

Neutral pion screening mass in a magnetized medium

High energy physics seminar, ICN-UNAM, IF-UNAM

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Overview

- ▶ Introduction (pole and screening masses)
- ▶ LQCD and NJL results for the screening mass of the neutral pion
- ▶ Linear sigma model with quarks (LSMq)
- ▶ Calculation of the neutral pion self-energy in the LSMq.
- ▶ Results
- ▶ Summary and perspectives

Interactions and parameters

Interactions $\rightarrow$ modification of parameters (mass, charge, vertex, etc.)

This modifications are parametrized in self energy diagrams

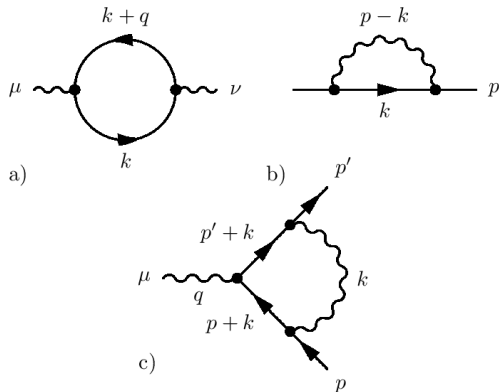


Figure: One loop diagrams

Neutral pion self energy

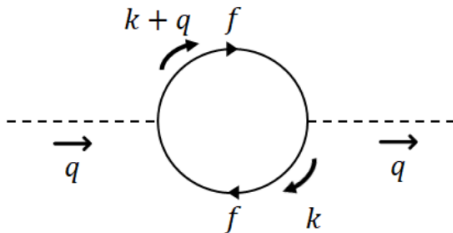


Figure: Neutral pion self-energy

Pole and screening masses ($B \neq 0, T = 0$)

- ▶ Pole mass

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B) \Big|_{p_3^2 = p_\perp^2 = 0} = 0$$

- ▶ Screening mass B breaks Lorentz invariance and defines $\parallel$ and $\perp$

- ▶ Longitudinal

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B) \Big|_{p_0^2 = p_\perp^2 = 0} = 0$$

- ▶ Transverse

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B) \Big|_{p_0^2 = p_3^2 = 0} = 0$$

Pole and screening masses ($B \neq 0, T = 0$)

Relations between pole and screening masses:

$$m_{\pi^0,scr.\perp} = \frac{m_{\pi^0,pole}}{u_{\perp}}$$

$$m_{\pi^0,scr.\parallel} = \frac{m_{\pi^0,pole}}{u_{\parallel}}$$

$$m_{\pi^0,pole} = m_{\pi^0,scr.\parallel} < m_{\pi^0,scr.\perp}$$

Comparison with the NJL model

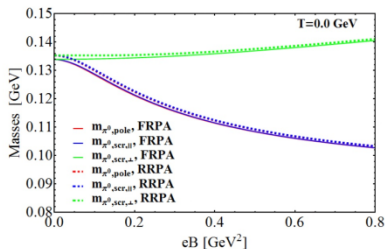
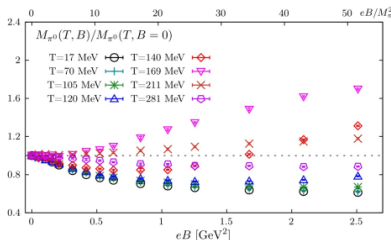


Figure: H. T. Ding, S. T. Li, J. H. Liu, and X. D. Wang, Phys. Rev D105, 034514 (2022), and B. Sheng, Y. Wang, X. Wang, and L. Yu, Phys. Rev. D103 (2021) 9, 094001.

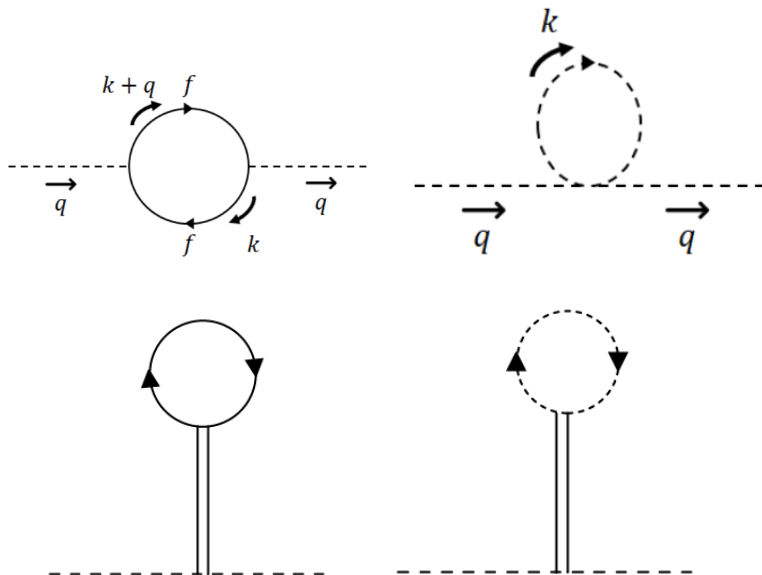
LSMq Lagrangian and features

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu\partial_\mu\psi \\ - ig\bar{\psi}\gamma^5\vec{\psi}\cdot\vec{\pi} - g\bar{\psi}\psi\sigma.$$

- ▶ it implements the SSB of: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$.
- ▶ $m_\pi(v) = \sqrt{\lambda v^2 - a^2} = 0$ at VEV.
- ▶ $m_f(v) = gv$.
- ▶ $m_\sigma(v) = \sqrt{3\lambda v^2 - a^2}$

$\mathcal{L} \rightarrow \mathcal{L} + h(\sigma + v)$ in order to give the correct vacuum pion mass.

Relevant Feynman diagrams



Feynman rules for the fermionic contribution to $\pi_{f\bar{f}}$ (vertices and propagator)

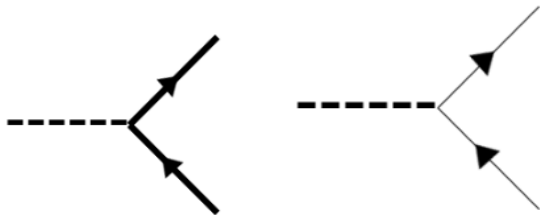


Figure: π_0 - u quark vertex= $g\gamma^5$; π_0 - d quark vertex= $-g\gamma^5$

$$S(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp \left[is \left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon \right) \right]$$

$$\times \left\{ (m_f + \not{p}_\parallel) \left(\cos(qBs) + \gamma^1 \gamma^2 \sin(qBs) \right) - \frac{\not{p}_\perp}{\cos(qBs)} \right\}$$

$$S(p) \rightarrow i \frac{(m_f + \not{p})}{p^2 - m_f^2 + i\epsilon}$$

Parametrizations of denominators

Feynman parametrization:

$$\frac{1}{AB} = \int_0^1 \frac{du}{[uA + (1-u)B]^2}$$

Schwinger parametrization:

$$\frac{i}{A + i\epsilon} = \int_0^\infty du e^{iu(A+i\epsilon)}$$

Neutral pion self-energy (fermion contribution)

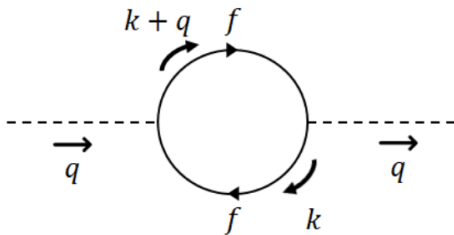


Figure: Neutral pion self-energy

$$-i\pi_{f\bar{f}} = -g^2 \int \frac{d^4 k}{(2\pi)^4} \text{Tr} [\gamma^5 S(k) \gamma^5 S(k + q)] + \text{c.c.}.$$

Neutral pion self-energy (fermion contribution)

$$\begin{aligned}
 -i\pi_{f\bar{f}} = & -4g^2 \int_0^\infty \int_0^\infty \frac{ds ds'}{\cos(qBs) \cos(qBs')} \\
 & \times \int \frac{d^4 k}{(2\pi)^4} e^{is \left(k_\parallel^2 - k_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon \right)} e^{is' \left((k+p)_\parallel^2 - (k+p)_\perp^2 \frac{\tan(qBs')}{qBs'} - m_f^2 + i\epsilon \right)} \\
 & \left\{ \cos[qB(s+s')][m_f^2 - k_\parallel \cdot (k+p)_\parallel] + \frac{k_\perp \cdot (k_\perp + p_\perp)}{\cos(qBs) \cos qBs'} \right\}
 \end{aligned}$$

Important integrals and convenient change of variables

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a} \sqrt{\frac{\pi}{a}}$$

$$u = s + s'$$

$$s = u(1 - v)$$

$$s' = uv$$

$$\frac{\partial(s, s')}{\partial(u, v)} = u$$

Neutral pion self-energy (fermion contribution)

$$\begin{aligned} \pi_{\bar{f}f} = & \frac{-4g^2 qB}{(4\pi)^2} \int_0^1 dv \int_0^\infty du \exp(-ix) \exp(-u\epsilon) \left[\frac{m_f^2}{\tan(qBu)} \right. \\ & - \frac{qB}{\sin^2(qBu)} \left(\frac{p_\perp^2}{|qB|} \frac{\sin(qBu(1-v)) \sin(qBuv)}{\sin(qBu)} \right) - \frac{v(1-v)(p_3^2 - p_0^2)}{\tan(qBu)} \\ & \left. - \frac{iqB}{\sin(qBu)} - \frac{i}{u \tan(qBu)} \right] \end{aligned}$$

where x is given by:

$$x = \frac{p_\perp^2}{qB} \frac{\sin(qBu(1-v)) \sin(qBuv)}{\sin(qBu)} + p_3^2 uv(1-v) - p_0^2 uv(1-v) + m_f^2 u$$

$B \rightarrow 0$ limit of $\pi_{\bar{f}f}$

$$\lim_{B \rightarrow 0} \pi_{\bar{f}f} = -\frac{4g^2}{(4\pi)^2} \int_0^1 dv \int_0^\infty \frac{du}{u} e^{-ix_0} e^{-u\epsilon} \\ \times \left\{ m_f^2 - v(1-v)(p_\perp^2 - p_3^2) + v(1-v)p_0^2 - \frac{2i}{u} \right\}$$

where

$$x_0 = uv(1-v)(p_\perp^2 + p_3^2) - p_0^2 uv(1-v) + m_f^2$$

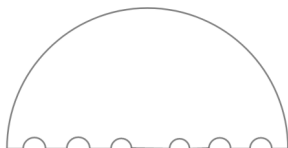
Analysis

$$f(p_0, p_\perp, p_\parallel, B) = \pi_{f\bar{f}} - \lim_{B \rightarrow 0} \pi_{f\bar{f}}$$

We start with the simplest case ($p_\perp^2 = p_0^2 = 0$). This is the 'longitudinal' screening mass which is found by solving the equation:

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B) \Big|_{p_0^2=p_\perp^2=0} = 0$$

Final expression



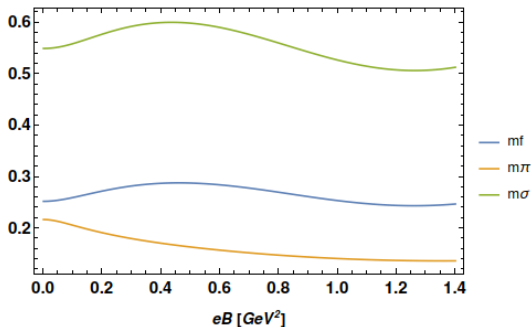
$$\begin{aligned} \Re f = \lim_{\epsilon \rightarrow 0} & \left(-\frac{4g^2}{(4\pi)^2} \int_0^1 dv \left\{ \left(-2v(1-v)p_3^2 \right) \right. \right. \\ & \times \left[\frac{\pi}{2} \frac{\sin(\frac{a\pi}{qB})}{\cosh(\frac{\epsilon\pi}{qB}) - \cos(\frac{a\pi}{qB})} - \tan^{-1} \left(\frac{\sin(\frac{a\pi}{qB}) e^{\left(\frac{-\epsilon\pi}{qB}\right)}}{1 - e^{\left(\frac{-\epsilon\pi}{qB}\right)} \cos(\frac{a\pi}{qB})} \right) \right] \\ & + qB \left[\frac{\epsilon\pi}{2qB} - \ln \sqrt{2 \cosh\left(\frac{\epsilon\pi}{qB}\right) - 2 \cos\left(\frac{a\pi}{qB}\right)} \right] \\ & \left. \left. - \frac{qB}{\pi} \left[\Re \left(Li_2 \left[e^{-(ia+\epsilon)\frac{\pi}{qB}} \right] \right) \right] \right\} \right) \end{aligned}$$

Parameters

Parameters at zero magnetic field

$$g = \frac{m_f}{v} \approx 1.5; \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2} \approx 10$$

Magnetic field dependence of masses



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¹S. S. Avancini, R. Farias, M. B. Pinto, W. R. Tavares, Phys. Letters B 767 (2017) 247-252.

Fermionic contribution with fixed g

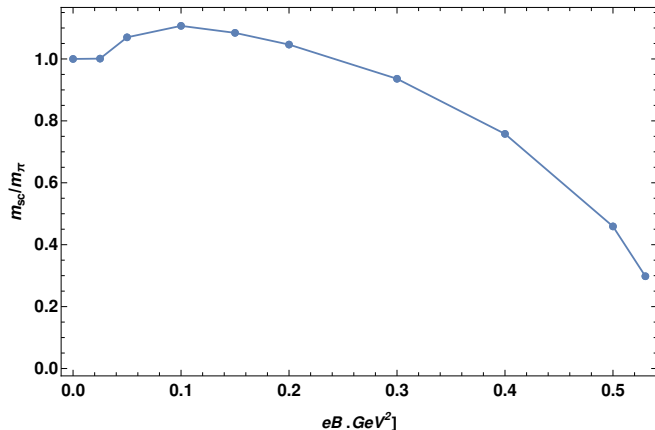


Figure: 'Longitudinal' SC mass as function of B , $g = 1.5$ (left).

Comparison with the NJL model

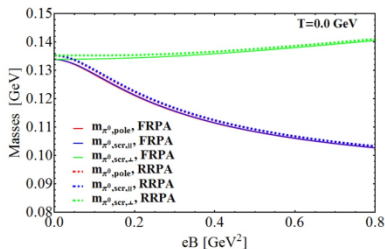
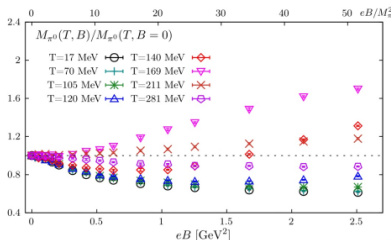


Figure: H. T. Ding, S. T. Li, J. H. Liu, and X. D. Wang, Phys. Rev D105, 034514 (2022), and B. Sheng, Y. Wang, X. Wang, and L. Yu, Phys. Rev. D103 (2021) 9, 094001.

Fermionic contribution with fixed g

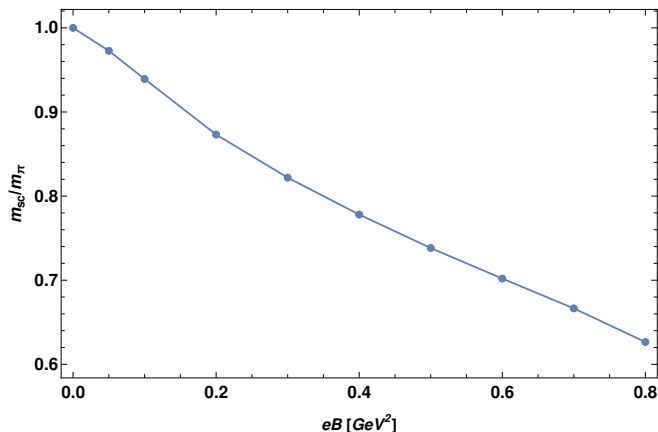


Figure: 'Longitudinal' SC mass as function of B , $g = 0.3$ (left).

Fermionic contribution with g_{eff}

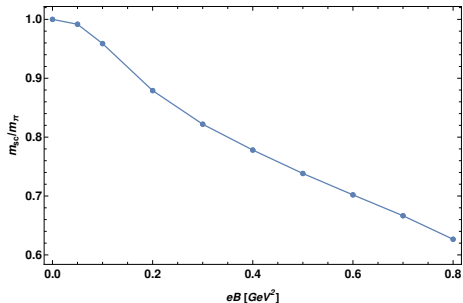
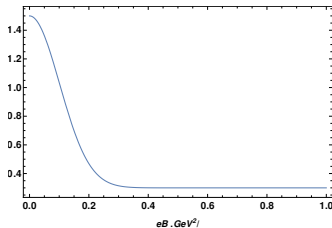
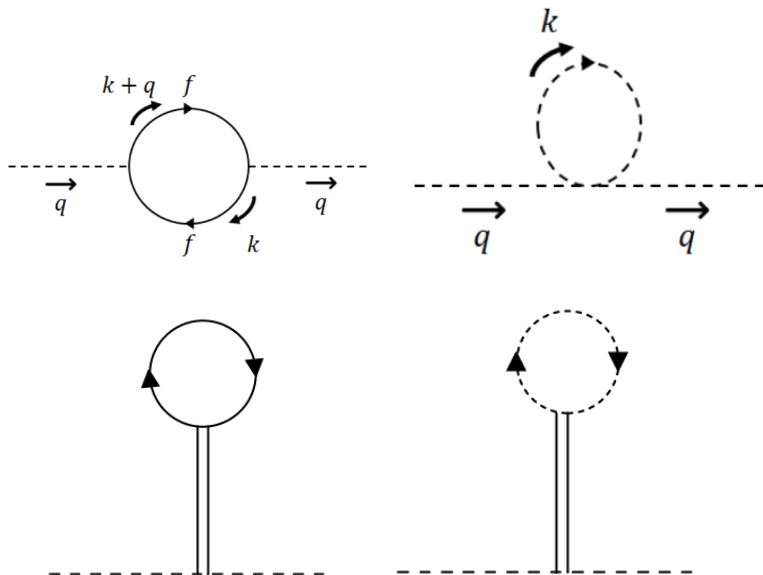


Figure: 'Longitudinal' SCM as function of B , $g_{\text{eff}}(B) = 0.3 + 1.2 \exp[-(7B)^2]$.



Relevant Feynman diagrams



Fermionic contribution + Tadpoles

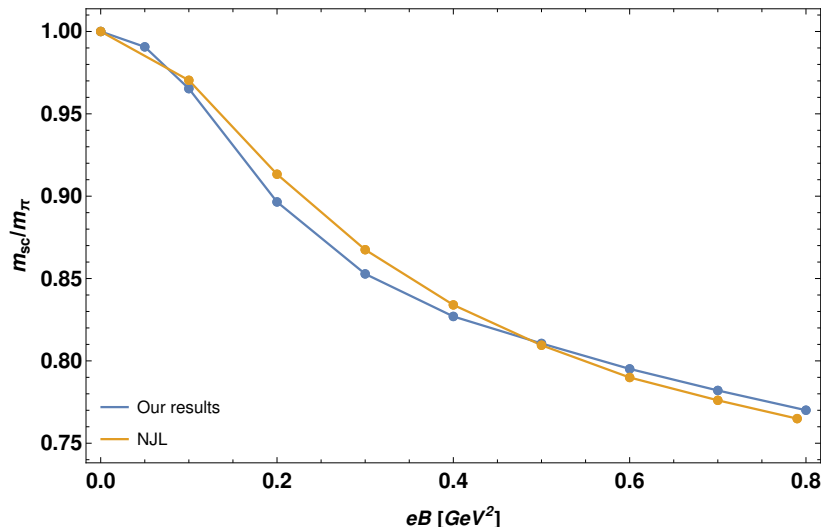


Figure: 'Longitudinal' Screening mass as a function of B ,
 $g_{eff}(B) = 0.3 + 1.2 \exp[-(7B)^2]$, $\lambda = 1.5$.

Fermionic contribution + Tadpoles

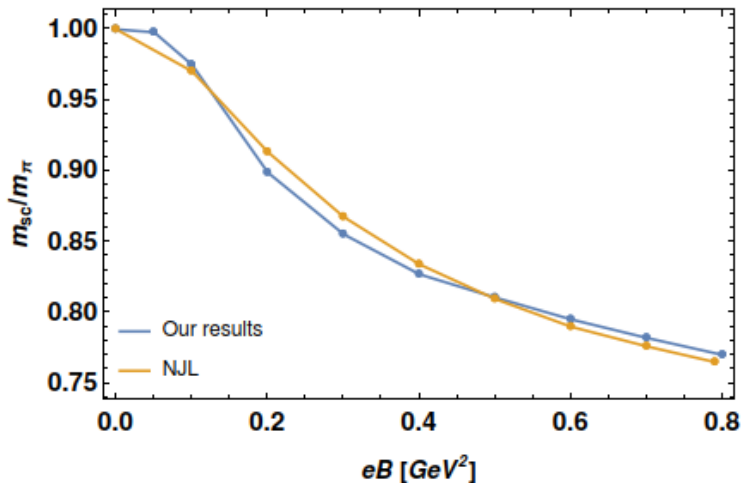


Figure: 'Longitudinal' Screening mass as a function of B ,
 $g_{\text{eff}}(B) = 0.3 + 1.2 \exp[-(7B)^2]$, $\lambda = 1.5 + 7 \exp[-(10B)^2]$.

Summary and perspectives

Summary:

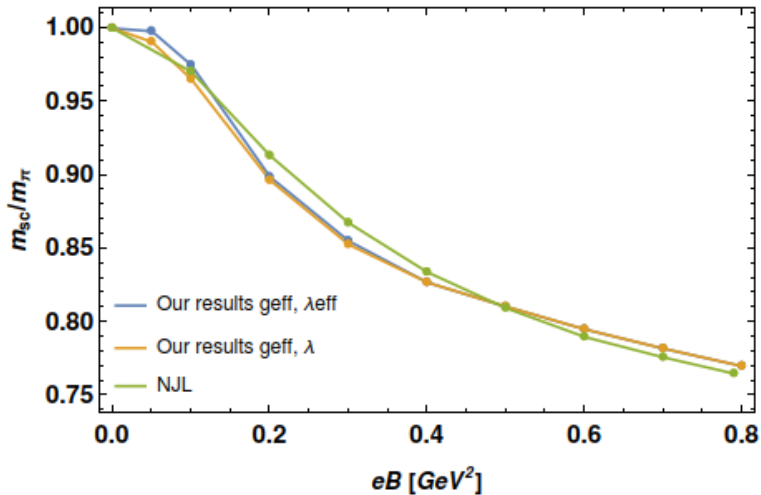
- ▶ We have calculated the neutral pion self-energy in the LSMq.
- ▶ We have obtained the 'longitudinal' screening mass as a function of B .
- ▶ We have compared our results with LQCD and NJL, and we have found a nice agreement **only when we have a magnetic field dependence on the couplings and masses.**

Perspectives:

- ▶ We will study the 'transverse' screening mass as a function of B .
- ▶ We will study the case where $T \neq 0$.

Thank You

Fermionic contribution g_{eff} and λ_{eff} as a function of B



Fermionic contribution with g_{eff} as a function of B

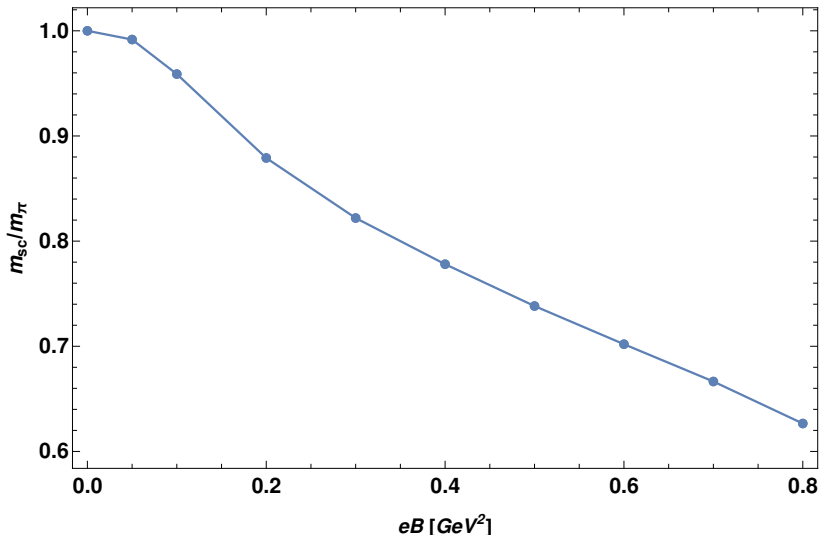


Figure: 'Longitudinal' Screening mass as a function of B ,
 $g_{eff}(B) = 0.3 + 1.2 \exp[-(7B)^2]$.

Fermionic contribution + Tadpoles

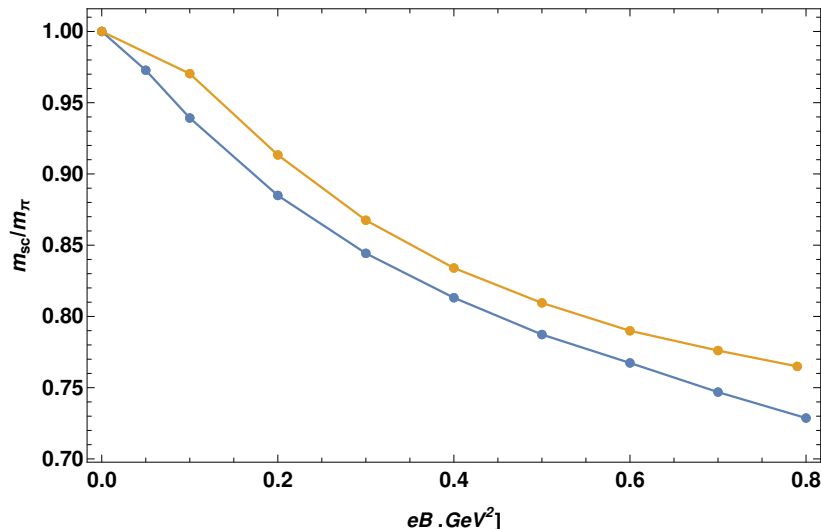


Figure: 'Longitudinal' Screening mass as a function of B vs NJL
 $g_{\text{eff}}(B) = 0.3, \lambda = 1$.

Preliminary results

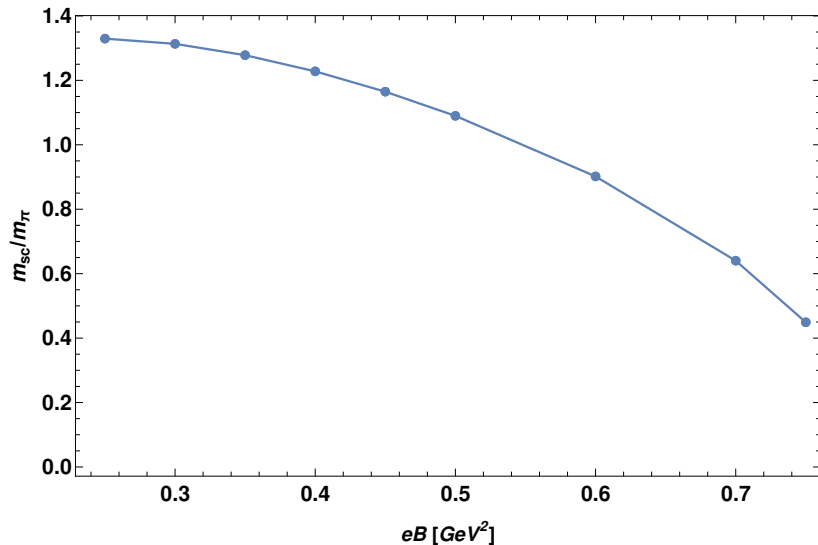


Figure: 'Longitudinal' Screening mass as a function of B .