INSTITUTO DE FÍSICA Y MATEMÁTICAS
U M S N H

Non-Perturbative Physics: Tools and Applications

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Distribution Functions of η and η'

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Outline

- SDE-BSE in a nutshell

- Algebraic Model

-Light-Front Wave Functions

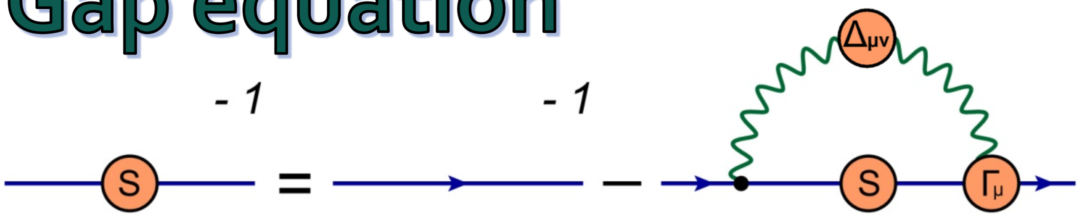
-GPDs: Overlap Representation

-PDFs and EFFs

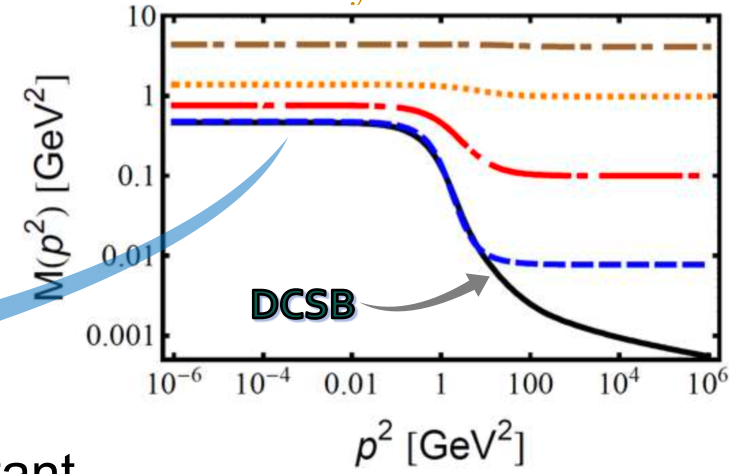
-AM implementation for $\eta - \eta'$

SDE-BSE approach

Gap equation



K. Raya PhD thesis



$$S_{q(\bar{h})}(k) = \left[-i\gamma \cdot k + M_{q(\bar{h})}\right] \Delta\left(k^2, M^2_{q(\bar{h})}\right)$$

where $\Delta(s,t) = (s+t)^{-1}$ and $M_{q(\bar{h})}$ is a constant.

Homogeneous BSE

Symmetry preserving

Pseudoscalar mesons:
4 covariant amplitudes

Leading amplitude: $n_{\rm M}\Gamma_{\rm M}(k,P) = i\gamma_5 \int_{-1}^{1} dw\, \rho_{\rm M}(w) \left[\hat{\Delta}\left(k_w^2, \Lambda_w^2\right)\right]^{\nu}$

$k_w = k + (w/2)P$

$\hat{\Delta}(s,t) = t\Delta(s,t)$

Spectral density
Determines BSA behavior

$\Lambda_w^2 \equiv \Lambda^2(w)$

Controls asymptotic behavior

BS Wave Function

The BSWF for a $q\bar{h}$ bound state is formally defined as

$$\chi_{\rm M}\left(k_{-},P\right)=S_{q}(k)\Gamma_{\rm M}\left(k_{-},P\right)S_{\bar{h}}\left(k-P\right)$$

with $k_{-}=k-P/2$ and $P^2=-m_{\rm M}^2$.

Using algebraic ansatzë for the propagator and the BSA:

$$n_{\rm M}\chi_{\rm M}(k_{-},P)=\mathcal{M}_{q,\bar{h}}(k,P)\int_{-1}^{1}dw\,\tilde{\rho}_{\rm M}^{\nu}(w)\mathcal{D}_{q,\bar{h}}^{\nu}(k,P)$$

Where $n_{\rm M}$ is a normalization constant and $\tilde{\rho}_{\rm M}^{\nu}(w)\equiv\rho_{\rm M}(w)\Lambda_{w}^{2\nu}$

$$\begin{aligned}\mathcal{M}_{q,\bar{h}}(k=p+P,P)\equiv-\gamma_5\big[&M_q\gamma\cdot P+\gamma\cdot k(M_{\bar{h}}-M_q)\\&+\sigma_{\mu\nu}k_\mu P_\nu-i\left(k\cdot p+M_qM_{\bar{h}}\right)\big]\end{aligned}$$

$$\mathcal{D}_{q,\bar{h}}^{\nu}(k,P)\equiv\Delta\left(k^2,M_q^2\right)\Delta\left(k_{w-1}^2,\Lambda_w^2\right)^{\nu}\Delta\left(p^2,M_{\bar{h}}^2\right)$$

Feynman parametrization for the denominators product

$$\mathcal{D}(k,P) = \frac{\Gamma(\nu+2)}{\Gamma(\nu)} \int_0^1 du \int_0^{1-u} dv \, \frac{u^{\nu-1}}{\sigma^{\nu+2}}$$

conveniently reexpressed by using $\beta = 1-u$ and $\alpha = v + \frac{1}{2}u(1-w)$

$$\mathcal{D}(k,P) = \frac{\Gamma(\nu+2)}{\Gamma(\nu)} \int_0^1 d\beta \int_{\frac{1}{2}(\beta-1)(w-1)}^{\frac{1}{2}[(w+1)\beta-(w-1)]} d\alpha \frac{u^{\nu-1}}{\sigma^{\nu+2}}$$

with $\sigma = [k-\alpha P]^2 + \Omega_M^2$ and

$$\begin{aligned} \Omega_M^2 &\equiv \beta M_q^2 + (1-\beta)\Lambda_w^2 + \left(M_{\bar{q}}^2 - M_q^2\right)\left[\alpha - \frac{1}{2}(1-\beta)(1-w)\right] \\ &+ \left[\alpha(\alpha-1) + \frac{1}{4}(1-\beta)(1-w^2)\right] m_M^2 \,. \end{aligned}$$

Cancelling β dependence:

$$\Lambda_w^2 \equiv \Lambda^2(w) = M_q^2 - \frac{1}{4}\left(1-w^2\right) m^2 + \frac{1}{2}(1-w)\left(M_{\bar{q}}^2 - M_q^2\right)$$

So that $\sigma = [k-\alpha P]^2 + \Lambda_{1-2\alpha}^2$ ← β-independent

BS Wave Function

$$n_{\mathrm{M}}\chi_{\mathrm{M}}(k_-,P)=\mathcal{M}_{q,\bar{h}}(k,P)\int_0^1 d\alpha\mathcal{F}_{\mathrm{M}}(\alpha,\sigma^{\nu+2})$$

with

$$\mathcal{F}_{\mathrm{M}}(\alpha,\sigma^{\nu+2})=2^{\nu}(\nu+1)\Big[\int_{-1}^{1-2\alpha}dw\left(\frac{\alpha}{1-w}\right)^{\nu}+\int_{1-2\alpha}^{1}dw\left(\frac{1-\alpha}{1+w}\right)^{\nu}\Big]\frac{\tilde{\rho}_{\mathrm{M}}^{\nu}(w)}{\sigma^{\nu+2}}\,.$$

We have implemented

$$\int_{-1}^{1}dw\int_0^1 d\beta\int_{\frac{1}{2}(\beta-1)(w-1)}^{\frac{1}{2}[(w+1)\beta-(w-1)]}d\alpha$$

$$=\int_0^1 d\alpha\left[\int_{-1}^{1-2\alpha}dw\int_{\frac{2\alpha}{w-1}+1}^{1}d\beta+\int_{1-2\alpha}^{1}dw\int_{\frac{2\alpha+(w-1)}{w+1}}^{1}d\beta\right]$$

Light-Front Wave Function

x = momentum fraction

$$\psi^q_{\mathrm{M}}\left(x, k_\perp^2\right) = \mathrm{tr}\int_{dk_\parallel} \delta(n\cdot k - x\, n\cdot P)\gamma_5\gamma\cdot n\,\chi_{\mathrm{M}}(k_-, P)$$

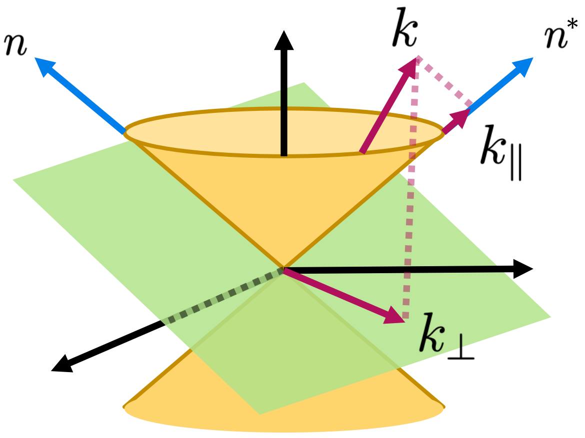
Perpendicular momentum dependence

Where n is a light-like four momentum such that

$$n^2 = 0$$

For a meson with mass m_M

$$n\cdot P = -m_{\mathrm{M}}$$



This allows us to decompose $k = k_\parallel + k_\perp$

which entails $n\cdot k = n\cdot k_\parallel$ and $n\cdot k_\perp = 0$

with $k^2 = k_\parallel^2 + k_\perp^2$

Meson mass restriction

LFWF Mellin moments

$$\langle x^m \rangle_{\psi_{\rm M}^q} = \int_0^1 dx\, x^m\, \psi_{\rm M}^q\left(x, k_\perp^2\right)$$

Dirac delta function allows simple x-integration

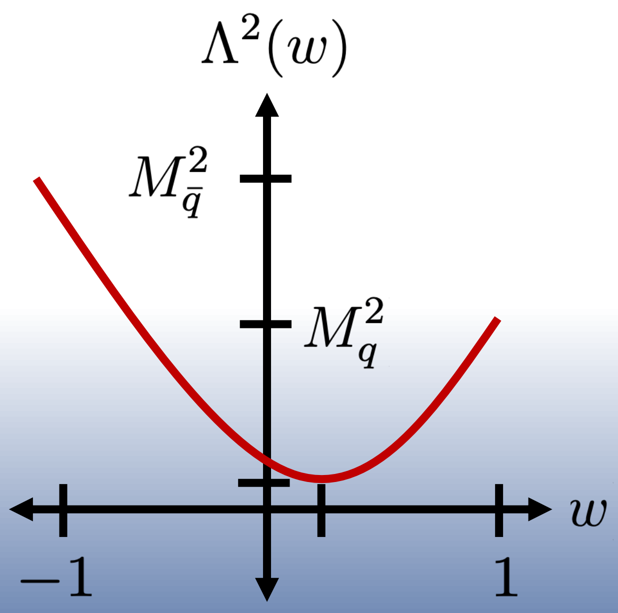
$$= \mathrm{tr} \frac{1}{n\cdot P}\int_{dk_\parallel}\left[\frac{n\cdot k}{n\cdot P}\right]^m \gamma_5 \gamma\cdot n \chi_{\rm M}(k_-, P)$$

$k_\parallel^2$-integration entails

$$\int_{dk_\parallel} \frac{1}{\left[k_\parallel^2 + k_\perp^2 + \Lambda_{1-2\alpha}^2\right]^{\nu+2}}$$

Positivity ensured if

$$\left| M_{\bar q} - M_q \right| \le m_M \le M_{\bar q} + M_q$$



LFWF: Integral expression

Provided the positivity of $\Lambda^2(w)$, $k_{\parallel}^2$-integration leads to

$$\langle x^m \rangle_{\psi_{\mathrm{M}}^q} = \int_0^1 d\alpha\alpha^m \left[\frac{12}{n_{\mathrm{M}}} \frac{\mathcal{F}_{\mathrm{M}}(\alpha, \sigma_\perp^{\nu+1})}{\nu+1} \right] (\alpha M_{\bar{h}} + (1-\alpha) M_q)$$

α = Feynman parameter

Compare to

x = momentum fraction

$$\langle x^m \rangle_{\psi_{\mathrm{M}}^q} = \int_0^1 dx\, x^m\, \psi_{\mathrm{M}}^q\left(x, k_\perp^2\right)$$

Uniqueness of Mellin moments implies the connection between α and x. Therefore

$$\psi_M(x, k_\perp^2) = \frac{2^\nu}{\pi F_N} \left[x^\nu \int_{-1}^{1-2x} dw \frac{\tilde{\rho}(w; \Lambda_w^\nu)}{(1-w)^\nu} + (1-x)^\nu \int_{1-2x}^{1} dw \frac{\tilde{\rho}(w; \Lambda_w^\nu)}{(1+w)^\nu} \right] \times \frac{[M_q + x(M_{\bar{q}} - M_q)]}{\left[k_\perp^2 + \Lambda_{1-2x}^2\right]^{\nu+1}},$$

Normalization factor

Parton Distribution Amplitude

$$f_{\rm M}\phi^q_{\rm M}(x) = \frac{1}{16\pi^3}\int d^2k_\perp \psi^q_{\rm M}\left(x, k_\perp^2\right) \quad \text{with} \quad \int_0^1 dx\,\phi_M(x) = 1$$

f_M = Decay constant

Normalization condition fixes F_N

Trivial integration over $k_\perp$ yields

$$\phi_M(y) = \frac{1}{2\nu F_N}\left[(1-y)^\nu \int_{-1}^{y} dw \frac{\tilde\rho(w;\Lambda_w^\nu)}{(1-w)^\nu} + (1+y)^\nu \int_y^1 dw \frac{\tilde\rho(w;\Lambda_w^\nu)}{(1+w)^\nu}\right] \times \frac{[(1+y)M_q + (1-y)M_{\bar q}]}{\Lambda_y^{2\nu}},$$

Analytical relation!
LFWF correlates x and $k_\perp$

A simple glance reveals

$$\psi^q_{\rm M}(x,k_\perp^2) = 16\pi^2 f_{\rm M}\frac{\nu\Lambda^{2\nu}_{1-2x}}{(k_\perp^2+\Lambda^2_{1-2x})^{\nu+1}}\phi^q_{\rm M}(x)$$

Factorization: Chiral limit ($m_M = 0$ and $M_q = M_{\bar q}$)

$$\psi^q_{\rm M}(x,k_\perp^2) = \left[16\pi^2 f_{\rm M}\frac{\nu M_q^{2\nu}}{(k_\perp^2+M_q^2)^{\nu+1}}\right]\phi^q_{\rm M}(x)$$

ρ_M – Differential equation

It is possible to cast the PDA integral equation into a differential equation for the spectral density

$$\eta_N\, \rho_{\mathrm{M}}(y) = \lambda_\nu^{(2)}(y)\varphi''(y) + \lambda_\nu^{(1)}(y)\varphi'(y) + \lambda_\nu^{(0)}(y)\varphi(y)$$

where $\varphi(y) \equiv \phi_M^q(\frac{1}{2}(1-y))$

and η_N ensures $\int_{-1}^{1} \rho_{\mathrm{M}}(y)dy = 1$

Normalization constants are related!

$$F_N = \tfrac{4}{3}\pi^2 f_{\mathrm{M}} n_{\mathrm{M}}\, \eta_N$$

Example: Asymptotic PDA: $\phi_{\mathrm{M}}^q(x) = \phi_{asy}(x) = 6x(1-x)$

$$\Longleftrightarrow \rho_{\mathrm{M}}(w) = \rho_{asy}(w) := \frac{3}{4}(1-w^2)$$

Generalized Parton Distribution

In the overlap representation of the LFWF, valence quark GPD is defined as

$$H_{\mathrm{M}}^{q}(x,\xi,t)=\int\frac{d^2k_\perp}{16\pi^3}\psi_{\mathrm{M}}^{q*}\left(x^-,(\mathbf{k}_\perp^-)^2\right)\psi_{\mathrm{M}}^{q}\left(x^+,(\mathbf{k}_\perp^+)^2\right)$$

where

$$x^\pm=\frac{x\pm\xi}{1\pm\xi}\qquad\qquad \mathbf{k}_\perp^\pm=k_\perp\mp\frac{\Delta_\perp}{2}\frac{1-x}{1\pm\xi}$$

$$-t=\Delta^2=(p-p')^2\qquad\qquad \Delta_\perp^2=\Delta^2(1-\xi^2)-4\xi^2m_{\mathrm{M}}^2$$

$$\xi=[-n\cdot\Delta]/[2n\cdot P]$$

In the AM

$$H_{\mathrm{M}}^{q}(x,\xi,t)=(16\pi^2 f_{\mathrm{M}}\nu)^2\phi_{\mathrm{M}}^{q}(x^+)\phi_{\mathrm{M}}^{q}(x^-)\Lambda_{1-2x^+}^{2\nu}\Lambda_{1-2x^-}^{2\nu}\times\int\frac{d^2k_\perp}{16\pi^3}\frac{1}{((\mathbf{k}_\perp^-)^2+\Lambda_{1-2x^-}^2)^{\nu+1}}\frac{1}{((\mathbf{k}_\perp^+)^2+\Lambda_{1-2x^+}^2)^{\nu+1}}$$

Parton Distribution Functions

Zero-skewness valence-quark GPD:

$$H_{\mathrm{M}}^{q}(x,0,t) \overset{t\to 0}{\approx} \mathcal{N}\frac{[\phi_{\mathrm{M}}^{q}(x)]^2}{\Lambda_{1-2x}^2}\left[1 - c_\nu^{(1)}(1-x)^2\left(\frac{-t}{\Lambda_{1-2x}^2}\right) + \ldots\right]$$

where

$$c_\nu^{(1)} = \frac{(1+\nu)(1+2\nu)}{2(3+2\nu)} \qquad \mathcal{N} = \left[\int_0^1 dx\, \frac{\phi_{\mathrm{M}}^2(x)}{\Lambda_{1-2x}^2}\right]^{-1}$$

GPD in the forward limit: **PDF**

$$q_{\mathrm{M}}(x) \equiv H_{\mathrm{M}}^{q}(x,0,0) = \mathcal{N}\frac{[\phi_{\mathrm{M}}^{q}(x)]^2}{\Lambda_{1-2x}^2}$$

Analytical connection between PDA and PDF

For $t \sim 0$, the GPD approaches an exponential behavior

$$H_{\mathrm{M}}^{q}(x,0,t) = q_{\mathrm{M}}(x)\,\exp[t\hat{f}_{\mathrm{M}}^{q}(x)] \qquad \hat{f}_{\mathrm{M}}^{q}(x) = \frac{c_\nu^{(1)}(1-x)^2}{\Lambda_{1-2x}^2}$$

Electromagnetic Form Factors

Zeroth moment of the GPD: $$F^q_{\rm M}(t) = \int_0^1 dx\; H^q_{\rm M}(x,0,t)$$

Complete meson EFF: $$F_{\rm M}(t) = e_q F^q_{\rm M}(t) + e_{\bar h} F^{\bar h}_{\rm M}(t)$$

Taylor expansión around $t \sim 0$ yields: $$F^q_{\rm M}(t) \overset{t\to 0}{\approx} 1 - \frac{(r^q_{\rm M})^2}{6}(-t) + \ldots$$

q-contibution to charge radius: $$(r^q_{\rm M})^2 \;=\; -6\frac{dF^q_{\rm M}(t)}{dt}\bigg|_{t=0}$$

$$r^2_M = \frac{3}{4}\frac{(\nu+1)(2\nu+1)}{(2\nu+3)}\left\{\frac{\int_{-1}^1 dy \frac{\phi^2(y)(1+y)^2}{\Lambda_y^4}}{\int_{-1}^1 dy \frac{\phi^2(y)}{\Lambda_y^2}}\right\}$$

Meson charge radius: $$r^2_{\rm M} = e_q (r^q_{\rm M})^2 + e_{\bar h}(r^{\bar h}_{\rm M})^2$$

Results for π and K

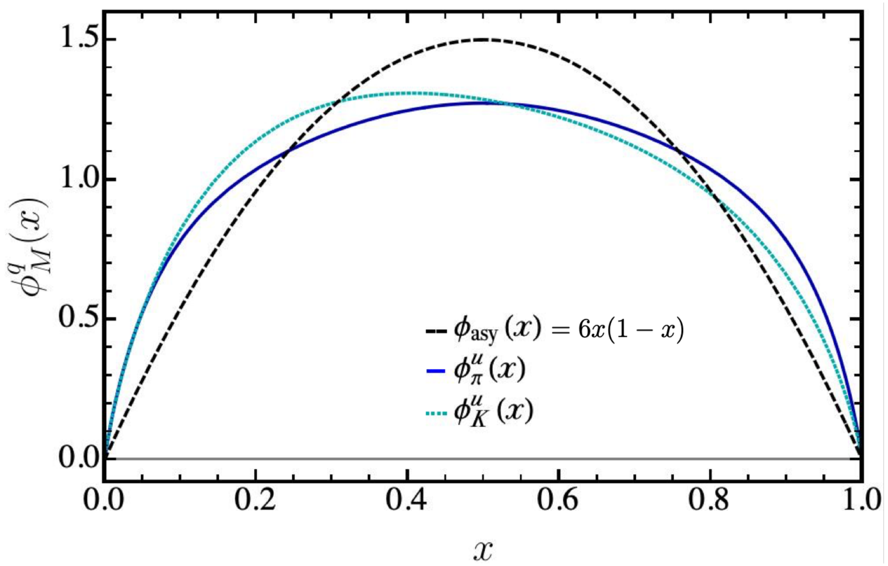
Inputs

Z.F. Cui et al
EPJ A 57 (2021) 1, 5

PDAS:

$$\phi_\pi^u(x) = 20.226\, x\bar{x}\left[1 - 2.509\sqrt{x\bar{x}} + 2.025 x\bar{x}\right],$$
$$\phi_K^u(x) = 18.04\, x\bar{x}\left[1 + 5x^{0.032}\bar{x}^{0.024} - 5.97x^{0.064}\bar{x}^{0.048}\right],$$

with $\bar{x} = 1 - x$.

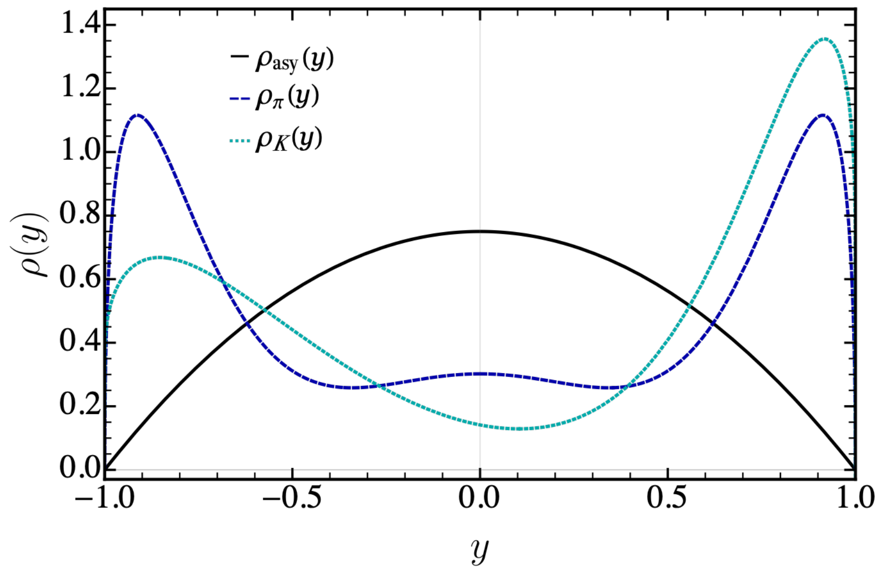


Parameters: $\nu = 1$
and

Meson	m_M (GeV)	r_M (fm)	Quark	M_q (GeV)
π	0.14	0.659	u, d	0.317
K	0.49	0.600	s	0.574

$\rho(\omega)$ and decay constants

Spectral density:



Decay constants:

$$F_N = \frac{4}{3}\pi^2 f_M n_M \eta_N$$

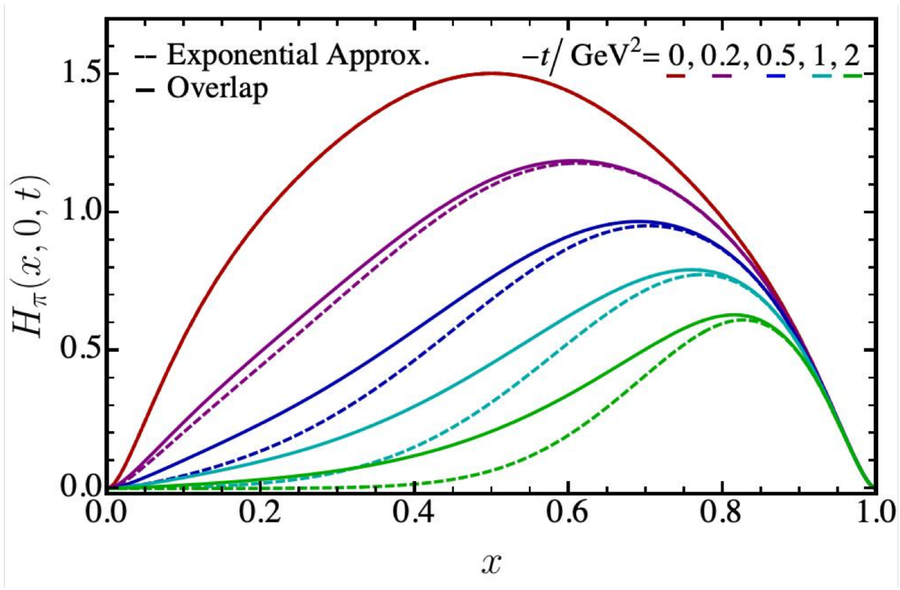
Meson	f_M (GeV)	Exp
π	0.092	0.092
K	0.11	0.11

where

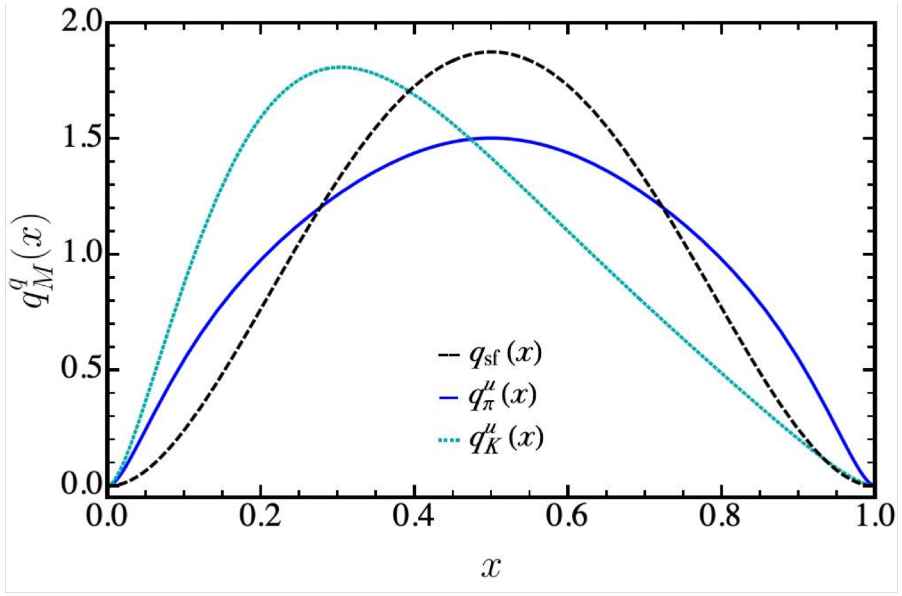
$$F_N = \frac{1}{4\nu}\int_{-1}^{1} dy \left[(1-y)^\nu \int_{-1}^{y} dw \frac{\tilde{\rho}(w;\Lambda_w^\nu)}{(1-w)^\nu} + (1+y)^\nu \int_{y}^{1} dw \frac{\tilde{\rho}(w;\Lambda_w^\nu)}{(1+w)^\nu}\right]$$

$$\times \frac{[(1+y)M_q + (1-y)M_{\bar{q}}]}{\Lambda_y^{2\nu}}$$

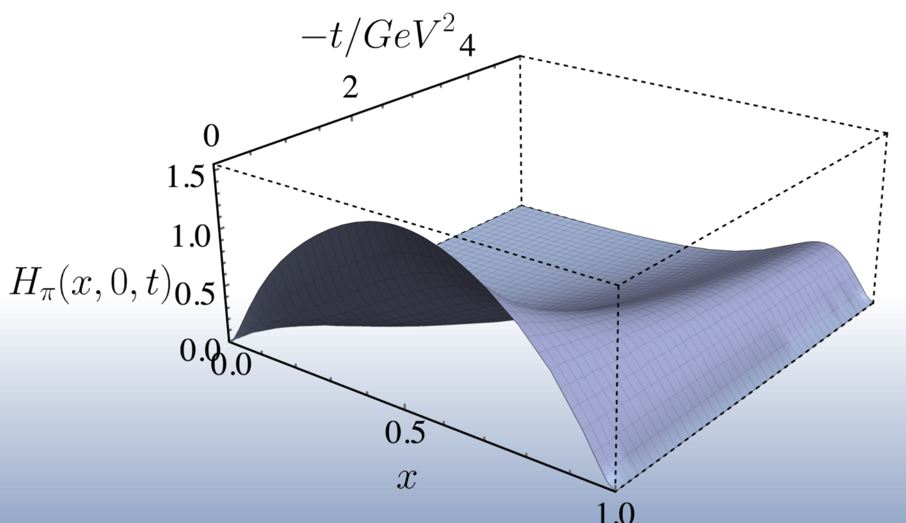
Pion



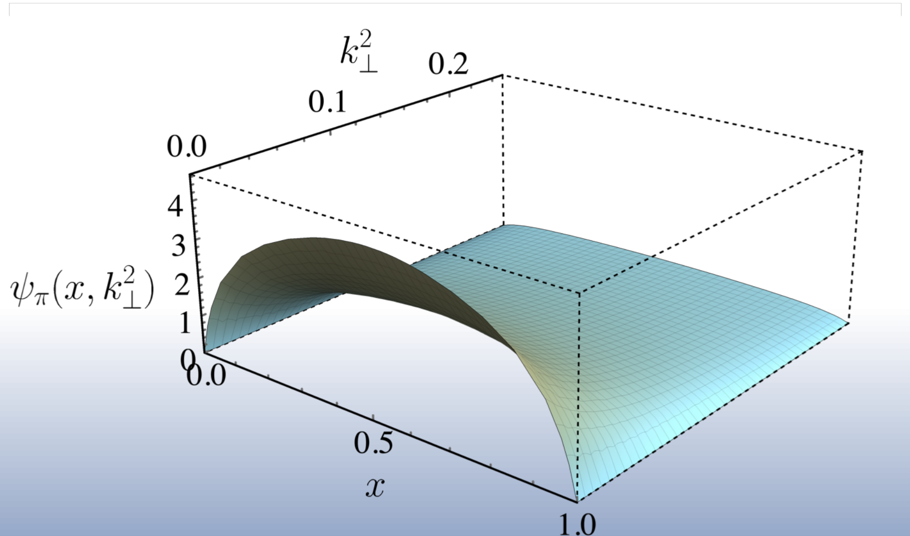
-- Exponential Approx.
— Overlap
$-t/\ \mathrm{GeV}^2 = 0, 0.2, 0.5, 1, 2$
$H_\pi(x,0,t)$
x



$q^q_M(x)$
$q_{\rm sf}(x)$
$q^u_\pi(x)$
$q^u_K(x)$
x

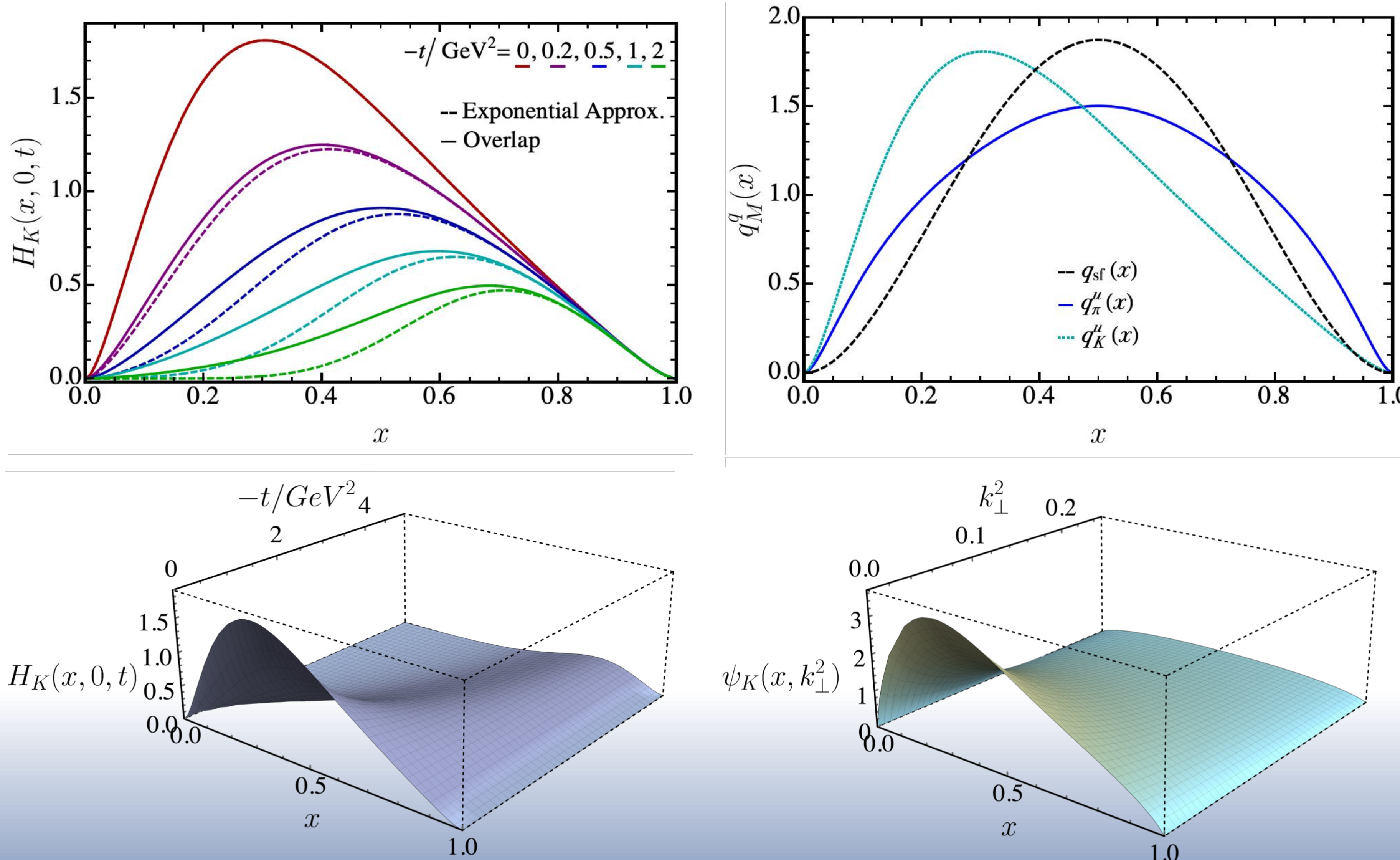


$-t/GeV^2$
$H_\pi(x,0,t)$
x



$k_\perp^2$
$\psi_\pi(x,k_\perp^2)$
x

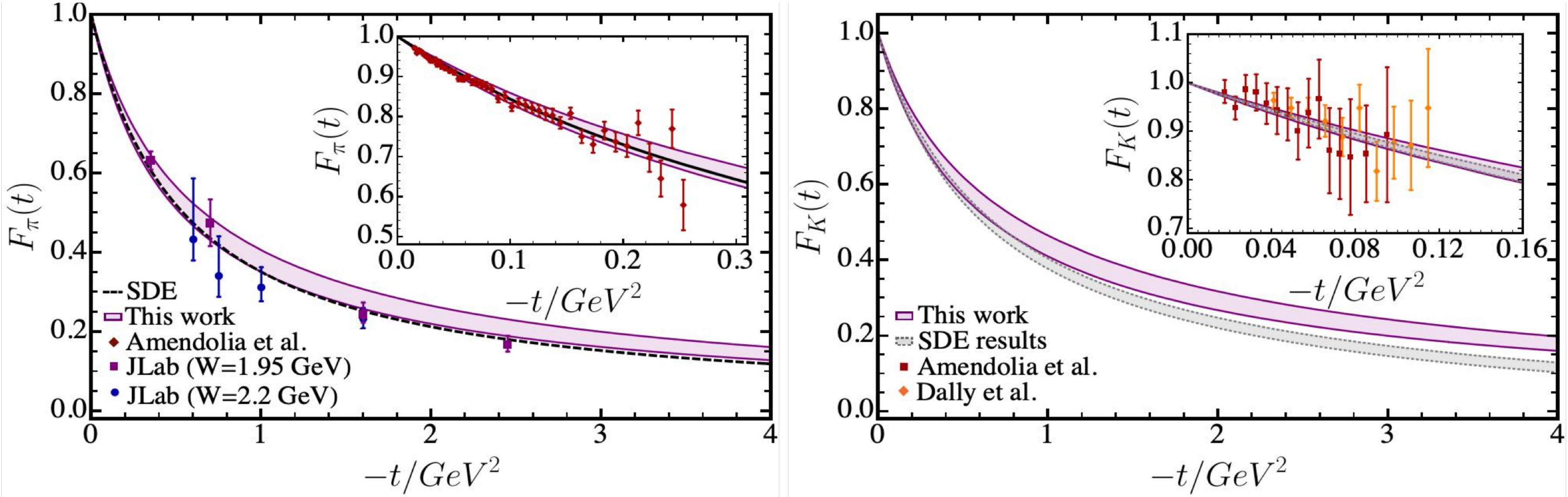
Kaon



$-t/\,\mathrm{GeV}^2 = 0, 0.2, 0.5, 1, 2$
-- Exponential Approx.
— Overlap
$H_K(x,0,t)$
x
$q^q_M(x)$
-- $q_{\rm sf}(x)$
— $q^u_\pi(x)$
··· $q^u_K(x)$
$-t/GeV^2$
$\psi_K(x,k_\perp^2)$
$k_\perp^2$

Electromagnetic Form Factors

Pion **Kaon**



Left panel: The (purple) band represents our pion result. The band width accounts for a 5% variation of the benchmark radius. Dashed (black) line is the pion SDE result [PRL 111, 141802 (2013).]. Right panel: Analogous results. Lower (gray) band is the kaon SDE result [PLB 797, 134855 (2019)]. Diamonds, rectangles and circles represent experimental data [NP B 277, 168 (1986). PRL 86, 1713 (2001). PRL 97, 192001 (2006)].

$\eta - \eta'$ mesons

BSA: Flavor basis

Flavor mixing conveniently addressed with an $U(N_f = 3)$ basis:

$$\chi_{\eta,\eta'}(k;P) = \mathbb{F}^l \chi^l_{\eta,\eta'}(k;P) + \mathbb{F}^s \chi^s_{\eta,\eta'}(k;P)$$

where

$$\mathbb{F}^l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbb{F}^s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

The (BSWFs) coefficients $\chi^{l,s}_{\eta,\eta'}(k;P)$ describe the momentum space correlations on the light and the strange sector in η and η'. In terms of BSAs:

$$\chi^{l,s}_{\eta,\eta'}(k;P) = S_{l,s}(k_+)\Gamma^{l,s}_{\eta,\eta'}(k;P)S_{l,s}(k_-)$$

Meson masses: $m_\eta \sim 550$ GeV $m_{\eta'} \sim 960$ GeV

Drawback: $m_{\eta'} \geq 2\, M_{u,d}$ $\Rightarrow$ $\Lambda^2(\omega) < 0$ for some ω.

$$\psi^q_{\mathrm{M}}(x,k_\perp^2) = 16\pi^2 f_{\mathrm{M}} \frac{\nu \Lambda^{2\nu}_{1-2x}}{(k_\perp^2 + \Lambda^2_{1-2x})^{\nu+1}} \phi^q_{\mathrm{M}}(x)$$

Pole

Mixing angle

T. Feldmann et al
PRD 58, 114006 (1998)
PLB 449 (1999), 339-346

Physical meson states can be related to orthogonal quark-flavor basis states:

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = U(\theta) \begin{pmatrix} |\eta_l\rangle \\ |\eta_s\rangle \end{pmatrix} \quad \text{with} \quad U(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

At first order Fock states:

Parton states

$$|\eta_l\rangle = \psi_l \left| u\bar{u} + d\bar{d} \right\rangle / \sqrt{2} + \ldots \qquad |\eta_s\rangle = \psi_s \, |s\bar{s}\rangle + \ldots$$

In this flavor basis, decay constants follow the pattern

$$\begin{pmatrix} f_\eta^l & f_\eta^s \\ f_{\eta'}^l & f_{\eta'}^s \end{pmatrix} = U(\theta) \begin{pmatrix} f_l & 0 \\ 0 & f_s \end{pmatrix} = \begin{pmatrix} f_l \cos\theta & -f_s \sin\theta \\ f_l \sin\theta & f_s \cos\theta \end{pmatrix}$$

State superposition

The decay constants pattern is readily stablished if

$$\psi_l(x,k_\perp^2) = \cos\theta\,\psi_\eta^l(x,k_\perp^2) + \sin\theta\,\psi_{\eta'}^l(x,k_\perp^2)$$

$$\psi_s(x,k_\perp^2) = -\sin\theta\,\psi_\eta^s(x,k_\perp^2) + \cos\theta\,\psi_{\eta'}^s(x,k_\perp^2)$$

Assuming that each independent wave function entails

$$f_{\mathrm{M}}\phi_{\mathrm{M}}^q(x) = \frac{1}{16\pi^3}\int d^2k_\perp \psi_{\mathrm{M}}^q\left(x,k_\perp^2\right)$$

then

$$f_l\,\phi_l(x) = \cos\theta\, f_\eta^l\phi_\eta^l(x) + \sin\theta\, f_{\eta'}^l\phi_{\eta'}^l(x)$$

$$f_s\,\phi_s(x) = -\sin\theta\, f_\eta^s\phi_\eta^s(x) + \cos\theta\, f_{\eta'}^s\phi_{\eta'}^s(x)$$

Therefore

$$\begin{pmatrix} f_l & 0 \\ 0 & f_s \end{pmatrix} = U^{-1}(\theta)\begin{pmatrix} f_\eta^l & f_\eta^s \\ f_{\eta'}^l & f_{\eta'}^s \end{pmatrix}$$

Procedure

1) PDAs definition

$$\phi_l(x) = \cos^2\theta\,\phi^l_\eta(x) + \sin^2\theta\,\phi^l_{\eta'}(x)$$

$$\phi_s(x) = \sin^2\theta\,\phi^s_\eta(x) + \cos^2\theta\,\phi^s_{\eta'}(x)$$

2) Fitting f_M^q requires fixing input parameters: θ, $M_{l,s}$ and $m_{\eta_{l,s}}$.

3) AM is indeed applicable

$$\psi_{l,s}(x,k_\perp^2) = 16\pi^2\nu_{l,s}\frac{\Lambda_{l,s}^{2\nu_{l,s}}(x)}{\left(k_\perp^2+\Lambda_{l,s}^2(x)\right)^{\nu_{l,s}+1}} f_{l,s}\phi_{l,s}(x)$$

with

$$\Lambda_{l,s}^2(x) = M_{l,s}^2 - x(1-x)m_{\eta_{l,s}}^2$$

Positivity ensured: $m_{\eta_l} \leq 2M_{u,d}$ and $m_{\eta_s} \leq 2M_s$ $\Rightarrow$ $\Lambda^2(\omega) > 0$

4) Identification of

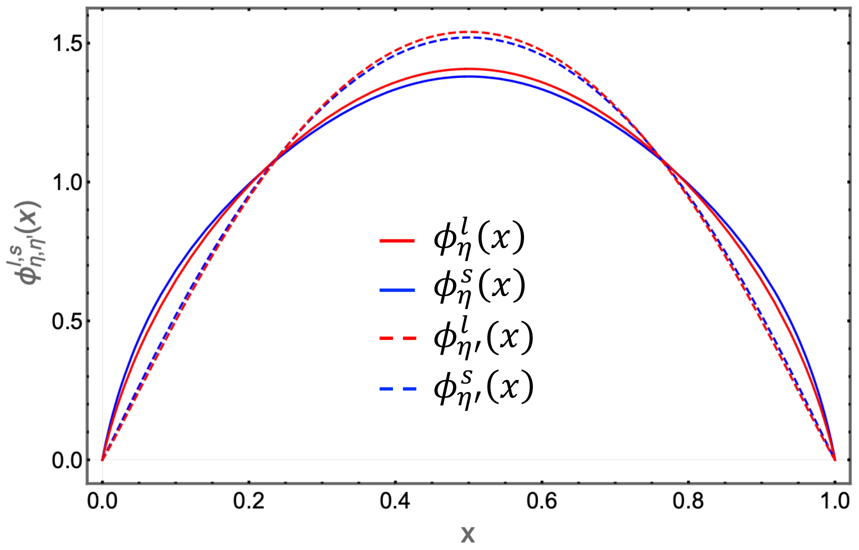
$$\psi^{l,s}_{\eta,\eta'}(x,k_\perp^2) = 16\pi^2\nu_{l,s}\frac{\Lambda_{l,s}^{2\nu_{l,s}}(x)}{\left(k_\perp^2+\Lambda_{l,s}^2(x)\right)^{\nu_{l,s}+1}} f^{l,s}_{\eta,\eta'}\phi^{l,s}_{\eta,\eta'}(x)$$

5) Computation of Distribution functions

Input

M. Ding, K. Raya et al
PRD 99 (2019) 1, 014014

PDAs:



Parameters: $\nu = 1$
and

Quark	M_q (GeV)
u, d	0.317
s	0.574

Meson	m_M (GeV)
η_l	0.450
η_s	0.840

Preliminar results

"Ideal" decay constants (GeV)

	f_l	f_s	θ
Herein	0.102	0.137	42.95
M. Ding	0.101	0.138	42.8

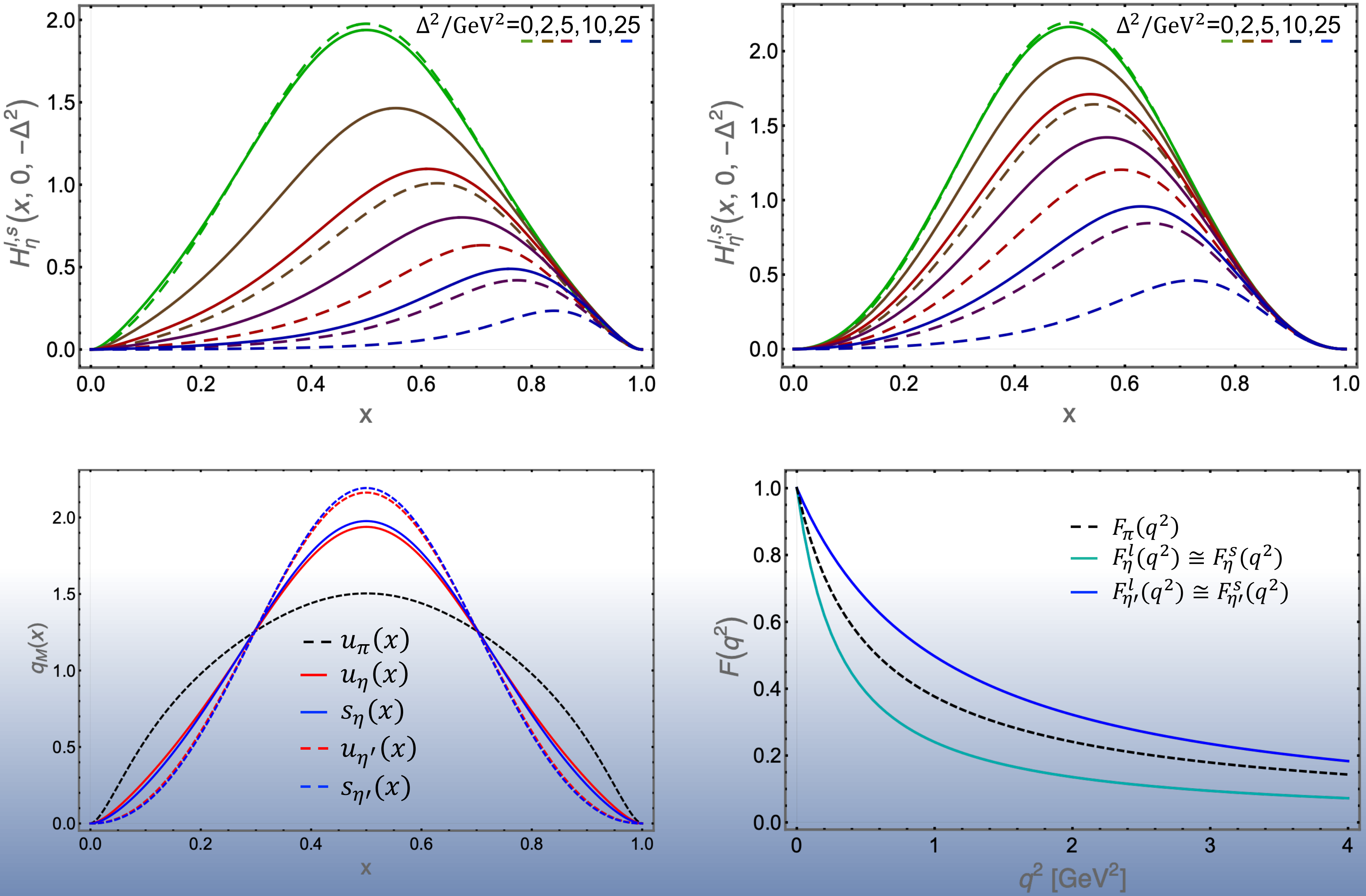
$\eta - \eta'$ decay constants (GeV)

	f_η^l	f_η^s	$f_{\eta'}^l$	$f_{\eta'}^s$
Herein	0.074	-0.092	0.069	0.099
M. Ding	0.074	-0.092	0.068	0.101

Charge radius (fm)

Meson	r_M^l	r_M^s
η	0.840	0.840
η'	0.472	0.472

Preliminary results



Conclusions

- We developed a unified AM for pseudoscalar meson BSWFs which satisfactorily describes the internal structure of pion and kaon.
- We computed their GPDs, built through the overlap representation of their LFWFs.
- We deduced the corresponding PDFs as well as the EFFs.
- We ensure most calculations continue to be analytic by promoting $\Lambda^2 \to \Lambda^2(x)$.
- We showed that in this model, the spectral density can be extracted unequivocally through the knowledge of the PDA.
- We have implemented a well-known quark-flavor basis decomposition to tackle the problem of $\eta - \eta'$ mixing: We have obtained the corresponding LFWFs, decay constants (in agreement with SDE results), GPDs and PDFs.

External input required by the model:

- PDAs
- Fitted parameters: f_M and r_M
- Free parameters: ν and quark masses

Model predictions: $\rho(\omega)$, LFWFs, GPDs, PDFs, EFFs

Thanks