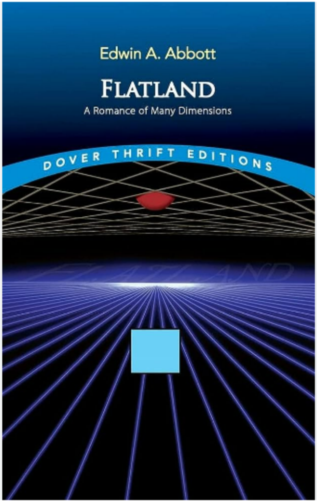
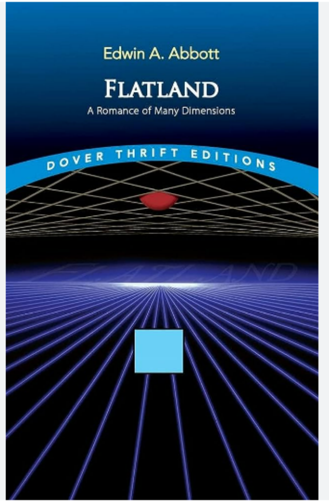
# Reduced QED - Anomalous magnetic moment of charged fermions

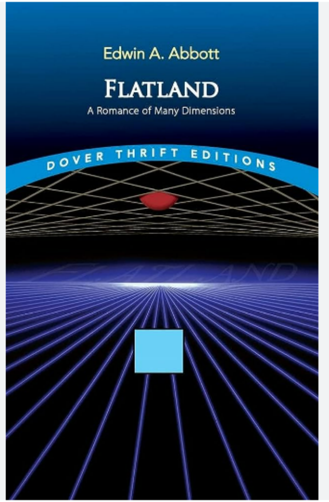
Victor M. Banda, UPSLP

**04 Sep 2023**


Edwin A. Abbott
FLATLAND
A Romance of Many Dimensions
DOVER THRIFT EDITIONS

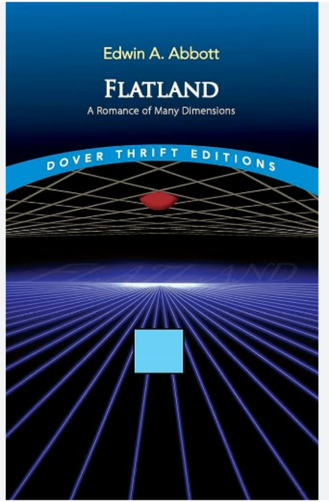



How does an object in a higher dimension interact with an object in a lower dimension?




How does an object in higher dimensions interact with an object in lower dimensions?

How does a photon in higher dimensions interact with an electron in lower dimensions?




How does an object in higher dimensions interact with an object in lower dimensions?

How does a photon in higher dimensions interact with a fermion in lower dimensions?

**Reduced QED theories**

What are the physical differences between standard $QED_4$ and $QED_3$ with $RQED_{4,3}$?

What are the physical differences between standard $\mathrm{QED}_4$ and $\mathrm{QED}_3$ with $\mathrm{RQED}_{4,3}$?

For example. What are the differences in the anomalous magnetic moment?

## Reduced QED - Anomalous magnetic moment of charged fermions

- RQED theories
- Combined second and first order formalism of QED
- Tensor reduction algorithm for Feynman integrals
- Dirac and Pauli form factors
- Pauli form factor for $\mathrm{RQED}_{4,3}$

## Reduced QED - Anomalous magnetic moment of charged fermions

- RQED theories

  Definition

- Combined second and first order formalism of QED

- Tensor reduction algorithm for Feynman integrals

- Dirac and Pauli form factors

- Pauli form factor for $\mathrm{RQED}_{4,3}$

In the RQED action, the electrons and photons resides in different dimensions

$$S_{\mathsf{RQED}} = \int d^{d_\gamma}x \Big[ -\frac{1}{4}F_{mn}F^{mn} - \frac{1}{2\xi}(\partial_m A^m)^2 \Big] + \int d^{d_e}x \overline{\psi}(i\partial\!\!\!/ - m)\psi + \text{ Interaction}$$

The interaction in RQED occurs in

the space-time dimensions of the electron

$$S_{\mathsf{RQED}} = \int d^{d_\gamma}x \Big[ -\frac{1}{4}F_{mn}F^{mn} - \frac{1}{2\xi}(\partial_m A^m)^2 \Big] + \int d^{d_e}x \Big[ \overline{\psi}(i\partial\!\!\!/ - m)\psi - e\overline{\psi}\gamma^\mu \psi A_\mu \Big]$$

From the definition of the RQED action,

we obtain the following generating function

$$Z(\eta,\overline{\eta},J^m) = N\ \mathsf{Exp}\Bigg[-ie\int d^{d_\gamma}x\,\delta(\overline{x})\delta^{m\mu} \times\left(\frac{1}{i}\frac{\delta}{\delta J^m(x)}\right)\left(i\frac{\delta}{\delta\eta(x_e)}\right)\gamma_\mu\left(\frac{1}{i}\frac{\delta}{\delta\overline{\eta}(x_e)}\right)\Bigg]Z_0$$

$$\begin{aligned}\delta(\overline{x}) &= \delta(x_{d_e+1})\ldots\delta(x_{d_\gamma})\,,\\ \delta^{m\mu} &= \begin{cases}1 & \text{if } m=\mu\,,\quad \text{for } \mu=0,\ldots,d_e-1\\ 0 & \text{otherwise}\,,\end{cases}\end{aligned}$$

From the definition of the RQED action,

we obtain the following generating function

$$Z(\eta, \overline{\eta}, J^m) = N \,\mathsf{Exp}\Bigg[ -ie \int d^{d_\gamma} x\, \delta(\overline{x}) \delta^{m\mu} \times \left( \frac{1}{i} \frac{\delta}{\delta J^m(x)} \right) \left( i \frac{\delta}{\delta \eta(x_e)} \right) \gamma_\mu \left( \frac{1}{i} \frac{\delta}{\delta \overline{\eta}(x_e)} \right) \Bigg] Z_0$$

$$Z_0 = \mathsf{Exp}\Bigg[ -i \int d^{d_e} x d^{d_e} y\, \overline{\eta}(x) S(x,y) \eta(y) + \frac{i}{2} \int d^{d_\gamma} x d^{d_\gamma} y\, J^m(x) \Delta_{mn}(x,y) J^n(y) \Bigg]$$

Photon and electron propagators in coordinate space

$$S(x,y) = \int \frac{d^{d_e}p}{(2\pi)^{d_e}}\, e^{-ip(x-y)}\, \frac{\not p + m}{p^2 - m^2},$$

$$\Delta_{mn}(x,y) = \int \frac{d^{d_\gamma}k}{(2\pi)^{d_\gamma}}\, \frac{e^{-ik(x-y)}}{k^2} \left(\eta_{mn} - (1-\xi)\frac{k_m k_n}{k^2}\right)$$

The derivation of the Feynman rules proceeds as

in standard QED, with two exceptions

- On shell photons are described by $\epsilon_\lambda^\mu(k)$

  The kinematics is constrained to be in the electron space

The derivation of the Feynman rules proceeds as

in standard QED, with two exceptions

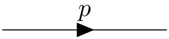
- The photon propagator is modified

  after we integrate out the non-interacting degrees of freedom

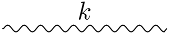
$$\tilde{\Delta}_{\mu\nu}(k) = \frac{1}{(4\pi)^{\epsilon_e}} \, \frac{\Gamma(1-\epsilon_e)}{(-k^2)^{1-\epsilon_e}} \left( \eta_{\mu\nu} - (1-\tilde{\xi}) \frac{k_\mu k_\nu}{k^2} \right)$$

$$\epsilon_e = \frac{d_\gamma - d_e}{2} \, , \qquad \tilde{\xi} = \epsilon_e + (1-\epsilon_e)\xi$$

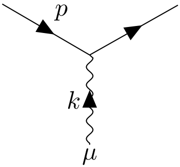
## Feynman rules




$$i\frac{\not p + m}{p^2 - m^2}$$




$$\frac{i}{(4\pi)^{\epsilon_e}}\frac{\Gamma(1-\epsilon_e)}{(-k^2)^{1-\epsilon_e}}\left(\eta^{\mu\nu} - (1-\tilde{\xi})\frac{k^\mu k^\nu}{k^2}\right)$$




$$-ie\gamma^\mu$$

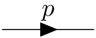
**Reduced QED - Anomalous magnetic moment of charged fermions**

- RQED theories
- Combined second and first order formalism of QED

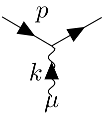
  One loop QED vertex in the combined formalism
- Tensor reduction algorithm for Feynman integrals
- Dirac and Pauli form factors
- Pauli form factor for $\mathrm{RQED}_{4,3}$

In the second order formalism of QED,

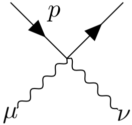
the scalar and spin vertex interaction is separated




$i(p^2 - m^2)^{-1}$




$-ie[(2p+k)^\mu + \sigma^{\mu\alpha}k_\alpha]$




$2ie^2\eta^{\mu\nu}$

■ Scalar ■ Spin

The second order rules arise from a Gordon-type decomposition of the fermion propagator and the first order vertex

$$e\, S(p+k)\, \gamma^\mu = e\, \frac{(\not p + \not k) + m}{(p+k)^2 - m^2}\, \gamma^\mu = \frac{A^\mu_{p,k}}{D_{p+k}}$$

$$\begin{aligned} A^\mu_{p,k} &= B^\mu_{p,k} + C^\mu_p \\ B^\mu_{p,k} &= e[(2p+k)^\mu + e\sigma^{\mu\alpha} k_\alpha] \\ C^\mu_p &= -e\gamma^\mu(\not p - m) \\ D_q &= q^2 - m^2 \end{aligned}$$

The second order rules arise from a Gordon-type decomposition of the fermion propagator and the first order vertex

$$e\,S(p+k)\,\gamma^\mu \quad = \quad e\,\frac{(\not{p}+\not{k})+m}{(p+k)^2-m^2}\,\gamma^\mu = \frac{A^\mu_{p,k}}{D_{p+k}}$$

$$\begin{aligned}
A^\mu_{p,k} &= B^\mu_{p,k} + C^\mu_p \\
B^\mu_{p,k} &= e[(2p+k)^\mu + e\sigma^{\mu\alpha}k_\alpha] \\
C^\mu_p &= -e\gamma^\mu(\not{p}-m) \\
D_q &= q^2-m^2
\end{aligned}$$

## Identity for $C_q$

$$C^{\mu}_{p+k}\,\frac{A^{\nu}_{p,k}}{D_{p+k}} \quad = \quad e^2\gamma^{\mu}\gamma^{\nu} = e^2\left(\eta^{\mu\nu} + \sigma^{\mu\nu}\right)$$

Upon an efficient combination of the first and second order formalism, the one loop vertex naturally decompose from the onset

$$V^\mu(p,p',k) = V_L^\mu(p,p',k) + V_T^\mu(p,p',k)$$

Transverse component: $k \cdot V_T(p,p',k) = 0$

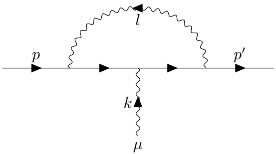
Longitudinal component: Satisfies the Ward-Takashi identity

$$k \cdot V_L(p,p',k) = e\left[\Sigma(\not{p}) - \Sigma(\not{p}')\right]$$

Upon an efficient combination of the first and second order formalism, the one loop vertex naturally decompose from the onset

Standard QED:

$$V^\mu(p',p) = \frac{e^3}{i}\int \frac{d^D l}{(2\pi)^D}\left[\gamma^\rho S(p'+l)\gamma^\mu S(p+l)\gamma^\nu\right]\Delta_{\nu\rho}(l),$$




$$S(q) = \frac{\not{q}+m}{q^2-m^2}$$

$$\Delta_{\nu\rho}(l) = \frac{1}{l^2}\left(\eta_{\nu\rho} - (1-\xi)\frac{l_\nu l_\rho}{l^2}\right)$$

Upon an efficient combination of the first and second order formalism, the one loop vertex naturally decompose from the onset

$$V^{\mu}(p',p) \quad = \quad e \int \frac{d^D l}{i(2\pi)^D}\,\gamma^{\rho}\left(\frac{B^{\mu}_{p+l,k}\,B^{\nu}_{p,l}}{D_{p'+l}D_{p+l}} + \frac{B^{\mu}_{p+l,k}\,C^{\nu}_{p}}{D_{p'+l}D_{p+l}}\right.$$
$$\left. + e^2\frac{\gamma^{\mu}\,\gamma^{\nu}}{D_{p'+l}}\right)\Delta_{\rho\nu}(l)$$

Upon an efficient combination of the first and second order formalism, the one loop vertex naturally decompose from the onset

$$V^\mu(p,p',k) = V_L^\mu(p,p',k) + V_T^\mu(p,p',k)$$

$$\begin{aligned} V_T^\mu \quad = \quad & e^3 \int \frac{d^D l}{i(2\pi)^D} \frac{1}{D_{p'+l} D_{p+l} l^2} \Big\{ - \Big[ (5 - D - \xi) \sigma^{\mu\alpha} k_\alpha \\ & + \frac{2(1-\xi)}{l^2} \left( l^\mu \not{k} \not{l} - k \cdot l \gamma^\mu \not{l} \right) \Big] (\not{p} - m) - 2\sigma^{\mu\alpha} k_\alpha \not{p} \\ & + 4(p \cdot k \gamma^\mu - p^\mu \not{k}) + 2\xi (k \cdot l \gamma^\mu - l^\mu \not{k}) \\ & - (1 - D + \xi) \sigma^{\mu\alpha} k_\alpha \not{l} - 4 \left( \gamma^\mu \not{k} \not{l} + k^\mu \not{l} \right) + 2(k \cdot l \gamma^\mu - l^\mu \not{k}) \\ & - 2(1-\xi) \frac{p \cdot l}{l^2} \sigma^{\mu\alpha} k_\alpha \not{l} + 4 \frac{1-\xi}{l^2} (p \cdot l \not{k} l^\mu - l \cdot k p \cdot l \gamma^\mu) \Big\} \end{aligned}$$

Upon an efficient combination of the first and second order formalism, the one loop vertex naturally decompose from the onset

$$V^\mu(p,p',k) = V_L^\mu(p,p',k) + V_T^\mu(p,p',k)$$

$$\begin{aligned} V_T^\mu \quad = \quad & e^3 \int \frac{d^D l}{i(2\pi)^D} \frac{1}{D_{p'+l} D_{p+l} l^2} \Bigg\{ - \Big[ (5-D-\xi)\sigma^{\mu\alpha} k_\alpha \\ & + \frac{2(1-\xi)}{l^2} \Big( l^\mu \not{k} \not{l} - k\cdot l \gamma^\mu \not{l} \Big) \Big] (\not{p} - m) - 2\sigma^{\mu\alpha} k_\alpha \not{p} \\ & + 4(p\cdot k \gamma^\mu - p^\mu \not{k}) + 2\xi(k\cdot l \gamma^\mu - l^\mu \not{k}) \\ & - (1-D+\xi)\sigma^{\mu\alpha} k_\alpha \not{l} - 4\Big(\gamma^\mu \not{k}\not{l} + k^\mu \not{l}\Big) + 2(k\cdot l\gamma^\mu - l^\mu \not{k}) \\ & - 2(1-\xi)\frac{p\cdot l}{l^2} \sigma^{\mu\alpha} k_\alpha \not{l} + 4\frac{1-\xi}{l^2}(p\cdot l \not{k} l^\mu - l\cdot k p\cdot l \gamma^\mu) \Bigg\} \end{aligned}$$

Upon an efficient combination of the first and second order formalism, the one loop vertex naturally decompose from the onset

$$V^\mu(p,p',k) = V_L^\mu(p,p',k) + V_T^\mu(p,p',k)$$

$$\begin{aligned} V_L^\mu \quad = \quad & e^3 \int \frac{d^D l}{i(2\pi)^D} \left\{ \frac{1}{D_{p'+l} D_{p+l} l^2} \left[ 2(p+p'+2l)^\mu \not{p} + (1-D+\xi)(p+p')^\mu \not{p} \right. \right. \\ & + 2(1-D+\xi) l^\mu \not{l} - 2(1-\xi)(p+p'+2l)^\mu \frac{p \cdot l \not{l}}{l^2} \\ & \left. \left. + (1-D-\xi) \left[ (p+p')^\mu + 2l^\mu \right] (\not{p} - m) \right] + \frac{D-3+\xi}{D_{p'+l}\, l^2} \gamma^\mu - 2 \frac{(1-\xi) l^\mu \not{l}}{D_{p'+l}\, l^4} \right\} \end{aligned}$$

We need to evaluate the following type of integrals

$$\int \frac{d^D l}{i\pi^{D/2}} \frac{l^\mu}{D_{p'+l}\, D_{p+l}\; l^{2\,a}}$$

$$\int \frac{d^D l}{i\pi^{D/2}} \frac{l^\mu l^\nu}{D_{p'+l}\, D_{p+l}\; l^{2\,a}}$$

## Reduced QED - Anomalous magnetic moment of charged fermions

- RQED theories
- Combined second and first order formalism of QED
- Tensor reduction algorithm for Feynman integrals

  One loop vertex in terms of scalar Feynman integrals
- Dirac and Pauli form factors
- Pauli form factor for $\mathrm{RQED}_{4,3}$

We can use the Schwinger parametrization to change a tensor integral into a scalar Feynman integral

Simple example:

$$J^\mu = \int \frac{d^d l}{\pi^{d/2}} \frac{l^\mu}{l^2[(l+p')^2 - m^2][(l+p)^2 - m^2]}$$

$p^2 = p'^2 = m^2$

We can use the Schwinger parametrization to change a tensor integral into a scalar Feynman integral

Simple example:

$$J^\mu = \int_0^\infty dxdydz\ (yp'^\mu + zp^\mu)\frac{\mathsf{e}^{-b^2/a}}{a^{\frac{d+2}{2}}}$$

$a = x + y + z$

$b^\mu = yp'^\mu + zp^\mu$

We can use the Schwinger parametrization to change a tensor integral into a scalar Feynman integral

Scalar integral:

$$J^D_{\alpha\beta} = \int \frac{d^D l}{\pi^{D/2}} \frac{1}{l^2[(l+p')^2 - m^2]^\alpha [(l+p)^2 - m^2]^\beta}$$

$p^2 = p'^2 = m^2$

We can use the Schwinger parametrization to change a tensor integral into a scalar Feynman integral

Scalar integral:

$$J^D_{\alpha\beta} = \int_0^\infty dxdydz \; \frac{(-1)^{\alpha+\beta+1}}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1} z^{\beta-1} \frac{\mathrm{e}^{-b^2/a}}{a^{D/2}}$$

$a = x + y + z$

$b^\mu = yp'^\mu + zp^\mu$

We can use the Schwinger parametrization to change a tensor integral into a scalar Feynman integral

Comparison:

$$J^D_{\alpha\beta} = \int_0^\infty dxdydz\; \frac{(-1)^{\alpha+\beta+1}}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1} z^{\beta-1} \frac{\mathrm{e}^{-b^2/a}}{a^{D/2}}$$

$$J^\mu = \int_0^\infty dxdydz\; (yp'^\mu + zp^\mu) \frac{\mathrm{e}^{-b^2/a}}{a^{\frac{d+2}{2}}}$$

We can use the Schwinger parametrization to change a tensor integral into a scalar Feynman integral

$$J^D_{\alpha\beta} = \int_0^\infty dxdydz\ \frac{(-1)^{\alpha+\beta+1}}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1} z^{\beta-1} \frac{\mathrm{e}^{-b^2/a}}{a^{D/2}}$$

$$J^\mu = \int_0^\infty dxdydz\ (yp'^\mu + zp^\mu) \frac{\mathrm{e}^{-b^2/a}}{a^{\frac{d+2}{2}}}$$

$J^\mu = p' J^{d+2}_{21} + p^\mu J^{d+2}_{12}$

We can use the Schwinger parametrization to change a tensor integral into a scalar Feynman integral

$$\int \frac{d^{d_e} l}{i\pi^{\frac{d_e}{2}}} \frac{l^\mu}{[(p'+l)^2 - m^2]\,[(p+l)^2 - m^2]\,(-l^2)^a} = -p^\mu \ J^{d_e+2}_{1,2,a} - p'^\mu \ J^{d_e+2}_{2,1,a}$$

$$\int \frac{d^{d_e} l}{i\pi^{\frac{d_e}{2}}} \frac{l^\mu l^\nu}{[(p'+l)^2 - m^2]\,[(p+l)^2 - m^2]\,(-l^2)^a} = -\frac{1}{2}\eta^{\mu\nu} \ J^{d_e+2}_{1,1,a} + 2p'^\mu p'^\nu \ J^{d_e+4}_{3,1,a} + 2p^\mu p^\nu \ J^{d_e+4}_{1,3,a} + (p^\mu p'^\nu + p^\nu p'^\mu) \ J^{d_e+4}_{2,2,a}$$

$$J^D_{a,b,c}(p,p') = \int \frac{d^D l}{i\pi^{\frac{D}{2}}} \frac{1}{[-(p'+l)^2 + m^2]^a\,[-(p+l)^2 + m^2]^b\,(-l^2)^c}$$

Longitudinal and transverse components of the one loop standard QED vertex in terms of scalar Feynman integrals

$$\begin{aligned}
V_L^\mu \quad = \quad & \frac{e^3}{(4\pi)^{\frac{d_e}{2}}}\Big\{m(1-\xi-d_e)(J^{d_e}_{1,1,1}-2J^{d_e+2}_{1,2,1})p^\mu + m(1-\xi-d_e)(J^{d_e}_{1,1,1}-2J^{d_e+2}_{2,1,1})p'^\mu \\
& +\Big[(2-d_e)J^{d_e+2}_{1,1,1}+(d_e-3+\xi)J^{d_e}_{1,0,1}+(m^2+p^2)(1-\xi)J^{d_e+2}_{1,1,2}\Big]\gamma^\mu \\
& -\Big[(2-d_e)\left(J^{d_e}_{1,1,1}-3J^{d_e+2}_{1,2,1}+4J^{d_e+4}_{1,3,1}\right)+(1-\xi)\Big(J^{d_e}_{1,1,1}-J^{d_e+2}_{1,1,2}-J^{d_e+2}_{1,2,1} \\
& +4m^2J^{d_e+4}_{1,3,2}+2p\cdot p' J^{d_e+4}_{2,2,2}\Big)\Big]p^\mu \not{p} - \Big[(d_e-2)\left(J^{d_e+2}_{2,1,1}-2J^{d_e+4}_{2,2,1}\right) \\
& +(1-\xi)\Big(J^{d_e+2}_{2,1,1}+2m^2J^{d_e+4}_{2,2,2}+4p\cdot p' J^{d_e+4}_{3,1,2}\Big)\Big]p^\mu \not{p}' \\
& -\Big[(2-d_e)\Big(J^{d_e}_{1,1,1}-J^{d_e+2}_{1,2,1}-2J^{d_e+2}_{2,1,1}+2J^{d_e+4}_{2,2,1}\Big) \\
& +(1-\xi)\Big(J^{d_e}_{1,1,1}-J^{d_e+2}_{1,1,2}+J^{d_e+2}_{1,2,1}+4p^2J^{d_e+4}_{1,3,2}-2J^{d_e+2}_{2,1,1} \\
& +2(m^2-p^2+p\cdot p')J^{d_e+4}_{2,2,2}\Big)\Big]p'^\mu \not{p} - \Big[(d_e-2)\left(J^{d_e+2}_{2,1,1}-4J^{d_e+4}_{3,1,1}\right) \\
& +(1-\xi)\Big(J^{d_e+2}_{2,1,1}+2p^2J^{d_e+4}_{2,2,2}+4(m^2-p^2+p\cdot p')J^{d_e+4}_{3,1,2}\Big)\Big]p'^\mu \not{p}'\Big\}
\end{aligned}$$

Longitudinal and transverse components of the one loop standard QED vertex in terms of scalar Feynman integrals

$$
\begin{aligned}
V_T^\mu \quad = \quad & \frac{e^3}{(4\pi)^{\frac{d_e}{2}}} \Big\{ \Big( 2(\xi-1) \Big[ J^{d_e+4}_{2,2,2} \left(p^\mu \not{k} - p\cdot k \gamma^\mu\right) \not{p}' + J^{d_e+4}_{2,2,2} \left(p'^\mu \not{k} - p'\cdot k\gamma^\mu\right) \not{p} \\
& + 2 J^{d_e+4}_{1,3,2} \left(p^\mu \not{k} - p\cdot k\gamma^\mu\right) \not{p} + 2 J^{d_e+4}_{3,1,2} \left(p'^\mu \not{k} - p'\cdot k\gamma^\mu\right) \not{p}' \Big] \\
& + \Big[ (4-d_e) J^{d_e}_{1,1,1} + (1-\xi)(J^{d_e}_{1,1,1} - 2 J^{d_e+2}_{1,1,2}) \Big] \sigma^{\mu\alpha} k_\alpha \Big) (-\not{p} + m) \\
& + \Big[ 2 J^{d_e}_{1,1,1} + (d_e - 6) J^{d_e+2}_{1,2,1} - (1-\xi) \Big( J^{d_e+2}_{1,1,2} - 2p\cdot p' J^{d_e+4}_{2,2,2} - 4p^2 J^{d_e+4}_{1,3,2} \\
& - J^{d_e+2}_{1,2,1} \Big) \Big] \sigma^{\mu\alpha} k_\alpha \not{p} - \Big[ (6-d_e) J^{d_e+2}_{2,1,1} - (1-\xi) \Big( 2p^2 J^{d_e+4}_{2,2,2} + 4p\cdot p' J^{d_e+4}_{3,1,2} \\
& + J^{d_e+2}_{2,1,1} \Big) \Big] \sigma^{\mu\alpha} k_\alpha \not{p}' + 2 \Big[ 2 J^{d_e}_{1,1,1} - 2 J^{d_e+2}_{1,2,1} - (1-\xi) \Big( J^{d_e+2}_{1,1,2} - 2p\cdot p' J^{d_e+4}_{2,2,2} \\
& - 4p^2 J^{d_e+4}_{1,3,2} - J^{d_e+2}_{1,2,1} \Big) \Big] (p^\mu \not{k} - k\cdot p\gamma^\mu) - 2 \Big[ 2 J^{d_e+2}_{2,1,1} - (1-\xi) \Big( 2p^2 J^{d_e+4}_{2,2,2} \\
& + 4p\cdot p' J^{d_e+4}_{3,1,2} + J^{d_e+2}_{2,1,1} \Big) \Big] (p'^\mu \not{k} - p'\cdot k\gamma^\mu) \Big\}
\end{aligned}
$$

For the case of RQED vertex we obtain the same expressions, but replacing:

$$e^3 \to e^3 \Gamma(\overline{\epsilon}_e) \qquad (4\pi)^{\frac{d_e}{2}} \to (4\pi)^{\epsilon_e + \frac{d_e}{2}}$$

$$J^D_{a,b,1} \to J^D_{a,b,\overline{\epsilon}_e} \qquad J^D_{a,b,2} \to J^D_{a,b,1+\overline{\epsilon}_e}$$

$$\overline{\epsilon}_e = 1 - \epsilon_e \qquad \xi \to \tilde{\xi}$$

$$\epsilon_e = \frac{d_\gamma - d_e}{2}\,, \qquad \tilde{\xi} = \epsilon_e + (1 - \epsilon_e)\xi$$

**Reduced QED - Anomalous magnetic moment of charged fermions**

- RQED theories
- Combined second and first order formalism of QED
- Tensor reduction algorithm for Feynman integrals
- Dirac and Pauli form factors

  One loop general expressions in RQED
- Pauli form factor for $\text{RQED}_{4,3}$

After we impose on-shell conditions $(\not{p} - m)u_s(p) = 0,\ \overline{u}_{s'}(p')(\not{p}' - m) = 0$, the longitudinal and transverse components of the vertex simplify to:

$$\overline{u}_{s'}(p')V_L^\mu u_s(p) = \frac{e^3\Gamma(\bar{\epsilon}_e)}{(4\pi)^{\epsilon_e+\frac{d_e}{2}}}\left\{ \left[(d_e + \tilde{\xi} - 3)J_{1,0,\bar{\epsilon}_e} + (2 - d_e)J^{d_e+2}_{1,1,\bar{\epsilon}_e} + 2m\, f_{2L}\right]\gamma^\mu + f_{2L}\sigma^{\mu\nu}k_\nu \right\}$$

$$\overline{u}_{s'}(p')V_T^\mu u_s(p) = \frac{e^3\Gamma(\bar{\epsilon}_e)}{(4\pi)^{\epsilon_e+\frac{d_e}{2}}}\left( f_{1T}\,\gamma^\mu - f_{2T}\,\sigma^{\mu\nu}k_\nu \right)$$

$$
\begin{aligned}
f_{2L} \quad = \quad & -2m\, J^{d_e}_{1,1,\overline{\epsilon}_e} + 4m\,(d_e-2) J^{d_e+4}_{3,1,\overline{\epsilon}_e} + 2m(4-d_e) J^{d_e+2}_{2,1,\overline{\epsilon}_e} - 2m(2-d_e) J^{d_e+4}_{2,2,\overline{\epsilon}_e} \\
& + m(1-\tilde{\xi}) \Big[ J^{d_e+2}_{1,1,1+\overline{\epsilon}_e} - 2(m^2 + p\cdot p') J^{d_e+4}_{2,2,1+\overline{\epsilon}_e} - 4(m^2 + p\cdot p') J^{d_e+4}_{3,1,1+\overline{\epsilon}_e} \\
& - 2 J^{d_e+2}_{2,1,\overline{\epsilon}_e} \Big] ,
\end{aligned}
$$

$$
\begin{aligned}
f_{1T} \quad = \quad & -2(4-d_e) k\cdot p'\, J^{d_e+2}_{2,1,\overline{\epsilon}_e} - 4k\cdot p\, J^{d_e}_{1,1,\overline{\epsilon}_e} + 4k\cdot p\, J^{d_e+2}_{2,1,\overline{\epsilon}_e} \\
& + 2k\cdot p(1-\tilde{\xi}) \Big[ J^{d_e+2}_{1,1,1+\overline{\epsilon}_e} - 2p\cdot p' J^{d_e+4}_{2,2,1+\overline{\epsilon}_e} - 4m^2\, J^{d_e+4}_{3,1,1+\overline{\epsilon}_e} - J^{d_e+2}_{2,1,\overline{\epsilon}_e} \Big] ,
\end{aligned}
$$

$$
\begin{aligned}
f_{2T} \quad = \quad & 2m(6-d_e)\, J^{d_e+2}_{2,1,\overline{\epsilon}_e} - 2m\, J^{d_e}_{1,1,\overline{\epsilon}_e} \\
& + m(1-\tilde{\xi}) \Big[ J^{d_e+2}_{1,1,1+\overline{\epsilon}_e} - 2(m^2 + p\cdot p') J^{d_e+4}_{2,2,1+\overline{\epsilon}_e} - 4(m^2 + p\cdot p') J^{d_e+4}_{3,1,1+\overline{\epsilon}_e} \\
& - 2 J^{d_e+2}_{2,1,\overline{\epsilon}_e} \Big] .
\end{aligned}
$$

Using the previous results, we obtain general expressions for the Pauli and Dirac form factors for RQED in arbitrary gauge and dimensions

$$\overline{u}_{s'}(p')V^\mu u_s(p) = e\overline{u}_{s'}(p')\left[F_1(k^2)\,\gamma^\mu - \frac{1}{2m}F_2(k^2)\sigma^{\mu\nu}k_\nu\right]u_s(p)\,,$$

$$\begin{aligned}
F_1(k^2) \quad &= \quad \frac{e^2\Gamma(\overline{\epsilon}_e)}{(4\pi)^{\epsilon_e+\frac{d_e}{2}}}\Big\{(d_e-2)J^{d_e}_{1,0,\overline{\epsilon}_e} + (2-d_e)J^{d_e+2}_{1,1,\overline{\epsilon}_e} - 2(2m^2-k^2)J^{d_e}_{1,1,\overline{\epsilon}_e} \\
&+ \left[4(4-d_e)m^2 - (6-d_e)k^2\right]J^{d_e+2}_{2,1,\overline{\epsilon}_e} + 4m^2(d_e-2)\left(J^{d_e+4}_{2,2,\overline{\epsilon}_e} + 2J^{d_e+4}_{3,1,\overline{\epsilon}_e}\right) \\
&-(1-\tilde{\xi})\Big[J^{d_e}_{1,0,\overline{\epsilon}_e} - (2m^2-k^2)J^{d_e+2}_{1,1,1+\overline{\epsilon}_e} + (4m^4+(2m^2-k^2)^2)J^{d_e+4}_{2,2,1+\overline{\epsilon}_e} \\
&+8m^2(2m^2-k^2)J^{d_e+4}_{3,1,1+\overline{\epsilon}_e} + (4m^2-k^2)J^{d_e+2}_{2,1,\overline{\epsilon}_e}\Big]\Big\}
\end{aligned}$$

$$F_2(k^2) \quad = \quad \frac{4e^2m^2\Gamma(\overline{\epsilon}_e)}{(4\pi)^{\epsilon_e+\frac{d_e}{2}}}\left[2J^{d_e+2}_{2,1,\overline{\epsilon}_e} + (2-d_e)(2J^{d_e+4}_{3,1,\overline{\epsilon}_e} + J^{d_e+4}_{2,2,\overline{\epsilon}_e})\right]$$

**Reduced QED - Anomalous magnetic moment of charged fermions**

- RQED theories
- Combined second and first order formalism of QED
- Tensor reduction algorithm for Feynman integrals
- Dirac and Pauli form factors
- Pauli form factor for $\mathrm{RQED}_{4,3}$

  and the $g-2$-factor

After Feynman parametrization the Pauli form factor takes a simple integral form

$$F_2(k^2) = \frac{4e^2m^2\Gamma\left(3-\epsilon_e-\frac{d_e}{2}\right)}{(4\pi)^{\epsilon_e+\frac{d_e}{2}}} \times \int_0^1 dx \int_0^{1-x} dy \, \frac{2x+(2-d_e)(x^2+xy)}{(1-x-y)^{\epsilon_e} A^{3-\epsilon_e-\frac{d_e}{2}}}$$

$$A \equiv (x+y)^2m^2 - xyk^2 + (1-x-y)m_\gamma^2$$

## Results

- Standard QED

$d_e = d_\gamma = 4$

$c = k^2/m^2$

$$
\begin{aligned}
F_2(k^2) &= \frac{\alpha}{2\pi}\int_0^1 \frac{dy}{1+cy(1-y)} \\
F_2(0) &= \frac{\alpha}{2\pi}
\end{aligned}
$$

## Results

■ Standard QED

$d_e = d_\gamma = 4$

$$\begin{aligned} F_2(k^2) &= \frac{\alpha}{2\pi}\int_0^1 \frac{dy}{1+cy(1-y)} \\ F_2(0) &= \frac{\alpha}{2\pi} \end{aligned}$$

■ Standard QED

$d_e = d_\gamma = 3$

$$\begin{aligned} F_2(k^2) &= \frac{e^2}{4\pi m}\int_0^1 dx \int_0^{1-x} dy\, \frac{x(2-x-y)}{A^{3/2}} \\ F_2(0) &= \frac{e^2}{8\pi m}\left[3(\kappa-1) + \left(2-\frac{3}{2}\kappa\right)\ln\left(\frac{2+\kappa}{\kappa}\right)\right] \end{aligned}$$

$$A = (x+y)^2 + cxy + (1-x-y)\kappa^2$$

$c = k^2/m^2$ $\kappa = m_\gamma/m$

## Results

- Standard QED

$d_e = d_\gamma = 4$

$$F_2(k^2) = \frac{\alpha}{2\pi}\int_0^1 \frac{dy}{1+cy(1-y)}$$
$$F_2(0) = \frac{\alpha}{2\pi}$$

- Standard QED

$d_e = d_\gamma = 3$

$$F_2(k^2) = \frac{e^2}{4\pi m}\int_0^1 dx \int_0^{1-x} dy \frac{x(2-x-y)}{A^{3/2}}$$
$$F_2(0) = \frac{e^2}{8\pi m}\left[3(\kappa-1) + \left(2-\frac{3}{2}\kappa\right)\ln\left(\frac{2+\kappa}{\kappa}\right)\right]$$

$$A = (x+y)^2 + cxy + (1-x-y)\kappa^2$$

- RQED

$d_\gamma = 4$ and $d_e = 3$

$$F_2(k^2) = \frac{8}{3}\left[\frac{\alpha}{2\pi}\int_0^1 \frac{dy}{1+cy(1-y)}\right]$$
$$F_2(0) = \frac{4\alpha}{3\pi}$$

$c = k^2/m^2$ $\qquad\qquad \kappa = m_\gamma/m$

## Summary and conclusions

- RQED is the quantum electromagnetic field theory where a photon in a higher dimension interacts with an electron in a lower dimension.

- We have generalized the results of the literature about one loop massless calculations to the massive case.

## Summary and conclusions

- We have combined the second order formalism of QED with the first order formalism to obtain a natural decomposition of the one loop vertex into transverse and longitudinal components form the onset.

- Using the tensor reduction algorithm we obtained a general expression for the one-loop RQED vertex in terms of Feynman scalar integrals in arbitrary gauge and dimensions.

## Summary and conclusions

- $\mathsf{RQED}_{4,3}$ is more like standard QED in 4 dimensions, than standard QED in 3 dimensions.

- The response of the gyromagnetic ratio $g$ of the electron to a magnetic field in RQED gets augmented in comparison with standard $\mathsf{QED}_4$.

$$\boxed{F_2^{\mathsf{RQED}_{4,3}} = \frac{8}{3}\, F_2^{\mathsf{QED}_4}}$$

## References

V. M. B. Guzmán and A. Bashir. One-loop fermion-photon vertex in arbitrary gauge and dimensions: A novel approach. Phys.Rev.D 107 (2023) 7, 073008.

V. M. B. Guzmán, A. Bashir, L. Albino and D. Rodríguez-Tzintzun. One loop reduced QED for massive fermions within an innovative formalism.