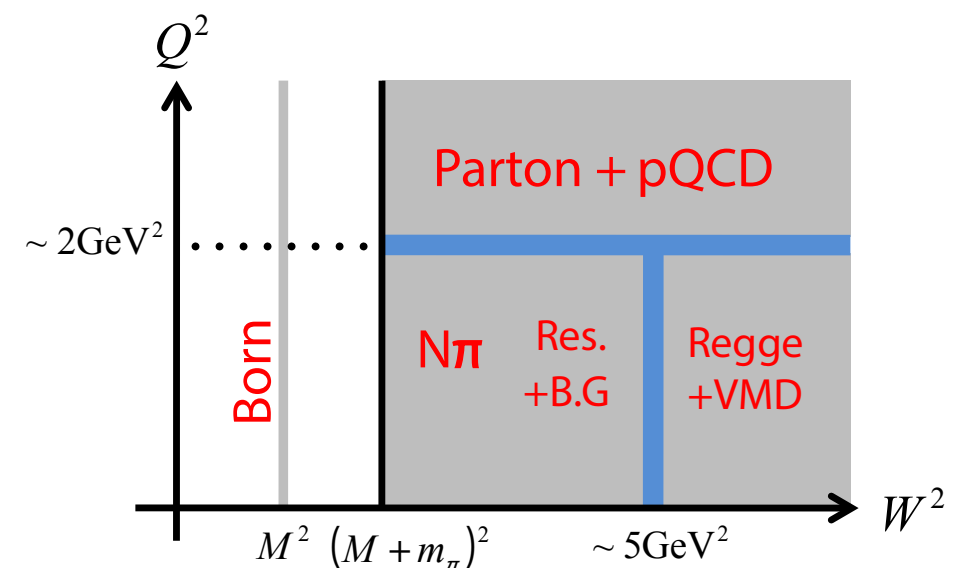
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Structure functions are measurable or may be related to data

Dispersion in energy:  
scanning hadronic intermediate states



Dispersion in  $Q^2$ :  
scanning dominant physics picture



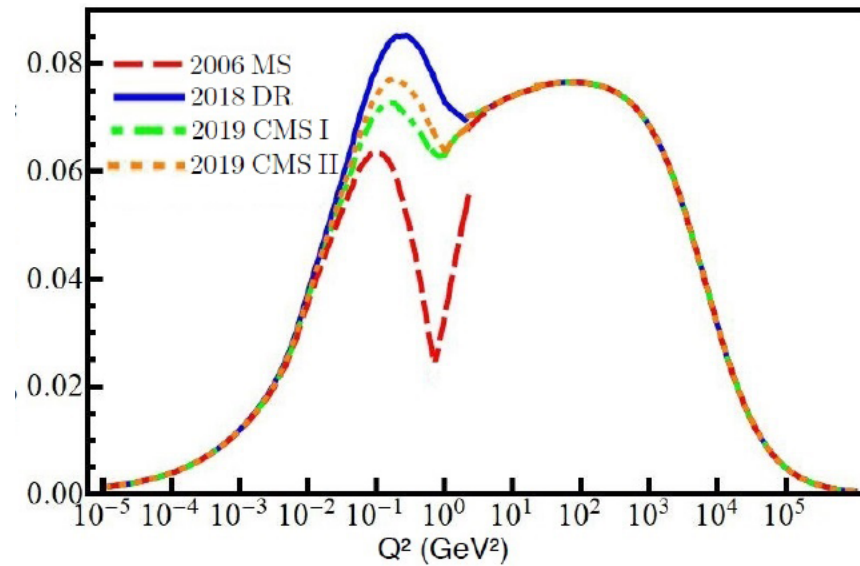
# Input into dispersion integral - $\nu/\bar{\nu}$ data

Isospin symmetry: Mixed CC-NC  $\gamma W$  SF (no data)  $\longleftrightarrow$  Purely CC WW SF (inclusive neutrino data)

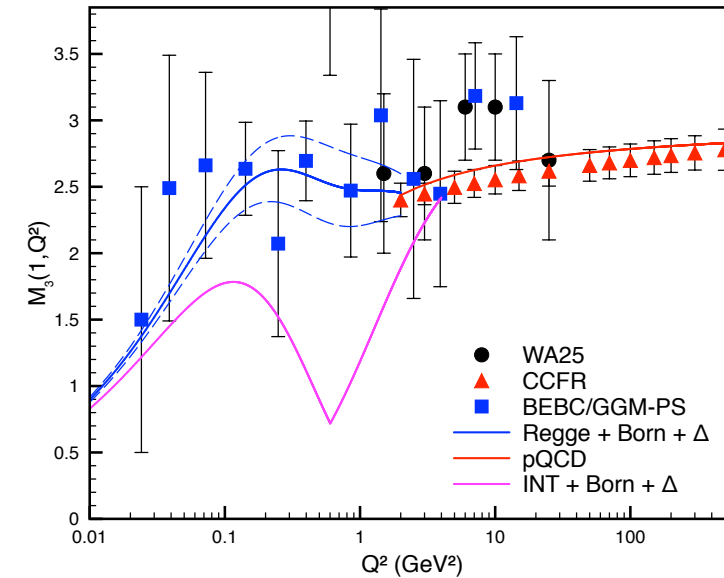
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Free neutron  $\gamma W$  box



Neutrino scattering data



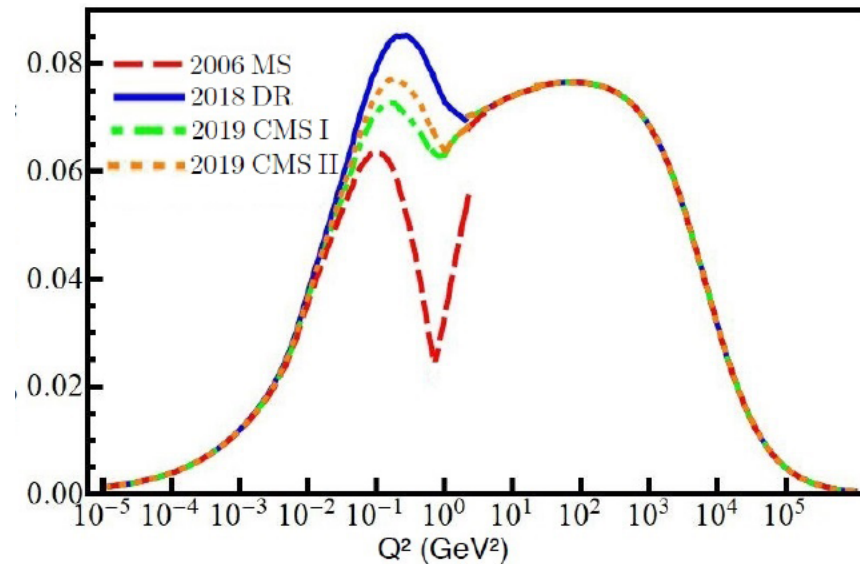
Marciano, Sirlin 2006:  $\Delta_R^V = 0.02361(38) \rightarrow |V_{ud}| = 0.97420(10)_{F_t(18)_{RC}}$

DR (Seng et al. 2018):  $\Delta_R^V = 0.02467(22) \rightarrow |V_{ud}| = 0.97370(10)_{F_t(10)_{RC}}$

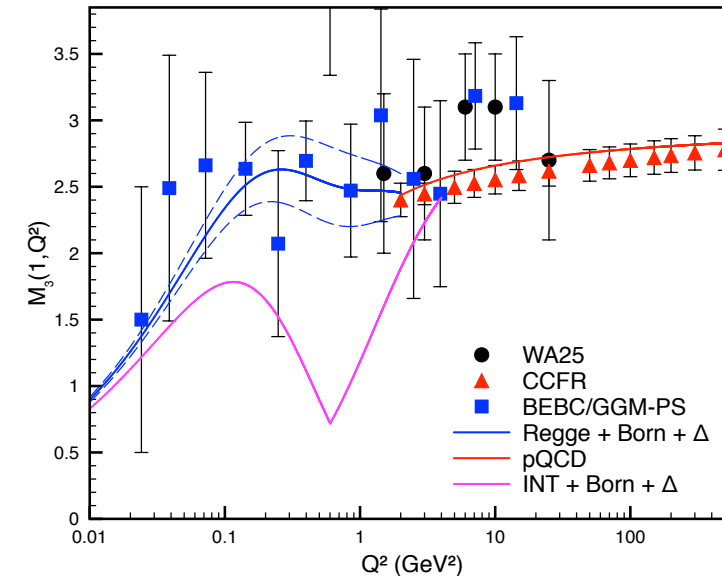
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DR (Seng et al. 2018):  $\Delta_R^V = 0.02467(22) \rightarrow |V_{ud}| = 0.97370(10)_{Ft}(10)_{RC}$

Shift upwards by  $3\sigma$  + reduction of uncertainty by factor 2

Confirmed by lattice QCD:

LQCD on pion + pheno:  $\Delta_R^V = 0.02477(24)_{\text{LQCD}^\pi + \text{pheno}}$

**Seng, MG, Feng, Jin, 2003.11264**  
**Yoo et al, 2305.03198**

LQCD on neutron:  $\Delta_R^V = 0.02439(19)_{\text{LQCD}^n}$

**Ma, Feng, MG et al 2308.16755**

# EFT: scale separation for free n

*Cirigliano et al, 2306.03138*

Effective Field Theory: explicit separation of scales + RGE running between

SM  $\rightarrow$  LEFT (no H,t,Z,W)  $\rightarrow$  ChPT  $\rightarrow$  NR QED

Formal consistency built in, RGE, transparent error estimation (naturalness)

Precision limited by matching (LEC) and HO — relies on inputs (e.g.  $\gamma W$ -box from DR)

To improve: need to go to higher order — new LECs, still tractable?

At present: order  $O(\alpha, \alpha\alpha_s, \alpha^2)$  — realistic to go beyond?

$$\frac{d\Gamma_n}{dE_e} = \frac{G_F^2 |V_{ud}|^2}{(2\pi)^5} (1 + 3\lambda^2) p_e E_e (E_0 - E_e)^2 [g_V(\mu_\chi)]^2 F_{NR}(\beta) \left(1 + \delta_{RC}(E_e, \mu_\chi)\right) \left(1 + \delta_{\text{recoil}}(E_e)\right)$$

$\lambda = g_A/g_V$   
 Extract from  
 Experiment

vector  
 coupling

$\pi^2, 1/\beta$   
 Enhanced

$\mathcal{O}(\alpha)$   
 [no logs]

$\mathcal{O}(m_e/m_N)$

Total RC:  $1 + \Delta_{\text{TOT}} = 1.07761(27) \%$

Good agreement within errors!

Total RC from DR:  $1 + \Delta_{\text{TOT}} = 1.07735(27) \%$



Nuclear-Structure RC  $\delta_{NS}$

# History of $\delta_{NS}$ : $\gamma W$ -box on nuclei

Jaus, Rasche 1990

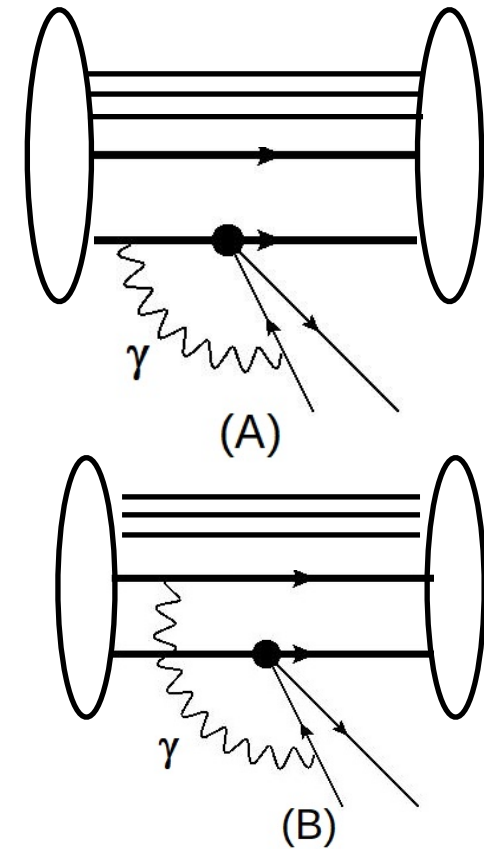
$\gamma$  and  $W$  on same nucleon  $\rightarrow$  already in  $\Delta_R^V$ : drop!

Towner 1994

Nucleons are bound — free-nucleon RC modified:  $\delta_{NS}^A$

Jaus, Rasche 1990; Hardy, Towner 1992-2020

$\gamma$  and  $W$  on distinct nucleons  $\rightarrow$  only in nuclei:  $\delta_{NS}^B$



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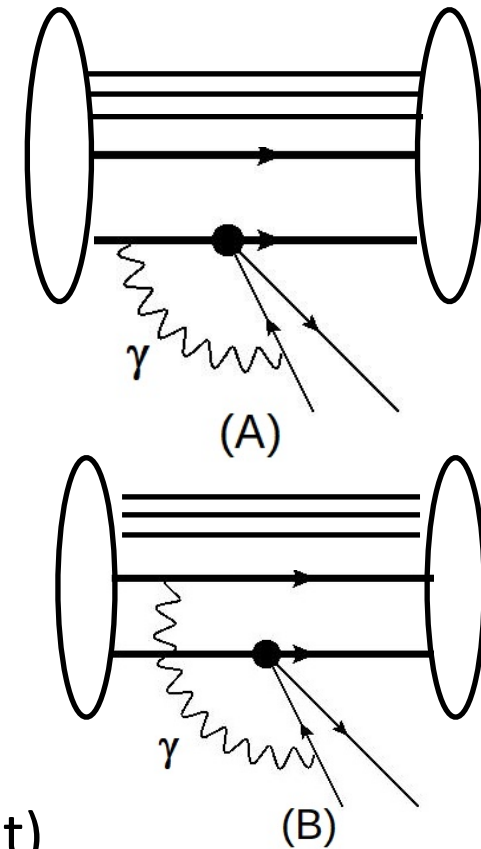
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Seng et al. 2018: continuum contribution (quasielastic nucleon knockout)

— can check the  $\delta_{NS}^A$  calculation explicitly in dispersion theory in Fermi gas model

MG 2019: energy dependence non negligible:  $\mathcal{F}t = ft(1 + \delta_{NS}^A + \delta_{NS}^{E\text{-dep}} + \dots)$



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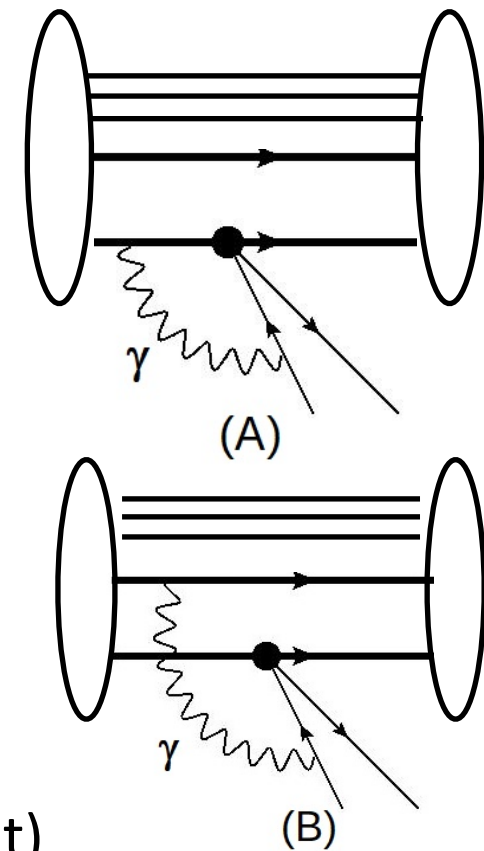
MG 2019: energy dependence non negligible:  $\mathcal{F}t = ft(1 + \delta_{NS}^A + \delta_{NS}^{E\text{-dep}} + \dots)$

Old estimate:  $\delta\mathcal{F}t = -(1.8 \pm 0.4)s + (0 \pm 0)s$

New estimate:  $\delta\mathcal{F}t = -(3.5 \pm 1.0)s + (1.6 \pm 0.5)s$

Two effects cancel but introduce 100% uncertainty:  $\mathcal{F}t = (3072.1 \pm 0.7)s \rightarrow \mathcal{F}t = (3072 \pm 2)s$

Has to be checked in modern ab-initio nuclear theory



# Dispersion theory of $\delta_{NS}$ with ab-initio input

Low-momentum part of the loop: account for nucleon d.o.f. only

Ab-initio methods:

*A. Ekström et al, **2212.11064***

NN interaction from chiral EFT: systematically improvable, self consistent

Solve many-body QM problem with potential derived from  $\chi$ EFT

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First case study:  $^{10}\text{C} \rightarrow ^{10}\text{B}$  in No-Core Shell Model (NCSM)

NCSM result for  $^{10}\text{C} \rightarrow ^{10}\text{B}$ :

$$\delta_{NS} = -0.406(39) \%$$

M. Gennari et al, **2405.19281**

Compare to Hardy-Towner

$$\delta_{NS} = -0.347(35) \% \quad (2014)$$

$$\delta_{NS} = -0.400(50) \% \quad (2020)$$

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Second case study:  $^{14}\text{O} \rightarrow ^{14}\text{N}$ : EFT + Green's Function Monte Carlo method (GFMC)

GFMC result for:  $^{14}\text{O} \rightarrow ^{14}\text{N}$

$$\delta_{NS}^{(0)} = - (1.76 + 0.11 \pm 0.88) \cdot 10^{-3}$$

*V. Cirigliano et al, 2405.18469*

Compare to Hardy-Towner

$$\delta_{NS,B} = -1.96(50) \cdot 10^{-3}$$

Large error: unknown contact term

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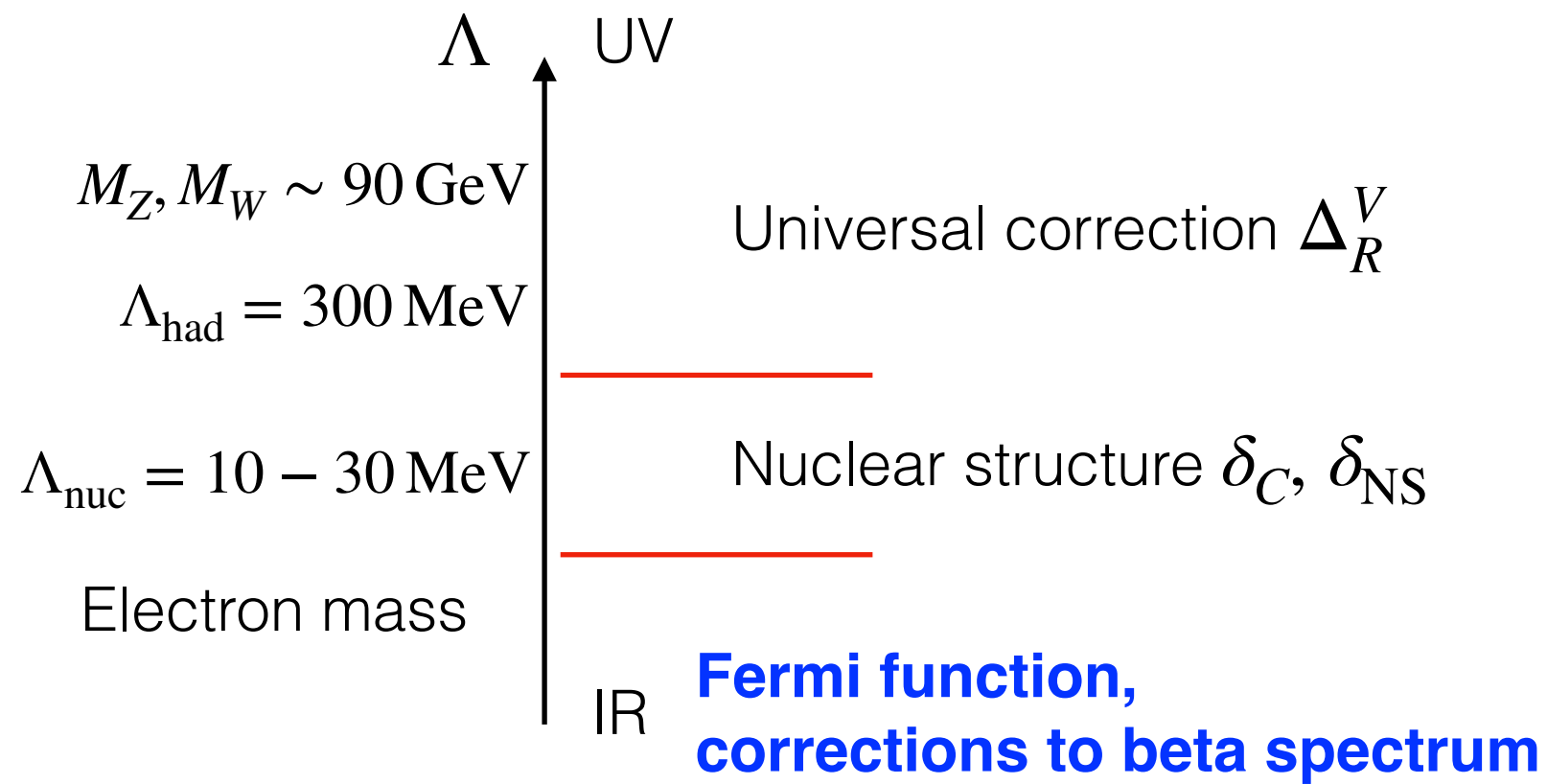
Compare to Hardy-Towner

$$\delta_{NS,B} = -1.96(50) \cdot 10^{-3}$$

Large error: unknown contact term

More to come in the near future!





# Nuclear Corrections

# QED: Corrections to Decay Spectrum

$$f = m_e^{-5} \int_{m_e}^{E_0} dE_e |\vec{p}_e| E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)$$

Unperturbed beta spectrum

Fermi function:  $e^+$  in Coulomb field of daughter nucleus

**Shape factor: spatial distribution of decay**

Atomic screening and overlap corrections

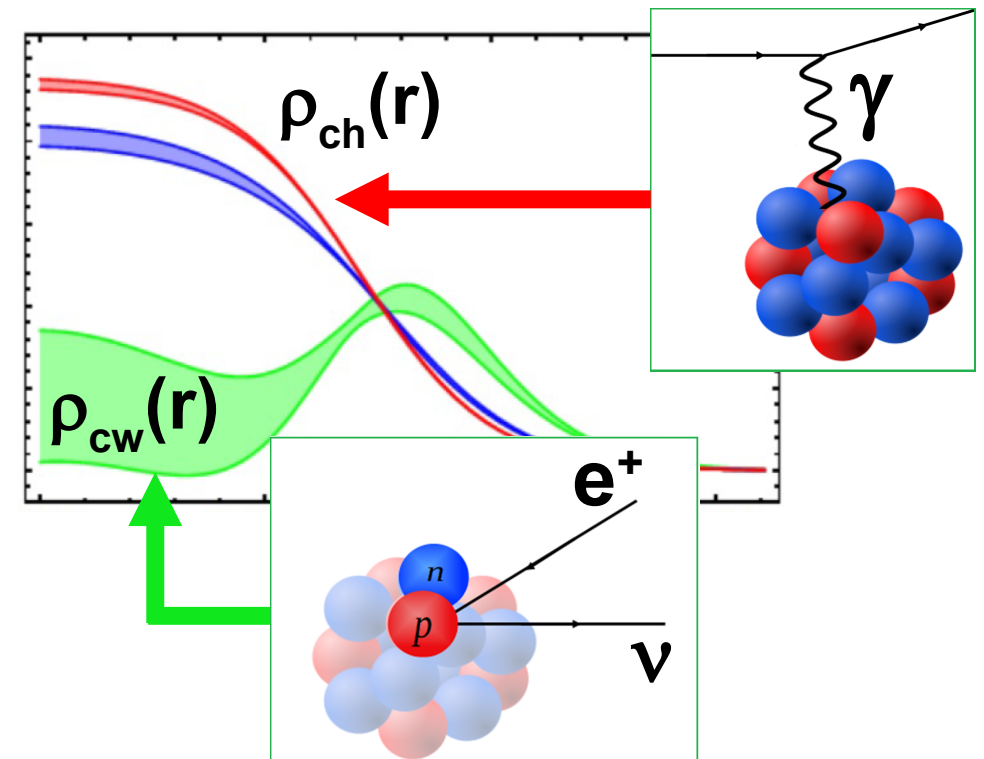
Recoil correction

Traditionally: assumed that decay probability is equally distributed across the entire nucleus

Recent development:  
isospin symmetry + known charge distributions

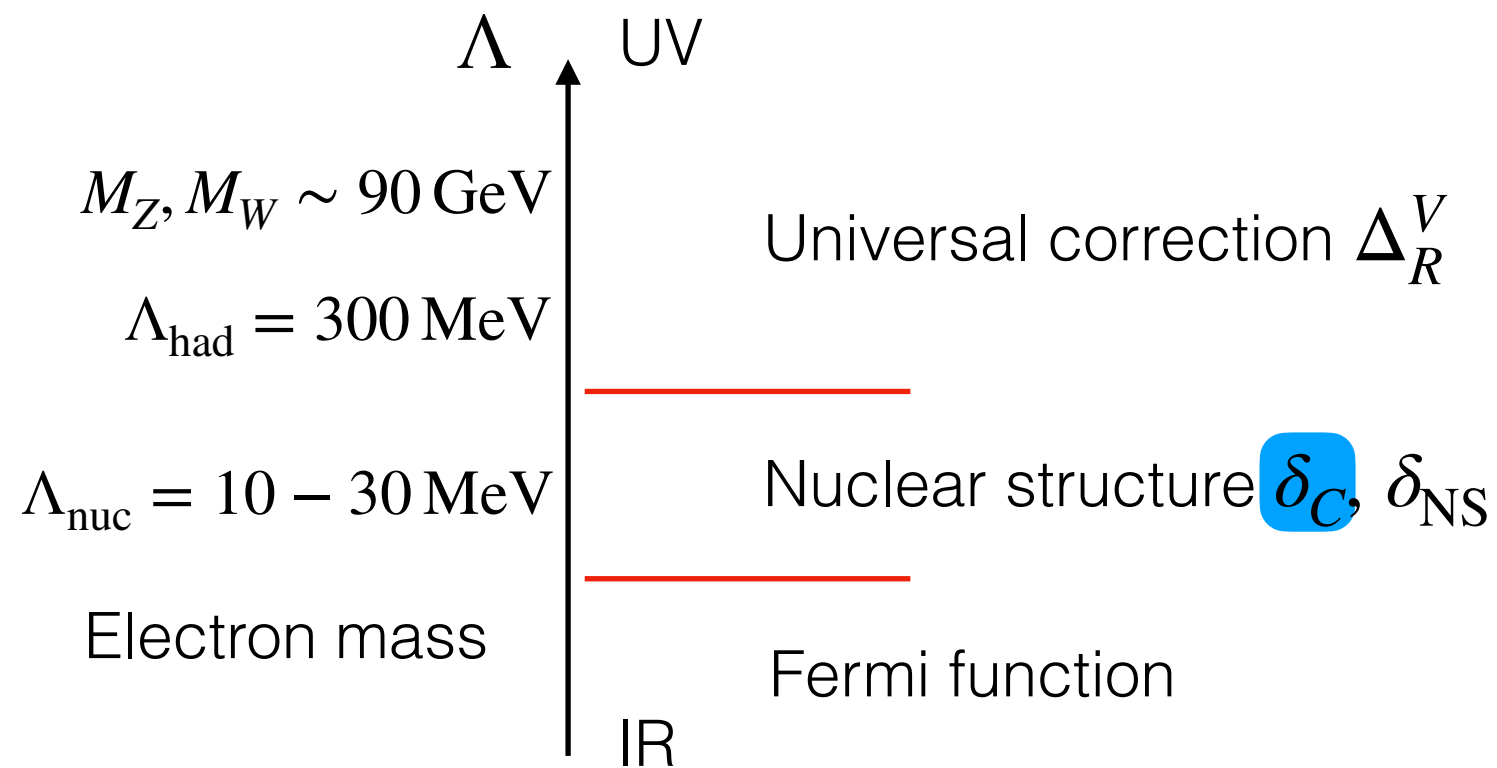
Only the outer protons can decay:  
all neutron states in the core occupied

Photon probes the entire nuclear charge



**Relative shift in f-values downwards of 0.01-0.1%**  
**Non-negligible given the precision goal 0.01%**

Seng, 2212.02681  
MG, Seng 2311.16755



Isospin breaking in nuclear WF:  $\delta_C$   
 Tree-level effect — ISB “large”

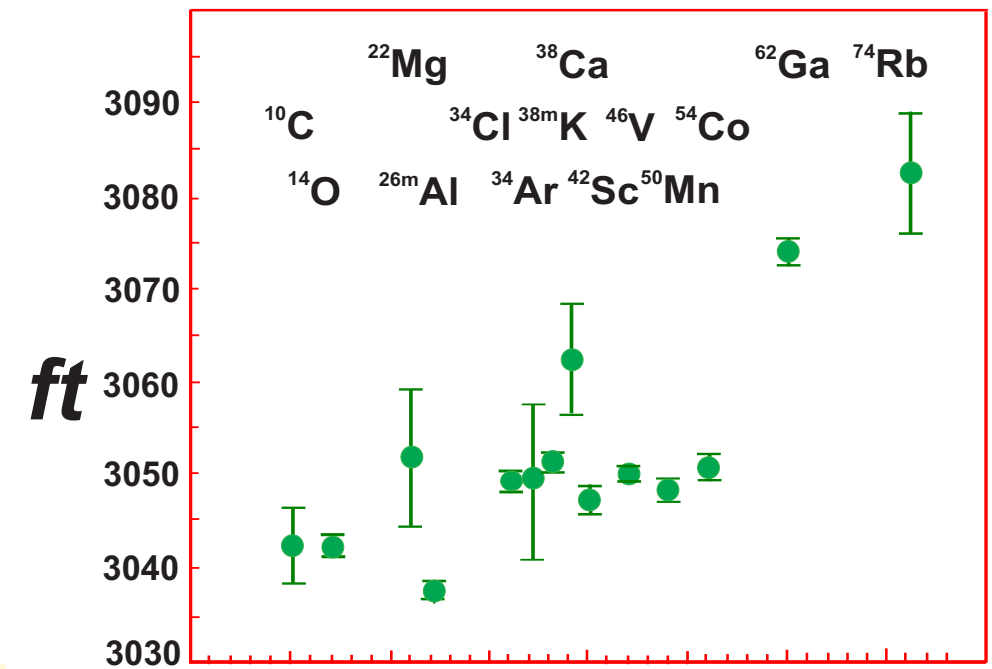
# Isospin symmetry breaking in superallowed $\beta$ -decay

Tree-level Fermi matrix element

$$M_F = \langle f | \tau^+ | i \rangle$$

$\tau^+$  — Isospin operator

$|i\rangle, |f\rangle$  — members of T=1 isotriplet



# Isospin symmetry breaking in superallowed $\beta$ -decay

Tree-level Fermi matrix element

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$\tau^+$  — Isospin operator

$|i\rangle, |f\rangle$  — members of T=1 isotriplet

If isospin symmetry were exact,  $M_F \rightarrow M_0 = \sqrt{2}$

Isospin symmetry is broken in nuclear states  
(e.g. Coulomb, nucleon mass difference, ...)

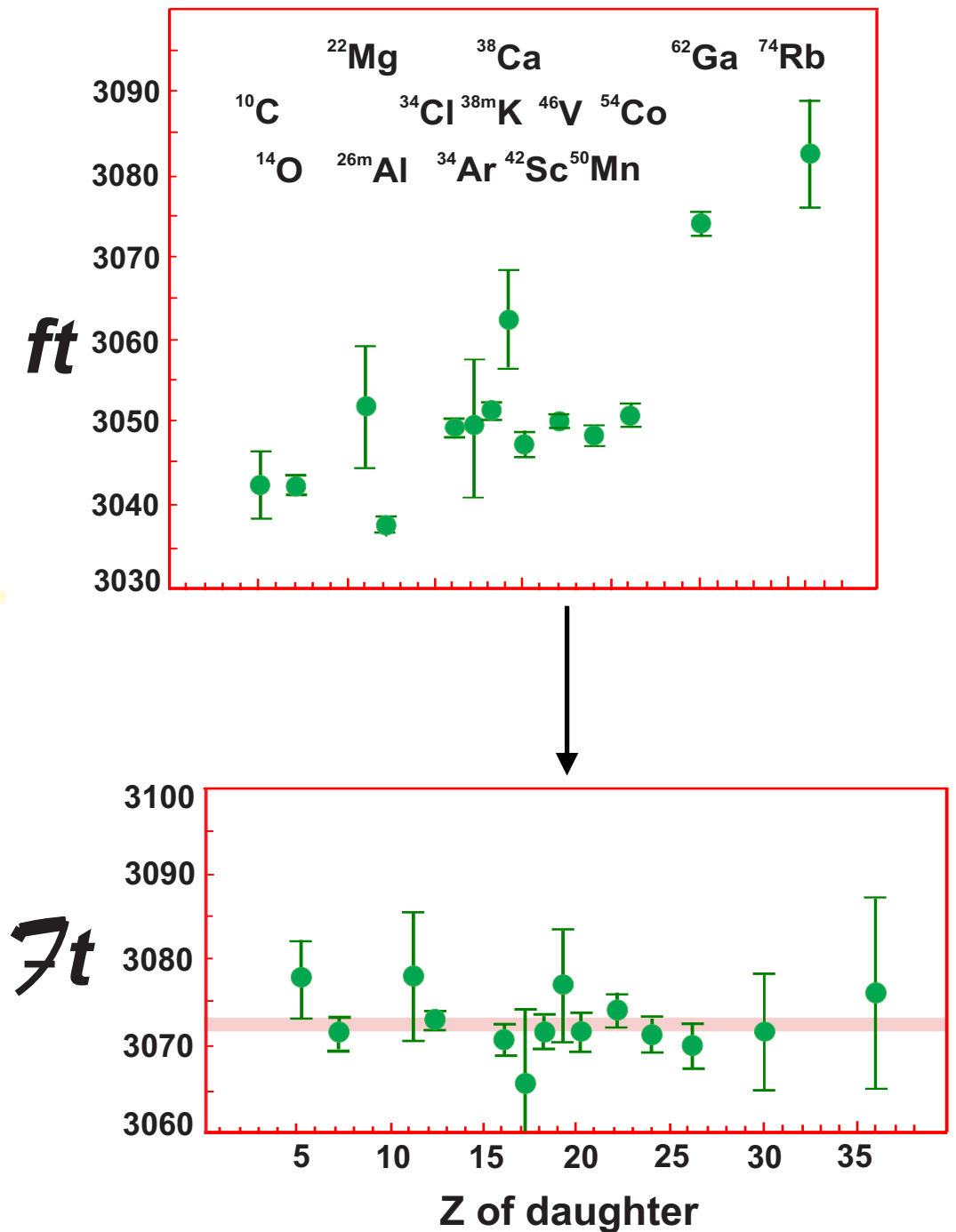
In presence of isospin symmetry breaking (ISB):

$$|M_F|^2 = |M_0|^2 (1 - \delta_C) \quad \text{MacDonald 1958}$$

ISB correction almost singlehandedly aligns ft-values!

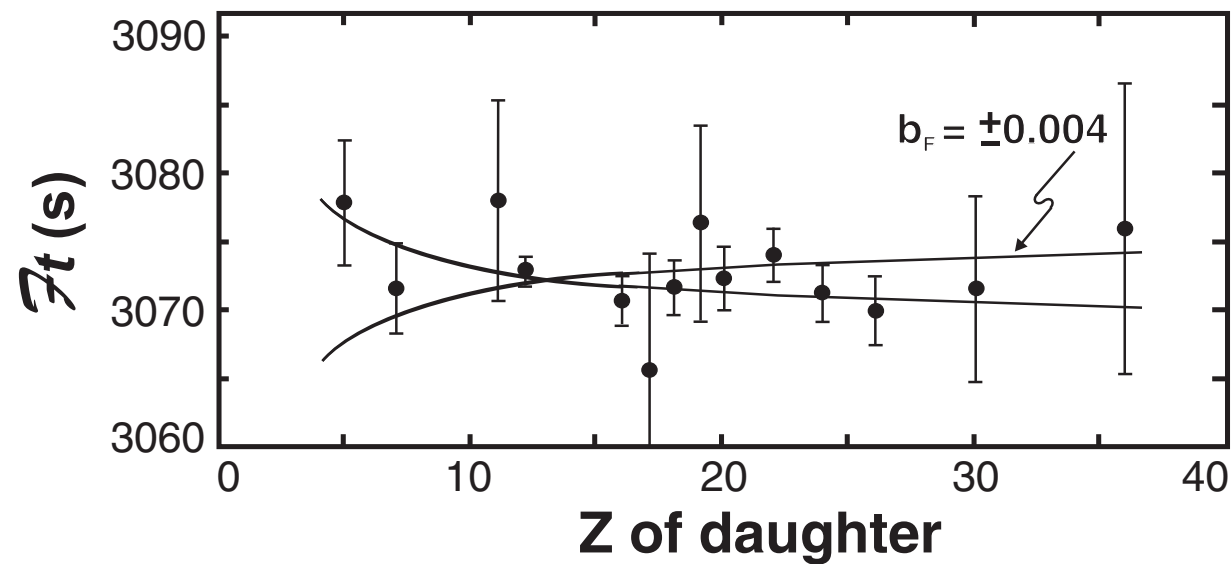
$$\delta_C \sim 0.17\% - 1.6\%!$$

Crucial for  $V_{ud}$  extraction



*J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501*

# BSM searches with superallowed beta decays



Induced scalar CC  $\rightarrow$  Fierz interference  $b_F$

$$Ft^{SM} \rightarrow Ft^{SM} \left( 1 + b_F \frac{m_e}{\langle E_e \rangle} \right)$$

$$b_F = -0.0028(26) \sim \text{consistent with } 0$$

Independently of  $V_{ud}$  and CKM unitarity: internal consistency of the data base with SM!

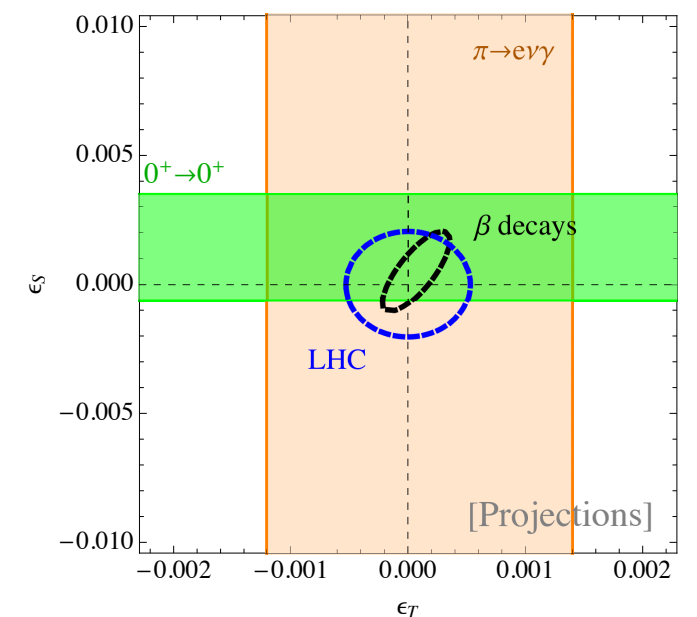
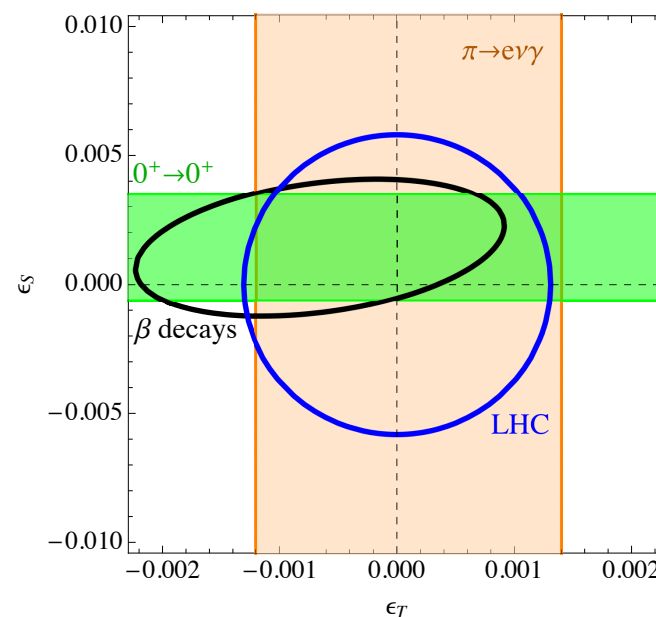
Like  $\Delta_{CKM}^{(3)}$  does not require other experimental inputs to make a statement on (B)SM

Entangled with nuclear theory uncertainties — a global effort of nuclear theory community needed

S, T interaction flips helicity:  
Suppressed at high energy

Beta decay vs. LHC on S,T  
Complementarity now and in the future!

*Gonzalez-Alonso et al 1803.08732*

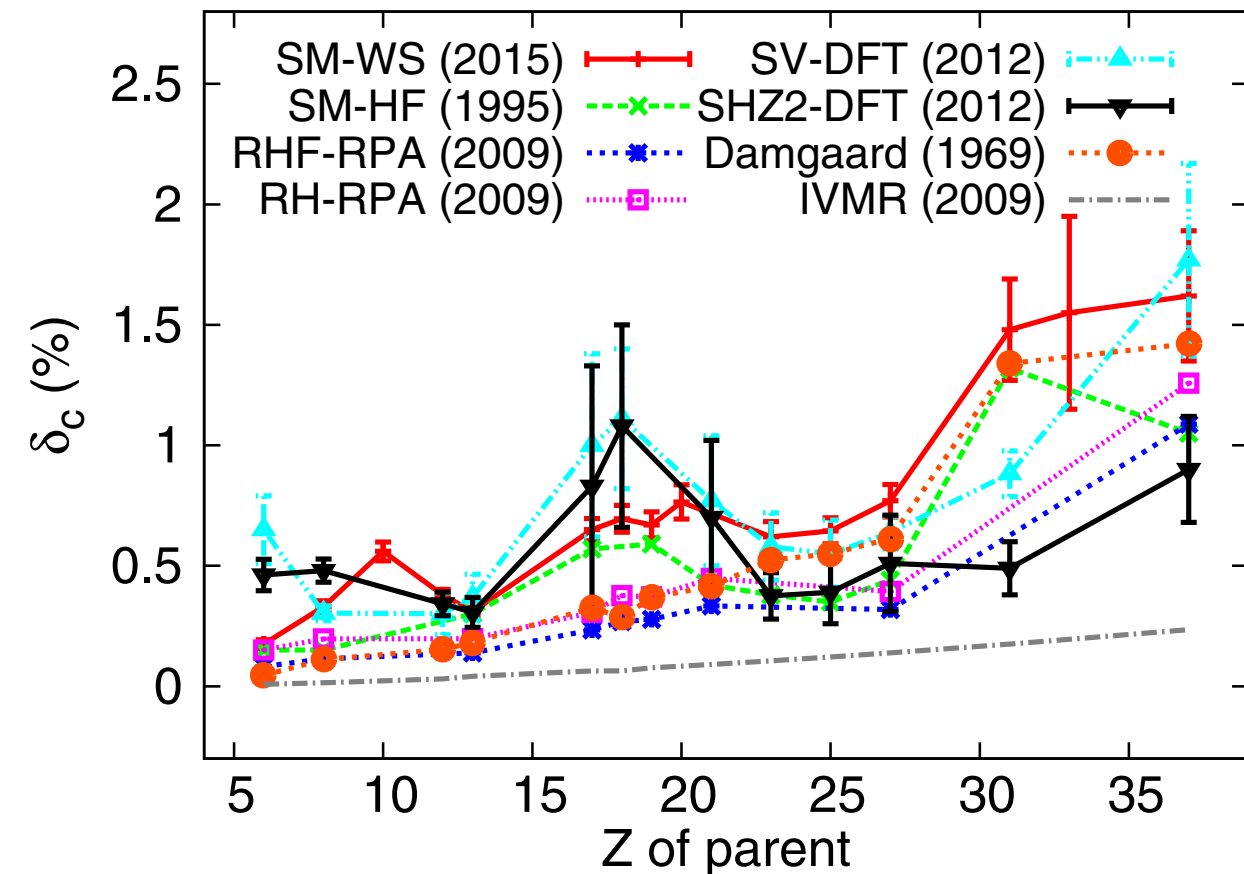


# Nuclear model dependence of $\delta_C$

*J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501*

	RPA					IVMR <sup>a</sup>	DFT
	SM-WS	SM-HF	PKO1	DD-ME2	PC-F1		
$T_z = -1$							
<sup>10</sup> C	0.175	0.225	0.082	0.150	0.109	0.147	0.650
<sup>14</sup> O	0.330	0.310	0.114	0.197	0.150		0.303
<sup>22</sup> Mg	0.380	0.260					0.301
<sup>34</sup> Ar	0.695	0.540	0.268	0.376	0.379		
<sup>38</sup> Ca	0.765	0.620	0.313	0.441	0.347		
$T_z = 0$							
<sup>26m</sup> Al	0.310	0.440	0.139	0.198	0.159		0.370
<sup>34</sup> Cl	0.650	0.695	0.234	0.307	0.316		
<sup>38m</sup> K	0.670	0.745	0.278	0.371	0.294	0.434	
<sup>42</sup> Sc	0.665	0.640	0.333	0.448	0.345		0.770
<sup>46</sup> V	0.620	0.600					0.580
<sup>50</sup> Mn	0.645	0.610					0.550
<sup>54</sup> Co	0.770	0.685	0.319	0.393	0.339		0.638
<sup>62</sup> Ga	1.475	1.205					0.882
<sup>74</sup> Rb	1.615	1.405	1.088	1.258	0.668		1.770
$\chi^2/\nu$	1.4	6.4	4.9	3.7	6.1		4.3 <sup>b</sup>

*L. Xayavong, N.A. Smirnova, Phys.Rev. C 97 (2018), 024324*



HT:  $\chi^2$  as criterion to prefer SM-WS;  $V_{ud}$  and limits on BSM strongly depend on nuclear model

Nuclear community embarked on ab-initio  $\delta_C$  calculations (NCSM, GFMC, CC, IMSRG)  
Especially interesting for light nuclei accessible to different techniques!

# Data-Driven $\delta_C$ from nuclear radii

ISB-sensitive combinations of radii can be constructed

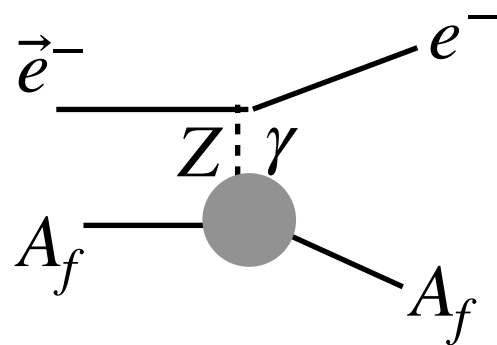
Seng, MG 2208.03037; 2304.03800; 2212.02681

$$\Delta M_B^{(1)} \equiv \frac{1}{2} \left( Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2 \right) - Z_0 R_{p,0}^2$$

$\Delta M_B^{(1)} = 0$  used for ft-value in isospin limit

$$\Delta M_A^{(1)} \equiv - \langle r_{CW}^2 \rangle + \left( \frac{N_1}{2} \langle r_{n,1}^2 \rangle - \frac{Z_1}{2} \langle r_{p,1}^2 \rangle \right)$$

Neutron radius: measurable with PV e- scattering!



$$A^{PV} = - \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{Q_W}{Z} \frac{F_{NW}(Q^2)}{F_{Ch}(Q^2)}$$

$$R_{NW} \approx R_n$$

Z-boson couples to neutrons, photon - to protons;

PV asymmetry at low  $Q^2$  sensitive to the difference  $\langle r_{n,1}^2 \rangle - \langle r_{p,1}^2 \rangle$  - neutron skin

Extensive studies in neutron rich nuclei (PREX, CREX)  $\rightarrow$  input to physics of neutron stars

Upcoming exp. program at Mainz (MREX)

Neutron skins of stable daughters (e.g. Mg-26, Ca-42, Fe-54)

PV asymmetry on C-12 for a sub-% measurement of  $R_n$

**N. Cargioli, MG et al, 2407.09743**

Unexpected connections via neutron skins:

ISB for precision tests vs. EoS of neutron-rich matter



# Summary & Outlook

Cabibbo Angle Anomaly at  $2-3\sigma$

Future experiments:

Neutron: UCN $\tau$ ,  $\tau$ SPECT ( $\delta\tau_n : 0.4 \rightarrow 0.1s$ ); PERC, Nab ( $\delta g_A : 4 \rightarrow 1 \times 10^{-4}$ )

Competitive! But: resolve existing discrepancies (e.g. “beam-bottle” lifetime)

Kaon decays: NA62, BELLE II  $K\ell 3$  vs  $K\mu 2$  (+ Lattice effort!)

Pion:  $\pi^+ \rightarrow \pi^0 e^+ \nu$  PIONEER @ PSI ( $\delta BR: 0.3\% \rightarrow 0.03\%$ ) but 2033+

Nuclear uncertainties under scrutiny:  $\delta_{NS}$  in ab-initio and EFT

$\delta_C$  &  $\delta_{NS}$  for 15 decays from  $^{10}C$  to  $^{74}Rb$  — Community effort required!

Nuclear charge radii across superallowed isotriplets

Stable:  $\mu$ -atoms @ PSI, radii of unstable nuclei @ ISOLDE, TRIUMF

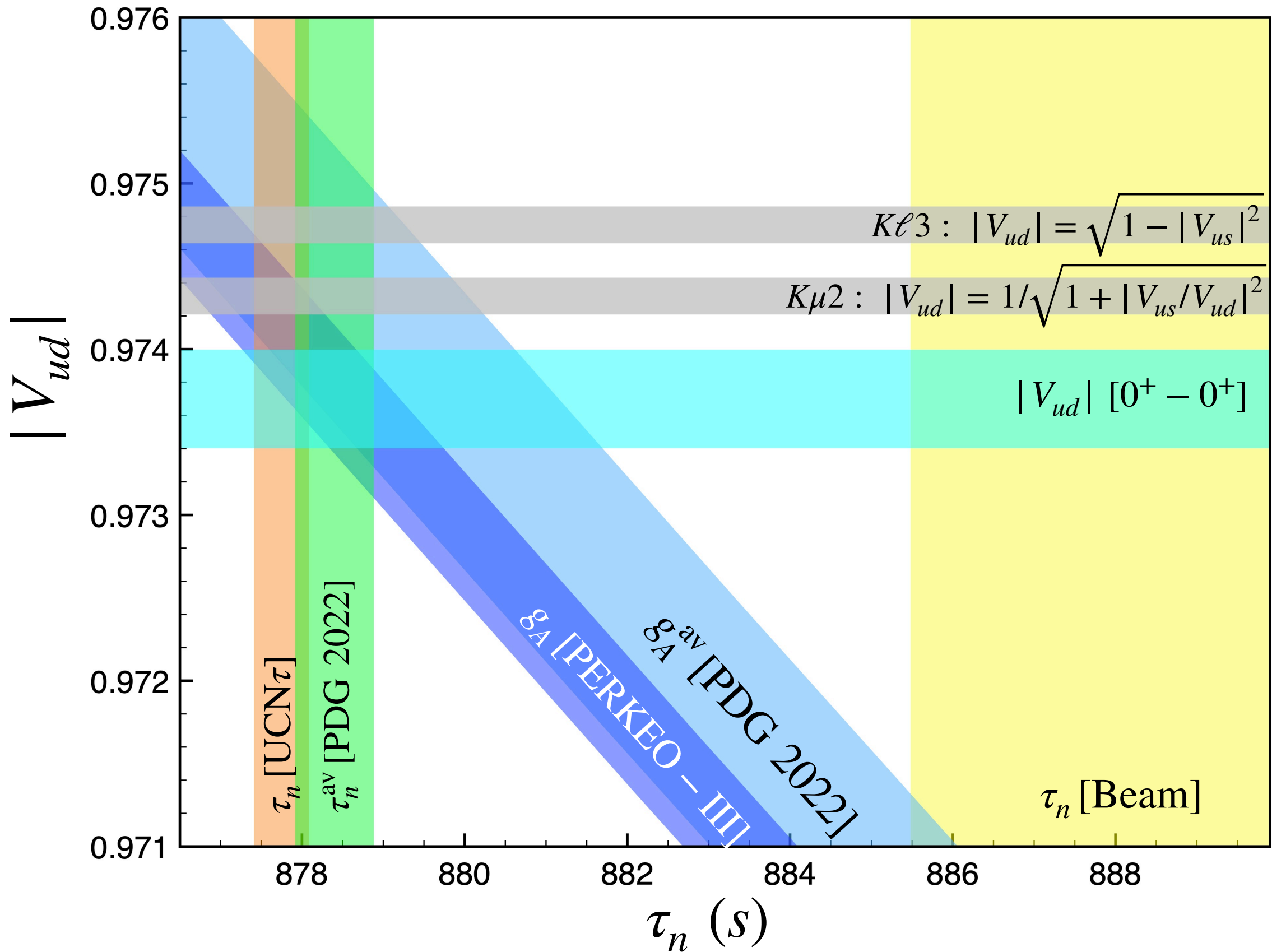
Neutron skins of stable daughters with PVES @ MESA

Interplay with the nuclear EoS program: neutron skin via symmetry energy vs. ISB

BSM: Cabibbo anomaly(ies) and superallowed dataset consistency

Backup

# Status of Cabibbo Unitarity



# $\gamma W$ -box from DR + Lattice QCD input

Currently available neutrino data at low  $Q^2$  - low quality;

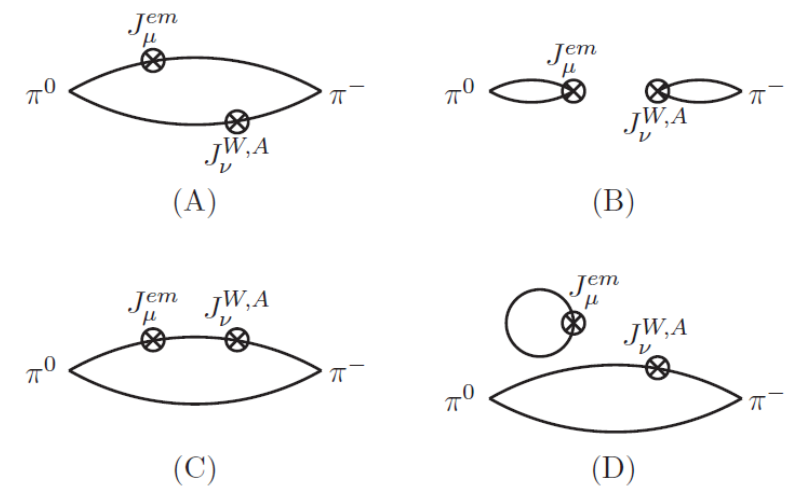
Look for alternative input — compute Nachtmann moment  $M_3^{(0)}$  on the lattice

First direct LQCD computation  $\pi^- \rightarrow \pi^0 e^- \nu_e$

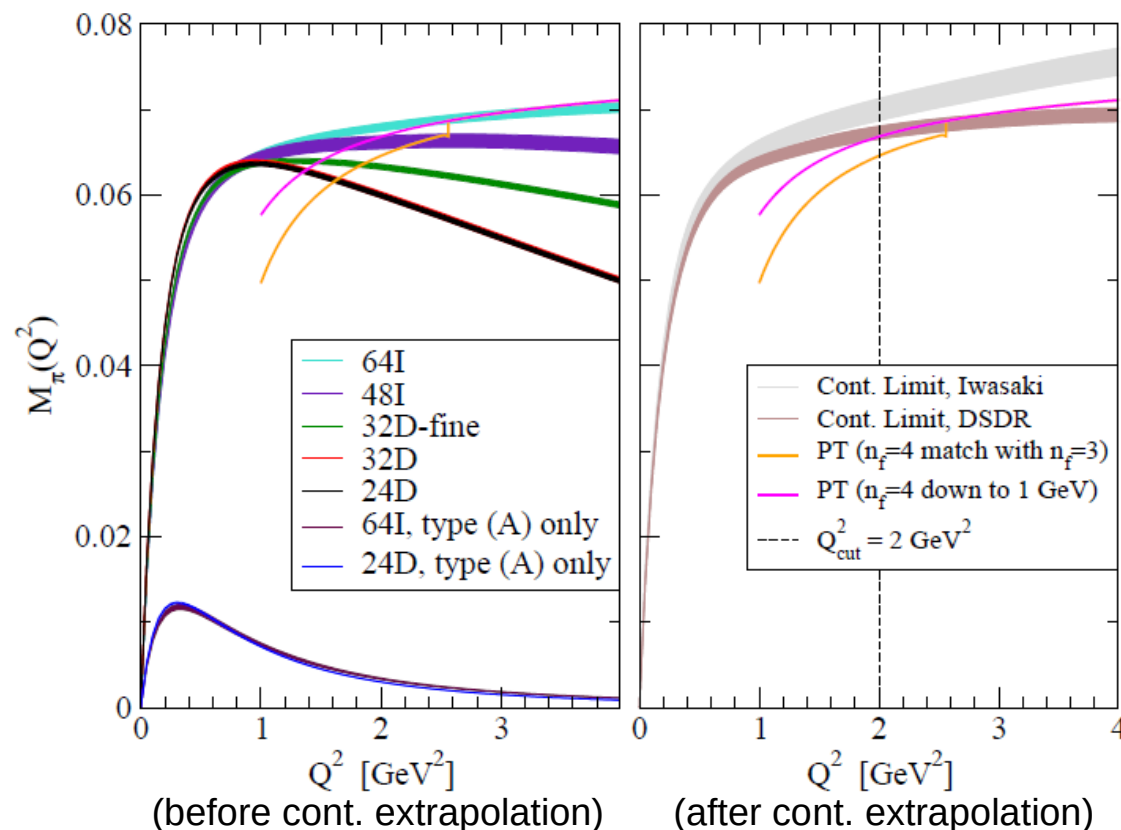
Feng, MG, Jin, Ma, Seng 2003.09798

5 LQCD gauge ensembles at physical pion mass  
Generated by RBC and UKQCD collaborations  
w. 2+1 flavor domain wall fermion

Match onto pQCD at  $Q^2 \sim 2 \text{ GeV}^2$



Quark contraction diagrams



$$\square_{\gamma W}^{VA, \pi} = 2.830(11)_{\text{stat}}(26)_{\text{sys}}$$

Independent calculation by Los Alamos group

Yoo et al, 2305.03198

$$\square_{\gamma W}^{VA, \pi} = 2.810(26)_{\text{stat+sys}}$$

# First lattice QCD calculation of $\gamma W$ -box

Direct impact for pion decay  $\pi^+ \rightarrow \pi^0 e^+ \nu_e$

Previous calculation of  $\delta$  — in ChPT

Significant reduction of the uncertainty!

Indirectly constrains the free neutron  $\gamma W$ -box  
— requires some phenomenology  
Based on Regge universality & factorization

Independent confirmation

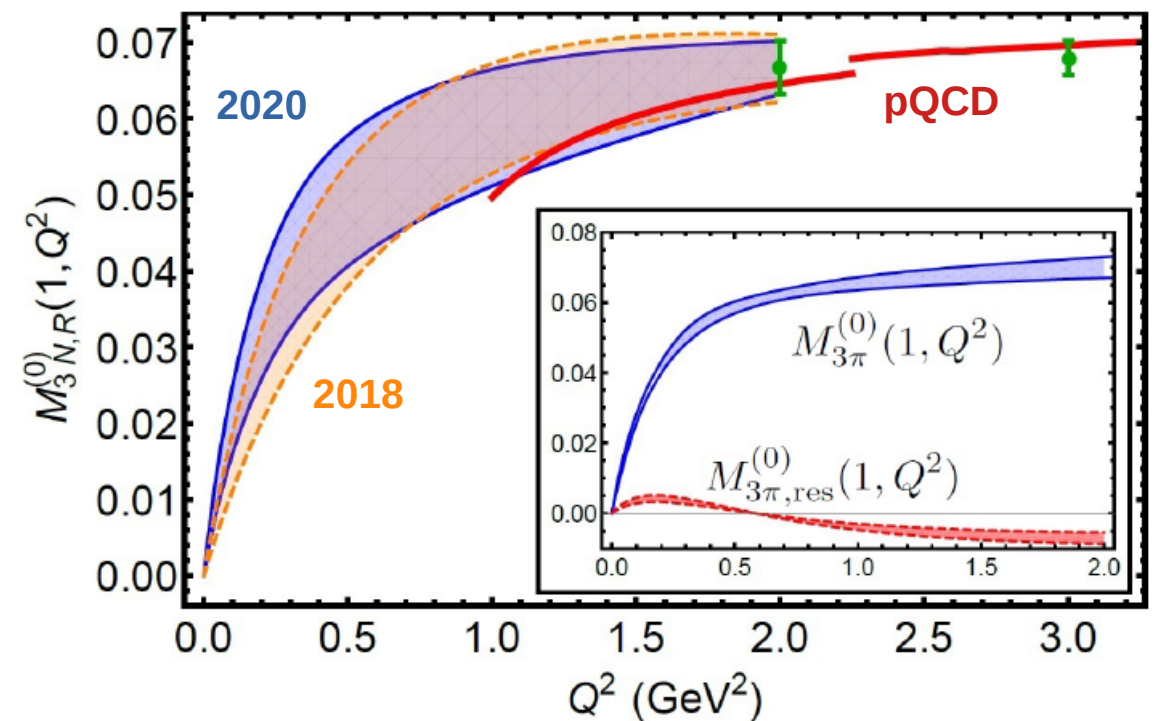
$$\Delta_R^V = 0.02467(22)_{\text{DR}} \rightarrow 0.02477(24)_{\text{LQCD+DR}}$$

Seng, MG, Feng, Jin, 2003.11264

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)} \frac{\Gamma_{\pi\ell 3}}{0.3988(23) \text{ s}^{-1}}$$

*Cirigliano, Knecht, Neufeld and Pichl, EPJC 2003*

$$\delta : 0.0334(10)_{\text{LEC}}(3)_{\text{HO}} \rightarrow 0.0332(1)_{\gamma W}(3)_{\text{HO}}$$



# First LQCD calculation of $\gamma W$ -box on the neutron

Much more challenging than pion:

Numerically heavier

Excited state contamination requires longer time

Large contribution from low  $Q \sim g_A \mu^V$  absent for pion

Split into long/short distance separated by  $t_s$

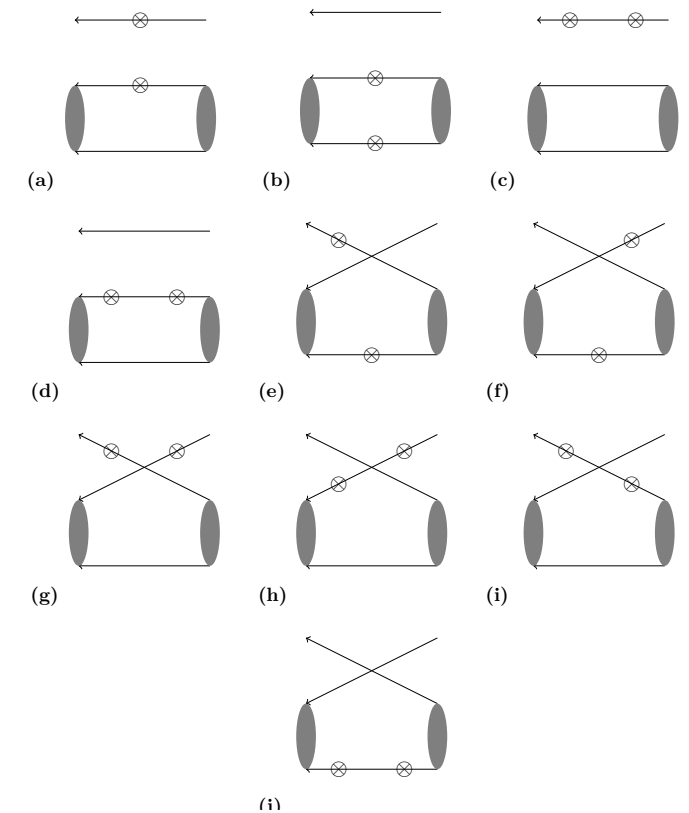
$$M_n(Q^2) = M_n^{\text{SD}}(Q^2, t_s) + M_n^{\text{LD}}(Q^2, t_s, t_g)$$

RBC/UKQCD 2+1 domain wall fermion

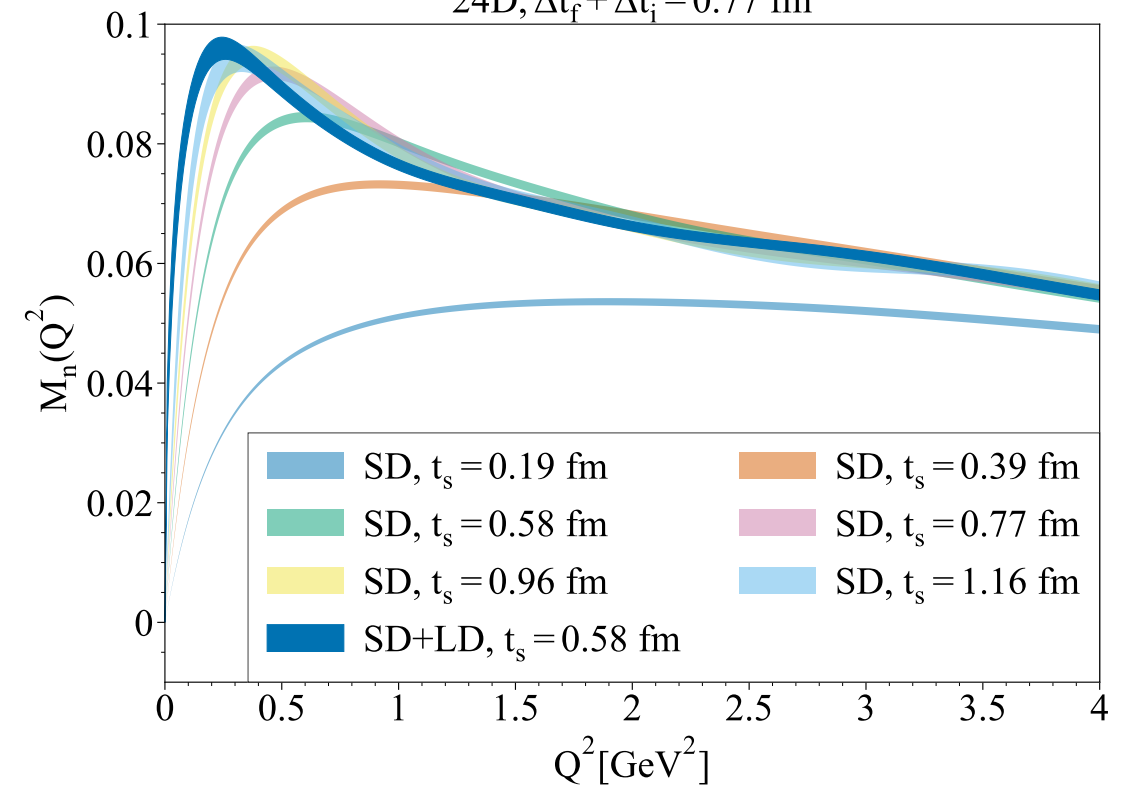
Ensemble	$m_\pi$ [MeV]	$L$	$T$	$a^{-1}$ [GeV]	$N_{\text{conf}}$
24D	142.6(3)	24	64	1.023(2)	207
32D-fine	143.6(9)	32	64	1.378(5)	69

$$\Delta_R^V = 0.02439(19)_{\text{LQCD}} \text{ vs } 0.02467(22)_{\text{DR}}$$

The result slightly lower than DR;  
Finer lattice calculations underway

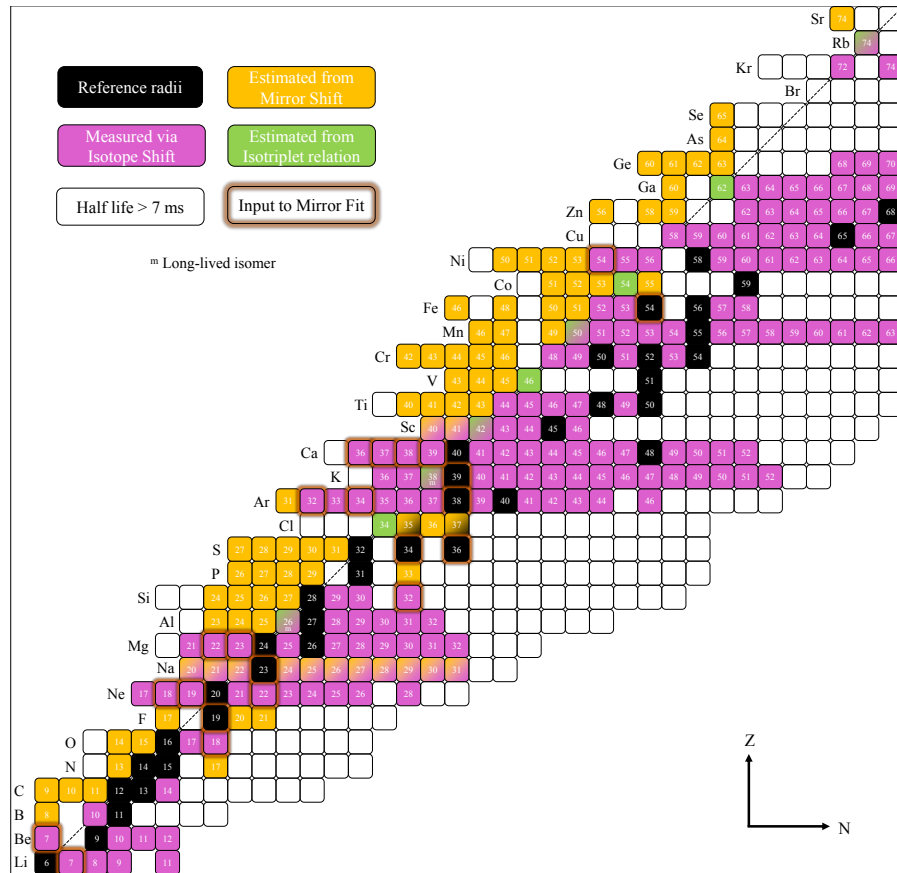


24D,  $\Delta t_f + \Delta t_i = 0.77$  fm



Ma, Feng, MG et al 2308.16755

# Global fit to charge radii off mirror nuclei



B. Ohayon, 2409.08193

Experimental input:

reference radii from muonic atoms;  
measured isotope shifts

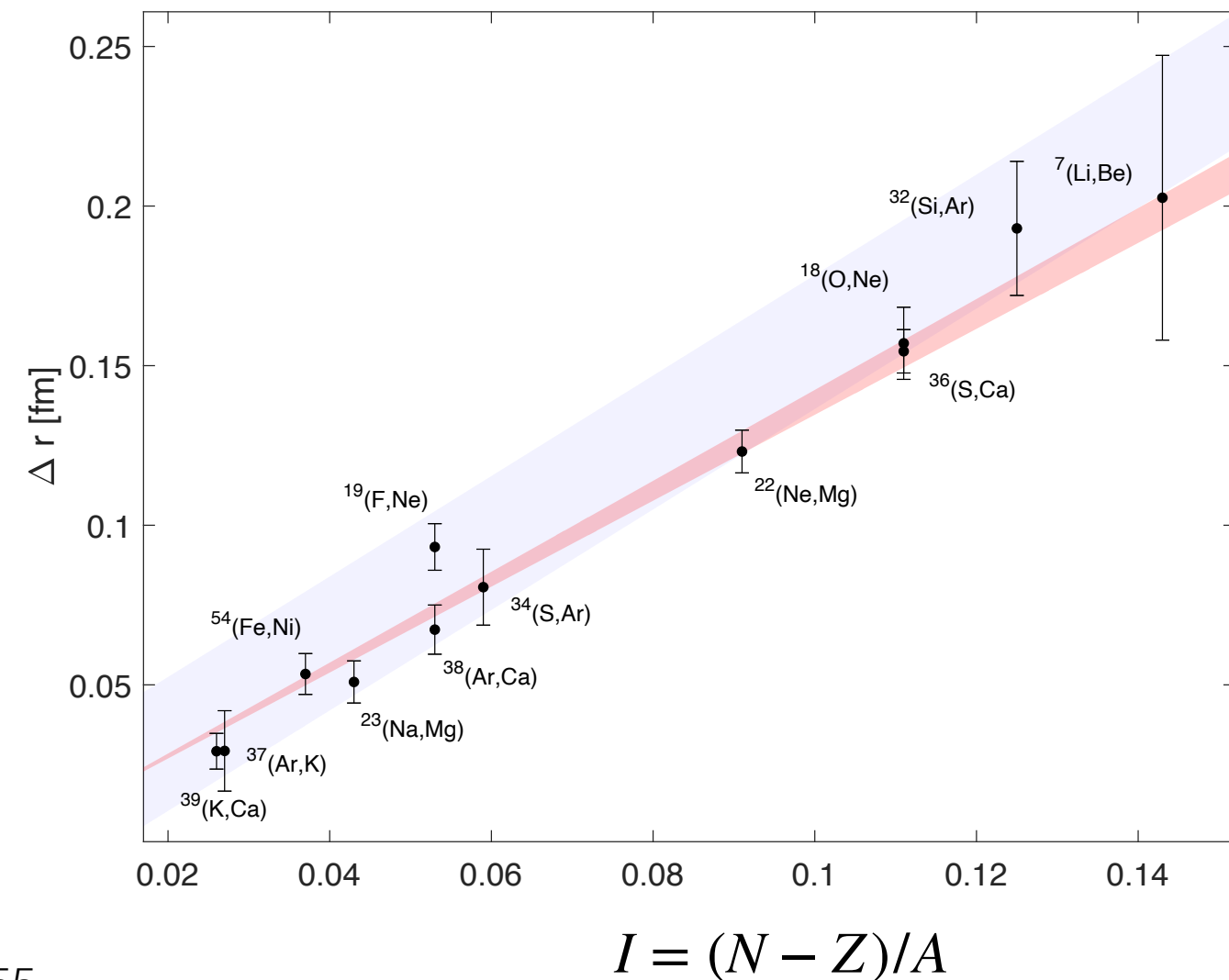
$$\Delta_I = r_{N,Z}(I) - r_{Z,N}(I) = 1.382(34) \times I \text{ fm}$$

$$I = (N - Z)/A$$

Linear fit to known radii in isotriplets (red)  
predictions for unknown radii

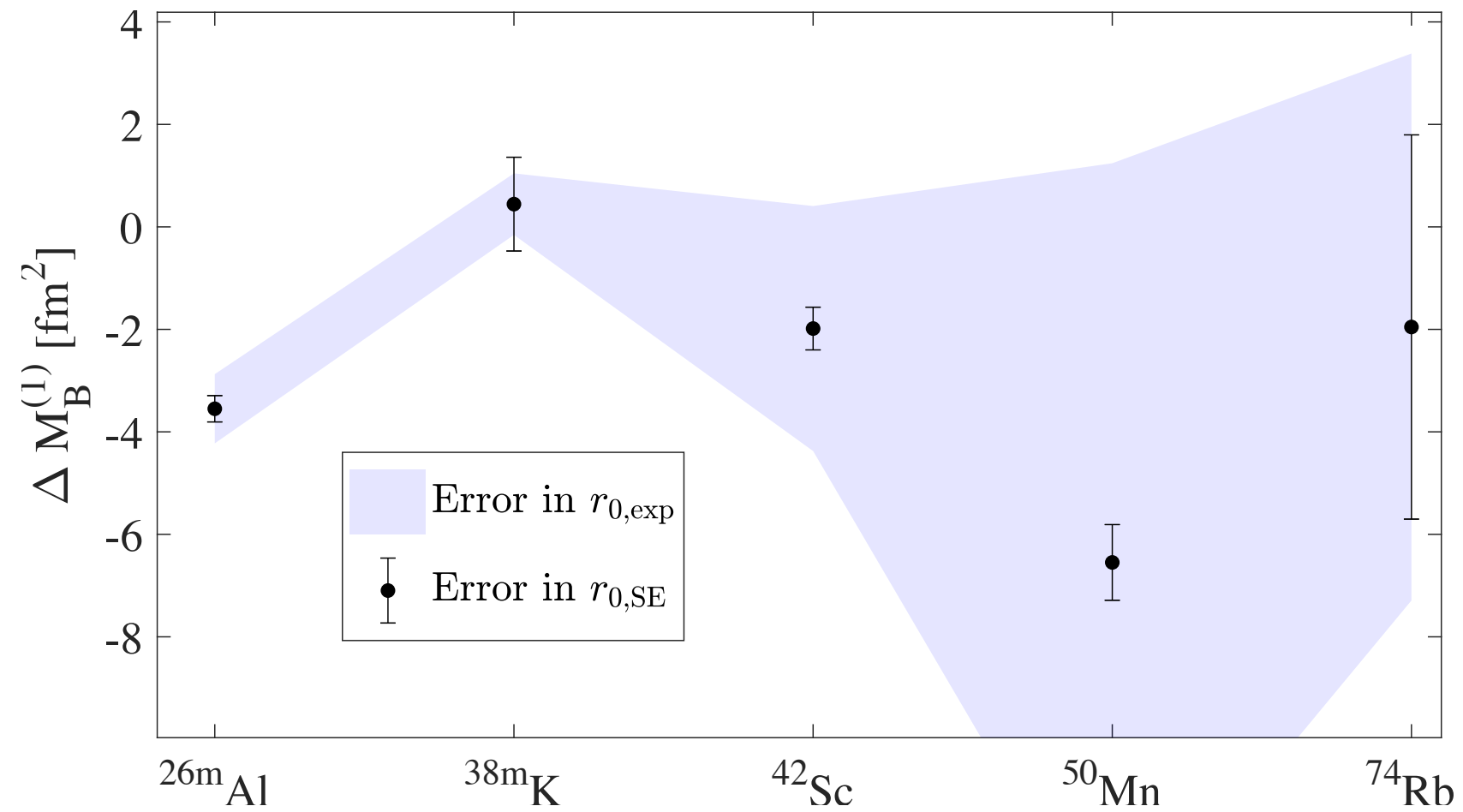
Compare to ab-initio theory estimate (blue)

Navario et al, Phys.Rev.Lett 130



# Global fit to charge radii off mirror nuclei

B. Ohayon, 2409.08193





# $\delta_{NS}$ in ab-initio nuclear theory

M. Gennari, M. Drissi, MG, P. Navratil, C.-Y. Seng, arXiv: **2405.19281**

Low-momentum part of the loop: account for nucleon d.o.f. only

First case study:  $^{10}\text{C} \rightarrow ^{10}\text{B}$  in No-Core Shell Model (NCSM)

Many-body problem in HO basis with separation  $\Omega$  and up to  $N = N_{max} + N_{Pauli}$

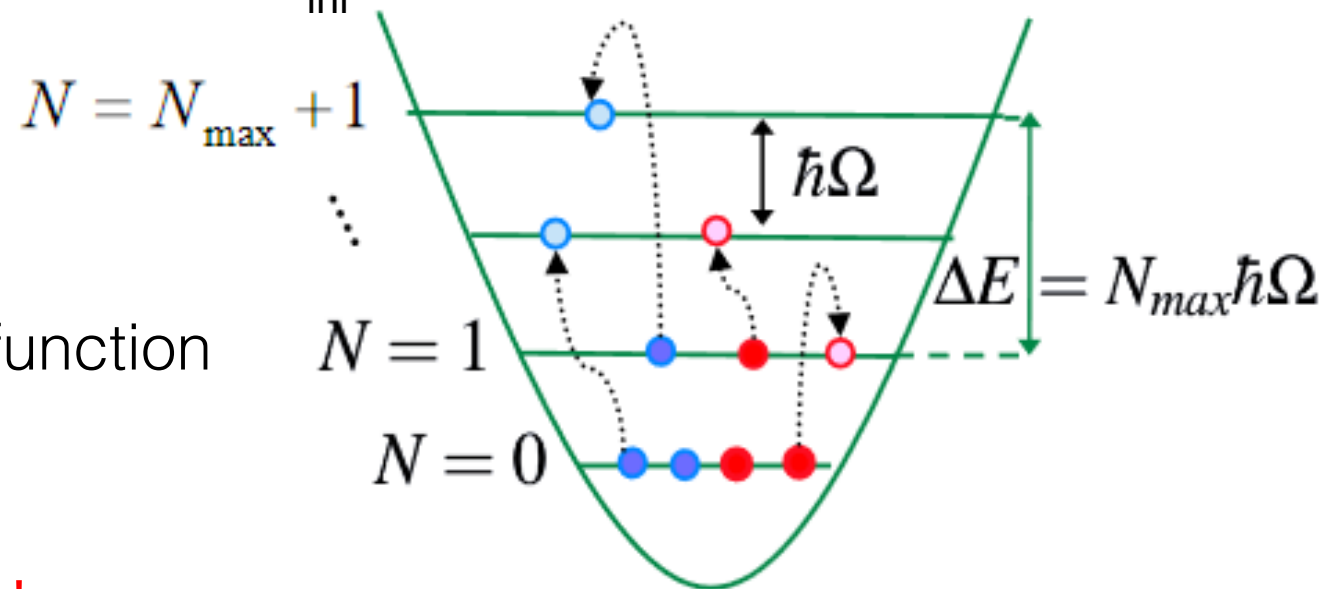
## ➤ Nuclear interactions from Chiral EFT:

- NN- $N^4\text{LO}+3N_{\text{Inl}}$
- NN- $N^4\text{LO}+3N_{\text{Inl}}^*$

*Entem, Machleidt and Nosyk, 2017 PRC;*

*Gysbers et al., 2019 Nature;*

*Kravvaris, Navrátil, Quaglioni, Hebberorn and Hupin, 2023 PLB*



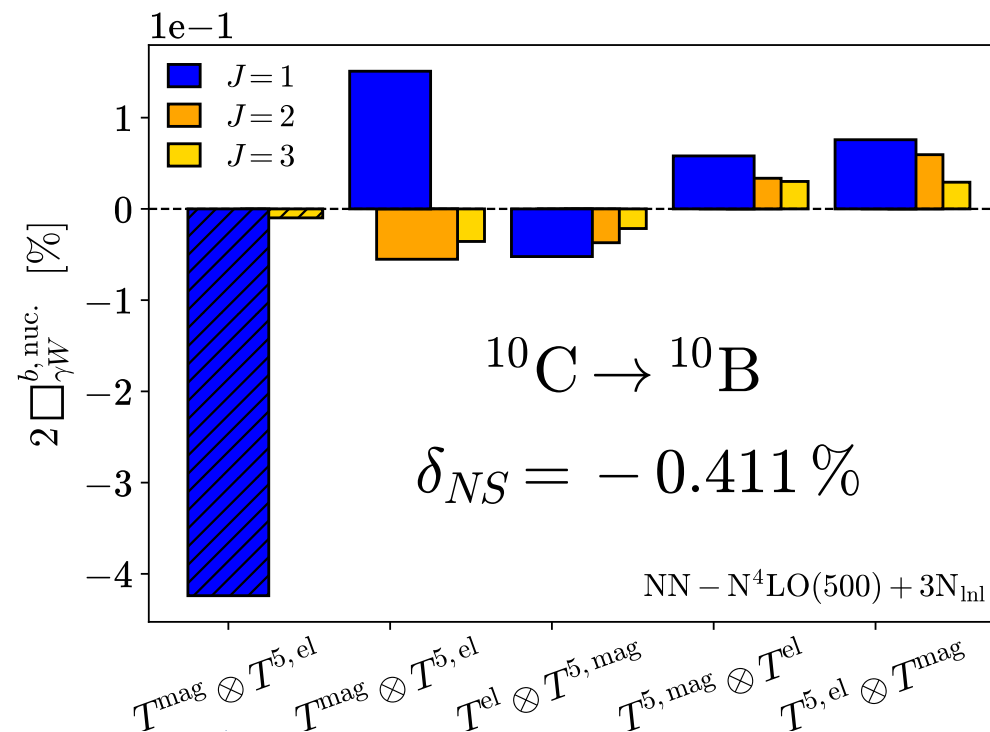
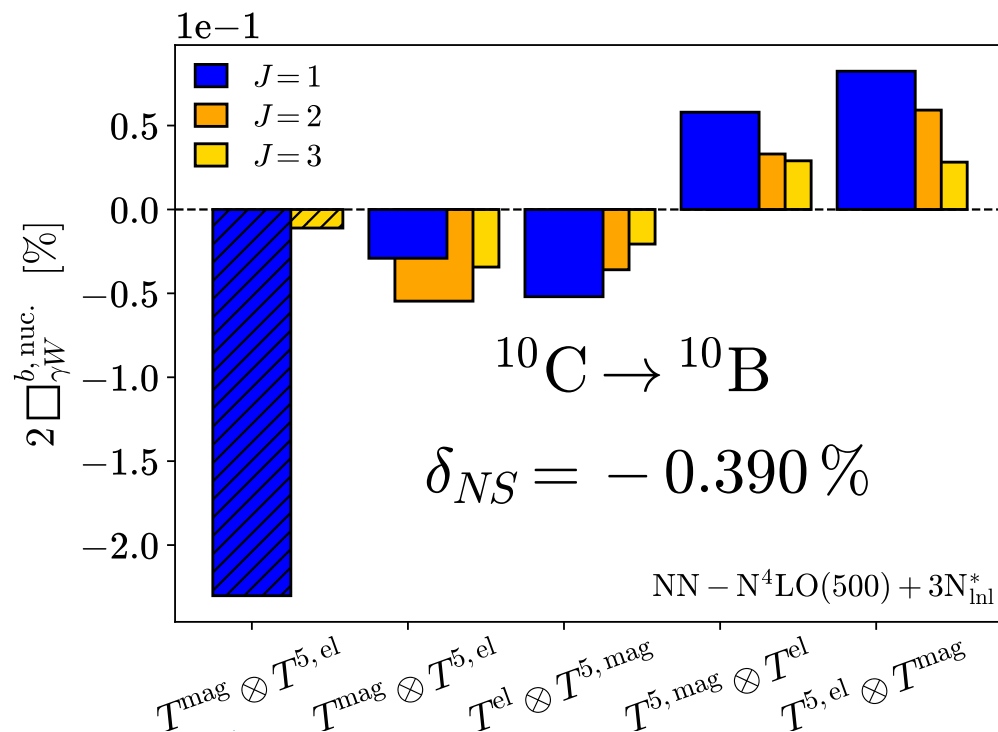
Evaluate the m.e. of nuclear Green's function

$$G(z) \equiv \frac{1}{z - H_0}$$

**Difficulty:**  
Inverting a  
large matrix!

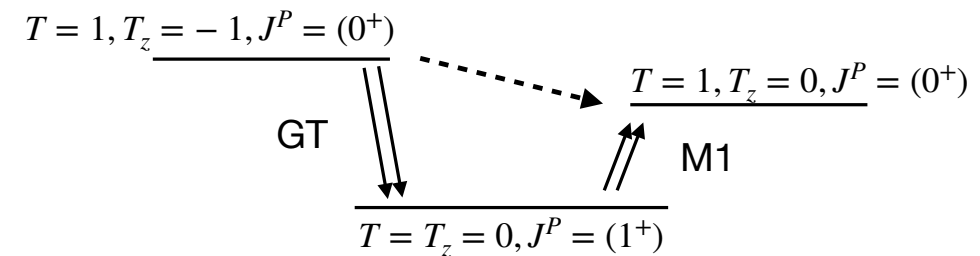
Lanczos continuous fraction method

# Ab-initio $\delta_{NS}$ : numerical results

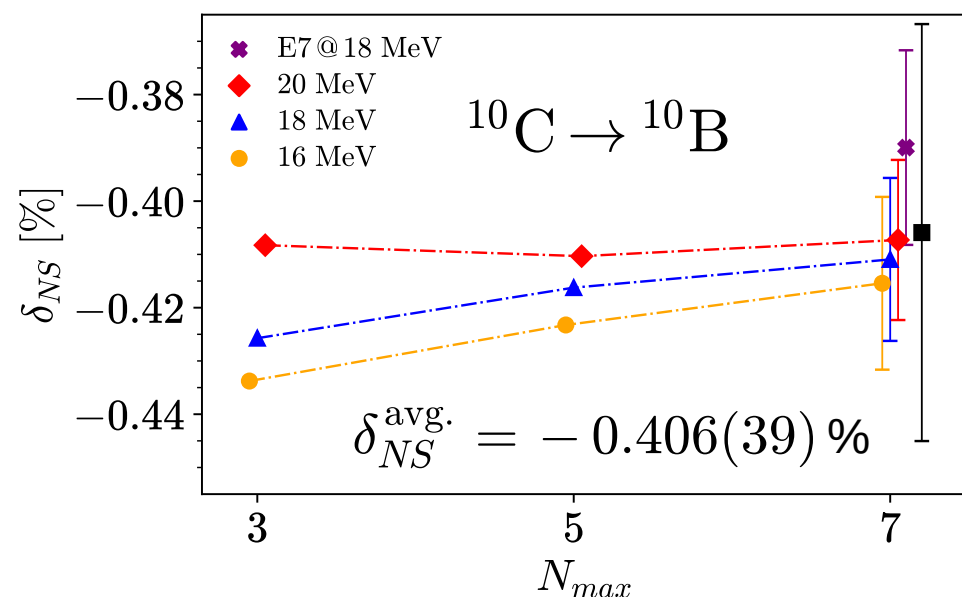


Large negative contribution: low-lying 1<sup>+</sup> level in <sup>10</sup>B

Large GT and M1 rates favor a two-step process



Check **Ω-independence** and **convergence w.r.t. N<sub>max</sub>**



Final result for  $10\text{C} \rightarrow 10\text{B}$ :

$$\delta_{NS} = -0.406(39)\%$$

arXiv: **2405.19281**

Compare to Hardy-Towner (old-fashion SM)

$$\delta_{NS} = -0.347(35)\% \quad (2014)$$

$$\delta_{NS} = -0.400(50)\% \quad (2020)$$

# Ab-initio $\delta_{NS}$ in EFT:

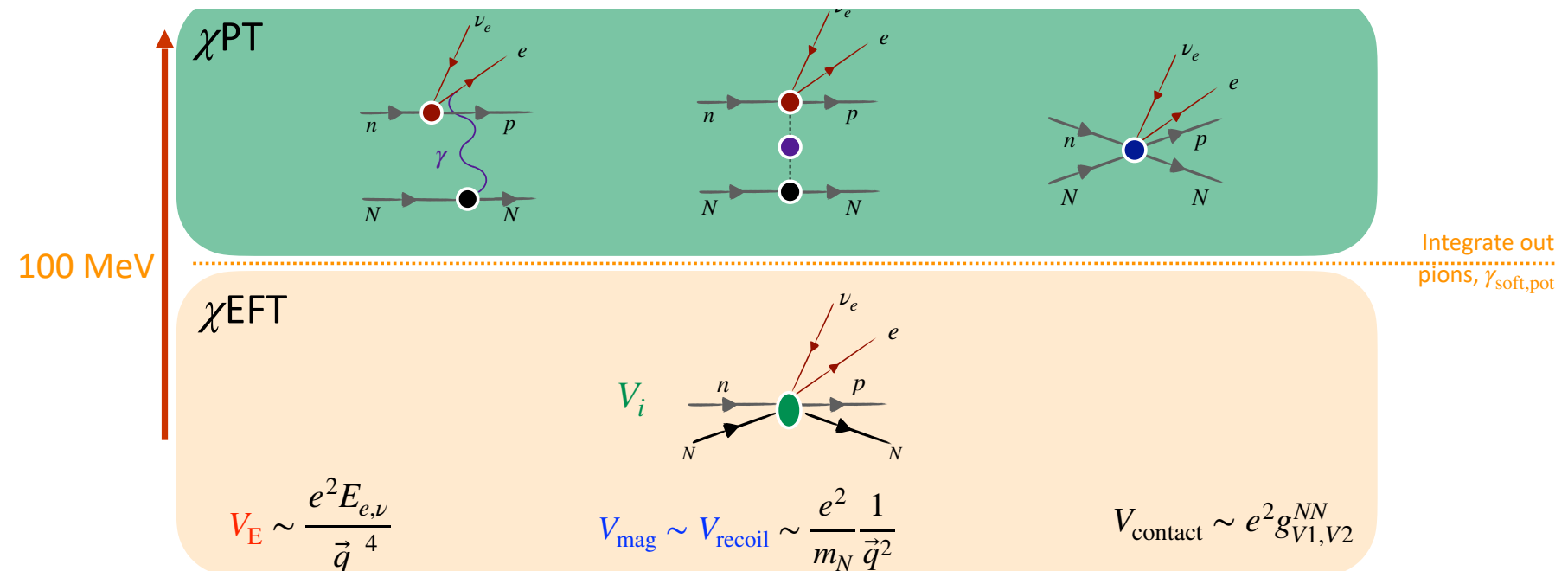
## $^{14}\text{O} \rightarrow ^{14}\text{N}$ with Variational Monte Carlo

V. Cirigliano et al, arXiv: **2405.18469**

$$\delta_{NS}^{(0)} = - (1.76 + 0.11 \pm 0.88) \cdot 10^{-3}$$

Uncertainty:  
assuming unknown  
counter term to be of  
“natural size”

$$g_{V1,V2}^{NN} = 1/(4m_N F_\pi^2)$$



Compare to Hardy-Towner 2020:  $\delta_{NS,B} = - 1.96(50) \cdot 10^{-3}$

Promising avenue: all logs under control and are consistent

Downside: EFT non-renormalizable  $\rightarrow$  unknown counter terms external to the theory

Need extra input (dispersion theory; explicit modeling; fit to data)