



# Sizing the double pole resonant enhancement in $e^+e^- \rightarrow \pi^0\pi^0\gamma$ and $\tau^- \rightarrow \pi^-\pi^0\nu_\tau\gamma$



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Based on  
hep-ph: 2308.02766  
and PRD 107 056006 (2023)  
Gustavo Ávalos, Leonardo Esparza,  
Antonio Rojas, Marxil Sánchez and GT

# Motivation

- ◆ The importance of two resonant states in:
- ◆  $e^+e^- \rightarrow \pi^0\pi^0\gamma$  cross section
- ◆ muon  $g-2$  ISB from  $\tau^- \rightarrow \pi^-\pi^0\nu_\tau\gamma$



# Motivation

## The importance of resonant states in muon $g-2$ ISB from $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau \gamma$

**Table 1** Contributions to  $a_\mu^{\text{had,LO}}[\pi\pi, \tau]$  ( $\times 10^{-10}$ ) from the isospin-breaking corrections discussed in Sect. 3. Corrections shown in two separate columns correspond to the Gounaris–Sakurai (GS) and Kühn–Santamaria (KS) parametrisations, respectively

Source	$\Delta a_\mu^{\text{had,LO}}[\pi\pi, \tau]$ ( $10^{-10}$ )	
	GS model	KS model
$S_{\text{EW}}$	$-12.21 \pm 0.15$	
$G_{\text{EM}}$	$-1.92 \pm 0.90$	
FSR	$+4.67 \pm 0.47$	
$\rho$ - $\omega$ interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$
$m_{\pi^\pm} - m_{\pi^0}$ effect on $\sigma$	$-7.88$	
$m_{\pi^\pm} - m_{\pi^0}$ effect on $\Gamma_\rho$	$+4.09$	$+4.02$
$m_{\rho^\pm} - m_{\rho_{\text{bare}}^0}$	$+0.20^{+0.27}_{-0.19}$	$+0.11^{+0.19}_{-0.11}$
$\pi\pi\gamma$ , electrom. decays	$-5.91 \pm 0.59$	$-6.39 \pm 0.64$
Total	$-16.07 \pm 1.22$	$-16.70 \pm 1.23$
	$-16.07 \pm 1.85$	

M. Davier, et al, Eur. Phys. J. C 66, 127-136 (2010)

Uncertainties quoted to incorporate the deviation from

V. Cirigliano, G. Ecker and H. Neufeld, JHEP. 08 (2002) 002

$\Delta a_\mu^{\text{HVP,LO}} _{G_{\text{EM}}(t)}$ ( $\times 10^{-11}$ )	ChPT $\mathcal{O}(p^4)$	$R_\chi T$ $\mathcal{O}(p^4)$	$R_\chi T$ $\mathcal{O}(p^6)$
	-10	$-15.9^{+5.7}_{-16.0}$	$-76 \pm 46$

J. A. Miranda and P. Roig, Phys. Rev. D 102, 114017 (2020)

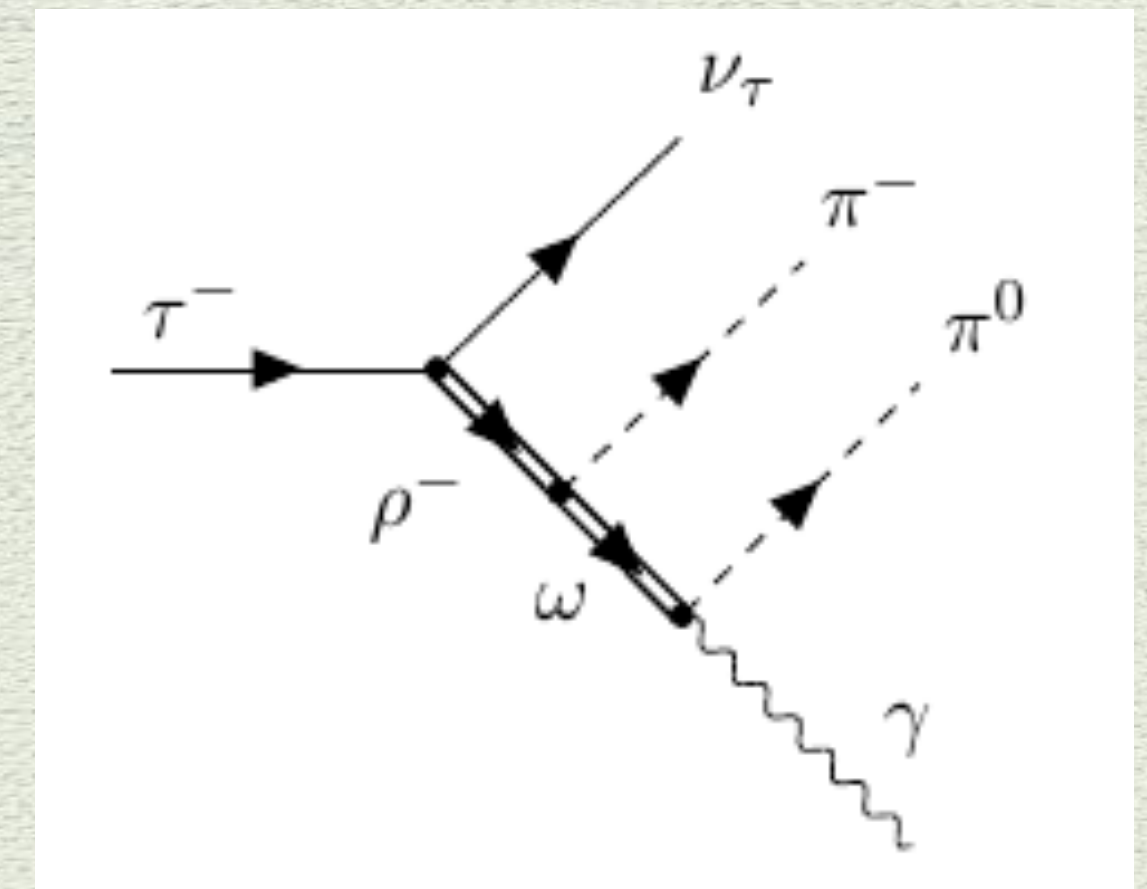
### VMD approach

A. Flores-Tlalpa, G. Lopez Castro and GT,  
Phys. Rev. D 72, 113003 (2005)

F. Flores-Baez, A. Flores-Tlalpa, G. Lopez Castro and GT,  
Phys. Rev. D 74, 071301 (2006)

Flores-Tlalpa, F. Flores-Baez, G. Lopez Castro and GT,  
Nucl. Phys. B Proc. Suppl. 169, 250-254 (2007)

G. Lopez Castro, P. Roig and GT,  
Nucl. Part. Phys. Proc. 260, 70-74 (2015)



In VMD this model dependent (MD) channel found to be the main responsible for the deviation from purely SI result

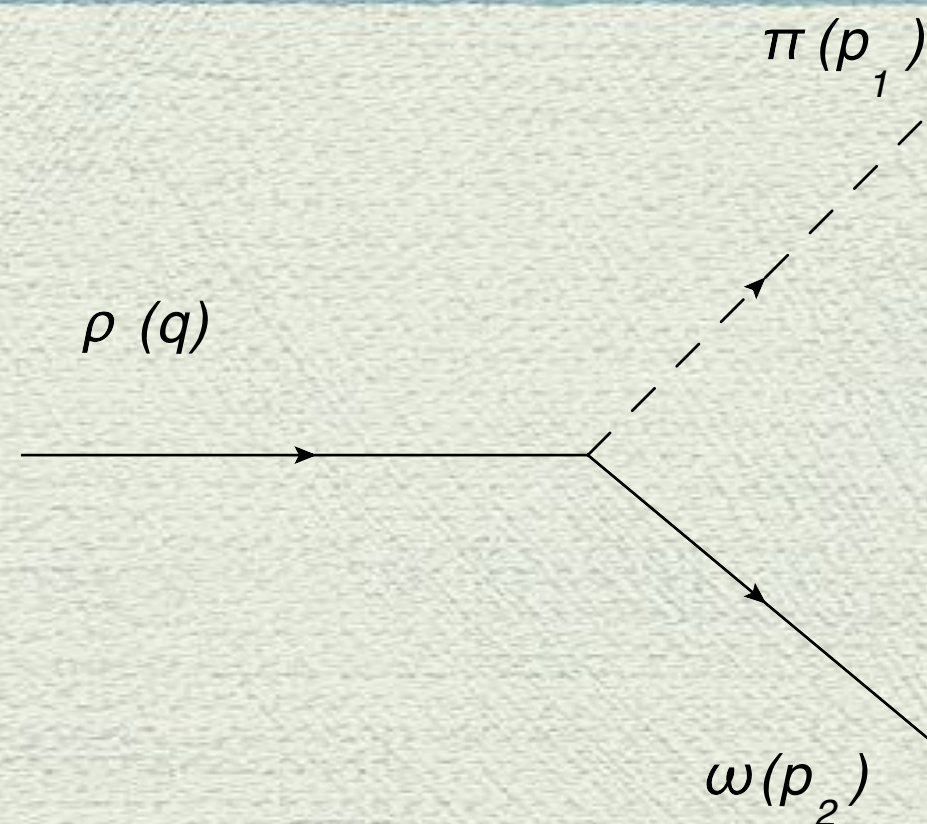
Not considered in estimates using BELLE data (removed but its interference)

# In this work

- ◆ General description of the two poles carrying different energy
- ◆ Explore the  $e^+e^- \rightarrow \pi^0\pi^0\gamma$  differential cross section, to identify the double pole resonant enhancement features
- ◆ Perform an analysis of the parameters involved, in base of experimental data available, to identify their robustness
- ◆ Perform an analysis of the  $\tau^- \rightarrow \pi^-\pi^0\nu_\tau\gamma$  omega channel, to exhibit the analogies to the previous case
- ◆ Analyse the muon g-2 ISB correction from  $\tau^- \rightarrow \pi^-\pi^0\nu_\tau\gamma$
- ◆ Conclusions

# Two poles carrying different energy

L. Esparza-Arellano, Antonio Rojas and GT hep-ph: 2308.02766



$$s \equiv (p_1 + p_2)^2$$

$$s_2 \equiv p_2^2$$

At least one of them off-shell

corresponding form factors

$$f_\rho[s] \equiv \frac{m_\rho^2}{m_\rho^2 - s + im_\rho\Gamma_\rho}$$

$$f_\omega[s_2] \equiv \frac{m_\omega^2}{m_\omega^2 - s_2 + im_\omega\Gamma_\omega}$$

$$s_2 = s + m_\pi^2 - 2\sqrt{s}E_\pi.$$

minimal energy for omega on-shell

$$\sqrt{s} = m_\omega + m_\pi = 0.92 \text{ GeV} \quad m_\rho + \Gamma_\rho.$$

Can we characterize when the maximum enhancement takes place, in terms of the resonance properties, in a particular process?

$\pi^\pm$

$$I^G(J^P) = 1^-(0^-)$$

Mass  $m = 139.57039 \pm 0.00018 \text{ MeV}$  ( $S = 1.8$ )  
 Mean life  $\tau = (2.6033 \pm 0.0005) \times 10^{-8} \text{ s}$  ( $S = 1.2$ )

$\rho(770)$

$$I^G(J^{PC}) = 1^+(1^{--})$$

See the review on "Spectroscopy of Light Meson Resonances."  
 T-Matrix Pole  $\sqrt{s} = (761-765) - i(71-74) \text{ MeV}$   
 Mass (Breit-Wigner) =  $775.26 \pm 0.23 \text{ MeV}$   
 Full width (Breit-Wigner) =  $149.1 \pm 0.8 \text{ MeV}$

$\omega(782)$

$$I^G(J^{PC}) = 0^-(1^{--})$$

Mass  $m = 782.66 \pm 0.13 \text{ MeV}$  ( $S = 2.0$ )  
 Full width  $\Gamma = 8.68 \pm 0.13 \text{ MeV}$

# Description of low energy processes

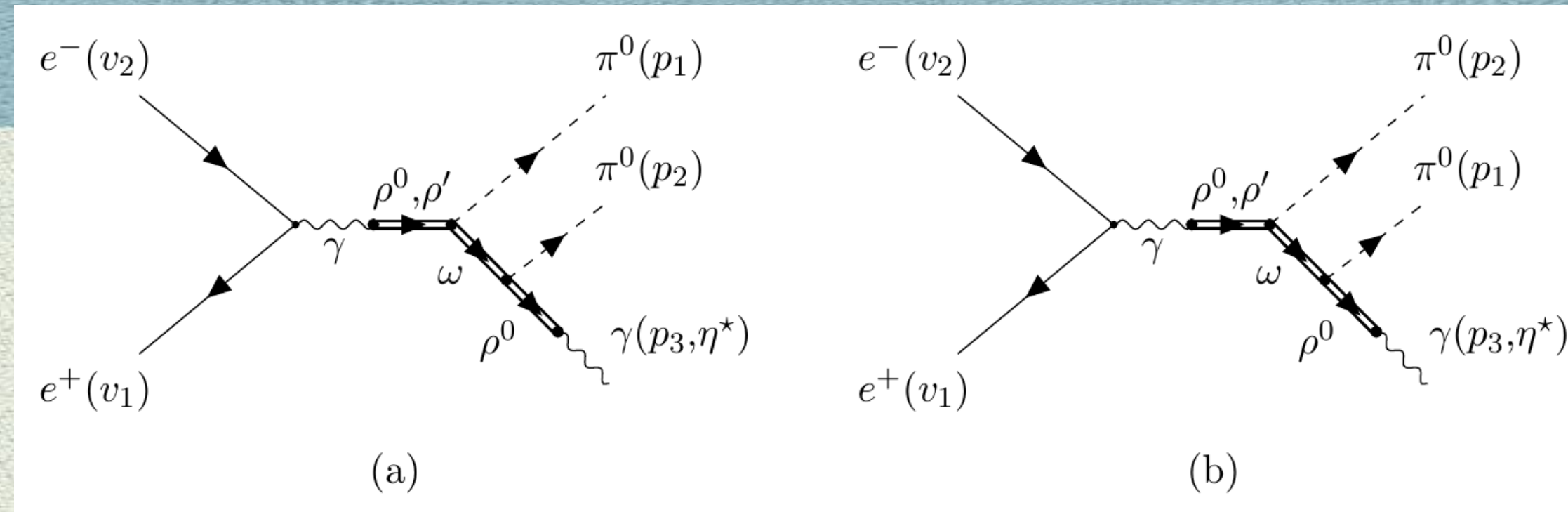
Vector meson dominance approach and effective interactions

$$\begin{aligned} \mathcal{L} = & \sum_{V=\rho, \rho'} g_{V\pi\pi} \epsilon_{abc} V_\mu^a \pi^b \partial^\mu \pi^c + \sum_{V=\rho, \rho'} g_{\omega V\pi} \delta_{ab} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \omega_\nu \partial_\lambda V_\sigma^a \pi^b \\ & + g_{3\pi} \epsilon_{abc} \epsilon^{\mu\nu\lambda\sigma} \omega_\mu \partial_\nu \pi^a \partial_\lambda \pi^b \partial_\sigma \pi^c + \sum_{V=\rho, \rho', \omega} \frac{e m_V^2}{g_V} V_\mu A^\mu. \end{aligned}$$

Consider hadrons as the relevant degrees of freedom at low energies.

Couplings are free parameters to be determined from experiment.

# $e^+e^- \rightarrow \pi^0\pi^0\gamma$ differential cross section



Amplitude

$$\mathcal{M}_{(a)} = \frac{e^2}{q^2} \left( C_{\rho^0} + e^{i\theta} C_{\rho'} \right) D_\omega(q - p_1) \epsilon_{\mu\sigma\epsilon\lambda} q^\sigma (q - p_1)^\epsilon \epsilon_{\alpha\lambda\beta\nu} (q - p_1)^\alpha p_3^\beta \eta^{*\nu} l^\mu,$$

$$C_\rho = \left( \frac{g_{\omega\rho\pi}}{g_\rho m_\omega} \right)^2 f_\rho[s] f_\omega[s_1], \quad C_{\rho'} = \frac{g_{\omega\rho'\pi} g_{\omega\rho\pi}}{g_{\rho'} g_{\rho'} m_\omega^2} f_{\rho'}[s] f_\omega[s_1], \quad l^\mu = -ie\bar{v}(v_1)\gamma^\mu u(v_2),$$

The differential cross section for a given angle of one of the pions emission wrt the collision axis

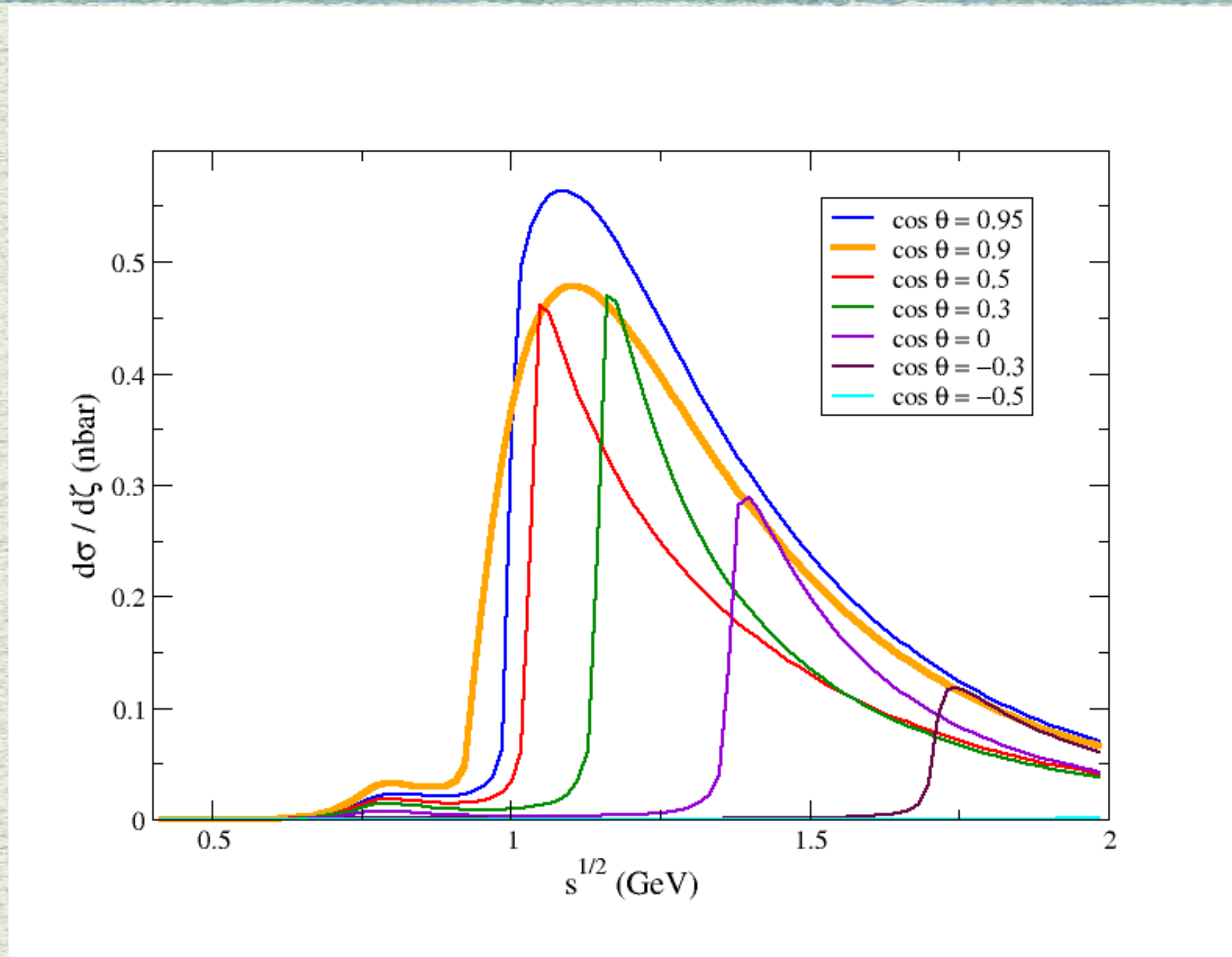
$$\frac{d\sigma(e^+e^- \rightarrow 2\pi^0\gamma)}{d\zeta} = \int \frac{|\bar{\mathcal{M}}|^2}{4(2\pi)^5 |\mathbf{v}_1| |\mathbf{v}_2|} \cdot \frac{\pi |(s_{1+} - s_{1-})(u_{1+} - u_{1-})(t_{1+} - t_{1-})|}{4s \lambda(s, m_\pi^2, u_1) \sqrt{(1 - \xi_1^2)(1 - \eta_1^2)(1 - \zeta_1^2)}},$$

$$\zeta = \cos \theta = \mathbf{p}_1 \cdot \mathbf{v}_1 / |\mathbf{p}_1| |\mathbf{v}_1|$$



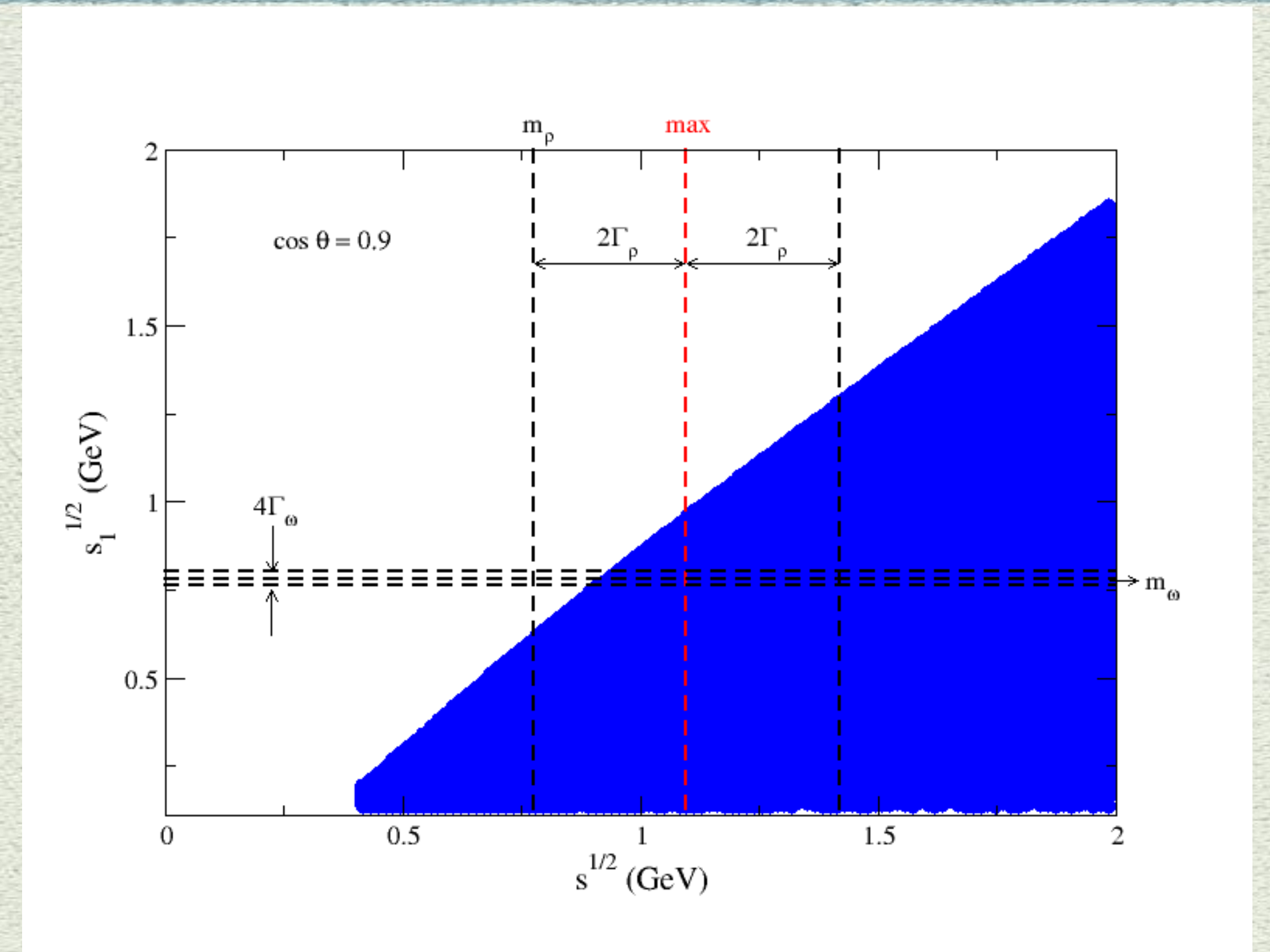
# $e^+e^- \rightarrow \pi^0\pi^0\gamma$ differential cross section

Including only  $\rho$  and the  $\omega$  resonant features



$e^+e^- \rightarrow \pi\pi\gamma$  differential cross section

both  $\rho$  and  $\omega$  particles resonant features combine to give a maximal enhancement

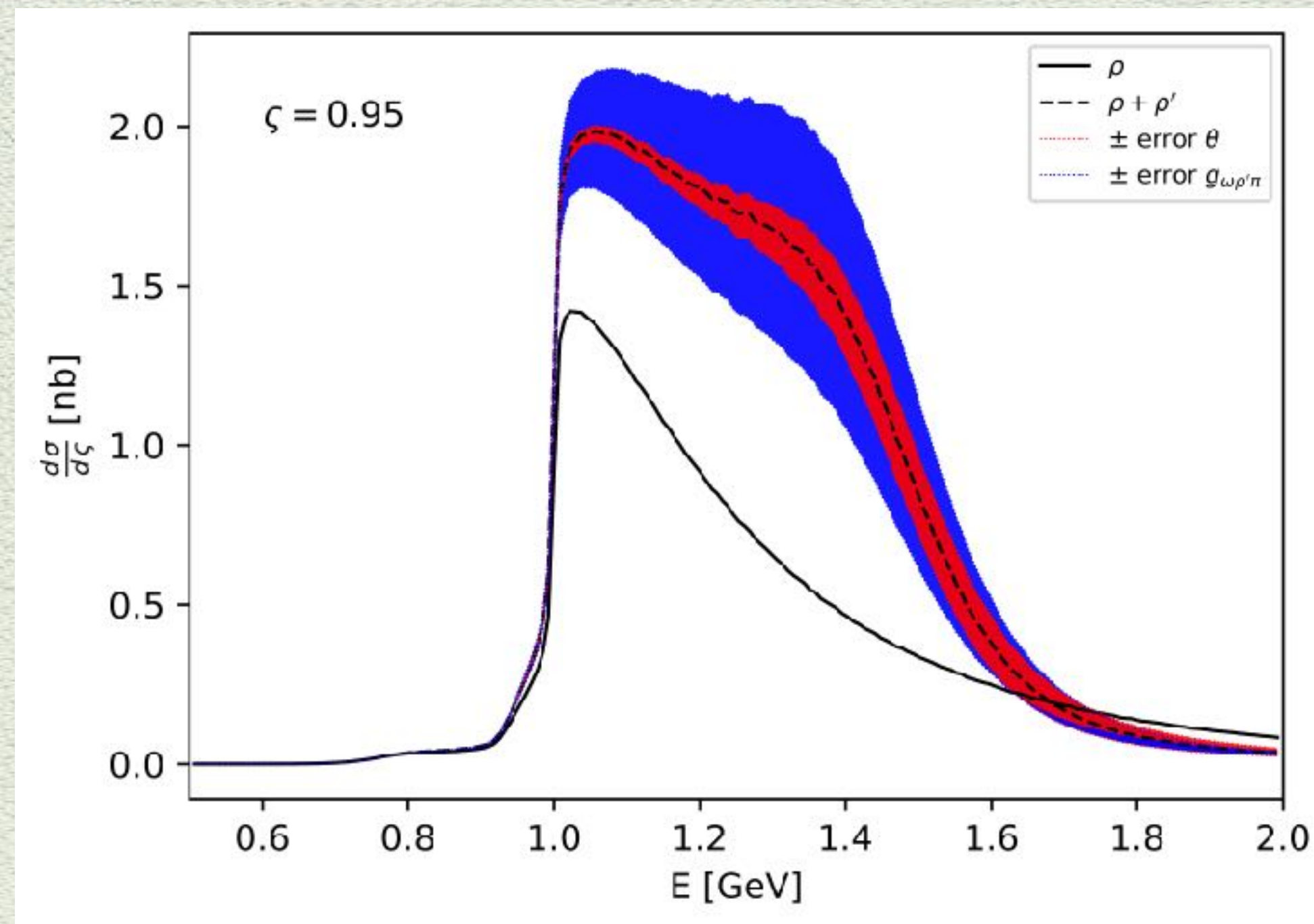


Dalitz region

The resonances around the maximum are twice their corresponding decay width away from its pole mass value

# $e^+e^- \rightarrow \pi^0\pi^0\gamma$ differential cross section

Including both  $\varrho$  and  $\varrho'$  with the  $\omega$  resonant features



Including only  $\varrho$  has small uncertainty (as we show later)

Observable strongly dependent on the  $\varrho'$  parameters  
Measurement of this differential cross section can shed light on the  $\varrho'$  effect

# Parameters analysis. From decay modes to cross section data

G. Avalos, A. Rojas, M Sanchez, GT  
Phys Rev D 107 056006 (2023)

We minimize the function

$$\chi^2(\theta) = \sum_{i=1}^N \frac{(y_i - \mu(x_i; \theta))^2}{E_i^2},$$

considering the couplings as free parameters, for the following data:

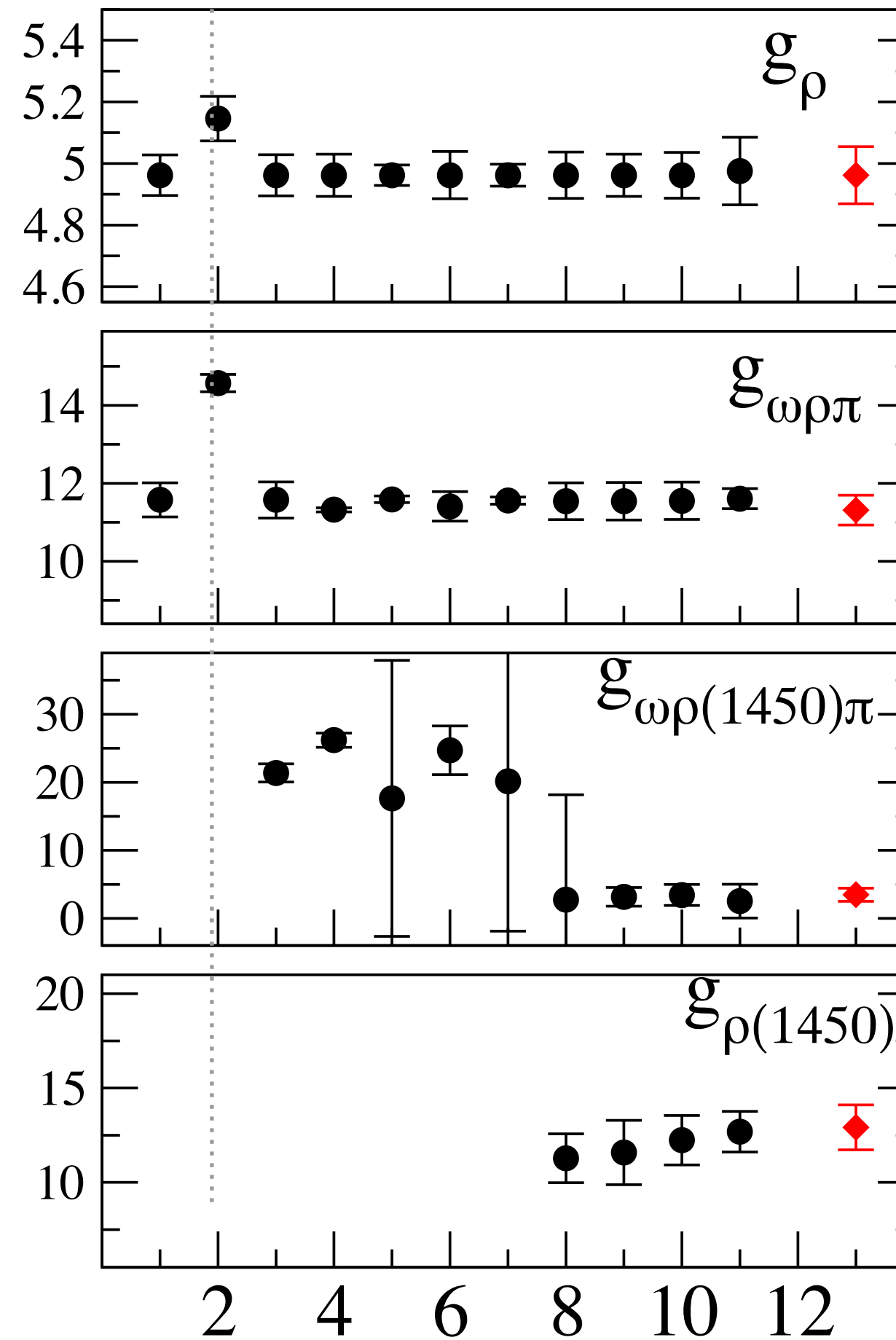
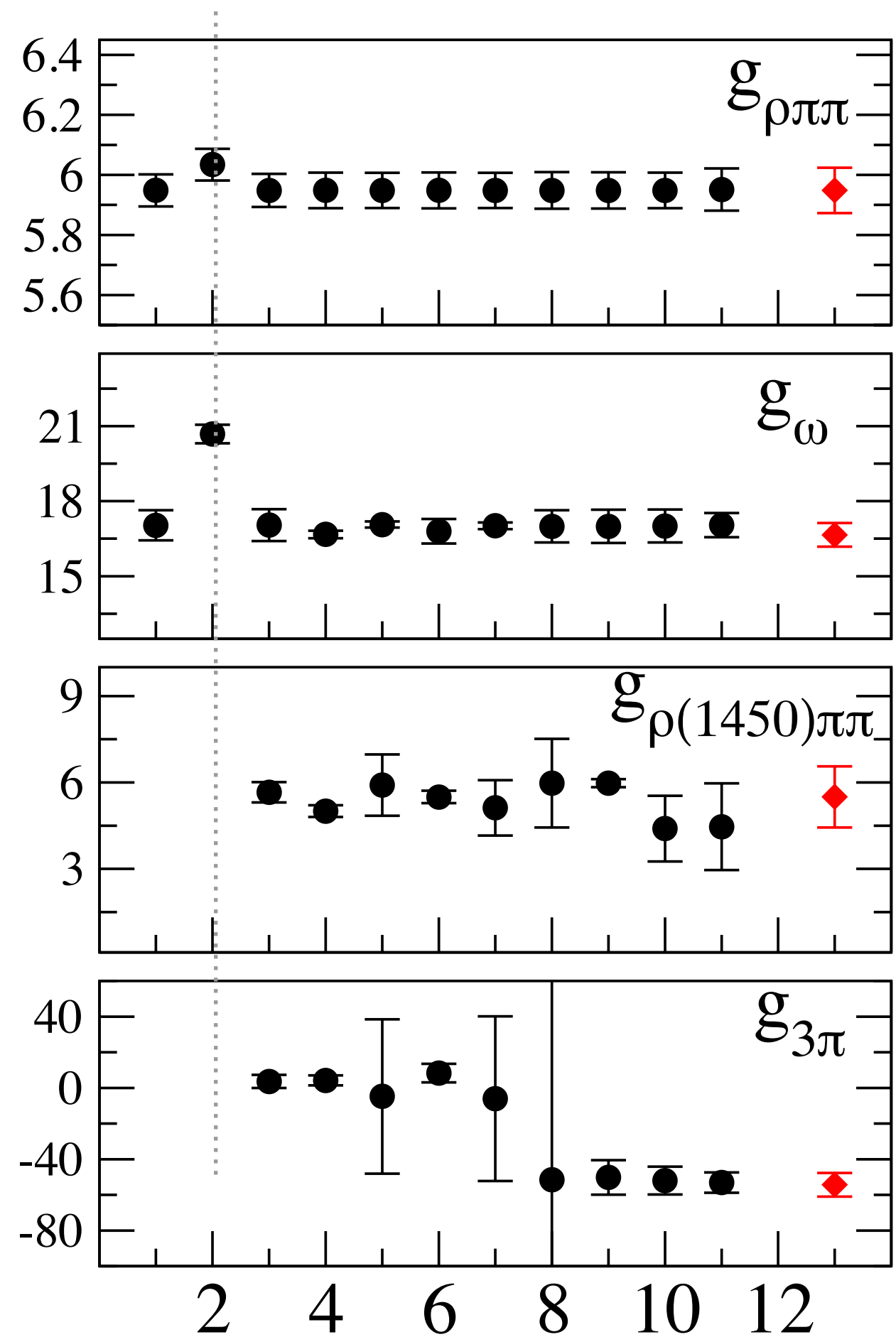
(a) 10 decay modes:  $\rho \rightarrow \pi\pi$   $\rho^0 \rightarrow e^+e^-, \mu^+\mu^-$   $\omega \rightarrow e^+e^-, \mu^+\mu^-$   
 $\rho \rightarrow \pi\gamma$   $\omega \rightarrow \pi^0\gamma$   $\pi^0 \rightarrow \gamma\gamma$ .

11 decay modes: (a) +  $\omega \rightarrow 3\pi$

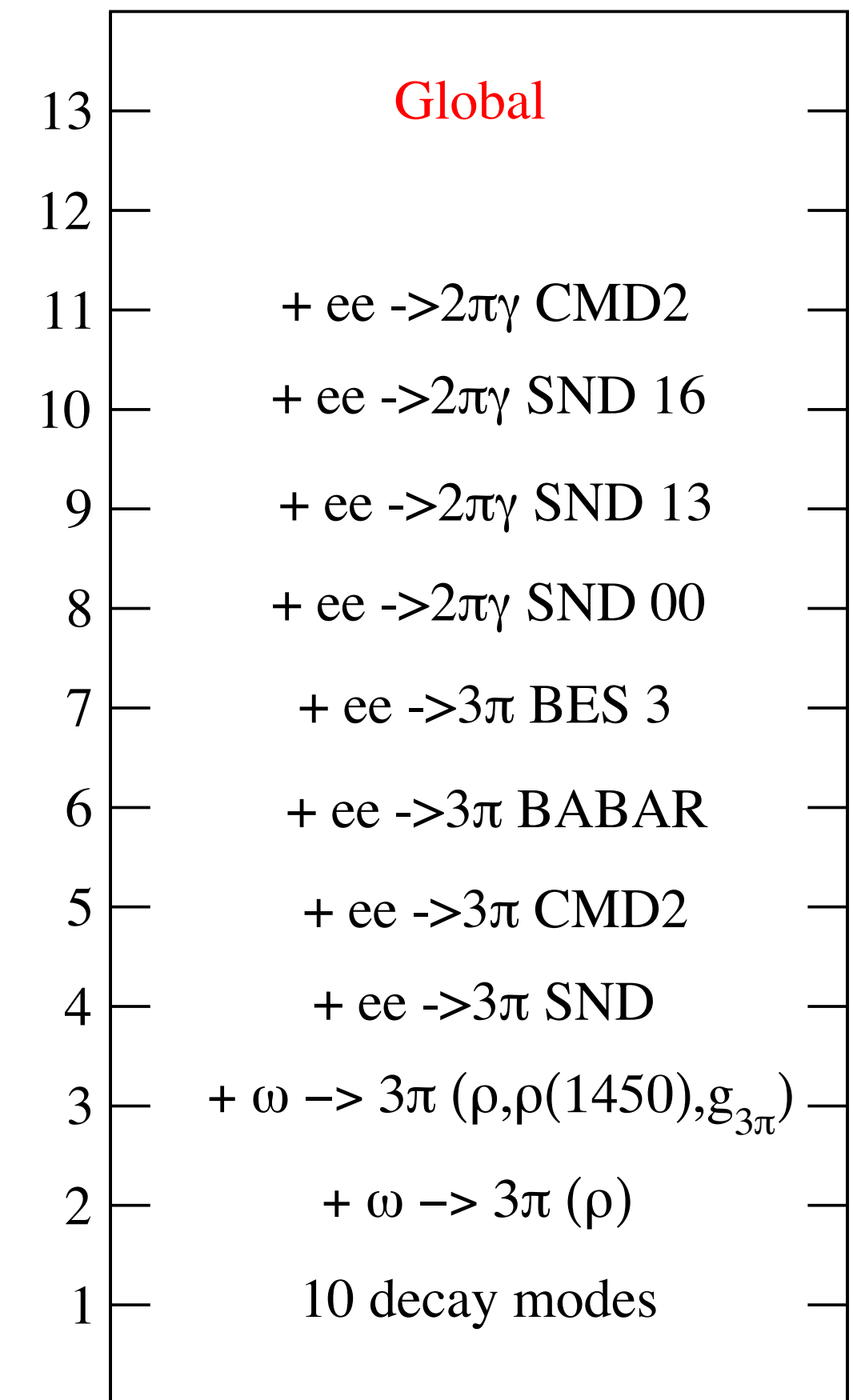
(b) 11 decay modes +  $e^+e^- \rightarrow \pi^0\pi^0\gamma$  SND (00), (13), (16), CMD2

(c) 11 decay modes +  $e^+e^- \rightarrow 3\pi$  SND, BABAR, CMD2, BES 3

# Couplings behavior for individual data



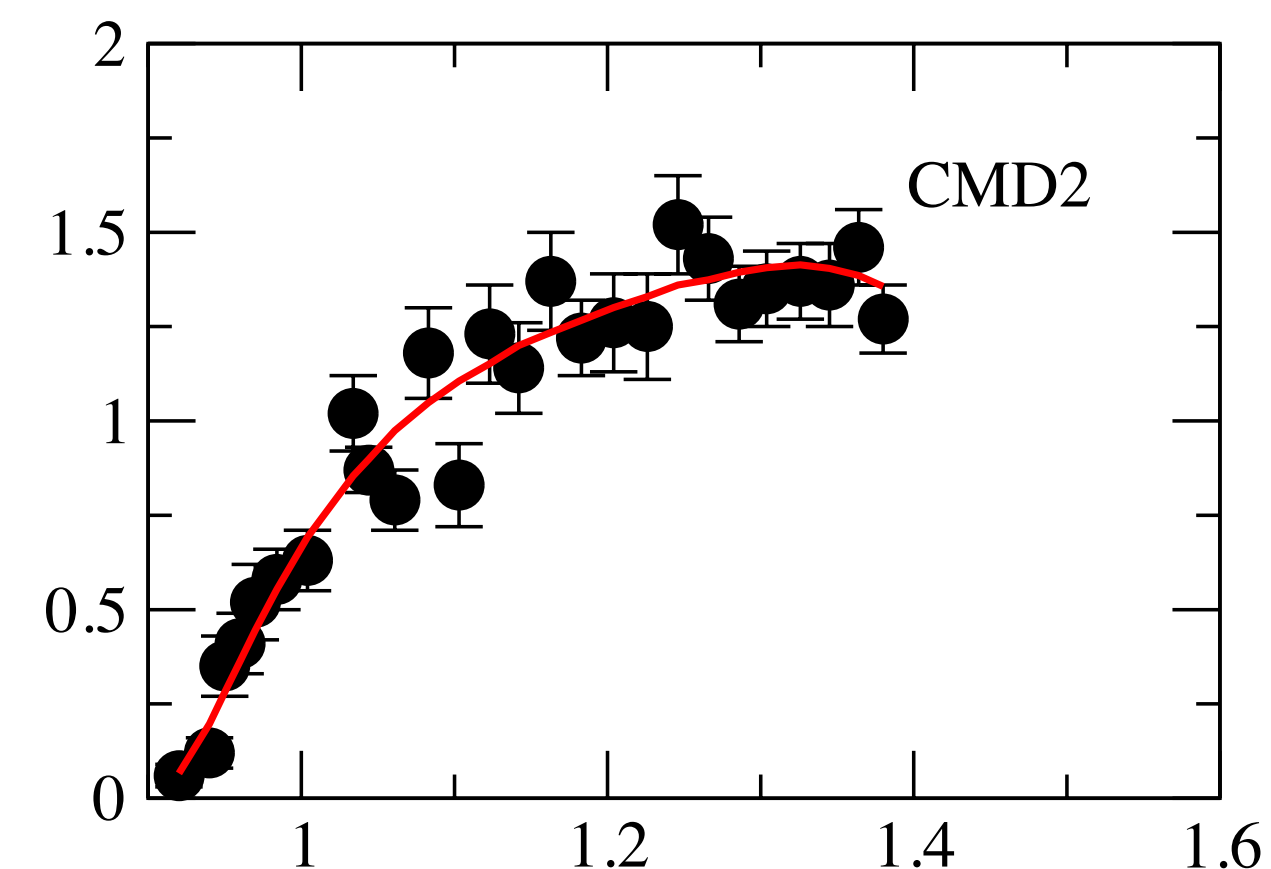
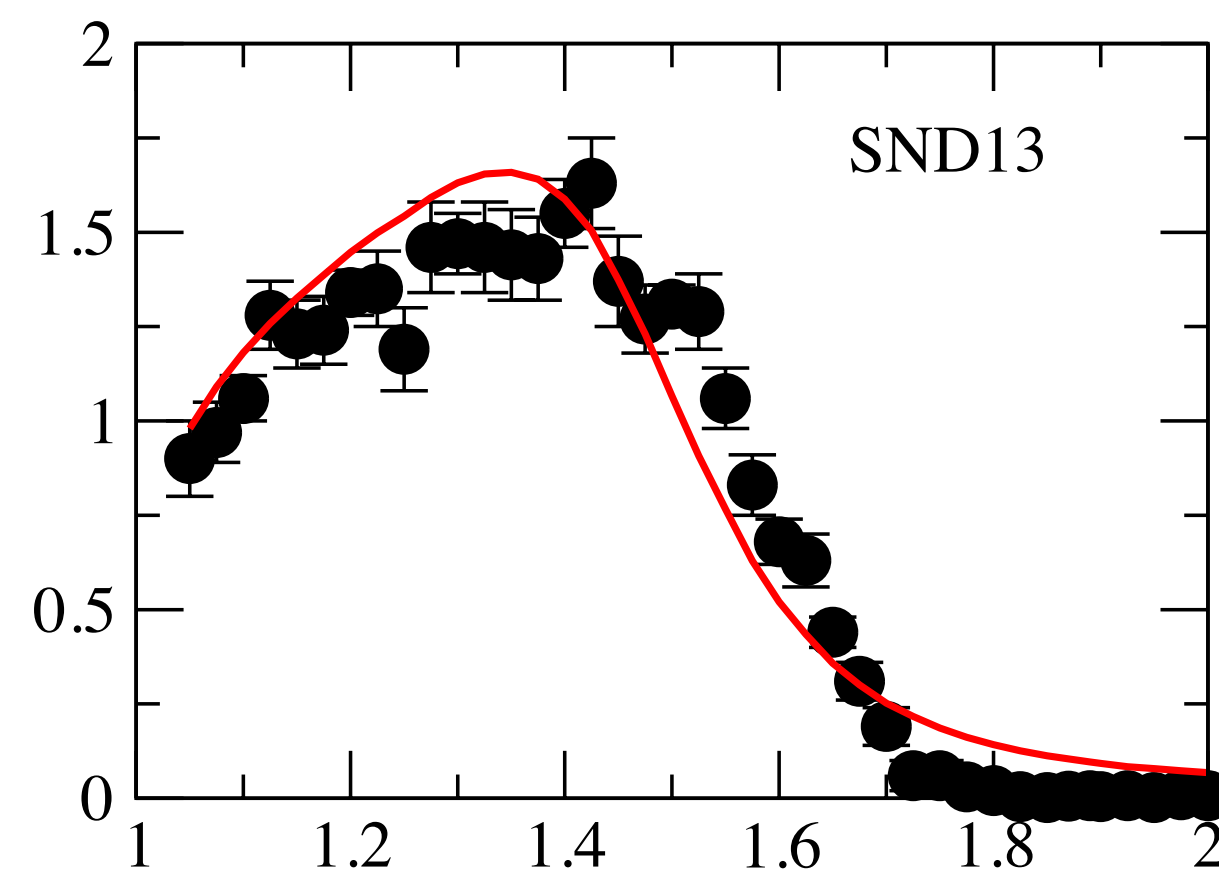
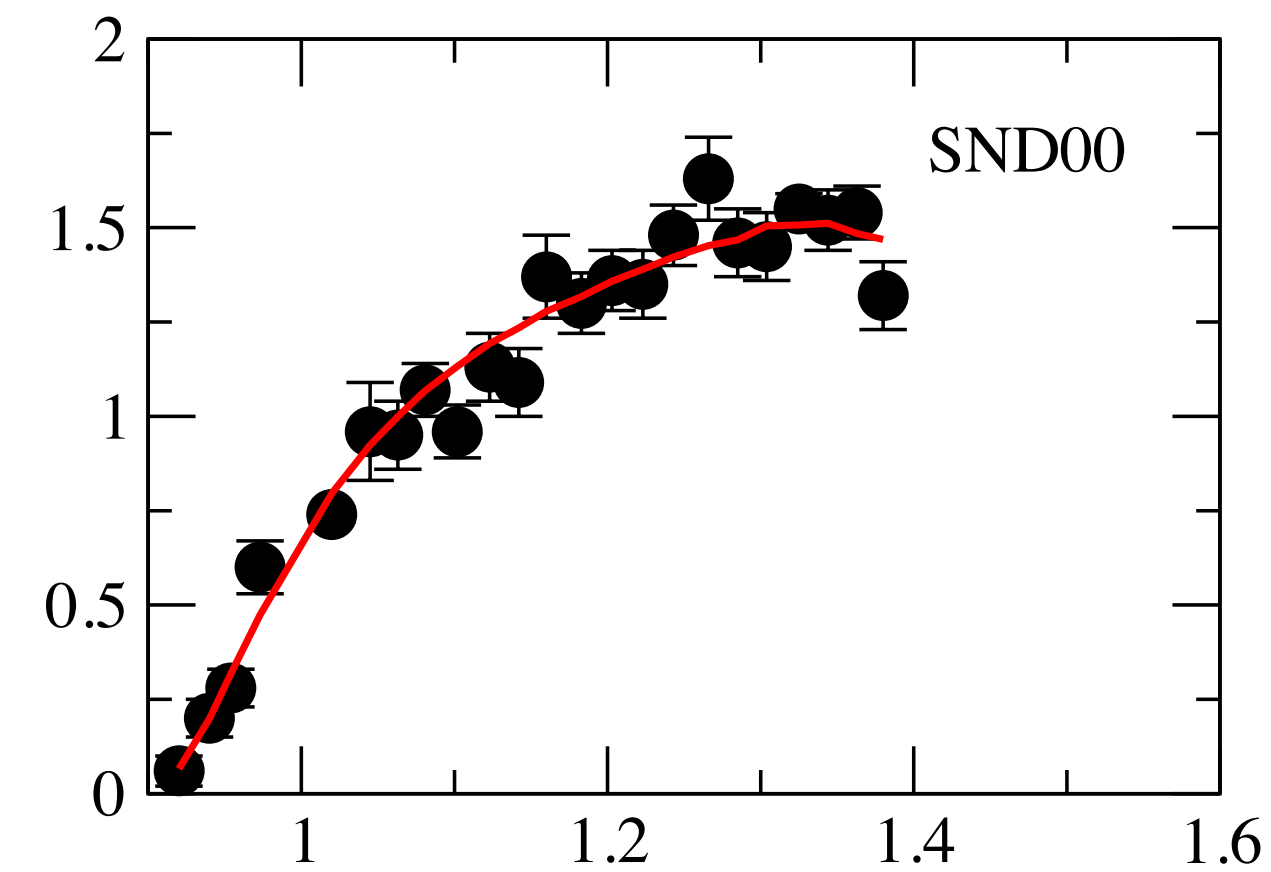
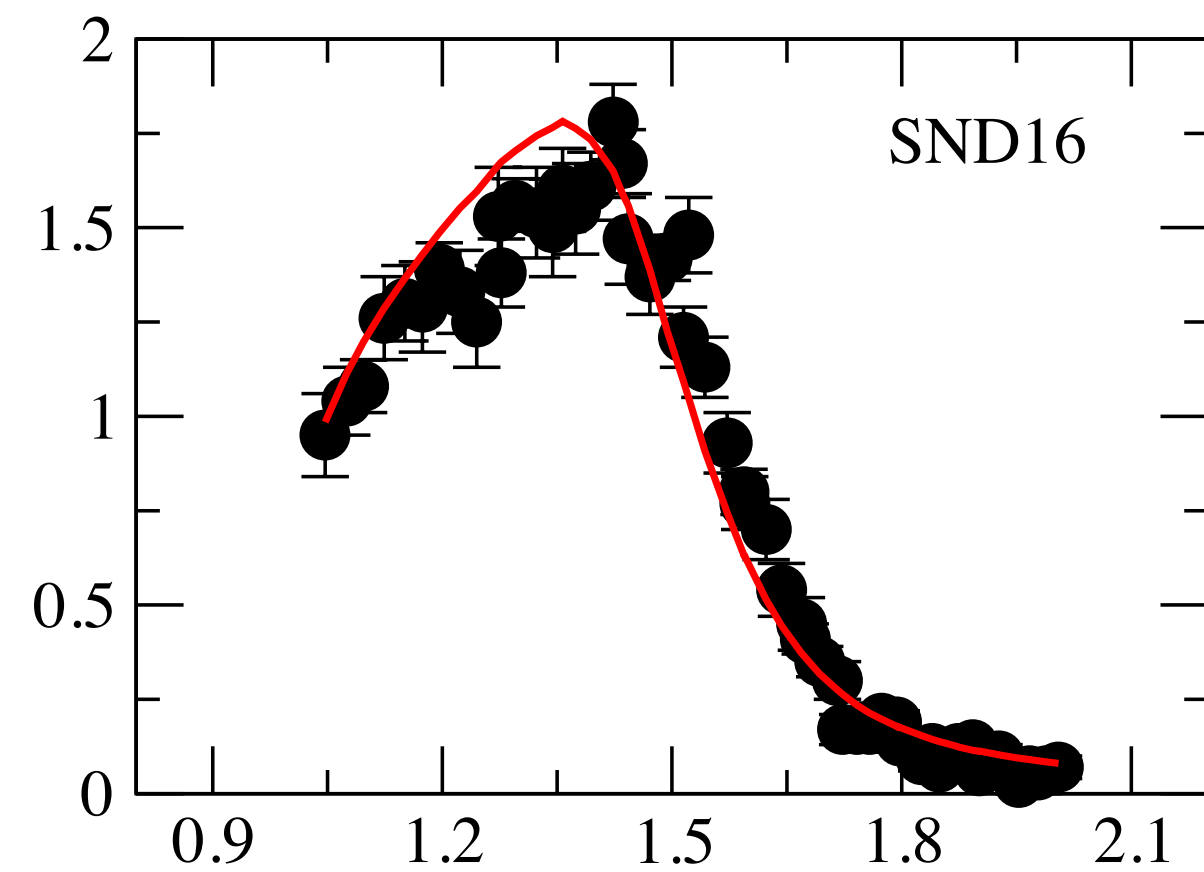
X axis labels



# Cross section description



$\sigma(e^+e^- \rightarrow 2\pi\gamma)$  (nb)



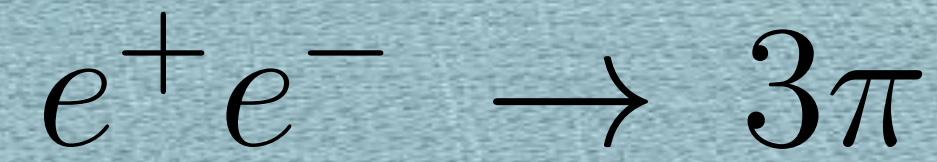
- M. N. Achasov, et al.,  
• Phys. Rev. D 94, 112001 (2016).

- M. N. Achasov, et al.,  
• Phys. Lett. B 486, 29 (2000).

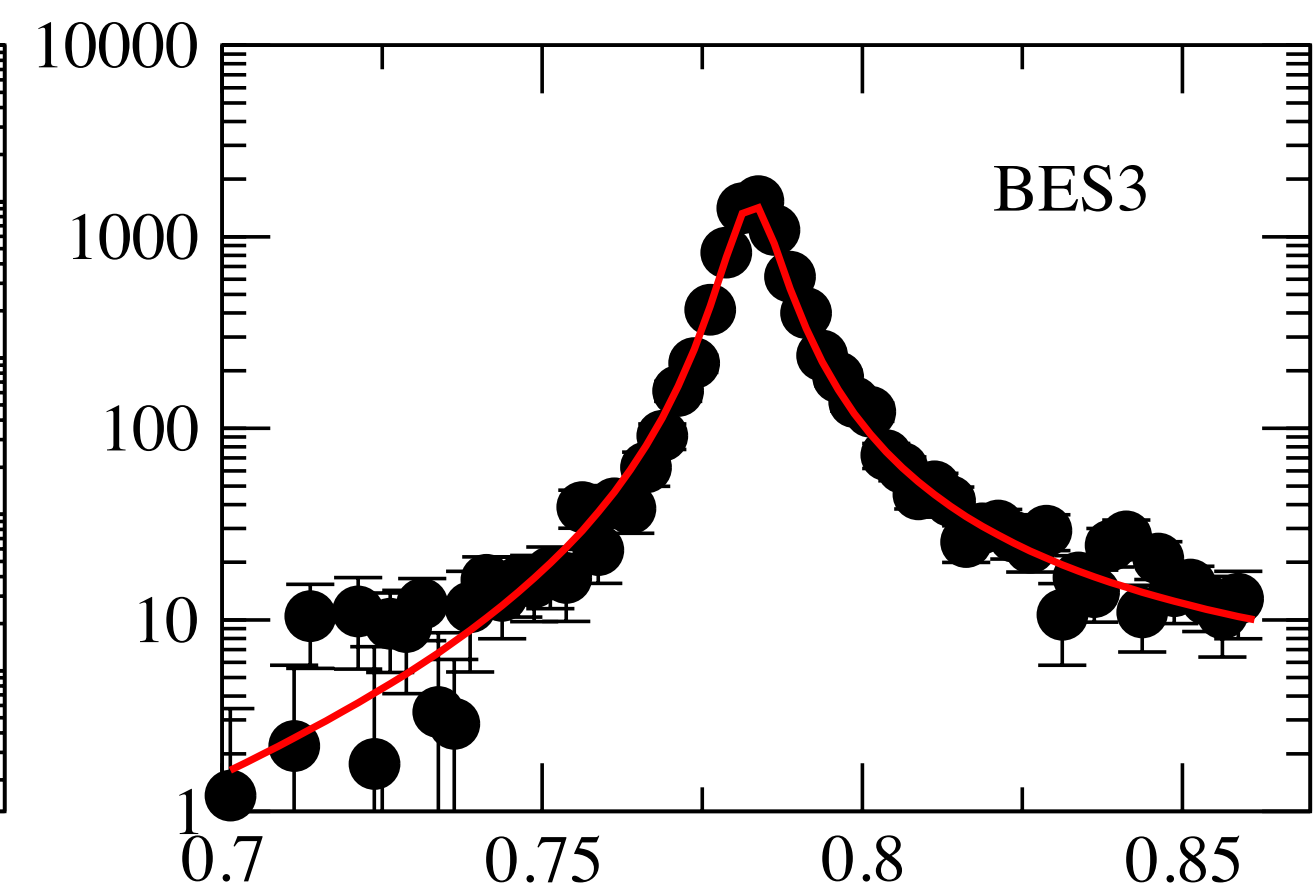
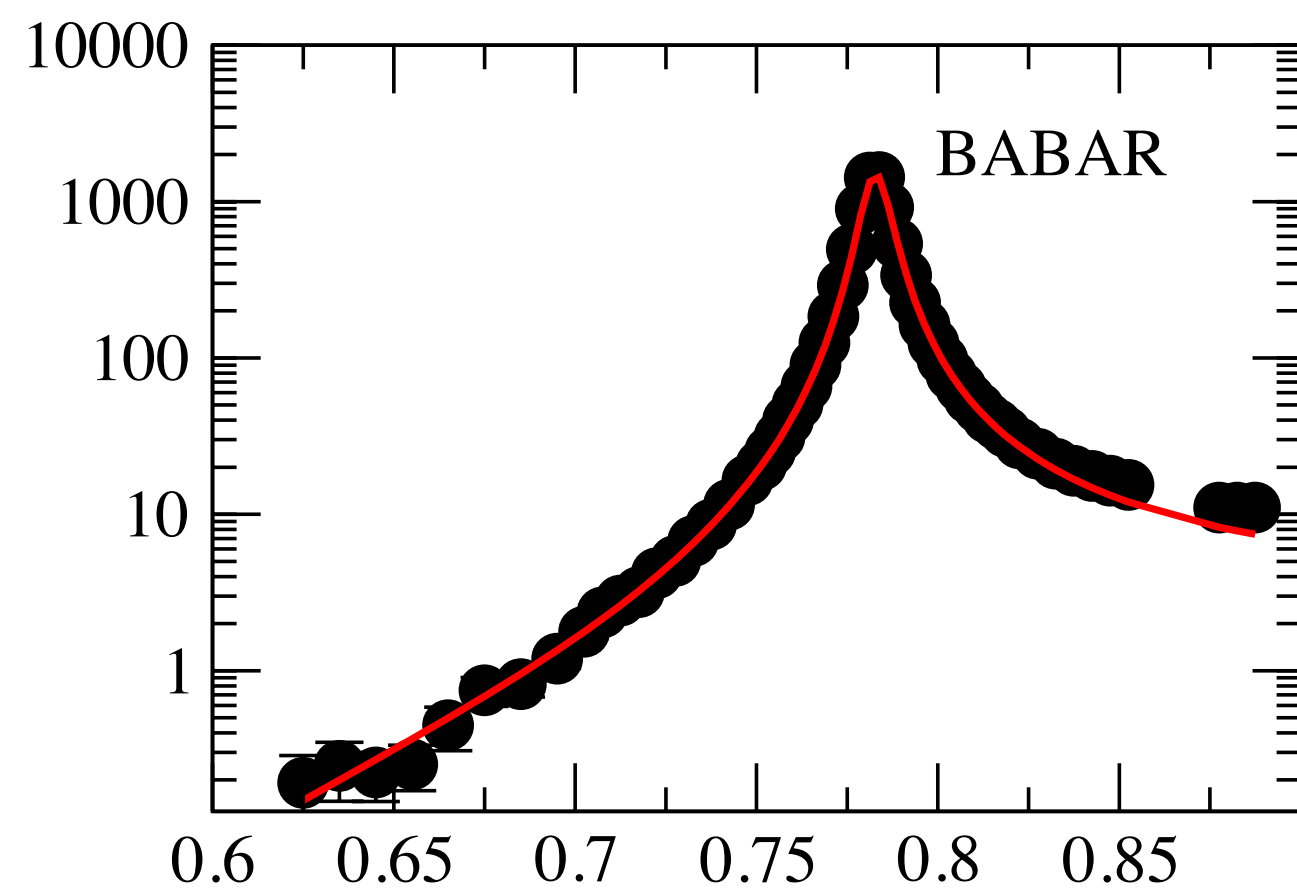
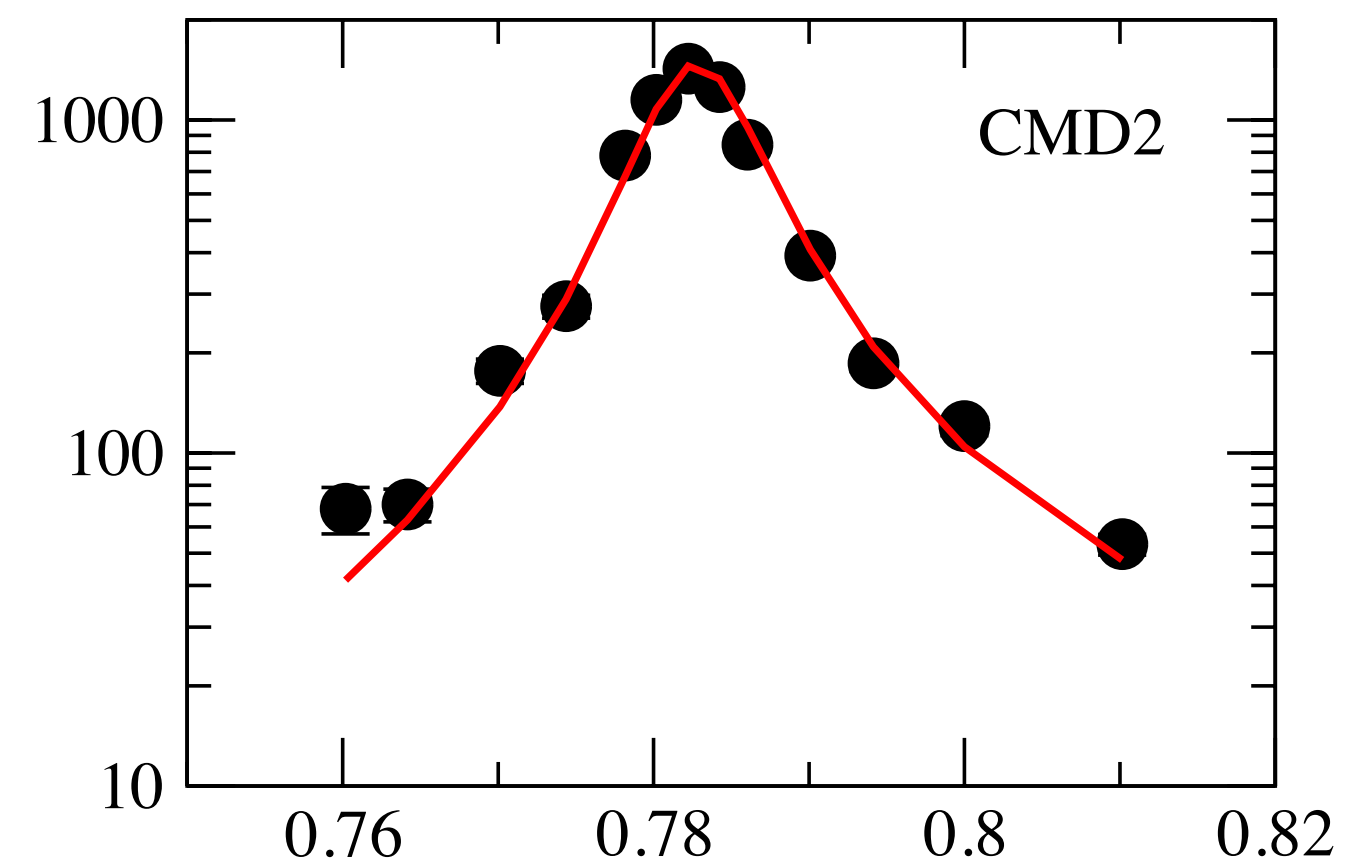
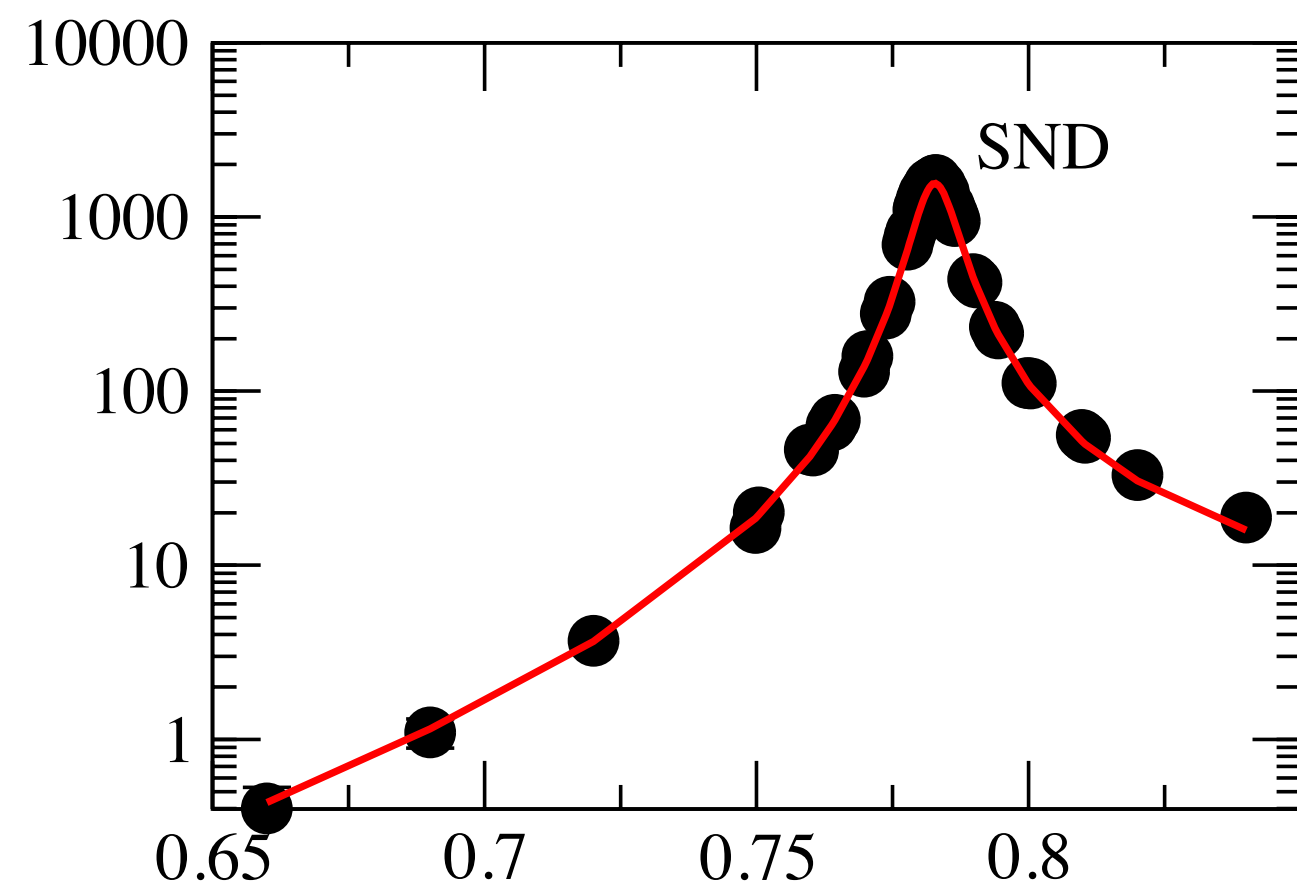
- M. N. Achasov, et al.,  
• Phys. Rev. D 88, 054013 (2013).

- R. R. Akhmetshin et al. (CMD-2 Collaboration),  
• Phys. Lett. B 562, 173 (2003).

# Cross section description



$\sigma(e^+ e^- \rightarrow 3\pi)$  (nb)



- M. N. Achasov, et al.,  
• Phys. Rev. D 68, 052006 (2003).
- R. R. Akhmetshin et al. (CMD-2 Collaboration),  
• Phys. Lett. B 578, 285 (2004).
- J. P. Lees et al. (BABAR Collaboration),  
• Phys. Rev. D 104, 112003 (2021).
- M. Ablikim et al. (BESIII Collaboration),  
• arXiv:1912.11208.

# $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau \gamma$

## General description

The general structure of the amplitude

$$\mathcal{M}_T = eG_F V_{ud}^* \epsilon^{*\mu} \left[ F_\nu \bar{u}(q) \gamma^\nu (1 - \gamma_5) (m_\tau + \not{p} - \not{k}) \gamma_\mu u(p) + (V_{\mu\nu} - A_{\mu\nu}) \bar{u}(q) \gamma^\nu (1 - \gamma_5) u(p) \right],$$

where

$$V_{\mu\nu} = -f_+[t'] \frac{p_{-\mu}}{p_- \cdot k} (\bar{Q} - k)_\nu - f_+[t'] g_{\mu\nu} + \frac{f_+[t'] - f_+[t]}{k \cdot k_-} k_{-\mu} \bar{Q}_\nu + \hat{V}_{\mu\nu},$$

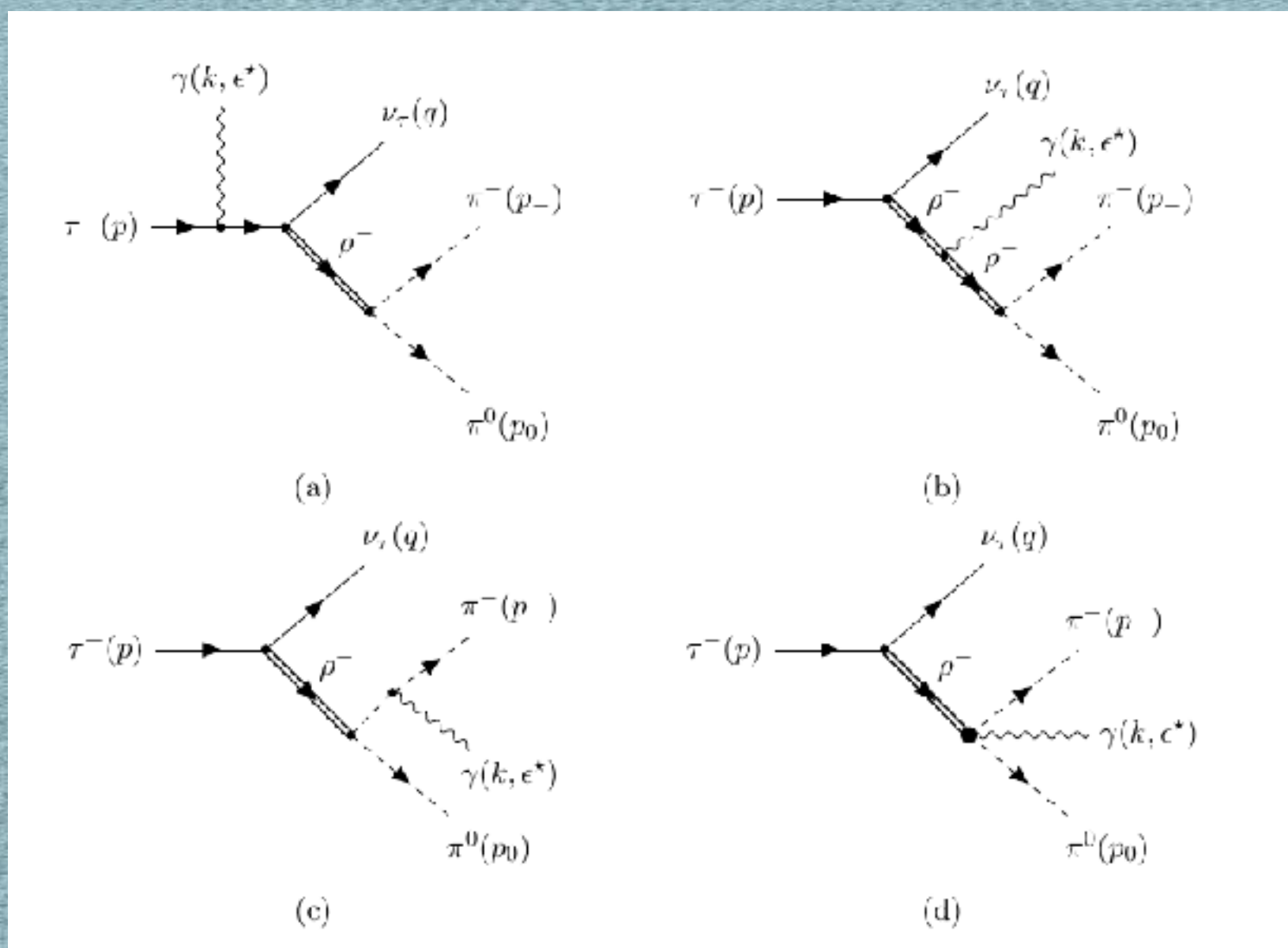
$$\hat{V}_{\mu\nu} \equiv v_1 p_- \cdot k F_{\mu\nu}(p_-) + v_2 p_0 \cdot k F_{\mu\nu}(p_0) + v_3 p_0 \cdot k p_- \cdot k L_\mu(p_-, p_0) p_{-\nu} + v_4 p_0 \cdot k p_- \cdot k L_\mu(p_-, p_0) k_{+\nu},$$

$$L_\mu(a, b) \equiv \frac{a_\mu}{a \cdot k} - \frac{b_\mu}{b \cdot k},$$

$$L(a, b) = L_\mu(a, b) \epsilon^\mu,$$

$$F_{\mu\nu}(a) \equiv g_{\mu\nu} - \frac{a_\mu k_\nu}{a \cdot k}.$$

# The $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau \gamma$ soft photon approximation



Diagrams SI

Model independent amplitude (MI) ( up to  $O(k^0)$ )

$$\mathcal{M}_{Low} = eG_F V_{ud}^* \left\{ f_+[t] L(p, p_-) \bar{Q}_\nu + 2 p_0 \cdot k L(p_0, p_-) \frac{df_+[t]}{dt} \bar{Q}_\nu - \frac{f_+[t]}{2 p \cdot k} \epsilon^\mu \left[ F_{\mu\nu}(\bar{Q}) \bar{Q} \cdot k + i \bar{Q}^\alpha k^\beta \epsilon_{\nu\alpha\beta\mu} \right] - f_+[t] \epsilon^\mu F_{\mu\nu}(p_-) \right\} l^\nu,$$

The  $\rho$  MDM fig. (b) introduces model dependent terms

$$v_1 = -v_2 = \beta_0 \frac{[f_+(t') - f_+(t)]}{2 k_- \cdot k},$$

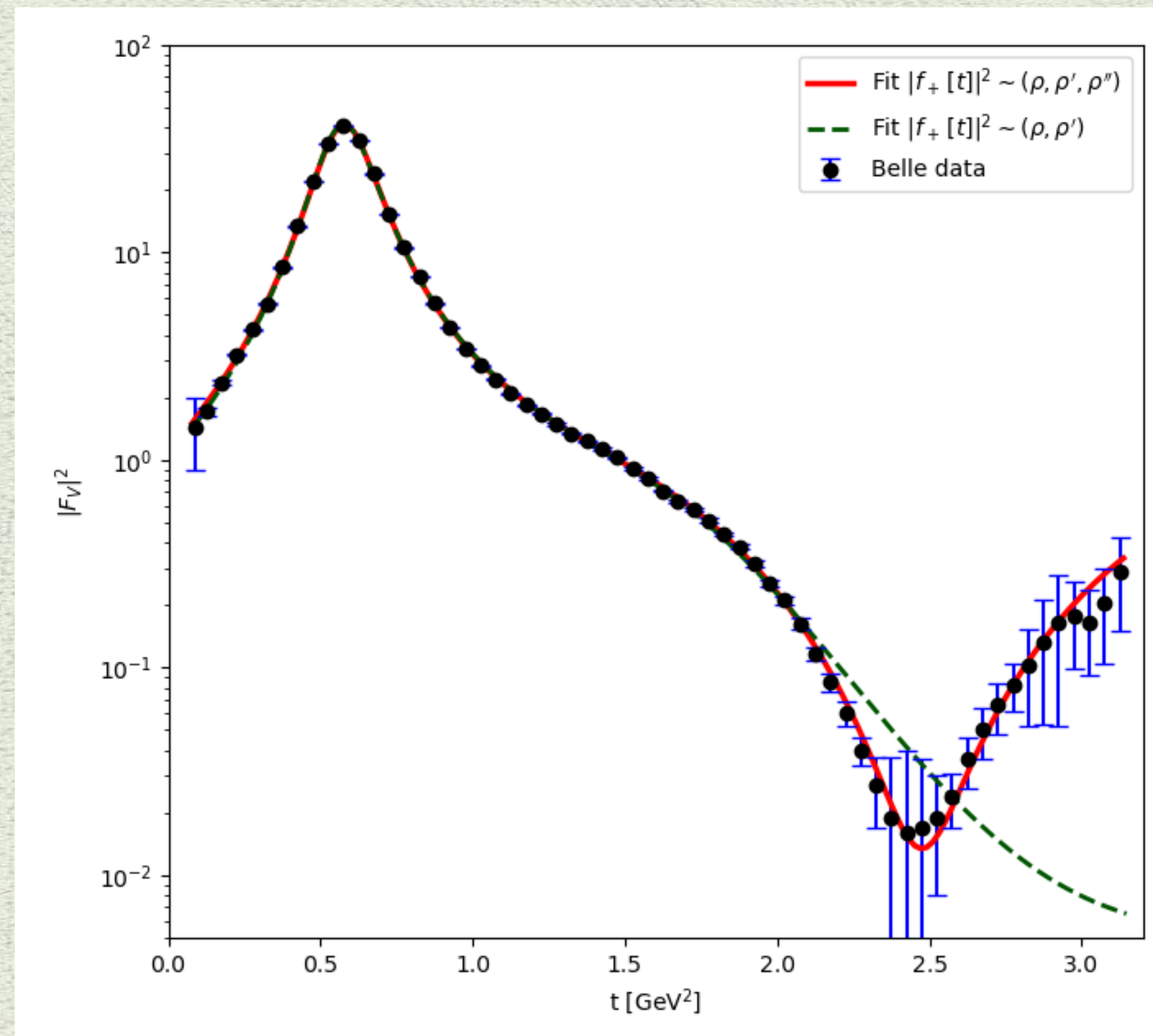
$$v_3 = 0,$$

$$v_4 = 2 \left( \frac{\beta_0}{2} - 1 \right) \frac{(1 + i\gamma) [f_+(t') - f_+(t)]}{m_\rho^2 k_- \cdot k}.$$

Adding these two contributions is known as the structure independent amplitude (SI)



# The vector form factor



Fit to Belle data

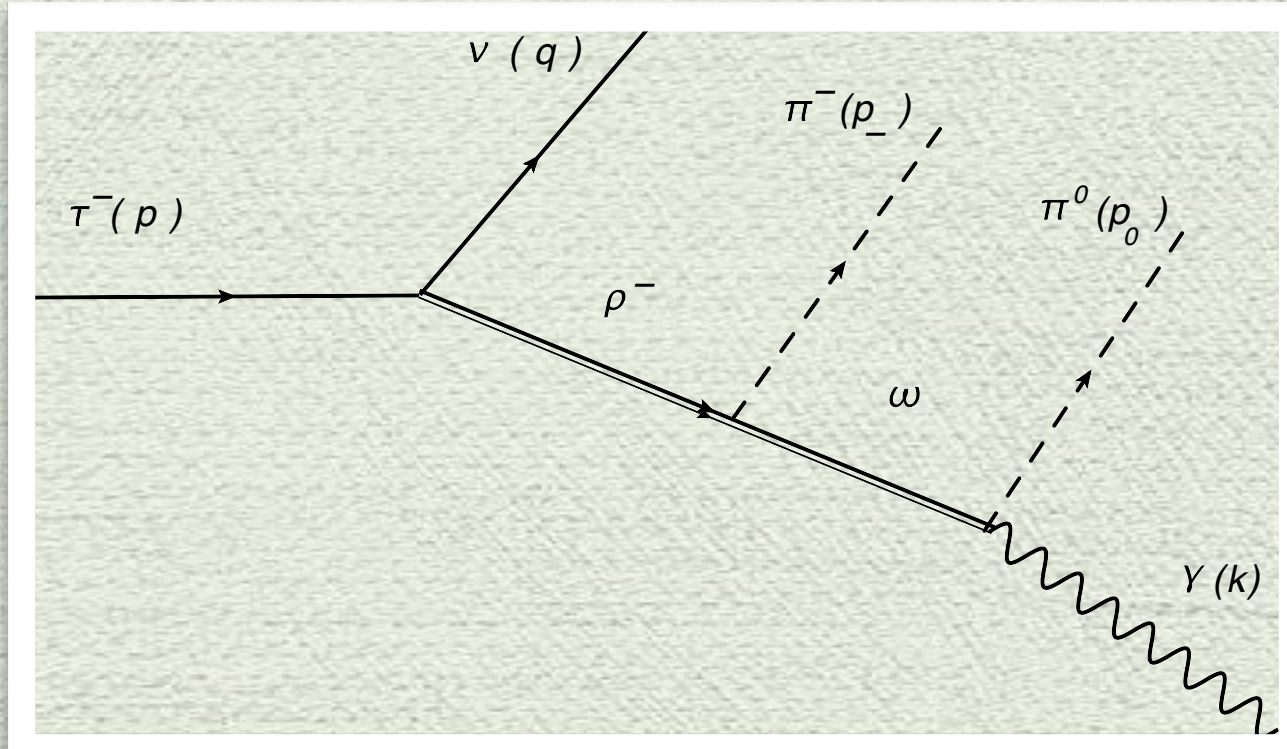
$$f_+[t] = \frac{1}{1 + \beta + \gamma} \left\{ f_\rho[t] + \beta f_{\rho'}[t] + \gamma f_{\rho''}[t] \right\},$$

Parameter	Value	Parameter	Value
$m_\rho$	0.7747 GeV	$\Gamma_\rho$	0.14612 GeV
$m_{\rho'}$	1.3832 GeV	$\Gamma_{\rho'}$	0.5653 GeV
$m_{\rho''}$	1.868 GeV	$\Gamma_{\rho''}$	0.3941 GeV
$B_0$	-0.4028	$f_b$	1.1321
$G_0$	-0.1725	$f_g$	$4.3756 \times 10^{-8}$

TABLE II: Parameters obtained from a fit to the Belle data form factor  $f_+[t]$ .

*Phys.Rev.D* 78 (2008) 072006

# The $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau \gamma$ omega channel



Model dependent amplitude  $O(k)$

$$\mathcal{M}_\omega = eG_F V_{ud}^* \frac{G_\rho}{\sqrt{2}} \frac{g_{\omega\rho\pi}^2}{g_\rho m_\rho^2 m_\omega^2} f_\omega[(p_0 + k)^2] f_o[t'] \epsilon_{\alpha\sigma\mu}^\lambda \epsilon_{\phi\lambda\chi}^\nu k^\sigma p_0^\alpha (p_0 + k)^\phi p_-^\chi \epsilon^{*\mu} \ell^\nu,$$

Including both  $\rho$  and  $\rho'$  coupled to the  $\omega$   $f_o[t'] \equiv \frac{1}{1 + B_1 e^{i\theta}} \{f_\rho[t'] + B_1 e^{i\theta} f_{\rho'}[t']\}.$

The amplitude set in the general structure form

$$\mathcal{M}_\omega = eG_F V_{ud}^* \epsilon^{*\mu} \hat{V}_{\mu\nu}^{(\omega)} \ell^\nu,$$

with  $v_1^\omega = -C_\omega f_\omega[(p_0 + k)^2] f_o[t'] (p_0 + 2k) \cdot p_0,$

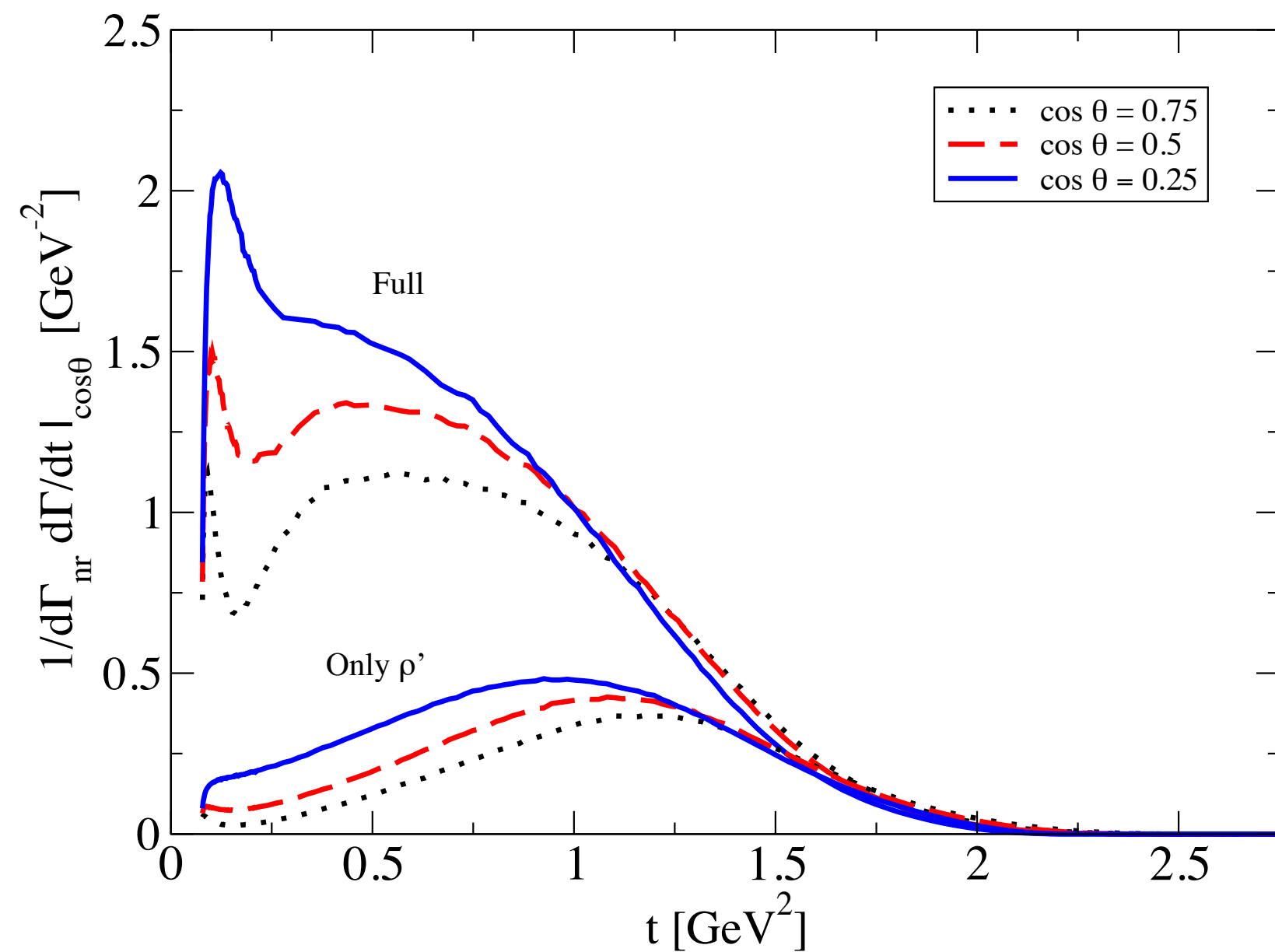
$$v_2^\omega = C_\omega f_\omega[(p_0 + k)^2] f_o[t'] (p_0 + k) \cdot p_-,$$

$$v_3^\omega = C_\omega f_\omega[(p_0 + k)^2] f_o[t'],$$

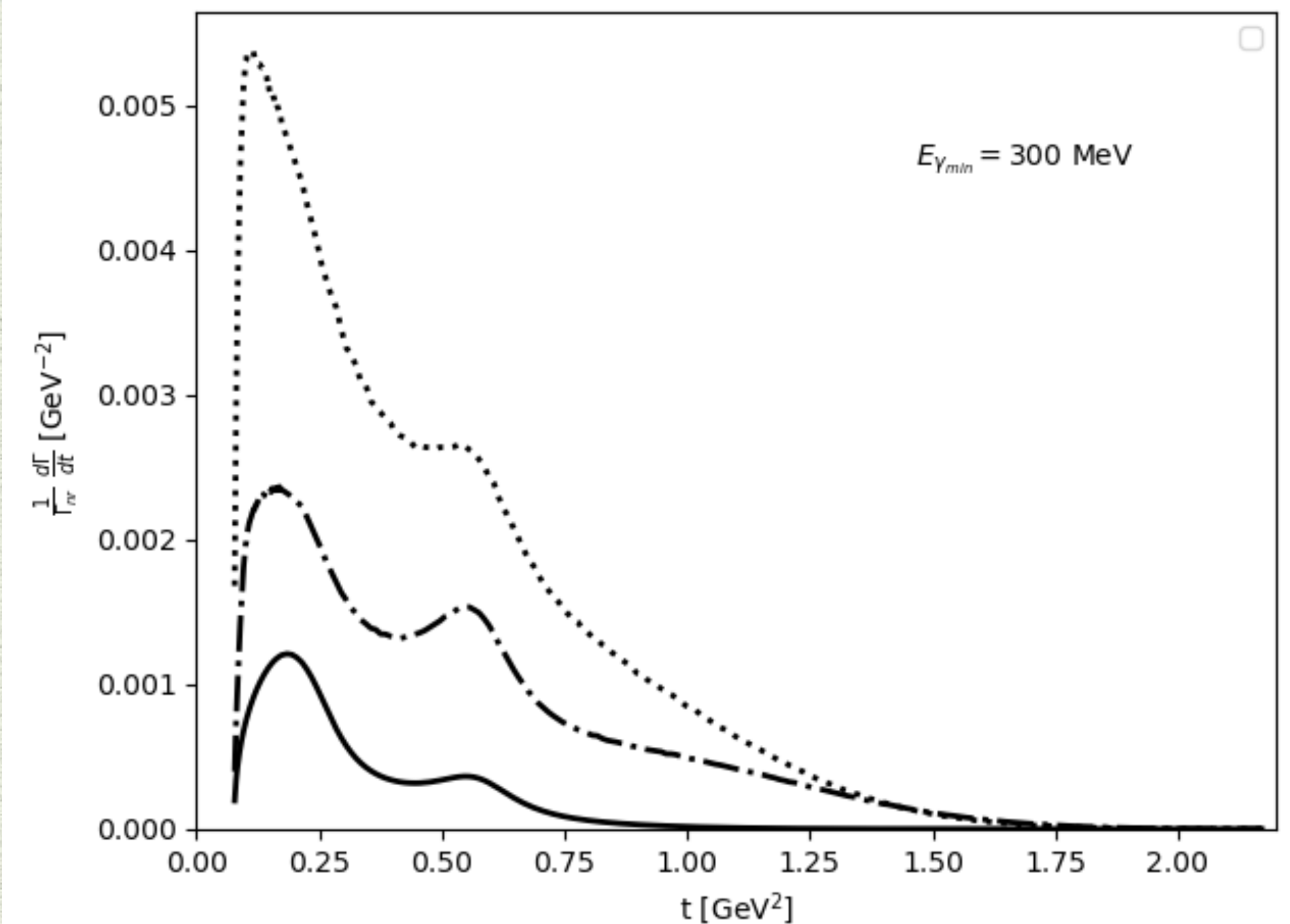
$$v_4^\omega = -C_\omega f_\omega[(p_0 + k)^2] f_o[t'],$$

$$C_\omega = \frac{g_{\omega\rho\pi}^2}{m_\omega^2 g_\rho g_{\rho\pi\pi}}.$$

# The $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau \gamma$ omega channel



Dipion invariant mass distribution due to the  $\omega$  channel, normalized to the non-radiative decay width ( $\Gamma_{nr}$ ) for several angles of the charged pion emission wrt dipion momentum

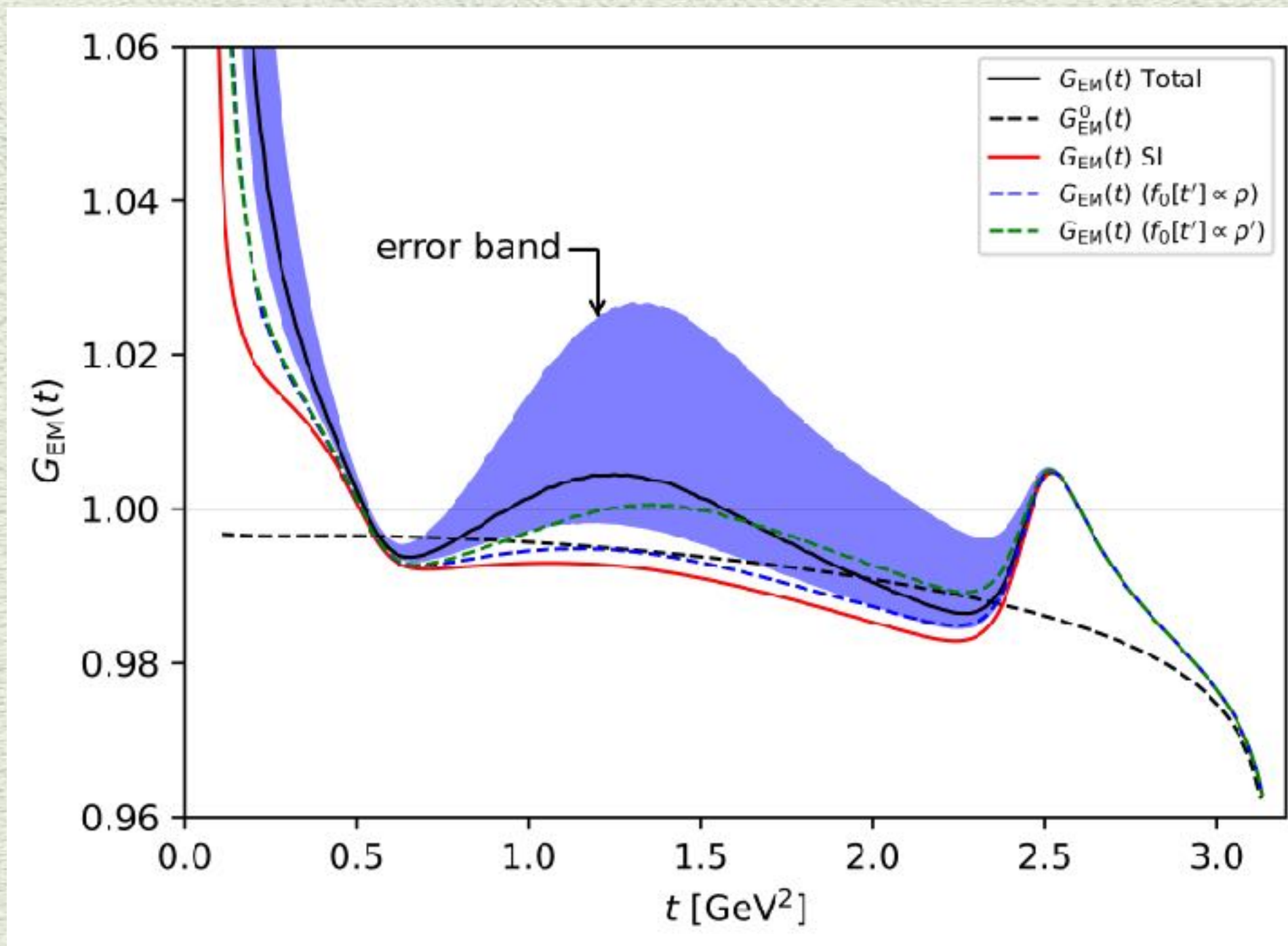


Dipion invariant mass distribution. The dotted line corresponds to the total, the dot-dashed line is the contribution excluding the  $\rho'$  in the omega channel, and the solid line is the SI contribution

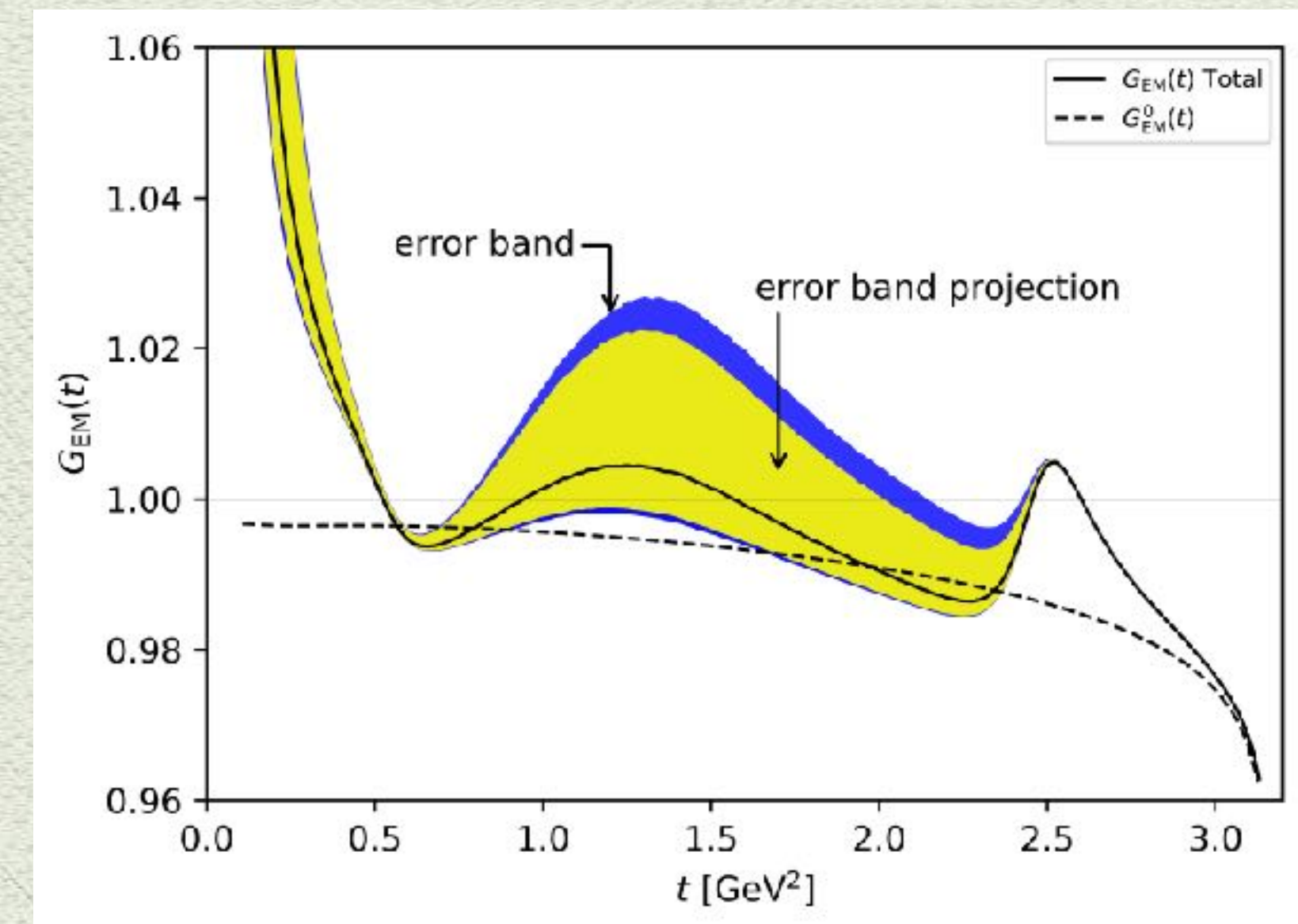
# The $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ radiative correction

Electromagnetic function  $\frac{d\Gamma_{2\pi(\gamma)}}{dt} = \frac{d\Gamma_{2\pi}^0}{dt} G_{EM}(t), \quad G_{EM}(t) = G_{EM}^0(t) + G_{EM}^{rest}(t),$

$G_0(t)$  accounts for the virtual and real contribution up to  $O(k^{-2})$ ,  
 $G_{rest}(t)$ , includes the remaining higher order contributions from the real part



GEM (t) function including several contributions (the error band corresponds to the inclusion of the rho')



Projection on rho(1450)

# muon g-2 ISB correction from $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau (\gamma)$

Isospin symmetry breaking correction

$$R_{IB}(t) = \frac{FSR(t)}{G_{EM}(t)} \frac{\beta_{\pi^+\pi^-}^3}{\beta_{\pi^+\pi^0}^3} \left| \frac{F_V(t)}{f_+(t)} \right|^2.$$

$$\Delta a_\mu^{(HVP,LO)}|_{G_{EM}(t)} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} dt K(t) \frac{K_\sigma(t)}{K_\Gamma(t)} \frac{d\Gamma_{2\pi(\gamma)}}{dt} \times \left[ \frac{1}{G_{EM}(t)} - 1 \right],$$

$$K_\Gamma(t) = \frac{G_F^2 |V_{ud}|^2 m_\tau^3}{384\pi^3} \left(1 - \frac{t}{m_\tau^2}\right) \left(1 + \frac{2t}{m_\tau^2}\right), \quad K_\sigma = \frac{\pi\alpha^2}{3t},$$

	$G_{EM}(t)$	$\Delta a_\mu^{(HVP,LO)} (\times 10^{-11})$
(i)	$G_{EM}^0(t)$	18
(ii)	$G_{EM}(t)$ (MI)	-12
(iii)	$G_{EM}(t)$ (SI)	-15
(iv)	$G_{EM}(t)$ (Full)	$-38.3^{+2.8}_{-3.2}$
(v)	$G_{EM}(t)$ (Full+ $\rho'$ )	$-94.2^{+32.3}_{-91.9}$
(vi)	$G_{EM}(t)$ (Projection)	$-94.2^{+28.2}_{-66}$

Consistent with previous estimates

Anomalously large

$\Delta a_\mu^{HVP,LO} _{G_{EM}(t)} (\times 10^{-11})$	ChPT $\mathcal{O}(p^4)$	$R_\chi T \mathcal{O}(p^4)$	$R_\chi T \mathcal{O}(p^6)$
	-10	$-15.9^{+5.7}_{-16.0}$	$-76 \pm 46$

# muon g-2 ISB correction from $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau (\gamma)$

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$\rho$ - $\omega$ interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$
$m_{\pi^\pm} - m_{\pi^0}$ effect on $\sigma$	$-7.88$	
$m_{\pi^\pm} - m_{\pi^0}$ effect on $\Gamma_\rho$	$+4.09$	$+4.02$
$m_{\rho^\pm} - m_{\rho_{\text{bare}}^0}$	$+0.20^{+0.27}_{-0.19}$	$+0.11^{+0.19}_{-0.11}$
$\pi\pi\gamma$ , electrom. decays	$-5.91 \pm 0.59$	$-6.39 \pm 0.64$
Total	$-16.07 \pm 1.22$	$-16.70 \pm 1.23$
	$-16.07 \pm 1.85$	

M. Davier, et al, Eur. Phys. J. C 66, 127-136 (2010)

from  $-(16.07 \pm 1.85) \times 10^{-11}$  to  $-(17.98 \pm 1.64) \times 10^{-10}$  considering only the  $\rho$  in the  $\omega$  channel, and  $-(23.57^{+3.67}_{-9.33}) \times 10^{-10}$  when adding the  $\rho'$ .

## muon g-2 experimental result 2023

$a_\mu = 116\,592\,057(25) \times 10^{-11}$  (0.21 ppm). Combining this result with our previous result from the 2018 data, we obtain  $a_\mu(\text{FNAL}) = 116\,592\,055(24) \times 10^{-11}$  (0.20 ppm). The new experimental world average is  $a_\mu(\text{Exp}) = 116\,592\,059(22) \times 10^{-11}$  (0.19 ppm), which represents a factor of two improvement in precision.

# Conclusions

We explored the  $e^+e^- \rightarrow \pi^0\pi^0\gamma$  differential cross section, and identified the double pole resonant enhancement features.

Performed an analysis of the parameters involved, in base of experimental data, to identify their robustness.  $g_{\rho\omega\pi} = 11.314 \pm 0.383 \text{ GeV}^{-1}$  was found to be consistent with all the relevant observables.

Performed an analysis of the  $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau \gamma$  omega channel, exhibiting the analogies to the previous case

The muon g-2 ISB correction from  $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau(\gamma)$  was found to be consistent with previous calculations using the same approach.

Considering only the rho, the uncertainties are well grounded.

The inclusion of rho(1450) calls for revision as the large effects observed may signal departure from soft photon approximation and structure dependence.

The link between the  $e^+e^- \rightarrow \pi^0\pi^0\gamma$  process and the  $\omega$  channel of the  $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau \gamma$  decay, can be used to gain further insight into the description of such processes.

**THANK YOU**  
**for your attention**

