Top-down rank reduction from heterotic non-Abelian orbifolds

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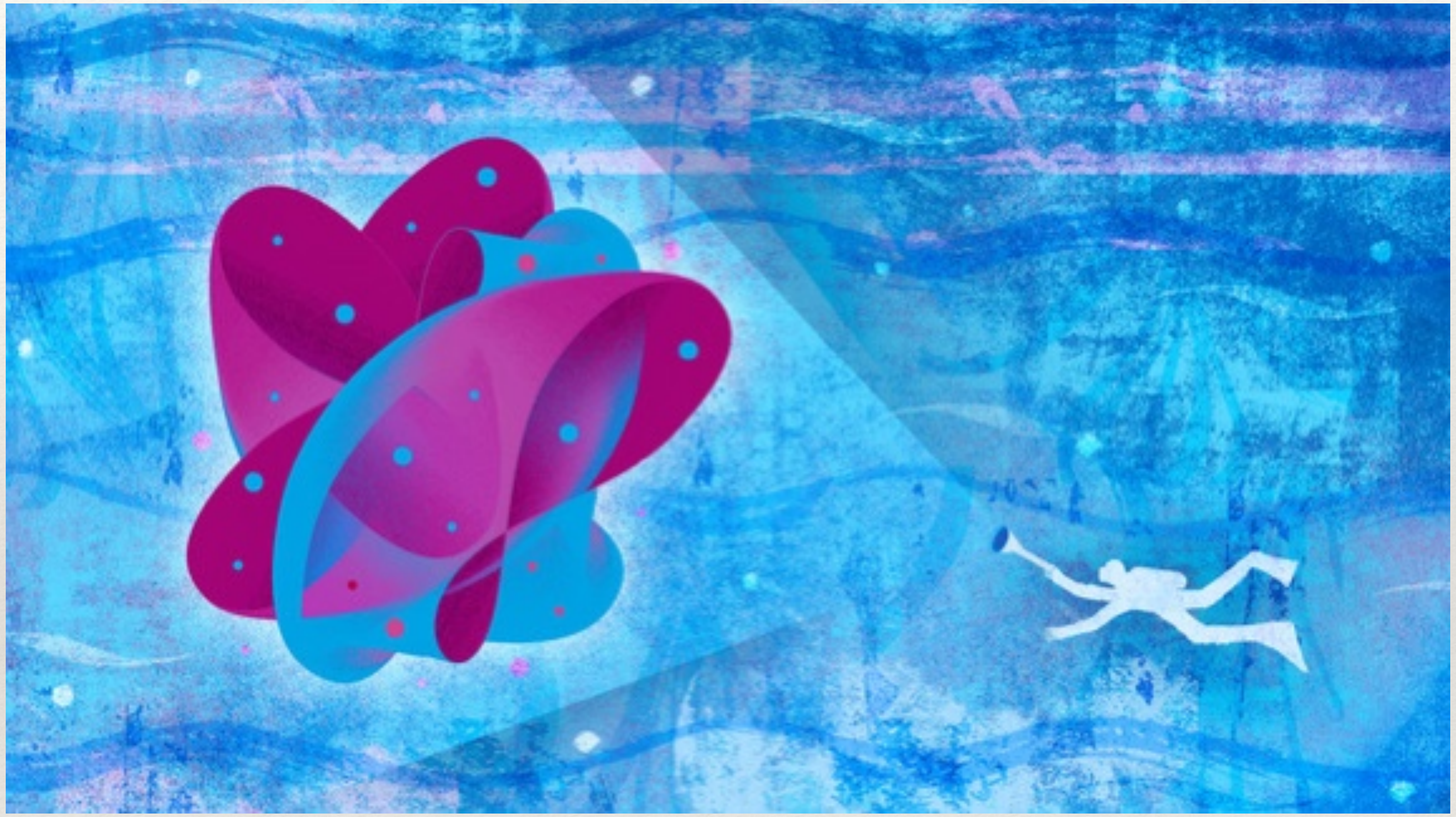
Overview

- Brief description of the heterotic string
- Orbifolds: a geometric description
- Why to focus on the non-Abelian case?
- Generalization of the Abelian formalism
- Results on specific geometries: S_3, D_4 and $\left(\mathbb{Z}_4 \times \mathbb{Z}_2\right) \rtimes \mathbb{Z}_2$
- Conclusions and future work

Motivation



[The power of mirror symmetry, Robbert Dijkgraaf]

Framework: Heterotic string theory

We have:

- 10 dimensional theory
- Supersymmetric theory
- Fixed gauge group: $E_8 \times E_8$ o $SO(32)$

We desire:

- 4 dimensional theory
- Supersymmetric theory (?)
- SM gauge group: $SU(3) \times SU(2) \times U(1)$

The spacetime dimension and the rank of the gauge groups does not match!

[Ibañez Uranga, String theory and particle physics]

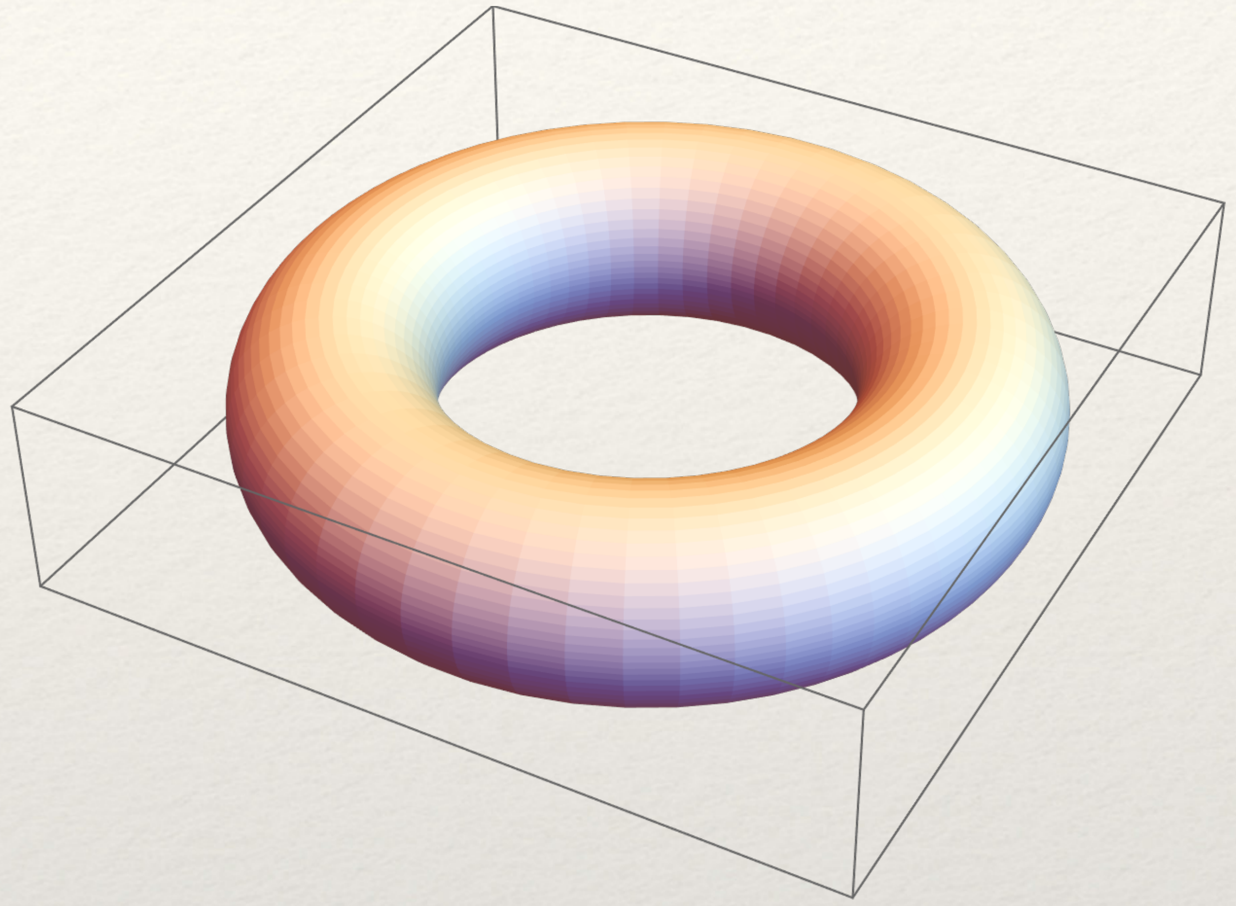
Dimensional reduction

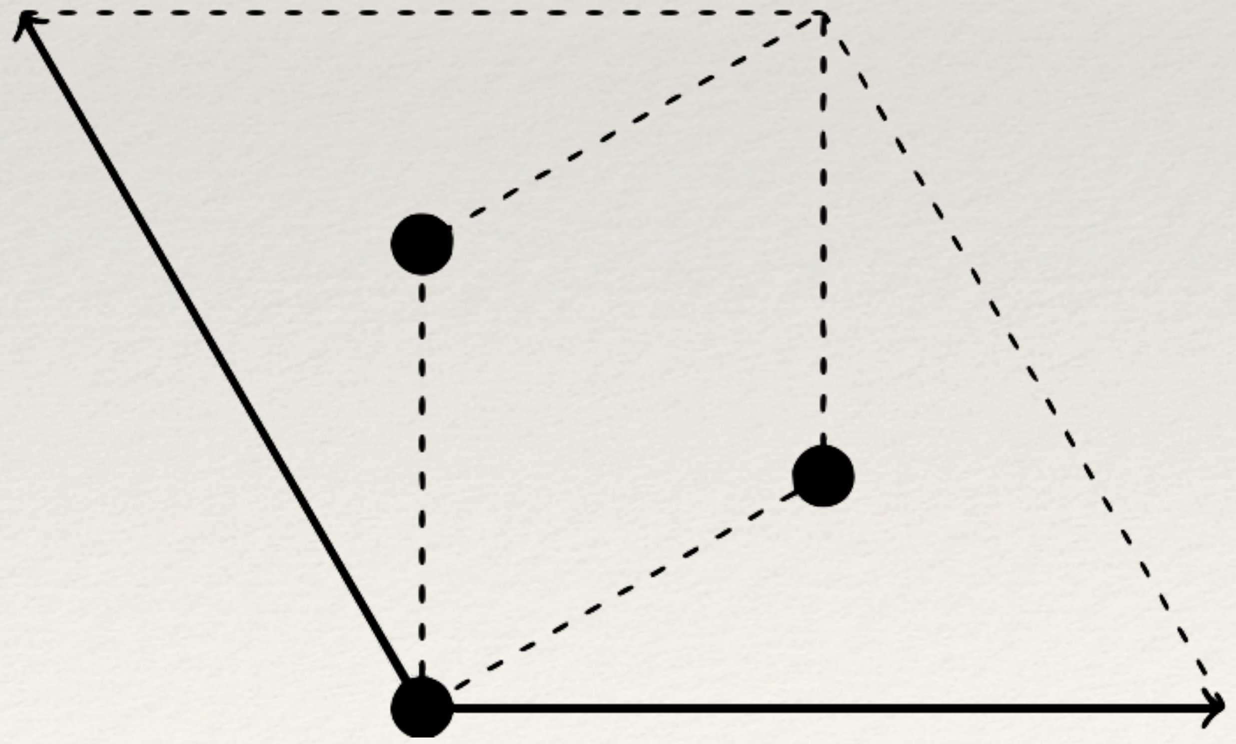
- Need to go from a 10 dim theory to a 4 dim one
- Ansatz $\mathscr{M}_{10} \to \mathscr{M}_4 \times X_6$
- In general X_6 should be a Calabi-Yau manifold
- In this talk, X_6 is a toroidal orbifold

[Ibañez Uranga, String theory and particle physics]

Orbifolds: a geometric description

- An orbifold is defined as the quotient
  $\mathcal{O} = M/P$
- We are interested on the case
  $M = \mathbb{T}^6, P \subset SU(3)$
- $\mathbb{T}^6 = \mathbb{R}^6/\Gamma$
- Hence, $\mathcal{O} = \mathbb{R}^6/S, \quad S = P \ltimes \Gamma$





Why non-Abelian P?

- From the bottom-up perspective, it is possible to obtain rank reduction from non-Abelian twists **[Hebecker-Ratz 2003]**
- Rank reduction evidence from the top-down approach **[Konopka 2013]**

Which non-Abelian groups?

- There are 35 inequivalent point groups compatibles with 4 dim. SUSY $\mathcal{N} = 1$,

S_3	$\mathbb{Z}_3 \times S_3$	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	$\mathbb{Z}_3 \times (\mathbb{Z}_3 \rtimes \mathbb{Z}_4)$	$\mathbb{Z}_3 \times S_4$
D_4	$\Delta(27)$	$SL(2,3) - I$	$\mathbb{Z}_3 \times A_4$	$\Delta(96)$
A_4	$\mathbb{Z}_4 \times S_3$	$\mathbb{Z}_3 \times SL(2,3)$	$\mathbb{Z}_6 \times S_3$	$SL(2,3) \rtimes \mathbb{Z}_4$
D_6	$(\mathbb{Z}_6 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$	$(\mathbb{Z}_4 \times \mathbb{Z}_4) \rtimes \mathbb{Z}_2$	$\Delta(48)$	$\Sigma(36\phi)$
$\mathbb{Z}_8 \rtimes \mathbb{Z}_2$	$\mathbb{Z}_3 \times D_4$	$\mathbb{Z}_3 \times ((\mathbb{Z}_6 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2)$	$GL(2,3)$	$\Delta(108)$
QD_{16}	$\mathbb{Z}_3 \rtimes Q_8$	$\Delta(216)$	$SL(2,3) \rtimes \mathbb{Z}_2$	$PSL(3,2)$
$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$	Frobenius T_7	S_4	$\Delta(54)$	$\Sigma(72\phi)$

[Fischer, Ratz, Torrado, Vaudrevange 2013]

- 331 non equivalent geometries arise from them

Abelian vs non-Abelian

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- In the Abelian case, P is a subgroup of the $SO(6)$ Cartan subalgebra

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- This can't happen in the non-Abelian scenario 😔

Abelian vs non-Abelian

- In the Abelian case, P is a subgroup of the $SO(6)$ Cartan subalgebra
- This can't happen in the non-Abelian scenario 😔
- This fact gives rise to rank reduction 😁

Generalization to the non-Abelian case

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- Tasks:
  Embed P in the geometric degrees of freedom $SO(6) \subset SO(8)$ and in the gauge degrees of freedom $SO(32)$
  Compute the 4 dim spectrum

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  Embed P in the geometric degrees of freedom $SO(6) \subset SO(8)$ and in the gauge degrees of freedom $SO(32)$
  Compute the 4 dim spectrum

- Difficulties:
  Write P elements as rotations (block diagonal matrices)
  Treat simultaneously with different choices for the Cartan basis (different roots systems)

$P \hookrightarrow SO(6)$

- To achieve the embedding, we have to assign a twist vector $v = (0, v_1, v_2, v_3) \in \Lambda_{SO(8)}$ to each $g \in P$ $\quad$ v is in the $SO(8)$ Cartan basis
- The components of the twist vector of a given g, are such that $g = \exp\left[2\pi i v_k J_k\right]$, for some J_k ($SO(6)$ generators)
- So, we need a basis β_g, such that $g = \exp\left[2\pi i v_k J_k\right]$
- Successfully done for S_3, D_4 y $\left(Z_4 \times Z_2\right) \rtimes Z_2$

Block diagonalization

- We developed an algorithm for this task

$$\begin{pmatrix} E_p & F_{q\times p} \\ G_{p\times q} & H_q \end{pmatrix} \to \begin{pmatrix} D_p & 0_{q\times p} \\ 0_{p\times q} & D_q \end{pmatrix},$$

by solving the equations

$$R(E+FR) = G + HR, \qquad (E+FR)X - X(H-RF) = -F,$$

for R and X. **[Eisenfeld 76]**

Block diagonalization

- If there are solutions R and X, the transformation that block diagonalize our original matrix is

$$W = \begin{pmatrix} \mathbb{1}_p & X \\ R & XR + \mathbb{1}_q \end{pmatrix} .$$ **[Eisenfeld 76]**

This W was found in 12 cases

$$\tilde{P} = \{S_3, D_4, A_4, D_6, (\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2, \mathbb{Z}_4 \rtimes S_3, S_4, \\ (\mathbb{Z}_4 \times \mathbb{Z}_4) \rtimes \mathbb{Z}_2, \mathbb{Z}_3 \times (\mathbb{Z}_3 \rtimes \mathbb{Z}_4), \Delta(27), \Delta(54), \Delta(96)\} .$$

Block diagonalization

- One last step

$$\begin{pmatrix} E_p & F_{q\times p} \\ G_{p\times q} & H_q \end{pmatrix} \to \begin{pmatrix} D_p & 0_{q\times p} \\ 0_{p\times q} & D_q \end{pmatrix} \to \begin{pmatrix} R(\theta)_2 & 0 & 0 \\ 0 & R(-\theta)_2 & 0 \\ 0 & 0 & \mathbb{I}_2 \end{pmatrix},$$

- Say that the full transformation is Q, we restricted to the case where Q is orthogonal. This condition reduced our previous list to $P \in \{S_3, D_4, (\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2\}$.

4 dim. gauge group G

4 dim. gauge group G

- To achieve $P \hookrightarrow SO(32)$, we have to assign a shift vector $V = (V_1, V_2, \ldots, V_{16}) \in \Lambda_{SO(32)}$ to each $g \in P$ $\quad$ V is in the $SO(32)$ Cartan basis

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- Solution: $V = (v_1, v_2, v_3, 0, \ldots, 0)$ [Standard embedding]

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- This gives
  $SO(32) \to SO(6) \times SO(26) \to G \times SO(26)$

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- Solution: $V = (v_1, v_2, v_3, 0, \ldots, 0)$ [Standard embedding]
- This gives
  $SO(32) \to SO(6) \times SO(26) \to G \times SO(26)$
- G, such that $\mathrm{rank}(G) < 3$.
  Therefore
  $\mathrm{rank}(G \times SO(26)) < 16$ 😁

Spectrum: untwisted sector

- Work with the $SO(8)$ weights, $|q\rangle \in \left\{ \left(\underline{(\pm 1)^2, 0^2} \right) \right\}$ and $SO(32)$ roots, $|p\rangle \in \left\{ \left(\underline{(\pm 1)^2, 0^{14}} \right) \right\}$
- We build states $|q\rangle \otimes |p\rangle$, such that
  $p \cdot V_g - q \cdot v_g = 0, \ \text{mod } 1 \quad \forall g \in S$
- This require us to work with different Cartan bases simultaneously
  Trouble!

Spectrum: twisted sectors

- We look for $|q\rangle$ in the $SO(8)$ weight lattice, and $|p\rangle$ in the $SO(32)$ root lattice such that, they satisfy the physical condition for their respective equivalence class $[g]$ and its centralizer $\mathscr{C}_S(g)$
- It reduces to the Abelian techniques (if $\mathscr{C}_S(g)$ is Abelian)
  Fine!

Dealing with different Cartan bases

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  $H = \{H_1, H_2, H_3\}$ y $H' = \{H'_1, H'_2, H'_3\}$ ordered bases

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- H_i and H'_i have identical roles
- H and H' give rise to different root systems
  $R = \{R_1, R_2, \ldots, R_6\}$ and $R' = \{R'_1, R'_2, \ldots, R'_6\}$

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  $R = \{R_1, R_2, \ldots, R_6\}$ and $R' = \{R'_1, R'_2, \ldots, R'_6\}$
- $\forall H_i,$ there are $R_l,\ R_m \in R$ such that R_l is the raising operator and R_m is the lowering operator for H_i

Dealing with different Cartan bases

- Take two different $SO(6)$ Cartan bases,
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  $R = \{R_1, R_2, \ldots, R_6\}$ and $R' = \{R'_1, R'_2, \ldots, R'_6\}$
- $\forall H_i,$ there are $R_l,\ R_m \in R$ such that R_l is the raising operator and R_m is the lowering operator for H_i
- This is also true for some $R'_l,\ R'_m \in R'$ for each H'_i

Dealing with different Cartan bases

- Solution: We propose a *bijection* $R_l \sim R_l' \quad R_m \sim R_m'$
- With this, we can manipulate twist and shift vectors in different basis!😁

Specific models

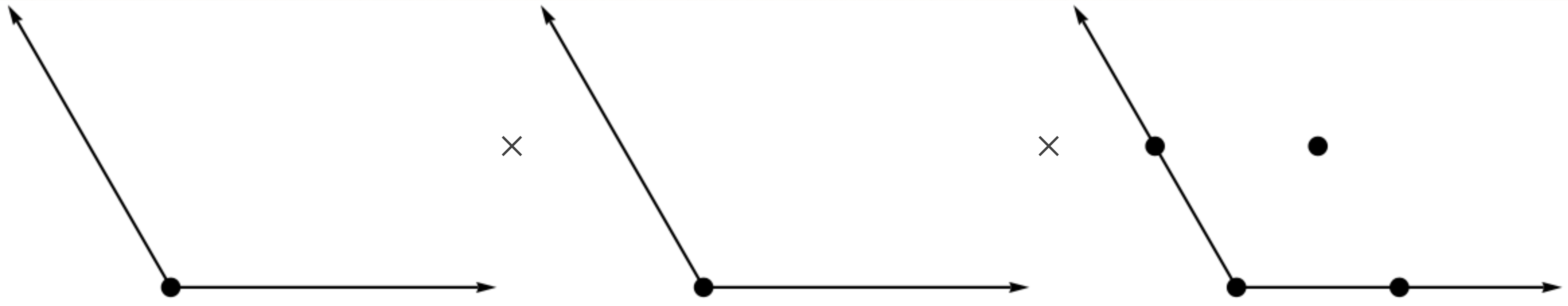
- We studied the geometries S_3, D_4 y $\left(Z_4 \times Z_2\right) \rtimes Z_2$
- These are completely inequivalent geometries, with different number of twisted sectors and fixed points.
  Therefore, their study will allow us to check our method in truly different scenarios and verify that rank reduction is indeed a feature of every non-Abelian orbifold

S_3 orbifold

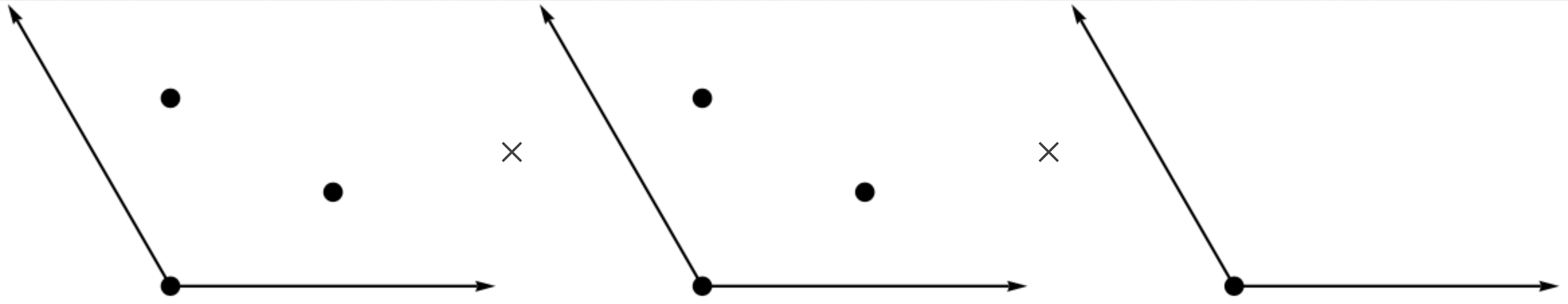
- S_3 has two generators, of order 2 and 3 respect. Say $\{\theta, \omega\}$
- S_3 is the symmetry group of an equilateral triangle
- It has two non trivial conjugation classes: $[\theta]$ y $[\omega]$
- This orbifold has 13 fixed points, 4 related to the $[\theta]$ sector and 9 for the $[\omega]$ sector

S_3 orbifold

- Fixed points



theta sector fixed points



omega sector fixed points

-

S_3 orbifold

- Through one single basis transformation, we found

$$\theta = \exp\left[\frac{2\pi i}{2}\left(J_{4,6} - J_{7,8}\right)\right], \quad \omega = \exp\left[\frac{2\pi i}{3}\left(J_{3,4} - J_{5,6}\right)\right]$$

S_3 orbifold

- 4 dim gauge group: $G = U(1) \times U(1) \times SO(26)$
- 4 modules (in agreement with [Fischer, Ramos, Vaudrevange 2013])
- We found the **26** irrep with the following multiplicity in each sector
  $4 \in [e], \quad 8 \in [\theta], \quad 18 \in [\sigma]$ (generalizes $E_8 \times E_8$ results [Fischer, Ramos, Vaudrevange 2013])

S_3 orbifold

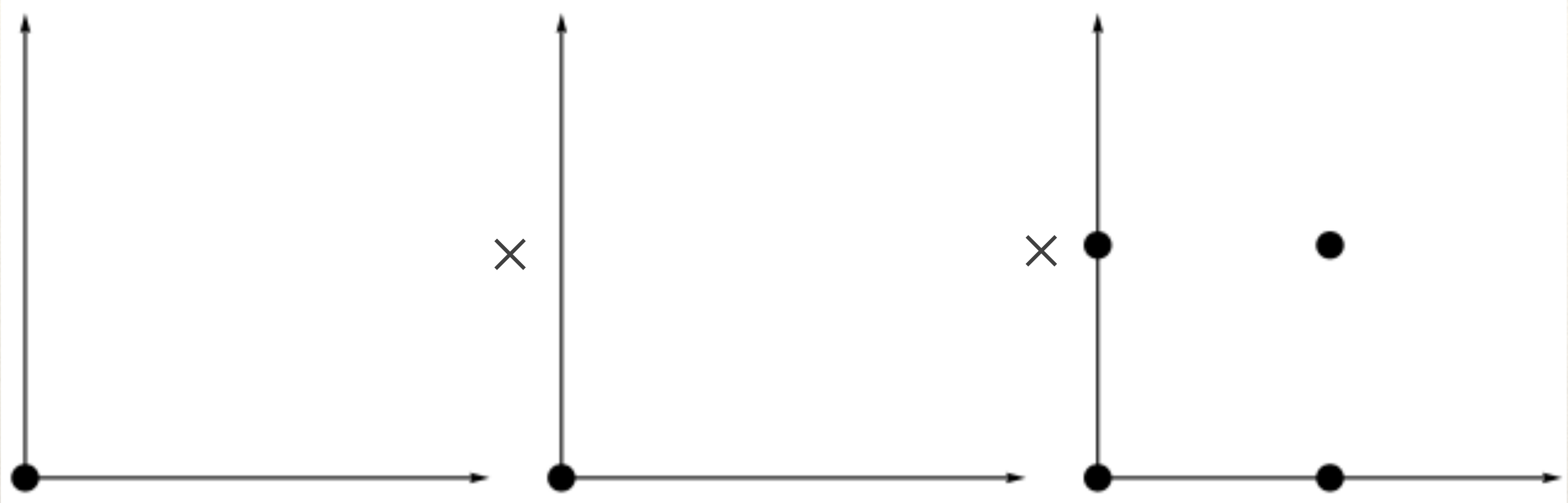
$SO(26) \times U(1) \times U(1)$ irrep.	$[e]$ sector	$[\vartheta]$ sector	$[\omega]$ sector
$\mathbf{26}_{(0,0)}$	4	8	0
$\mathbf{26}_{(1,0)}$	0	0	9
$\mathbf{26}_{(-1,0)}$	0	0	9
$\mathbf{1}_{(1,0)}$	1	16	0
$\mathbf{1}_{(-1,0)}$	1	16	0
$\mathbf{1}_{(0,1)}$	0	8	0
$\mathbf{1}_{(0,-1)}$	0	8	0
$\mathbf{1}_{(1,1)}$	0	0	9
$\mathbf{1}_{(1,-1)}$	0	0	9
$\mathbf{1}_{(-1,1)}$	0	0	9
$\mathbf{1}_{(-1,-1)}$	0	0	9

4 dimensional spectrum

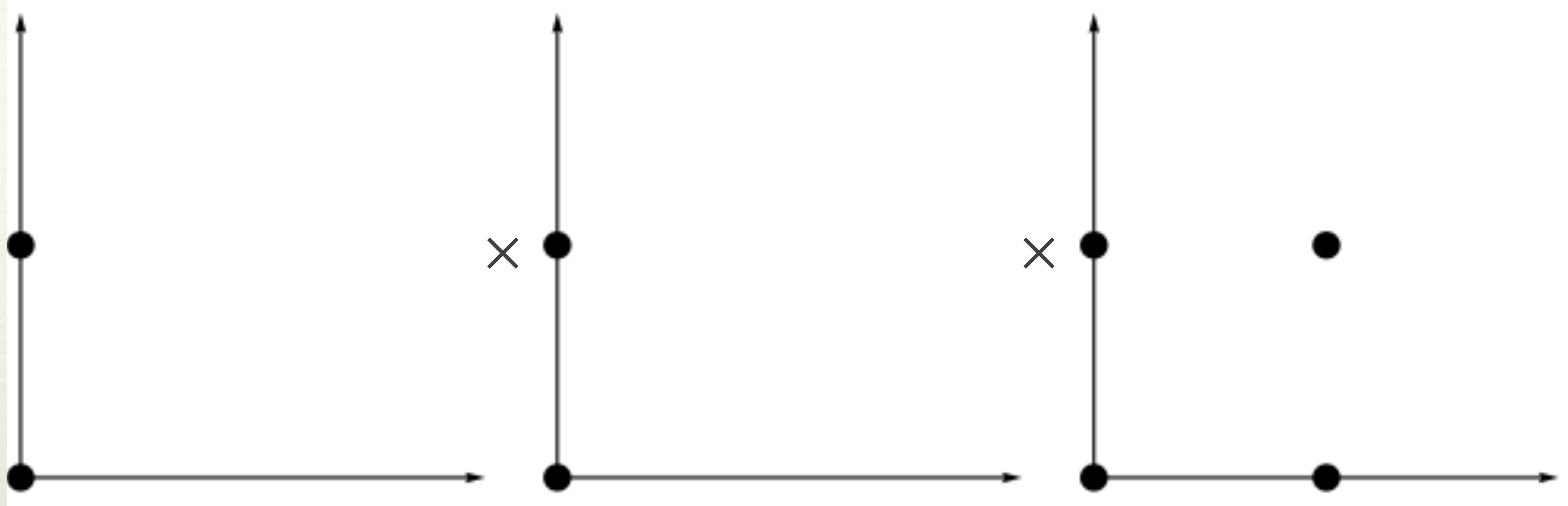
D_4 orbifold

- D_4 has 2 order 2 generators, $\{\theta, \omega\}$
- D_4 is the symmetry group of a square
- 4 non trivial conjugacy classes: $[\theta],\ [\omega],\ [\theta\omega],\ [\theta\omega\theta\omega]$
- 34 fixed points

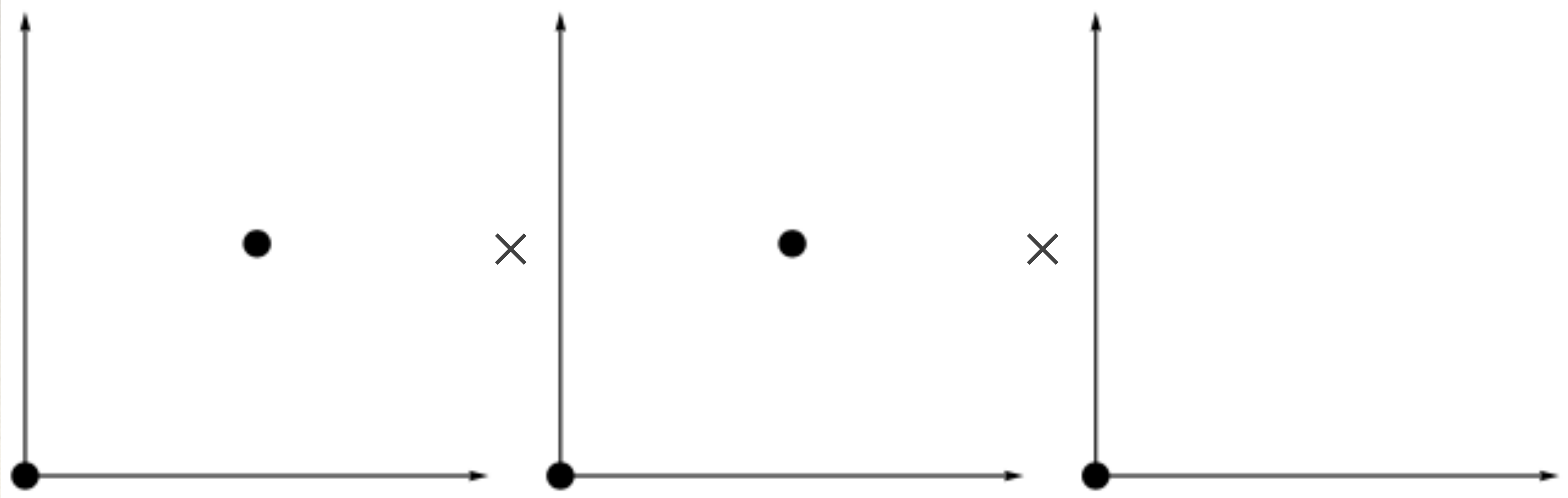
D_4 orbifold



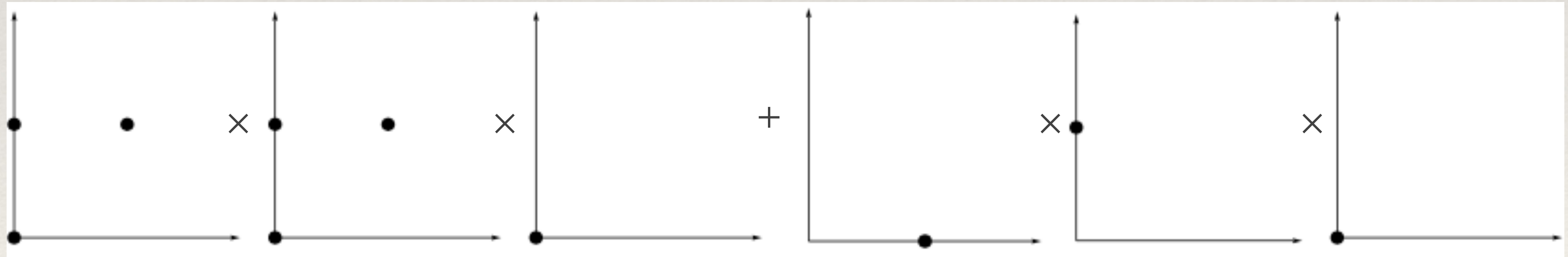
theta sector fixed points



omega sector fixed points



theta*omega sector fixed points



theta*omega*theta*omega sector fixed points

D_4 orbifold

- We require two different transformaciones to arrive to the following expressions

$$\theta = \exp\left[\frac{2\pi i}{2}\left(J_{3,6} - J_{7,8}\right)\right], \qquad \omega = \exp\left[\frac{2\pi i}{3}\left(J_{4,6} - J_{7,8}\right)\right],$$

$$\theta\omega = \exp\left[\frac{2\pi i}{4}\left(-J_{3,4} + J_{5,6}\right)\right], \; \theta\omega\theta\omega = \exp\left[\frac{2\pi i}{2}\left(J_{3,4} - J_{5,6}\right)\right].$$

D_4 orbifold

- 4 dim gauge group $G = U(1) \times SO(26)$
- 4 modules (in agreement with [Fischer, Ramos, Vaudrevange 2013])
- **26** irreps with the following multiplicity in each sector:
  $4 \in [e], \quad 4 \in [\theta], \quad 16 \in [\omega], \quad 8 \in [\theta\omega], \quad 10 \in [\theta\omega\theta\omega]$
  (generalizes results for $E_8 \times E_8$ [Fischer, Ramos, Vaudrevange 2013])

D_4 orbifold

$SO(26) \times U(1)$ irrep	$[e]$	$[\vartheta]$	$[\omega]$	$[\vartheta\omega]$	$[(\vartheta\omega)^2]$
$\mathbf{26}_0$	2	4	16	0	10
$\mathbf{26}_1$	1	0	0	4	0
$\mathbf{26}_{-1}$	1	0	0	4	0
$\mathbf{1}_0$	2	8	0	12	10
$\mathbf{1}_{-1}$	2	0	16	4	20
$\mathbf{1}_1$	2	0	16	4	20

4 dimensional spectrum

$\left(\mathbb{Z}_4 \times \mathbb{Z}_2\right) \rtimes \mathbb{Z}_2$ orbifold

- 3 generators with order 4, 2 and 2 respect. $\{\rho, \theta, \omega\}$
- This group can be understood as a discrete version of $SU(2)$
- 8 non trivial conjugacy classes: $[\rho],\ [\theta],\ [\omega],\ [\theta\omega],\ [\theta\rho],\ [\omega\rho],\ [\theta\omega\rho^3],\ [\rho^2]$
- 35 fixed points

$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$ orbifold

- We found 5 different transformations that lead us to

$$\rho = \exp\left[\frac{2\pi i}{4}\left(J_{5,6} - J_{4,7}\right)\right], \qquad \theta = \exp\left[\frac{2\pi i}{2}\left(J_{3,4} - J_{7,8}\right)\right],$$

$$\omega = \exp\left[\frac{2\pi i}{2}\left(J_{3,7} - J_{4,8}\right)\right], \qquad \theta\omega = \exp\left[\frac{2\pi i}{4}\left(-J_{4,5} - J_{6,7} + 2J_{3,8}\right)\right],$$

$$\theta\rho = \exp\left[\frac{2\pi i}{4}\left(J_{3,4} + J_{6,7} - 2J_{5,8}\right)\right], \qquad \omega\rho = \exp\left[\frac{2\pi i}{2}\left(J_{3,4} - J_{6,8}\right)\right],$$

$$\theta\omega\rho^3 = \exp\left[\frac{2\pi i}{4}\left(J_{4,5} - J_{6,7}\right)\right], \qquad \rho^2 = \exp\left[\frac{2\pi i}{2}\left(J_{5,6} - J_{4,7}\right)\right].$$

$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$ orbifold

- 4 dim gauge group $G = SO(26)$ (max rank reduction from standard embedding)
- 3 modules (in agreement with [Fischer, Ramos, Vaudrevange 2013])
- **26** irreps with the following multiplicity in each sector
  $3 \in [e], \quad 4 \in [\theta], \quad 4 \in [\omega], \quad 10 \in [\rho], \quad 2 \in [\theta\omega],$
  $2 \in [\theta\rho], \quad 2 \in [\omega\rho], \quad 6 \in [\theta\omega\rho^3], \quad 3 \in [\rho^2]$
  (generalizes results for $E_8 \times E_8$ [Fischer, Ramos, Vaudrevange 2013])

$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$ orbifold

SO(26) irrep	$[e]$	$[\vartheta]$	$[\omega]$	$[\rho]$	$[\vartheta\omega]$	$[\vartheta\rho]$	$[\omega\rho]$	$[\vartheta\omega\rho^3]$	$[\rho^2]$
26	3	4	4	10	2	2	2	6	5
1	3	8	16	30	8	4	10	24	20

4 dimensional spectrum

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  S_3: $SO(32) \to U(1) \times U(1) \times SO(26)$

  D_4: $SO(32) \to U(1) \times SO(26)$

  $\left(\mathbb{Z}_4 \times \mathbb{Z}_2\right) \rtimes \mathbb{Z}_2$: $SO(32) \to SO(26)$ (max rank reduction for standard embedding)

Conclusions

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  S_3: $SO(32) \to U(1) \times U(1) \times SO(26)$
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  $\left(\mathbb{Z}_4 \times \mathbb{Z}_2\right) \rtimes \mathbb{Z}_2$: $SO(32) \to SO(26)$ (max rank reduction for standard embedding)
- We found less modules that the usual for the Abelian case [Ibañez, Lust 92]

Future work

- Take into account Wilson loops
- Compute the flavor symmetries that arise from non-Abelian geometries (geometric and modular ones)
- Extend the formalism to non standard embedding Model building from our non-Abelian geometries (interesting gauge groups and flavor symmetries (modular))

Thanks for your attention!

Space group S

- Conjugacy classes $[g] = \{hgh^{-1} | h \in S\}$
- Fixed points, for each $[g]$ there are some z such that $gz = z$
- In general, for each $g \in S$ we solve $z = (\mathbb{1} - g)^{-1} n_\alpha e_\alpha$
- Abelian case also need to embed P in $SO(6)$

Abelian techniques

- P is contained in the $SO(6)$ Cartan subalgebra H, i.e. every $g \in P$ is an exponential map of linear combinations of elements of H, say
$g = \exp\left[2\pi i \alpha_j H_j\right]$.

- Every conjugación class is define by a twist vector
$v = (\alpha_1, \alpha_2, \alpha_3)$, v is in the $SO(6)$ Cartan basis
such that
$\alpha_1 + \alpha_2 + \alpha_3 = 0.$

- This define the embedding of P in the geometric dof

Abelian techniques

- To embed S in the gauge dof, we map
  $\left(\theta^k, n_\alpha e_\alpha\right) \rightarrow \left(kV, n_\alpha A_\alpha\right)$,
  V is the so called shift vector, and A_α are Wilson loops
  V is such that $NV \in \Lambda$, Λ the $SO(32)$ weight lattice

- Modular invariance requires
  $N\left(V^2 - v^2\right) = 0 \mod 2$, (no Wilson loops)

  Simplest solution: standard embedding
  $V = \left(v^1, v^2, v^3, 0^{13}\right)$. V is in the G Cartan basis

Abelian techniques

- For computing the spectrum, there are two cases
- Untwisted sector $[e]$

  States $|q\rangle_R \otimes |p\rangle_L$, such that $0 = \frac{q^2}{2} + N - 1/2, \quad 0 = \frac{p^2}{2} + \tilde{N} - 1.$

  Solutions if $N = 0$ y $q^2 = 1$

  while $\tilde{N} = 1$ y $p = (0^{16})$ (Cartan generators, sugra multiplet, modules)

  or $\tilde{N} = 0$ y $p^2 = 2$ (every other gauge group generators)

  Physical states, those that $p \cdot V_g - q \cdot v_g = 0, \mod 1 \quad \forall g \in S$

Abelian techniques

- Twisted sectors $[g]$

  States $\left|q_{sh}\right\rangle_R \otimes \left|p_{sh}\right\rangle_L$ such that $\frac{q_{sh}^2}{2} - \frac{1}{2} + \delta_g = 0, \quad \frac{p_{sh}^2}{2} - 1 + \tilde{N} + \delta_g = 0,$

  $q_{sh} = q + v_g, \quad p_{sh} = p + V_g\,.$

  q and p in $\Lambda_{SO(8)}$ and $\Lambda_{SO(32)}$ respect.

- Physical states if

  $p_{sh} \cdot V_h - R \cdot v_h = 0, \text{ mod } 1 \quad \forall g \in \mathcal{C}_S(g),$

  with $R^i = q^i_{sh} - \tilde{N}^i + \tilde{N}^{*\bar{i}}, \quad i \in \{0,1,\ldots,3\}\,.$

  States in this sectors are matter fields