First radial excitation of the pion and its box contribution to the muon g-2

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Work in collaboration with:
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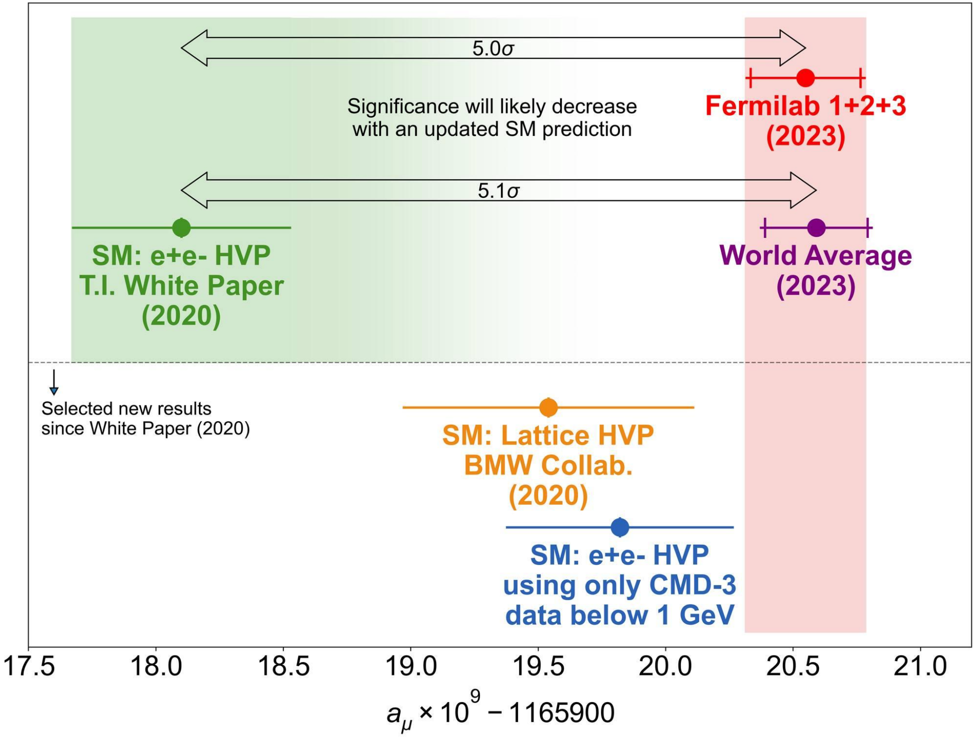
Outline

- Introduction
- Box contributions
- Formalism
  - Excited states
  - Form factor (Preliminary)
  - Contribution to g-2 (Preliminary)
- Outlook

Introduction

- The muon's g-2 can be both measured and predicted with high precision. Therefore, any discrepancy between the measured and predicted values might hint at the presence of new, previously unobserved physics phenomena or particles.
- The **new results** from the Muon g-2 experiment were unveiled in a scientific seminar on August 10, 2023

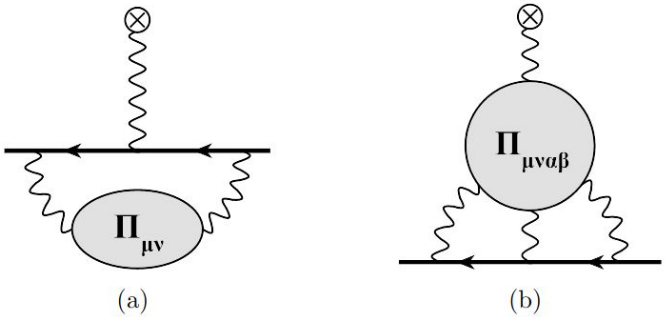


Hadronic contributions

Precision tests of the Standard Model (SM) require theoretical predictions taking into account higher-order quantum corrections. There are two hadronic contributions that dominate the SM uncertainty:

- **Hadronic vacuum polarization:** is the dominant hadronic correction, making its precise understanding essential for interpreting the results of the muon g-2 experiment.
- **Hadronic light by light scattering:** this is a subdominant but essential quantum correction for consistency.

Given the low characteristic scale, these contributions have to be calculated with non-perturbative methods.



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Current Status

- Ongoing works to reduced uncertainties for the HLBL and HVP contributions.
- Next steps are being discussed in 6th plenary workshop of the Muon TI in Bern, September 4-8, 2023.

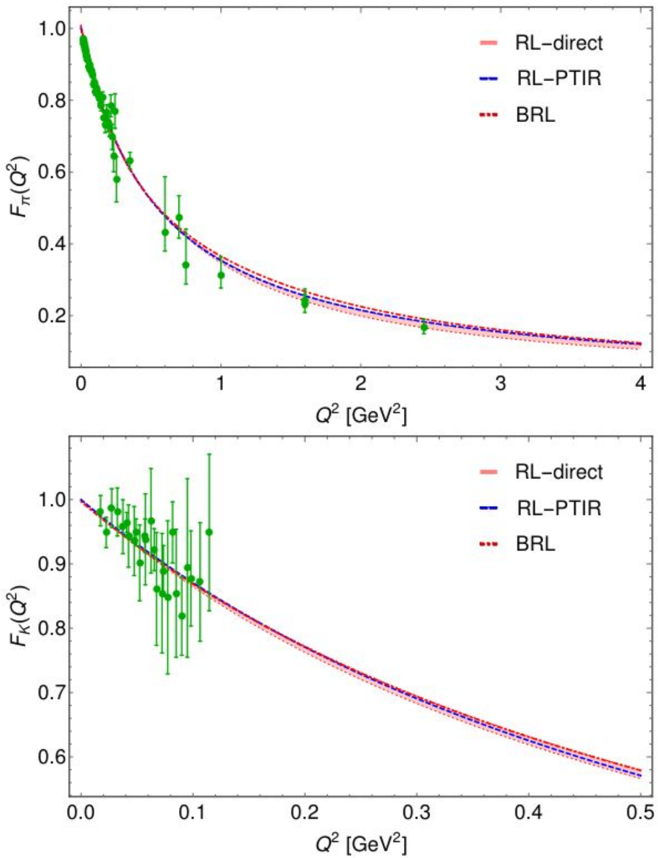
Box contributions to HLBL

One of the intricate contributions within the HLBL correction comes from box diagrams, or "box contributions".

$$a_\mu^{\mathbf{P}-box} = \frac{\alpha_{em}^3}{432\pi^2}\int_\Omega \sum_i^{12} T_i(Q_1,Q_2,\tau)\bar{\Pi}_i^{\mathbf{P}-box}(Q_1,Q_2,\tau),$$

$$a_\mu^{\pi^\pm-\text{box}} = -(15.4 \pm 0.3)\times 10^{-11} \ \ [\text{RL-direct}]\ ,$$
$$a_\mu^{\pi^\pm-\text{box}} = -(15.6 \pm 0.3)\times 10^{-11} \ \ [\text{RL-PTIR}]\ ,$$
$$a_\mu^{\pi^\pm-\text{box}} = -(15.7 \pm 0.2)\times 10^{-11} \ \ [\text{BRL}]\ .$$

$$a_\mu^{K^\pm-\text{box}} = -(0.47 \pm 0.03)\times 10^{-11} \ \ [\text{RL-direct}]\ ,$$
$$a_\mu^{K^\pm-\text{box}} = -(0.48 \pm 0.03)\times 10^{-11} \ \ [\text{RL-PTIR}]\ ,$$
$$a_\mu^{K^\pm-\text{box}} = -(0.48 \pm 0.02)\times 10^{-11} \ \ [\text{BRL}]\ .$$



A.S. Miramontes, Khépani Raya, Adnan Bashir and Pablo Roig **Phys.Rev.D** 105 (2022) 074013

$\pi(1300)$ box contribution

Among the different mesons, the role of the first radial excitation of the pion in the box contribution to the muon's anomalous magnetic moment warrants further exploration and analysis.

The calculation of the pion's first radial excitation and its contribution is a nontrivial task.

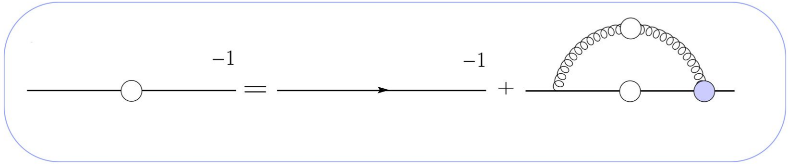
1. Bethe-Salpeter amplitude for the corresponding meson
2. Electromagnetic form factor
3. Box contribution

Formalism

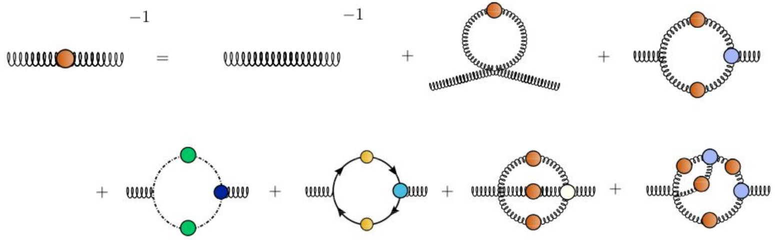
Formalism: DSE/BSE

Dyson-Schwinger equations

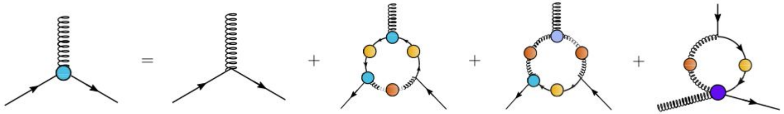
- They are the equations of motion of a quantum field theory
  - Propagators
  - Vertices
- Exact equations derived from the path integral.
- Infinite set of integral equations.



Gluon Propagator



Quark-Gluon Vertex

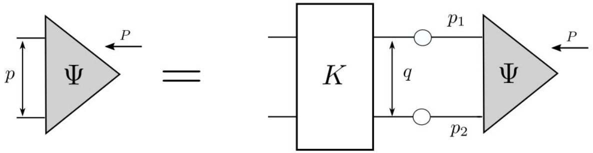


$$S^{-1} = S_0^{-1} - Z_{1f} \int_q \gamma_\mu S(q) \Gamma_\nu^{qgl}(q,k) D_{\mu\nu}(k) \; ,$$

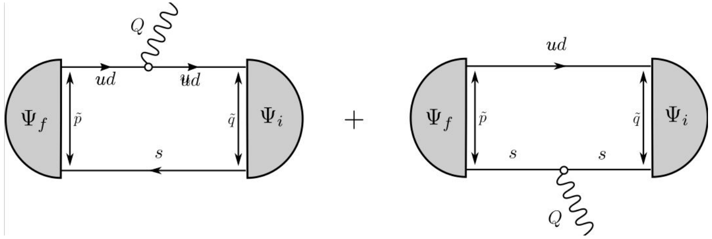
Formalism: DSE/BSE

Bethe-Salpeter equations

- ❑ Encode the information about the hadron
  - ❑ Mass
  - ❑ Decay constants
  - ❑ Decay width
- ❑ When coupled to an external field, we can have access to form factors
- ❑ Also an infinite set of diagrams.



$$(\Gamma)_{a\alpha,b\beta}(p,P) = \int_q K^{r\rho,s\sigma}_{a\alpha,b\beta}(P,p,q) \times S_{r\rho,e\epsilon}(k_1)\,(\Gamma)_{e\epsilon,n\nu}(q,P)\,S_{n\nu,s\sigma}(k_2)$$

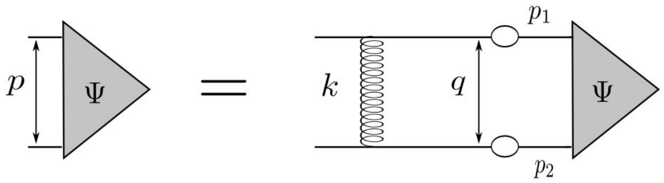


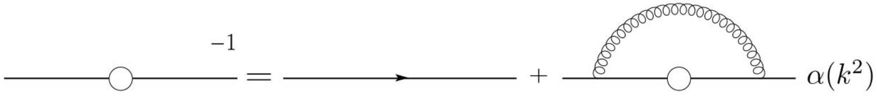
Ingredients:

- ❑ Quark propagator
- ❑ Meson Bethe-Salpeter amplitudes
- ❑ Interaction kernel
- ❑ Quark-photon vertex

Rainbow-ladder truncation

- Simples truncation to preserve chiral symmetry.
- One collects all the correction of the gluon propagator and quark gluon vertex into an effective coupling.
- A model to describe this effective coupling will be necessary.
- We employ the Maris-Tandy model, it includes two free parameters to be fitted to reproduce the pion mass and decay constant.

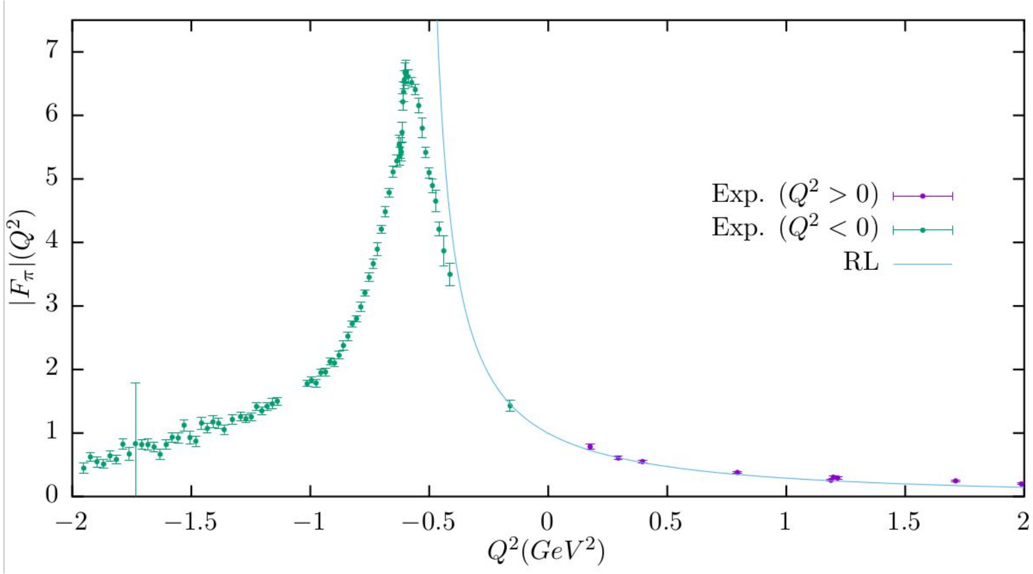




$$\alpha(q^2) = \pi\eta^7 \left(\frac{q^2}{\Lambda^2}\right)^2 e^{-\eta^2 \frac{q^2}{\Lambda^2}} + \frac{2\pi\gamma_m (1 - e^{-q^2/\Lambda_t^2})}{\ln[e^2 - 1 + (1 + q^2/\Lambda_{QCD}^2)^2]}$$

P. Maris and P. C. Tandy, Phys. Rev. C 60 (1999) 055214
doi:10.1103/PhysRevC.60.055214

Problems with R-L truncation

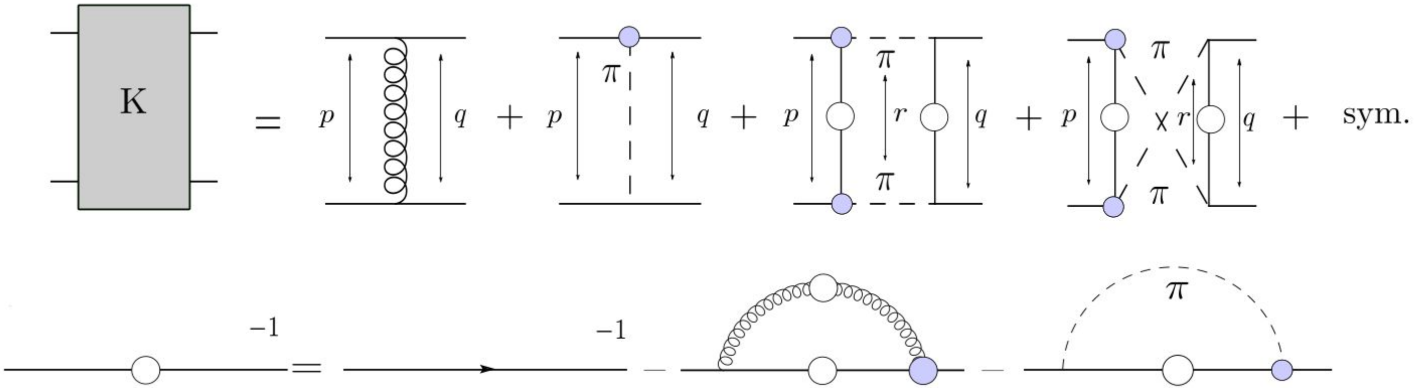


- The solutions are stable bound states, do not describe the decay width.
- Describes quite well the data for a space-like photon.
- Fails to reproduce the experimental data for a time-like photon.
- Not a good description of excited states.

R. R. Akhmetshin et al. [CMD-2], Phys. Lett. B 648, 28-38 (2007) doi:10.1016/j.physletb.2007.01.073 [arXiv:hep-ex/0610021 [hep-ex]].
C. J. Bebek, C. N. Brown, M. Herzlinger, S. D. Holmes, C. A. Lichtenstein, F. M. Pipkin, S. Raither and L. K. Sisterson, Phys. Rev. D 13 (1976), 25 doi:10.1103/PhysRevD.13.25

Beyond the Rainbow: Meson cloud effects

- Mesonic effects are considered by including explicit mesons as degrees of freedom.
- Besides the gluon contribution, a meson loop and meson exchange will appear on the DSE/BSE system.



C. S. Fischer, D. Nickel and R. Williams, Eur. Phys. J. C 60 (2009) 47 doi:10.1140/epjc/s10052-008-0821-1

A. S. Miramontes and H. Sanchis-Alepuz, Eur. Phys. J. A 55 (2019) no.10, 170 doi:10.1140/epja/i2019-12847-6

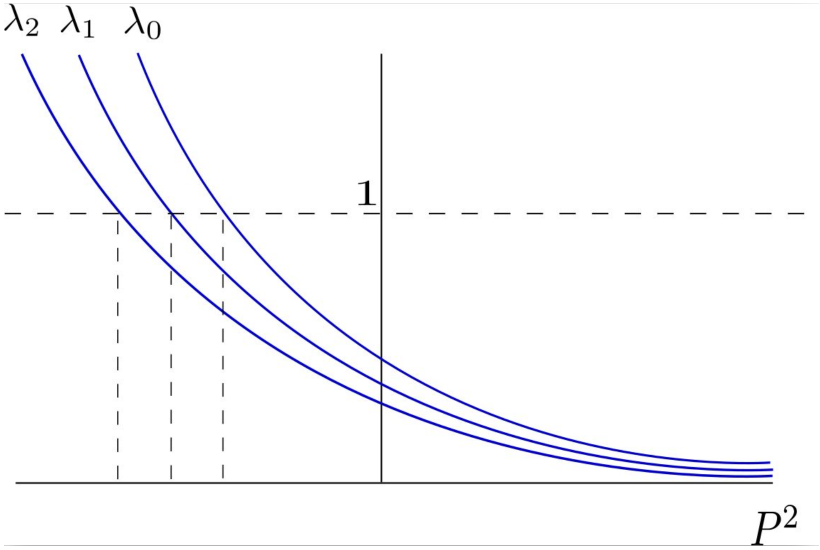
Radial excitations

Numerical setup

- The Bethe-Salpeter equation is usually solved as an eigenvalue problem with solutions

$$P_n^2 = -M_n^2$$

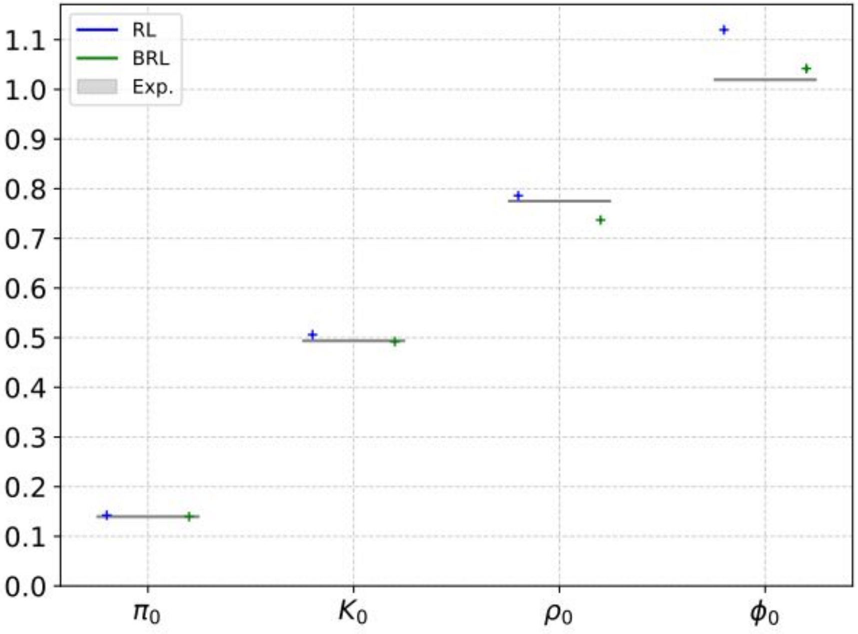
- Rainbow-ladder truncation has problems to describe ground and excited states using the same set of parameters.

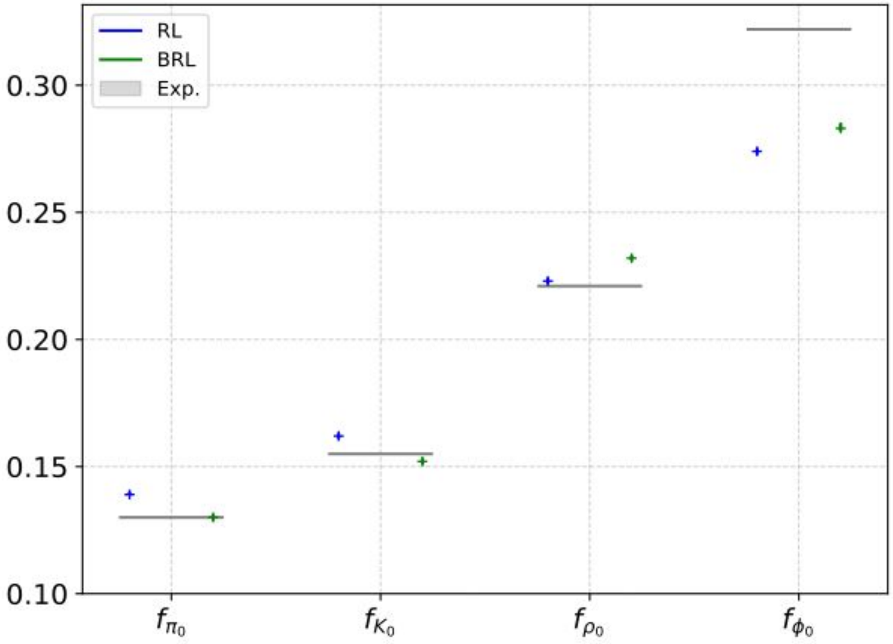


Numerical Setup

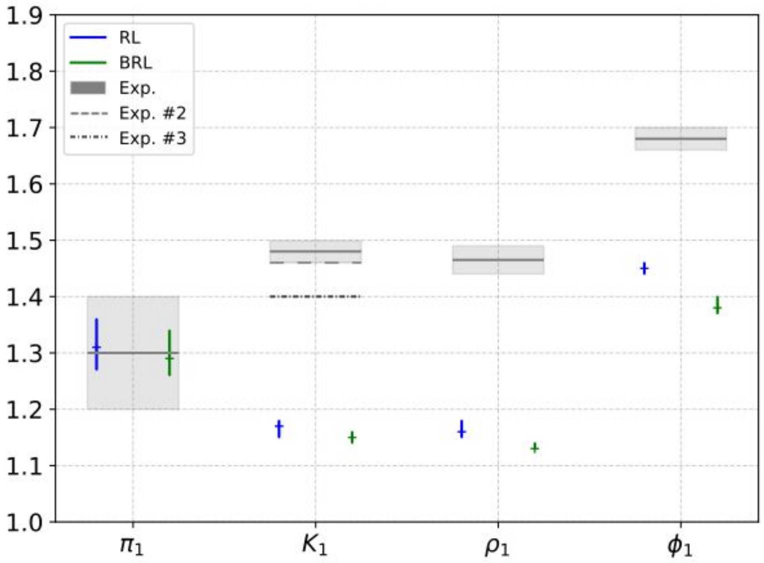
- The observables have been calculated in the RL and BRL truncations employing three different sets of free parameters in the MT interaction.
- We maintain the parameter $\Lambda = 0.78$ fixed while varying η=1.60,1.65 and 1.70
- The quark masses have been fixed to be m_u= 3.7 MeV and m_s = 85 MeV
- For the calculation of all meson BSAs we have employed a total of six Chebyshev polynomials.

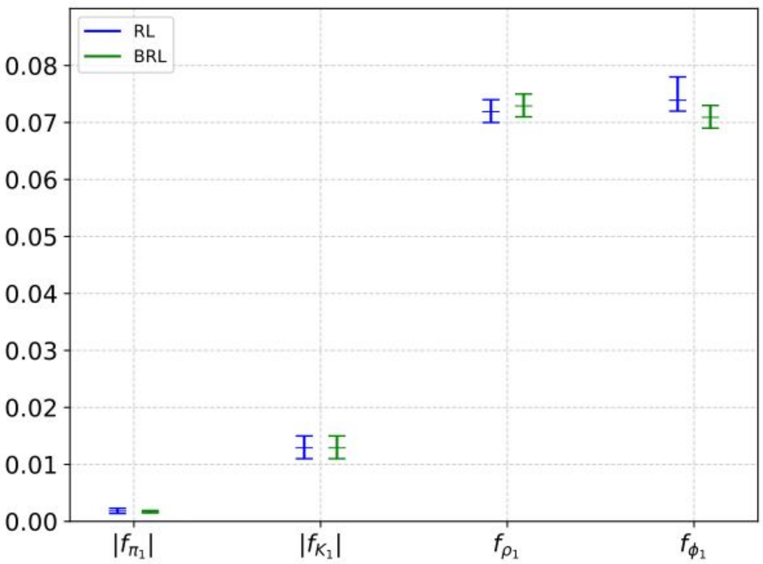
Ground states





First Radial excitation

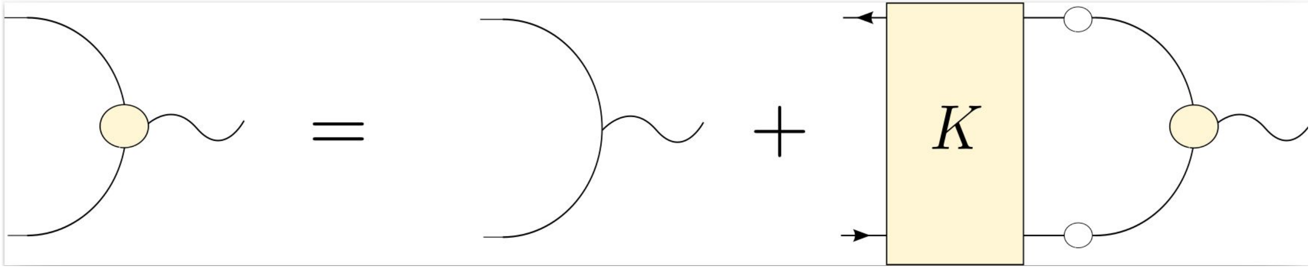




Electromagnetic Form Factors

Quark-photon vertex

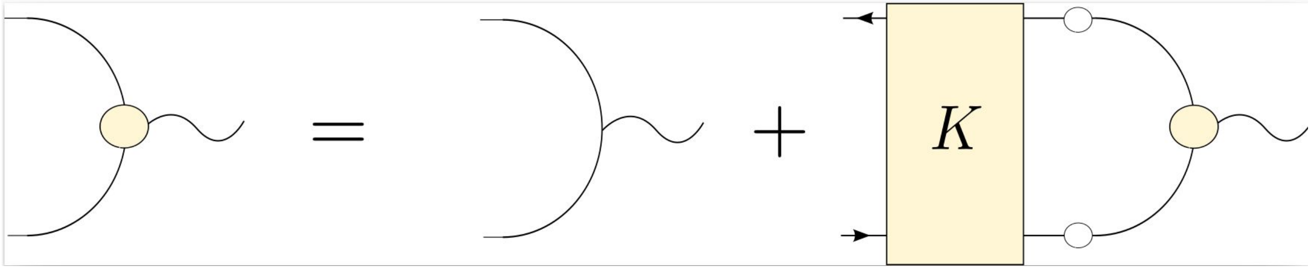
The internal structure of a meson can be probed through a photon interacting with its constituents. The fully-dressed QPV which characterizes the interaction between quarks and photons, can be described via the following inhomogeneous BSE:



Quark-photon vertex

- The full vertex can be expanded in transverse and non-transverse components.
- The transverse components encode the resonance information. Resonance poles will appear in the time-like domain

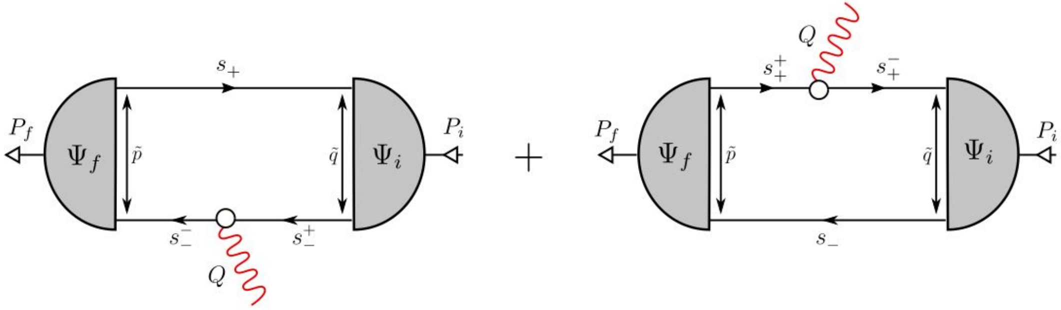
$$\Gamma^\mu(Q,p) = \left[\lambda_1(Q,p)\gamma^\mu + 2p^\mu\left(\lambda_2(Q,p)\not{p} + i\lambda_3(Q,p)\right)\right] + \Lambda_T^\mu(Q,p)$$



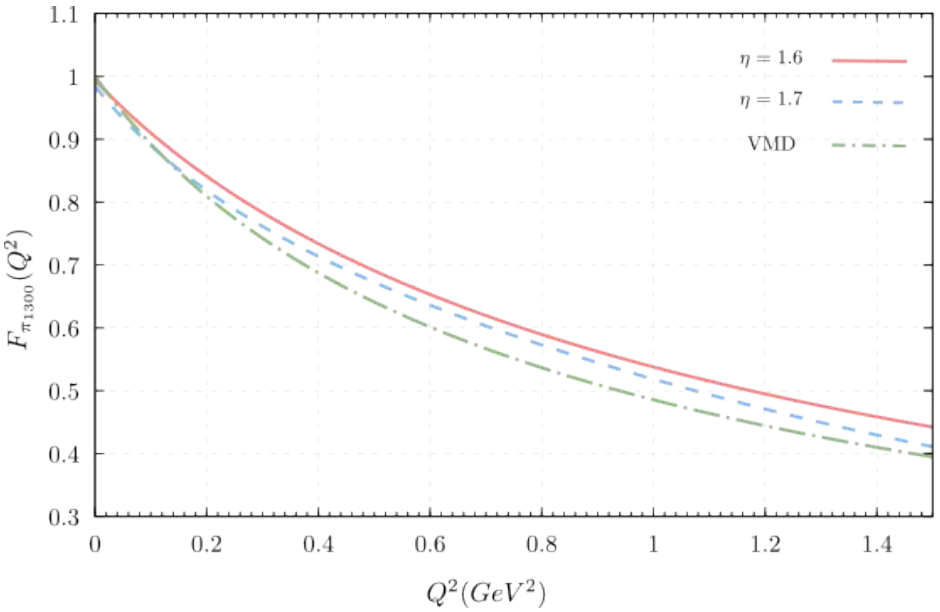
Electromagnetic form factors

To investigate Form factors in the framework of BSE we start from the electromagnetic current, then we relate this current with the form factor.

$$J^\mu = \bar{\psi}_f G_0 (\boldsymbol{\Gamma}^\mu - K^\mu) G_0 \psi_i \qquad \langle P_f | \, J^\mu(0) \, | P_i \rangle = (P_f + P_i)^\mu F(k^2)$$



π(1300) space-like EFF (Preliminary results)



Box contribution (Preliminary results)

Once the π(1300) amplitude has been calculated together with its electromagnetic form factor, the box contribution can be computed.

$$a_\mu^{\pi_1-\text{box}} = (-1.74 \pm 0.10) \times 10^{-13}$$

$$a_\mu^{\pi^\pm-\text{box}} = -(15.4 \pm 0.3) \times 10^{-11} \quad \text{[RL-direct]}\,,$$
$$a_\mu^{\pi^\pm-\text{box}} = -(15.6 \pm 0.3) \times 10^{-11} \quad \text{[RL-PTIR]}\,,$$
$$a_\mu^{\pi^\pm-\text{box}} = -(15.7 \pm 0.2) \times 10^{-11} \quad \text{[BRL]}\,.$$

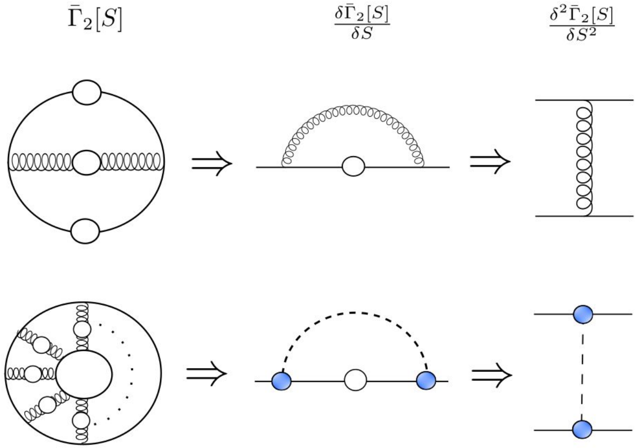
Ground state results

Outlook

- We have employed the formalism of the DSE/BSE to compute the amplitude for the $\pi(1300)$, its EFF and box contribution to the muon g-2.
- We have the first calculation of the $\pi(1300)$ space-like electromagnetic form factor. At the present time, there are not results from other approaches.
- We also have the first $\pi(1300)$ box contribution.
- Project ongoing on the HVP contribution.

Truncations for DSE/BSE

How do we «derive» interaction kernels for BSE/DSEs? A way of defining truncations is using effective action or nPI techniques.



- Diagrammatic representation of the leading and subleading contribution to the effective action.
- The leading diagram leads to RL truncation while the subleading to meson exchange