Pseudoscalar mesons: how and why?

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In collaboration with:

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Daniele Binosi
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Craig D. Roberts
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and many more…

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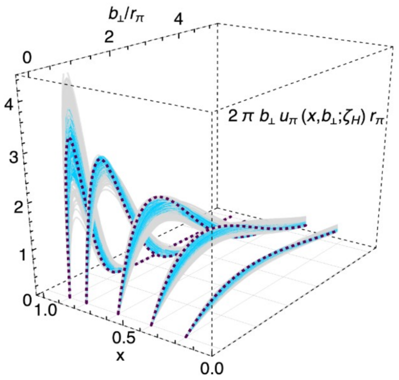
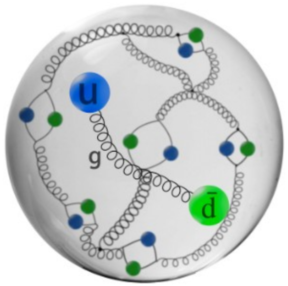




Non-Perturbative Physics: Tools and Applications

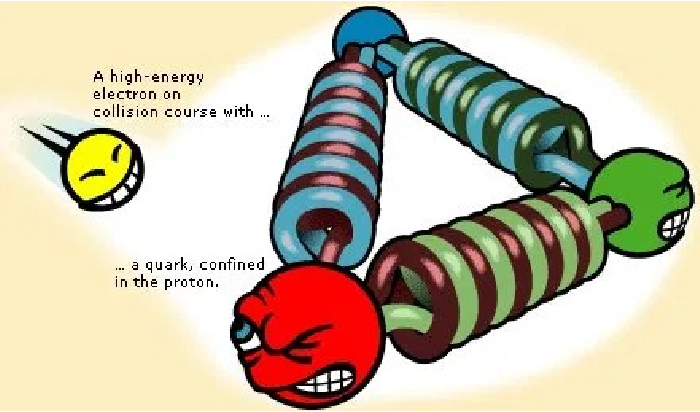
Sep 4-8, 2023. UMSNH

QCD: Emergent Phenomena

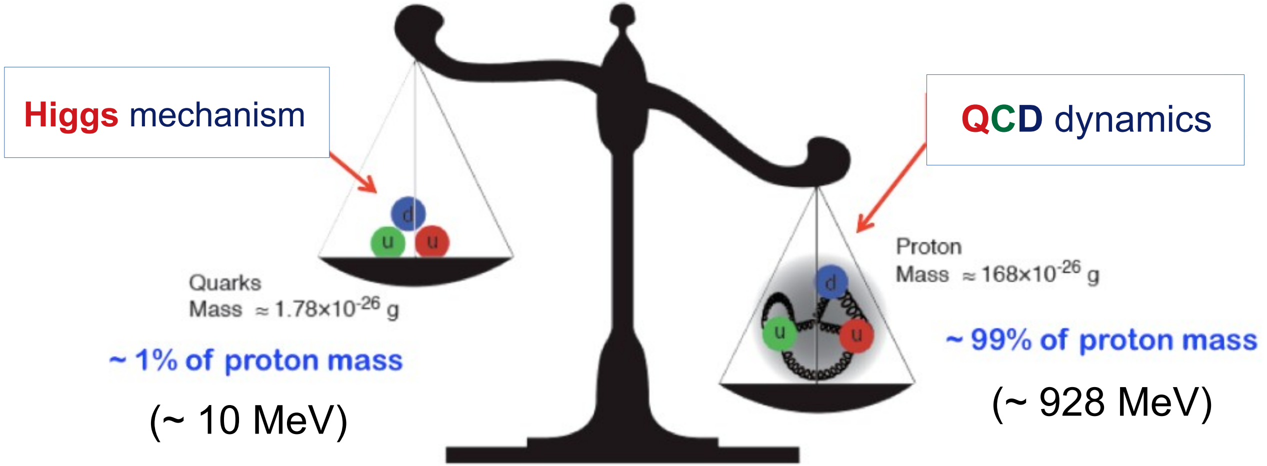
➢ **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\frac{1}{2}\lambda^a A^a_\mu\,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"
  - **1-fm scale** size of hadrons?



- Emergence of hadron masses (EHM) from QCD **dynamics**



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Can we trace them down to fundamental d.o.f?

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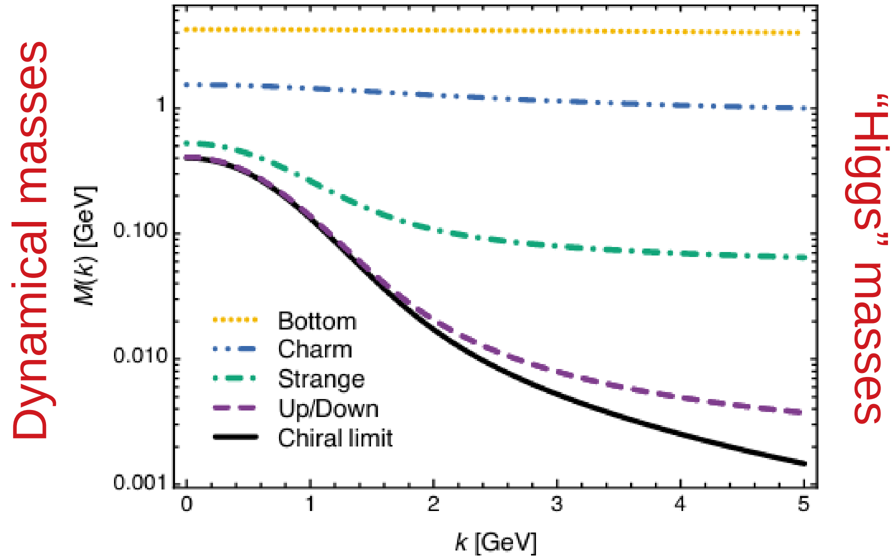
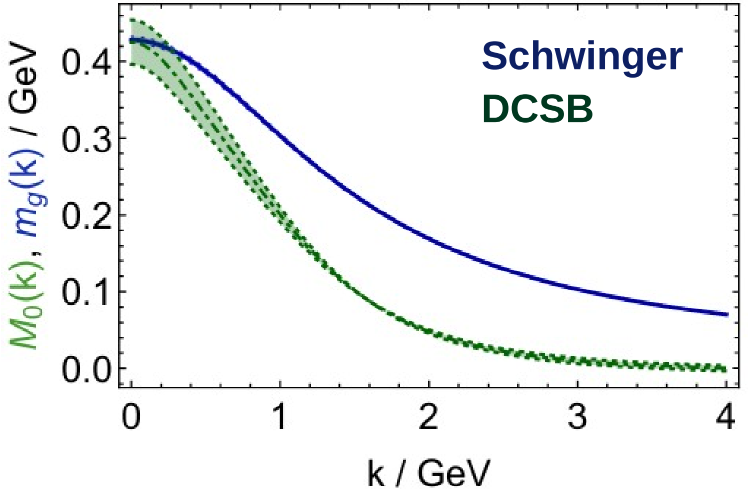


$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$



Gluon and quark *running masses*

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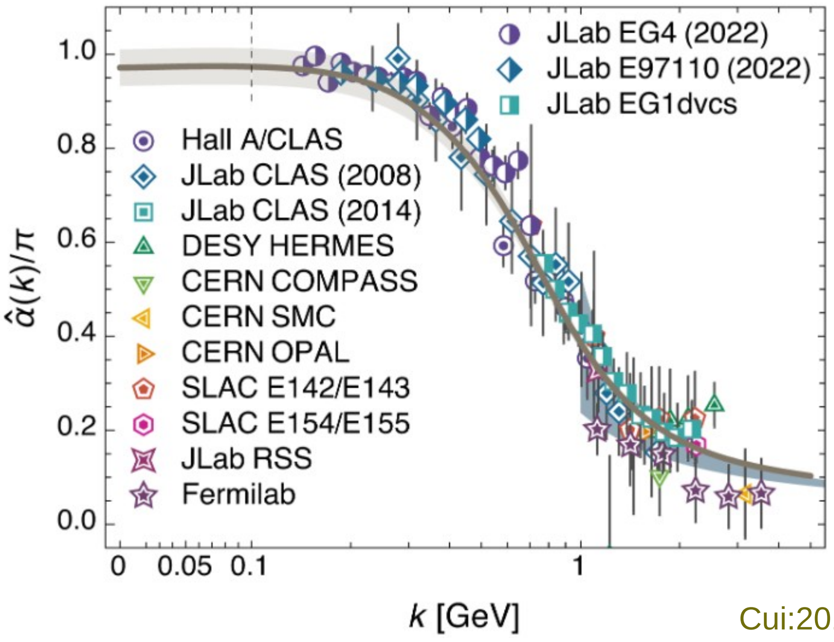
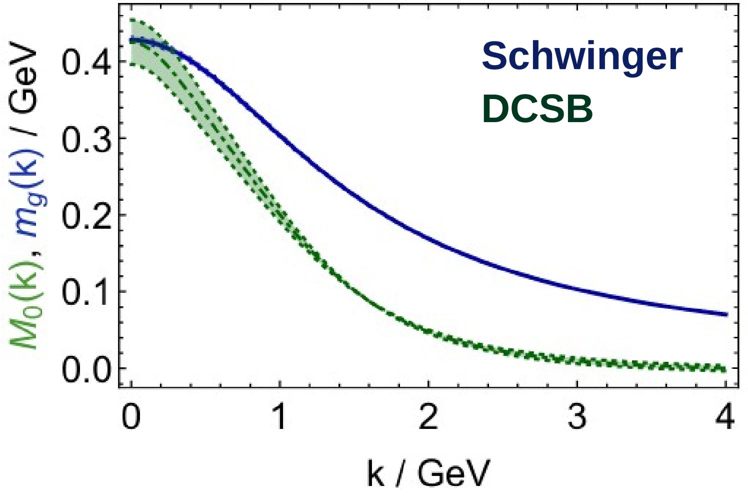


Cui:2019dwv



Gluon and quark *running masses*

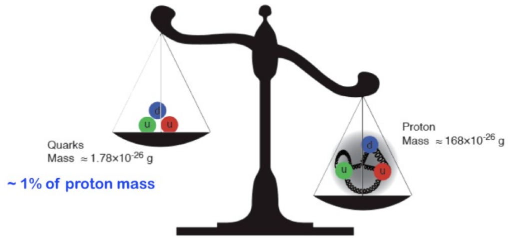
Mass **Budgets**

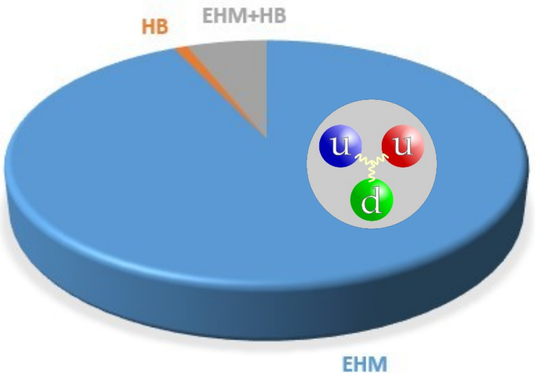
$$M_{u/d} \approx 0.3\,\text{GeV}$$

- What is the origin of **EHM**?

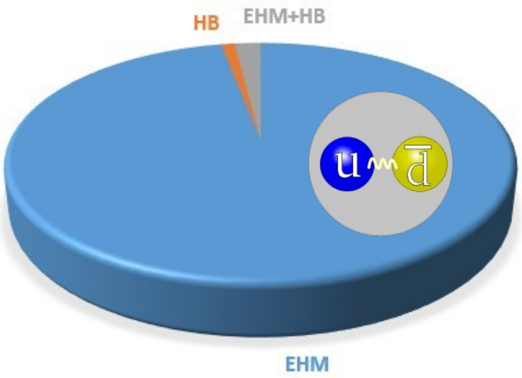
  … its connection with *e.g.* **confinement** and **DCSB**?

> - **Most** of the **mass** in the visible universe is contained within **nucleons**
>   - Which remain **pretty massive** whether there is Higgs mechanism or not...





Proton mass budget



Rho meson mass budget

$$m_p = 0.938\,\text{GeV} \approx 2M_u + M_d$$

$$m_\rho = 0.775\,\text{GeV} \approx M_u + M_d$$

Proton and **rho** meson mass budgets are practically **identical**

Mass **Budgets**

$$M_{u/d} \approx 0.3\,\text{GeV} \qquad \begin{array}{l} m_s/m_u \sim 20 \\ M_s/M_u \sim 1.2 \end{array}$$

- What is the origin of **EHM**?

  … its connection with e.g. **confinement** and **DCSB**?

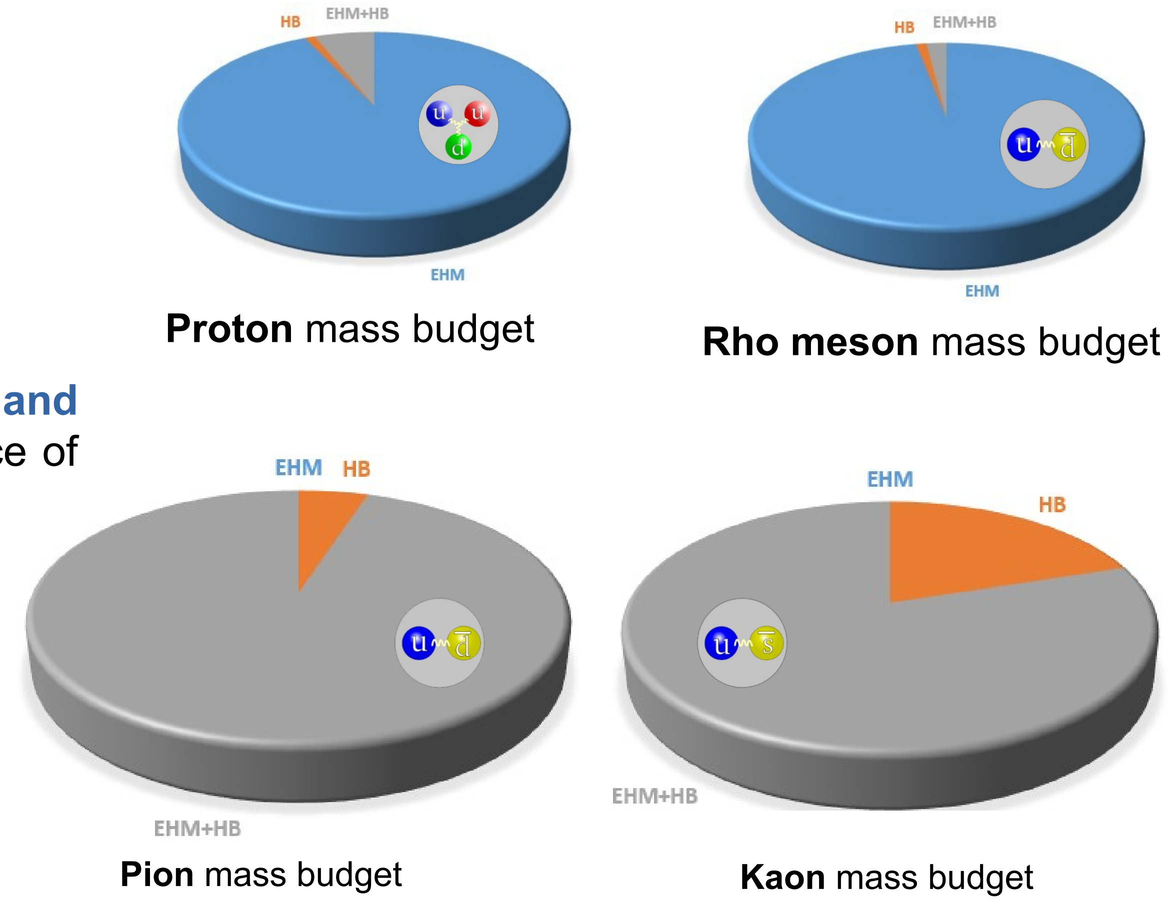
  And **Nambu-Goldstone** bosons?

- Unlike *e.g.* proton and ρ meson, **pion and Kaon** would be **massless** in the absence of **Higgs** mass generation.

  - And **structurally alike**.

$$m_\pi = 0.14\,\text{GeV} \neq M_u + M_d$$

$$m_K = 0.49\,\text{GeV} \neq M_u + M_s$$



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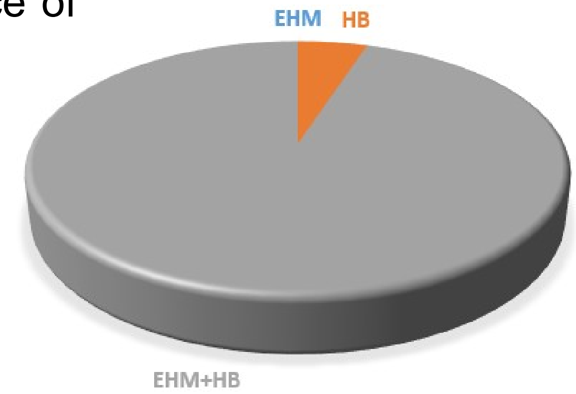
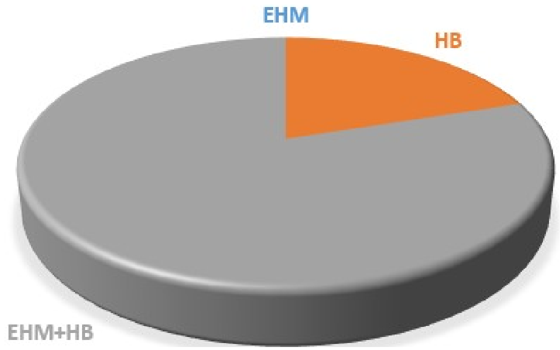
Pion & Kaon

- Both quark-antiquark **bound-states** and **NG bosons**
  - Their mere **existence** is connected with **mass** generation in the **SM**



Pion mass budget



Kaon mass budget

Mass **Budgets**

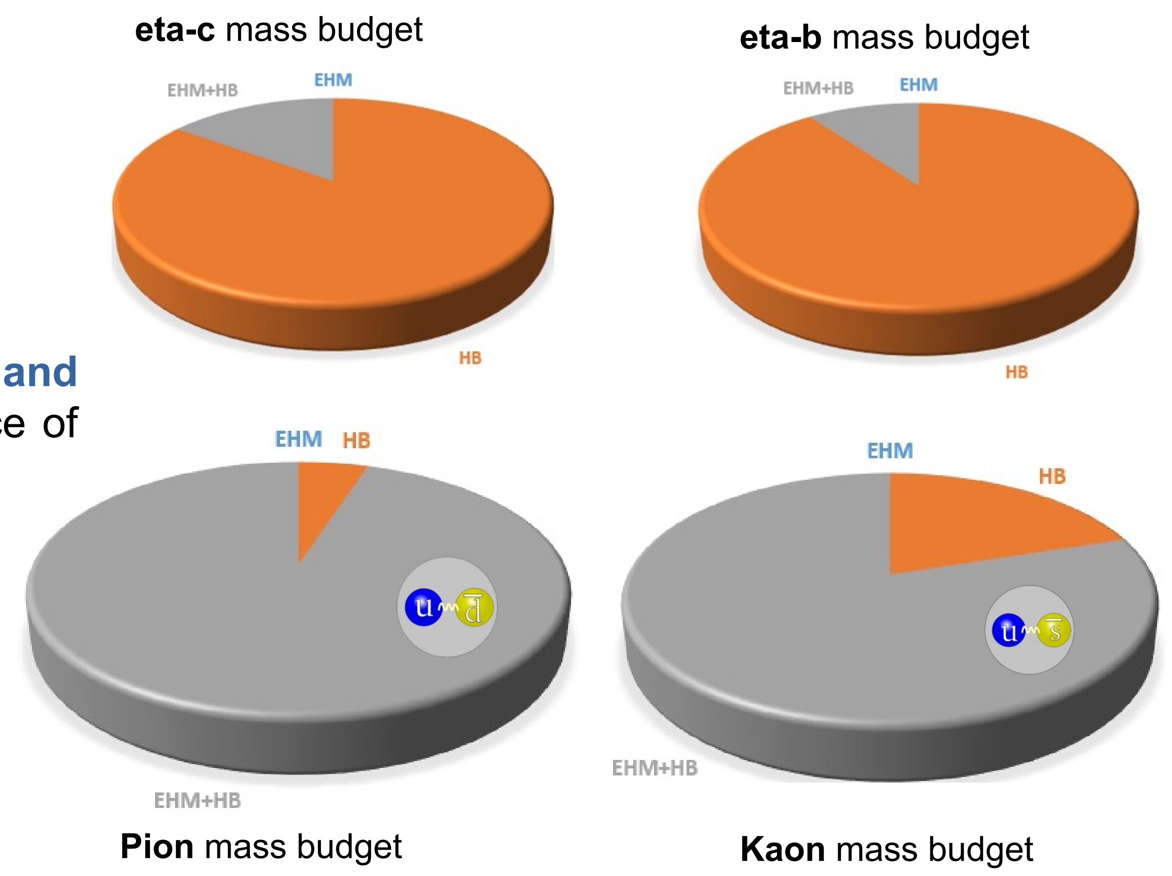
$M_{u/d} \approx 0.3\,\text{GeV}$ $m_s/m_u \sim 20$ $M_s/M_u \sim 1.2$

- What is the origin of **EHM**?

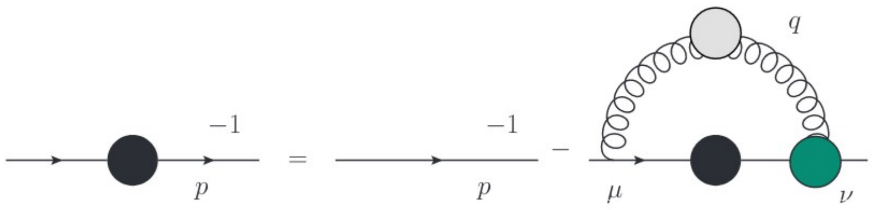
  … its connection with e.g. **confinement** and **DCSB**?

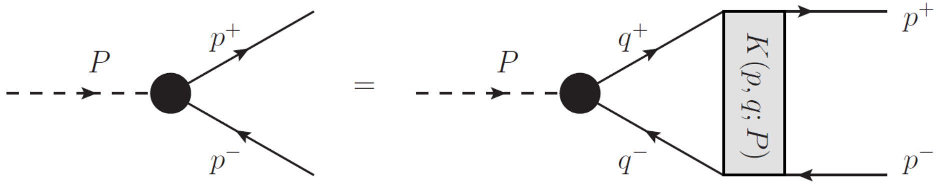
  And **Nambu-Goldstone** bosons?

- Unlike *e.g.* proton and ρ meson, **pion and kaon** would be **massless** in the absence of **Higgs** mass generation.
  - And **structurally alike**.
- The scrutiny of their heavier counterparts reveals the role of **weak mass generation** on the hadron **structural properties**.



Continuum Schwinger Methods
(CSM)





Dyson-Schwinger Equations

- Equations of motion of a **quantum field theory**
- Relate Green functions with higher-order Green functions

  → • **Infinite** tower of coupled equations.
  ✗ Systematic **truncation** required

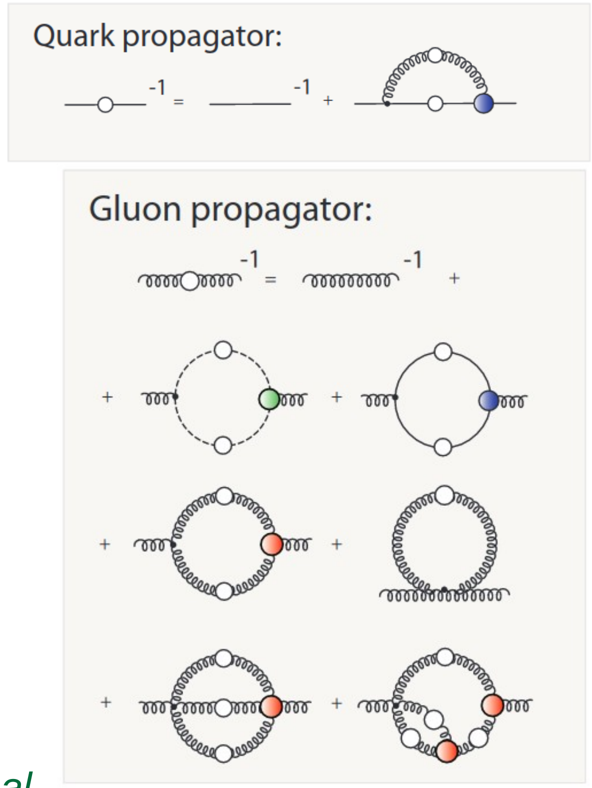
✓ **No assumptions** on the **coupling** for their derivation.

  → ✓ Capture both **perturbative** and **non-perturbative** facets of **QCD**

✓ **Not limited** to a certain domain of current **quark masses**

✓ Maintain a **traceable connection** to QCD.

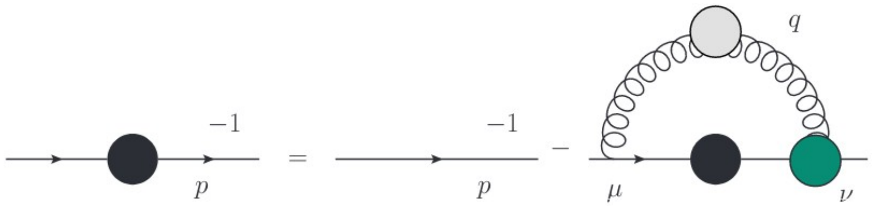
Example DSEs



C.D. Robert and A.G. Williams,
Prog.Part.Nucl.Phys. 33 (1994) 477-575

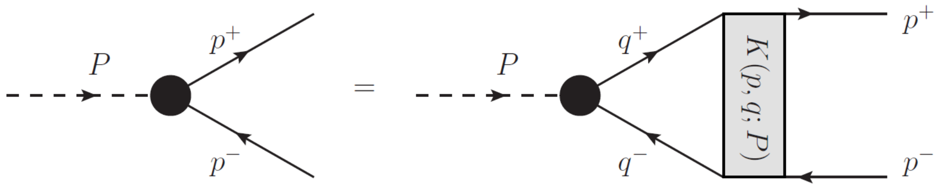
G. Eichmann, H. Sanchis-Alipus *et al.*
Prog.Part.Nucl.Phys. 91 (2016) 1-100

DSE-BSE approach



Quark DSE

- Relates the quark propagator with **QGV** and **gluon propagator**.



Meson BSE

- Contains **all interactions** between the valence quark and antiquark

- Any sensible truncation must preserve the **Goldstone's Theorem**, whose most fundamental expression is captured in:

"**Pions** exists, if and only if, **DCSB** occurs."

$$f_\pi E_\pi(k; P=0) = B(k^2)$$

Leading BSA

"***Mass*** Function"

Unveiling the pseudoscalars

L. Chang *et al.*,
Phys.Rev.Lett. 110 (2013) 13, 132001

Valence-quark distribution amplitudes (PDAs)

$$f_M \phi_M^q(x) = \mathrm{tr} \int_{dk} \delta_n^x(k_M) \gamma_5 \gamma \cdot n \chi_M(k_-, P)$$

Light-front momentum fraction

Written in terms of **BSWF**

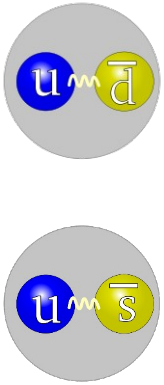
- Analogous with **quantum mechanic's** wave function (sort of).
- Clear **probe of EHM**, related with hard **exclusive processes**, etc.

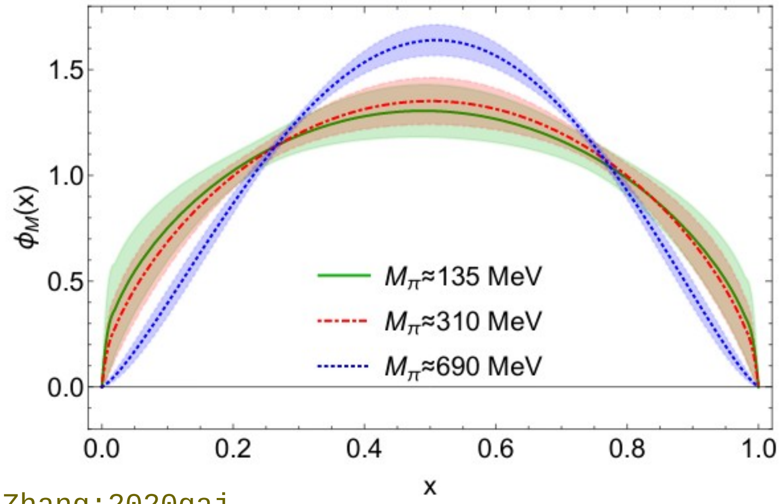
π-K **PDAs**

$m_s/m_u \sim 20$
$M_s/M_u \sim 1.2$

- **Broad** and concave **PDAs**.
- Dominated by **QCD dynamics**.
- **Mild** skewing in **Kaon**: **strong** & **weak** interplay.

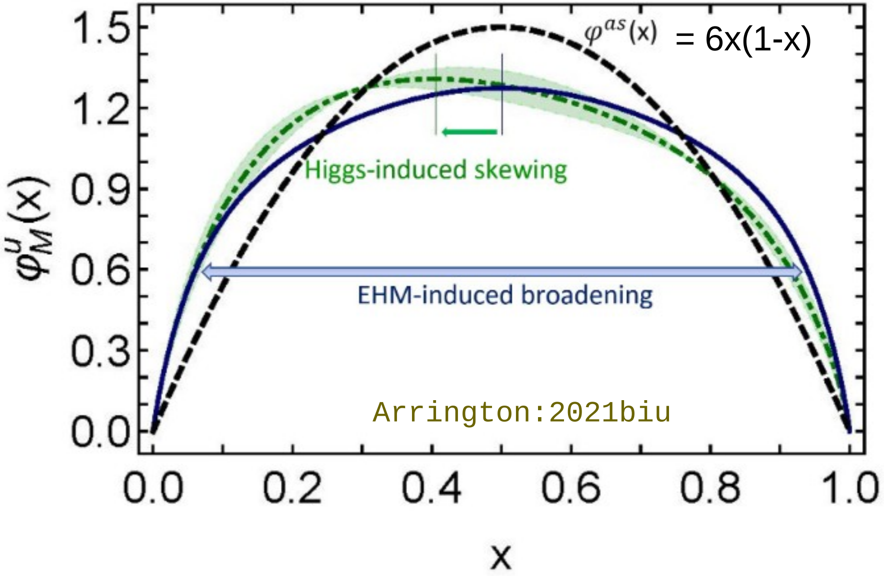
@ real life
(order of GeVs)





Zhang:2020gaj

[Blue]: **Pion** [Green]: **Kaon**



- Lattice QCD supports those findings:

Zhang:2020gaj Bali:2019dqc Segovia:2013eca

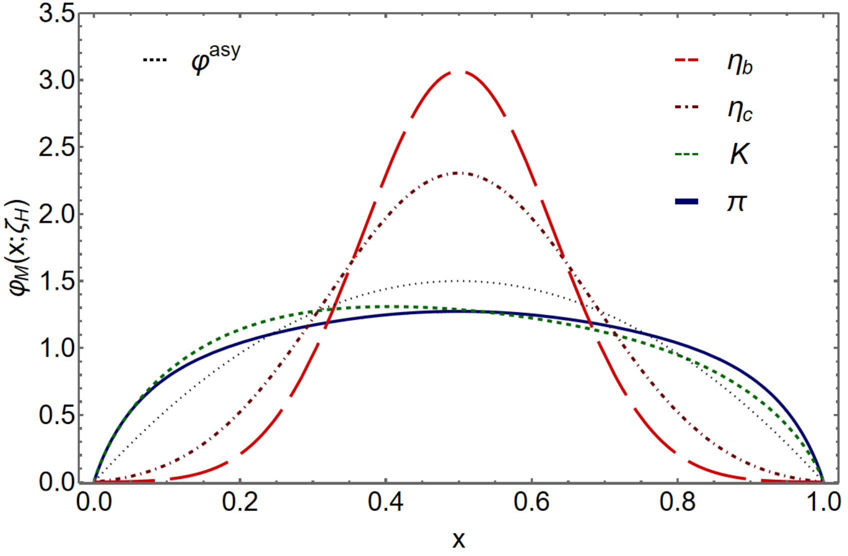
Pointwise form of the **PDA**!.

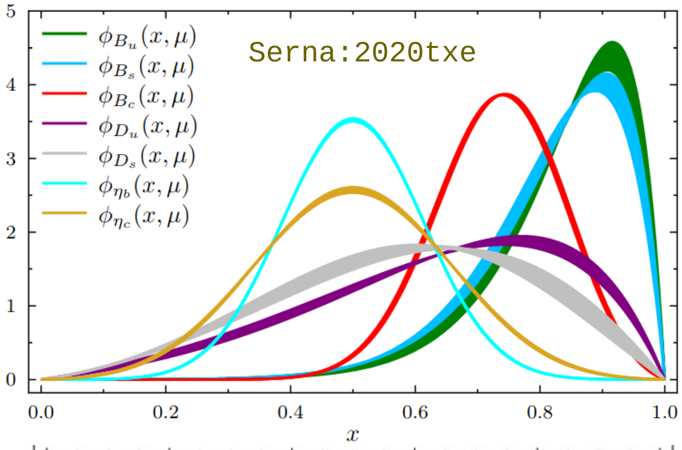
Heavy mesons PDAs

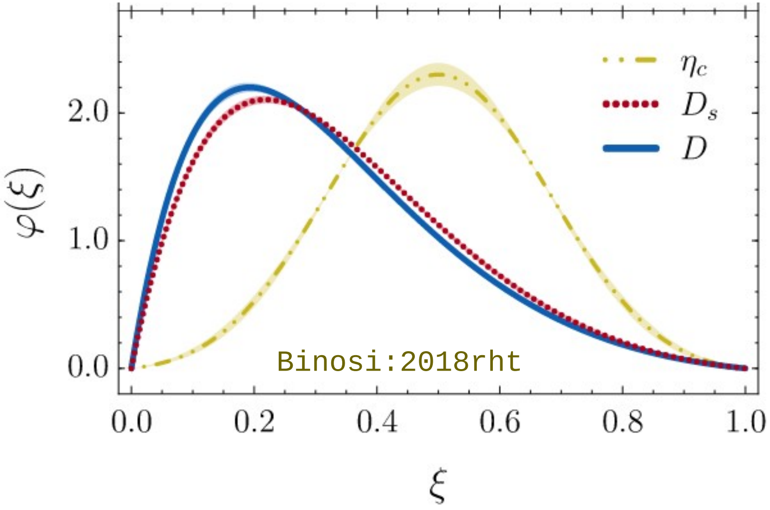
- Largely influenced by **Higgs** mass generation.
  - ➔ **Narrow** PDAs.

@ real life
(order of GeVs)

- Narrowness also observed in **heavy-light** mesons.



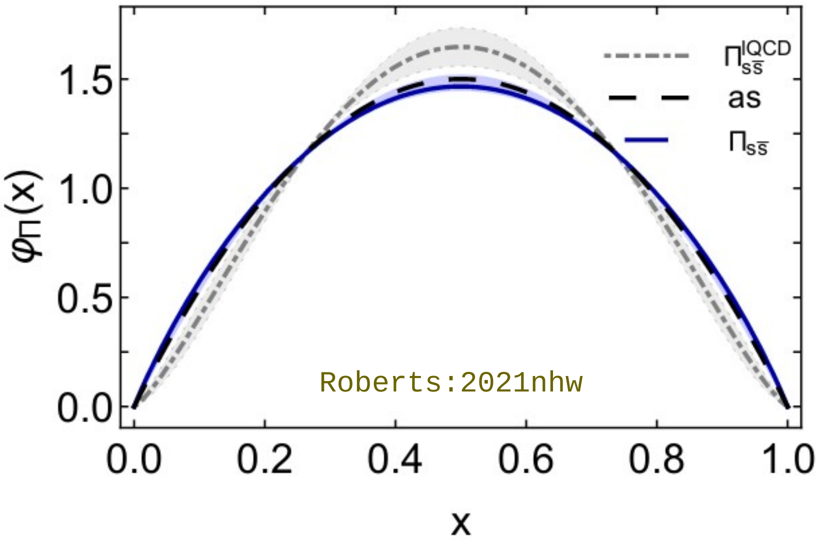




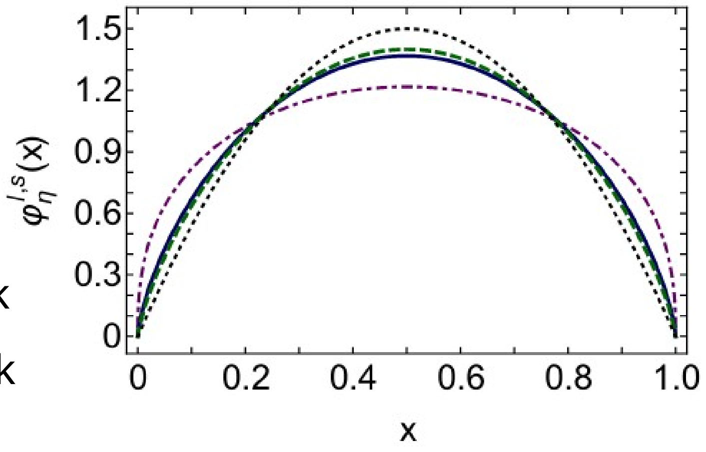
‘Strange’ PDAs

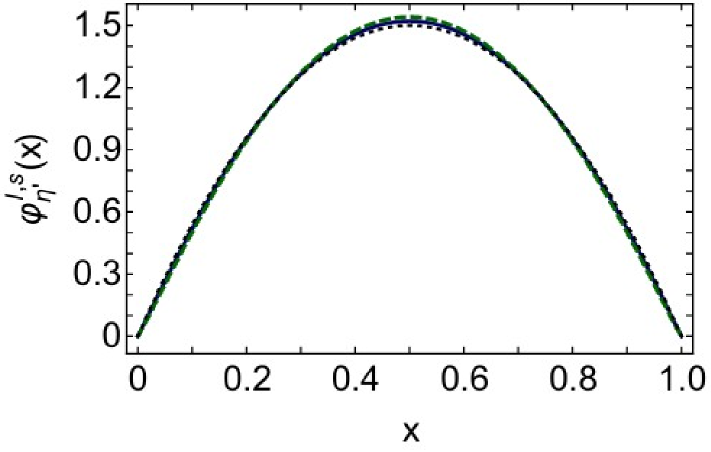
M. Ding, KR *et al.*
Phys.Rev.D 99 (2019) 1, 014014

- **s-quark** mass: **interplay** between **strong** and **Higgs** mass-generation.
- **PDAs** lie **near** the **asymptotic distribution**. @ any scale



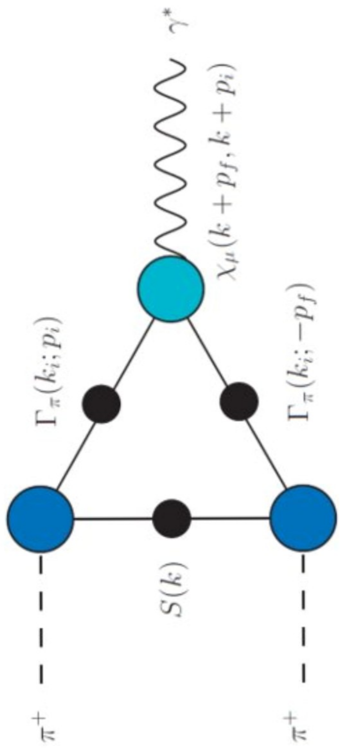
[**Blue**]: u-quark
[**Green**]: s-quark
[**Purple**]: Pion
[**Black**]: 6x(1-x)





L. Chang *et al.*,
Phys.Rev.Lett. 111 (2013) 14, 141802

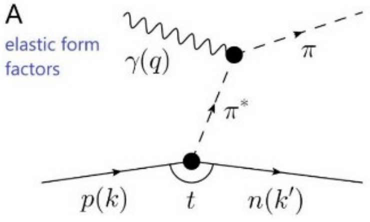
Electromagnetic Elastic Form Factors (EFFs)



$$K_\mu F_M(Q^2) = N_c \mathrm{tr} \int_{dk} \chi_\mu(k+p_f, k+p_i)\Gamma_M(k_i;p_i)S(k)\gamma_M(k_f;-p_f)$$

All can be written in terms of **propagators** and **vertices**

- Gives information on **momentum**/**charge** distribution.
- **Pion EFF** highly relevant for contemporary physics.

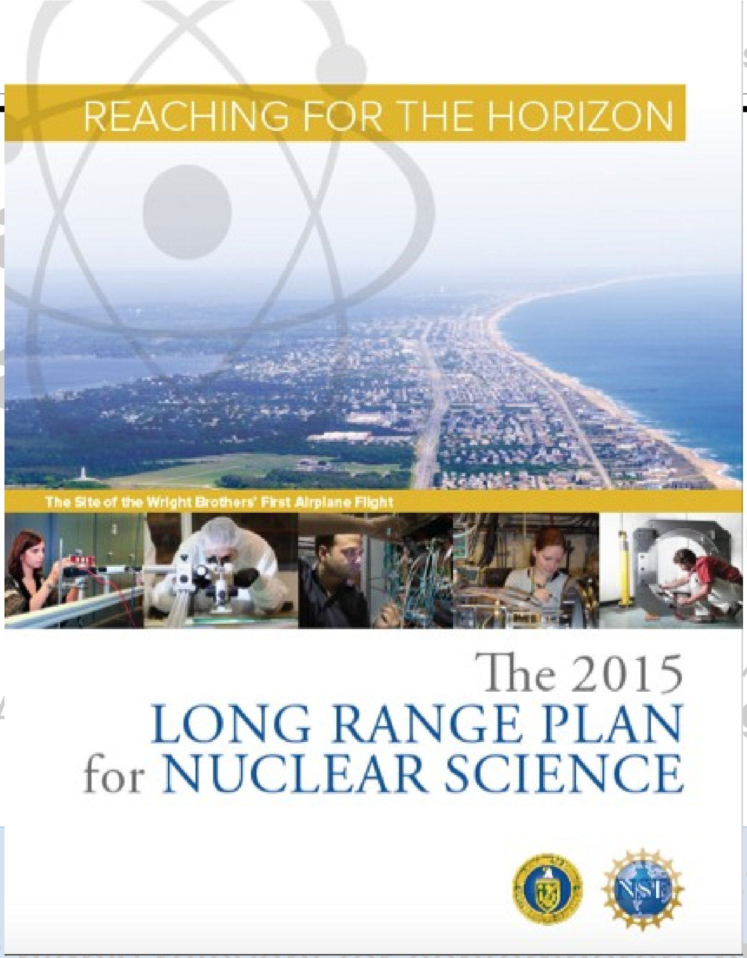


L. Chang *et al.*,
s.Rev.Lett. 111 (2013) 14, 141802

Electroma ic Form Fa

$K_\mu F_M(Q^2)$ ... $(k_i;p_i)S(k)\gamma_M(k_f;-p_f)$

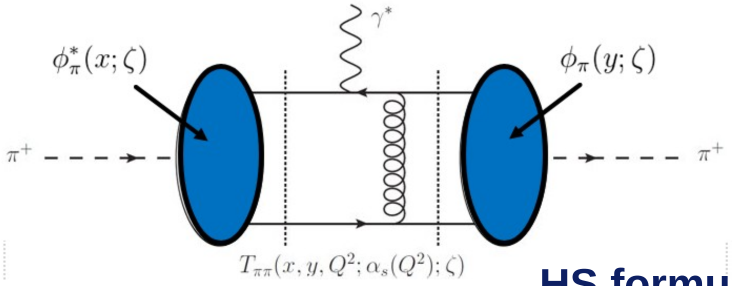
A gators and vertices



- Gives info tribution.
- **Pion EFF** highly relevant for contemporary physics.

Elastic Form Factors

- Clear probe of the hadron's structure.
  - Structure manifests in $F(Q^2)$!= constant
- **Connected** with the **PDA**:



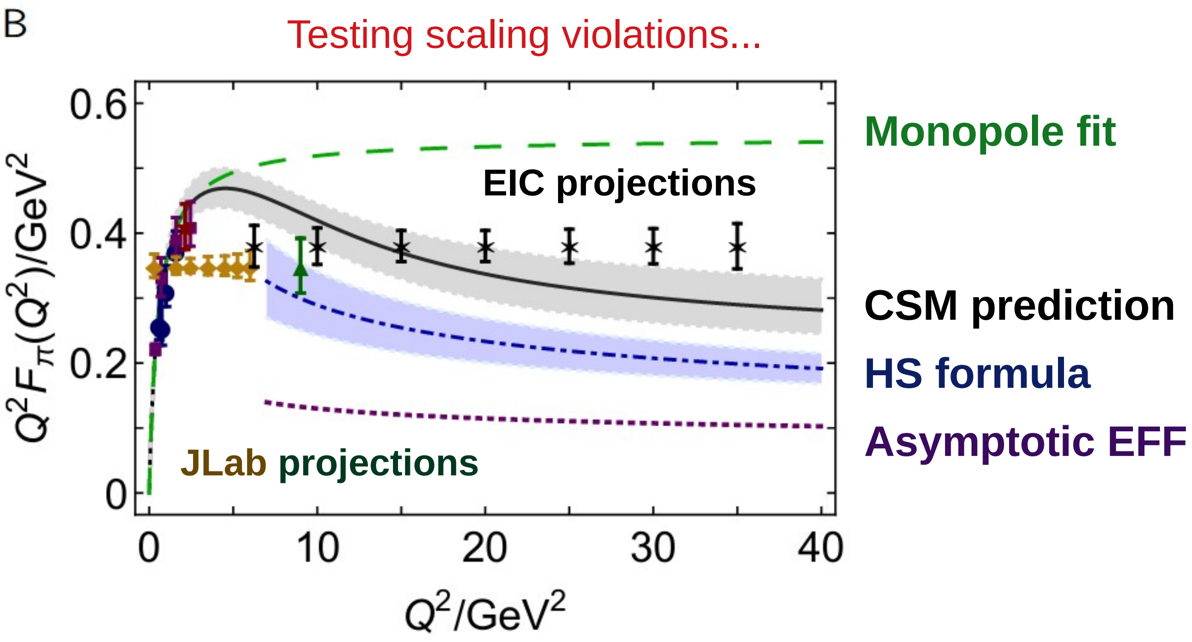
HS formula

(at sufficiently large Q^2)

- **Factorization** is a proof of the validity of **QCD itself.**

Lepage:1980fj

Testing scaling violations...



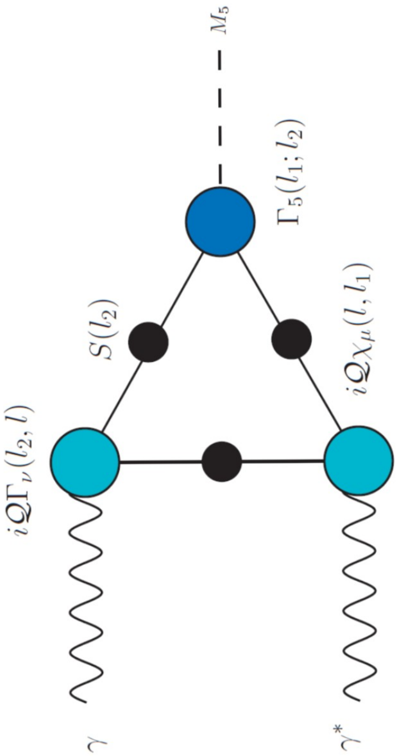
$$\boxed{\exists Q_0 > \Lambda_{\rm QCD} \mid Q^2 F_\pi(Q^2) \overset{Q^2>Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 w_\varphi^2,}$$

$$w_\varphi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_\pi(x)$$

PDA

KR, L. Chang *et al.*,
Phys.Rev.D 93 (2016) 7, 074017

Two-photon Transition Form Factors (TFFs)

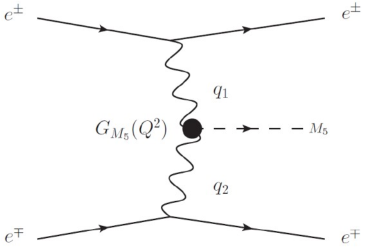


$$T_{\mu\nu}(k_1,k_2) = \epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} G_{M_5}(k_1^2, k_1\cdot k_2, k_2^2) \ ,$$

$$T_{\mu\nu}(k_1,k_2) = \mathrm{tr}\int \frac{d^4 l}{(2\pi)^4} i\mathcal{Q}\chi_\mu(l,l_1)\Gamma_{M_5}(l_1,l_2)S(l_2)i\mathcal{Q}\Gamma_\nu(l_2,l)$$

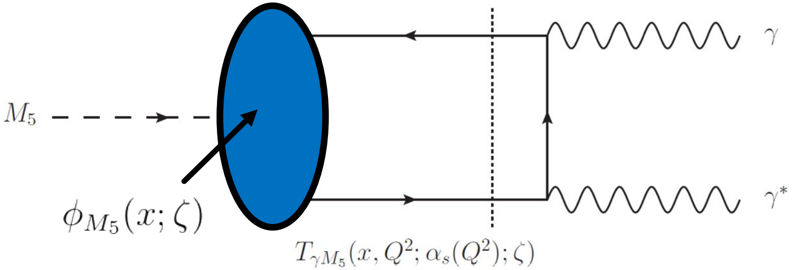
All can be expressed in terms of **propagators** and **vertices**

- Gives information on **momentum**/**charge** distribution.
- **Pion TFF** highly relevant for contemporary physics.



Two-photon TFFs

- Clear probe of the hadron's structure.
    - Structure manifests in $G(Q^2)$!= constant
- **Connected** with the **PDA**:



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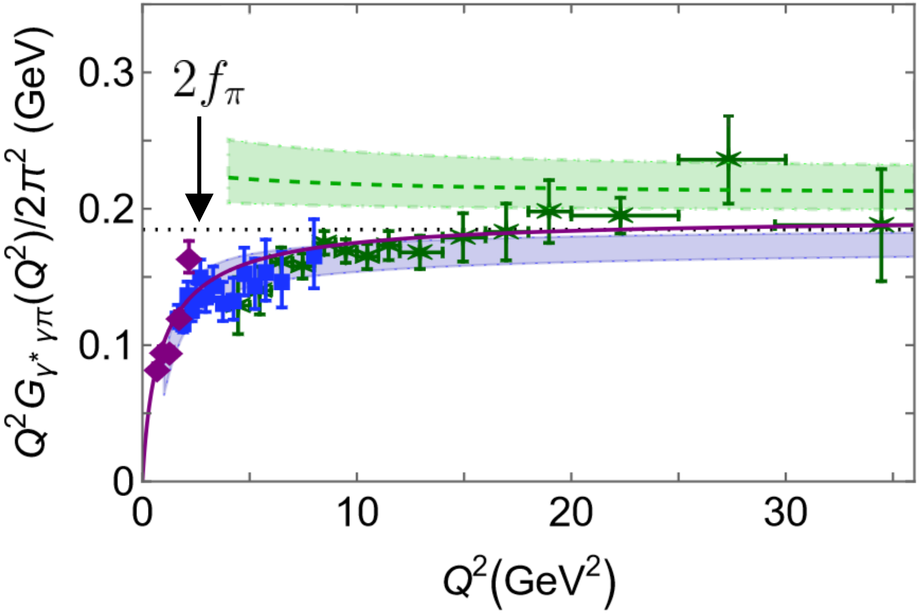


$$\boxed{\exists \tilde{Q}_0 > \Lambda_{\mathrm{QCD}} | Q^2 G_5(Q^2) \overset{Q^2 > \tilde{Q}_0^2}{\sim} f_5 \, w_\phi}$$

$$w_\varphi = \frac{1}{3}\int_0^1 dx \, \frac{1}{x} \varphi_\pi(x)$$

PDA

Two-photon TFFs

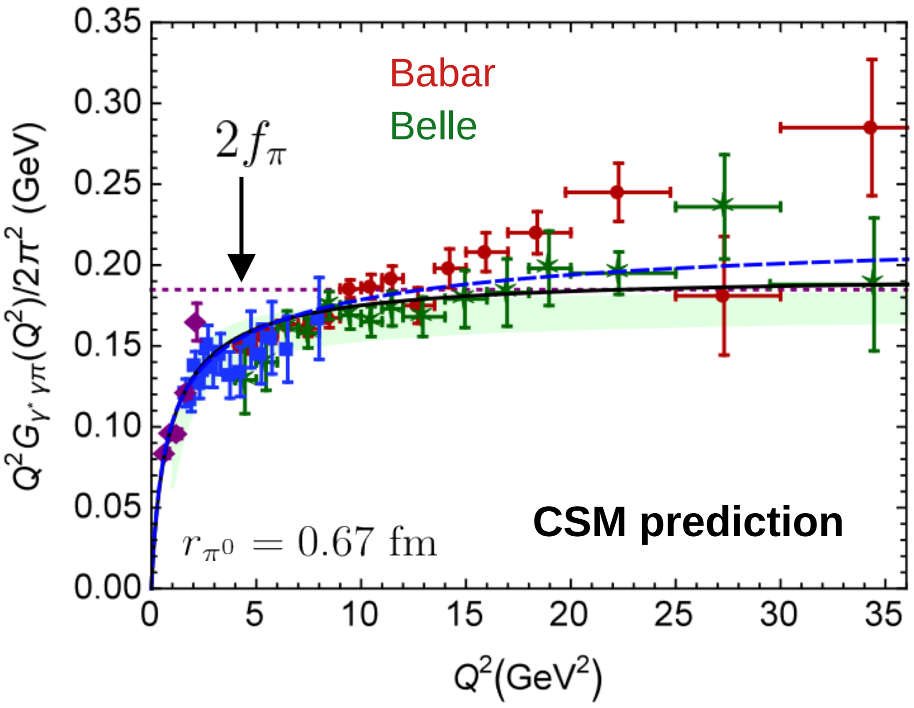
KR, L. Chang, A. Bashir *et al.*,
Phys.Rev.D 93 (2016) 7, 074017

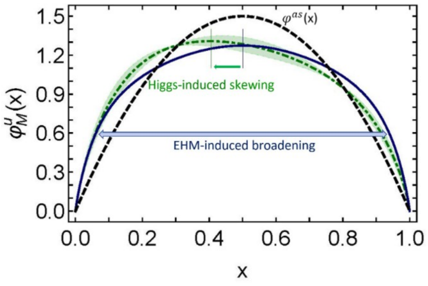
- The **CSM prediction** satisfies the **Abelian anomaly**, while being faithfully recovering the **asymptotic limit**.

- A **dilated+concave** **PDA**, at the hadronic scale, connects both pion **EFF** and **TFF**.

- Precise **agreement** with ***all*** experimental data; except for Babar at large **Q^2**.

  → Cannot conciliate with Babar...





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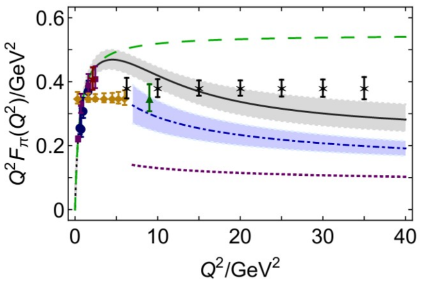

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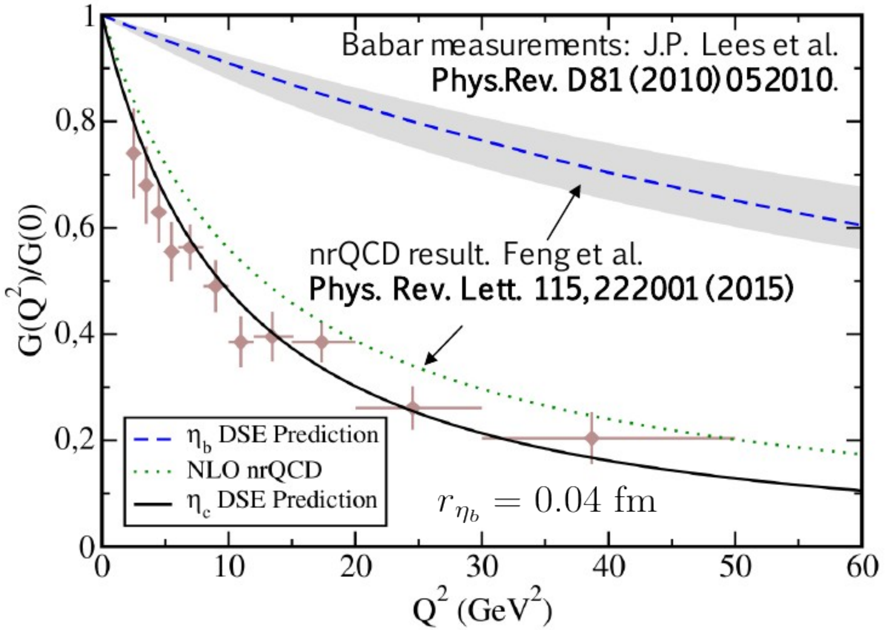
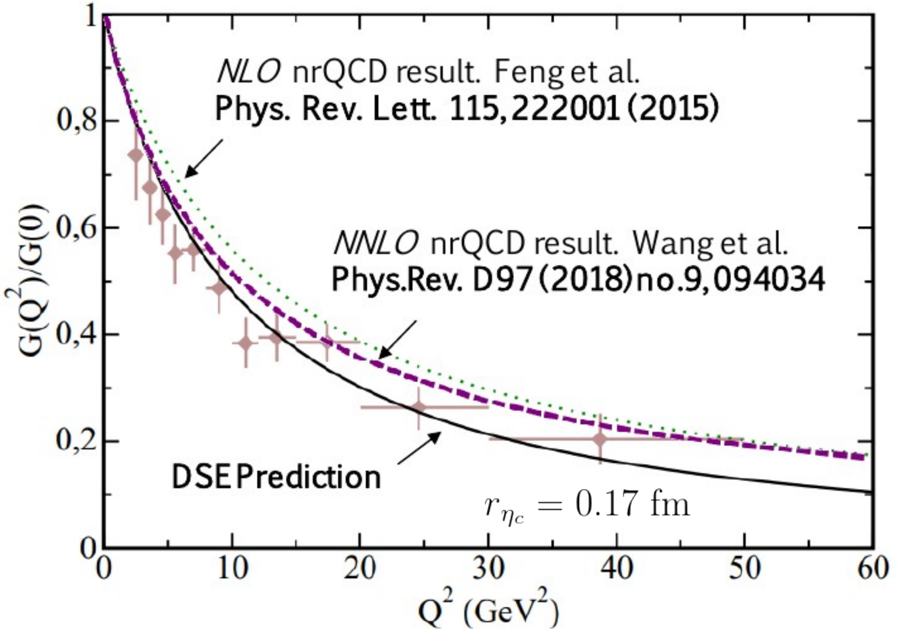
Two-photon TFFs

KR, M. Ding *et al.*
Phys.Rev.D 95 (2017) 7, 074014

- ***All*** two-photon **TFFs** involving ground-state neutral pseudoscalars are within reach:
  - Invariably, **agreement** with the **experimental** data is found, with the exception of the large-Q^2 Babar data for the pion.

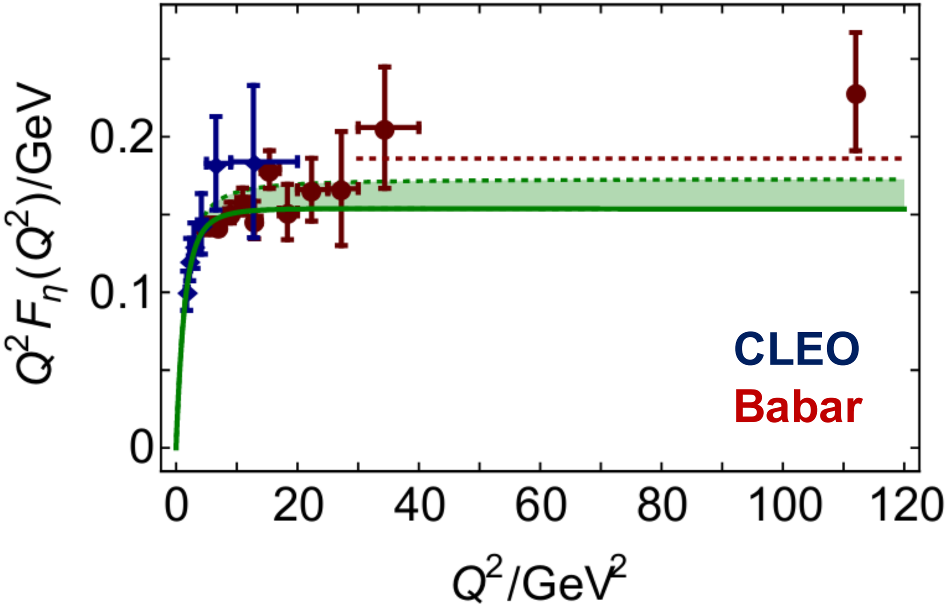


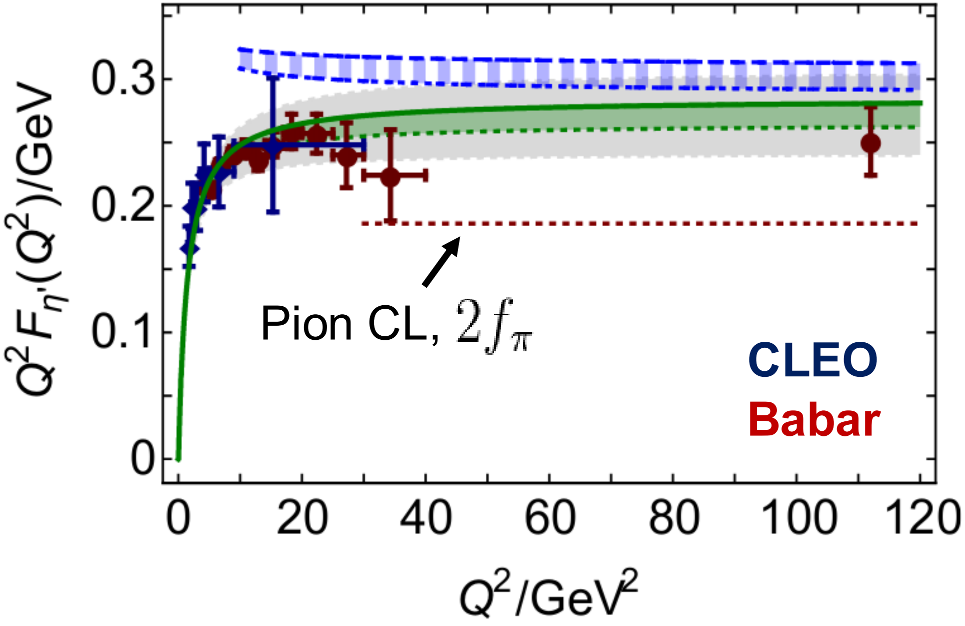


Two-photon **TFFs**

KR, M. Ding *et al.*
Phys.Rev.D 95 (2017) 7, 074014

- ***All*** two-photon **TFFs** involving ground-state neutral pseudoscalars are within reach:
  - Invariably, **agreement** with the **experimental** data is found, with the exception of the large-Q^2 Babar data for the pion.





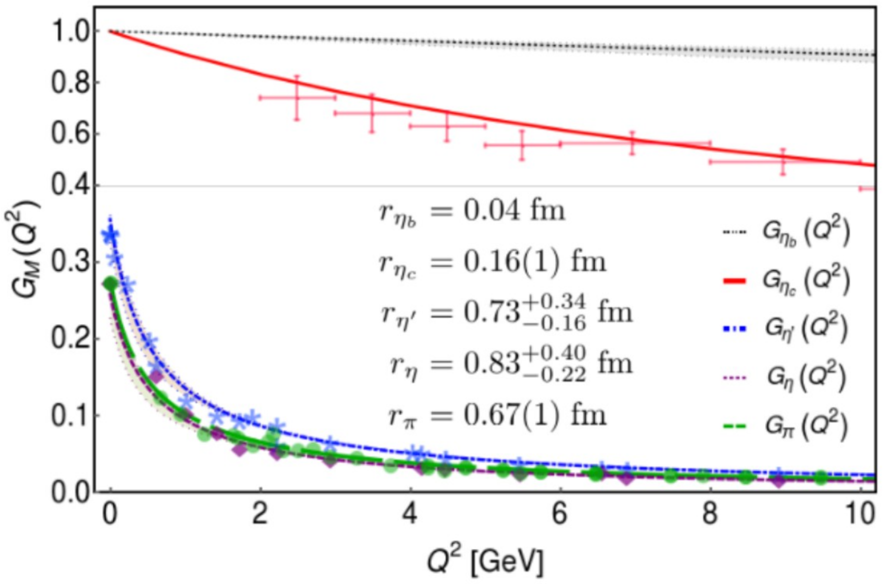
Muon g-2 endeavors

KR, A. Bashir, P. Roig
Phys.Rev.D 101 (2020) 7, 074021

- With ***All*** two-photon **TFFs** at hand we ventured into **muon g-2** related endeavors and become part of the White Paper appearing in Physics Reports:

‘The anomalous magnetic moment of the muon in the Standard Model’

T. Aoyama *et al.* Phys.Rept. 887 (2020) 1-166



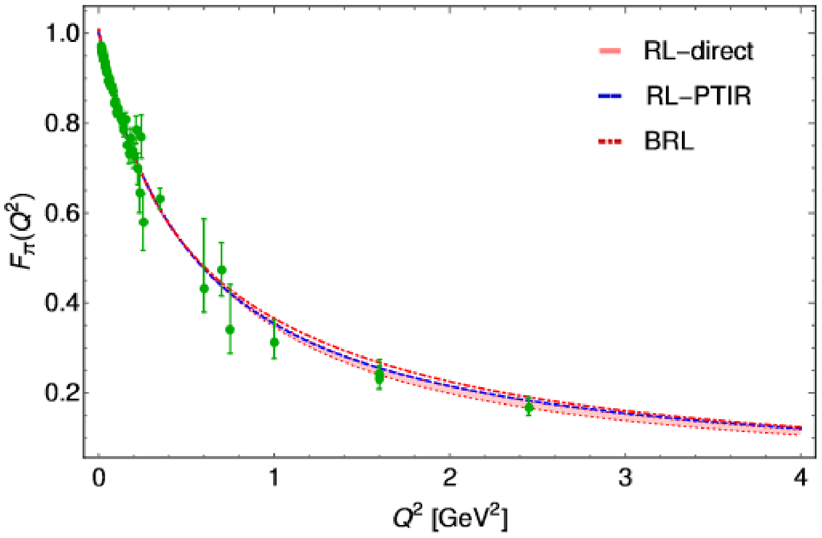
$$\begin{aligned}
a_\mu^{\pi^0-\text{pole}} &= (6.14 \pm 0.21)\cdot 10^{-10}\,,\\
a_\mu^{\eta-\text{pole}} &= (1.47 \pm 0.19)\times 10^{-10}\,,\\
a_\mu^{\eta'-\text{pole}} &= (1.36 \pm 0.08)\times 10^{-10}\,,\\
a_\mu^{\eta_c-\text{pole}} &= (0.09 \pm 0.01)\times 10^{-10}\,,\\
a_\mu^{\eta_b-\text{pole}} &= (0.26 \pm 0.01)\times 10^{-13}\,.
\end{aligned}$$

$$a_\mu^{\text{p-pole}} = (9.06 \pm 0.49) \times 10^{-10}$$

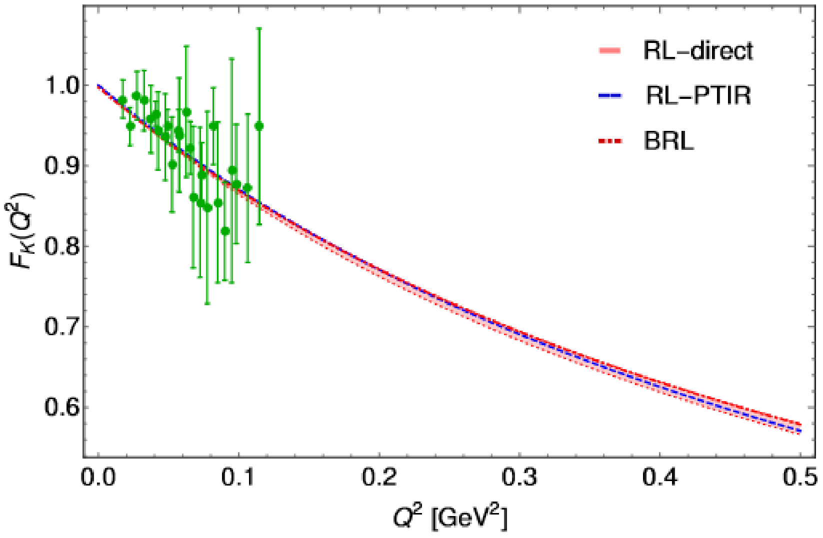
Muon g-2 endeavors

A. Miramontes, A. Bashir, KR, P. Roig
Phys.Rev.D 105 (2022) 7, 074013

- We continue on this path through the computation of the pion and kaon **box contributions**, stemming from their corresponding elastic electromagnetic form factors.



$$a_\mu^{\pi^\pm-box} = -(15.6 \pm 0.2) \times 10^{-11}$$



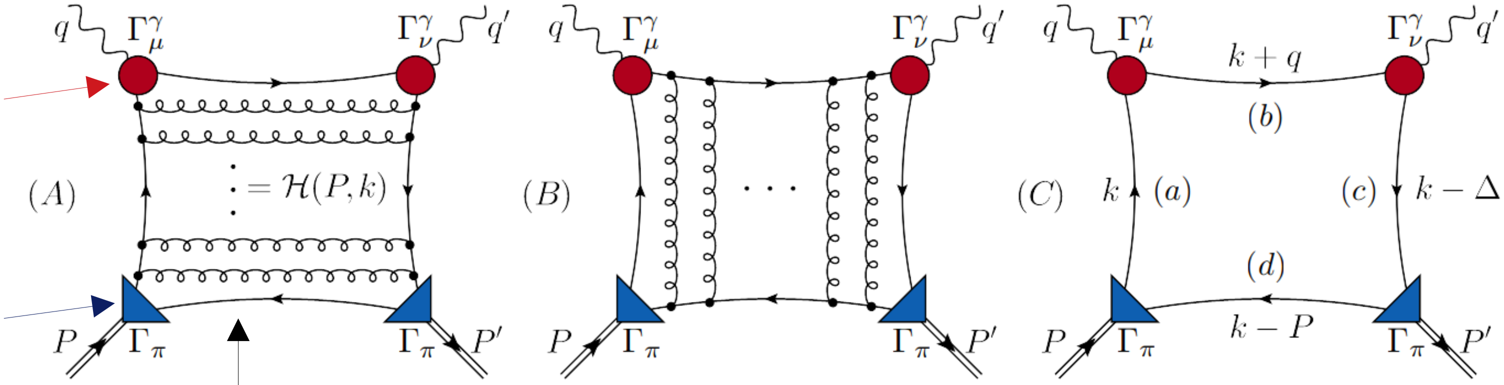
$$a_\mu^{K^\pm-\mathrm{box}} = -(0.48 \pm 0.02) \times 10^{-11}$$

M. Ding *et al.*, PRD 101 5, 054014 (2020)
Z-F Cui *et al.*, EPJC 80 11, 1064 (2021)

Distribution functions (PDFs)

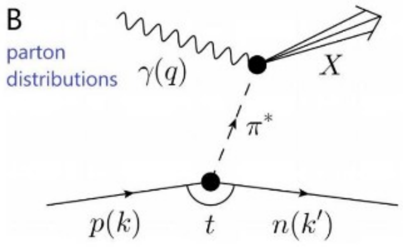
Insertion

BSA



Propagator

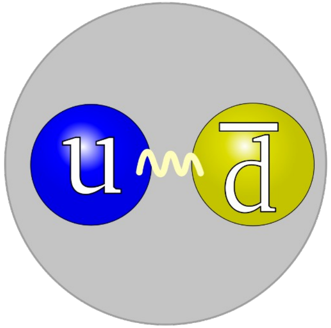
Diagrams that contribute to the (**valence**) **PDF**



- Yields information on **momentum** distribution.
- Evolution disentangles **valence**, **sea** and **gluon** contributions.

M. Ding *et al.*, PRD 101 5, 054014 (2020)
Z-F Cui *et al.*, EPJC 80 11, 1064 (2021)

Distribution functions (PDFs)

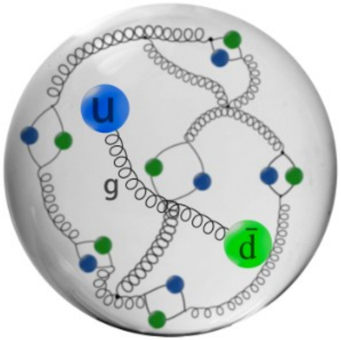


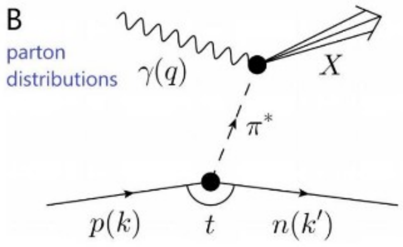
ζ_H

Resolution Scale

Evolution equations

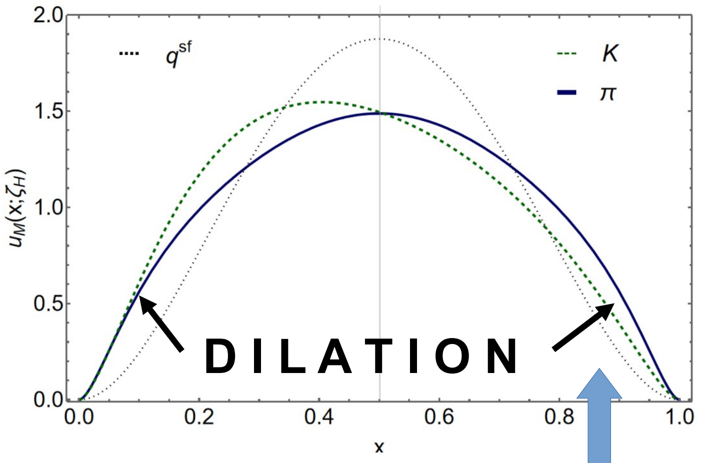
$\zeta > \zeta_H$

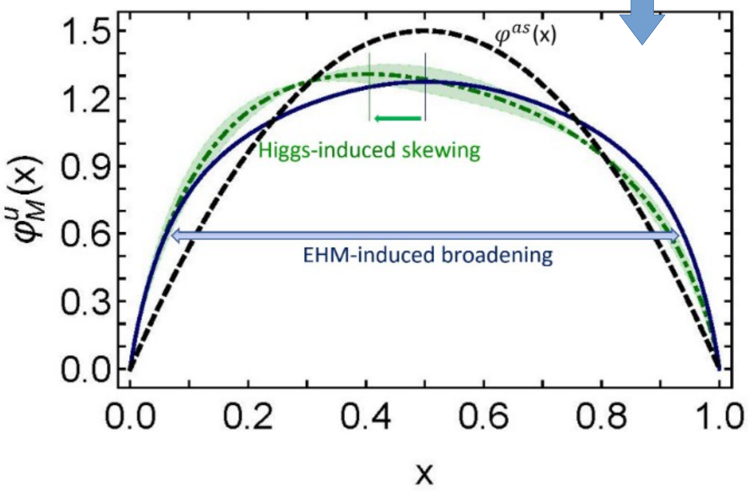




- Yields information on **momentum** distribution.
- Evolution disentangles **valence**, **sea** and **gluon** contributions.

π-K PDFs: Hadronic Scale





- As the **PDAs**, the **π-K PDFs** are **dilated**.
- The kaon distributions are only-shifted by a **few-percentage**.
  - ➔ QCD's EHM is still dominant. $m_s/m_u \sim 20$ $M_s/M_u \sim 1.2$
  - ➔ The momentum fractions at ζ_H:

$$< x >_u^\pi = 0.5 \qquad < x >_u^K = 0.47\,, \; < x >_s^K = 0.53$$

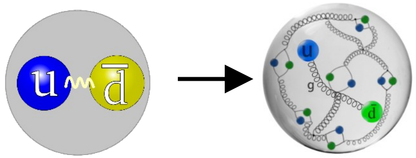
- The **bridge** between **PDA** and **PDF** is the **light-front wavefunction**:

$$f_P \varphi_P^u(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3} \psi_P^u\left(x, k_\perp^2; \zeta_{\mathcal{H}}\right)$$

$$u^P(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3} \left|\psi_P^u\left(x, k_\perp^2; \zeta_{\mathcal{H}}\right)\right|^2$$

EHM manifests in **PDAs, PDFs, LFWFs...**

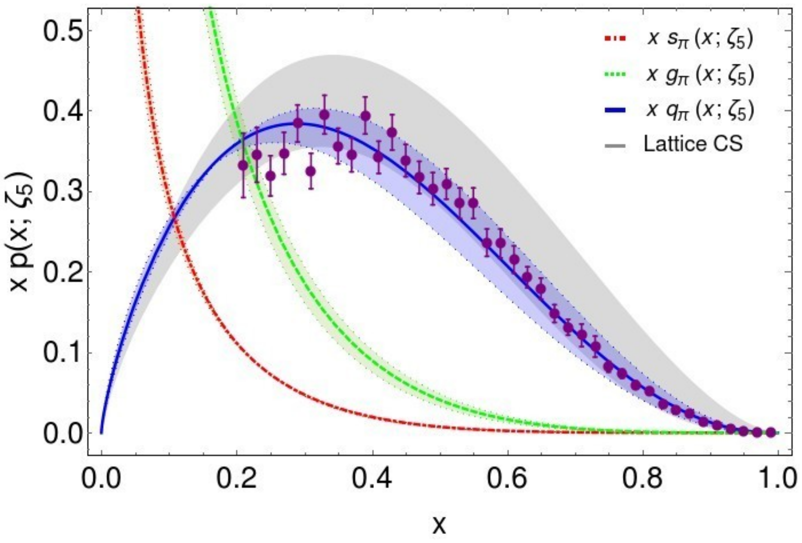
Pion PDFs: Lattice & Experiment



- At **5.2 GeV**, the experimental scale, our predictions matches that from Aicher *et al.*

Aicher:2010cb

$$<x_{\text{gluon}}> = 0.45(1)\ ,\ <x_{\text{sea}}> = 0.14(2)$$



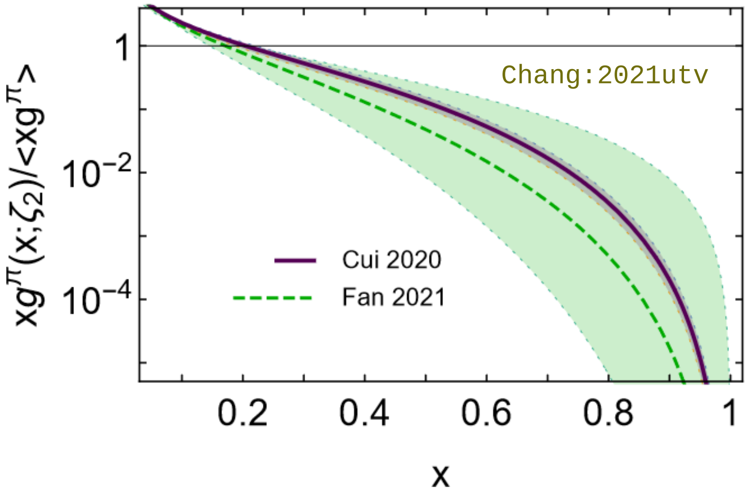
- An agreement with novel **lattice** "Cross Section" results is also obtained.

Sufian:2019bol

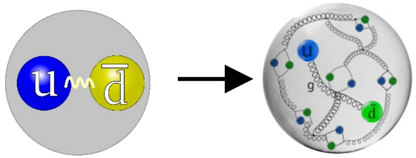
- At **2 GeV**, the **valence DF** shows agreement with lattice moments:

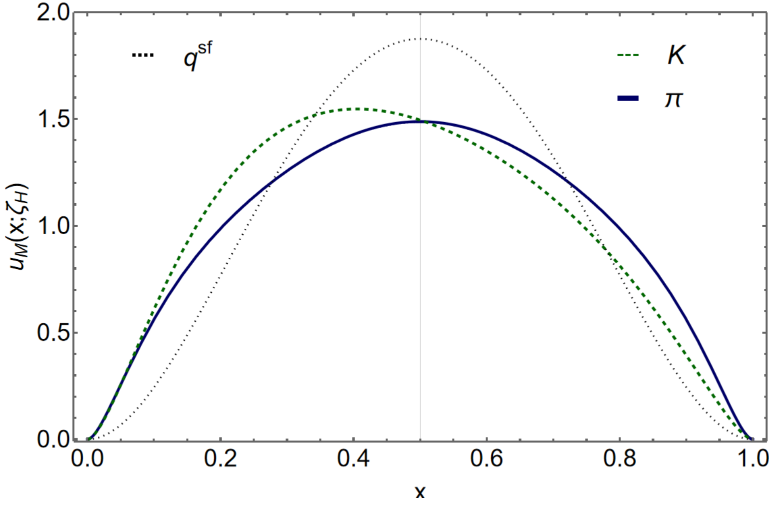
ζ_2	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
Ref. [34]	0.24(2)	0.09(3)	0.053(15)
Ref. [35]	0.27(1)	0.13(1)	0.074(10)
Ref. [36]	0.21(1)	0.16(3)	
Herein	0.24(2)	0.098(10)	0.049(07)

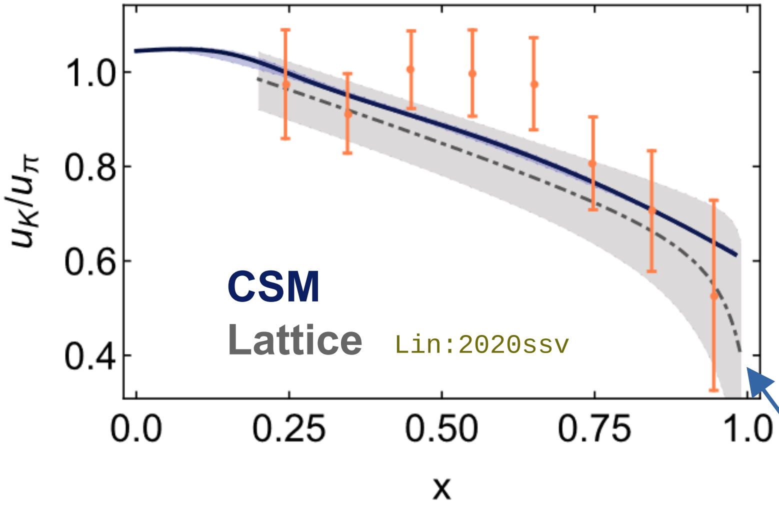
- The **Gluon DF** profiles matches **lattice** expectations:



Kaon PDFs: Lattice & Experiment







- Ratio is good but **too forgiving!**
- Besides, there are only **few data points**

➢ At the hadronic scale ζ_H, one has:

$$< x >^{\pi}_{u} = 0.5 \qquad < x >^{K}_{u} = 0.47\,, \; < x >^{K}_{s} = 0.53$$

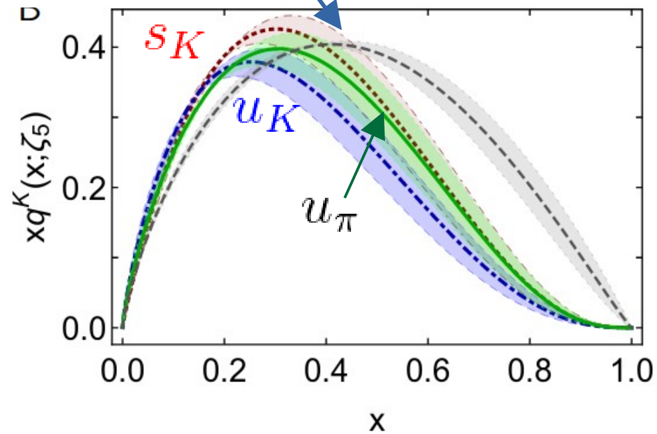
➢ **Above $\zeta > \zeta_H$**, there is slightly more valence content in K.

Massiveness of the s-quark is considered in the evolution equations.

$$<\mathbf{x}>^{\text{val}}_{\pi} = 0.41(4)$$

$$<\mathbf{x}>^{\text{val}}_{K} = 0.43(4)$$

$$\zeta = 5.2\,\text{GeV}$$



Pion DF vs Proton DF

Y. Lu *et al.*
Phys.Lett.B 830 (2022) 137130

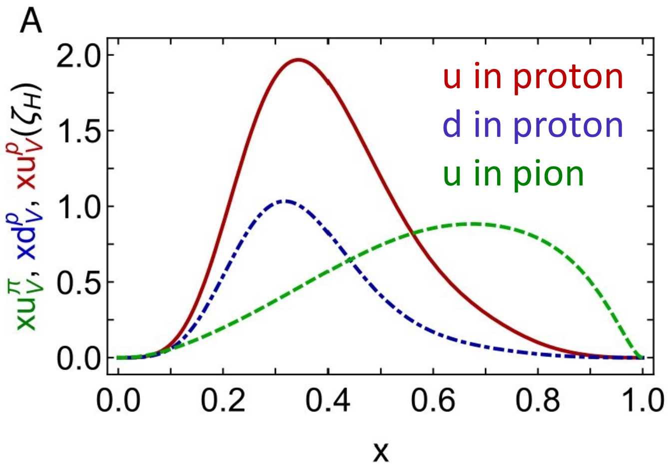
- The (nearly) massless **pion DFs** differs vastly from the massive **proton**. For instance:
  - The momentum fractions at ζ_H: $(M_u = M_d)$

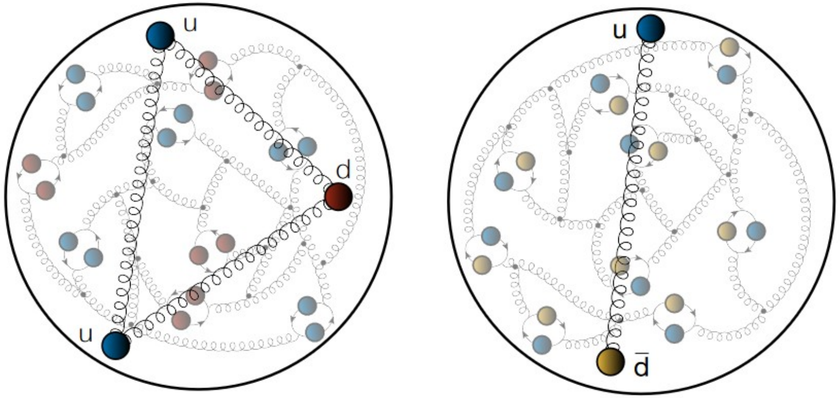
$$\langle x\rangle_{u_p}^{\zeta_\mathcal{H}} = 0.687\,,\ \langle x\rangle_{d_p}^{\zeta_\mathcal{H}} = 0.313\,,\ \langle x\rangle_{u_\pi}^{\zeta_\mathcal{H}} = 0.5$$

$$\Rightarrow u_V\,(x) \neq 2d_V(x)$$

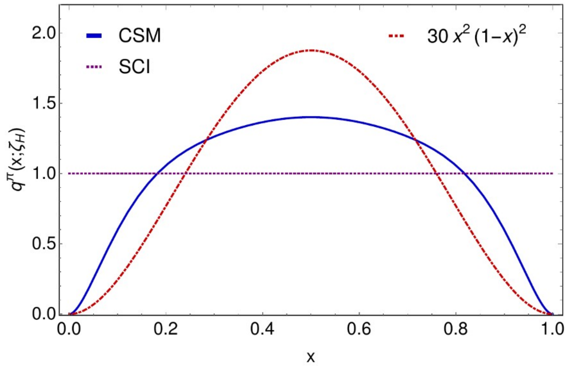
EHM-induced diquark correlations inside the proton:

- No equitable distribution of momentum!





- Marked dilation of the **pion PDF** at ζ_H.



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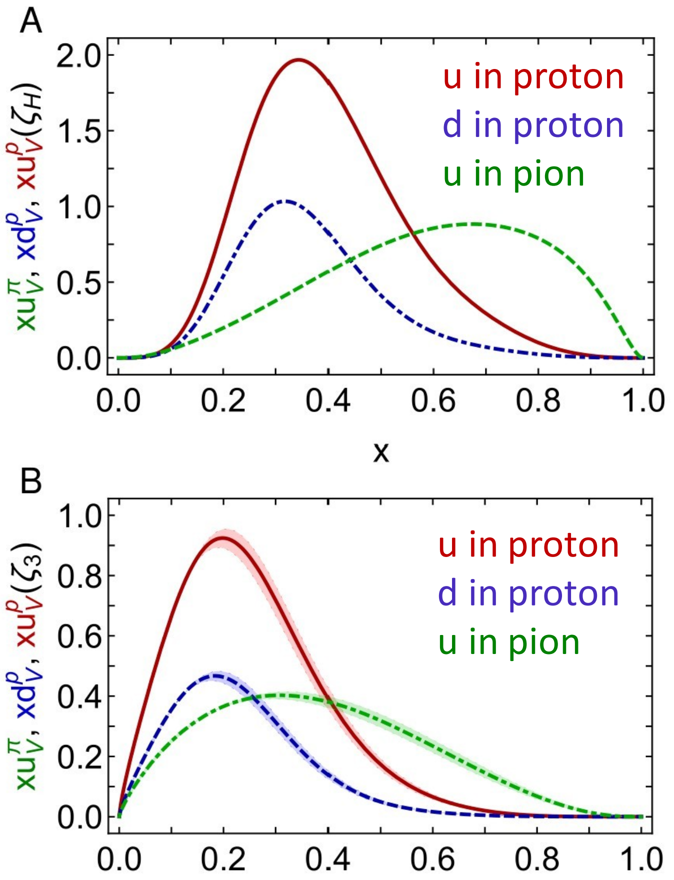
EHM induced diquark correlations inside the proton:

- → No equitable distribution of momentum!

  - Counting rules entail large-x behaviors **(1-x)²** and **(1-x)³** for the **pion** and **proton**, respectively.
  - Marked dilation of the **pion PDF** at ζ_H.
  - Differences are **preserved** after evolution.



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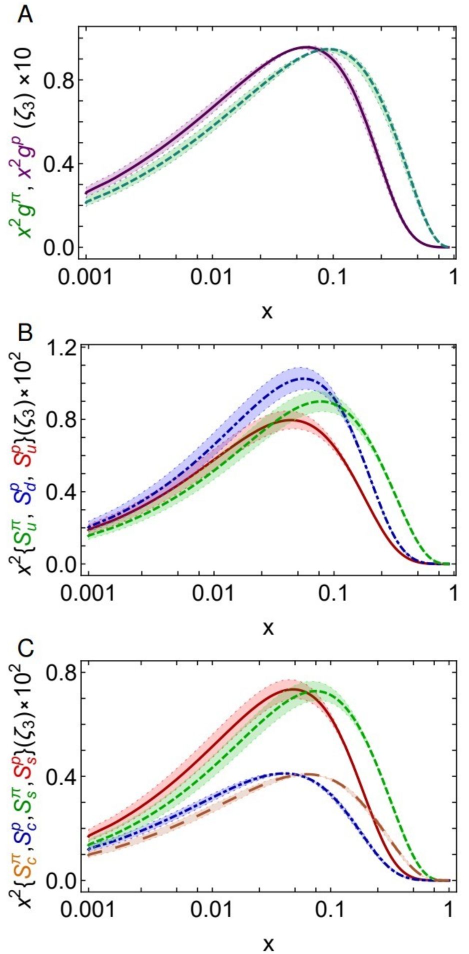
- Marked dilation of the **pion PDF** at ζ_H.
- Differences are **preserved** after evolution.
  - which results in different profiles of glue and sea PDFs.



Gravitational FFs (GFFs)

$$\Lambda_{\mu\nu}(P,Q) = N_c \int_{dk} \mathrm{Tr}\left[\Gamma_\pi\left(k-\frac{Q}{4}, P-\frac{Q}{2}\right) S\left(k-\frac{P}{2}\right)\Gamma_\pi\left(k+\frac{Q}{4}, P+\frac{Q}{2}\right) S\left(k+\frac{P}{2}+\frac{Q}{2}\right)\boxed{\Gamma_{\mu\nu}\left(k+\frac{P}{2}, Q\right)} S\left(k+\frac{P}{2}-\frac{Q}{2}\right)\right]$$

+ beyond **I.A.**

Quark-tensor vertex

- Yields information on **mass and pressure** distributions.
- Evolution disentangles **valence**, **sea** and **gluon** contributions.

Gravitational form factors

- For a given parton class, the **spin-0** energy-momentum tensor (**EMT**) can take the following form:

$$\underbrace{\Lambda^{a}_{\mu\nu}(P,Q)}_{\langle P_f|T_{\mu\nu}(0)|P_i\rangle} = 2P_\mu P_\nu \theta^{a}_2(Q^2) + \frac{1}{2}\left(Q^2 g_{\mu\nu} - Q_\mu Q_\nu\right)\theta^{a}_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\bar{c}^{a}(Q^2)$$

With: $P = [P_f + P_i]/2 \text{ and } Q = P_f - P_i$

- Such that $\theta_{1,2}(Q^2),\ \bar{c}(Q^2)$ define the so called **gravitational form factors** (**GFFs**).

(these are extracted by sensible projection operators)

$$\int d^3r\, T_q^{00}(\vec{r}) = m_\pi \Theta_{2,q}(0) \Longrightarrow \theta_2(Q^2)$$

Is connected with the **mass** distribution inside the hadron

$$T_q^{ij}(\vec{r}) = p_q(r)\,\delta_{ij} + s_q(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) \Longrightarrow \theta_1(Q^2)$$
$$(i,\, j = 1,\, 2,\, 3)$$

Is connected with the **mechanical** properties of the hadron

p(r) : pressure
s(r) : shear forces

Gravitational **form factors**

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With: $P = [P_f + P_i]/2 \text{ and } Q = P_f - P_i$

- Such that $\theta_{1,2}(Q^2),\ \bar{c}(Q^2)$ define the so called **gravitational form factors** (**GFFs**).

- Energy-momentum **conservation** entail the following **sum rules**:

$$\sum_{q,g}\theta_2(0) = 1 \qquad \sum_{q,g}\bar{c}(t) = 0$$

At the hadronic scale, all is contained within the valence quarks

- While, in the **chiral limit**, the **soft-pion theorem** constraints:

$$\sum_{q,g}\theta_1(0) = 1$$

Deviations of this results are due to the mass of the pseudoscalar.

Quark-tensor **vertex**

- The interaction of a **quark** with a **spin-2 probe** is encoced in the **QTV**, $\Gamma^{\mu\nu}$
- As the **quark-photon** vertex (**QPV**), the **QTV** obeys a **DSE**:

$$i\Gamma^{\mu\nu}(P,Q) = \underbrace{i\Gamma_0^{\mu\nu}(P,Q)}_{\text{Tree level QTV}} + \int \underbrace{K^{(2)}(P,Q|P',Q')}_{\text{IA kernel}}\, i\Gamma^{\mu\nu}(P',Q') + \underbrace{\Delta^{\mu\nu}(P,Q)}_{\text{Symmetry restoring term}}$$

$$i\Gamma_0^{\mu\nu}(P,Q) = i\gamma^{\mu} P_i^{\nu} - g^{\mu\nu} S_0^{-1}(P_i)$$

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- Again, as the **QPV**, symmetry principles (**WGTIs**) partially constraint its structure:

$$iQ_\mu\Gamma^{\mu\nu}(P,Q) = P_i^\nu S^{-1}(P_f) - P_f^\nu S^{-1}(P_i) \qquad \text{(QTV WGTI)}$$

Quark-tensor vertex

- The interaction of a **quark** with a **spin-2 probe** is encoded in the **QTV**, $\Gamma^{\mu\nu}$
- As the **quark-photon** vertex (**QPV**), the **QTV** obeys a **DSE**:

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- Again, as the **QPV**, symmetry principles (**WGTIs**) partially constraint its structure:

$$iQ_\mu\Gamma^{\mu\nu}(P,Q) = P_i^\nu S^{-1}(P_f) - P_f^\nu S^{-1}(P_i)$$ (QTV WGTI)

- Thus, the **QTV** can be expressed as:

$$i\Gamma_{\mathrm{G}}^{\mu\nu}(Q,K) = i\Gamma_{\mathrm{WI}}^{\mu\nu}(Q,K) + i\Gamma_{\mathrm{T}}^{\mu\nu}(Q,K)$$

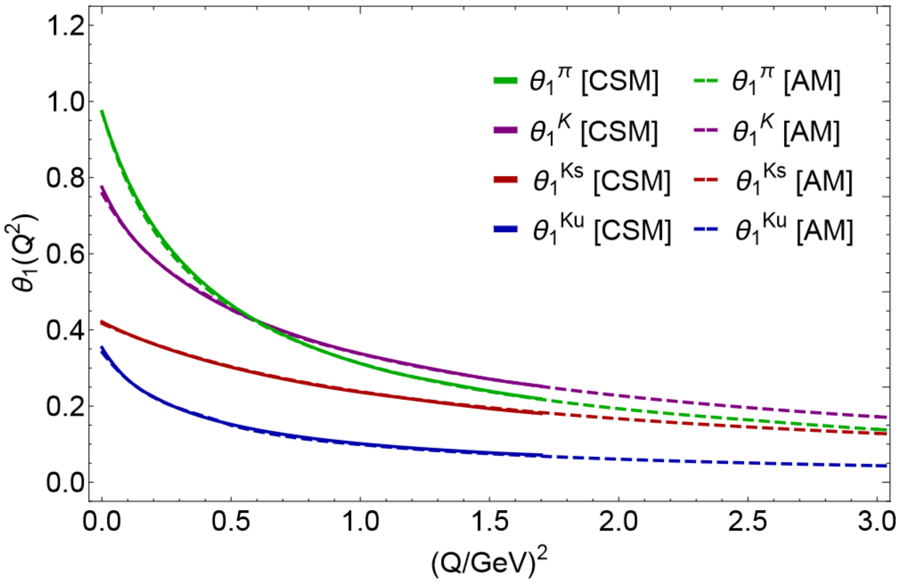
$$Q_\mu\Gamma_T^{\mu\nu} = 0$$

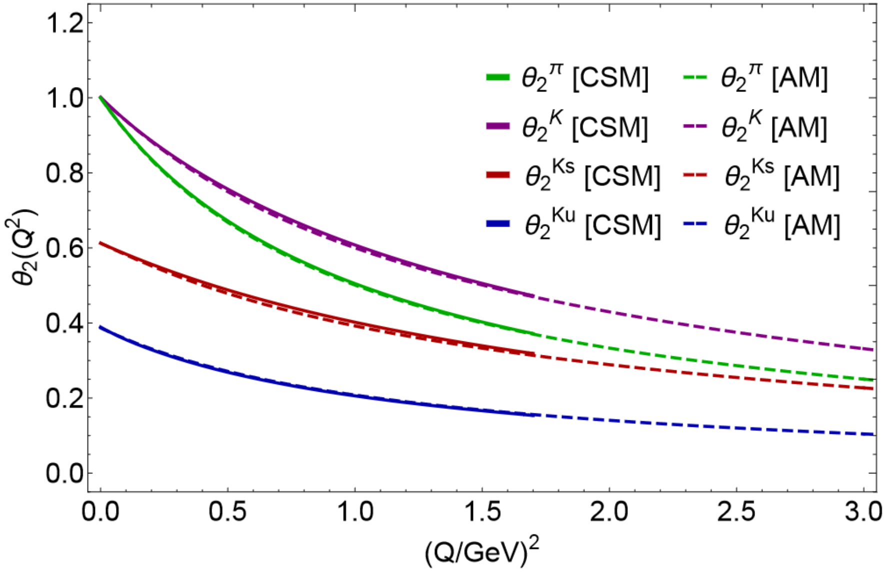
Further related with the **QPV**, but with an **axial-vector** pole

Transverse part containing the **scalar** pole, thus fixing $\theta_1(0)$

Gravitational form factors

- The produced **gravitational form factors:**





One finds:

$$r^{\theta_1}/r_\pi = 1.17\,,\; r^{\theta_2}/r_\pi = 0.75$$

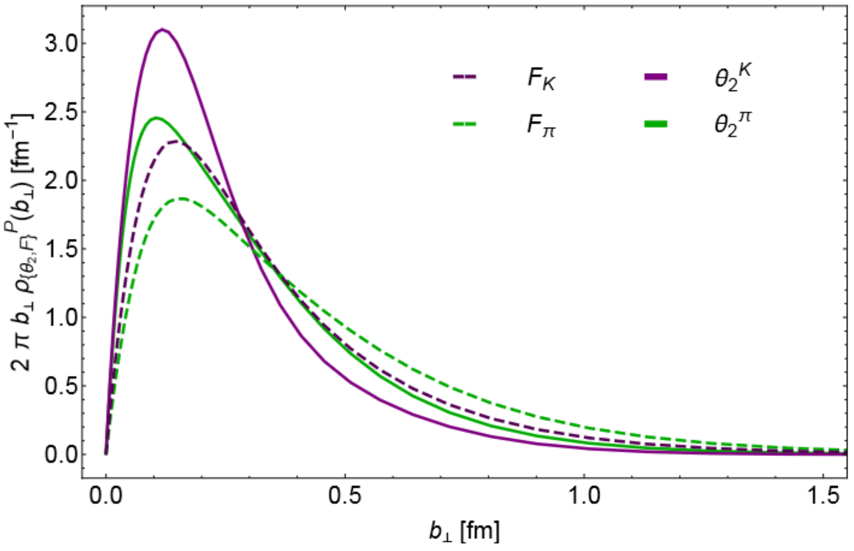
$$r^{\theta_1}/r_K = 1.17\,,\; r^{\theta_2}/r_K = 0.7$$

$$r_{mech} > r_{charge} > r_{mass}$$

The radii **hierarchy** indicates how **mass**, **charge** and the impact of **mechanical** properties spatially **extend**.

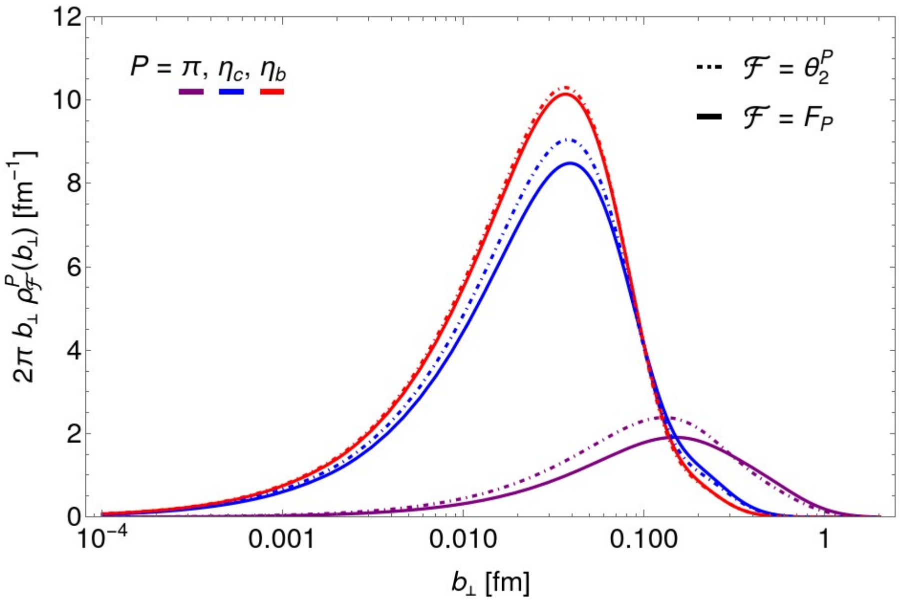
Charge and Mass **distributions**

$$\rho^{\pi,K}_{\{\theta_2,F\}}(b) = \int_0^\infty \frac{d\Delta}{2\pi}\Delta J_0(\Delta b)\{F_{\pi,K},\theta_2^{\pi,K}\}(\Delta^2)$$



	$\langle b^2\rangle_{\theta_2}$ [fm]	$\langle b^2\rangle_F$ [fm]
π	0.39	0.52
K	0.32	0.43

- **Charge** effect span over a **larger domain** than **mass** effects.
  - More **massive** hadron
    - More **compressed**



π-K: Pressure profiles

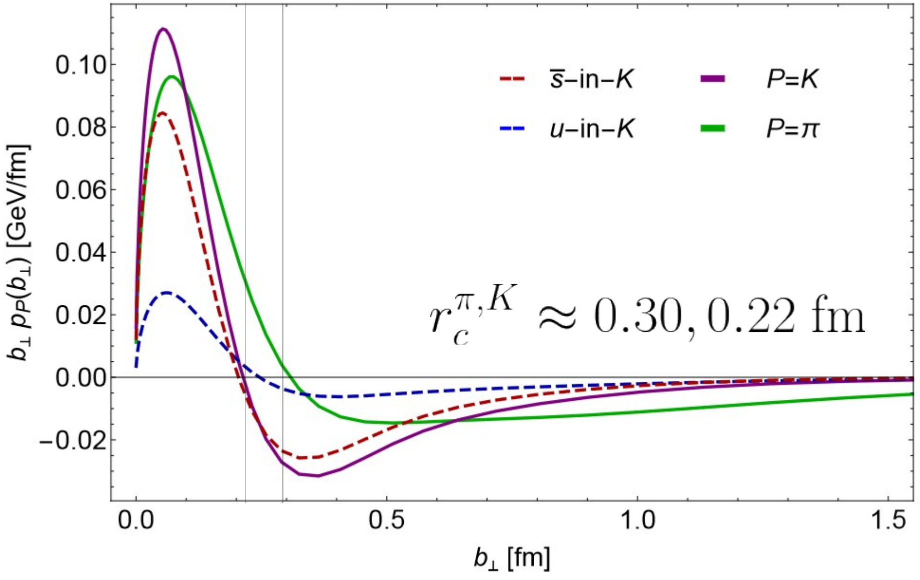
$$p_{\pi,K}(b) = \frac{1}{6\pi}\int_0^\infty d\Delta\, \frac{\Delta}{2E_{\pi,K}(\Delta)}\, J_0(\Delta b)[\Delta^2\theta_1^{\pi,K}(\Delta^2)]$$

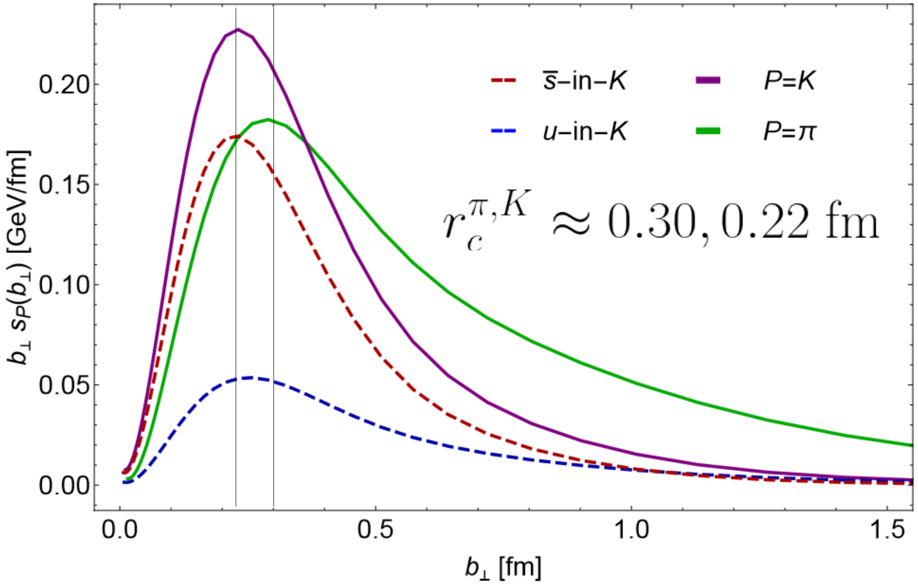
$$s_{\pi,K}(b) = \frac{9}{16\pi b}\int_0^\infty d\Delta\, \frac{1}{2E_{\pi,K}(\Delta)}\, J_1(\Delta b)\,[\Delta^2\theta_1^{\pi,K}(\Delta^2)] - \frac{9}{4}p_{\pi,K}(b)$$

"Isotropic"
Quark attraction/repulsion

CONFINEMENT

Deformation QCD forces
"Shear"





KR, Z-F Cui *et al.*,
Chin.Phys.C 46 (2022) 1, 013105

Light-front wave functions (LFWF)

$$\psi_{\mathrm{M}}^{q}\left(x, k_{\perp}^{2}\right)=\operatorname{tr} \int_{d k_{\|}} \delta_{n}^{x}\left(k_{\mathrm{M}}\right) \gamma_{5} \gamma \cdot n\, \chi_{\mathrm{M}}\left(k_{-}, P\right)$$

Bethe-Salpeter wave function

"One ring to rule them all"

- **Intrinsic** of the hadron's nature.
- Yields a **variety** of **distributions**.

LFWFs: Connecting the dots

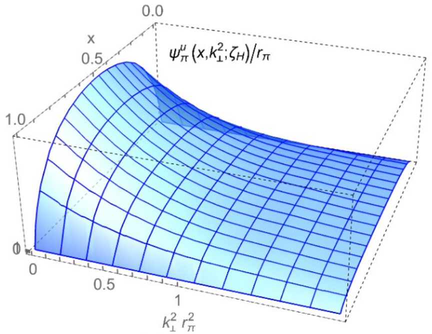
The idea: Connect ***everything*** through the **LFWF.**

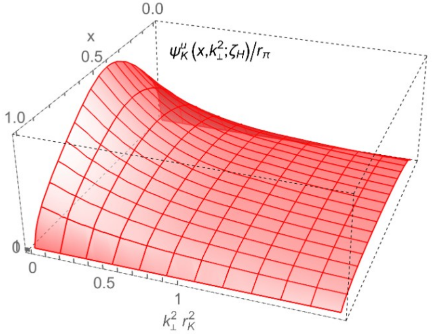
LFWFs → **PDAs**

$\sim \psi^* \psi$

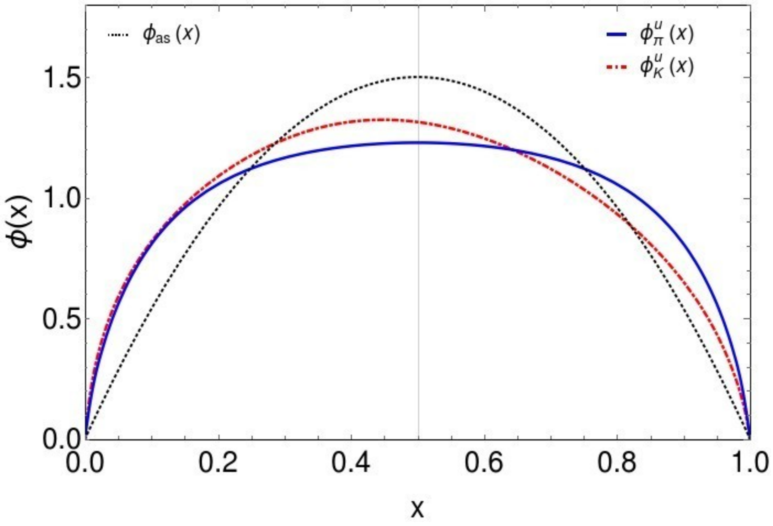
TMDs

$$f_{\mathrm{P}} \varphi_{\mathrm{P}}^{u}(x, \zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3} \psi_{\mathrm{P}}^{u}\left(x, k_\perp^2; \zeta_{\mathcal{H}}\right)$$





- **1-D projection** of the hadron's **LFWF**.

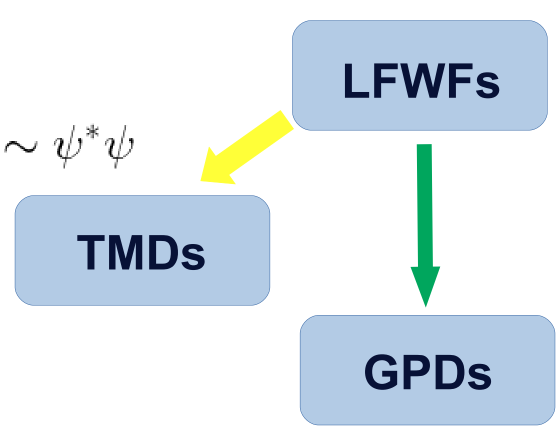


$\int dx$ $\int dk_\perp$

$t = 0, \xi = 0$

LFWFs: Connecting the dots

The idea: Connect ***everything*** through the **LFWF.**



$$f_{\mathrm{P}}\varphi_{\mathrm{P}}^u(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3}\psi_{\mathrm{P}}^u\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)$$

Overlap representation:

$$H_{\mathrm{M}}^q(x,\xi,t) = \int \frac{d^2k_\perp}{16\pi^3}\psi_{\mathrm{M}}^{q*}\left(x^-,(\mathbf{k}_\perp^-)^2\right)\psi_{\mathrm{M}}^q\left(x^+,(\mathbf{k}_\perp^+)^2\right)$$

Sufficient to sketch charge, mass and spatial distributions...

- ✓ ***Valid*** in the **DGLAP** region
- ✓ ***Positivity*** fulfilled
- ✓ Can be ***extended*** to the **ERBL** region Chavez:2021llq

$\int dx$ $\int dk_\perp$

$t=0, \xi=0$

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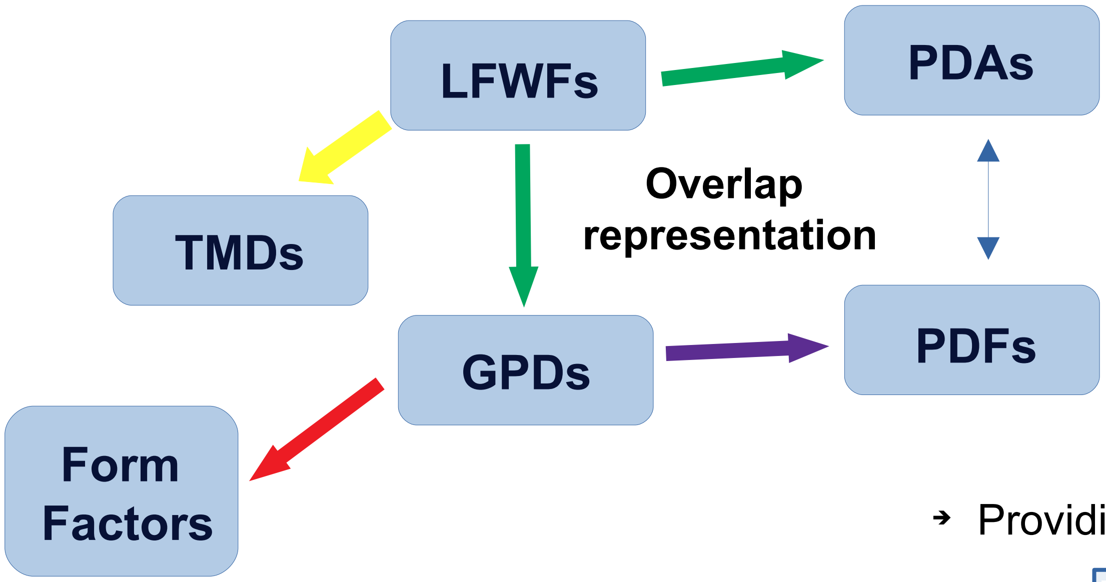


$$f_{\mathsf{P}}\varphi_{\mathsf{P}}^{u}(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3}\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)$$

PDFs are understood as the **forward limit** of the **GPD**

$$q(x) = H(x,0,0)$$

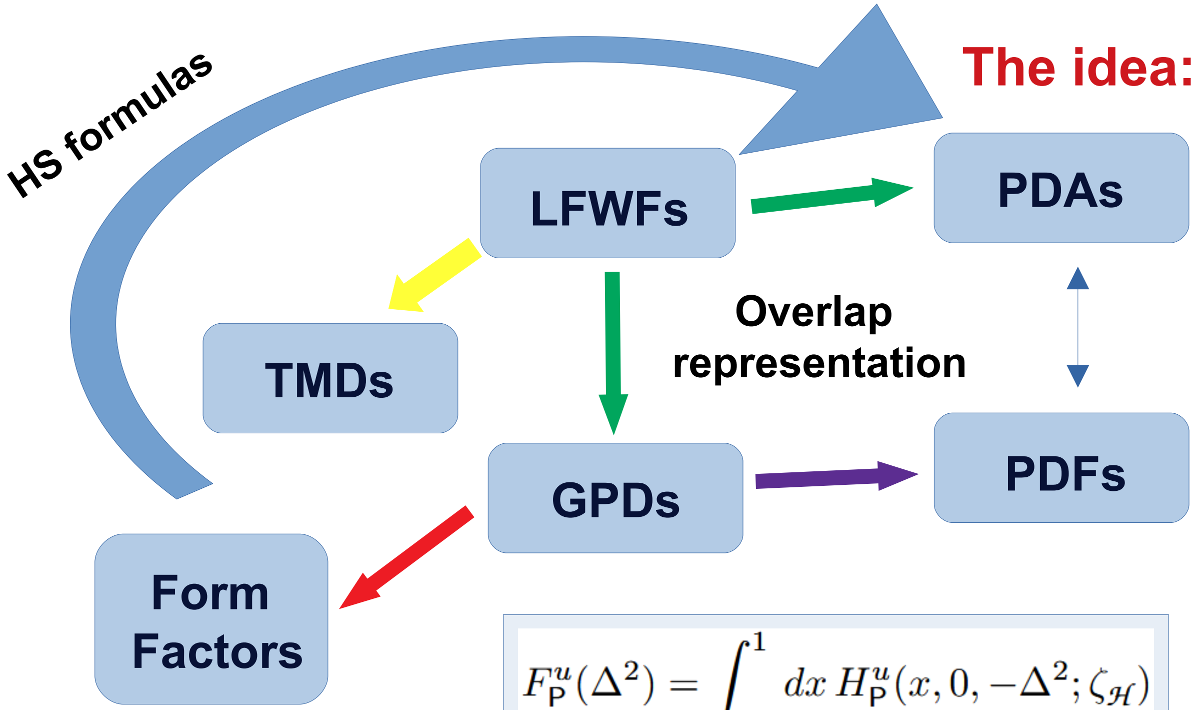
→ Providing another **connection** with the **PDA**.

$$u^{\mathsf{P}}(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\left|\psi_{\mathsf{P}}^{u}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)\right|^2$$

$\int dx$ $\quad$ $\int dk_\perp$

$t=0, \xi=0$

LFWFs: Connecting the dots



The idea: Connect ***everything*** through the **LFWF.**

$$f_{\mathsf{P}}\varphi^u_{\mathsf{P}}(x,\zeta_{\mathcal{H}}) = \int \frac{dk^2_\perp}{16\pi^3}\psi^u_{\mathsf{P}}\left(x,k^2_\perp;\zeta_{\mathcal{H}}\right)$$

PDFs are understood as the **forward limit** of the **GPD**

$$q(x) = H(x,0,0)$$

$$u_M(x;\zeta_H) \sim \int d^2k_\perp \psi^*\psi$$

Form factors are expressed as **sum rules** of the **GPD**

Electromagnetic FF

$$F^u_{\mathsf{P}}(\Delta^2) = \int_{-1}^{1} dx\, H^u_{\mathsf{P}}(x,0,-\Delta^2;\zeta_{\mathcal{H}})$$

Gravitational FFs

$$\int_{-1}^{1} dx\, x\, H^q_{\mathsf{P}}(x,\xi,-\Delta^2;\zeta_{\mathcal{H}}) = \theta^{\mathsf{P}}_2(\Delta^2) - \xi^2\theta^{\mathsf{P}}_1(\Delta^2)$$

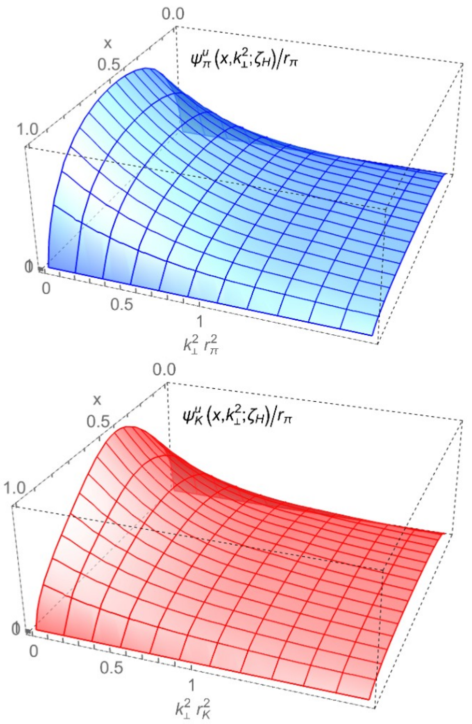
$\int dx$ $\quad$ $\int dk_\perp$

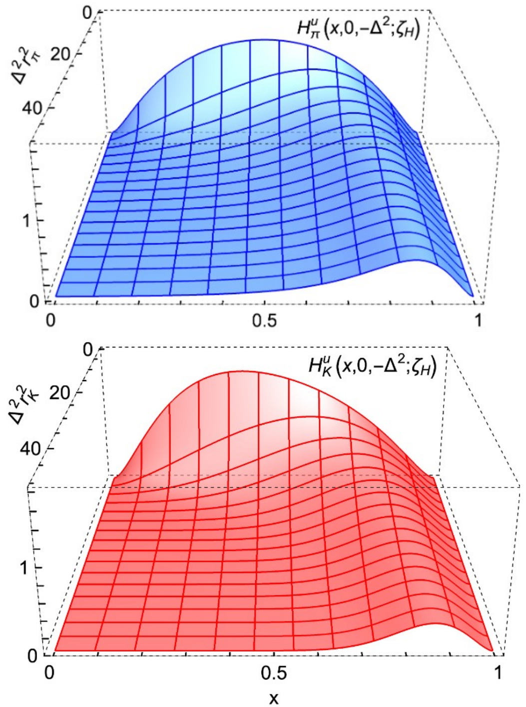
$t=0, \xi=0$

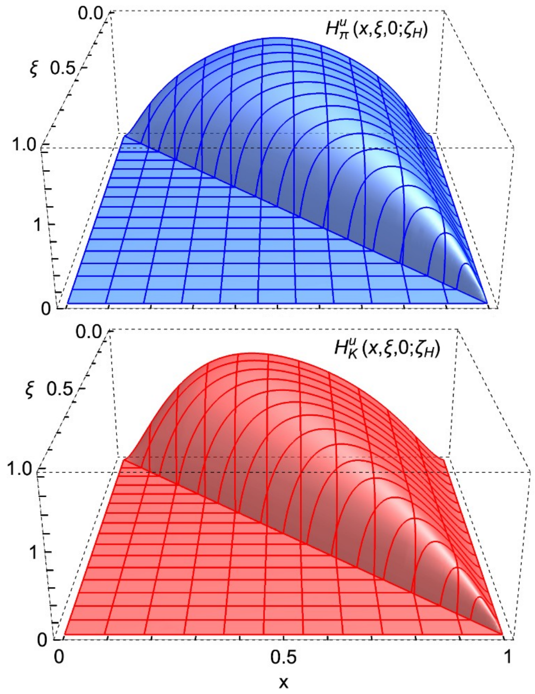
π-K: LFWFs & GPDs

LFWFs → GPDs

$$H^q_{\mathrm{M}}(x,\xi,t) = \int \frac{d^2k_\perp}{16\pi^3}\psi^{q*}_{\mathrm{M}}\left(x^-,(\mathbf{k}^-_\perp)^2\right)\psi^q_{\mathrm{M}}\left(x^+,(\mathbf{k}^+_\perp)^2\right)$$







Impact parameter space GPDs

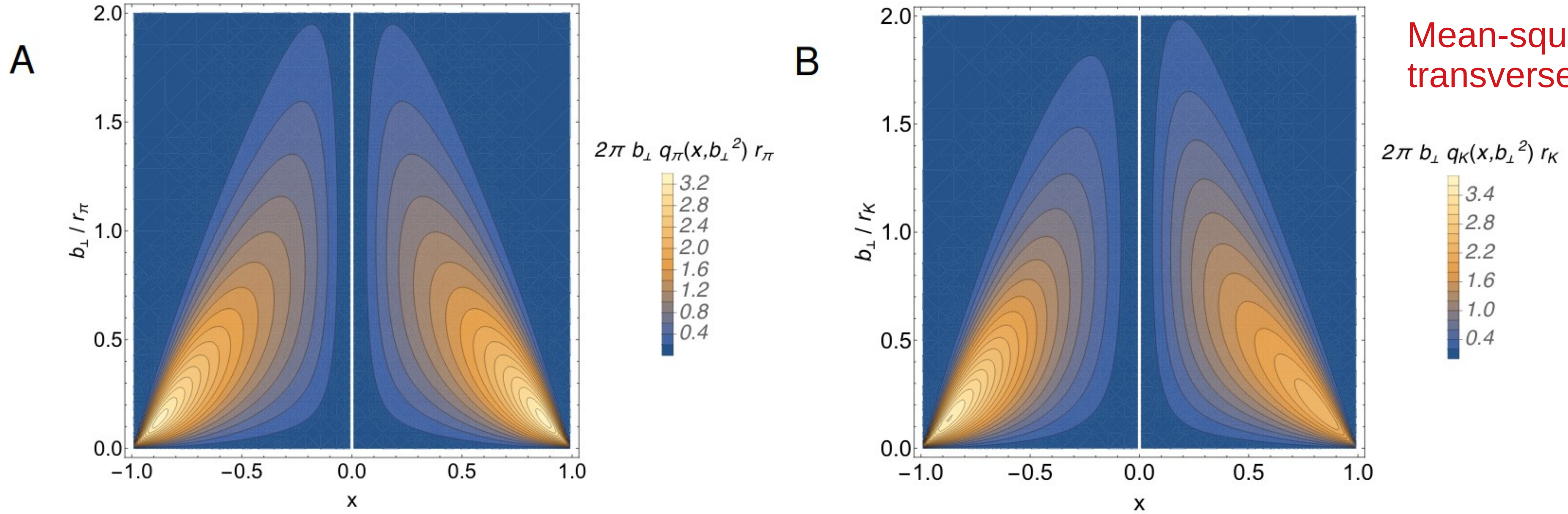
$$u^{\mathrm{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathrm{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

Algebraic derivation!

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^\pi = \boxed{\frac{2}{3}} r_\pi^2 = \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{d}}^\pi, \approx [r_\pi^{\theta_2}]^2$$

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^K = 0.71 r_K^2, \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{s}}^K = 0.58 r_K^2 .$$

Mean-square transverse extent



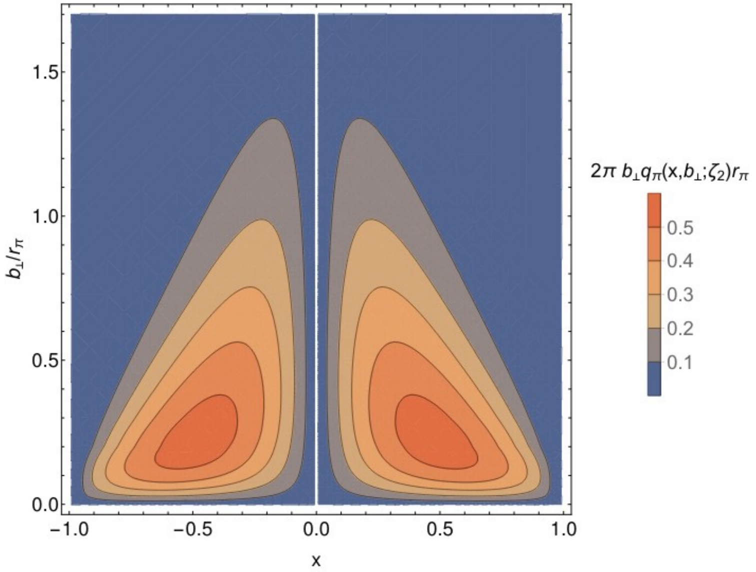
- Likelihood of finding a valence-**quark** with **momentum fraction x**, at **position b**.

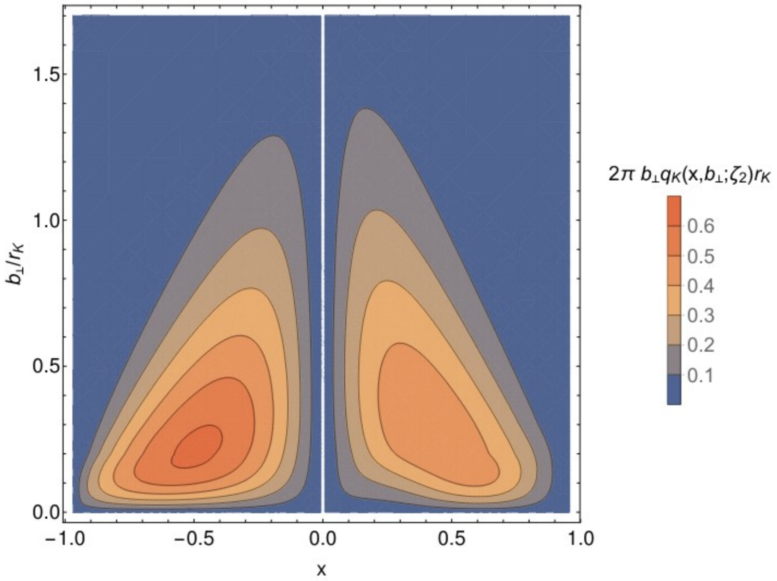
Evolved IPS-GPDs

$\zeta_H \to \zeta = 2\ \text{GeV}$

$$u^{\mathrm{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathrm{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

- **Valence-quarks** become **less dressed**:
  - Peaks **broaden** and **maximum drifts.**





- Likelihood of finding a valence-**quark** with **momentum fraction x**, at **position b**.

Pion GPD: Empirical determination

Pion GPD: Empirical determination

- **Question**:
  From the empirical knowledge of 1-dimensional distributions (**EFF** and **PDF**), can we obtain the 3-dimensional **GPD**?

$$u^\pi(x;\zeta_{e/l})\,,\; F_\pi(\Delta^2) \longrightarrow H_\pi(x,\xi,-\Delta^2;\zeta)$$ **???**

- **Answer**:

$$u^\pi(x;\zeta_{e/l})$$

All orders evolution

DGLAP GPD

$$H^u_\pi(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

Factorized LFWF

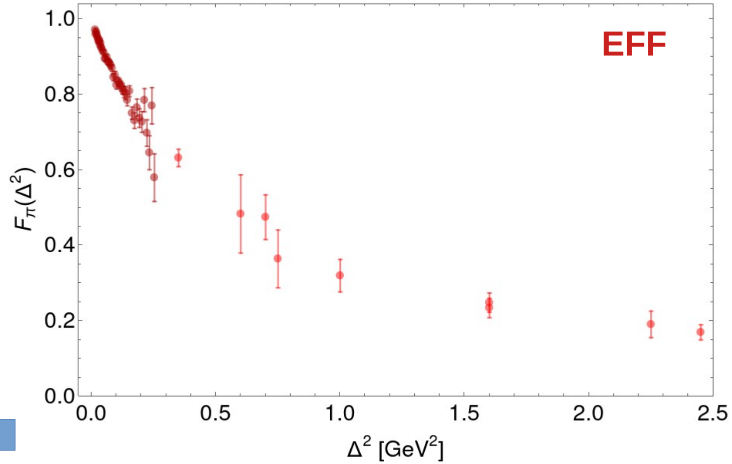
Sum rule

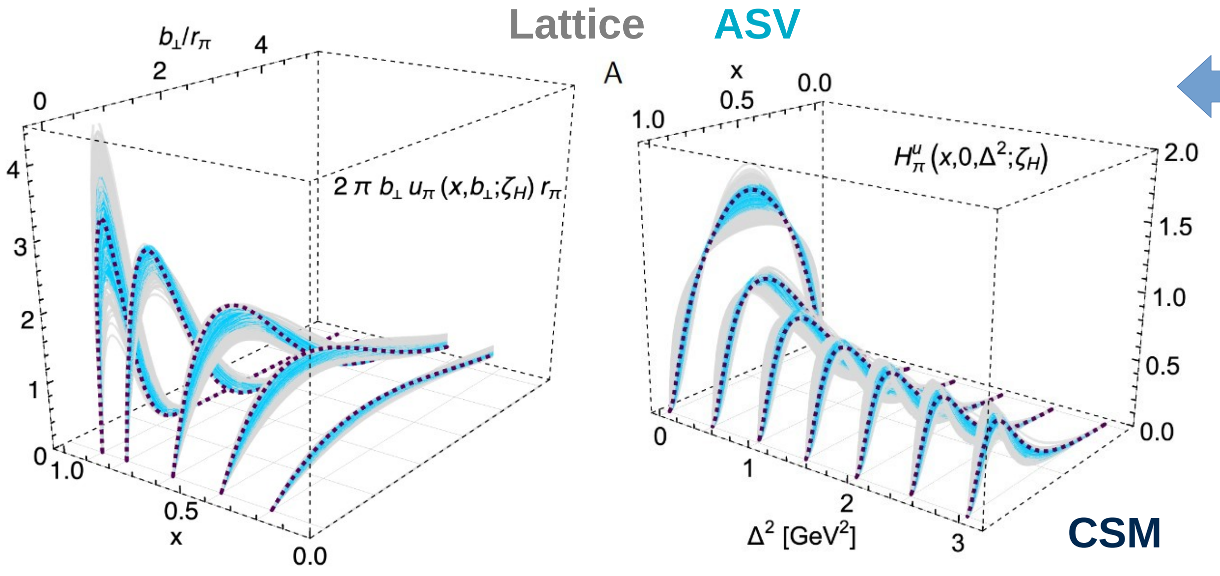
$$H^u_\pi(x,\xi,-\Delta^2;\zeta_H) \sim \int_{k_\perp} \psi^*\psi$$

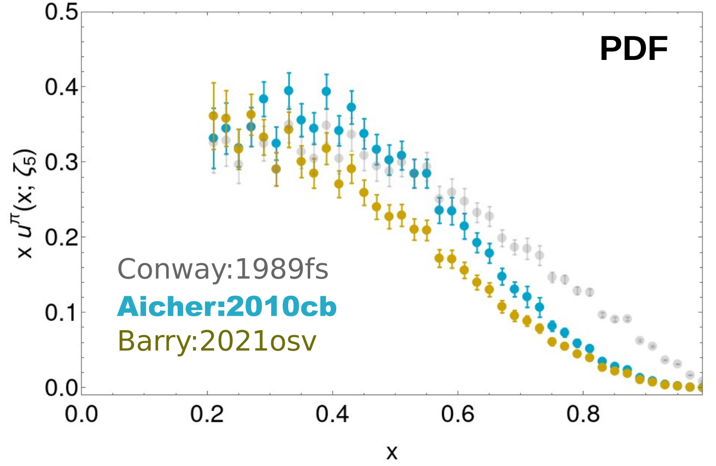
$$F_\pi(\Delta^2) = \int_0^1 dx\, H^u_\pi(x,0,-\Delta^2)$$

Pion GPD: Empirical determination

- The connection of **GPDs** with PDFs and EFFs enable us to use existing data on those quantities to **reconstruct** the **pion GPD**.
- Using a chi^2-based probabilistic selection procedure, an ensemble of **representations** for the **pion GPD** is generated.

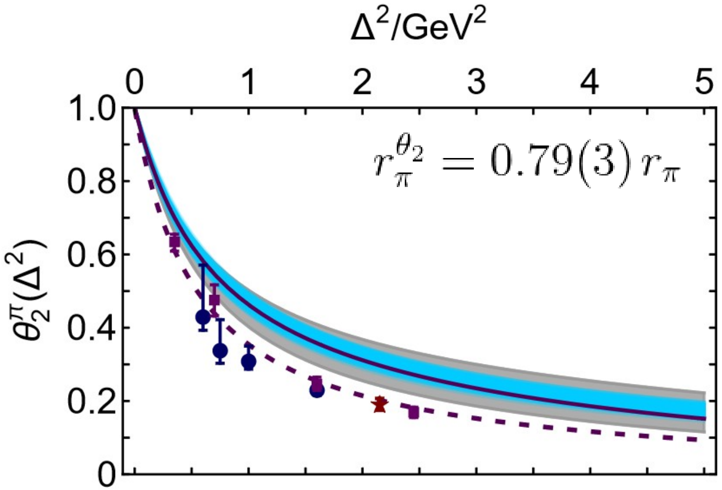






Pion GPD: Empirical determination

- The connection of **GPDs** with PDFs and EFFs enable us to use existing data on those quantities to **reconstruct** the **pion GPD**.
- Using a chi^2-based probabilistic selection procedure, an ensemble of **representations** for the **pion GPD** is generated.
- The produced ensemble turns out to be in **agreement** with previous **CSM predictions**.



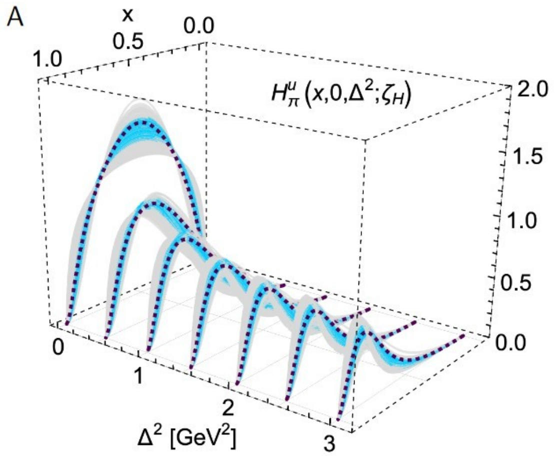
- Proving, once again, that θ_2 is harder than the EFF:

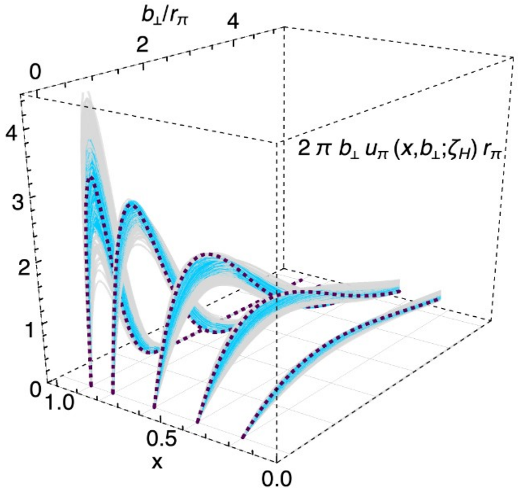
  i.e. the **mass distribution** is **more compact** than the **charge** one.

The physical boundaries:

$$\frac{1}{\sqrt{2}} \leq r_\pi^{\theta_2}/r_\pi \leq 1$$

Xu, KR *et al.*
Chin.Phys.Lett. 40 (2023) 4, 041201





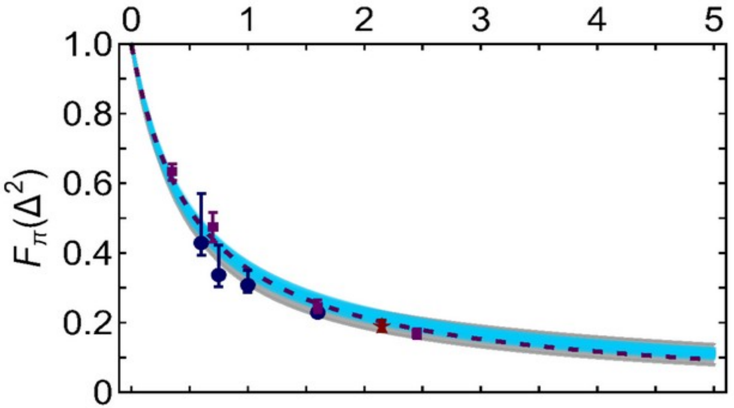
Pion GPD: Empirical determination

- With the **EFF** determined from experimental data, and further validated by a **completely independent** observable (the **PDF**), we can safely rely on the produced ensamble to derive other quantities.

- Such is the case of the **pion-box** contribution to the **muon's** anomalous magnetic moment:

$$\alpha_{\mu}^{\mathbf{P}-box} = \frac{\alpha_{em}^3}{432\pi^2}\int_{\Omega}\sum_{i}^{12} T_i(Q_1,Q_2,\tau)\bar{\Pi}_i^{\mathbf{P}-box}(Q_1,Q_2,\tau),$$

$$\bar{\Pi}_i^{\mathbf{P}-box}(Q_1^2,Q_2^2,Q_3^2) = \underline{F_{\mathbf{P}}(Q_1^2)F_{\mathbf{P}}(Q_2^2)F_{\mathbf{P}}(Q_3^2)} \times \mathcal{I}_i$$



$$F_\pi(\Delta^2) = \int_0^1 dx\, H_\pi^u(x,0,-\Delta^2)$$

- An **exploratory** calculation yields: **(with P. Roig)**

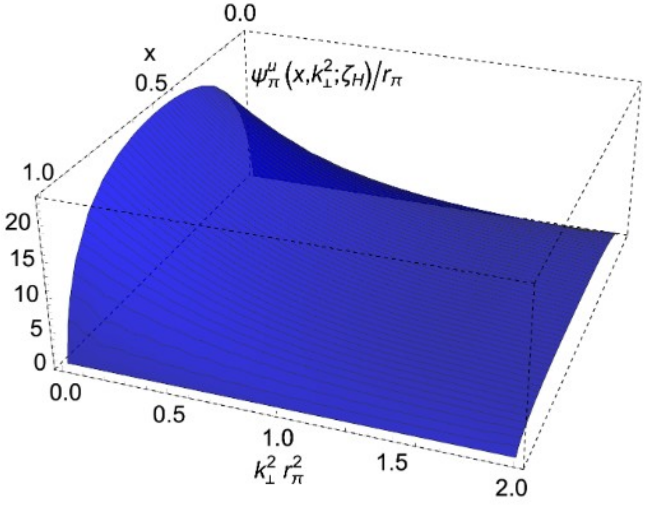
$$a_{\mu}^{\pi-\text{box}} = -(15.1)^{+0.5}_{-0.3}\times 10^{-11}$$

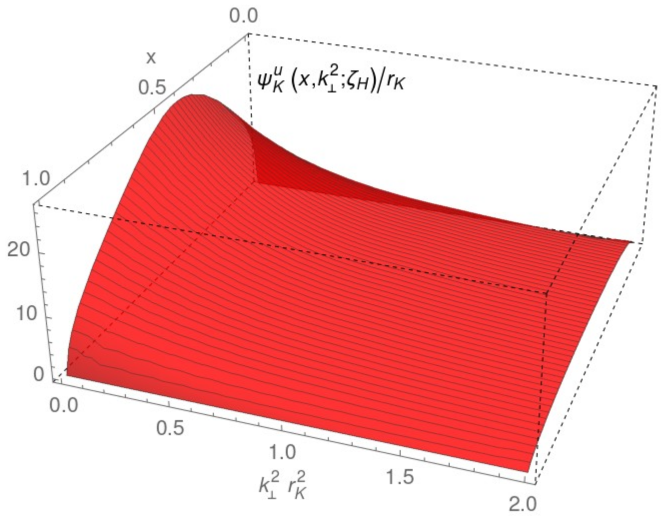
In fair agreement with modern estimates.

Eichmann:2019bqf
Miramontes:2021exi

$$H_\pi^u(x,\xi,-\Delta^2;\zeta_H) = \theta(x_-)\sqrt{u^\pi(x_-;\zeta_H)u^\pi(x_+;\zeta_H)}\,\Phi^\pi(z^2;\zeta_H)$$

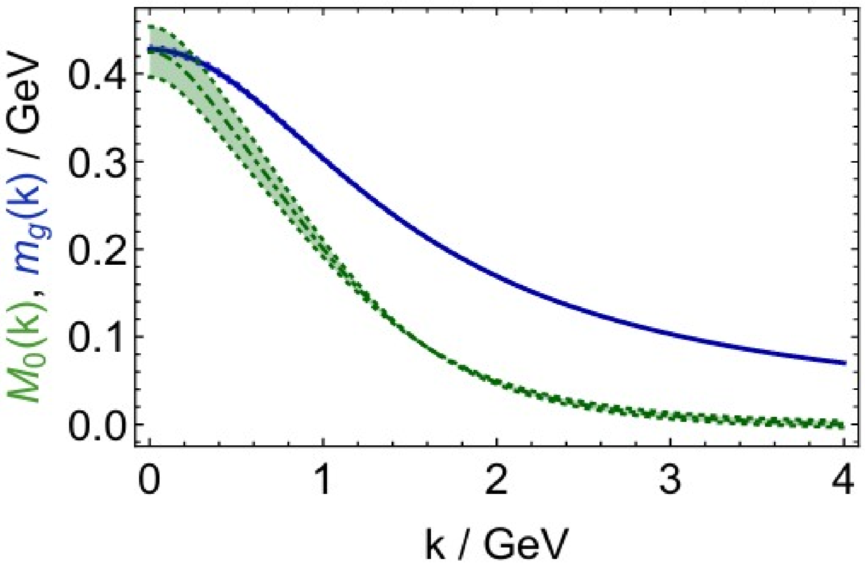
Final Highlights

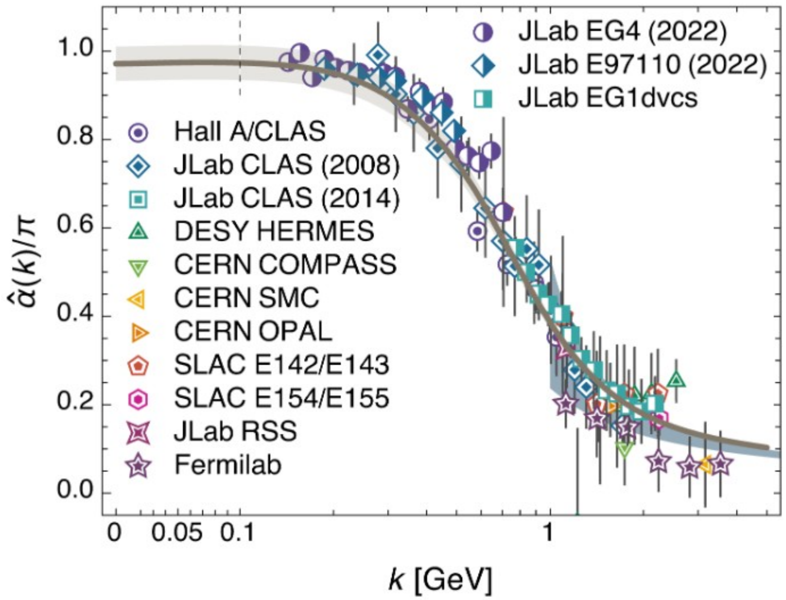




Final **Highlights**

- The **emergent phenomena** in **QCD** produces unique outcomes:
  - The degrees-of-freedom are not directly accessible, we get to observe hadrons (**confinement**).
  - Through their own mechanisms, **dynamical mass generation** is present in both **matter** and **gauge** sectors of QCD; the later yielding a running **coupling** that saturates at infrared momenta.





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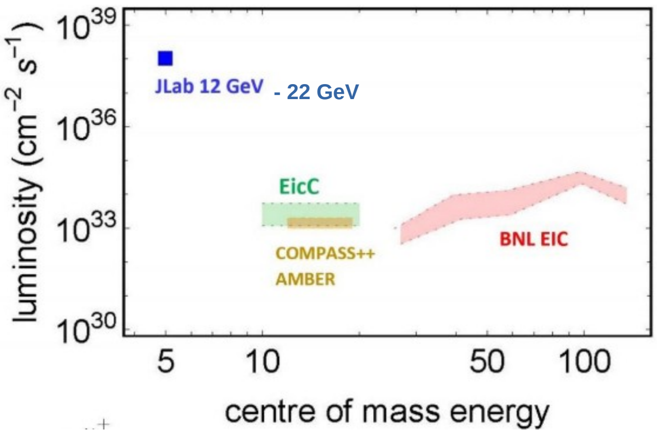
- **Pseudoscalar** mesons and **nucleons** are crucial to inquire on these facets of **QCD**:
  - Their mere **existence and properties** are connected with the **mass generation** in the Standard Model and, potentially, confinement.
  - Modern facilities are **capable** to address the properties of **NG bosons** and **nucleons**, and it's connection with QCD's emergent phenomena.

→ JLab, EIC, EicC, Amber@CERN, etc.



J. Arrington *et al.*
J.Phys.G 48 (2021) 7, 075106

A. Accardi *et al.*
2306.09360 [nucl-ex]

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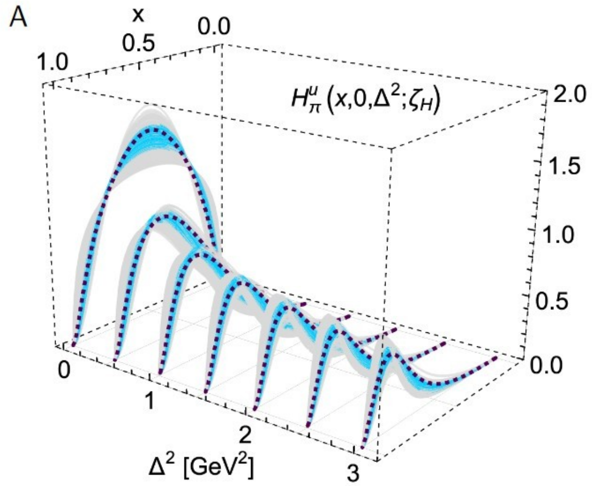


- Theory has evolved to the point where **all sorts of** parton **distributions** of pseudoscalar mesons are **within reach**.
  - → The same level of sophistication for the **nucleon** is yet to be achieved...

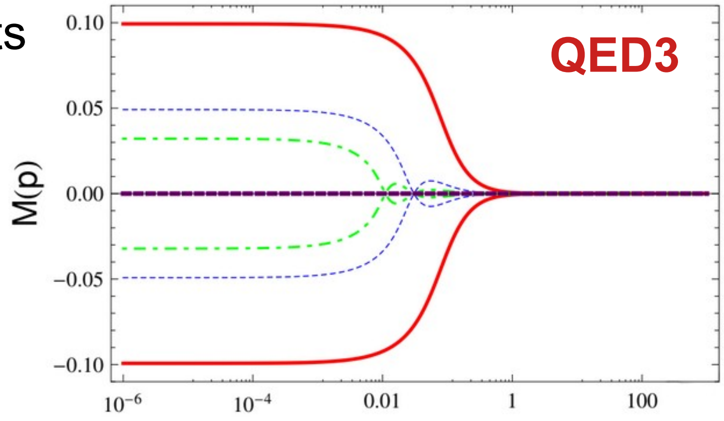
What is left in the inkwell?

QED3: A toy model of QCD

QED3: General Features

KR, A. Bashir *et al.*,
Phys.Rev.D 88 (2013) 9, 096003

- As **QCD**, planar quantum electrodynamics (**QED3**) exhibits non-perturbative features: **confinement and DCSB**.
  - There is no ***critical*** coupling for **DCSB** to arise.
  - In fact, the coupling sets the mass scale of all quantities.

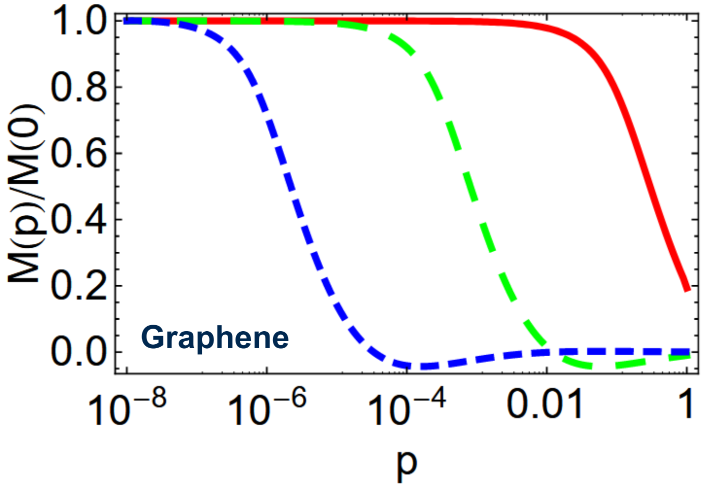


- An obvious application in a physical system is **graphene**.
  - In this case, the speed of light is replaced by the Fermi velocity;

$$c \approx 3 \times 10^8 \mathrm{ms}^{-1} \rightarrow v_F \approx 1 \times 10^6 \mathrm{ms}^{-1}$$

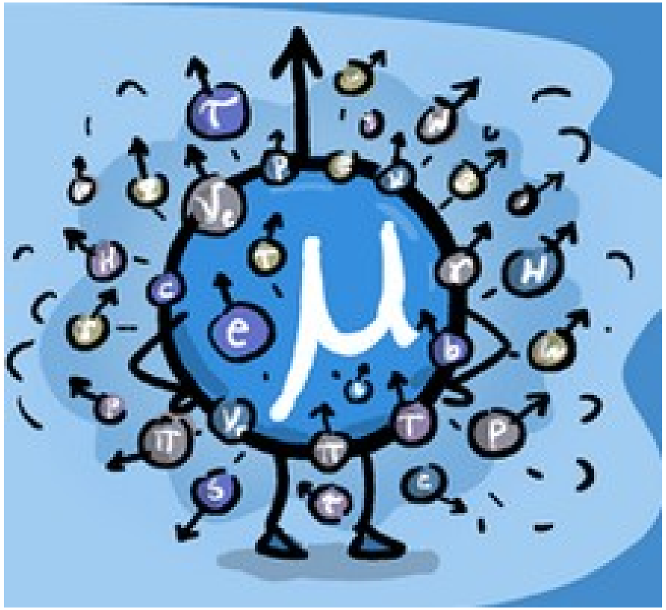
  - Thus, the interaction strength is strong enough to produce **DCSB**-induced solutions.

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137} \rightarrow \alpha_{\mathrm{eff}} = \frac{e^2}{\hbar v_{\mathrm{eff}}} \approx 2$$



Olivares:2021svj

Muon g-2 endeavors



Muon g-2 endeavors

- With ***<u>All</u>*** two-photon **TFFs** at hand we ventured into muon g-2 related endeavors and become part of the White Paper appearing in Physics Reports:

'The anomalous magnetic moment of the muon in the Standard Model'

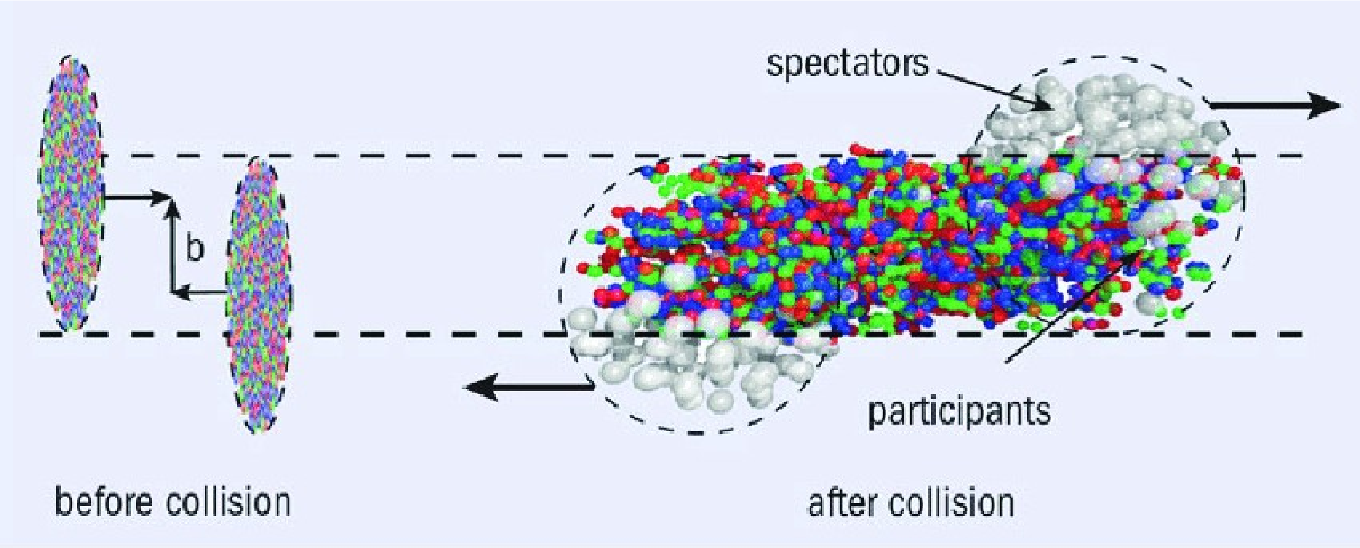
T. Aoyama *et al.* Phys.Rept. 887 (2020) 1-166

- We continue on this path through the computation of the pion and kaon **box contributions**, stemming from their corresponding elastic electromagnetic form factors.

- **What's left to do?** (via CSM framework)
  - Axial-vector mesons HLbL contributions.
  - Analogous for the excited π, η and η' cases.
  - $e^+e^- \rightarrow 3\pi$ HVP contribution.

+ ongoing PDF-based Data-Driven analysis

MUEC physics



MUEC physics

A. Ayala, L. A. Hernández, KR, R. Zamora
Phys.Rev.D 104 (2021) 3, 039901

- Collisions of heavy nuclei at high energies produce deconfined strongly interacting matter, dubbed as the **quark-gluon plasma** (QGP).

- When these collisions are **off-center**, the inhomogeneity of the matter distribution in the transverse plane causes the colliding region to **develop** an orbital **angular velocity**.

- From **first principles**, a.k.a. the Dirac equation in a rotating environment, we derived the corresponding **fermion propagator**.

$$\mathcal{O}^{\pm} \equiv \frac{1}{2}\left[1 \pm i\gamma^1\gamma^2\right]$$

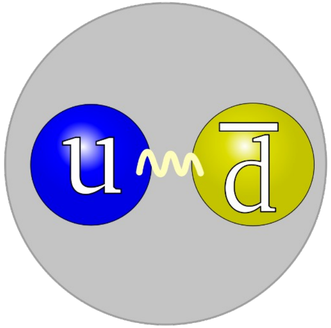
$$S(p) = \frac{[p_0 + \Omega/2 - p_z + ip_\perp]\gamma_0 + m}{(p_0 + \Omega/2)^2 - \vec{p}^2 - m^2 + i\epsilon}\mathcal{O}^+ + \frac{[p_0 - \Omega/2 + p_z - ip_\perp]\gamma_0 + m}{(p_0 - \Omega/2)^2 - \vec{p}^2 - m^2 + i\epsilon}\mathcal{O}^- .$$

- The overall goal is to perform an analogous calculation for the gauge boson case, and the fermion-boson vertex.

(aiming at, inter alia, estimate the relaxation time required for the alignment between the spin of quark/antiquark and the thermal vorticity.)

Backup slides

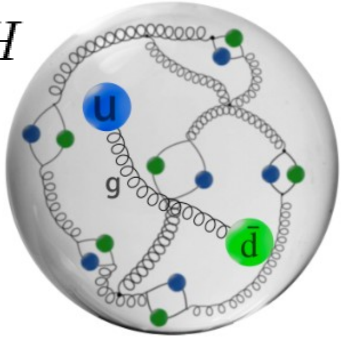
PARTON DISTRIBUTIONS



ζ_H

Resolution Scale

$\zeta > \zeta_H$



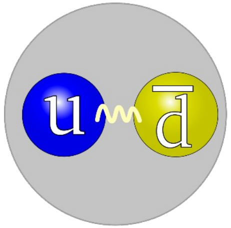
- **Fully-dressed valence quarks**

  (quasiparticles)

- **Unveiling of glue and sea d.o.f**

  (partons)

Pion PDF: hadronic scale



$(M_u = M_d)$

- **Fully-dressed valence quarks** (quasiparticles)

ζ_H : hadronic scale

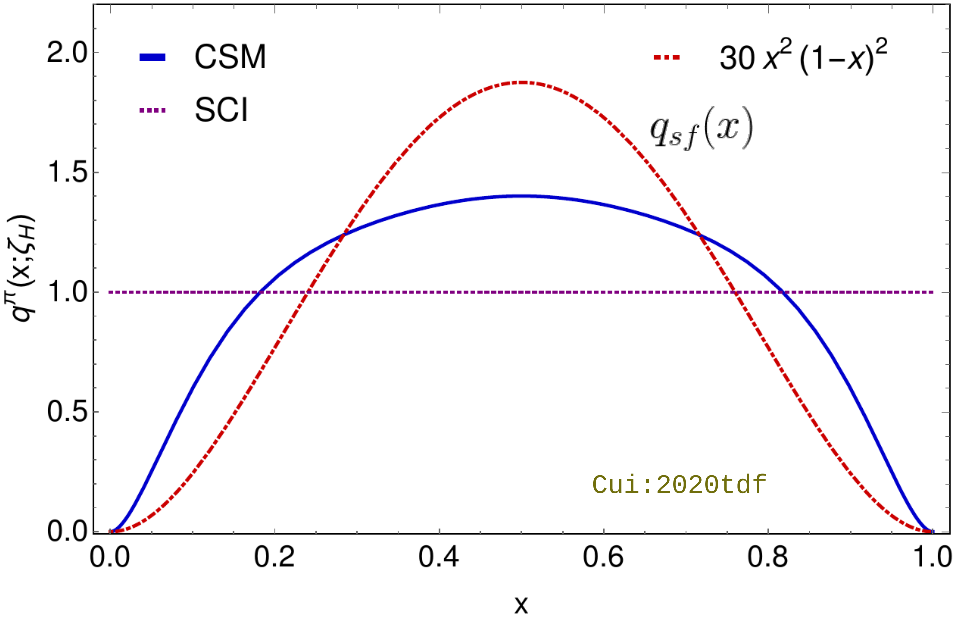
➢ At this scale, **all properties** of the hadron are contained within their valence quarks.

➔ Equally massive quarks means a **50-50** share of the total momentum:

$$<x(\zeta_H)>_q = 0.5$$

➔ This implies symmetric distributions:

$$q(x;\zeta_H) = q(1-x;\zeta_H)$$



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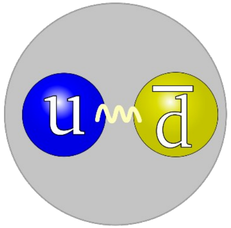

$(M_u = M_d)$

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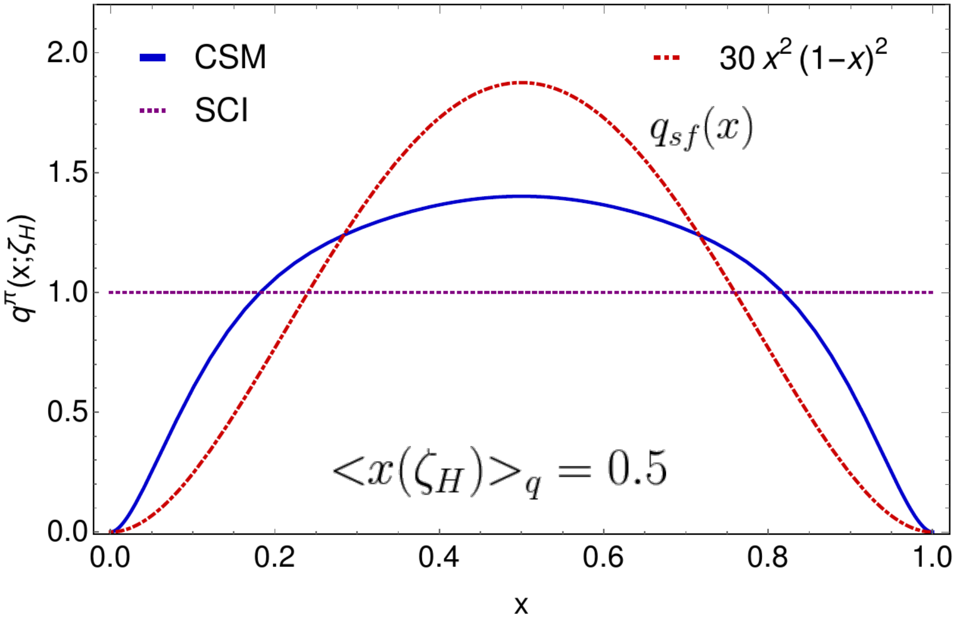
"**Physical**" boundaries:

$$\frac{1}{2^n} \overset{(i)}{\leq} \langle x^n \rangle_{u_\pi}^{\zeta_H} \overset{(ii)}{\leq} \frac{1}{1+n}$$

Produced by $q(x;\zeta_H) = \delta(x - 1/2)$ (infinitely heavy valence quarks)

Produced by $q(x;\zeta_H) = 1$ (massless SCI case)



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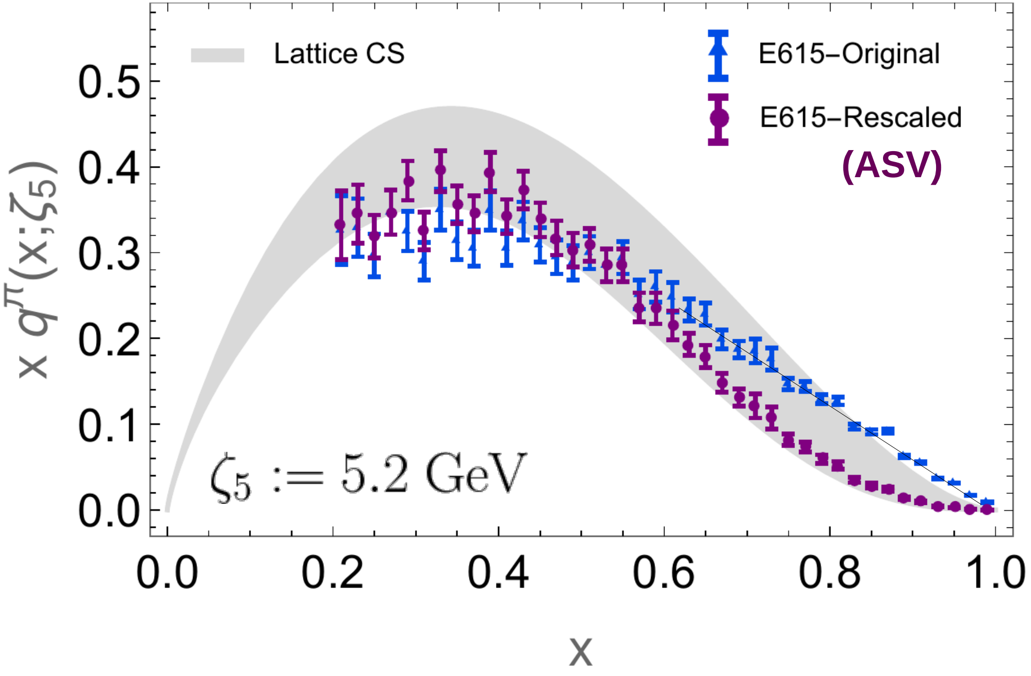
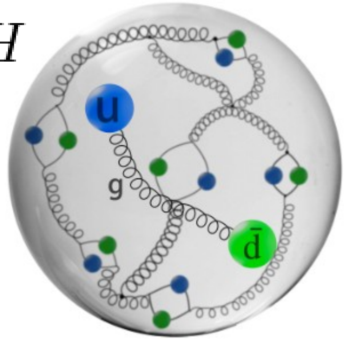
➔ This implies symmetric distributions.

Pion PDF: experimental scale

Conway:1989fs Aicher:2010cb Sufian:2020vzb



$$\zeta > \zeta_H$$

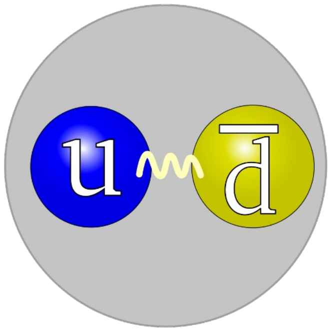


- **Unveiling of glue and sea d.o.f**
(partons)

- **Experimental** data is given **here**.
- **Lattice QCD** results are also quoted beyond the **hadronic scale**.
  - The interpretation of parton distributions from cross sections demands **special care**.

Barry:2021osv

Pion PDF: energy scales

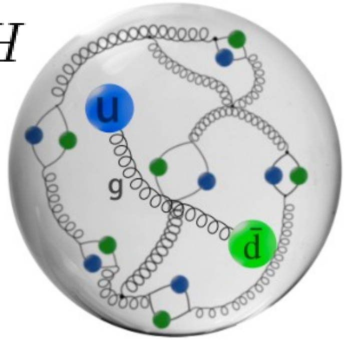


ζ_H

Resolution Scale

Evolution equations

$\zeta > \zeta_H$



- **Fully-dressed valence quarks**

  (quasiparticles)

  - Theoretical calculations are perfomed at *some* low energy scale.

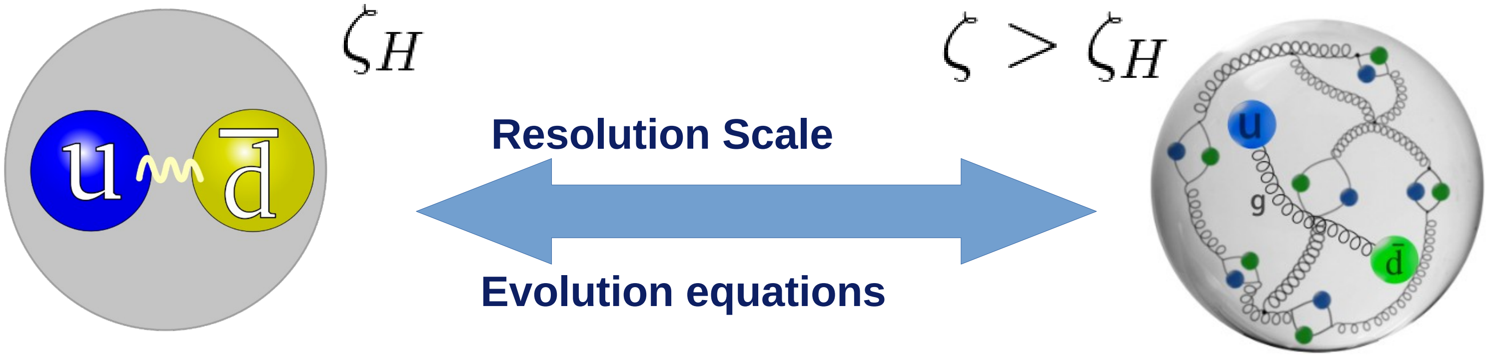
- **Unveiling of glue and sea d.o.f**

  (partons)

  - Then evolved via **DGLAP** equations to compare with experiment and lattice.

Pion PDF: energy scales

Rodriguez-Quintero:2019fyc



- **Fully-dressed valence quarks**

  (quasiparticles)

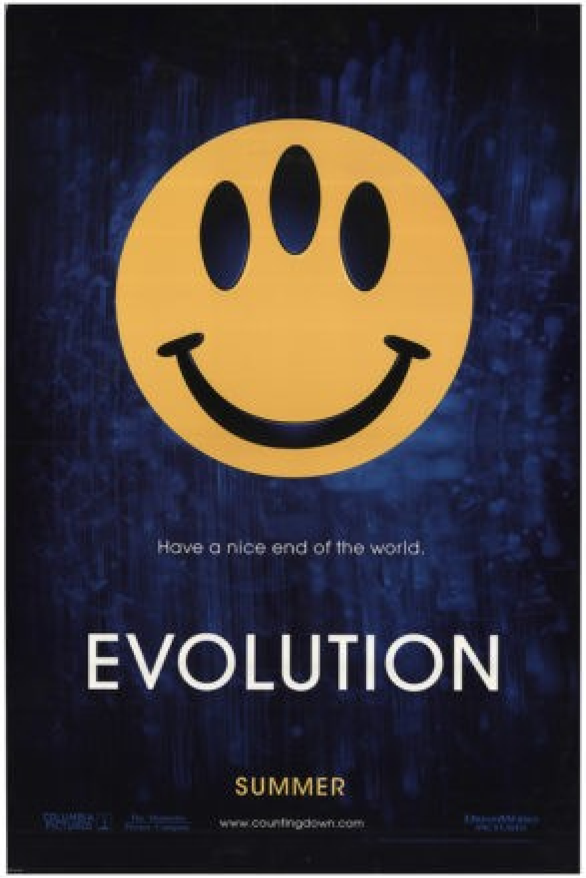
  - Theoretical calculations are perfomed at *some* low energy scale.

- **Unveiling of glue and sea d.o.f**

  (partons)

  - Then evolved via **DGLAP** equations to compare with experiment and lattice.

> - Following our **all orders** evolution, we can go **either way**.
> - Besides, the **hadronic scale** becomes unambigously **determined**.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DGLAP: All orders evolution

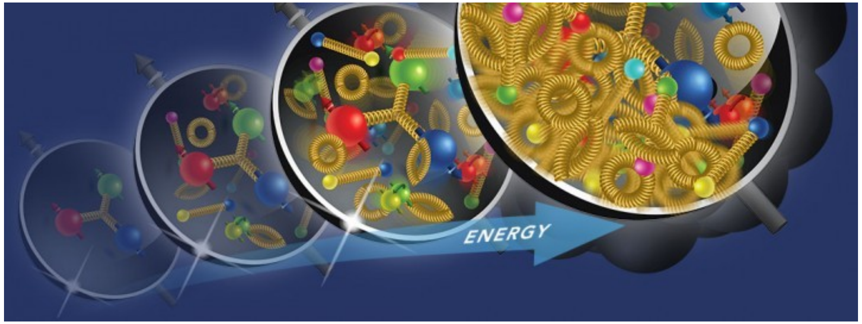
Idea. Define an **effective** coupling such that:

"All orders evolution"

Starting from fully-dressed **quasiparticles**, at ζ_H

Sea and **Gluon** content unveils, as prescribed by **QCD**

$$\left\{\zeta^2 \frac{d}{d\zeta^2}\int_0^1 dy\,\delta(y-x) - \boxed{\frac{\alpha(\zeta^2)}{4\pi}}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}^{\rm NS}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^{\rm S}\left(\frac{\mathbf{x}}{\mathbf{y}}\right)\end{pmatrix}\right\}\begin{pmatrix} H_\pi^{\rm NS,+}(y,t;\zeta) \\ \mathbf{H}_\pi^{\rm S}(y,t;\zeta)\end{pmatrix} = 0$$



- **Not** the **LO** QCD coupling but an **effective** one.
- Making this equation **exact**.
- And connecting with the **hadron scale.**

Raya:2021zrz Cui:2020tdf

DGLAP: All orders evolution

Implication 1:

$$\boxed{\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q}$$

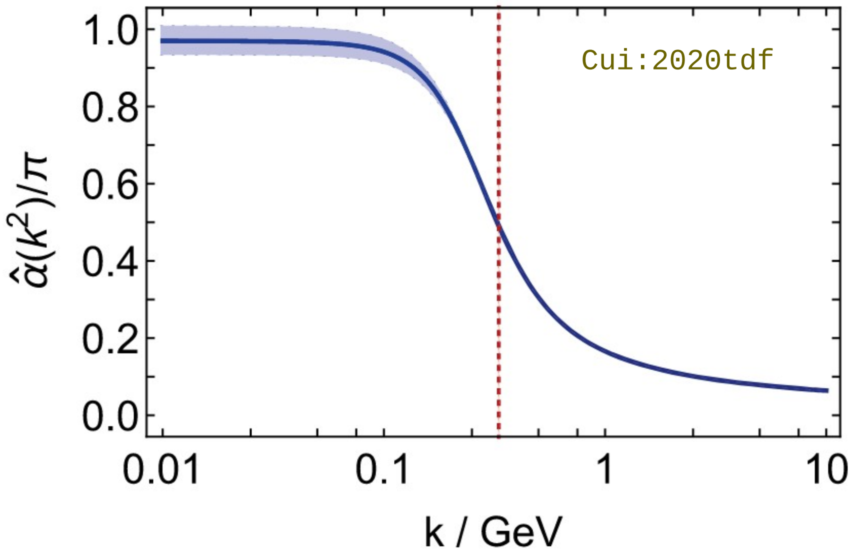
$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\,\alpha(t)$$

Explicitly depending on the **effective charge**

$$\langle x^n(t;\zeta)\rangle_F = \int_0^1 dx\, x^n\, F(x,t;\zeta)$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

- The **QCD PI effective charge** is our best candidate to accommodate our **all orders scheme**.

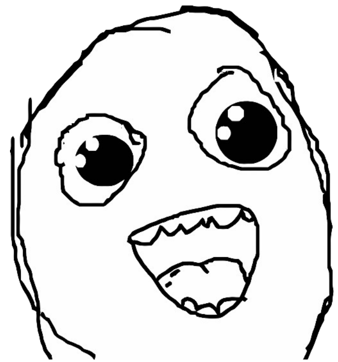


$$\hat{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{M}^2(k^2)}{\Lambda_{\rm QCD}^2}\right]} \Rightarrow \boxed{\zeta_H = 0.331\ \text{GeV}}$$

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$



$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\,\alpha(t)$$

This contains, ***implicitly***, the information of the **effective charge**

- No actual ***need*** to know it. <u>Assuming its existence is **sufficient**.</u>
- ***Unambiguous*** definition of the **hadron scale**:

$$\langle x(\zeta_H)\rangle_q = 0.5 \Rightarrow \boxed{\langle x^n(\zeta_f)\rangle_q = \langle x^n(\zeta_H)\rangle_q\left(\langle 2x(\zeta_f)\rangle_q\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}}$$

(flavor symmetric case)

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \underbrace{\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}}_{\text{Information on the charge is here}}$$

- Details of the effective charge are **encoded** in the ratio of first moments.
- Natural connection with the **hadron scale**.

Implication 2:

$$\begin{aligned}
\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q=u,\bar{d}\,; \\
\langle x(\zeta_f)\rangle_{\text{sea}} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - (\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}})\,, \\
&= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u \\
\langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1 - \langle 2x(\zeta_f)\rangle_u^{7/4}\right)\,;
\end{aligned}$$

- **Sea** and **gluon** determined from valence-quark moments

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \underbrace{\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)}_{\text{Information on the charge is here}}{}^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

- Can **jump** from one scale to another (both ways)
- Natural connection with the **hadron scale**.

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$$\begin{aligned}
\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}\cancel{S(\zeta_H,\zeta_f)}\right), \qquad q=u,\bar{d};\\
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\end{aligned}$$

- **Sea** and **gluon** determined from valence-quark moments
- **Asymptotic** (massless) limits are evident.

DGLAP: All orders evolution

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\end{aligned}$$

- **Sea** and **gluon** determined from valence-quark moments
- **Asymptotic** (massless) limits are evident.
- And, of course, the momentum **sum rule**:

$$\langle 2x(\zeta_f)\rangle_q + \langle x(\zeta_f)\rangle_{\rm sea} + \langle x(\zeta_f)\rangle_g = 1$$

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \underbrace{\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}}_{\text{Information on the charge is here}}$$

- Can **jump** from one scale to the another (even downwards)
- Natural connection with the **hadron scale**.

Implication 3: Recurrence relation

$$\langle x^{2n+1}\rangle^{\zeta}_{u_\pi} = \frac{(\langle 2x\rangle^{\zeta}_{u_\pi})^{\gamma_0^{2n+1}/\gamma_0^1}}{2(n+1)} \times \sum_{j=0,1,\ldots}^{2n} (-)^j \binom{2(n+1)}{j} \langle x^j\rangle^{\zeta}_{u_\pi} (\langle 2x\rangle^{\zeta}_{u_\pi})^{-\gamma_0^j/\gamma_0^1}\,.$$

- Since **isospin symmetry** limit implies:

$$q(x;\zeta_H) = q(1-x;\zeta_H)$$

- **Odd** moments can be expressed in terms of previous **even** moments.
- Thus arriving at the recurrence **relation** on the left.

DGLAP: All orders evolution

Implication 1:

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_H,\zeta_f)\right)\langle x^n(\zeta_H)\rangle_q = \langle x^n(\zeta_H)\rangle_q \underbrace{\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}}$$

Information on the charge is here

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	$\langle x^n\rangle^{\zeta_5}_{u_\pi}$	
n	Lattice input	Recurrence relation
1	0.230(3)(7)	0.230
2	0.087(5)(8)	0.087
3	0.041(5)(9)	0.041
4	0.023(5)(6)	0.023
5	0.014(4)(5)	0.015
6	0.009(3)(3)	0.009
7		0.0078

Gravitational form factors

Gravitational **form factors**

- For a given parton class, the **spin-0** energy-momentum tensor (**EMT**) can take the following form:

$$\underbrace{\Lambda^{a}_{\mu\nu}(P,Q)}_{\langle P_f|T_{\mu\nu}(0)|P_i\rangle} = 2P_\mu P_\nu \theta^{a}_2(Q^2) + \frac{1}{2}\left(Q^2 g_{\mu\nu} - Q_\mu Q_\nu\right)\theta^{a}_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\bar{c}^{a}(Q^2)$$

With: $P = [P_f + P_i]/2 \text{ and } Q = P_f - P_i$

- Such that $\theta_{1,2}(Q^2),\ \bar{c}(Q^2)$ define the so called **gravitational form factors** (**GFFs**).

(these are extracted by sensible projection operators)

$$\int d^3r\, T_q^{00}(\vec{r}) = m_\pi \Theta_{2,q}(0) \Rightarrow \theta_2(Q^2)$$

Is connected with the **mass** distribution inside the hadron

$$T_q^{ij}(\vec{r}) = p_q(r)\,\delta_{ij} + s_q(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) \Rightarrow \theta_1(Q^2)$$

$(i,\, j = 1,\, 2,\, 3)$

Is connected with the **mechanical** properties of the hadron

p(r) : pressure
s(r) : shear forces

Gravitational **form factors**

- For a given parton class, the **spin-0** energy-momentum tensor (**EMT**) can take the following form:

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- Energy-momentum **conservation** entail the following **sum rules**:

$$\sum_{q,g}\theta_2(0) = 1 \qquad \sum_{q,g}\bar{c}(t) = 0$$

- While, in the **chiral limit**, the **soft-pion theorem** constraints:

$$\sum_{q,g}\theta_1(0) = 1$$

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- While, in the **chiral limit**, the **soft-pion theorem** entails:

$$\sum_{q,\cancel{g}} \theta_1(0) = 1$$

- At the **hadronic scale**, ζ_H, all properties of the hadron are contained within the valence quarks.

Here we shall work...