Accelerating High-Energy collisions calculations using analytic models to describe effective PDFs

Salvador A. Ochoa Oregon

in colaboration with D. F. Rentería-Estrada, R. J. Hernández-Pinto, G. F. R. Sborlini and Pia Zurita.

XV American Symposium on High Enery Physics (SILAFAE)

CDMX, - 05.11.2024

CONFÍE CIENCIA PARA EL PROGRESO

FACULTAD DE CIENCIAS FÍSICO-MATEMÁTICAS UNIVERSIDAD AUTÓNOMA DE SINALOA

SILAFAE

Cinvestav

smf SOCIEDAD MEXICANA DE FÍSICA

iF Instituto de Física UNAM

Outlook

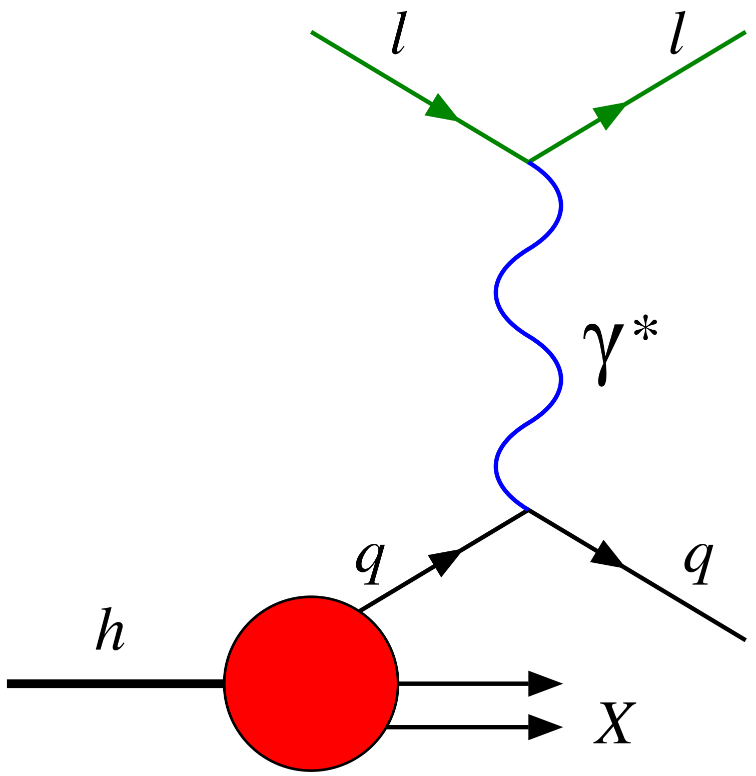
1. **Parton Distribution Functions**
    1.1 **Some history**
    1.2 **How to compute them**
2. **Motivation**
3. **Machine Learning for PDFs**
4. **Results**
5. **Conclusions**

Based on:

- S. A. Ochoa-Oregon, D. F. Rentería-Estrada, R. J. Hernández-Pinto, G. F.R. Sborlini and Pia Zurita, **"Using analytic models to describe effective PDFs",** Phys. Rev. D 110, 036019

Deep Inelastic Scattering

Parton distributions are present since 1960 when the determination of the cross section depends on the **structure functions** $\boldsymbol{F_2}$ and $\boldsymbol{F_L}$.



Kinematical variables

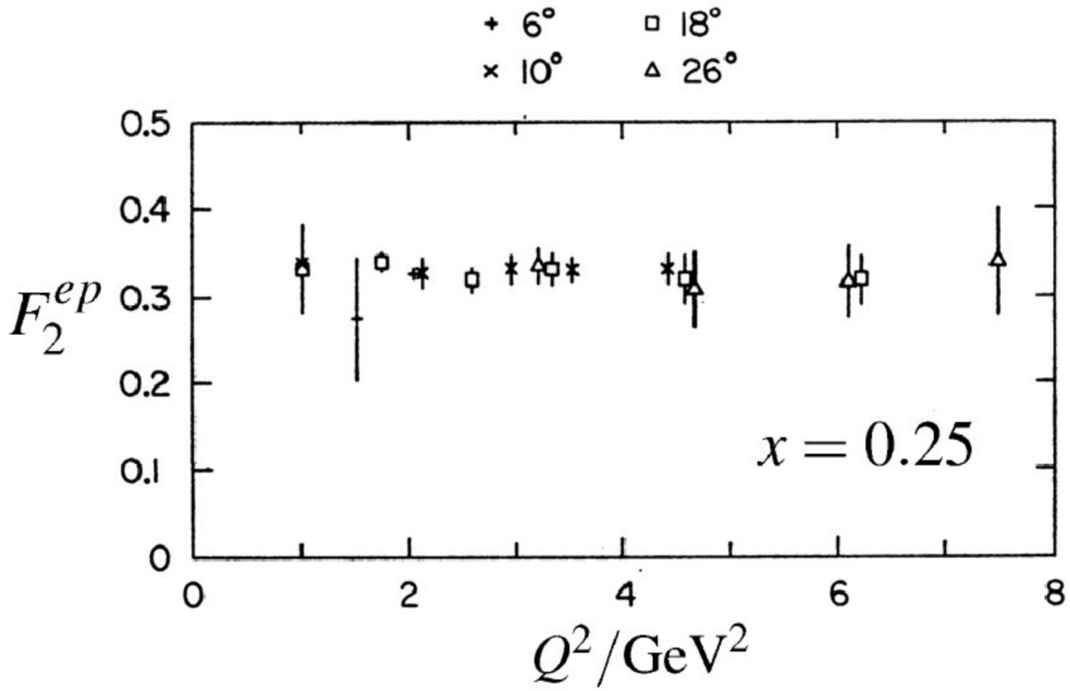
$$Q^2 = -(k - k')^2$$

$$x = \frac{Q^2}{2P \cdot Q} \qquad y = \frac{P \cdot Q}{P \cdot k}$$

$$\frac{d^2\sigma}{dx dQ^2} \approx F_2(x, Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x, Q^2)$$

Parton Distribution Functions

Experimental results of **structure functions** were



Then, a scaling was expected as

$$F_2^{LO}(x) = x \sum_{i=1}^{n_f} e_i^2 f_{i/h}(x)$$

J.T. Friedman and H.W. Kendall, Ann.Rev.Nucl.sci. 22(1972) 2013

Parton Distribution Functions

- Where $f_{i/h}$ is the probability density of **finding the parton i inside the hadron h** with a **fraction of momentum x**. They are called Parton Distribution Functions (PDFs).

Parton Distribution Functions

- Where $f_{i/h}$ is the probability density of **finding the parton i i** **he** **hadron h** with a **fraction of momentum x**. They are c
  Distribution Functions (PDFs).

Idea only
true at LO

- At higher orders in the expansion the scaling breaks down and a **dependece on the renormalization scale μ** appears.

Parton Distribution Functions

- Where $f_{i/h}$ is the probability density of **finding the parton i i** **he hadron h** with a **fraction of momentum x**. They are c Distribution Functions (PDFs).

Idea only true at LO

- At higher orders in the expansion the scaling breaks down and a **dependece on the renormalization scale μ** appears.

- The funcitonal form of the PDFs is not known from first principles. Nevertheless, **DGLAP gives the evolution with the scale**,

$$\mu^2 \frac{d}{d\mu^2} f_{i/h}(x,\mu) = \int_{\mu}^{1} \frac{d\xi}{\xi} \sum_b P_{a/b}\left(\frac{x}{\xi}, \alpha_s(\mu)\right) f_{b/h}(\xi,\mu)$$

with $P_{a/b}$ **the Altarelli-Parisi splitting functions**.

Altarelli, G., & Parisi, G. (1977). Asymptotic freedom in parton language. *Nuclear Physics B*, *126*(2), 298-318

One big asumption…

Universality holds for the PDFs, therefore, any process that is an inclusive hard scattering can be written as,

$$d\sigma^{\mathrm{DIS}} = \sum_i d\sigma^{l+i\to l'} \otimes f_i \quad \text{and} \quad d\sigma^{\mathrm{DY}} = \sum_{i,j} d\sigma^{i+j\to l+l'} \otimes f_i \otimes f_j$$

Factorization scheme and fixed order calculations

with the **same PDFs.** Also, the PDFs **contains the long distance structure of hadrons**.

We need global Fits

One big asumption…

Universality holds for the PDFs, therefore, any process that is an inclusive hard scattering can be written as,

$$d\sigma^{\mathrm{DIS}} = \sum_i d\sigma^{l+i\to l'} \otimes f_i \quad \text{and} \quad d\sigma^{\mathrm{DY}} = \sum_{i,j} d\sigma^{i+j\to l+l'} \otimes f_i \otimes f_j$$

Factorization scheme and fixed order calculations

with the **same PDFs.** Also, the PDFs **contains the long distance structure of hadrons**.

We need global Fits

Without PDFs there are no theorical predictions!

Global Fits

- Steps in general. Choose: **i)** a factorization scheme, **ii)** an order in pQCD, **iii)** a starting scale Q_0, **iv)** the data to be fitted, **v)** the heavy flavor scheme.

Global Fits

- Steps in general. Choose: **i)** a factorization scheme, **ii)** an order in pQCD, **iii)** a starting scale Q_0, **iv)** the data to be fitted, **v)** the heavy flavor scheme.

- **Parametrize partonic distributions**, in general,

$$xf_i(x, Q_0^2) = A_i x^{\alpha_i}(1 - x)^{\beta_i} P(x, c_i)$$

Global Fits

- Steps in general. Choose: **i)** a factorization scheme, **ii)** an order in pQCD, **iii)** a starting scale Q_0, **iv)** the data to be fitted, **v)** the heavy flavor scheme.

- **Parametrize partonic distributions**, in general,

$$xf_i(x, Q_0^2) = A_i x^{\alpha_i}(1 - x)^{\beta_i} P(x, c_i)$$

- **Solve the DGLAP equations** for the measured kinematics.

- Convolute PDFS and partonic Cross-sections.

Global Fits

- Steps in general. Choose: **i)** a factorization scheme, **ii)** an order in pQCD, **iii)** a starting scale Q_0, **iv)** the data to be fitted, **v)** the heavy flavor scheme.

- **Parametrize partonic distributions**, in general,

$$x f_i(x, Q_0^2) = A_i x^{\alpha_i}(1 - x)^{\beta_i} P(x, c_i)$$

- **Solve the DGLAP equations** for the measured kinematics.

- Convolute PDFS and partonic Cross-sections.

- **Minimize** distance between theorical predictions and experimental values.

- Use a method to estimate theorical error bands.

- Create **grids in x and Q^2**, and provide an **interpolator for the grid**.

Motivation

Process	NLO (CPU years)	NNLO (CPU years)	N3LO (CPU years)
pp → W/Z		0.6	160
pp → H		0.6	160
pp → $\gamma\gamma$		4.6	Process not available
pp → tt		20	Process not available
pp → $\gamma\gamma$+2jets		2.4	Process not available
pp → 2 jets		10	Process not available
pp → H+jet		57	Process not available
pp → $\gamma\gamma\gamma$		31	Process not available
pp → Z+jet		57	Process not available
pp → 3 jets		> 114	Process not available

don't worry

Cieri, L. (2024, june 19). *Precise theoretical predictions at colliders*. LHCPHENO 2024, IFIC, Valencia, España.

- All the running of MC codes **take a long time** to reach good precision.
- They carry a significant **environmental impact** (and to our pockets given the **cost of the CPU and then the electricity**)

*** One CPU year is equivalent to run the code in 1000 cores continuosly 9 hours.**

Process	NLO (CPU years)	NNLO (CPU years)	N3LO (CPU years)
pp → W/Z		0.6	160
pp → H		0.6	160
pp → γγ		4.6	Process not available
pp → tt		20	Process not available
pp → γγ+2jets		2.4	Process not available
pp → 2 jets		10	Process not available
pp → H+jet		57	Process not available
pp → γγγ		31	Process not available
pp → Z+jet		57	Process not available
pp → 3 jets		> 114	Process not available

don't worry

Cieri, L. (2024, june 19). *Precise theoretical predictions at colliders*. LHCPHENO 2024, IFIC, Valencia, España.

Motivation

- All the running of MC codes **take a long time** to reach good precision.
- They carry a significant **environmental impact** (and to our pockets given the **cost of the CPU and then the electricity**).
- Can we **speed up** the running time **without touching the code**?

* One CPU year is equivalent to run the code in 1000 cores continuosly 9 hours.

Motivation

Process	NLO (CPU years)	NNLO (CPU years)	N3LO (CPU years)
$pp \to W/Z$		0.6	160
$pp \to H$		0.6	160
$pp \to \gamma\gamma$		4.6	Process not available
$pp \to tt$		20	Process not available
$pp \to \gamma\gamma$+2jets		2.4	Process not available
$pp \to$ 2 jets		10	Process not available
$pp \to$ H+jet		57	Process not available
$pp \to \gamma\gamma\gamma$		31	Process not available
$pp \to$ Z+jet		57	Process not available
$pp \to$ 3 jets		> 114	Process not available

don't worry

Cieri, L. (2024, june 19). *Precise theoretical predictions at colliders*. LHCPHENO 2024, IFIC, Valencia, España.

- All the running of MC codes **take a long time** to reach good precision.
- They carry a significant **environmental impact** (and to our pockets given the **cost of the CPU and then the electricity**)
- Can we **speed up** the running time **without touching the code**?
- Most codes require **non perturbative** inputs (e.g. PDFs) and most of **them are provided as** grids and interpolate over them.

*** One CPU year is equivalent to run the code in 1000 cores continuosly 9 hours.**

Process	NLO (CPU years)	NNLO (CPU years)	N3LO (CPU years)
pp → W/Z		0.6	160
pp → H		0.6	160
pp → γγ		4.6	Process not available
pp → tt		20	Process not available
pp → γγ+2jets		2.4	Process not available
pp → 2 jets		10	Process not available
pp → H+jet		57	Process not available
pp → γγγ		31	Process not available
pp → Z+jet		57	Process not available
pp → 3 jets		> 114	Process not available

don't worry

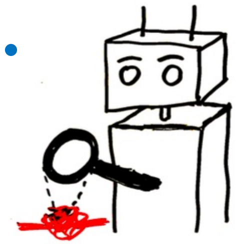
Cieri, L. (2024, june 19). *Precise theoretical predictions at colliders*. LHCPHENO 2024, IFIC, Valencia, España.

Motivation

- All the running of MC codes **take a long time** to reach good precision.
- They carry a significant **environmental impact** (and to our pockets given the **cost of the CPU and then the electricity**)
- Can we **speed up** the running time **without touching the code**?
- Most codes require **non perturbative** inputs (e.g. PDFs) and most of **them are provided as** grids and interpolate over them.
- A quick exploration shows that interpolation time could be **reduced 40-50%** if we had **analytical expressions for the PDFs**.

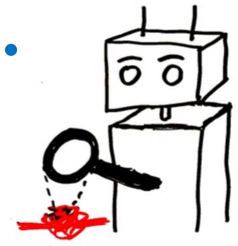
* One CPU year is equivalent to run the code in 1000 cores continuosly 9 hours.

Goal: Find an analytical x and Q^2 form for a set of proton PDFs.



Machine Learning for PDFs

Goal: Find an analytical x and Q^2 form for a set of proton PDFs.

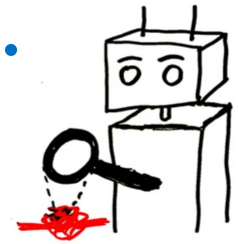


- How? Inpired by **functional form of `HERAPDF`**, we propose a general functional form

$$
\begin{aligned}
xf_i(x,Q^2) = {} & A_i(Q^2)x^{\alpha_i(Q^2)}(1-x)^{\beta_i(Q^2)}P\big(x,c_i(Q^2)\big) \\
& -\Theta(x_{c,i}-x)A'_i(Q^2)x^{\alpha'_i(Q^2)}(1-x)^{\beta'_i(Q^2)}P\big(x,c'_i(Q^2)\big)
\end{aligned}
$$

More details in: Phys. Rev. D 110, 036019 [hep-ph]

Goal: Find an analytical x and Q^2 form for a set of proton PDFs.



- How? Inpired by **functional form of `HERAPDF`**, we propose a general functional form

$$x f_i(x, Q^2) = A_i(Q^2) x^{\alpha_i(Q^2)} (1 - x)^{\beta_i(Q^2)} P\big(x, c_i(Q^2)\big)$$
$$- \Theta(x_{c,i} - x) A_i'(Q^2) x^{\alpha_i'(Q^2)} (1 - x)^{\beta_i'(Q^2)} P\big(x, c_i'(Q^2)\big)$$

- Our hypotesis is that the **Q^2-dependence** of the PDFs is given by **the parameters**.
- The `heavyside` function is used to give more flexibility at low x. (Inspired by `HERAPDF` and gradient boosting algorithm)
- Fitted using Machine Learning tools and techniques.

More details in: Phys. Rev. D 110, 036019 [hep-ph]

Fitting procedure

- We generate a grid of 5,000-10,000 random points in $\{x, Q^2\}$ using `HERAPDF20_NLO_EIG` (`HERAPDF 2.0`)

- The range was chosen in accordance to `HERAPDF`

- Furthermore, we use for light quarks and gluons $Q_{\mathrm{Min}} = Q_0 \approx 1.37$ GeV, whilst $Q_0 = 1.5$ GeV and $Q_0 = 4.5$ GeV for the charm and bottom quarks, respectively

Note: We are fitting the results of an existing fit.

More details in: Phys. Rev. D 110, 036019 [hep-ph]

Finding the best fit: cost function

- Since HEP phenomenology is **not only interested in the central value** of the PDF, we rather define the best fitting parameters through the **cost function**,

$$\Delta_i(Q^2) = \frac{I[f_i^{\mathrm{ML}}(x,Q^2),Q^2] - I[f_i^{\mathrm{HERA}}(x,Q^2),Q^2]}{I[f_i^{\mathrm{HERA}}(x,Q^2),Q^2]},$$

Inspired by Factorizaction Theorem

- which takes care of the integral error of the determination of the PDFs, through the integration operator,

$$I[f,Q^2] = \int_{10^{-4}}^{1} dx\, f(x,Q^2).$$

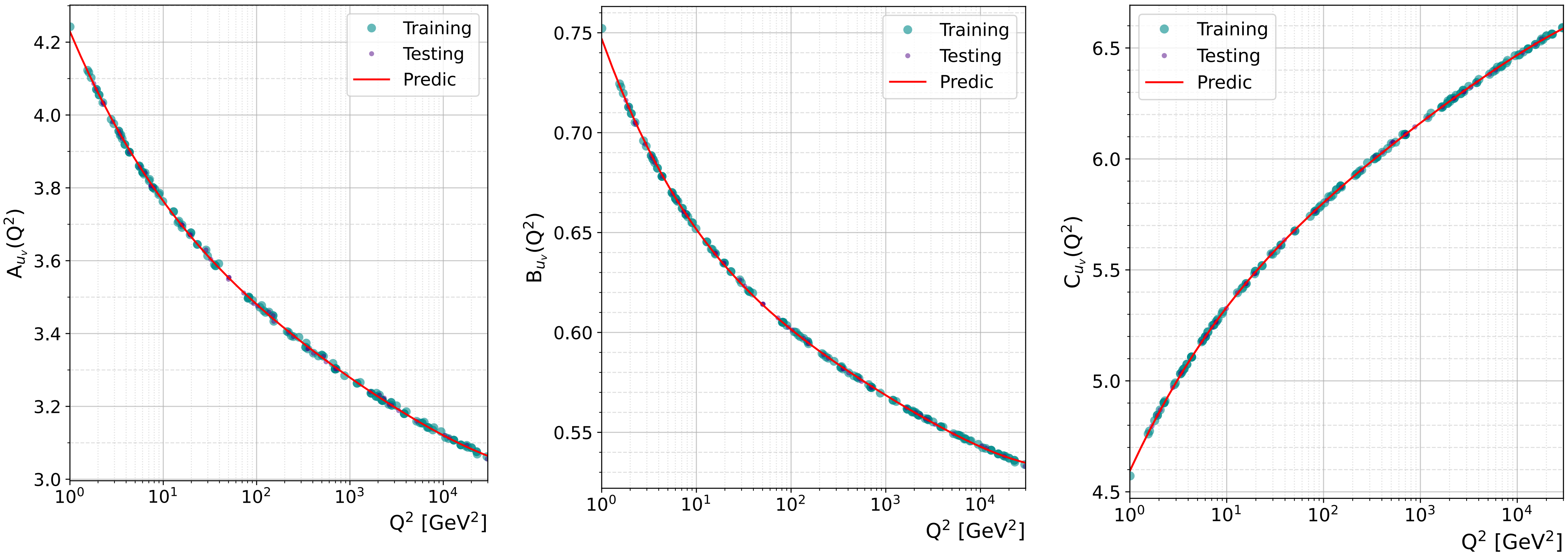
More details in: Phys. Rev. D 110, 036019 [hep-ph]

Machine Learning for PDFs

xu_v distribution coefficients

More details in: Phys. Rev. D 110, 036019 [hep-ph]

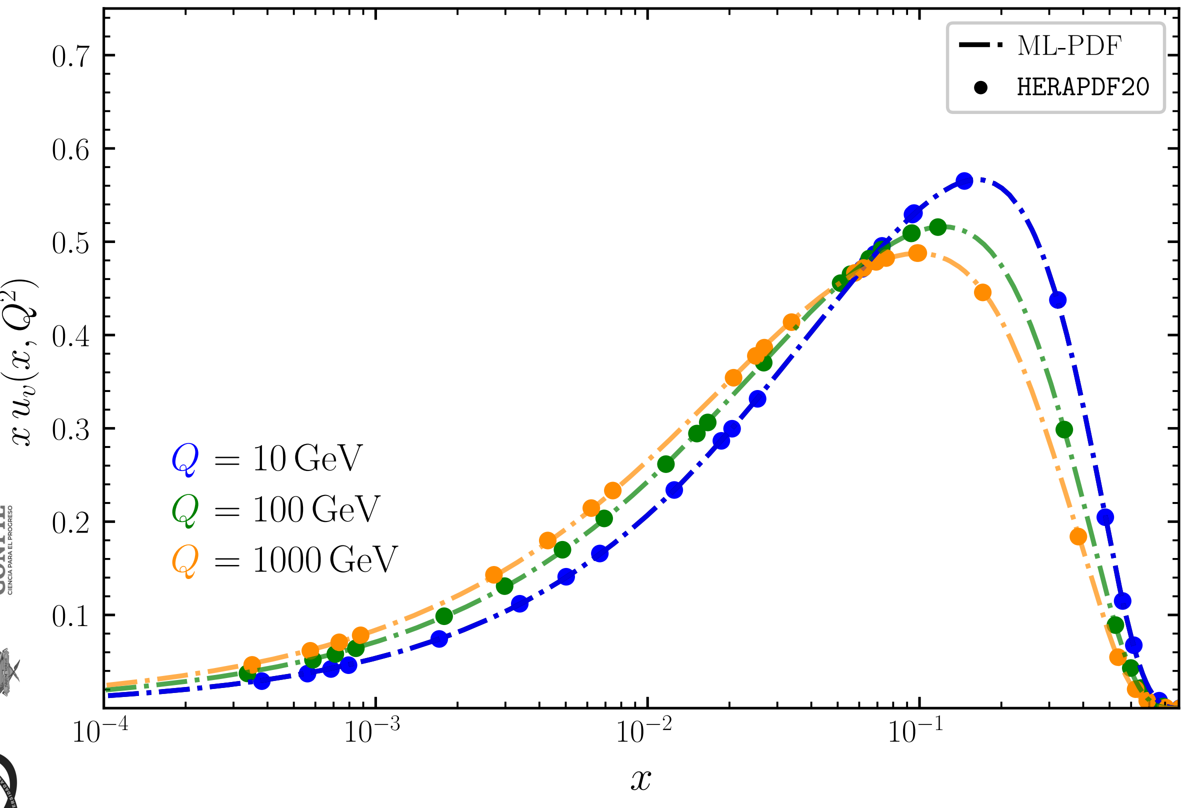
- We use ML to find all coefficients.

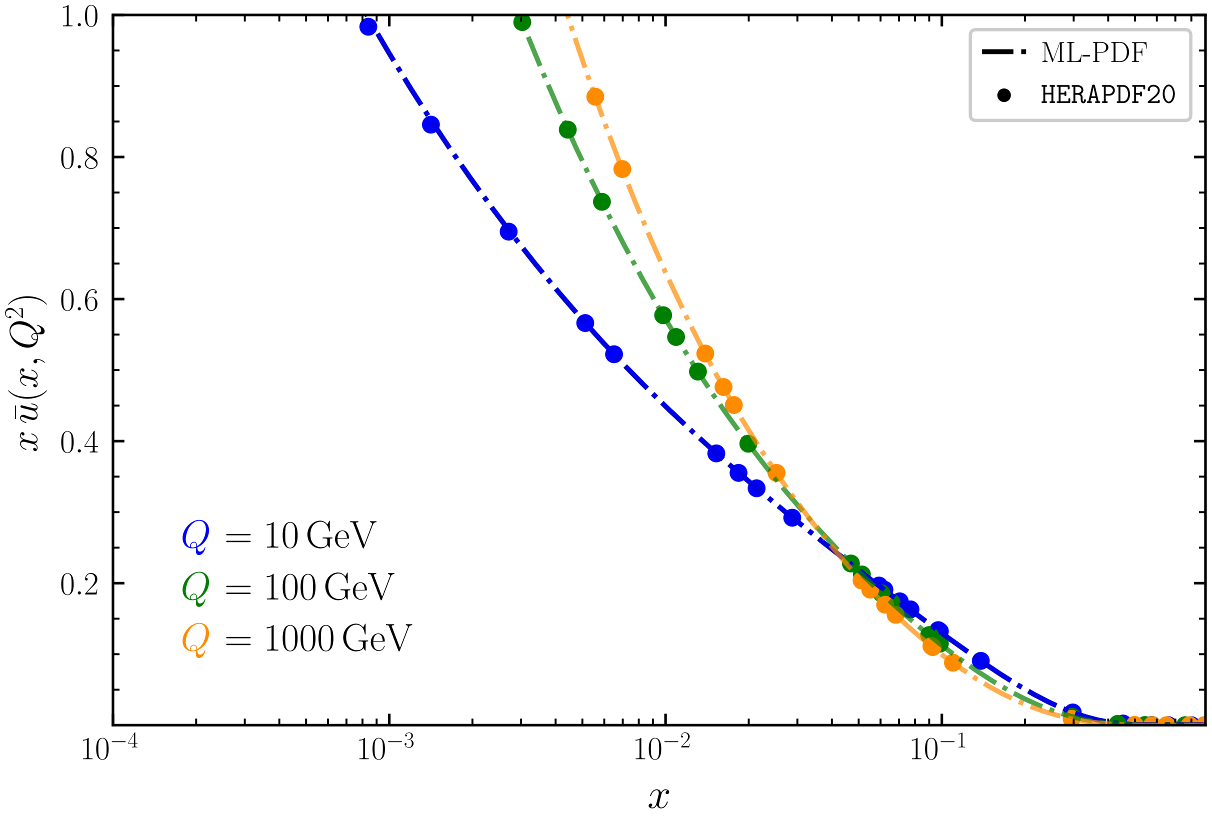


- For this particular case, a **polynomial function** was sufficient. In many cases it was necessary to propose a **more complicated basis.**

Machine Learning for PDFs

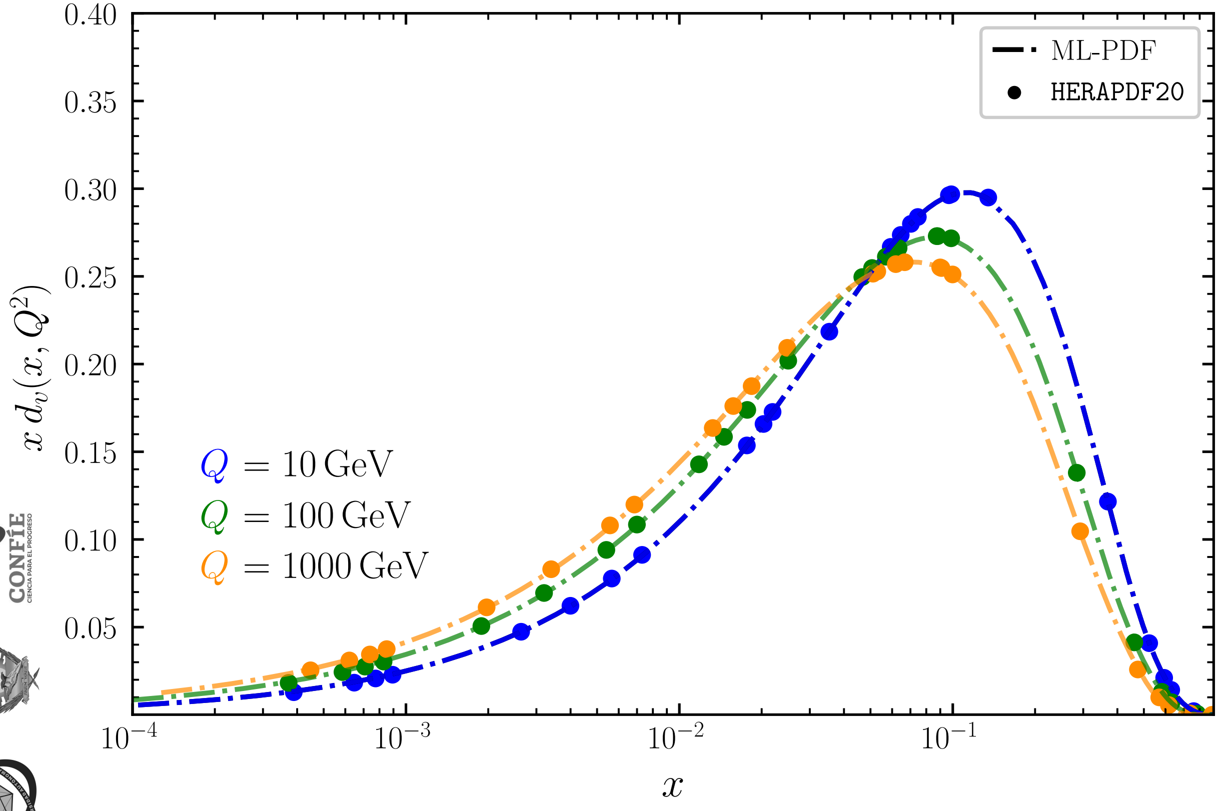
Up quark distributions

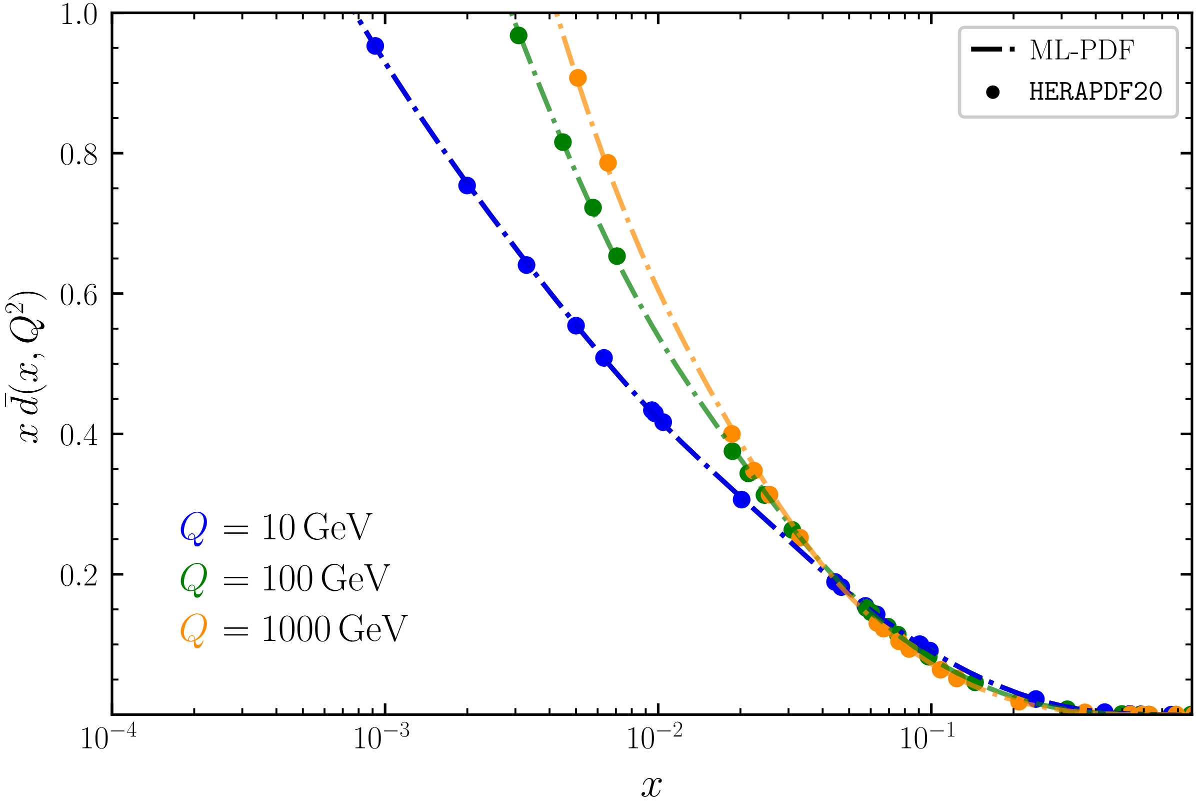




- We found a **good agreement** and only use **one region to $x\bar{u}$** and **two regions to xu_v**.
- We can obtain u-quark distribution by: $xu = x\bar{u} + xu_v$

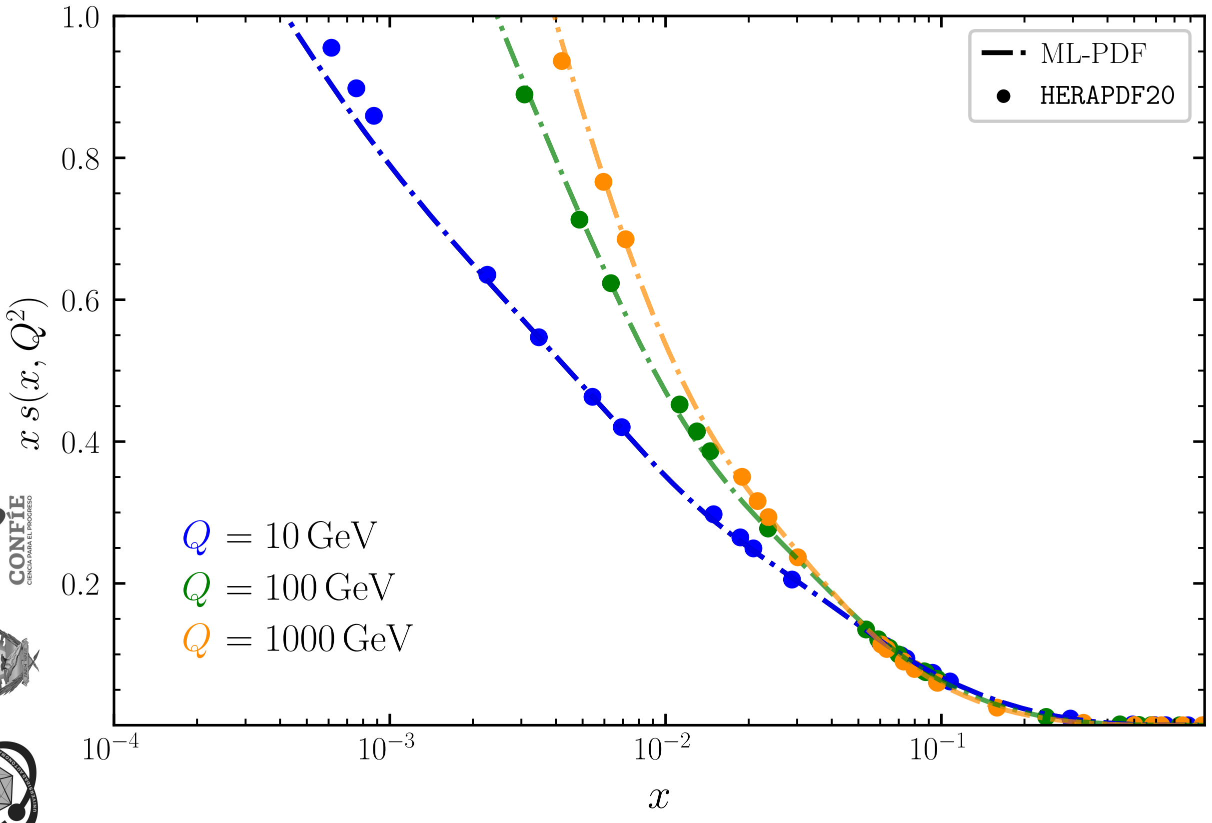
Down quark distributions

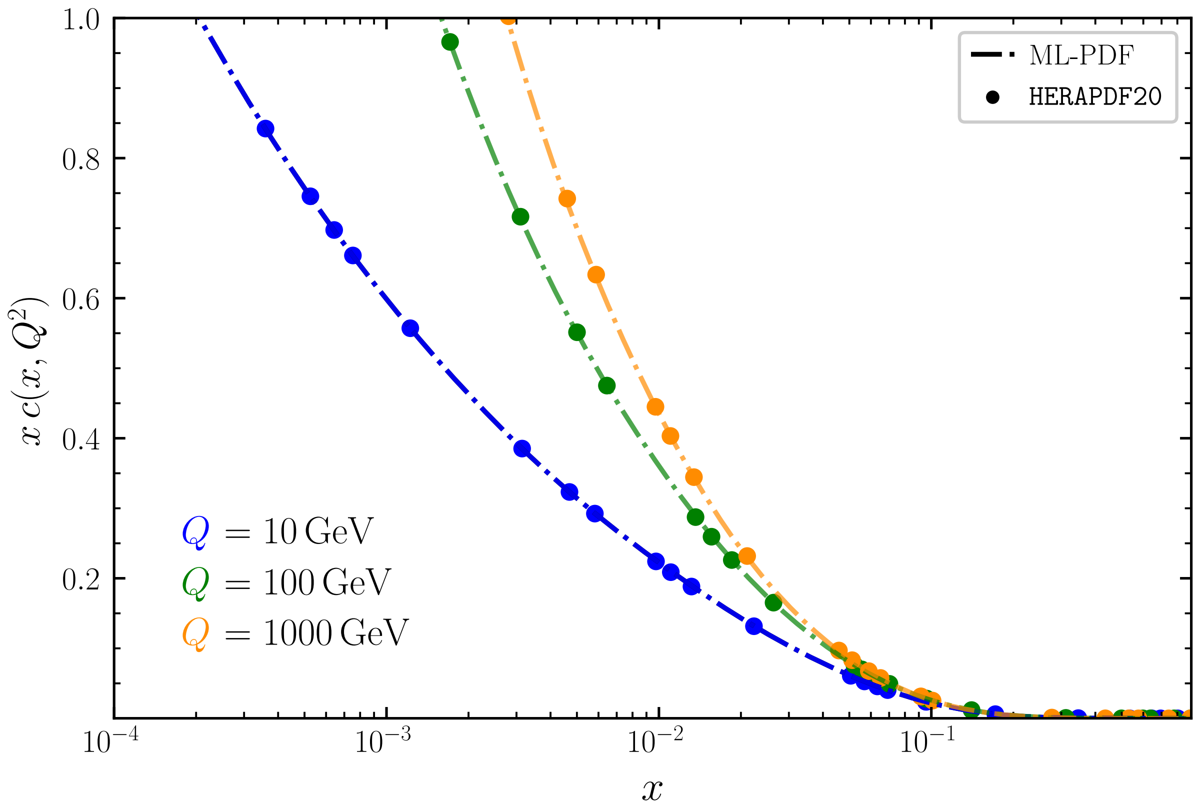




- Similar to the u-quark, we found a **good agreement** and in this case we only use **one region** to both distributions.
- Similarly, d-quark distribution by: $xd = x\bar{d} + xd_v$

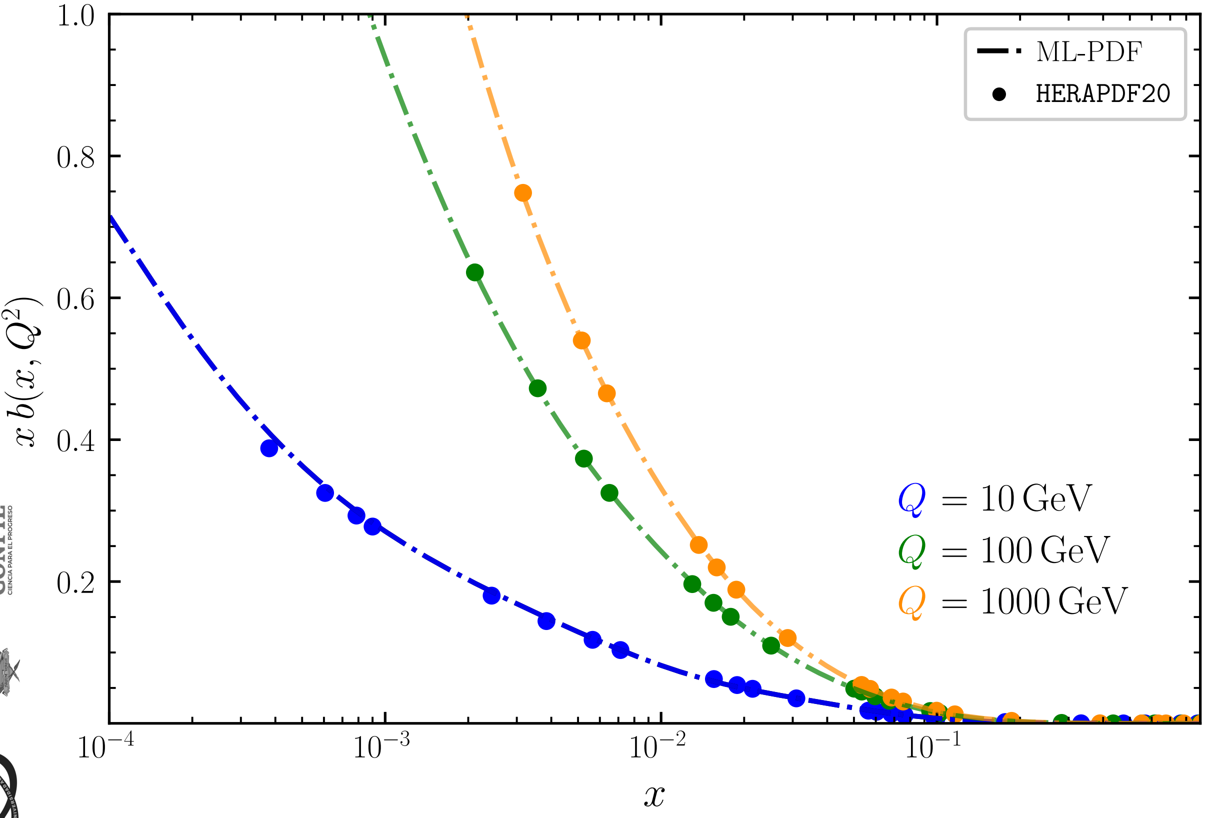
Strange and Charm quark distributions

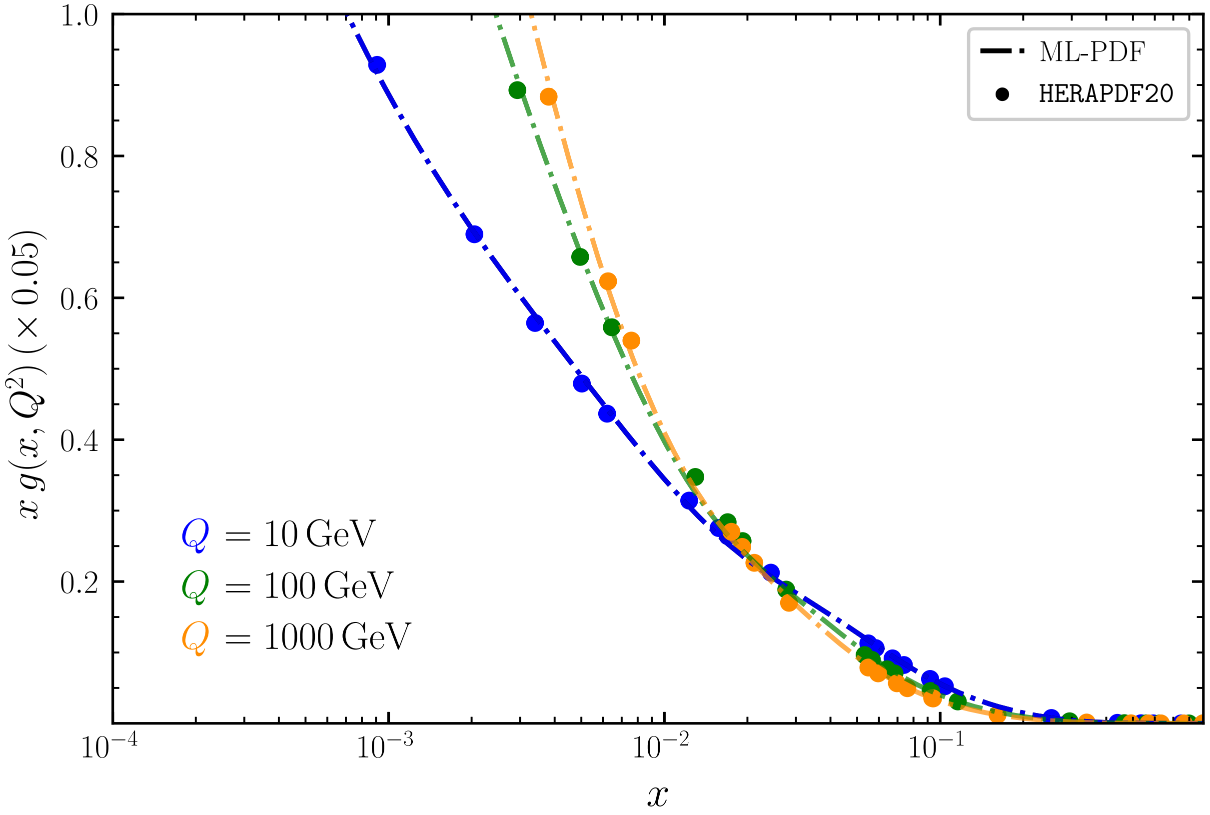




- For s-quark and c-quark we used **two regions.**
- Only **small discrepancies** for $x < 10^{-3}$ at $Q = 10$ GeV in s-quark distribution are present.

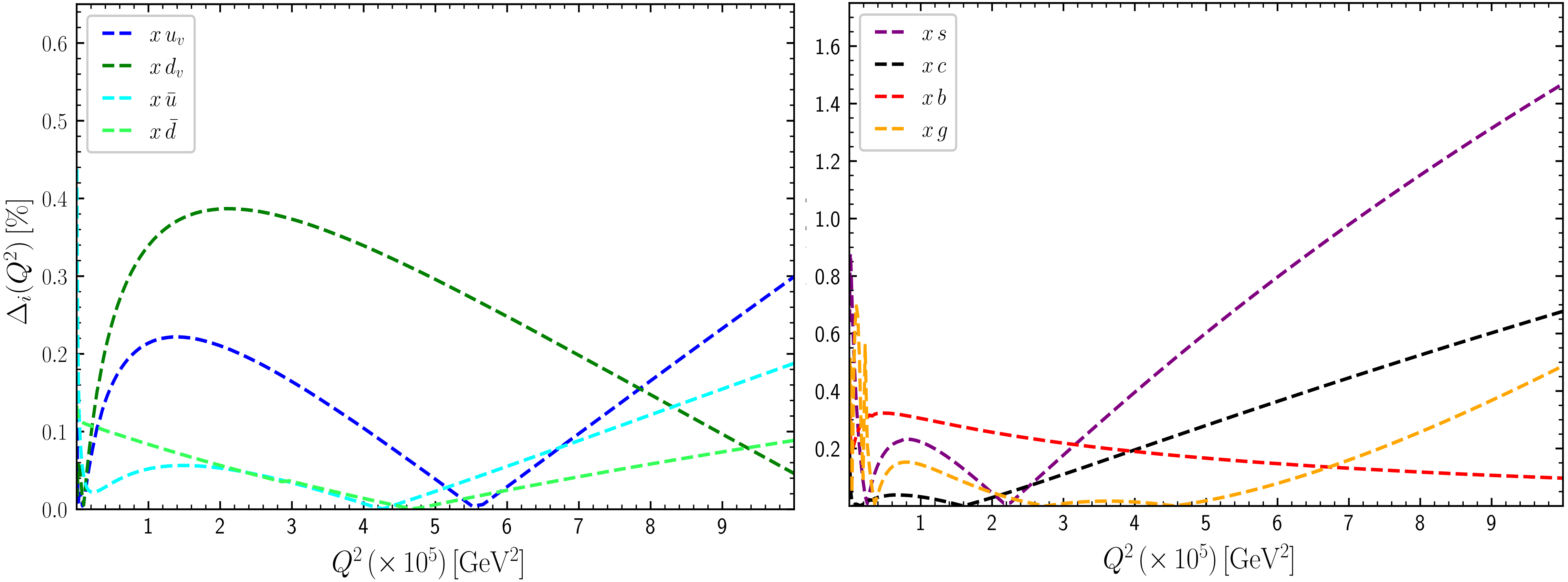
Bottom quark and Gluon distributions





- For the b-quark we split into **two regions** and for the gluon we split into **four regions**.
- **Small deviations** of the central value for $x < 10^{-3}$ in b-quark were found.

How good is good ?



- The estimator $\Delta_i(Q^2)$ shows **small integral error** (maximum of 1.5% for all partons)

S. A. Ochoa-Oregon et al., Phys. Rev. D 110, 036019 [hep-ph].

How fast is fast?

$$\text{Gain}(\%) = 100\times\frac{\text{time}_{LHAPDF} - \text{time}_{ML-PDF}}{\text{time}_{LHAPDF}}$$

N_{points}	LHAPDF (s)	ML-PDFs (s)	Gain(%)
10^3	$3.76 \cdot 10^{-2}$	$2.92 \cdot 10^{-4}$	99.22
10^4	$4.20 \cdot 10^{-2}$	$2.50 \cdot 10^{-3}$	94.05
10^5	$8.94 \cdot 10^{-2}$	$2.50 \cdot 10^{-2}$	72.10
10^6	0.56	0.25	55.46
10^7	5.25	2.50	52.49
10^8	52.04	24.92	52.11

- Comparison of the time (in seconds) required to compute N_{points} evaluations of `HERAPDF2.0` within `LHAPDF` framework, and our `ML-PDF` analytic approximation.
- The gains seems to approach a **plateau around 50%.**

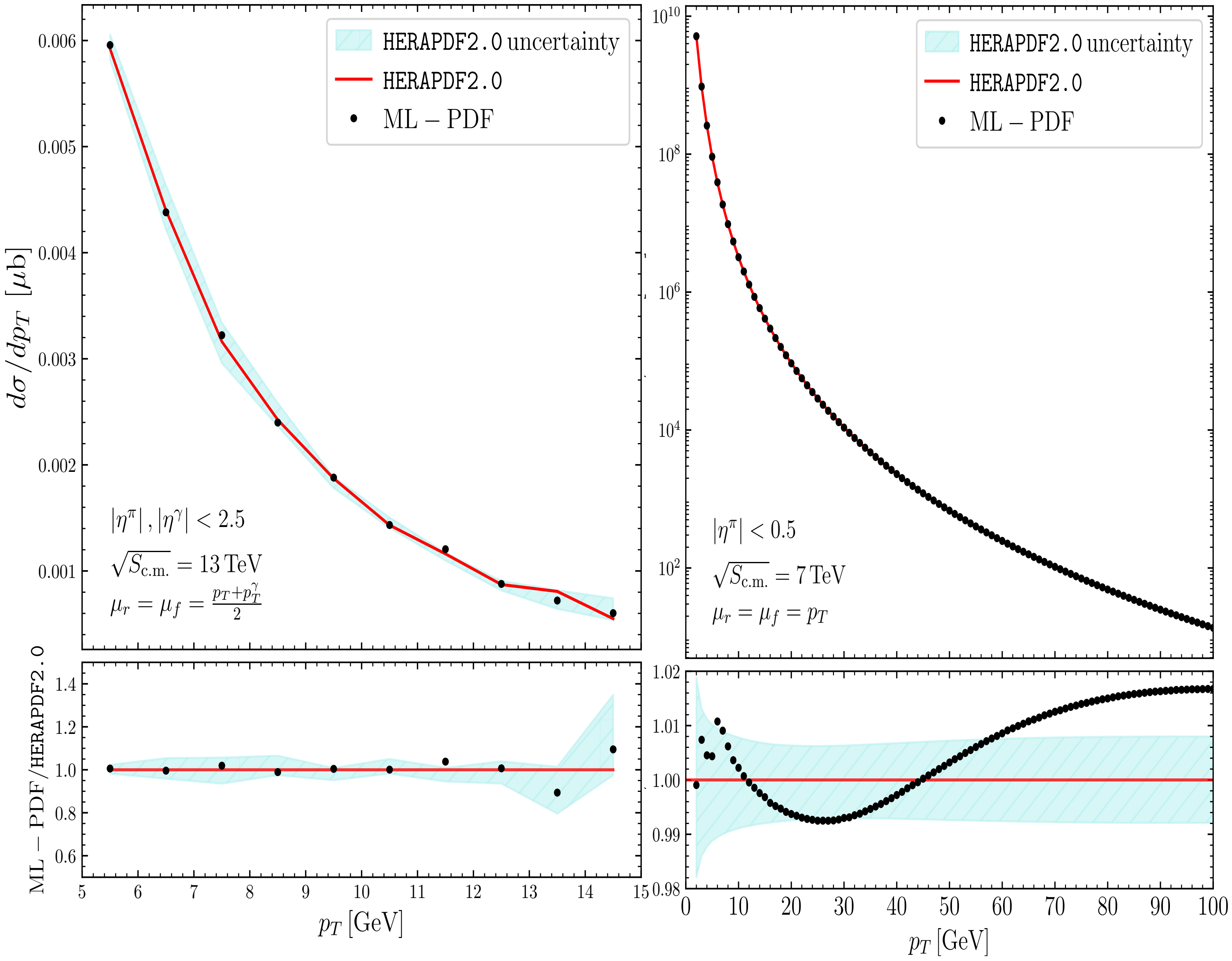
S. A. Ochoa-Oregon et al., Phys. Rev. D 110, 036019 [hep-ph].

Results

The final test ...

- We test our results with two observables:
  - **i)** $p + p \to \pi$ and
  - **ii)** $p + p \to \pi + \gamma$ at NLO.
- We find an almost perfect agreemen **(≈ 1% difference)**
- Time consumption is also improved!

Obs.	LHAPDF (s)	ML-PDFs (s)	Gain(%)
$p+p \to \pi$	628.320	558.854	11.06
$p+p \to \gamma+\pi$	12452.273	8671.827	30.36



S. A. Ochoa-Oregon et al., Phys. Rev. D 110, 036019 [hep-ph].

FF: DEHSS2014

Results

The final test ...

Published July 15, 2024 | Version v2

Computational notebook Open

Ancillary files for "Using analytic models to describe effective PDFs"

Ochoa-Oregon, Salvador (Researcher)[1];
Rentería-Estrada, David (Researcher)[2];
Hernández-Pinto, Roger (Researcher)[1]; Sborlini, German (Researcher)[3];
Zurita, Pía (Researcher)[4]

$p+p \to \pi$	628.320	558.854	11.06
$p+p \to \gamma+\pi$	12452.273	8671.827	30.36

S. A. Ochoa-Oregon et al arXiv:2404.15175 [hep-ph].

FF: DEHSS2014

Conclusions

- **PDFs** are **key ingredients** for any phenomenological theoretical predictions.

- Since MC simulations are taking the CPU cost to the extreme, we exploited ML to extract **analytical PDFs** to avoid running interpolations techniques.

- We compared our results w.r.t. LHAPDF within **two benchmarks** finding an **improvement in the CPU time** of around **11% for $p+p \to \pi$** and more than **30% for $p+p \to \pi+\gamma$**, both at NLO.

- The code (FORTRAN) with the PDFs can be found at **https://zenodo.org/records/12745978.**


THANKS!

BACKUP SLIDES

Regions

xu_v distribution

$R_1 = \{Q_0 \leq Q \leq 10 \text{ GeV}\}$

$R_2 = \{10\text{GeV} \leq Q \leq 1000 \text{ GeV}\}$

xs distribution

$R_1 = \{Q_0 \leq Q \leq 4.5 \text{ GeV}\}$

$R_2 = \{4.5 \text{ GeV} \leq Q \leq 1000 \text{ GeV}\}$

xb distribution

$R_1 = \{4.5 \leq Q \leq 15 \text{ GeV}\}$

$R_2 = \{15 \text{ GeV} \leq Q \leq 1000 \text{ GeV}\}$

xc distribution

$R_1 = \{1.47 \leq Q \leq 3 \text{ GeV}\}$

$R_2 = \{3 \text{ GeV} \leq Q \leq 1000 \text{ GeV}\}$

xg distribution

$R_1 = \{Q_0 \leq Q \leq 2.5 \text{ GeV}\}$

$R_2 = \{2.5 \text{ GeV} \leq Q \leq 5 \text{ GeV}\}$

$R_3 = \{5 \text{ GeV} \leq Q \leq 5 \text{ GeV}\}$

$R_4 = \{150 \text{ GeV} \leq Q \leq 1000 \text{ GeV}\}$

S. A. Ochoa-Oregon et al., Phys. Rev. D 110, 036019 [hep-ph].

Functional Forms

Up distributions

$$xu_v(x,Q^2) = A_{u_v}(Q^2)x^{B_{u_v}(Q^2)}(1-x)^{C_{u_v}(Q^2)} \times [1 + D_{u_v}(Q^2)x + E_{u_v}(Q^2)x^2 + F_{u_v}(Q^2)x^3 + G_{u_v}(Q^2)x^4 + H_{u_v}(Q^2)x^5]$$

For R_1:

$$x\bar{u}(x,Q^2) = A_{\bar{u}}(Q^2)x^{B_{\bar{u}}(Q^2)}(1-x)^{C_{\bar{u}}(Q^2)} \times [1 + D_{\bar{u}}(Q^2)x + E_{\bar{u}}(Q^2)x^2 + F_{\bar{u}}(Q^2)x^3 + G_{\bar{u}}(Q^2)x^4 + H_{\bar{u}}(Q^2)x^5]$$

For R_2:

$$x\bar{u}(x,Q^2) = A_{\bar{u}}(Q^2)x^{B_{\bar{u}}(Q^2)}(1-x)^{C_{\bar{u}}(Q^2)} \times [1 + D_{\bar{u}}(Q^2)x + E_{\bar{u}}(Q^2)x^2 + F_{\bar{u}}(Q^2)x^3 + G_{\bar{u}}(Q^2)x^4 + H_{\bar{u}}(Q^2)x^5]$$

Down distributions

$$xd_v(x,Q^2) = A_{d_v}(Q^2)x^{B_{d_v}(Q^2)}(1-x)^{C_{d_v}(Q^2)} \times [1 + D_{d_v}(Q^2)x^2 + E_{d_v}(Q^2)x^4 + F_{d_v}(Q^2)x^6],$$

$$x\bar{d}(x,Q^2) = A_{\bar{d}}(Q^2)x^{B_{\bar{d}}(Q^2)}(1-x)^{C_{\bar{d}}(Q^2)} - \Theta(x_{C,\bar{d}} - x)A'_{\bar{d}}(Q^2)x^{B'_{\bar{d}}(Q^2)}(1-x)^{C'_{\bar{d}}(Q^2)} \times [1 + D'_{\bar{d}}(Q^2)x^2 + E'_{\bar{d}}(Q^2)x^4],$$

S. A. Ochoa-Oregon et al., Phys. Rev. D 110, 036019 [hep-ph].

Functional Forms

Strange distribution

For R_1:

$$xs(x,Q^2) = A_s(Q^2)x^{B_s(Q^2)}(1-x)^{C_s(Q^2)} - \Theta(x_{C,s}(R_1)-x)A'_s(Q^2)x^{B'_s(Q^2)} \times (1-x)^{C'_s(Q^2)}$$

For R_2:

$$xs(x,Q^2) = A_s(Q^2)x^{B_s(Q^2)}(1-x)^{C_s(Q^2)} - \Theta(x_{C,s}(R_2)-x)A'_s(Q^2)x^{B'_s(Q^2)} \times (1-x)^{C'_s(Q^2)}[1+D'_s(Q^2)x^2]$$

Charm distribution

For R_1:

$$xc(x,Q^2) = A_c(Q^2)x^{B_c(Q^2)}(1-x)^{C_c(Q^2)}[1+D_c(Q^2)x^2] - \Theta(x_{C,c}(R_1)-x)A'_c(Q^2)x^{B'_c(Q^2)} \times (1-x)^{C'_c(Q^2)}[1+D'_c(Q^2)x^2]$$

For R_1, R_2:

$$xc(x,Q^2) = A_c(Q^2)x^{B_c(Q^2)}(1-x)^{C_c(Q^2)} \times (1+D_c(Q^2)x+E_c(Q^2)x^2) - \Theta(x_{C,c}(R_2)-x)A'_c(Q^2)x^{B'_c(Q^2)} \times (1-x)^{C'_c(Q^2)}(1+D'_c(Q^2)x^2)$$

S. A. Ochoa-Oregon et al., Phys. Rev. D 110, 036019 [hep-ph].

Functional Forms

Bottom distribution

For R_1:

$$xb(x,Q^2) = A_b(Q^2)x^{B_b(Q^2)}(1-x)^{C_b(Q^2)}[1+D_b(Q^2)x^2] - \Theta(x_{C,b}(R_1)-x)A'_b(Q^2)x^{B'_b(Q^2)} \times (1-x)^{C'_b(Q^2)}[1+D'_b(Q^2)x^2]$$

For R_2:

$$xb(x,Q^2) = A_b(Q^2)x^{B_b(Q^2)}(1-x)^{C_b(Q^2)} \times [1+D_b(Q^2)x+E_b(Q^2)x^2] - \Theta(x_{C,b}(R_2)-x)A'_b(Q^2)x^{B'_b(Q^2)} \times (1-x)^{C'_b(Q^2)}[1+D'_b(Q^2)x^2],$$

Gluon distribution

For R_1:

$$xg(x,Q^2) = A_g(Q^2)x^{B_g(Q^2)}(1-x)^{C_g(Q^2)} - \Theta(x_{C,g}-x)A'_g(Q^2)x^{B'_g(Q^2)} \times (1-x)^{C'_g(Q^2)}$$

For R_1, R_2 and R_3 :

$$xg(x,Q^2) = A_g(Q^2)x^{B_g(Q^2)}(1-x)^{C_g(Q^2)} - \Theta(x_{C,g}-x)A'_g(Q^2)x^{B'_g(Q^2)} \times (1-x)^{C'_g(Q^2)}$$

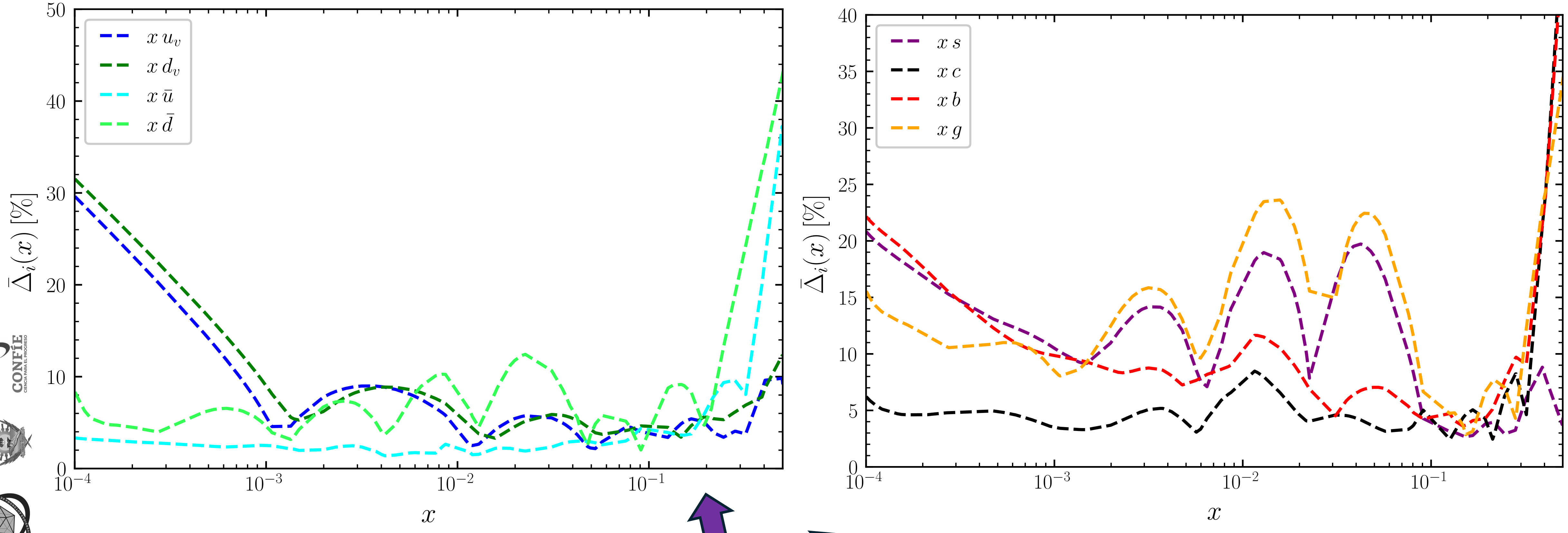
S. A. Ochoa-Oregon et al., Phys. Rev. D 110, 036019 [hep-ph].

CONFIE CIENCIA PARA EL PROGRESO

SIL FAE

Another Cost-Function

Error Shape



$N = 50{,}000$ and $Q \in \{10 \text{ GeV}, 1000 \text{ GeV}\}$

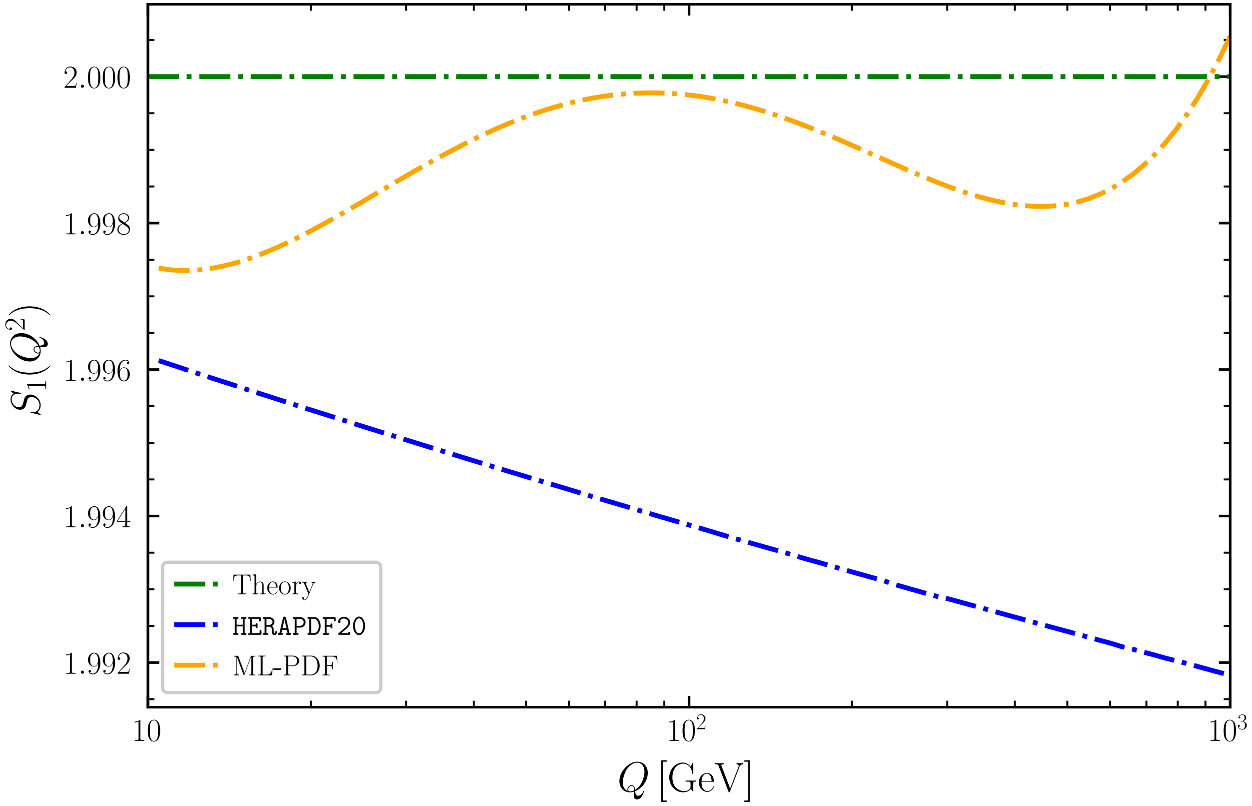
$$\bar{\Delta}_i(x) = \frac{1}{N}\sum_{j=1}^{N}\left|1 - \frac{f_i^{\mathrm{ML}}(x, Q_j^2)}{f_i^{\mathrm{HERA}}(x, Q_j^2)}\right|$$

S. A. Ochoa-Oregon et al., Phys. Rev. D 110, 036019 [hep-ph].

Validity of the sum rules

Sume rules for any energy scale Q

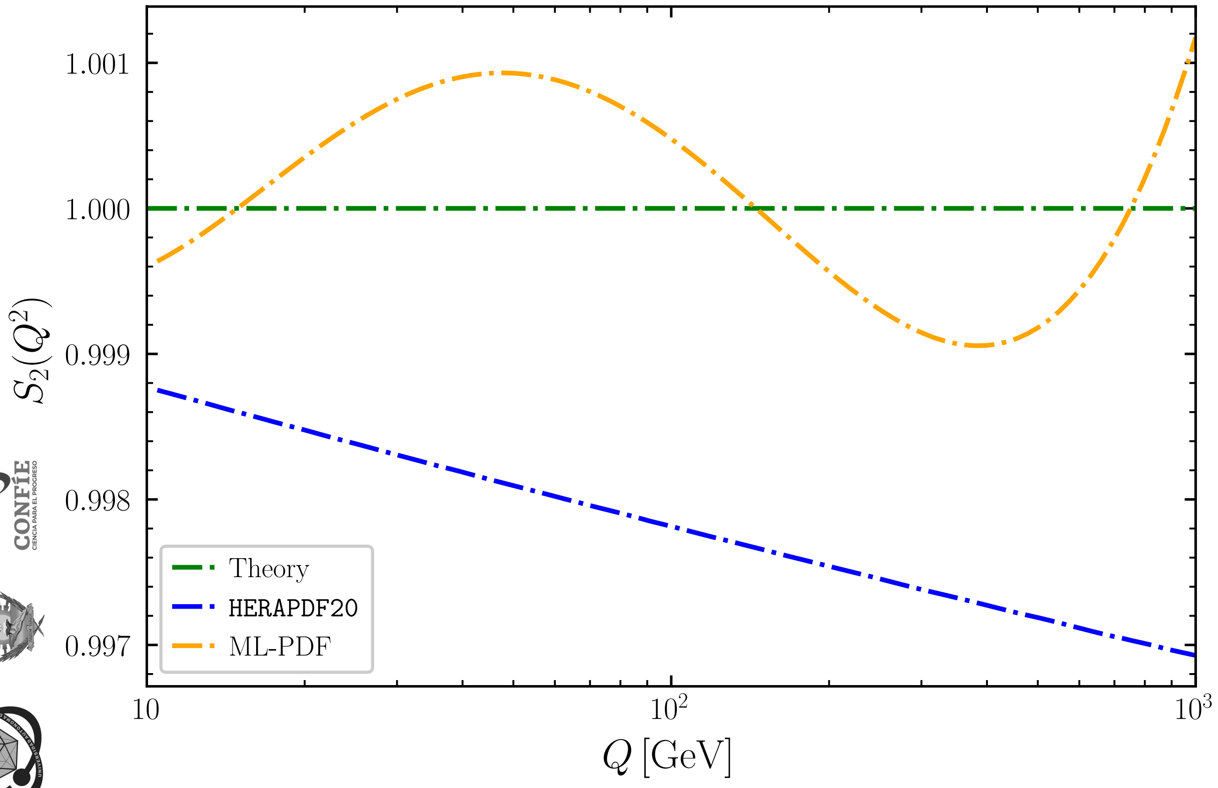
$$S_1(Q^2) = \int_0^1 dx\, u_v(x, Q^2) = 2 \quad S_2(Q^2) = \int_0^1 dx\, d_v(x, Q^2) = 1 \quad S_3(Q^2) = \int_0^1 dx\, x\left[g(x, Q^2) + \sum_{i\in\{q,\bar{q}\}} f_i(x, Q^2)\right] = 1$$

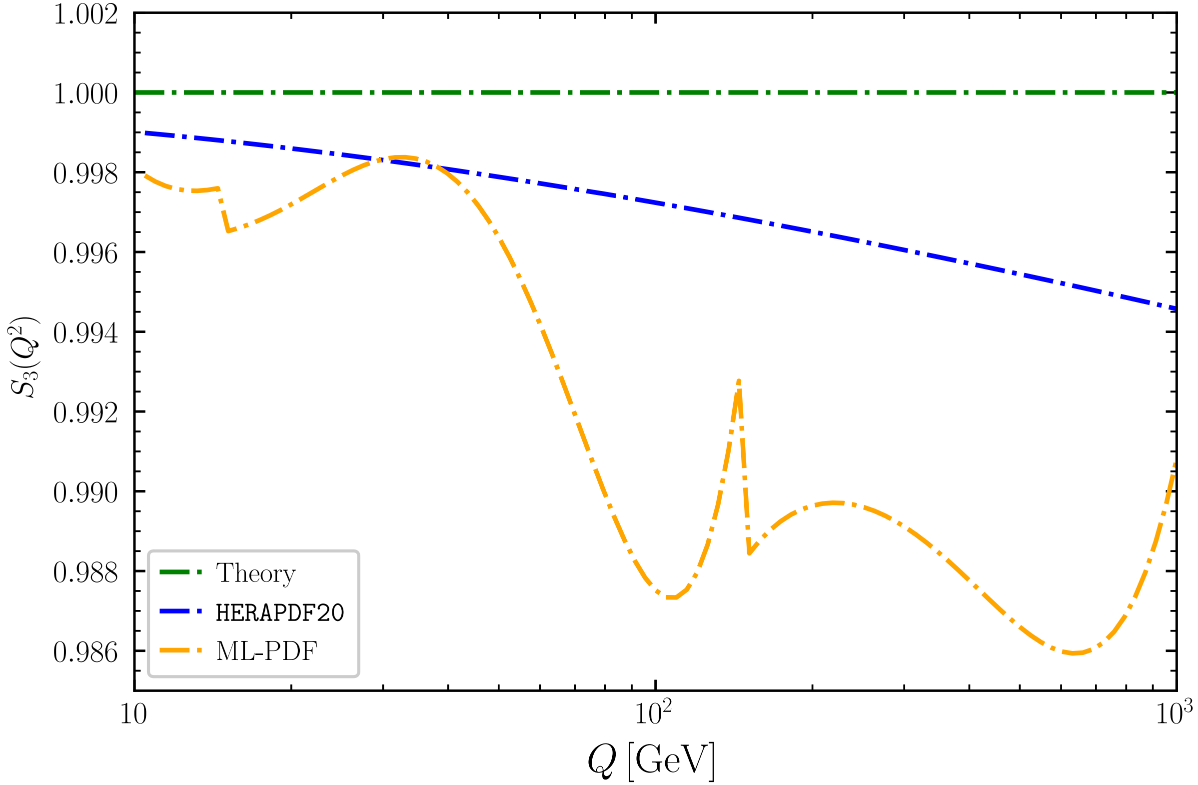


S. A. Ochoa-Oregon et al., Phys. Rev. D 110, 036019 [hep-ph].

Validity of the sum rules

Sume rules for any energy scale Q





S. A. Ochoa-Oregon et al., Phys. Rev. D 110, 036019 [hep-ph].