τ-data based evaluation of $a_\mu^{HVP,LO}$ & the significance of the discrepancy between measurement & SM

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Cinvestav (Mexico City)

Non-perturbative physics: tools & applications
Morelia Workshop, 4-8 September 2023

CONTENTS

- The measurement

- The SM prediction

- Their discrepancy

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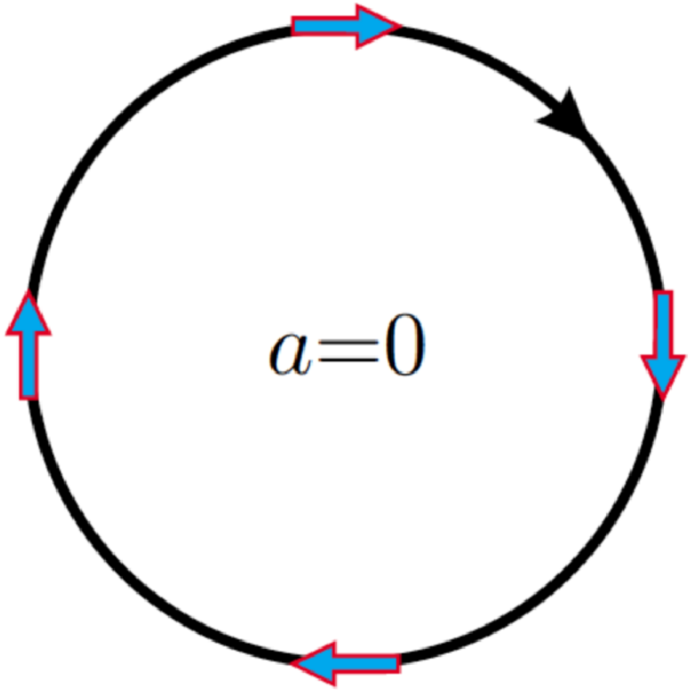
The a_μ measurement



There is no precession with g=2

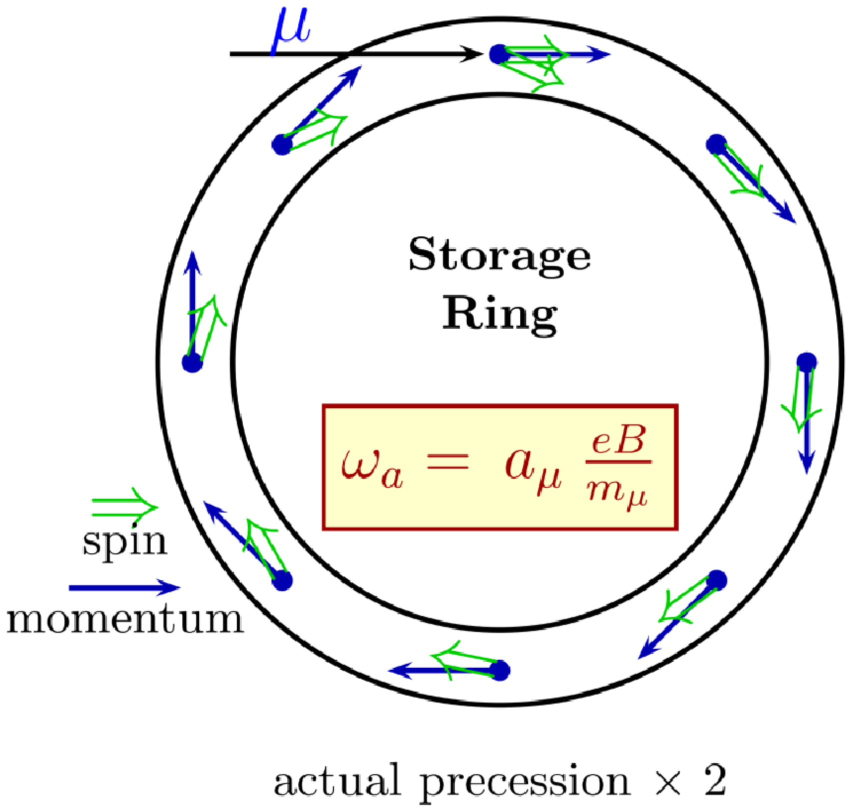


Fig. 3. Spin precession in the $g-2$ ring ($\sim 12^\circ$/circle).

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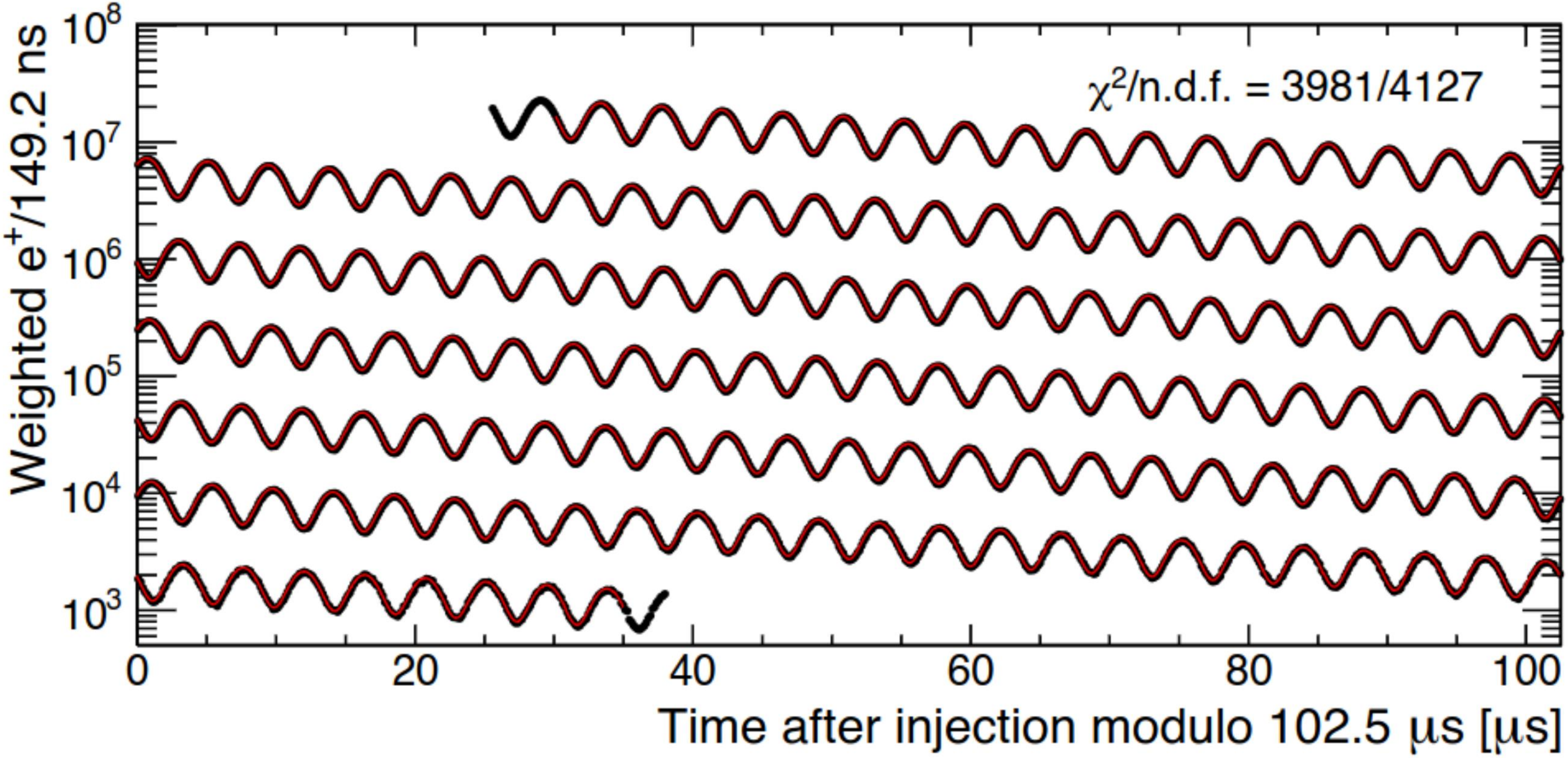
The a_μ measurement @ FNAL (Muon g-2 Coll.'23)



The a_μ measurement (Muon g-2 Coll.'23)

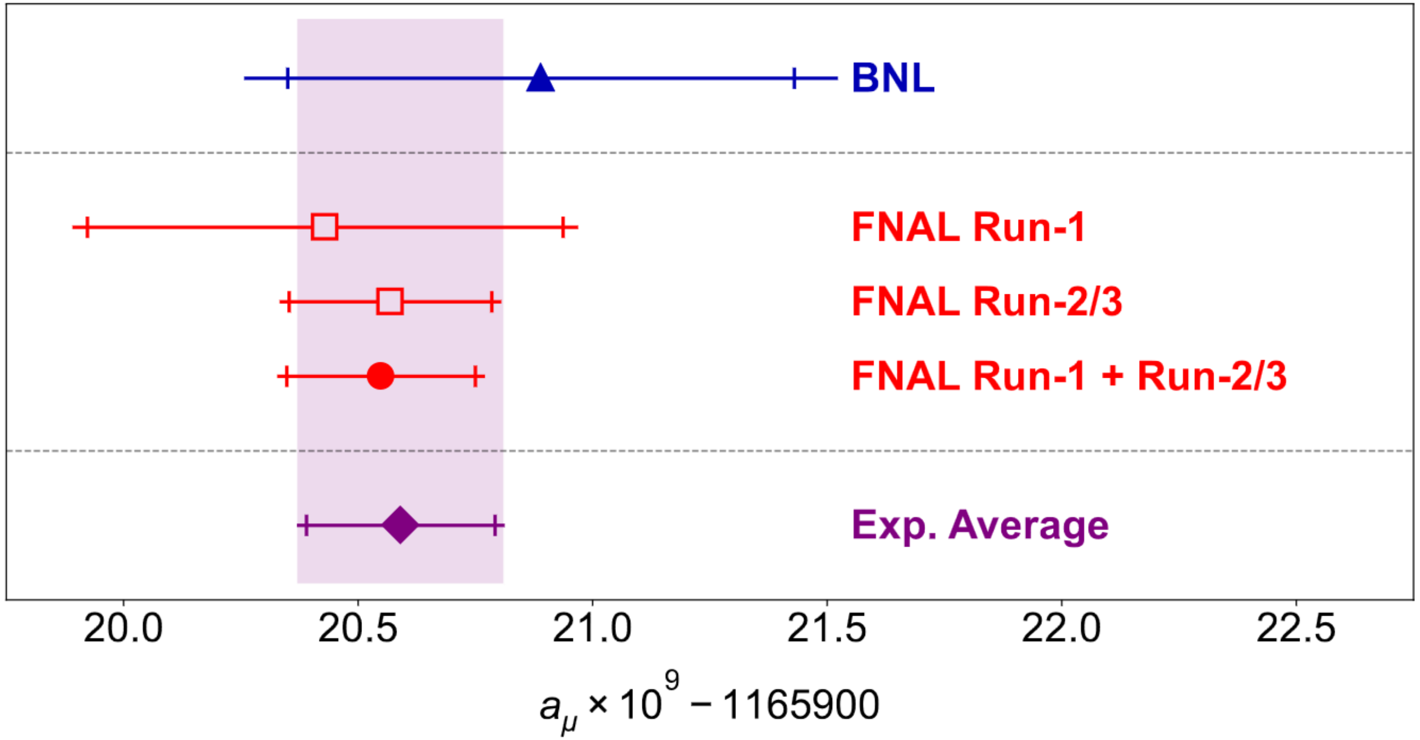


FIG. 3. Experimental values of a_μ from BNL E821 [8], our Run-1 result [1], this measurement, the combined Fermilab result, and the new experimental average. The inner tick marks indicate the statistical contribution to the total uncertainties.

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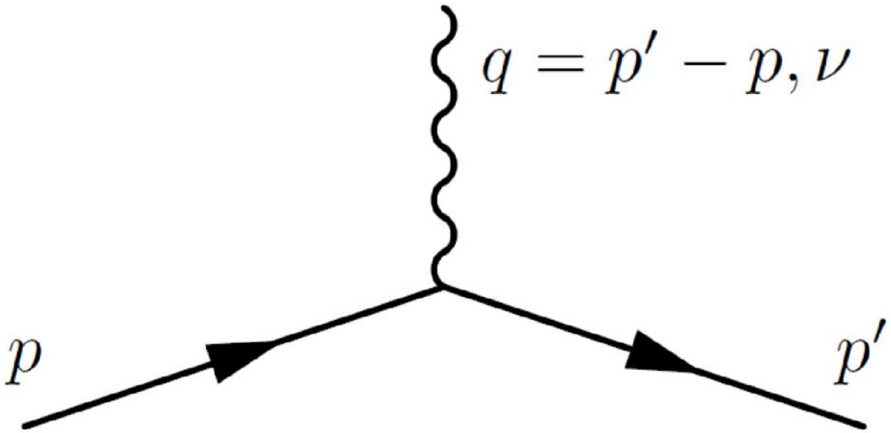
White Paper'20, Snowmass document and refs. therein

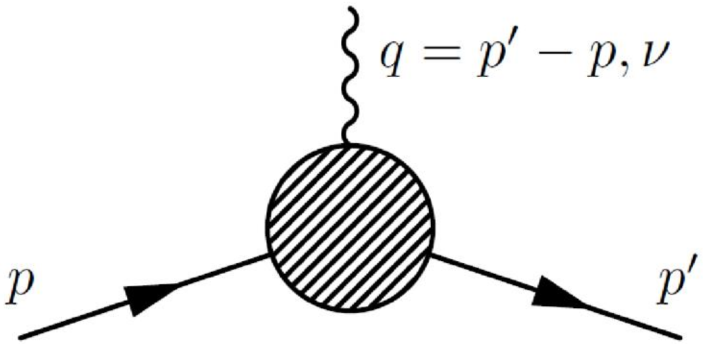
The SM



Dirac equation implies:

$$\bar{u}(p')\gamma_\nu u(p)$$

$$g = 2$$



$$\bar{u}(p') \left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)[\gamma_\nu, \gamma_\rho]q_\rho}{4m} \right) u(p)$$

$$a = F_2(q^2 = 0) = \frac{g-2}{2}$$

Quantum correction

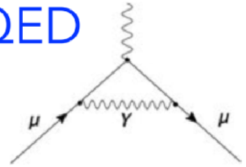
The SM

Muon g-2 Theory Initiative

$$a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})$$

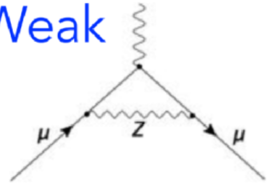
https://doi.org/10.1016/j.physrep.2020.07.006 (SM prediction, '**White Paper**')
https://muon-gm2-theory.illinois.edu

QED



$+\ldots$ $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak

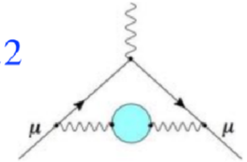


$+\ldots$ $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic… Dominate the error

…Vacuum Polarization (HVP)

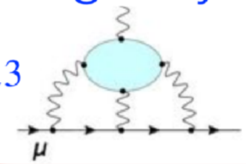
α^2



$+\ldots$ $6845\,(40) \times 10^{-11}$ $[0.6\%]$ 0.37 ppm

…Light-by-Light (HLbL)

α^3



$+\ldots$ $92\,(18) \times 10^{-11}$ $[20\%]$ 0.15 ppm

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The SM

Muon g-2 Theory Initiative

$$a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})$$

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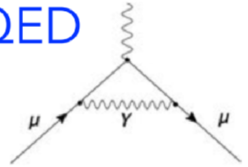
https://muon-gm2-theory.illinois.edu

QED



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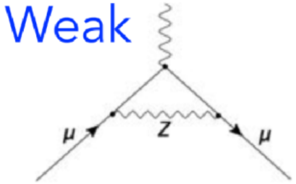

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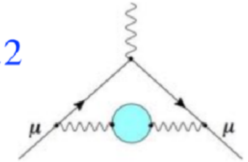
Hadronic… Dominate the error

Focus here ← …Vacuum Polarization (HVP)

α^2



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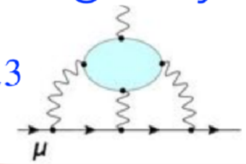

$+\ldots$ $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

See e.g. my talk @ HADRON'23

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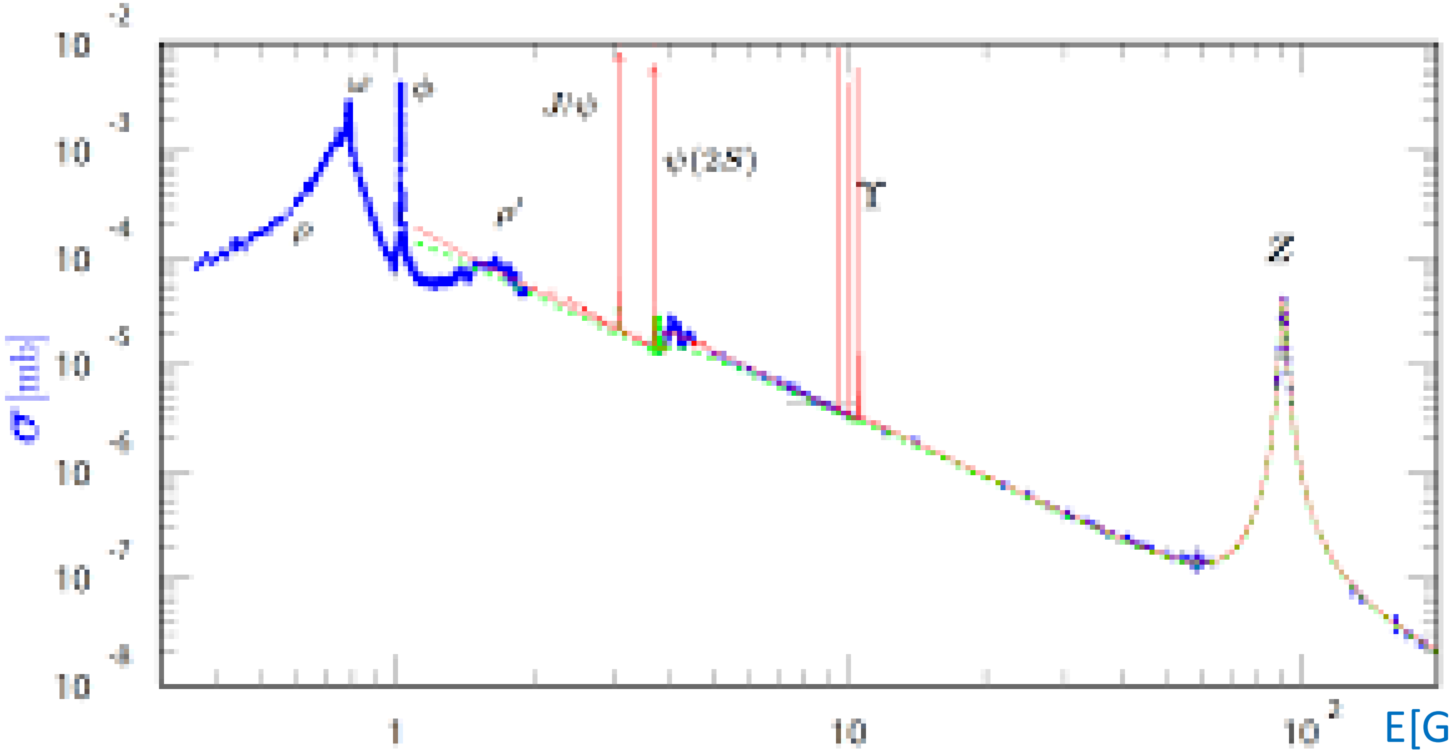
https://agenda.infn.it/event/33110/contributions/198273/attachments/106048/150321/Presentaci%C3%B3n3.pptx

The SM

White Paper'20, Snowmass document and refs. therein

$$a_\mu^{HVP,LO} = \frac{1}{4\pi^3}\int_{s_{thr}}^{\infty} ds\, K(s)\sigma^0_{e^-e^+\to hadrons}(s)$$

Both K & σ go as 1/s enhancing low-E contributions



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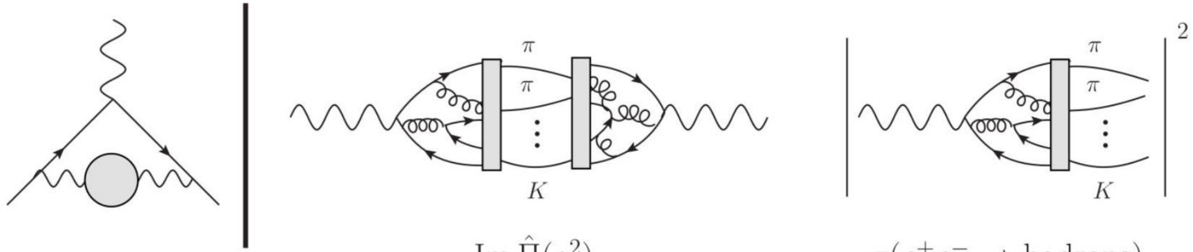
The SM

- Nature has solved QCD; use via the optical and Cauchy's th. to get $\hat{\Pi}(-Q^2)$

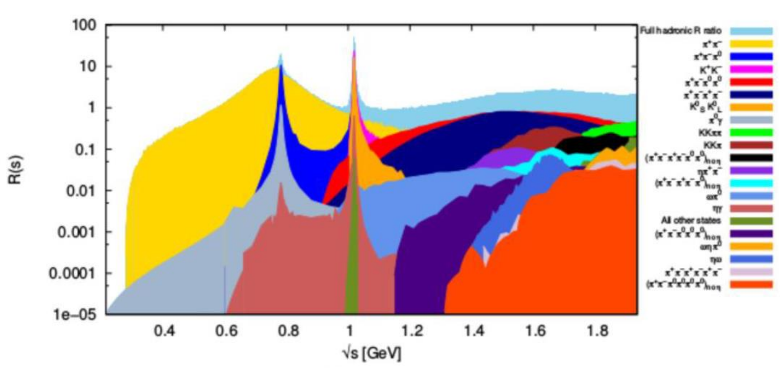


$$\mathrm{Im}\,\hat{\Pi}(q^2) \quad \sim \quad \sigma(e^+e^- \to \text{hadrons})$$

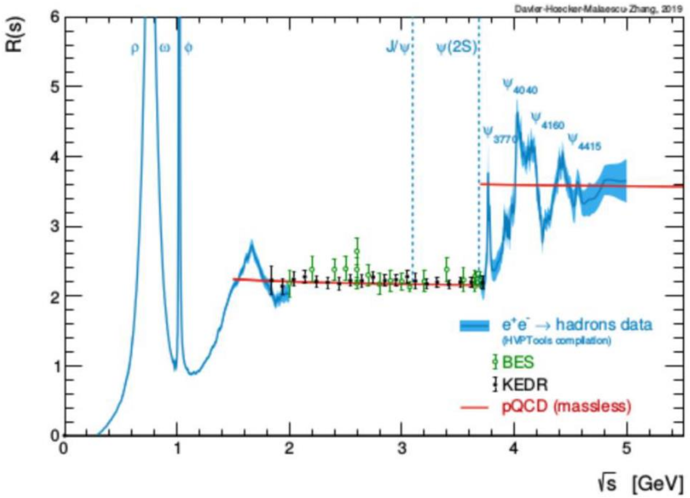
$$\int d^4x e^{iq\cdot x} \langle 0| \, T\{j^\mu(x) j^\nu(0)\} \, |0\rangle = i(q^2 g^{\mu\nu} - q^\mu q^\nu)\hat{\Pi}(q^2) \leftarrow \hat{\Pi}(0)$$

$$R = \frac{\sigma^{(0)}(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Oversimplifying: precise measurements for $e^+e^- \to \text{hadrons}$ or the R-ratio



(a) The hadronic R-ratio.



Figs in KNT'18/DHMZ'19 (left/right)

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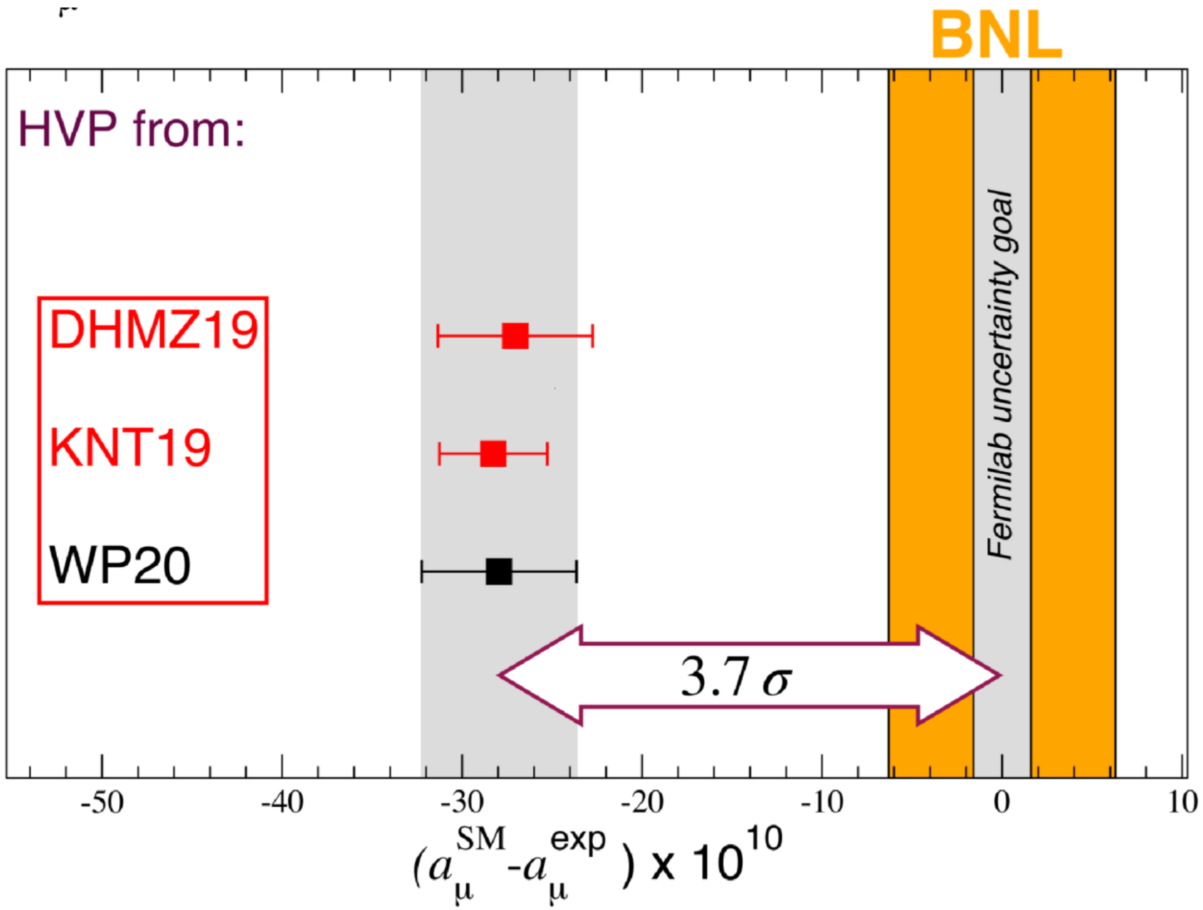
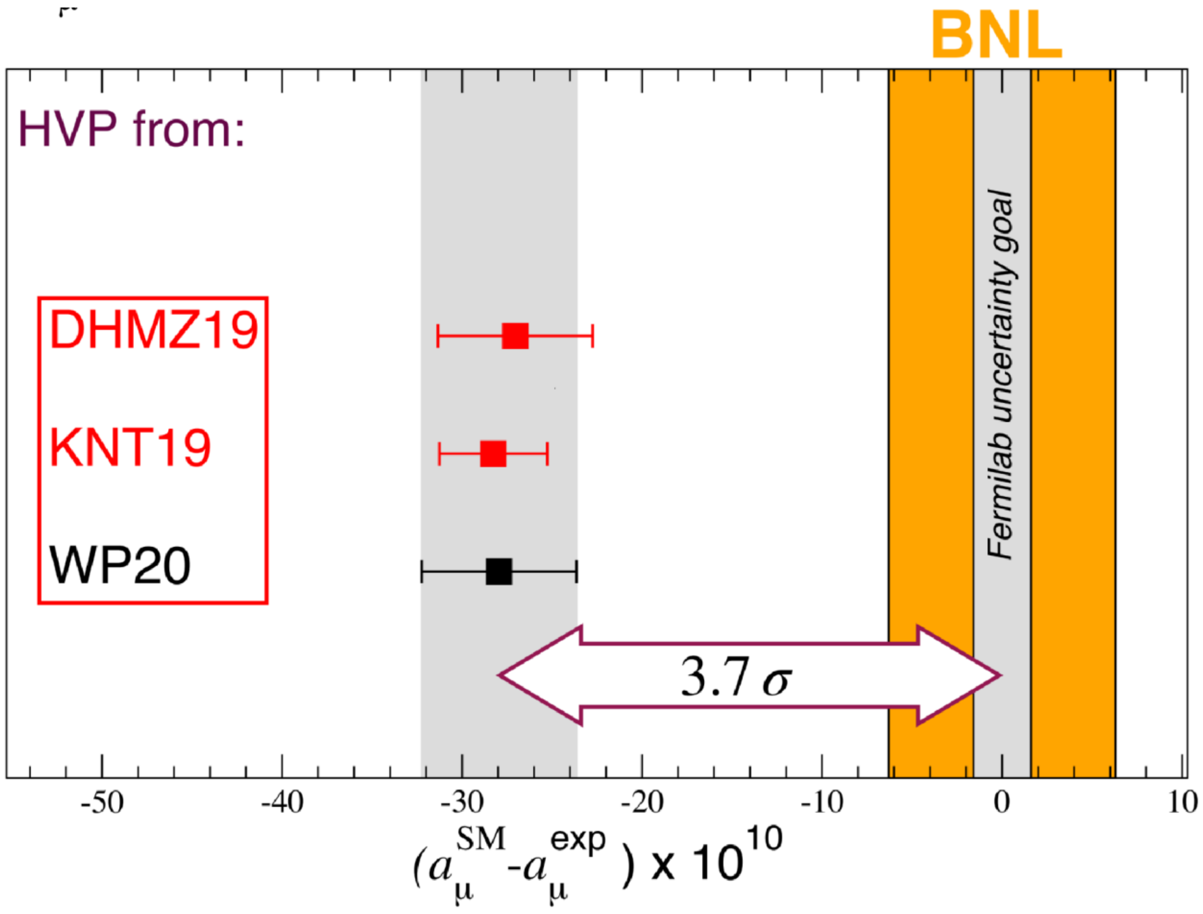
The SM



Davier-**H**öcker-**M**alaescu-**Z**hang'**19** & **K**eshavarzi-**N**omura-**T**eubner'**19** drive the **W**hite **P**aper'**20** combination

The SM



Davier-Höcker-Malaescu-Zhang'19 & Keshavarzi-Nomura-Teubner'19 drive the White Paper'20 combination

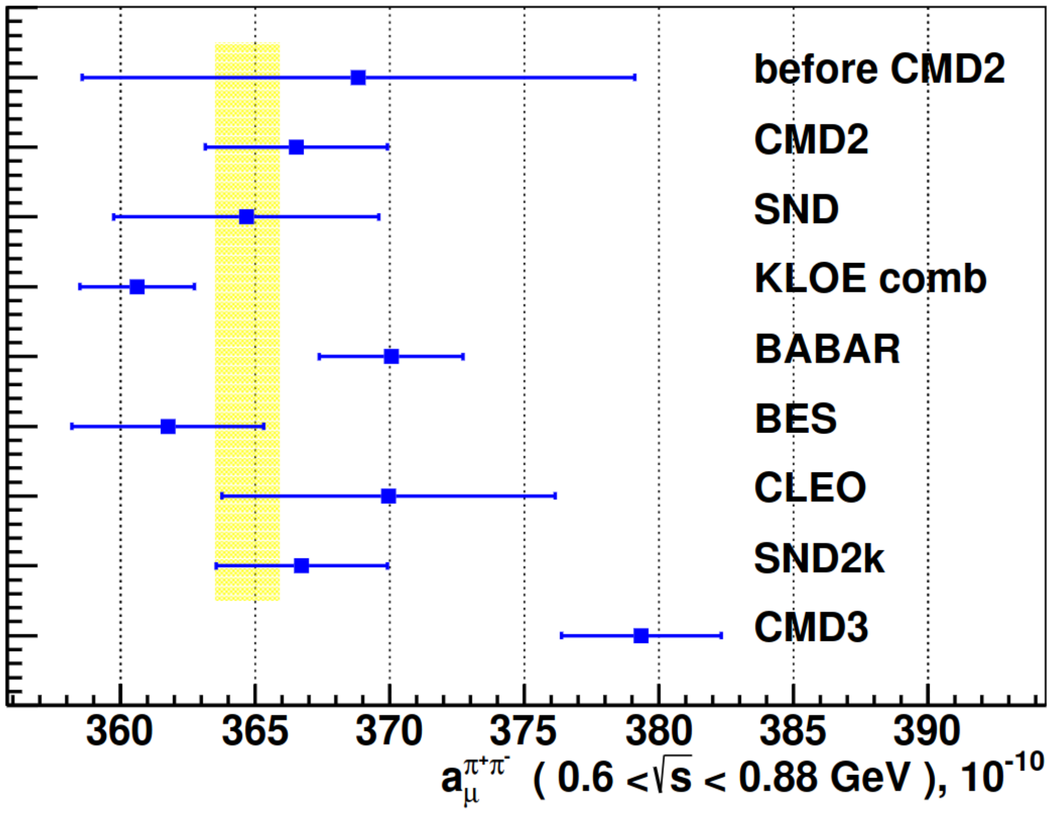


Figure 36: The $\pi^+\pi^-(\gamma)$ contribution to the $a_\mu^{had,LO}$ from the energy range $0.6 < \sqrt{s} < 0.88$ GeV obtained from the CMD-3 data and the results of the other experiments.

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The SM: Our τ-data based prediction

Miranda-Roig'20

$$a_\mu^{HVP,LO} = \frac{1}{4\pi^3}\int_{s_{thr}}^{\infty} ds\, K(s)\sigma^0_{e^-e^+\to hadrons}(s)$$

Both K & σ go as 1/s enhancing low-E contributions

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(Alemany, Davier, Höcker '97)

Alternative evaluation possible using semileptonic tau decay data, specifically 2π (4π) channel. Requires isospin breaking (IB).

$$\sigma^0_{\pi\pi} = \left[\frac{K_\sigma(s)}{K_\Gamma(s)}\frac{d\Gamma_{\pi\pi[\gamma]}}{ds}\right]\frac{R_{IB}(s)}{S_{EW}},$$

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Kinematics
& global cts.

Measurement

Short-distance EW RadCor

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$$R_{IB}(s) = \frac{FSR(s)}{G_{EM}(s)}\frac{\beta^3_{\pi^+\pi^-}}{\beta^3_{\pi^+\pi^0}}\left|\frac{F_V(s)}{f_+(s)}\right|^2$$

Kinematics & global cts. — Measurement — Short-distance EW RadCor

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The SM: Our τ-data based prediction

Miranda-Roig'20

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Kinematics & global cts.

Measurement

Short-distance EW RadCor

Final-state Rad

Kinematics

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Kinematics & global cts. — Measurement — Short-distance EW RadCor

Final-state Rad — Kinematics

(Cirigliano-Ecker-Neufeld '01)

The ratio of neutral to charged current di-pion form factors (**F_V/f_+**) and the long-distance em RadCor (**G_{EM}**) are challenging.

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The SM: Our τ-data based prediction

Miranda-Roig'20

$$\sigma^0_{\pi\pi} = \left[\frac{K_\sigma(s)}{K_\Gamma(s)}\frac{d\Gamma_{\pi\pi[\gamma]}}{ds}\right]\frac{R_{IB}(s)}{S_{EW}},$$

Kinematics & global cts. — Measurement — Short-distance EW RadCor

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Final-state Rad — Kinematics

The ratio of neutral to charged current di-pion form factors (**F_V/f_+**) and the long-distance em RadCor (**G_{EM}**) are challenging.

- The S_{EW} contribution $S_{EW} = 1.0201$ gives $\Delta a_\mu^{HVP,LO} = -103.1 \times 10^{-11}$, consistent with earlier determinations (using slightly different values of S_{EW}) and with a negligible error.
- The phase space (PS) correction induces $\Delta a_\mu^{HVP,LO} = -74.5 \times 10^{-11}$ (trivially in agreement with previous computations), again with tiny uncertainties.
- The final state radiation (FSR, which is formally NLO) yields $\Delta a_\mu^{HVP,LO} = +45.5(4.6) \times 10^{-11}$, in accord with ref. [67] (its value was not quoted in ref. [62]).

[62] Cirigliano-Ecker-Neufeld'02
[67] Davier-...-López Castro-...-Toledo et al.'09

The SM: Our τ-data based prediction

Miranda-Roig'20

$$\sigma^0_{\pi\pi} = \left[\frac{K_\sigma(s)}{K_\Gamma(s)}\frac{d\Gamma_{\pi\pi[\gamma]}}{ds}\right]\frac{R_{IB}(s)}{S_{EW}},$$

Kinematics & global cts. — Measurement — Short-distance EW RadCor

$$R_{IB}(s) = \frac{FSR(s)}{G_{EM}(s)}\frac{\beta^3_{\pi^+\pi^-}}{\beta^3_{\pi^+\pi^0}}\left|\frac{F_V(s)}{f_+(s)}\right|^2$$

Final-state Rad — Kinematics

The ratio of neutral to charged current di-pion form factors (**F_V/f_+**) and the long-distance em RadCor (**G_{EM}**) are challenging.

This correction was $+(61 \pm 26 \pm 3) \cdot 10^{-11}$ in [62] and $+(86 \pm 32 \pm 7) \cdot 10^{-11}$ in [67], in agreement (despite the big errors) with our FF2 and FF1 determinations, respectively.

$$\Delta a_\mu^{HVP,LO} = +40.9(48.9) \times 10^{-11} \qquad \Delta a_\mu^{HVP,LO} = +77.6(24.0) \times 10^{-11}$$

[62] Cirigliano-Ecker-Neufeld'02
[67] Davier-...-López Castro-...-Toledo et al.'09

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The SM: Our τ-data based prediction

Miranda-Roig'20

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Kinematics & global cts. Measurement Short-distance EW RadCor

Final-state Rad Kinematics

The ratio of neutral to charged current di-pion form factors (**F_V/f_+**) and the long-distance em RadCor (**G_{EM}**) are challenging.

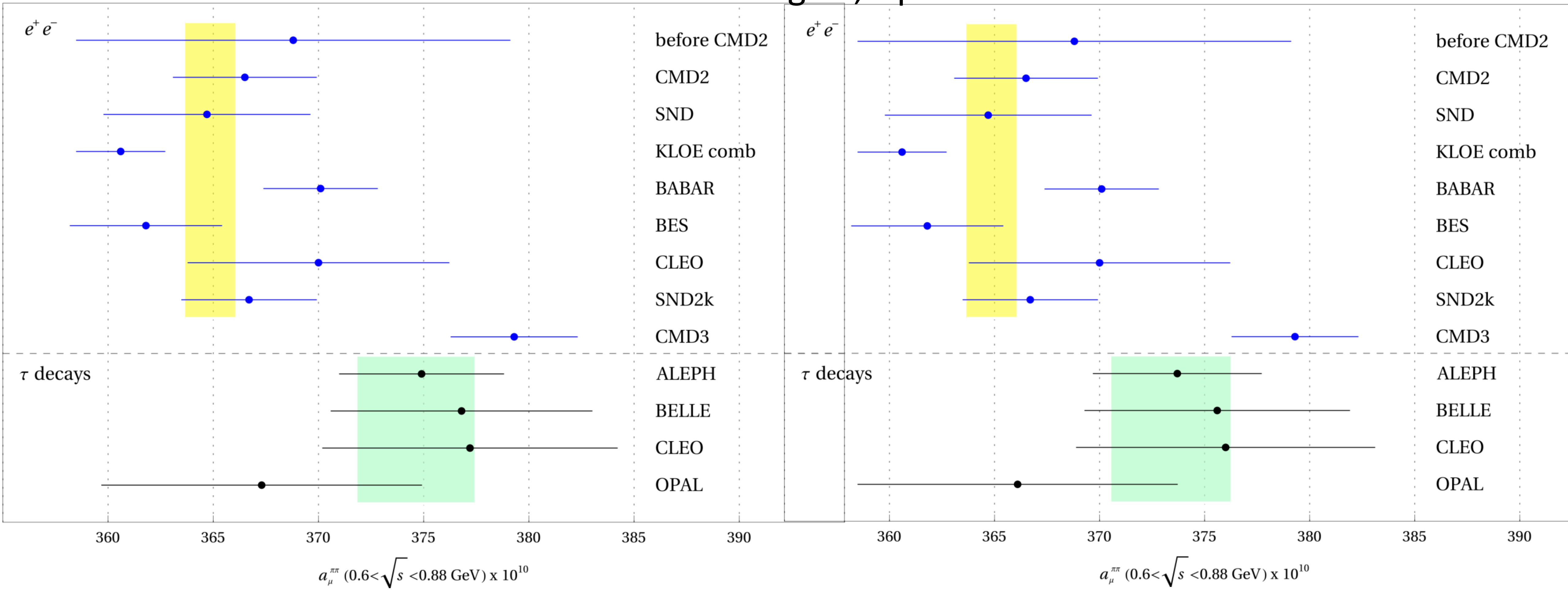
- Finally, we get $(-15.9^{+5.7}_{-16.0})\cdot 10^{-11}$ $((-76\pm 46)\cdot 10^{-11})$ for the $G_{EM}(s)$ correction at $\mathcal{O}(p^4)$ $(\mathcal{O}(p^6))$, versus $-10\cdot 10^{-11}$ in [62] and $-37\cdot 10^{-11}$ in [65] (from the last two results, $(-19.2\pm 9.0)\cdot 10^{-11}$ was used in [67]). ($\omega\to\pi^0\gamma$ contribution was subtracted from [65]'s)

[62] Cirigliano-Ecker-Neufeld'02
[67] Davier-...-López Castro-...-Toledo et al.'09

[65] Florez Baez- Flores Tlalpa-López Castro-Toledo '06
Consistent results found in Esparza-Arellano—Rojas—Toledo'23

The SM: Our τ-data based prediction

O(p^4) Miranda-Roig'20, updated O(p^6)



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The SM: Window quantities

Blum, Boyle, Gülpers, Izubuchi, Jin, Jung, Jüttner, Lehner, Portelli, Tsang (RBC, UKQCD),'18

$$\Theta_{\rm SD}(t) = 1 - \Theta(t, t_0, \Delta)\,,$$

$$\Theta_{\rm win}(t) = \Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)\,,$$

$$\Theta_{\rm LD}(t) = \Theta(t, t_1, \Delta)\,,$$

$$t_0 = 0.4\,\mathrm{fm}\,, \qquad t_1 = 1.0\,\mathrm{fm}\,, \qquad \Delta = 0.15\,\mathrm{fm}\,.$$

$$\Theta(t, t', \Delta) = \frac{1}{2}\left(1 + \tanh\frac{t - t'}{\Delta}\right),$$

The SM: Window quantities

Colangelo, El-Khadra, Hoferichter, Keshavarzi, Lehner, Stoffer, Teubner'22

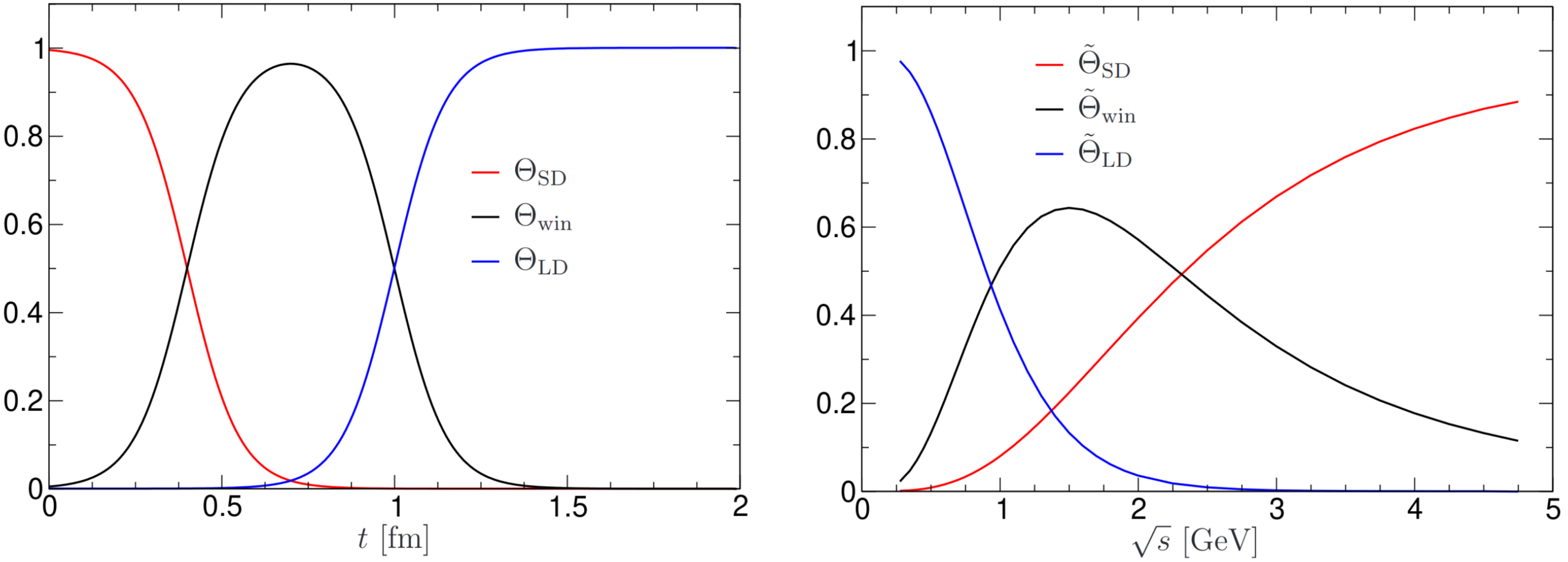
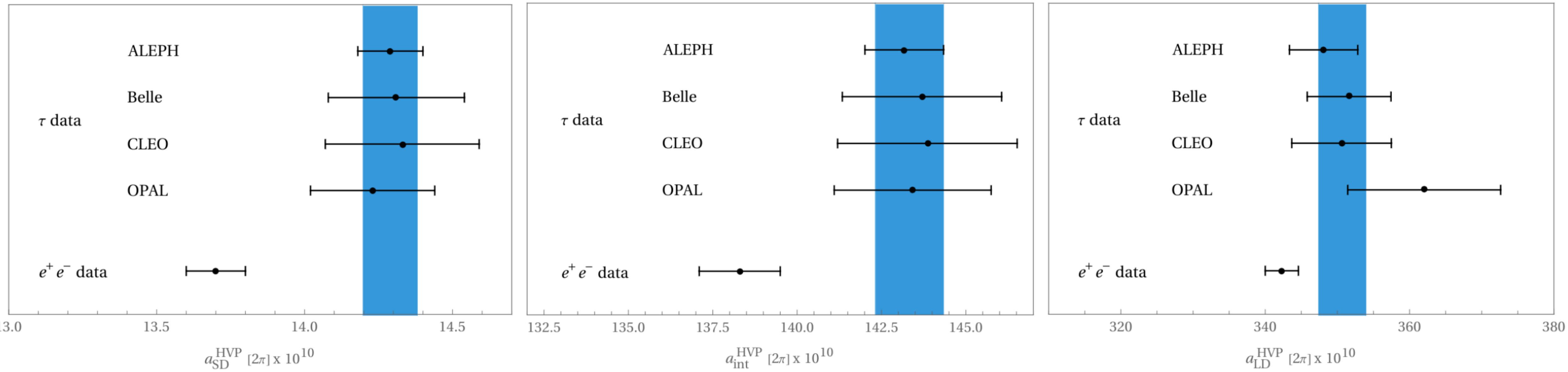


Figure 1: Short-distance, intermediate, and long-distance weight functions in Euclidean time (left), and their correspondence in center-of-mass energy (right).

The SM: Our τ-data based prediction

Masjuan-Miranda-Roig'23



Window quantities @O(p^4) below 1 GeV. Blue band is τ-data average. e^+e^- number taken from Colangelo et al.'22

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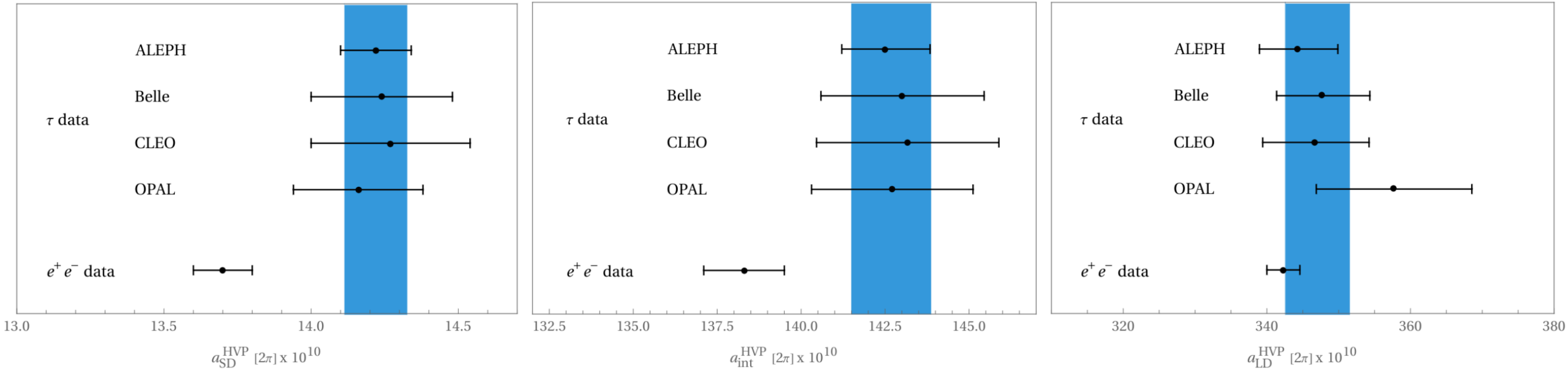
The SM: Our τ-data based prediction

Masjuan-Miranda-Roig'23



Window quantities @O(p^6) below 1 GeV. Blue band is τ-data average. e^+e^- number taken from Colangelo et al.'22

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The SM: Our τ-data based prediction

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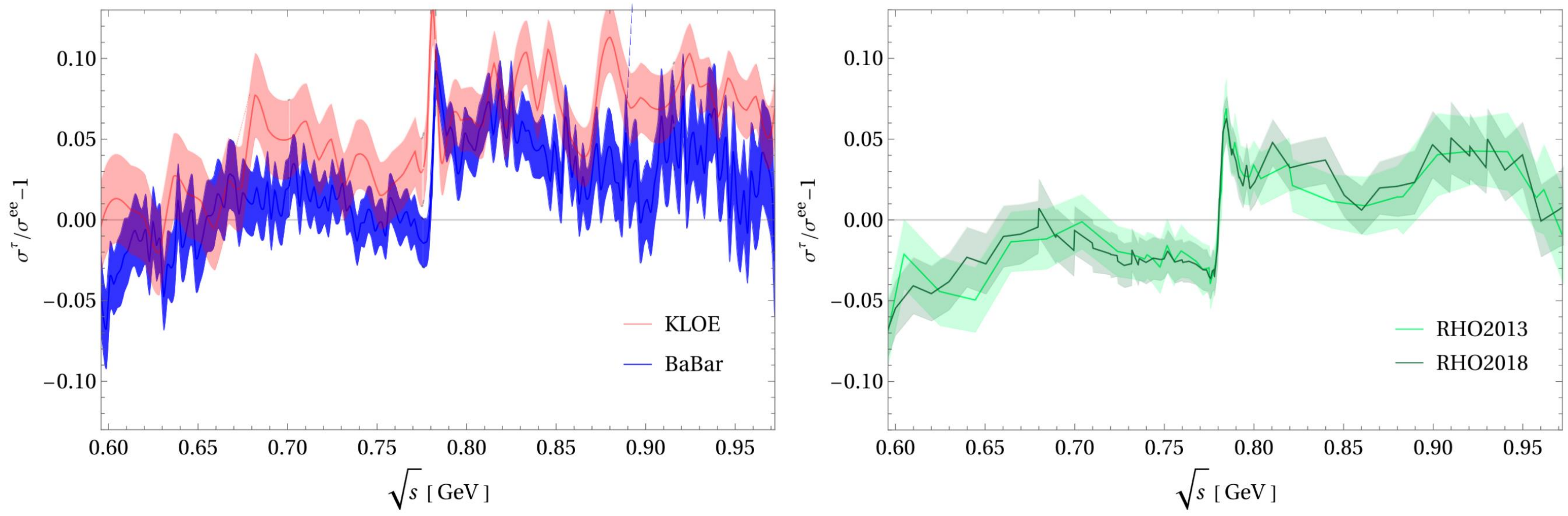

Figure 3: Comparison between the τ (after IB corrections) and $e^+e^- \to \pi^+\pi^-$ spectral function using the ISR measurements from BABAR [78] and KLOE [76] (left-hand) and the energy-scan measurements from CMD-3 [67] (right-hand).

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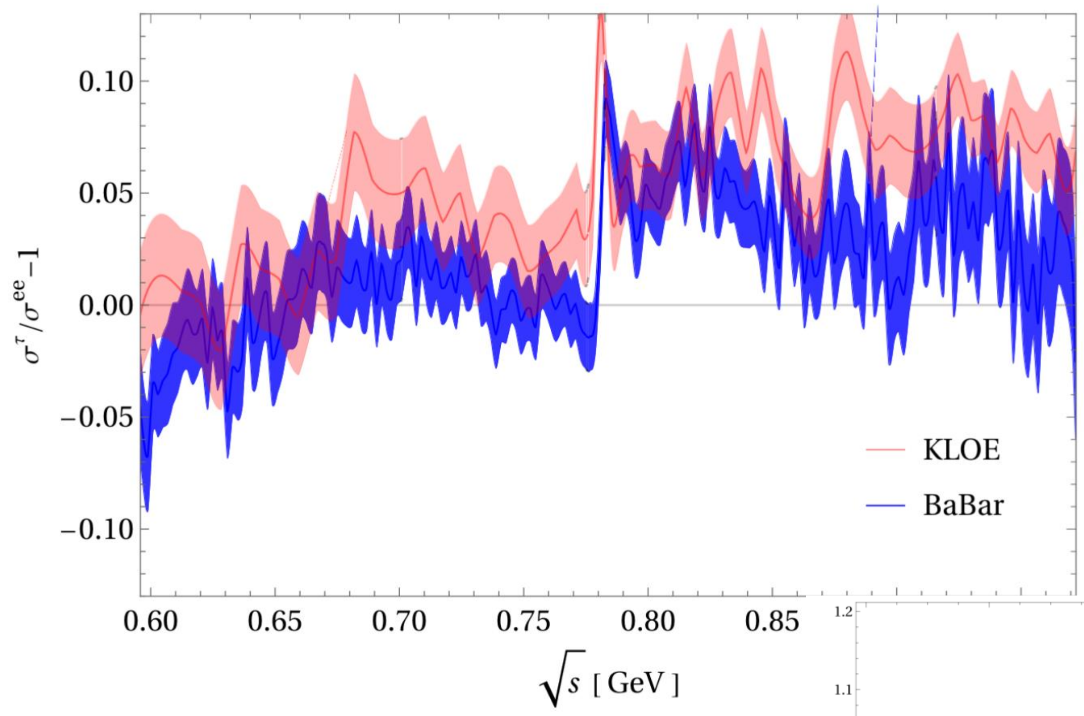
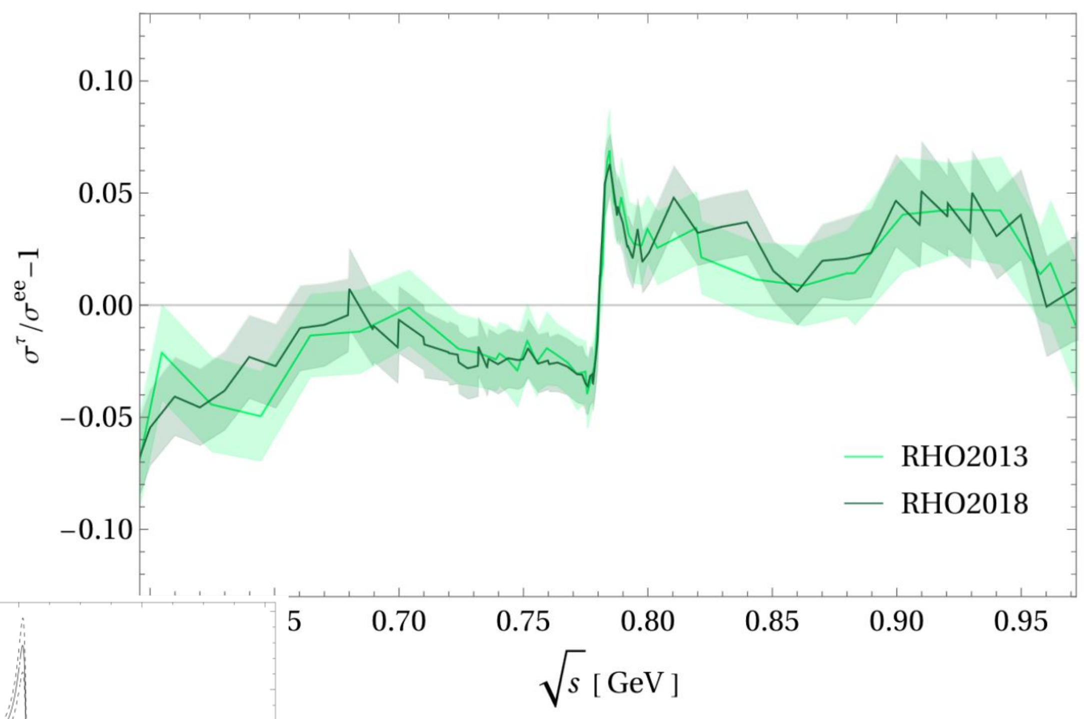
The SM: Our τ-data based prediction

Masjuan-Miranda-Roig'23





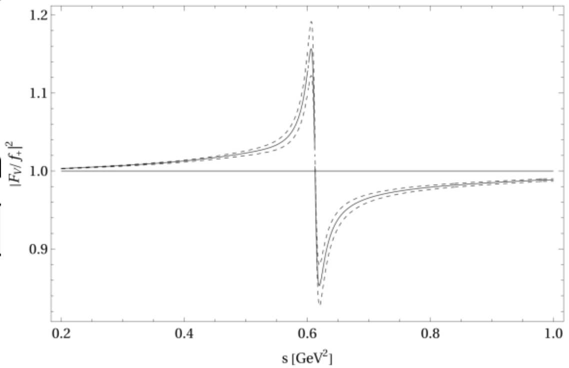


Figure 3: Comparison between the τ (a ... $e^+e^- \to \pi^+\pi^-$ spectral function using the ISR measurements from BABAR [... -hand) and the energy-scan measurements from CMD-3 [67] (right-hand).

Comparison with CMD-3 data seems to be dominated by the FFs ratio!!

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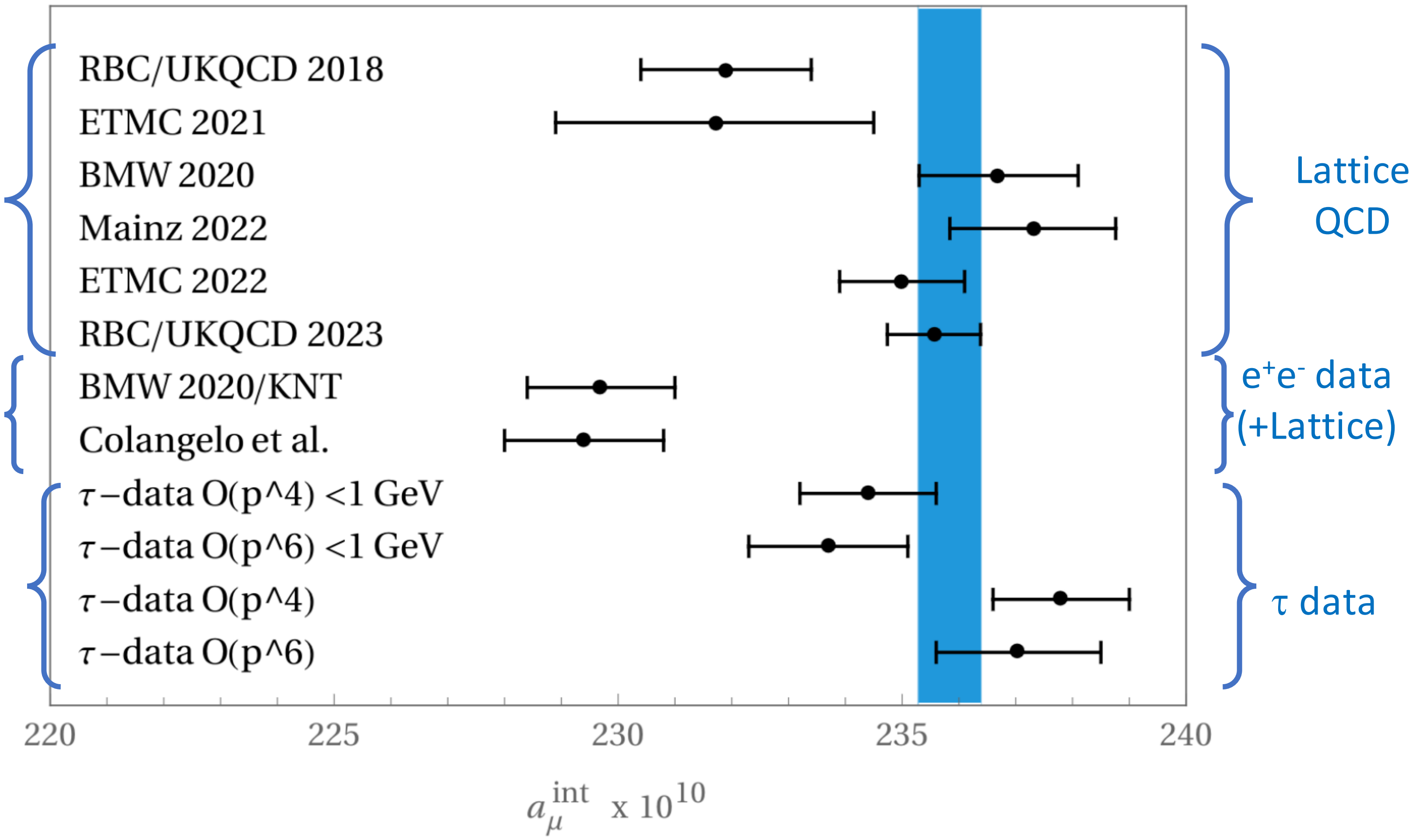
The SM: Our τ-data based prediction

Comparison of the total intermediate window contribution to $a_\mu^{HVP,LO}$. Blue band corresponds to lattice average excluding first two results.

Masjuan-Miranda-Roig'23



Pablo Roig (Cinvestav, Mexico City)

Their discrepancy (Updated using **M**iranda-**R**oig'20)

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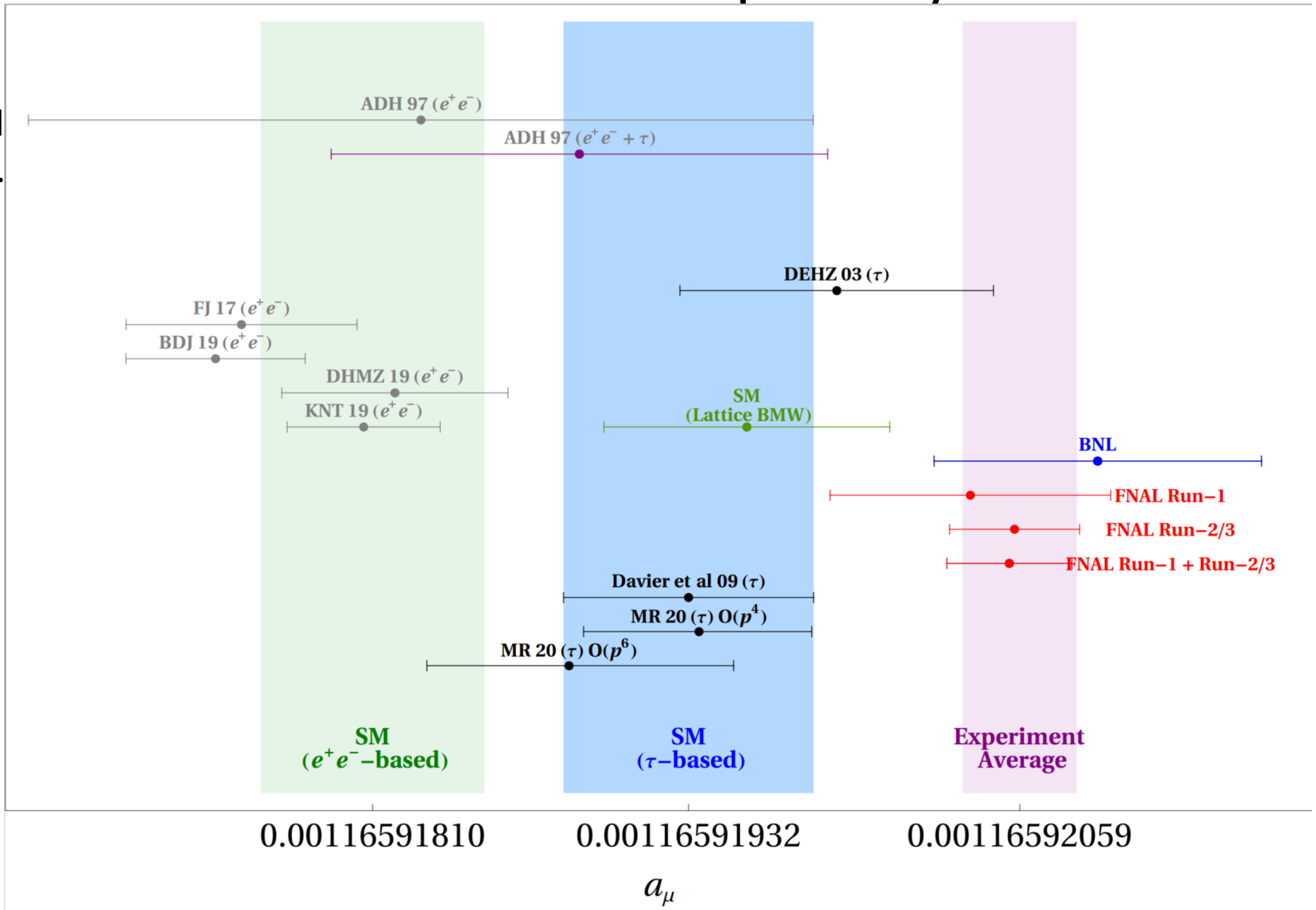

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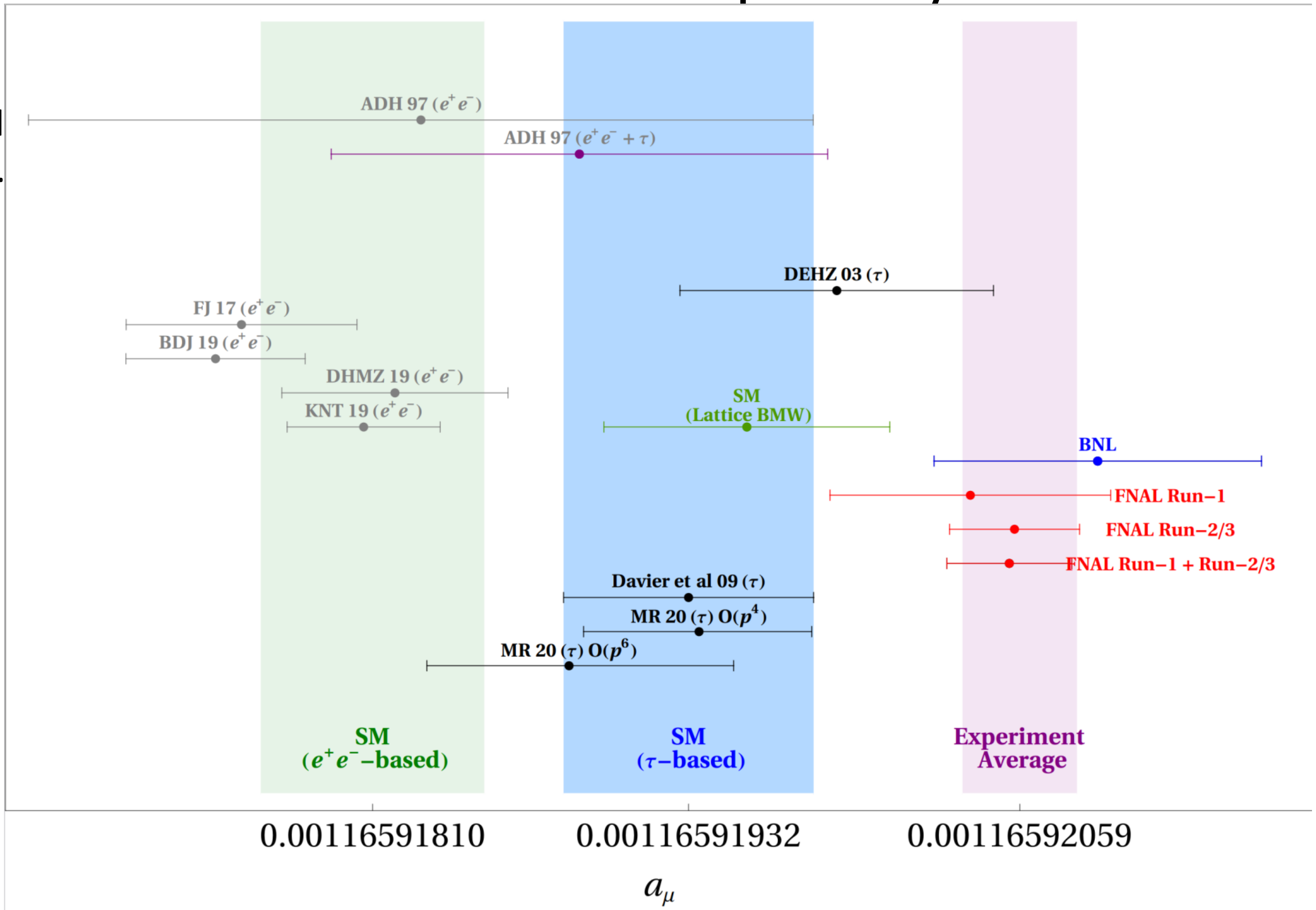

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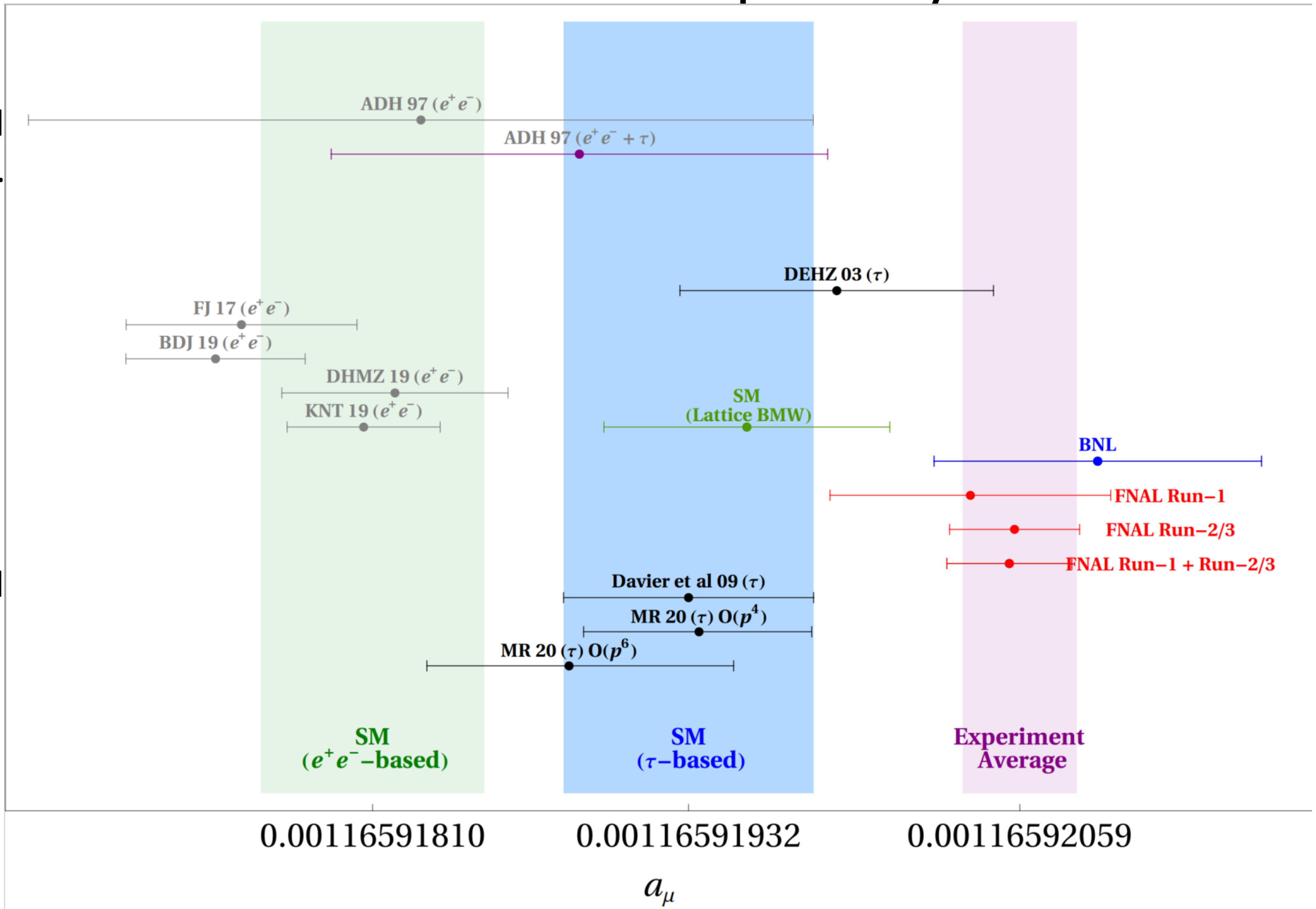

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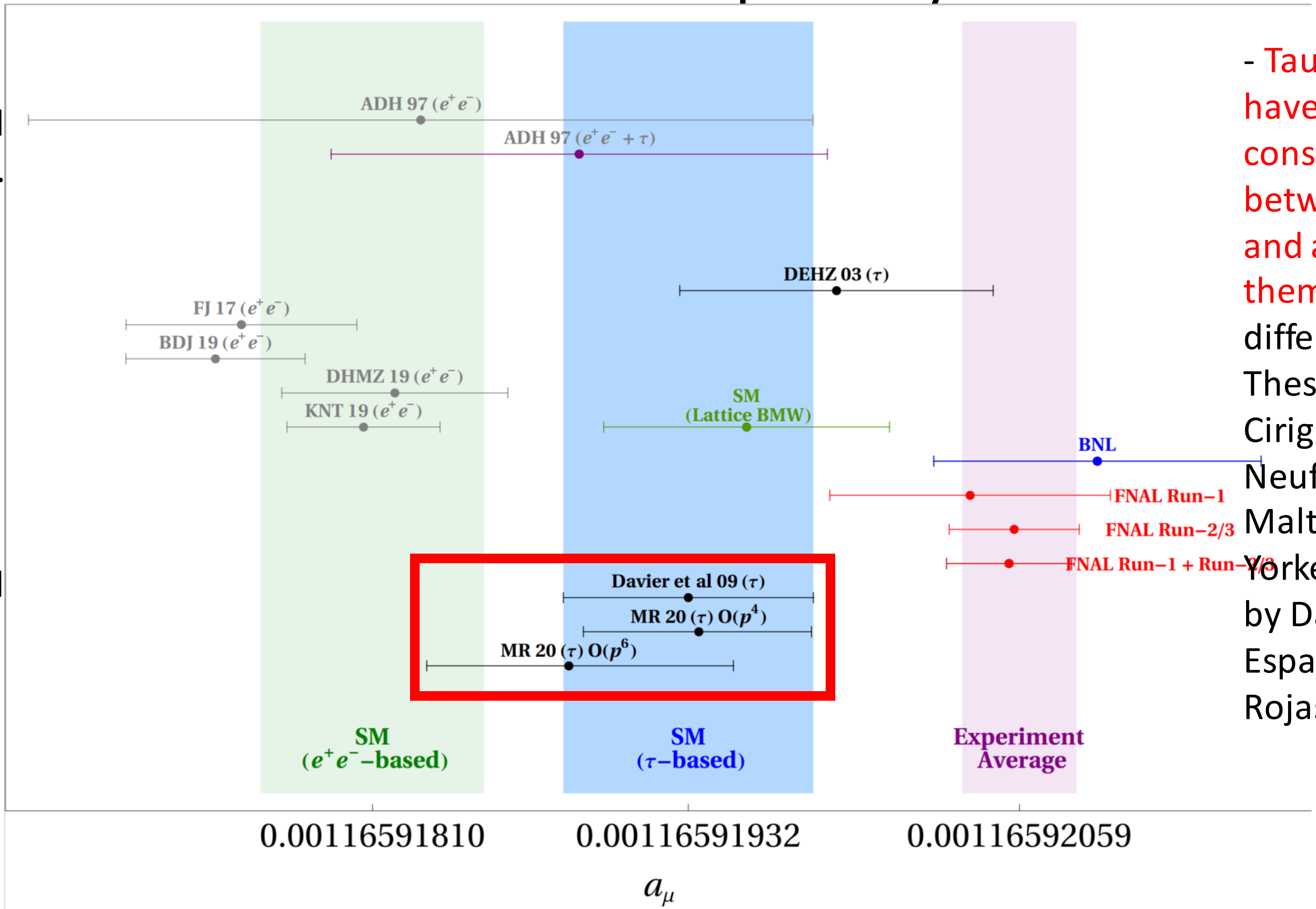

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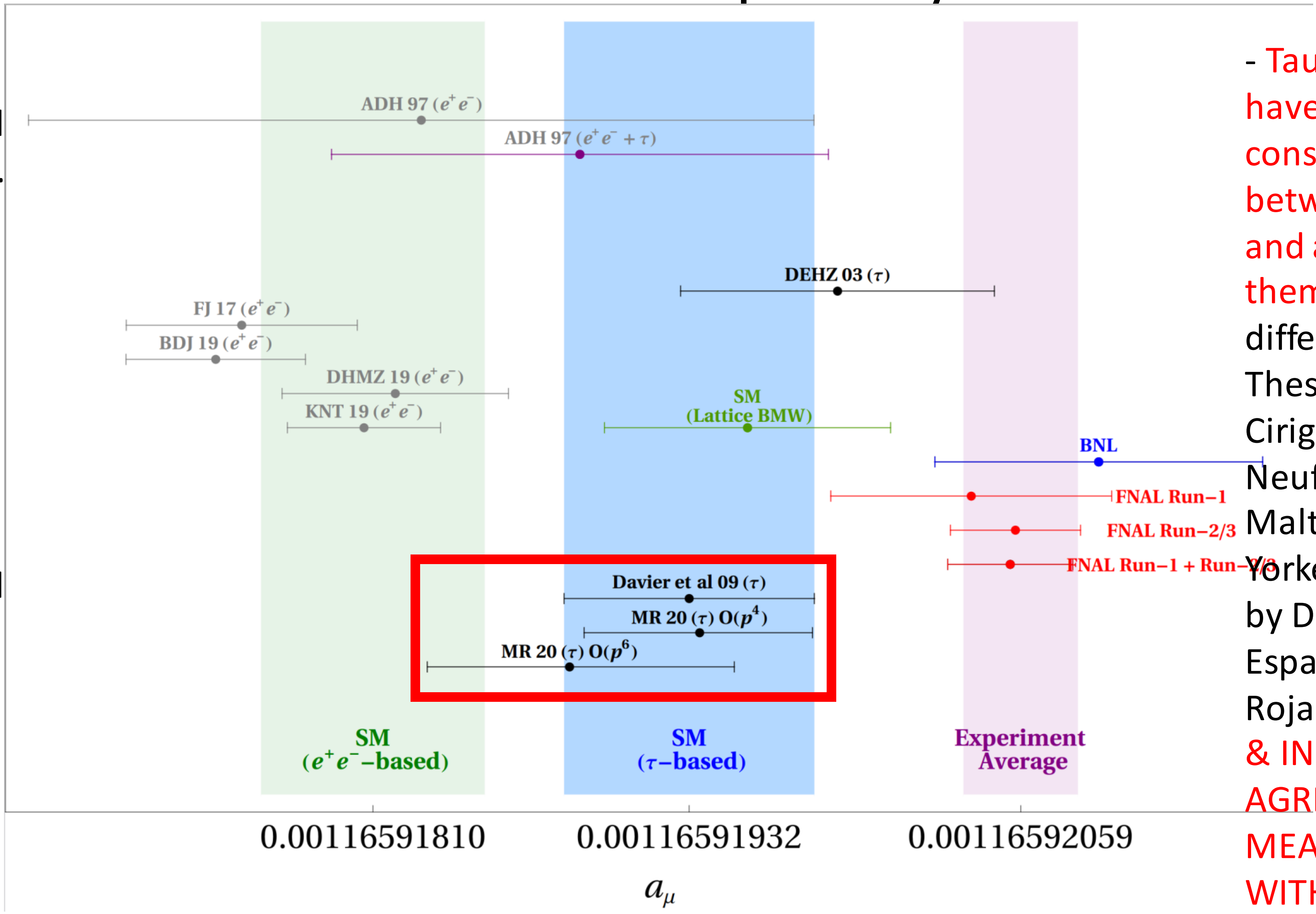

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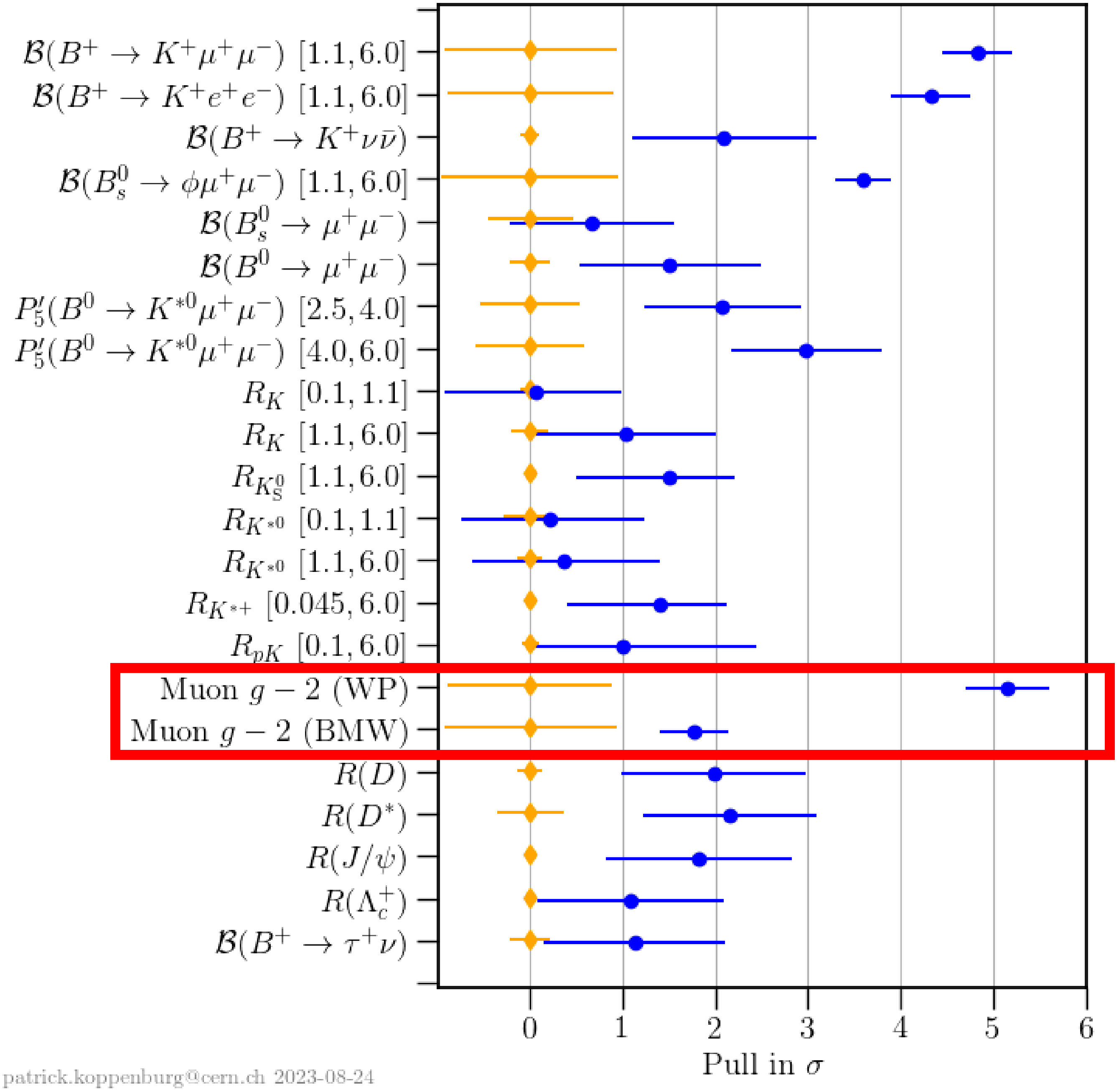
Pablo Roig (Cinvestav, Mexico City)

ANOMALIES & THEIR PULL



Pablo Roig (Cinvestav, Mexico City)

Outlook (Largely from Theory Initiatitive web)

- This event coincides with the **VI plenary workshop of the Muon g-2 Theory Initiative** in Bern, http://muong-2.itp.unibe.ch/ .
- A new analysis of the cross section based on the **full statistics** collected by the **BABAR** experiment is underway.
- The **SND** 2020 analysis of was based on 10% of the available statistics. An **update** using the full data set is in preparation.
- **Improved BES III** data between 2&5 GeV will be published.
- The **largest KLOE** data set ('04-'05) will be **analyzed**. Its statistics is 7 times larger than their published results together.
- Radiative corrections and Monte Carlo generators, in particular for the crucial di-pion channel, are being scrutinized. This includes the calculation and implementation of higher-order and structure-dependent corrections.
- New lattice-QCD results for the total HVP contribution and the long-distance window observable with a precision comparable to BMW and the data-driven approach will be available by 2025.
- **Other window quantities** and related observables will be analyzed.
- **Belle-II** will soon release their 3pi analysis & by 2025 the $\pi\pi$ one.
- The **MUonE** experiment at CERN will provide an independent and competitive method to compute the HVP contribution to the muon g-2, based on the high-precision measurement of the shape of the differential cross section of muon-electron elastic scattering as a function of the space-like squared momentum transfer. It would take data from 2026 on.
- **Lattice QCD** will make substantial progress in the evaluation of the **IB corrections** needed to use tau data for the HVP contribution.
- **Belle-II** shall improve the measurement of di-pion **tau** decays.
- ...

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