ELECTROMAGNETIC FORM FACTORS OF MESONS: A PHENOMENOLOGICAL APPROACH

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Non-perturbative Physics: Tools and Applications

MOTIVATION

- Hadron physics experiments probe the asymptotic predictions of Quantum Chromodynamics and its non-perturbative emergent phenomena of dynamical chiral symmetry breaking and confinement.

- The theoretical study of the dynamics of Hadrons represents a challenge since the extrapolation from fundamental particles to bound state systems is a hard task.

- The use of the SDE and BSE to study static and dynamic properties of Hadrons with the minimum number of input parameters is a long term goal.

- At leading-order in a symmetry preserving truncation of the SDEs, internal properties of pseudoscalar, vector, scalar and axial-vector mesons such as pion, rho, sigma and a_1 can be studied in a consistent manner.

- In this work we are interested on the determination of Elastic Meson Form Factors (EMFF).

- The calculation of all elastic meson form factors requires the computation of the quark propagator, the BSA of mesons, their masses as well as the knowledge of the quark-photon interaction at different probing momenta.

GAP EQUATION

- Dressed masses of quarks are obtained by use of the GAP equation,

$$S(p)^{-1} = \imath\gamma \cdot p + m_f + \Sigma(p)$$

- Where

$$\Sigma(p) = \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q)\gamma_\mu S(q)\Gamma_\nu(q,p)$$

- And m_f is the current quark mass and Γ_ν is the quark-vector boson vertex

C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. 33, 477 (1994).
C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. 45, S1 (2000).

- By means of the Contact Interaction (CI),

$$g^2 D_{\mu\nu}(q) = \delta_{\mu\nu} \frac{4\pi\alpha_{\rm IR}}{m_G^2}$$

$$\Gamma_\nu(q,p) = \gamma_\nu$$

- it is possible to find dressed masses of mesons M_f by,

$$M_f = m_f + M_f \frac{4\alpha_{\rm IR}}{3\pi m_G^2} \mathcal{C}(M_f^2)$$

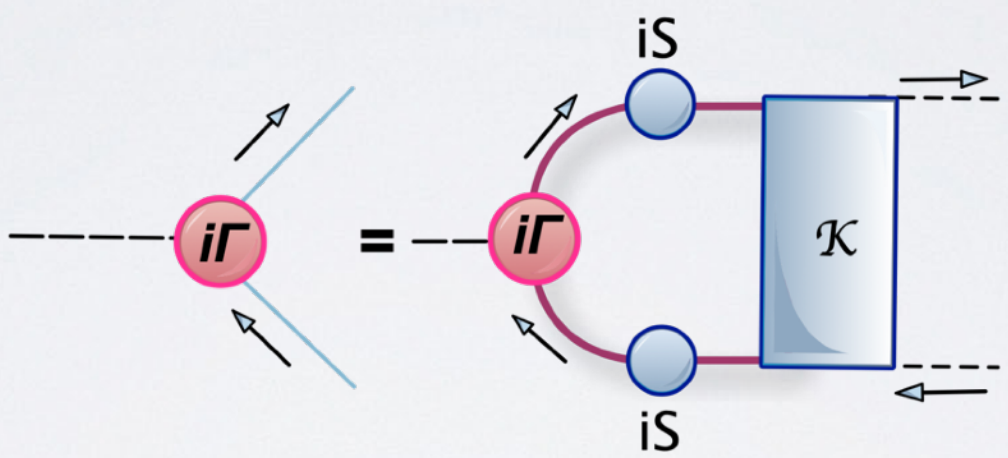
- Where,

$$\mathcal{C}(z)/z = \Gamma(-1, z\,\tau_{\rm UV}^2) - \Gamma(-1, z\,\tau_{\rm IR}^2)$$

L. X. Gutierrez-Guerrero, A. Bashir, I. C. Cloet, and C. D. Roberts, Phys. Rev. C81, 065202 (2010), 1002.1968.

BETHE-SALPETER EQUATION

- The bound-state problem for Hadrons characterized by two valence-fermions may be studied using the homogeneous BS equation,



- Where Γ the is the Bethe-Salpeter Amplitude (BSA); and explicitly is,

$$[\Gamma(k;P)]_{tu} = \int \frac{d^4q}{(2\pi)^4}[\chi(q;P)]_{sr}\mathcal{K}^{rs}_{tu}(q,k;P)\,,$$

- This equation has a solution when $P^2 = -M_M^2$ with M_M being the meson mass.

E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951)

- A general decomposition of the BSA in the CI for Scalar (S), Pseudoscalar (PS), Vector (V) and Axial-Vector (AV) mesons is given by,

$$\Gamma_{\rm S}(P) = i\, E_{\rm S}\,(P)\, I$$

$$\Gamma_{\rm PS}(P) = i\,\gamma_5\, E_{\rm PS}\,(P) + \frac{1}{2M_R}\gamma_5\,\gamma\cdot P\, F_{\rm PS}(P)$$

$$\Gamma_{{\rm V},\mu}(P) = \gamma_\mu^T\, E_{\rm V}\,(P)$$

$$\Gamma_{{\rm AV},\mu}(P) = \gamma_5\,\gamma_\mu^T\, E_{\rm AV}\,(P)$$

- Such as $P_\mu\gamma_\mu^T = 0$ and, the reduced mass with dressed quark masses is

$$M_R = M_{f_1}M_{f_2}/(M_{f_1}+M_{f_2})$$

C. H. Llewellyn-Smith, Annals Phys. 53, 521 (1969).

QUARK-PHOTON VERTEX

- Since the Elastic Form Factors of mesons shall be extracted from the process $\gamma M \to M$, the dressed quark-photon vertex is needed.

- Within the framework described in the CI, the quark-photon vertex is given by,

$$\Gamma_\mu(Q^2) = \gamma_\mu^{\rm L} \, P_{\rm L}(Q^2) + \gamma_\mu^{\rm T} \, P_{\rm T}(Q^2)$$

- Where, $P_{\rm L}(Q^2) = 1$ and $P_{\rm T}(Q^2) = (1 + K_\gamma(Q^2))^{-1}$, with

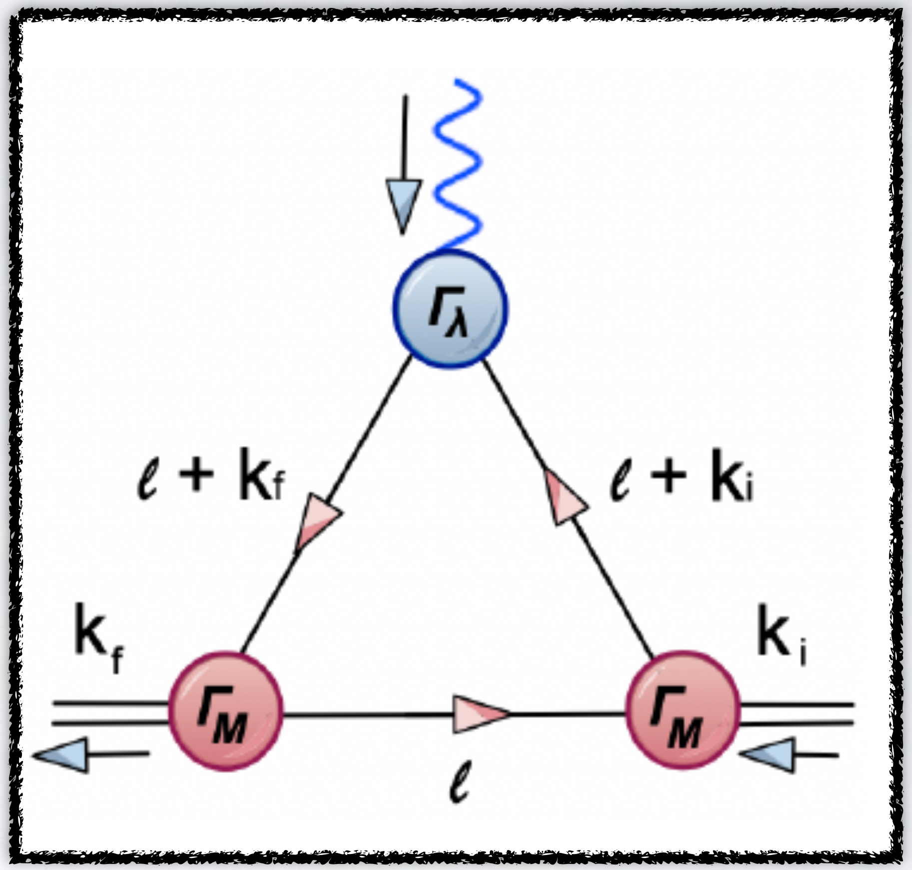
$$K_\gamma(Q^2) = \frac{4\alpha_{\rm IR}}{3\pi m_G^2} \int_0^1 d\alpha \alpha(1-\alpha) Q^2 \overline{\mathcal{C}}_1(\omega)$$

- And $\overline{\mathcal{C}}_1(z) = \Gamma(0, z\tau_{\rm UV}^2) - \Gamma(0, z\tau_{\rm IR}^2)$, $\omega(x,y,z) = x + y(1-y)z$.

H.L.L. Roberts, A. Bashir, L.X. Gutierrez-Guerrero, C.D. Roberts, D.J. Wilson, *Phys.Rev.C* 83 (2011) 065206

ELASTIC FORM FACTORS

- The kinematics of the process is given by the Feynman diagram



- And the standard momentum parametrization,

$$k_i = K - Q/2$$

$$k_f = K + Q/2$$

- Using Feynman rules, we find that for a general meson, this process can be written as,

$$\Lambda^{M,f_1} = N_c \int \frac{d^4\ell}{(2\pi)^4} \mathrm{Tr}\, \mathcal{G}^{M,f_1}$$

- where we have omitted the Dirac indices, and,

$$\begin{aligned} \mathcal{G}^{M,f_1} &= -\imath\, \Gamma_M(k_f)\, S(\ell + k_i, M_{f_1})\, \Gamma_\lambda(Q) \\ &\times S(\ell + k_f, M_{f_1})\, \overline{\Gamma}_M(-k_i)\, S(\ell, M_{f_2}) \end{aligned}$$

- The function Λ^{M,f_1} labels all M kind of mesons. Besides, the photon interacts with the quark with flavor f_1, and the fermion f_2 is a spectator.

- Λ^{M,f_1} can be written as a function of the Elastic Form Factors as,

Scalar	$\Lambda^{S,f_1} = -2K_\lambda F^{S,f_1}$
Pseudoscalar	$\Lambda^{PS,f_1} = -2K_\lambda F^{PS,f_1}$
Vector	$\Lambda^{V,f_1} = \sum_{i=1}^{3} T^{(i)}_{\lambda\mu\nu} F_i^{V,f_1}$
Axial-Vector	$\Lambda^{AV,f_1} = \sum_{i=1}^{3} T^{(i)}_{\lambda\mu\nu} F_i^{AV,f_1}$

- Where the tensor structure is given by

$$T^{(1)}_{\lambda\mu\nu} = 2K_\lambda \, \mathcal{P}^{\mathrm{T}}_{\mu\alpha}(p_i) \, \mathcal{P}^{\mathrm{T}}_{\alpha\nu}(p_f) \, ,$$

$$T^{(2)}_{\lambda\mu\nu} = \left(Q_\mu - p_{i,\mu} \frac{Q^2}{2m^2_{\langle q\bar{q}\,'\rangle}} \right) \mathcal{P}^{\mathrm{T}}_{\lambda\nu}(p_f) - \left(Q_\nu - p_{f,\nu} \frac{Q^2}{2m^2_{\langle q\bar{q}\,'\rangle}} \right) \mathcal{P}^{\mathrm{T}}_{\lambda\mu}(p_i) \, ,$$

$$T^{(3)}_{\lambda\mu\nu} = \frac{K_\lambda}{m^2_{\langle q\bar{q}\,'\rangle}} \left(Q_\mu - p_{i,\mu} \frac{Q^2}{2m^2_{\langle q\bar{q}\,'\rangle}} \right) \left(Q_\nu - p_{f,\nu} \frac{Q^2}{2m^2_{\langle q\bar{q}\,'\rangle}} \right) \, ,$$

- And $\mathcal{P}^{\mathrm{T}}_{\mu\nu}(p) = \delta_{\mu\nu} - p_\mu p_\nu / p^2$.

- In order to consider the total elastic form factor of mesons, we use,

$$F_M = e_{f_1} F^{M,f_1} + e_{\bar{f}_2} F^{M,\bar{f}_2}$$

- Furthermore, in the case of vector and axial-vector mesons, we present the results of the electric, magnetic and quadrupole form factors, defined as,

$$G_E = F_1 + \frac{2}{3}\eta G_Q$$
$$G_M = -F_2$$
$$G_Q = F_1 + F_2 + (1+\eta)F_3$$

- Where $\eta = Q^2/(4m^2_{\langle q\bar{q}'\rangle})$ and we label $\langle q\bar{q}'\rangle$ to the meson state.

- Analytical expressions for the Forms factors can be casted in the most general equation as,

$$F_i^{(j)}(Q^2) = \frac{3}{4\pi^2} P_T(Q^2) \int_0^1 d\alpha\, d\beta\, \alpha$$
$$\times \left(\mathcal{A}_i^{(j)} \overline{\mathcal{C}}_1(\omega_2) + (\mathcal{B}_i^{(j)} - \omega_2 \mathcal{A}_i^{(j)}) \overline{\mathcal{C}}_2(\omega_2)\right)$$

- where the $j = \{\mathrm{S}, \mathrm{PS}, \mathrm{V}, \mathrm{AV}\}$ and $i = 1{,}2{,}3$ is only present for in the case of vector and axial-vector mesons. Furthermore, this equation makes evident the dressing of the quark-photon vertex, and the triangular topology dependence in ω_2.

- Coefficients depends on: dressed masses of quarks, the meson mass, IR and UV cuts.

H-P, Gutierrez-Guerrero, Bashir, Bedolla, Higuera-Angulo, *Phys.Rev.D* 107 (2023) 5, 054002

ANALYSIS OF EFF

- The generation of dressed quark masses are,

$m_u = 0.007$ $M_u = 0.367$	$m_d = 0.007$ $M_d = 0.367$	$m_s = 0.17$ $M_s = 0.53$	$m_c = 1.08$ $M_c = 1.52$	$m_b = 3.92$ $M_b = 4.68$

- For the generation of masses of pseudoscalar and scalar mesons, it depends on the following parameters,

quarks	Z_H	$\Lambda_{\rm UV}$ [GeV]	$\hat{\alpha}_{\rm IR}$
u, d, s	1	0.905	4.57
c, u, s	3.034	1.322	1.50
c	13.122	2.305	0.35
b, u	11.273	3.222	0.41
b, s	17.537	3.574	0.26
b, c	30.537	3.886	0.15
b	129.513	7.159	0.035

$$\tau = 1/\Lambda$$

$$\Lambda_{\rm IR} = 0.24\,\text{GeV}$$

- With these parameters, masses of pseudoscalar mesons and BSA are,

	Mass[GeV]	E_{PS}	F_{PS}	$m_{PS}^{\exp}$[GeV]	error [%]
$u\bar{d}$	0.139	3.59	0.47	0.139	0.008
$u\bar{s}$	0.499	3.81	0.59	0.493	1.162
$s\bar{s}$	0.701	4.04	0.75	—	—
$c\bar{u}$	1.855	3.03	0.37	1.864	0.494
$c\bar{s}$	1.945	3.24	0.51	1.986	1.183
$u\bar{b}$	5.082	3.72	0.21	5.279	3.735
$s\bar{b}$	5.281	2.85	0.21	5.366	1.586
$c\bar{b}$	6.138	2.58	0.39	6.274	2.166
$c\bar{c}$	2.952	2.15	0.40	2.983	1.053
$b\bar{b}$	9.280	2.04	0.39	9.398	1.262

- PS meson masses are well determined by the parameters of the CI model.

H-P, Gutierrez-Guerrero, Bashir, Bedolla, Higuera-Angulo, *Phys.Rev.D* 107 (2023) 5, 054002

- For the scalar meson, we find the following parameters for masses and the BSA,

quarks	Z_H	Λ_{UV} [GeV]	$\hat{\alpha}_{IR}$
u, d, s	1	0.905	4.57
c, u	3.034	1.322	1.50
c, s	3.034	2.222	1.50
c	13.122	2.305	0.35
b, u	18.473	10.670	0.25
b, s	29.537	11.064	0.15
b, c	34.216	14.328	0.13
b	127.013	26.873	0.036

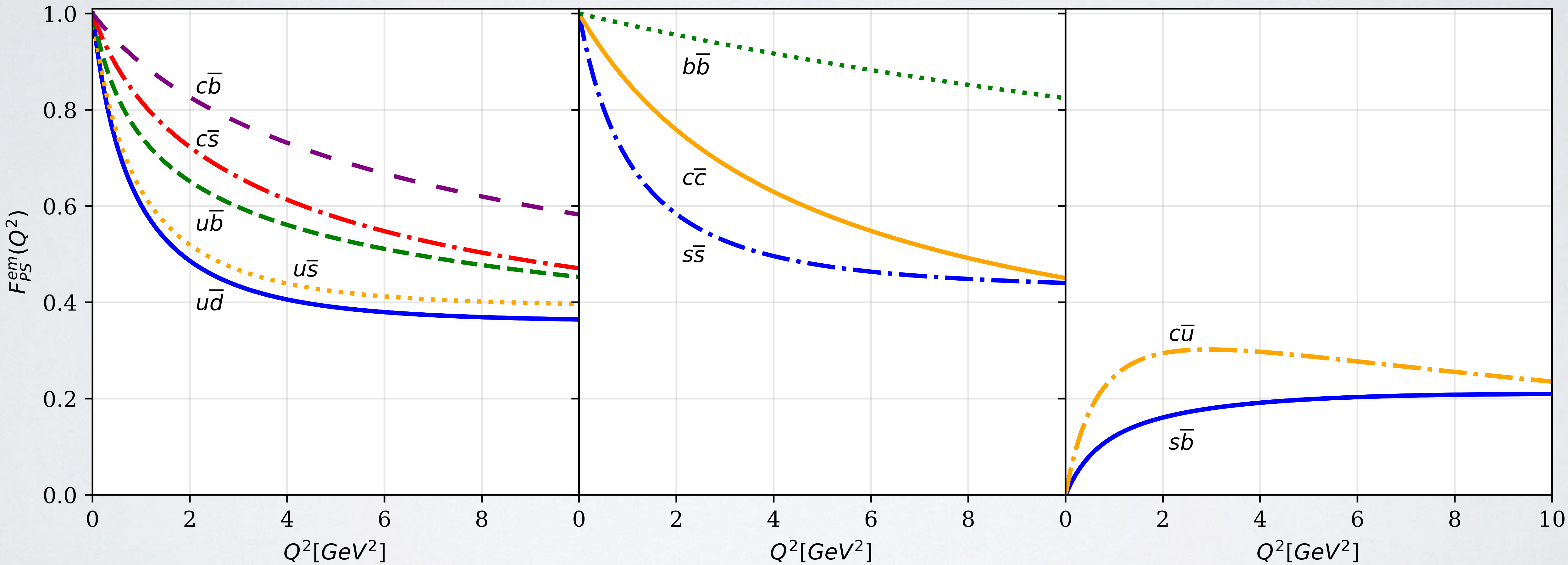
	Mass [GeV]	E_S	m_S^{exp} [GeV]	error [%]
$u\bar{d}$	1.22	0.66	—	—
$u\bar{s}$	1.38	0.65	—	—
$s\bar{s}$	1.46	0.64	—	—
$c\bar{u}$	2.31	0.39	2.30	0.19
$c\bar{s}$	2.42	0.42	2.32	3.54
$u\bar{b}$	5.30	1.53	—	—
$s\bar{b}$	5.64	0.26	—	—
$c\bar{b}$	6.36	1.23	6.71	5.26
$c\bar{c}$	3.33	0.16	3.42	2.73
$b\bar{b}$	9.57	0.69	9.86	2.95

- In this case, we have not all experimental values to compare with, however, we find a good agreement with the available data.

H-P, Gutierrez-Guerrero, Bashir, Bedolla, Higuera-Angulo, *Phys.Rev.D* 107 (2023) 5, 054002

- Analytical expressions for pseudoscalar mesons can be evaluated numerically in order to know the behaviors of the EMFF.

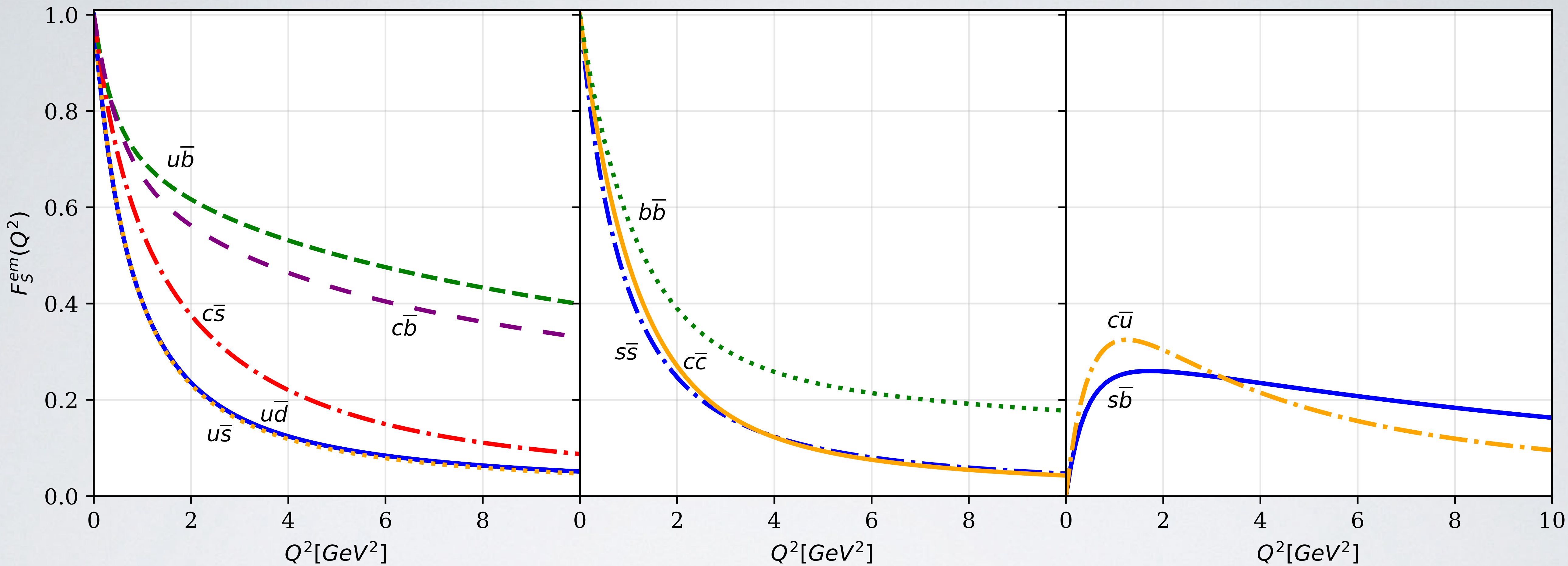
- For the ten pseudoscalar mesons, we find the following EMFF.



- Charged mesons Different quark flavors | Charged mesons Same quark flavors | Neutral mesons

H-P, Gutierrez-Guerrero, Bashir, Bedolla, Higuera-Angulo, *Phys.Rev.D* 107 (2023) 5, 054002

- For the ten scalar mesons, we find the following EMFF.



Charged mesons
Different quark flavors

Charged mesons
Same quark flavors

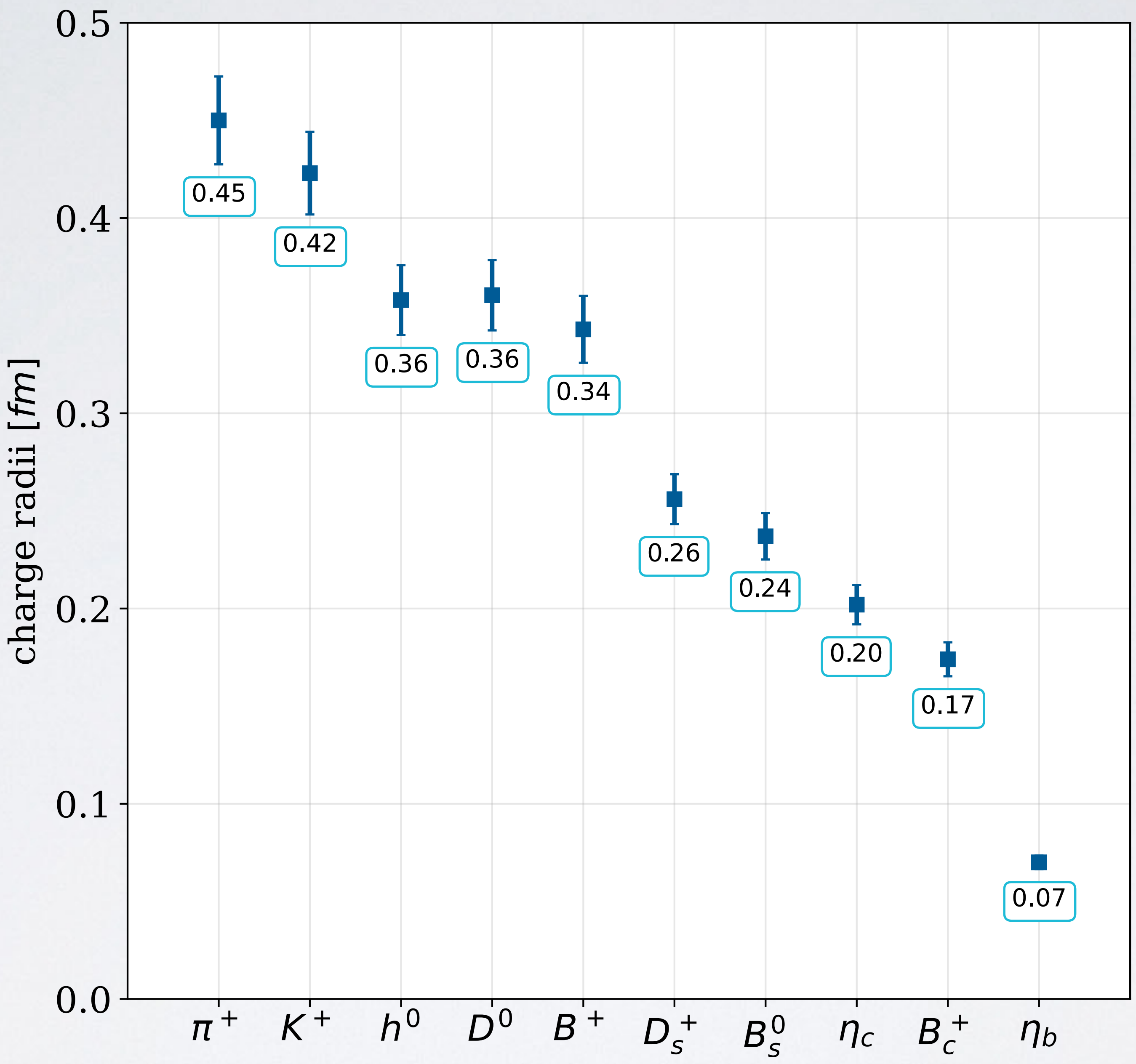
Neutral mesons

- The behavior at low Q^2 brings information about the charge radii of mesons. The definition is given by,

$$r^2_{\langle q\bar{q}'\rangle} = \left\| \left(6 \frac{d}{dQ^2} F_M \Big|_{Q^2 \to 0} \right) \right\|$$

H-P, Gutierrez-Guerrero, Bashir, Bedolla, Higuera-Angulo, *Phys.Rev.D* 107 (2023) 5, 054002

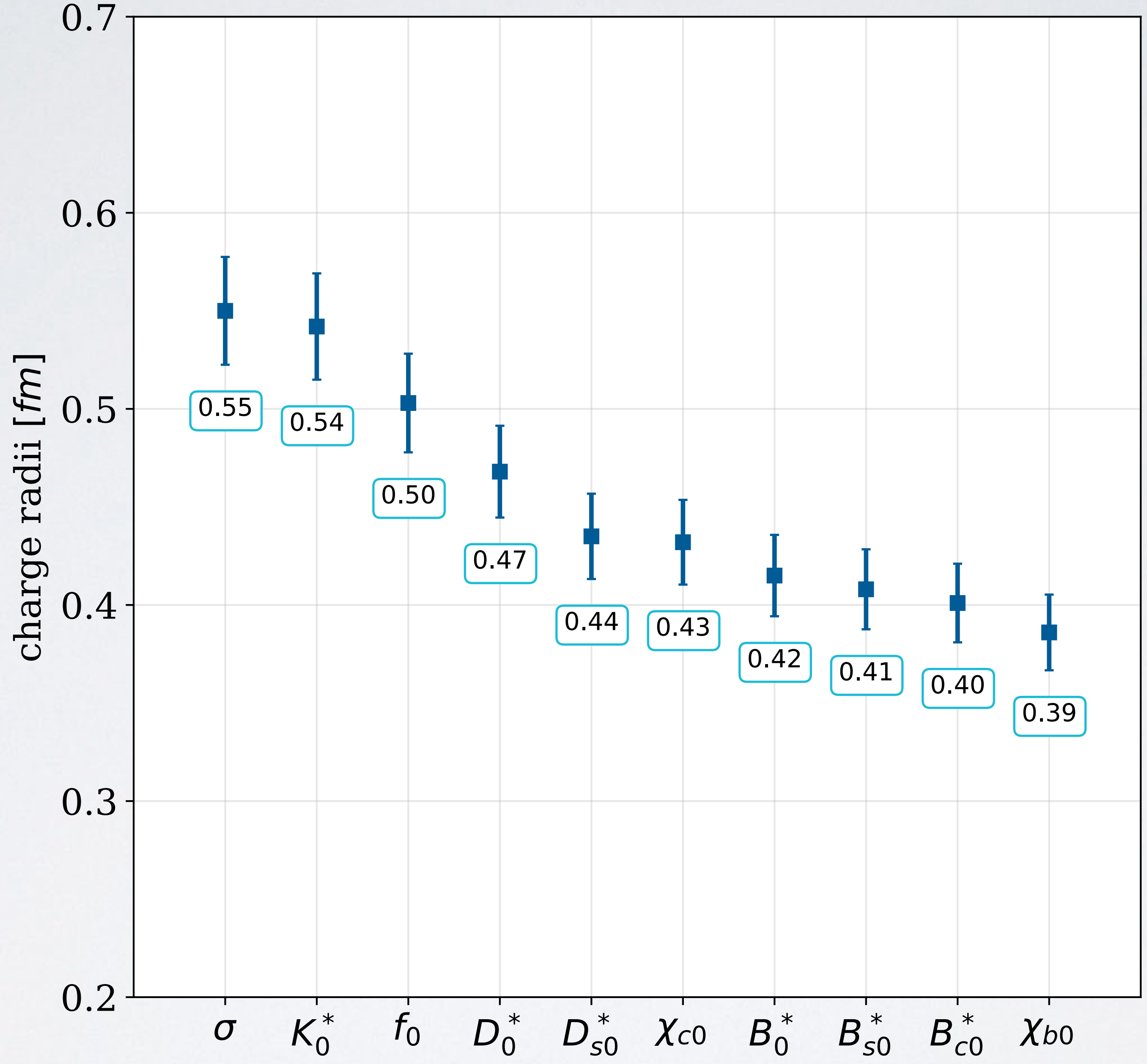
- For pseudoscalar mesons, we find the following charge radii,



- We find a hierarchy on the meson mass and the charge radii.

H-P, Gutierrez-Guerrero, Bashir, Bedolla, Higuera-Angulo, *Phys.Rev.D* 107 (2023) 5, 054002

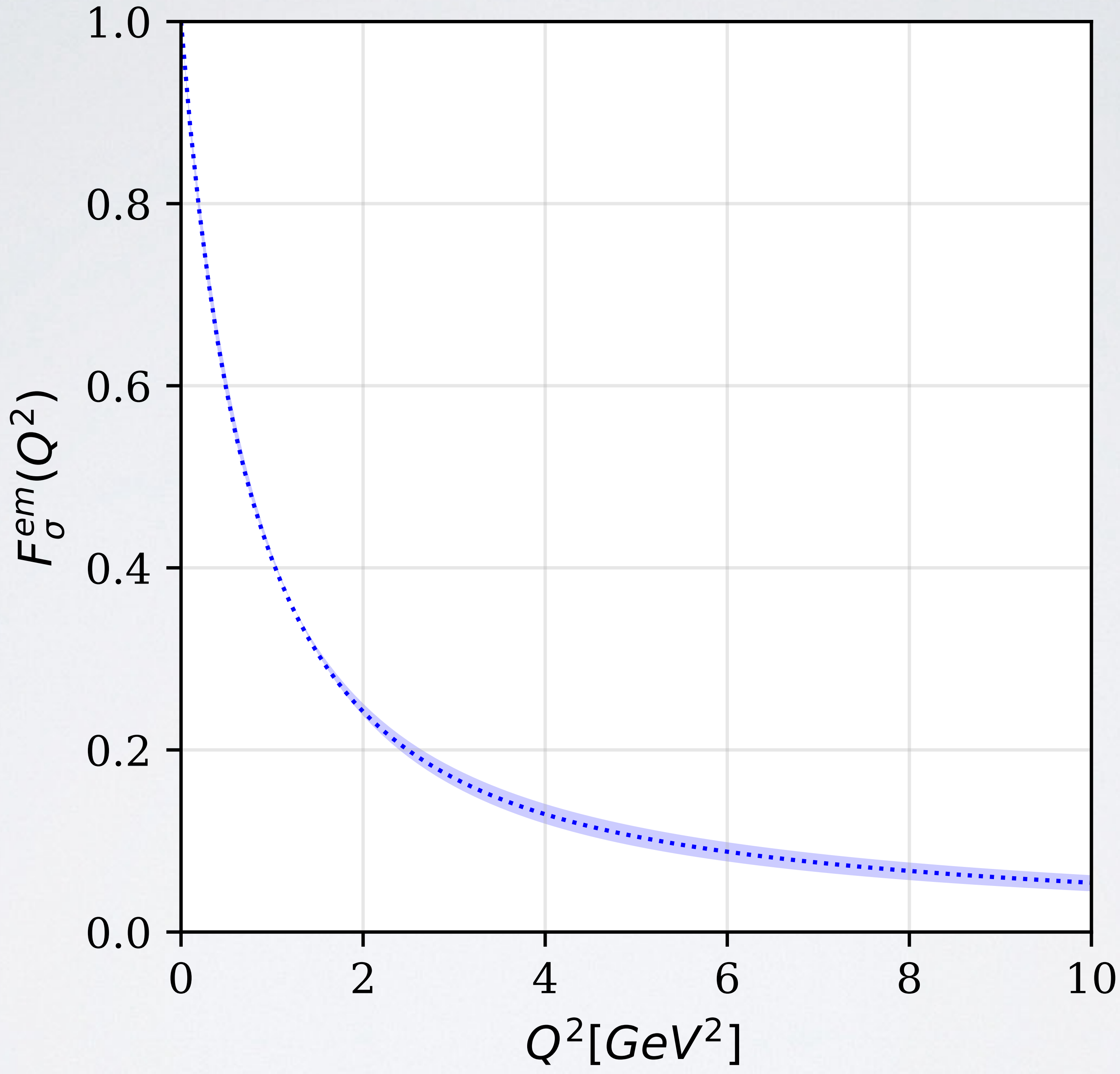
- A similar situation is found for the scalar mesons.



The general trend of decreasing charge radii with increasing constituent quark mass seems reassuring

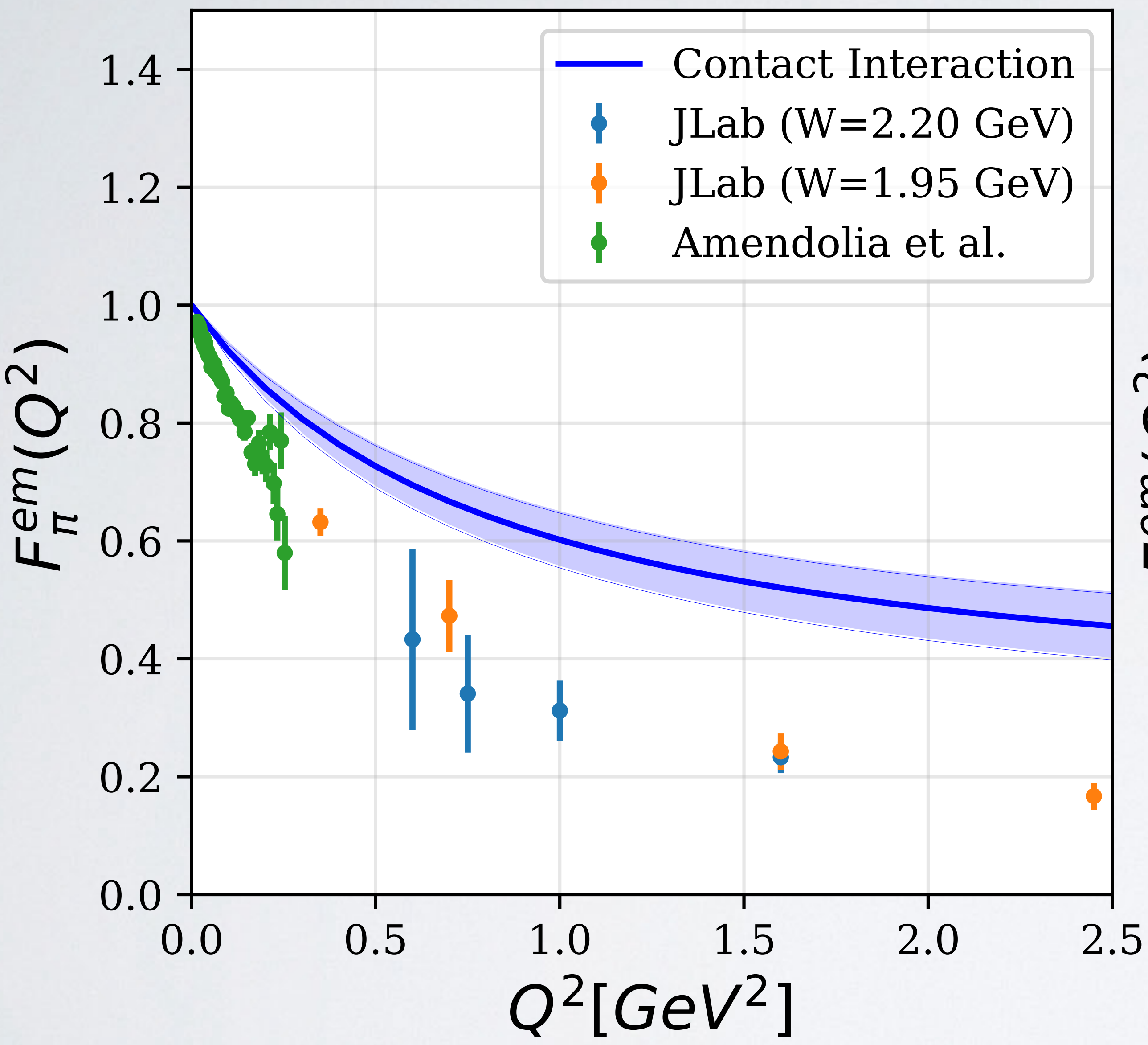
H-P, Gutierrez-Guerrero, Bashir, Bedolla, Higuera-Angulo, *Phys.Rev.D* 107 (2023) 5, 054002

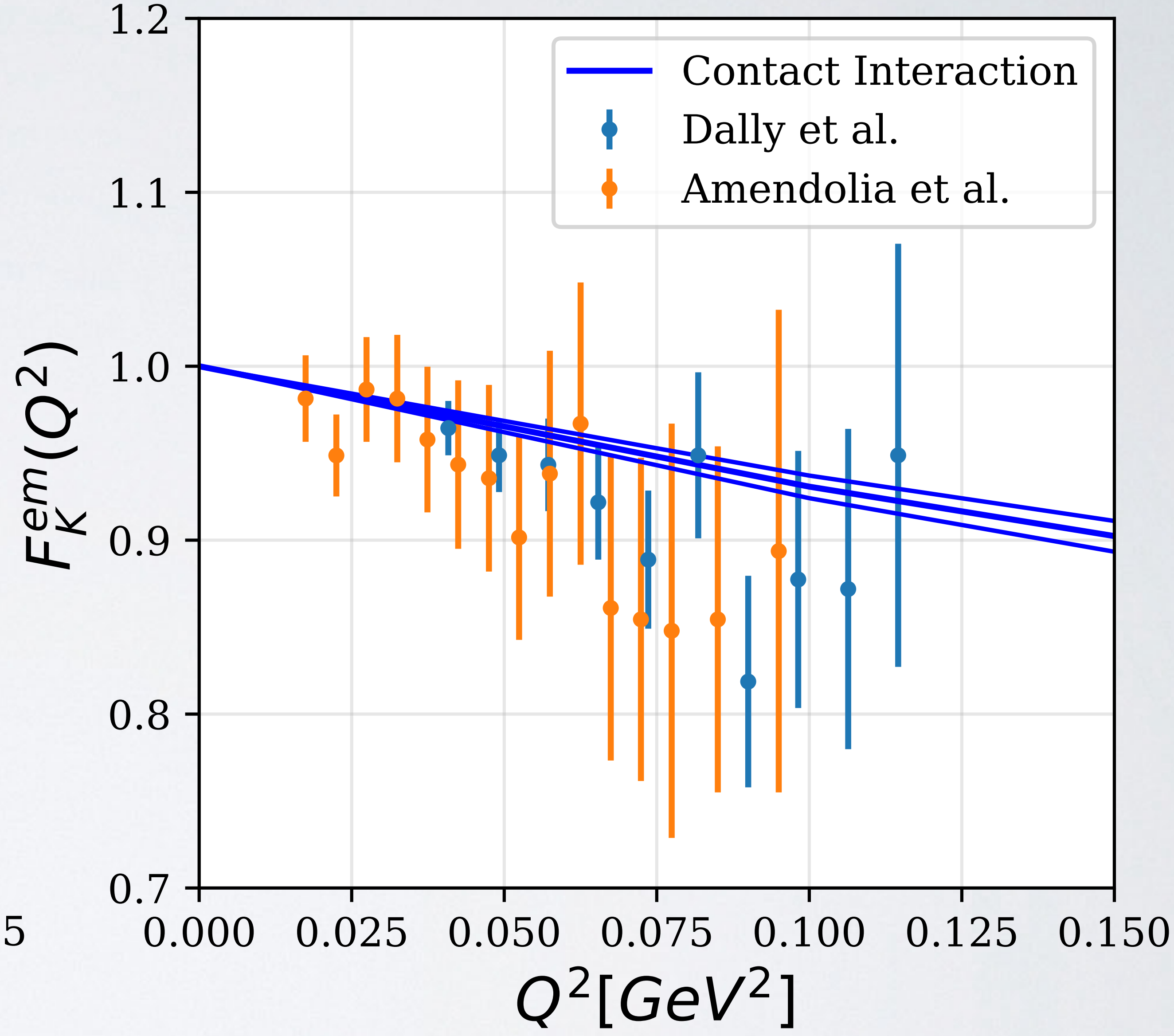
- The error bars presented in the previous plots has computed by allowing a variation of 5% over the charge radii of S and PS mesons.



- Variations of the charge radii in S mesons are in general small.

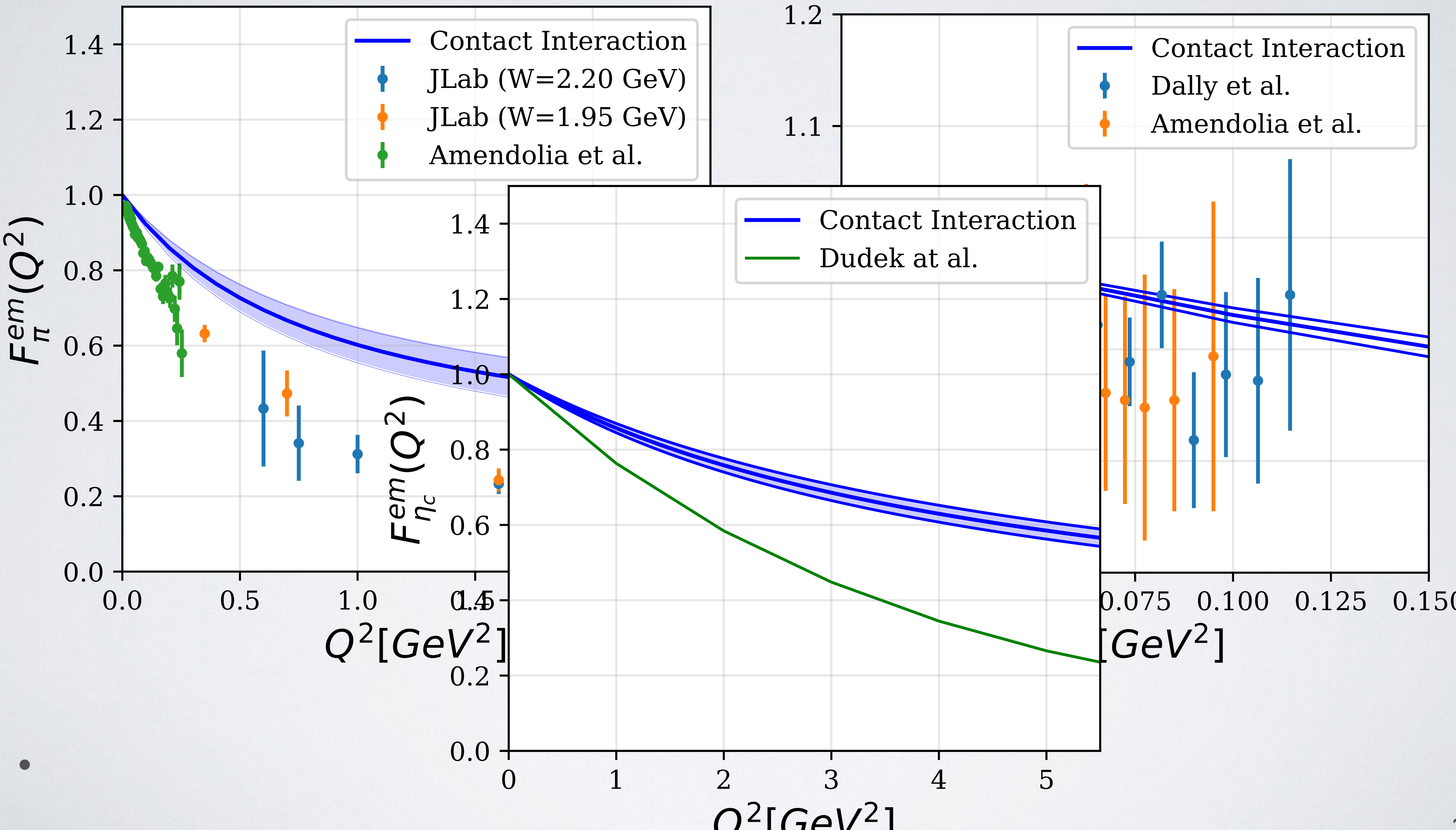
- For the lightest psedoscalar mesons and the η_c meson, error bars can be compared with available experimental and lattice results.





- Comparison with experimental data shows that the simplicity of the CI model underestimate the behavior of the EMFF for pion.

- For the lightest pseudoscalar mesons and the η_c meson, error bars can be compared with available experimental and lattice results.



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- For the practical utility and intuitive understanding of their large Q^2 behavior, we perform an interpolation for S and PS mesons EFF in the range $Q^2 \in [0,8M_M^2]$.

- We adopt the following functional form:

$$F^i(Q^2) = \frac{e_M + a_i\, Q^2 + b_i\, Q^4}{1 + c_i\, Q^2 + d_i\, Q^4}\,,$$

- Where $i \in \{S, PS\}$ and e_M is the electric charge of the corresponding meson.

- The results of our fit for S and PS mesons are

	a_S	b_S	c_S	d_S
$u\bar{d}$	0.286	0.003	1.543	0.617
$u\bar{s}$	0.266	0.002	1.486	0.629
$s\bar{s}$	0.217	0.001	1.271	0.542
$c\bar{u}$	0.759	−0.005	0.680	0.641
$c\bar{s}$	0.004	0.001	0.783	0.047
$u\bar{b}$	0.984	0.001	1.619	0.087
$s\bar{b}$	0.210	0.001	0.175	0.115
$c\bar{b}$	0.289	0.001	0.743	0.026
$c\bar{c}$	0.217	0.001	0.860	0.673
$b\bar{b}$	0.269	0.000	1.607	0.020

	a_{PS}	b_{PS}	c_{PS}	d_{PS}
$u\bar{d}$	0.330	0.029	1.190	0.068
$u\bar{s}$	0.335	0.029	1.092	0.065
$s\bar{s}$	0.328	0.040	0.874	0.092
$c\bar{u}$	0.616	−0.001	1.370	0.109
$c\bar{s}$	0.615	0.028	0.897	0.111
$u\bar{b}$	1.143	0.033	1.921	0.146
$s\bar{b}$	0.218	0.000	0.840	0.009
$c\bar{b}$	0.333	0.003	0.493	0.021
$c\bar{c}$	1.778	0.057	1.994	0.334
$b\bar{b}$	0.099	0.000	0.127	0.002

- 20 mesons. It means that we reproduce the well-known tendency, i.e., $\sim 1/Q^2$

- Finally, we compare our results with available data and models.

	$u\bar{d}$	$u\bar{s}$	$s\bar{s}$	$c\bar{u}$	$c\bar{s}$	$u\bar{b}$	$s\bar{b}$	$c\bar{b}$	$c\bar{c}$	$b\bar{b}$
Our Result	0.45	0.42	0.36	0.36	0.26	0.34	0.24	0.17	0.20	0.07
SDE [4, 20]	0.676 ± 0.002	0.593 ± 0.002	-	-	-	-	-	-	0.24	0.09
Lattice [7, 8, 57]	0.648 ± 0.141	0.566 (extracted)	-	-	-	-	-	-	0.25	-
Exp. [58]	0.659 ± 0.004	0.560 ± 0.031	-	-	-	-	-	-	-	-
r_M^t [54]	0.658	0.568	-	-	-	-	-	-	0.13	0.03
r_M^t(CI)	0.45	0.38	0.33	-	-	-	-	-	0.09	0.02

- There are not studies for all pseudoscalar mesons.
- Unfortunately, this is the same scenario for the scalar mesons
- Vector and Axial-Vector… to be continued.

CONCLUSIONS

- The determination of the internal structure of mesons can be understood from Form Factors.
- In this work, we present the study of Electromagnetic Form Factors of the elastic process $\gamma M \to M$, for scalar and pseudoscalar.
- By means of the Contact Interaction, we find meson masses, BSA, charge radii for all kinds of mesons.
- We find a good agreement for the determination of meson masses.
- Due to the simplicity of the CI, we performed an exhaustive analysis of the EMFF of scalar and pseudoscalar mesons.

CONCLUSIONS

- The small Q^2 dependency shows that the charge radii of scalar and pseudoscalar mesons present a hierarchy such that,

$$r_{u\bar{d}} > r_{u\bar{s}} > r_{c\bar{u}} > r_{u\bar{b}}$$

$$r_{u\bar{s}} > r_{s\bar{s}} > r_{c\bar{s}} > r_{s\bar{b}}$$

$$r_{c\bar{u}} > r_{c\bar{s}} > r_{c\bar{c}} > r_{c\bar{b}}$$

$$r_{u\bar{u}} > r_{s\bar{s}} > r_{c\bar{c}} > r_{b\bar{b}}$$

- On the other hand, we find that for large Q^2 the EMFF drops as $1/Q^2$ as predicted by QCD.

PERSPECTIVES

- We are now finishing the analysis for Electromagnetic Forma Factors of vector and axial vector mesons.

- A similar analysis can be done for Transition Form Factors for scalar, pseudoscalar, vector and axial vector mesons.

- A more involved but necessary analysis shall be implemented for baryons.

GRACIAS