

Continuum QCD Methods: Making Inroads into Hadron Structure

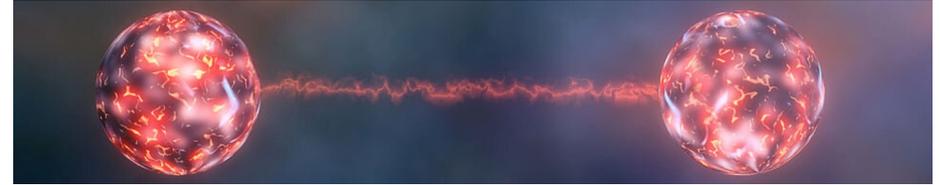
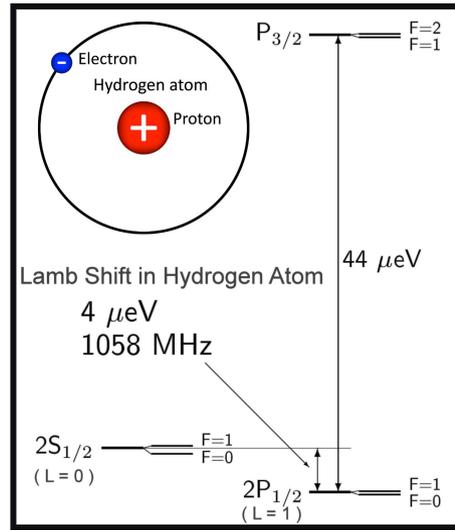
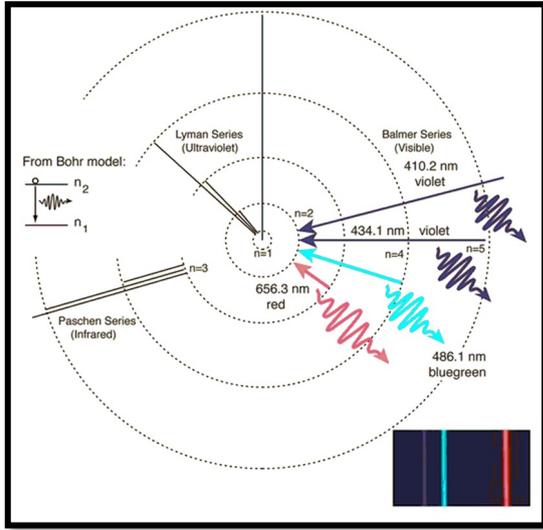
Adnan Bashir

Institute of Physics and Mathematics
University of Michoacán, Morelia, Michoacán, Mexico
Fulbright Visiting Scientist
Jefferson Laboratory, Newport News, Virginia, USA

September 4, 2023, University of Michoacan, Morelia, Mexico

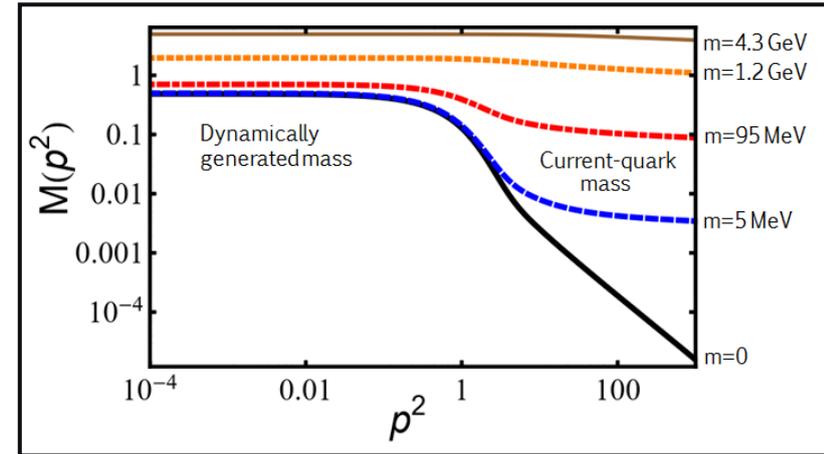


Two particle bound states

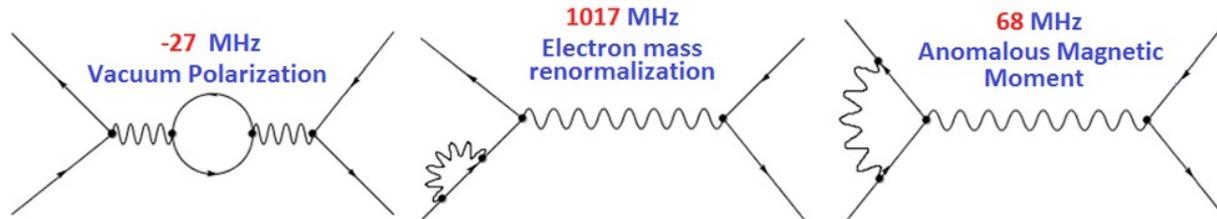


π and the **K** are the simplest two-body **bound states** in **QCD**. Unraveling their internal structure is a bigger challenge.

Confinement and emergent hadron mass

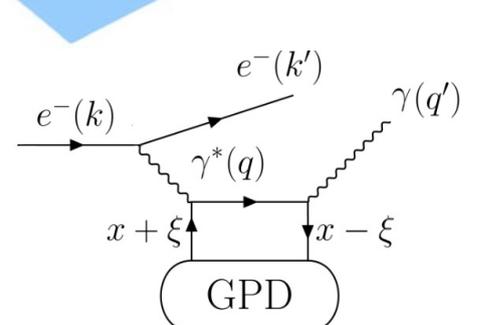
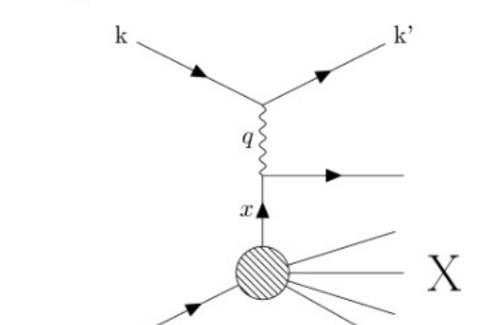
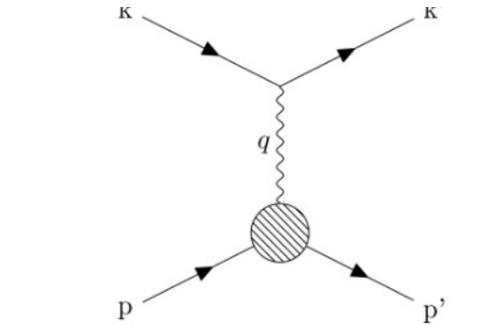
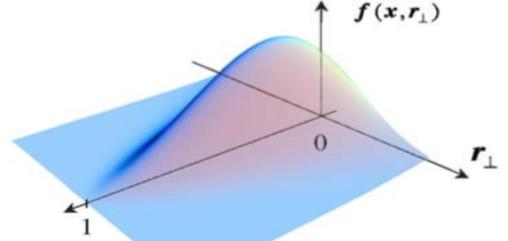
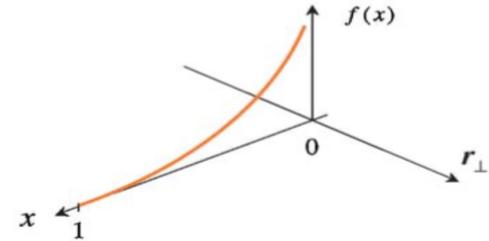
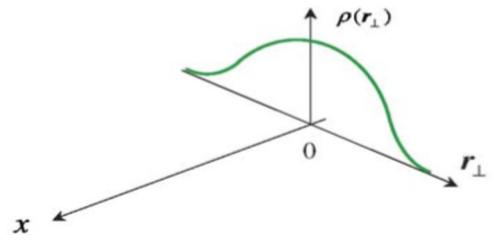
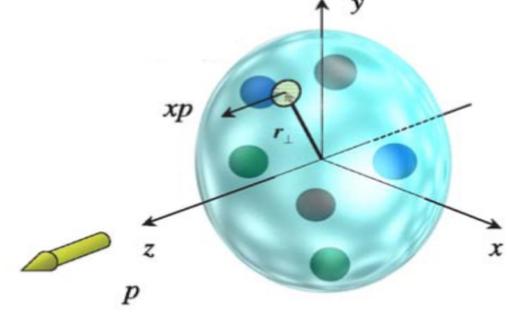
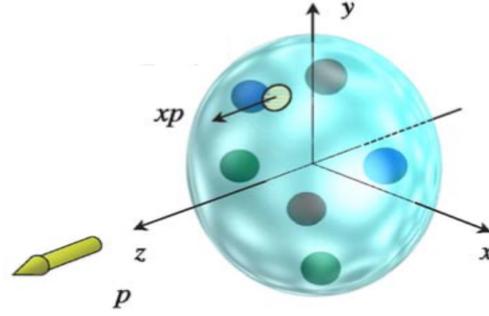
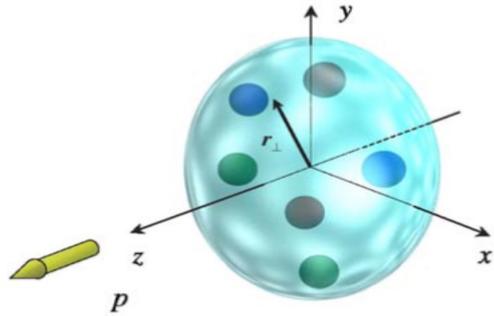


Dyson: If you don't understand the **Hydrogen atom** (in **QED**) you don't understand anything.



Renormalized QED

Three Dimensional Structure of Hadrons



Hofstadter, et. al., Phys. Rev. 91, 422 (1953)
 Hofstadter, Rev. Mod. Phys. 28, 214 (1956)

Feynman, Phys. Rev. Lett. 23, 1415 (1969)

GPDs: Footprints of QCD

3D Structure of Hadrons: **Global analysis** of **experimental data**, discrete **lattice studies**, **effective field theories** and continuum **Schwinger-Dyson equations**.

QCD is characterized by two **emergent** phenomena: **confinement** and **emergent hadron mass (EHM)**.

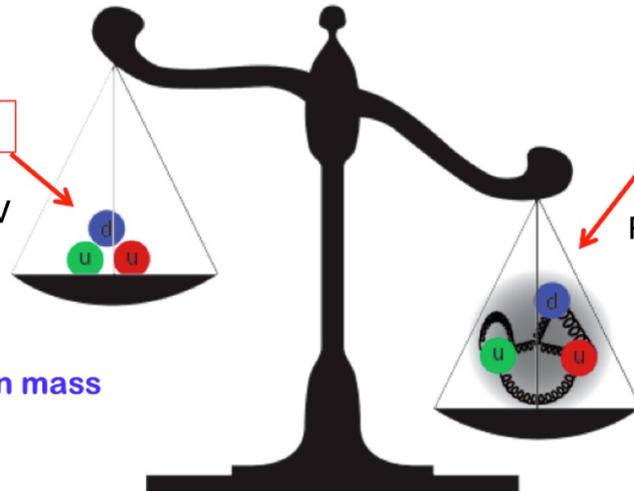
- Formation of color-singlet bound states: “**Hadrons**”



Higgs mechanism

Quark Mass ~ 3 MeV

~ 1% of proton mass



Dynamics of gluons

Proton Mass = 938.27 MeV

~ 99% of proton mass

$$\mathcal{L}_{\text{QCD}} = \sum_{j=u,d,s,\dots} \bar{q}_j [\gamma_\mu D_\mu + m_j] q_j + \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a,$$
$$D_\mu = \partial_\mu + ig \frac{1}{2} \lambda^a A_\mu^a,$$
$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc} A_\mu^b A_\nu^c,$$

Emergence of hadron masses (**EHM**) from QCD **dynamics**

QCD – Schwinger-Dyson Equations

Gauge Technique – Non Perturbative Solutions

- A. Salam, R. Delbourgo, Phys. Rev. 135 (1964) 6, B1398-B1427.

DCSB - Non-perturbative QED

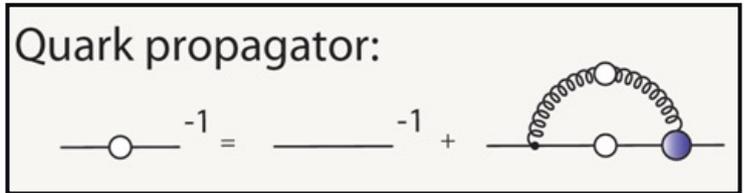
- P.I. Fomin, V.A. Miransky, Phys. Lett. B64 (1976) 166-168.

DCSB - Spectrum of PS-Mesons

- V.A. Miransky, P.I. Fomin, Phys. Lett. B105 (1981) 387-391

DCSB – MT Model - Vector Mesons

- P. Maris, P.Tandy, Phys. Rev. C60 (1999)



$$S(p^2, \mu^2) = \frac{Z(p^2, \mu^2)}{i \gamma \cdot p + M(p^2)}$$

PHYSICAL REVIEW VOLUME 135, NUMBER 6B 21 SEPTEMBER 1964

Renormalizable Electrodynamics of Scalar and Vector Mesons. II

ABDUS SALAM*
Imperial College, London, England

ROBERT DELBOURGO†
Imperial College, London, England

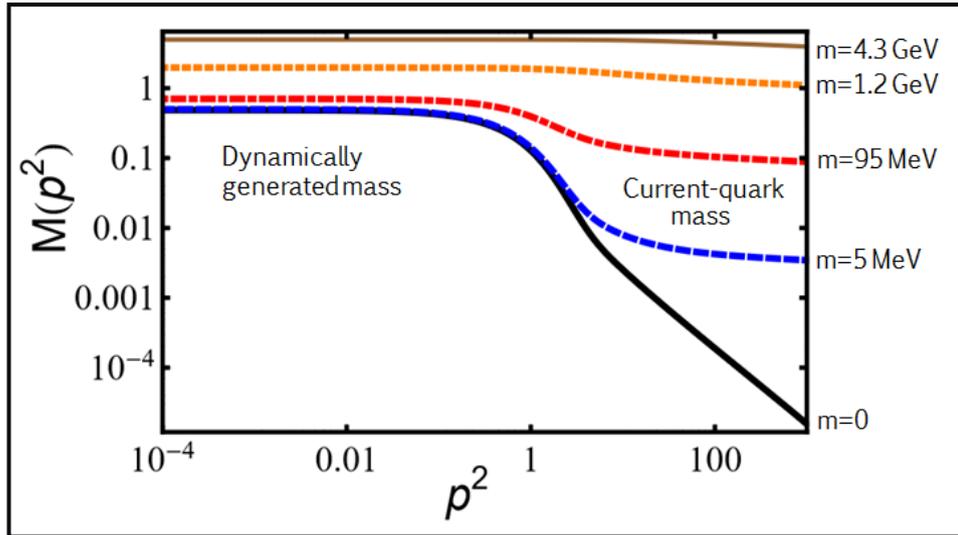
The "gauge technique" for solving field theories introduced in an earlier paper is applied to scalar and vector electrodynamics.

A. Dyson-Schwinger Set;

For a typical 3-field (e.g., electron-photon) interaction the well-known Dyson equations

$$S^{-1} = Z_2 S_0^{-1} + Z_1 e^2 \int \Gamma S \Gamma_0 D \quad \leftarrow \text{(I.1)}$$

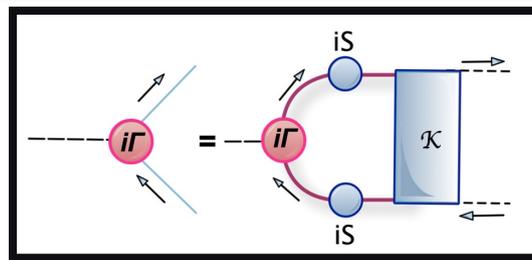
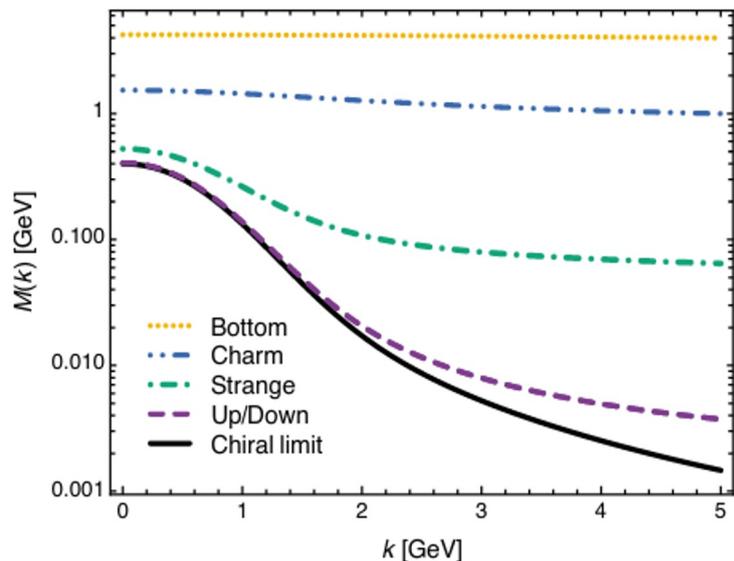
SDE: electron propagator

$$D^{-1} = Z_3 D_0^{-1} + Z_1 e^2 \int \Gamma S \Gamma_0 S \quad \text{(I.2)}$$


π and K : Bound States and Goldstone Bosons

The effects of the pattern of **DCSB** are traceable in the **Q^2 evolution** of the π and K **Bethe-Salpeter Amplitudes (BSAs)** and **form factors** explored and planned in the **JLab** and the **EIC**.

$$\Gamma_\pi(k, P) = \gamma_5 \left[iE_\pi(k; P) + \gamma \cdot P F_\pi(k; P) + \gamma \cdot k k \cdot P G_\pi(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k; P) \right]$$



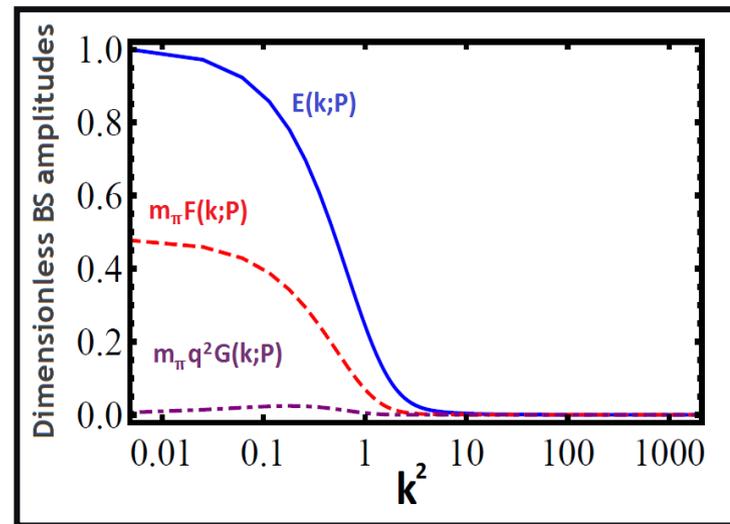
$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

$$f_\pi E_\pi(k; P = 0) = B(p^2)$$

$$F_R(k; 0) + 2 f_\pi F_\pi(k; 0) = A(k^2)$$

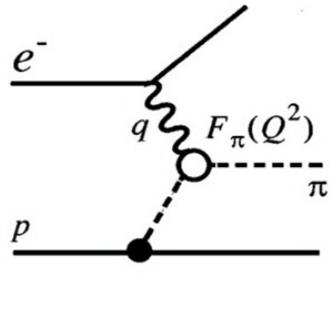
$$G_R(k; 0) + 2 f_\pi G_\pi(k; 0) = 2A'(k^2)$$

$$H_R(k; 0) + 2 f_\pi H_\pi(k; 0) = 0$$

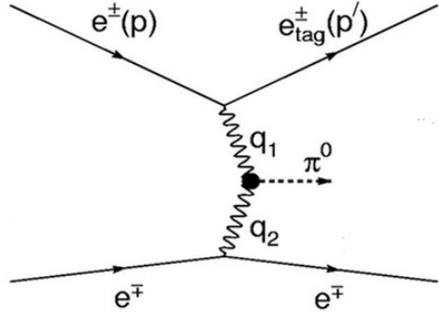


Mesons: Probing Quarks with Photons

In studying the **elastic** or **transition form factors**, it is the **photon** which probes the **dressed quarks** inside the **bound states**, highlighting the importance of the **quark-photon vertex**.



Pion Elastic Form Factor



Pion Transition Form Factor

$$\Gamma_{\mu}^L(p, k, q) = \sum_{i=1}^4 \lambda_i L_{\mu}^i(p, k)$$

$$L_{\mu}^1 = \gamma_{\mu}$$

$$L_{\mu}^2(p, k) = (\not{p} + \not{k})(p + k)_{\mu}$$

$$L_{\mu}^3(p, k) = -(p + k)_{\mu}$$

$$L_{\mu}^4(p, k) = -\sigma_{\mu\nu}(p + k)^{\nu}$$

$$\Gamma_{\mu}^T(p, k, q) = \sum_{i=1}^8 \tau_i(p^2, k^2, q^2) T_{\mu}^i(p, k)$$

$$T_{\mu}^1 = p_{\mu}(k \cdot q) - k_{\mu}(p \cdot q),$$

$$T_{\mu}^2 = [p_{\mu}(k \cdot q) - k_{\mu}(p \cdot q)](\not{p} + \not{k}),$$

$$T_{\mu}^3 = q^2 \gamma_{\mu} - q_{\mu} \not{q},$$

$$T_{\mu}^4 = q^2 [\gamma^{\mu}(\not{k} + \not{p}) - (k + p)^{\mu}]$$

$$+ 2(k - p)^{\mu} \sigma_{\nu\lambda} p^{\nu} k^{\lambda},$$

$$T_{\mu}^5 = -\sigma_{\mu\nu} q^{\nu},$$

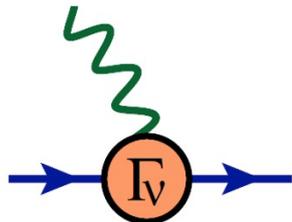
$$T_{\mu}^6 = \gamma_{\mu}(p^2 - k^2) + (p + k)_{\mu} \not{q},$$

$$T_{\mu}^7 = \frac{1}{2}(p^2 - k^2) [\gamma_{\mu}(\not{p} + \not{k}) - (p + k)_{\mu}]$$

$$- (p + k)_{\mu} \sigma_{\nu\lambda} p^{\nu} k^{\lambda},$$

$$T_{\mu}^8 = \gamma_{\mu} \sigma_{\nu\lambda} p^{\nu} k^{\lambda} - p_{\mu} \not{k} + k_{\mu} \not{p}.$$

Gauge covariance (WTI, TTI, LKFT),
kinematic singularities, perturbation
theory, multiplicative renormalizability

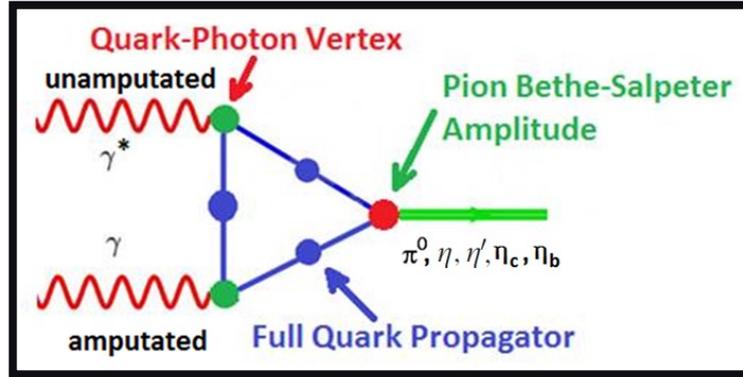
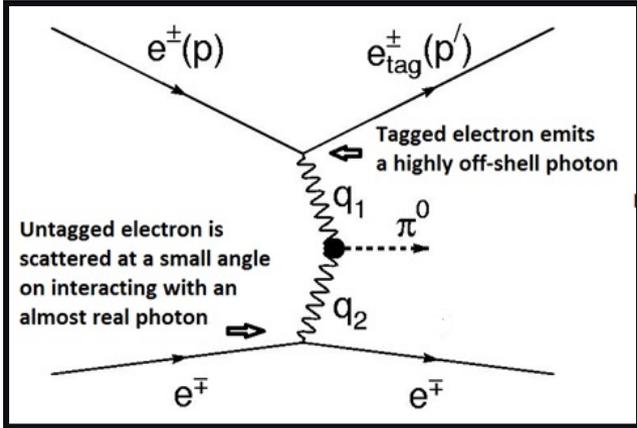


AB, M.R. Pennington, Phys. Rev. D50 7679 (1994)

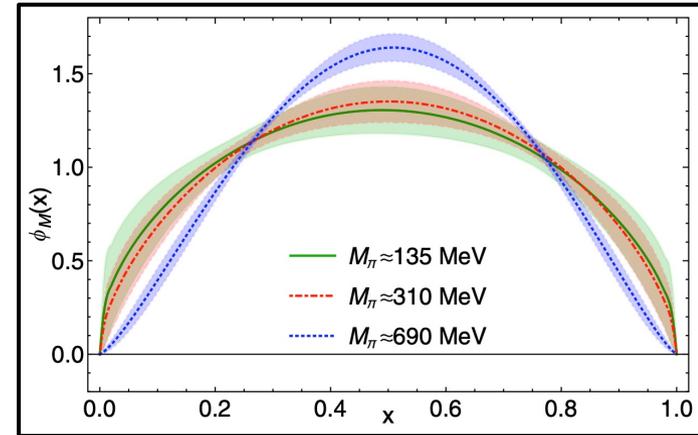
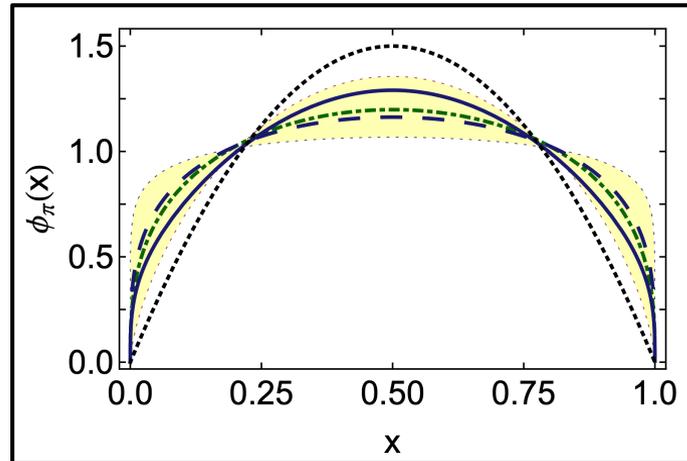
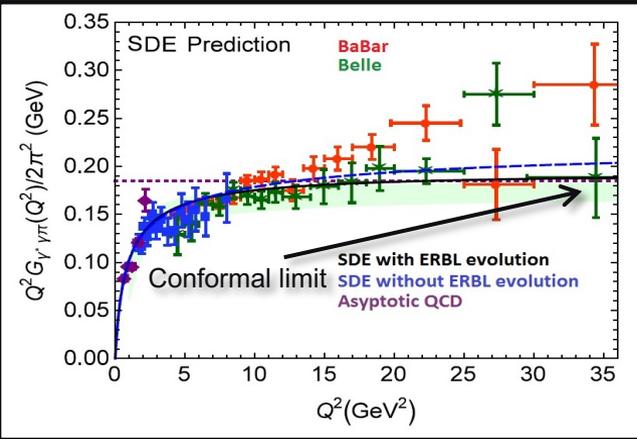
R. Bermudez et. al., Phys. Rev. C85, 045205 (2012)

V. Banda, AB, Phys. Rev. D107 073008 (2023)

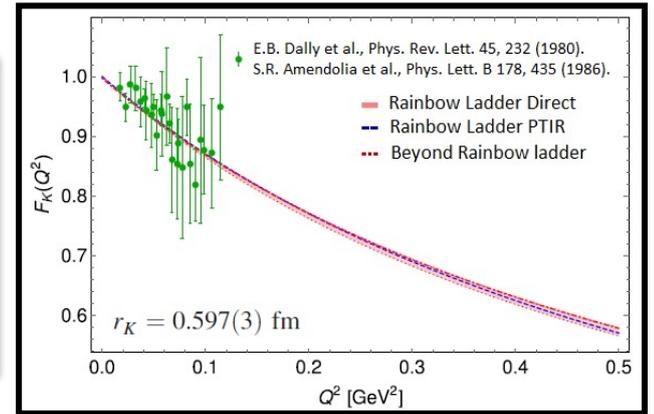
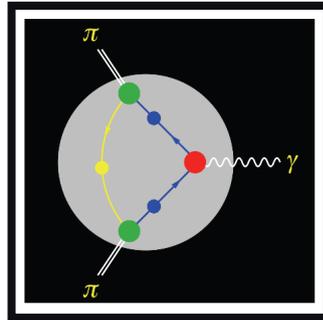
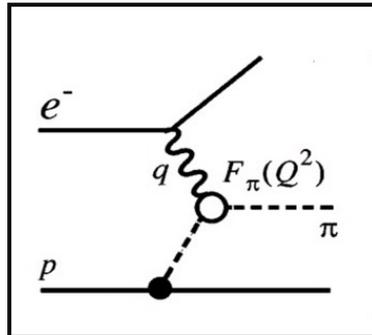
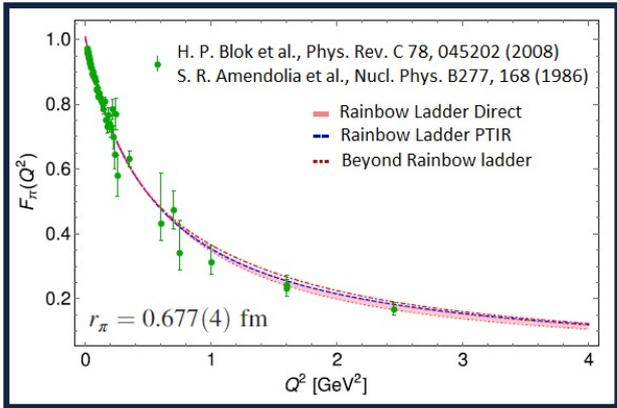
$\pi^0 \rightarrow \gamma^* \gamma$ Transition Form Factor



The pattern of **DCSB** dictates the **Q^2 evolution** of this **transition form factor**. Both experiment and **asymptotic QCD** for the largest Q^2 values provide verifications.



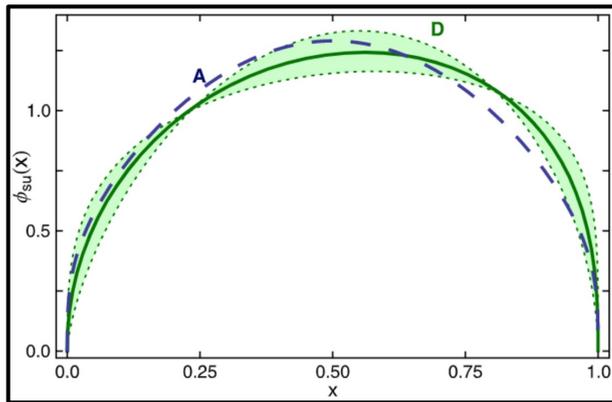
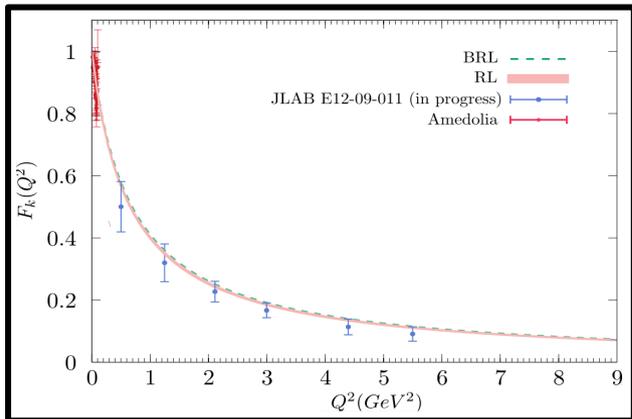
π^\pm and K^\pm Electromagnetic Form Factors



A. Miramontes et. al., Phys. Rev. D 105 (2022) 7, 074013

Pion Elastic Form Factor

Kaon Elastic Form Factor



J. Segovia et. al. , Phys. Lett. B 731 (2014) 13

The **electromagnetic form factors** of π and K can be measured till approximately $Q^2 \sim 10-40$ GeV^2 and $Q^2 \sim 10-20$ GeV^2 , respectively, at the **Electron-Ion Collider (EIC)**.

π and K Form Factors: Probing the Standard Model

A muon with spin s has a **magnetic moment**:

$$\mu = g \frac{e}{2m} s$$

The factor g is called the gyro-magnetic factor. The **Dirac equation** for a charged elementary fermion with spin $1/2$ implies $g = 2$.

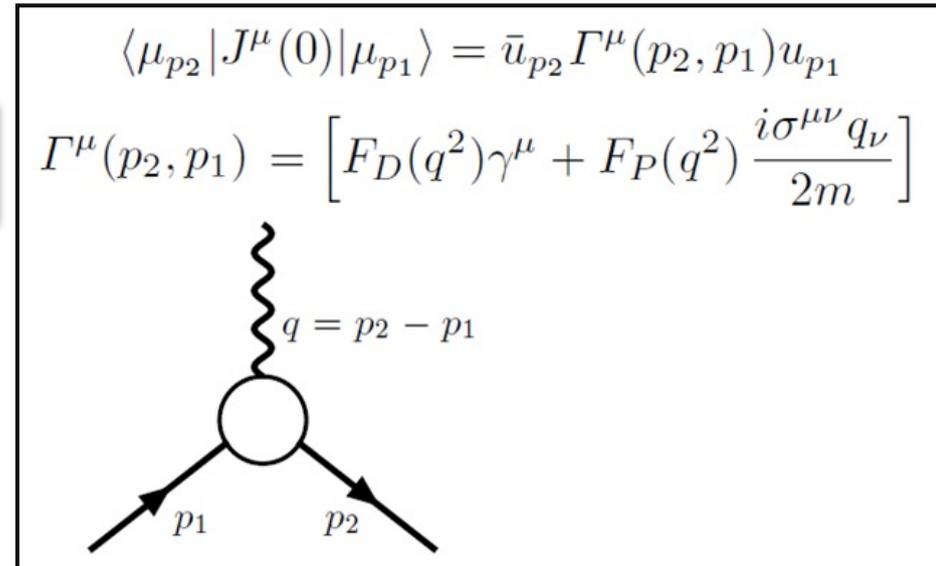
The **anomalous magnetic moment** is the deviation from $g = 2$, parameterized by $a_\mu = (g-2)/2$. It appears due to radiative corrections. **Renormalization** of **QED** was established in 1943 and 1947-1948 by Tomonaga, Schwinger and Feynman.

The leading contribution to a_μ , calculated by Schwinger in 1949, is:

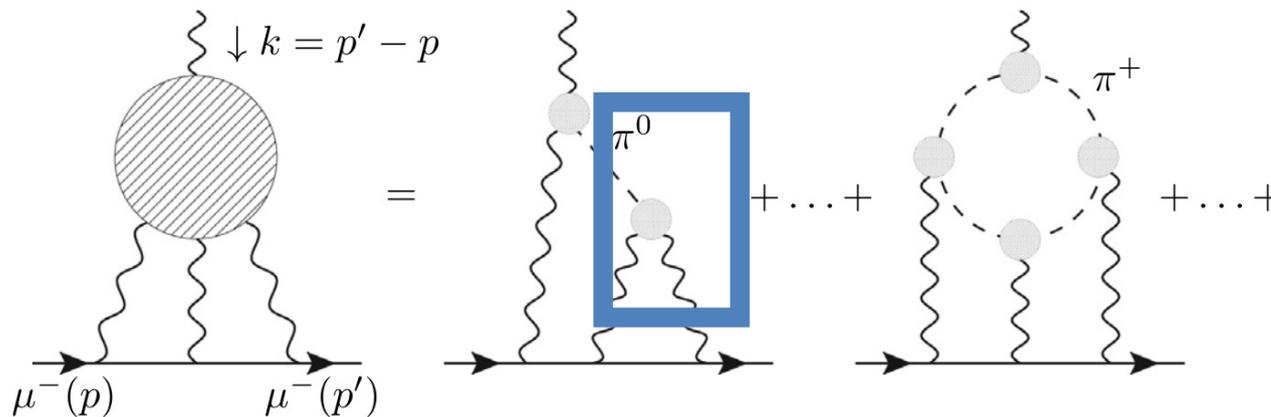
$$a_\mu = \frac{\alpha}{2\pi}$$

The **amplitude** of a muon scattering off an external electromagnetic field A is: ($q=p_2-p_1$):

$$\mathcal{M} = -ie \langle \mu_{p_2} | J^\mu(0) | \mu_{p_1} \rangle A_\mu(q)$$

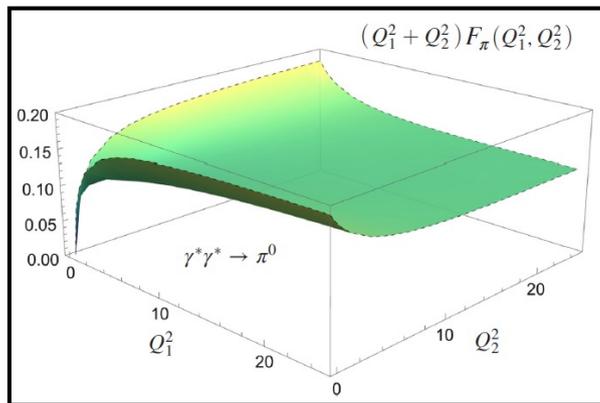
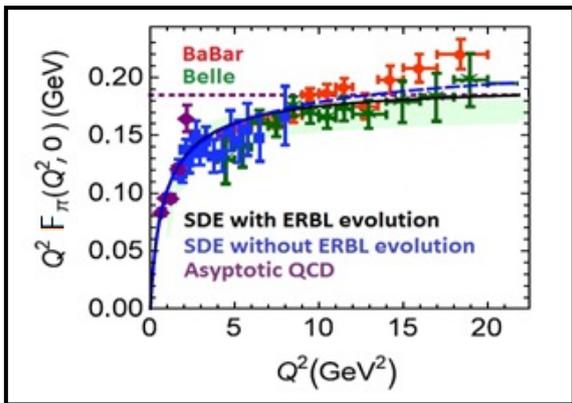


Neutral Pseudoscalar Pole Contributions



$a_{\mu}^{\pi^0\text{-pole}}$	$= (61.4 \pm 2.1) \times 10^{-11}$
$a_{\mu}^{\eta\text{-pole}}$	$= (14.7 \pm 1.9) \times 10^{-11}$
$a_{\mu}^{\eta'\text{-pole}}$	$= (13.6 \pm 0.8) \times 10^{-11}$
$a_{\mu}^{\eta_c\text{-pole}}$	$= (0.9 \pm 0.1) \times 10^{-11}$
$a_{\mu}^{\eta_b\text{-pole}}$	$= (0.0026 \pm 0.0001) \times 10^{-11}$
$a_{\mu}^{\text{PS-pole}}$	$= (90.6 \pm 4.9) \times 10^{-11}$

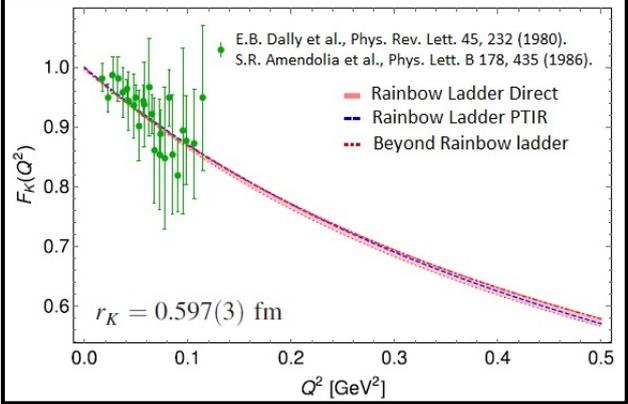
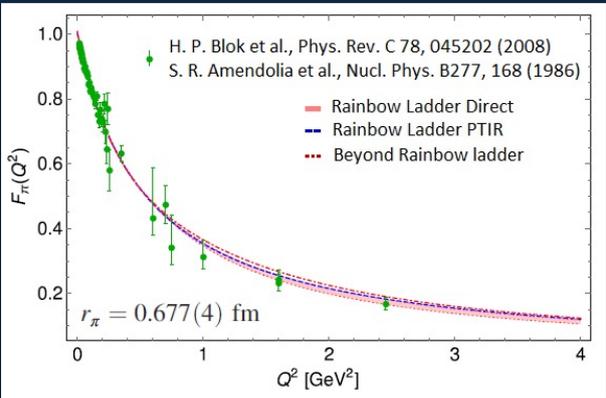
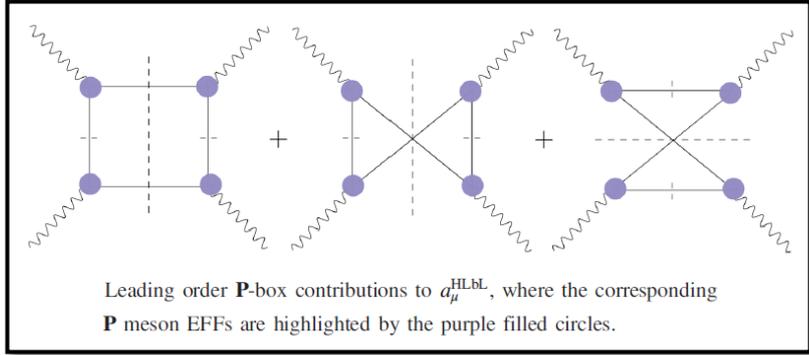
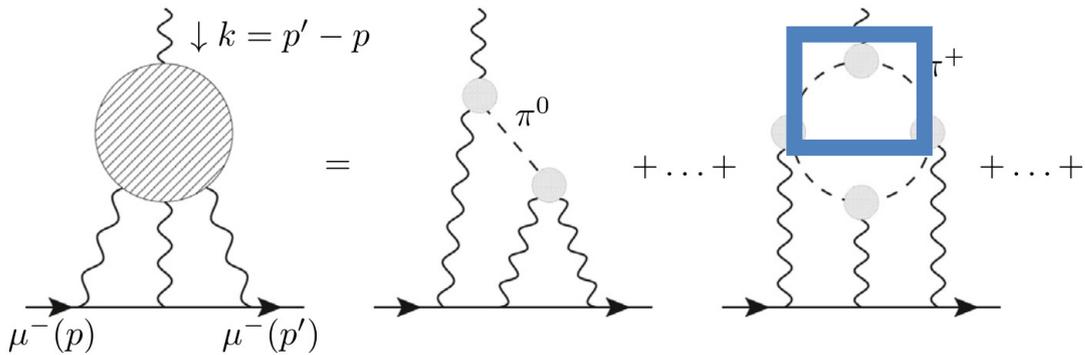
The transition form factor $\pi^0 \rightarrow \gamma^* \gamma$ extended to $\pi^0 \rightarrow \gamma^* \gamma^*$.



Dispersive methods:	$a_{\mu}^{\pi^0\text{-pole}} = 63.6(2.7) \times 10^{-11}$
	$a_{\mu}^{\eta\text{-pole}} = 16.3(1.4) \times 10^{-11}$
	$a_{\mu}^{\eta'\text{-pole}} = 14.5(1.9) \times 10^{-11}$
Lattice:	$a_{\mu}^{\pi^0\text{-pole}} = 59.7(3.6) \times 10^{-11}$

K. Raya, AB, P. Roig,
Phys. Rev. D 101 (2020) 7, 074021

π^\pm and K^\pm Box Contributions



A. Miramontes, AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013

Radial excitations of π and K :

A. Miramontes, et. al. In preparation (Preliminary)

$$a_\mu^{\pi_1\text{-box}} = (-3.2 \pm 0.6) \times 10^{-13}$$

$$a_\mu^{K_1\text{-box}} = -6.8 \times 10^{-14}$$

$$a_\mu^{K^+\text{-box,DSE}} = -0.48(2) \times 10^{-11}$$

Eichmann, et. al. Phys.Rev.D 101 (2020) 5, 054015

Dispersive methods:

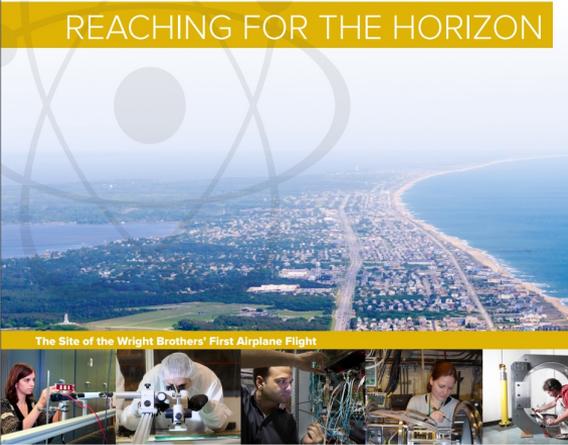
$$a_\mu^{\pi\text{-box}} = -15.9(2) \times 10^{-11}$$

$$a_\mu^{K^+\text{-box,VMD}} = -0.50 \times 10^{-11}$$

$$a_\mu^{\pi^\pm\text{-box}} = -(15.6 \pm 0.2) \times 10^{-11}$$

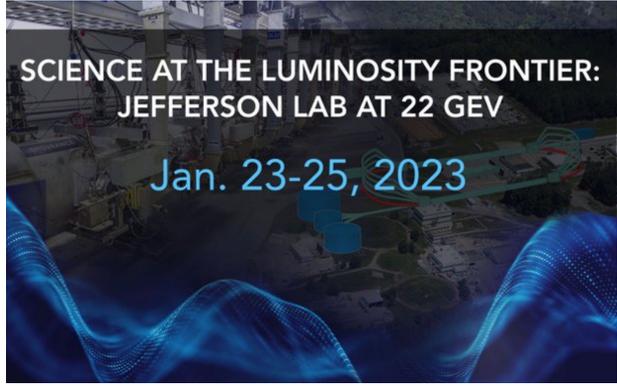
$$a_\mu^{K^\pm\text{-box}} = -(0.48 \pm 0.02) \times 10^{-11}$$

π and K Form Factors at Large Q^2



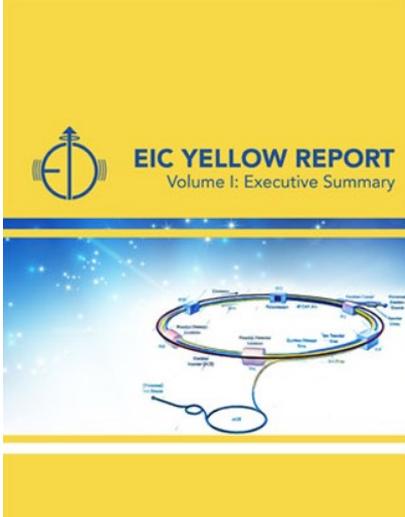
π and K form factors can be measured till $Q^2 \sim 6-8 \text{ GeV}^2$ & $Q^2 \sim 5 \text{ GeV}^2$ at the **12 GeV JLab** upgrade.

At **22 GeV upgrade**, π & K form factors can be measured till $Q^2 \sim 15$ & $Q^2 \sim 10 \text{ GeV}^2$ respectively.



The 2015
LONG RANGE PLAN
for **NUCLEAR SCIENCE**

The study of the **pion form factor** is one of the **flagship goals** of the **JLab 12-GeV** ... in which ... **QCD** begins a **transition** from **large-** to **short-distance** scale.



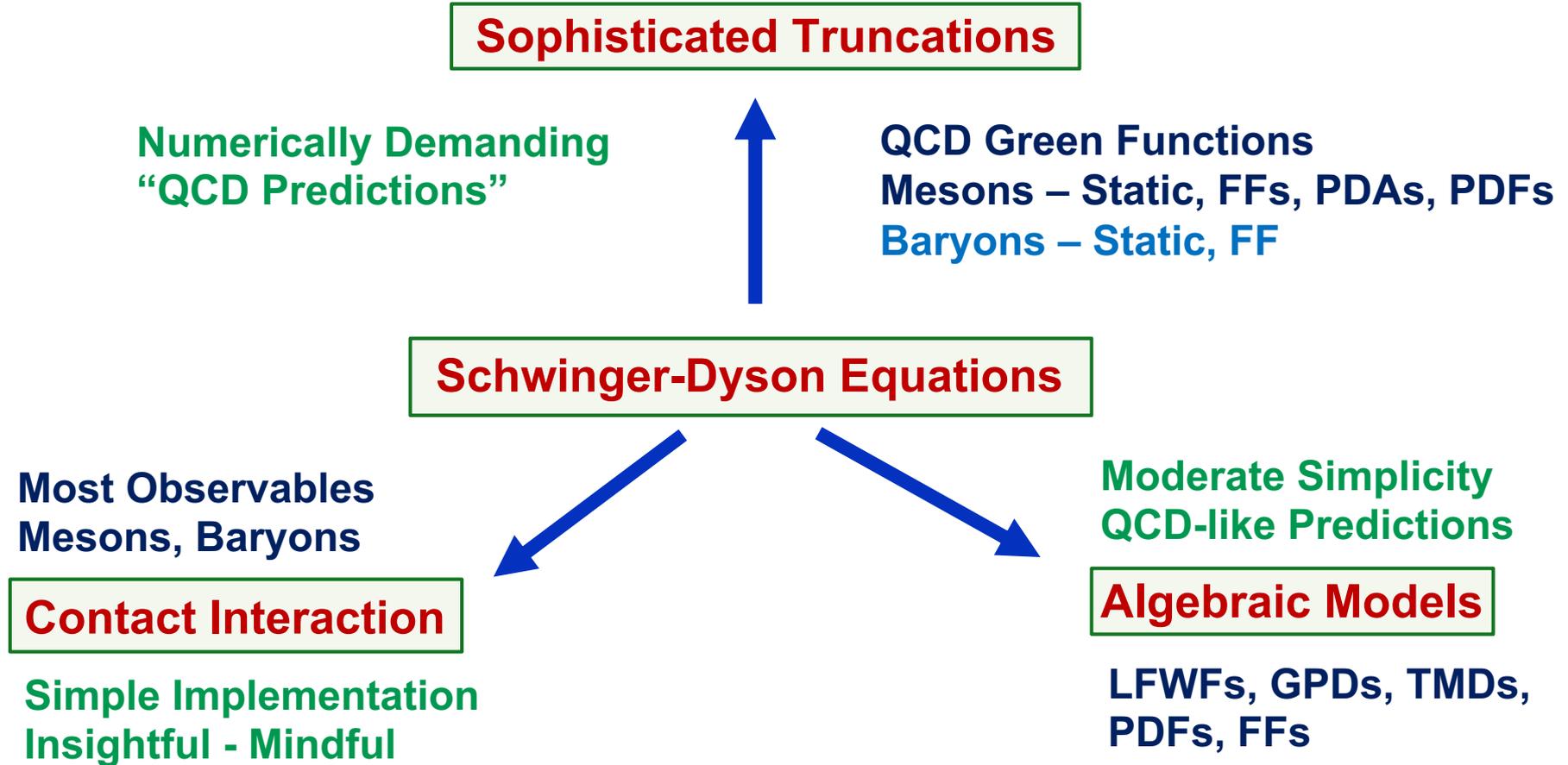
"Strong Interaction Physics at the Luminosity Frontier with 22 GeV Electrons at Jefferson Lab",
A. Accardi, et. al., e-Print: 2306.09360 [nucl-ex]

Science Question: emergent mass mechanism.
Key measurement: π form factor: 10-40 GeV^2 .

Science Question: Interference between emergent Higgs mass generation mechanism.
Key measurement: K form factor: 10-20 GeV^2

Towards The Three Dimensional Structure

Towards Algebraic Models



The Algebraic Model (AM)

The quark propagator:

$$S_{q(\bar{h})}(k) = [-i\gamma \cdot k + M_{q(\bar{h})}] \Delta(k^2, M_{q(\bar{h})}^2)$$

$$\Delta(s, t) = (s + t)^{-1}$$

Bethe-Salpeter Amplitude:

$$n_M \Gamma_M(k, P) = i\gamma_5 \int_{-1}^1 dw \rho_M(w) [\hat{\Delta}(k_w^2, \Lambda_w^2)]^\nu$$

$$\hat{\Delta}(s, t) = t\Delta(s, t), \quad k_w = k + (w/2)P$$

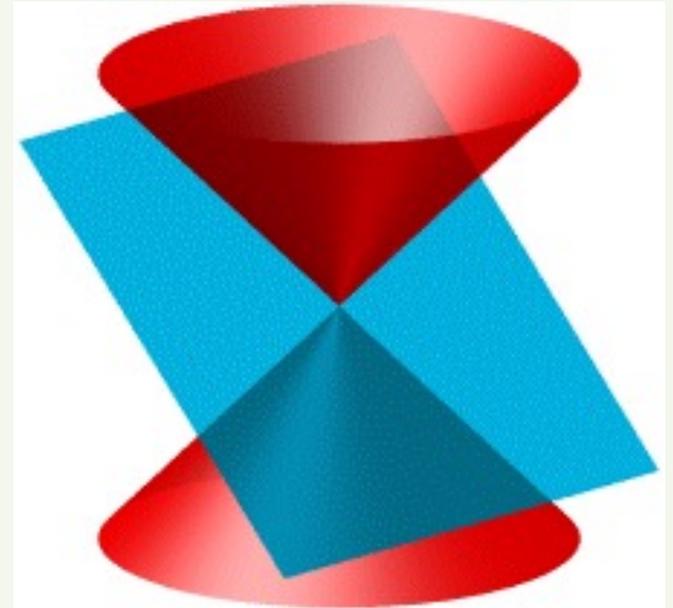
$$\Lambda^2(w) = M_q^2 - \frac{1}{4}(1 - w^2)m_M^2 + \frac{1}{2}(1 - w)(M_{\bar{h}}^2 - M_q^2)$$

The Algebraic Model:

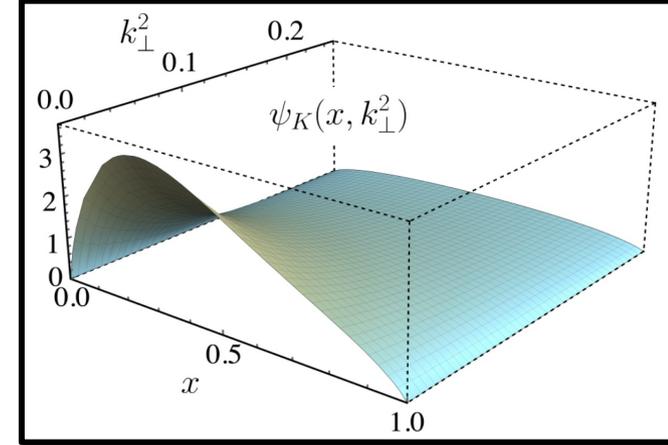
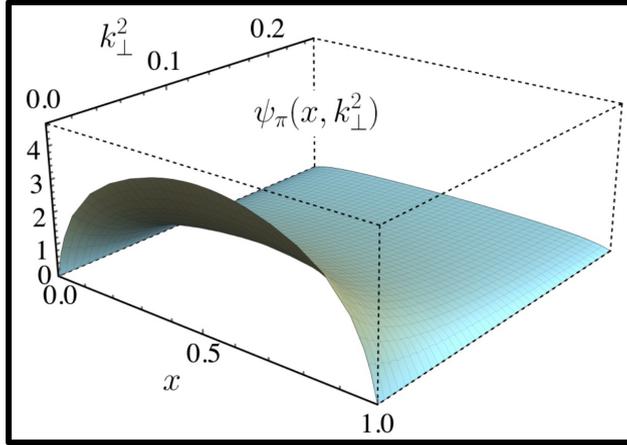
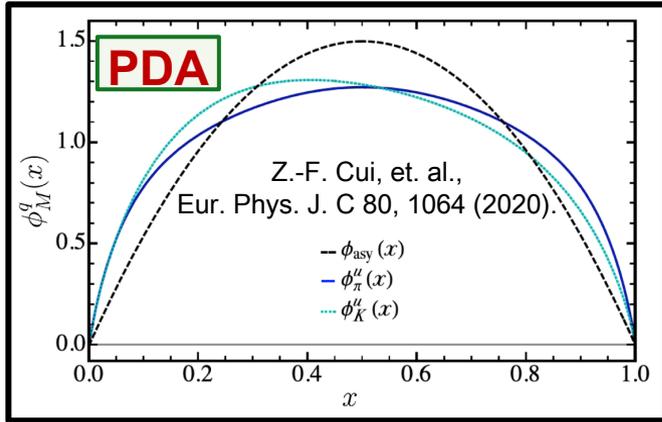
L. Albino, M. Higuera, K. Raya, AB, Phys. Rev. D 106 (2022) 3, 034003

$$\psi_M^q(x, k_\perp^2) = 16\pi^2 f_M \frac{\nu \Lambda_{1-2x}^{2\nu}}{(k_\perp^2 + \Lambda_{1-2x}^2)^{\nu+1}} \phi_M^q(x)$$

For a quark in pseudo-scalar meson **M**, the **leading twist** (2-particle) **light front wave function**, Ψ_M , can be obtained via the light front projection of the meson's **BSWF**.



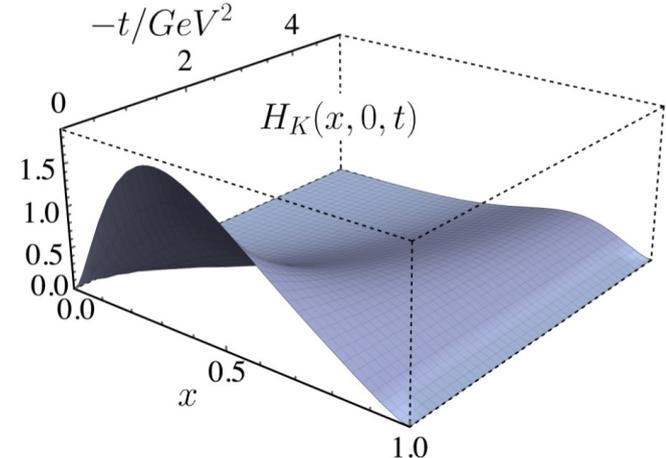
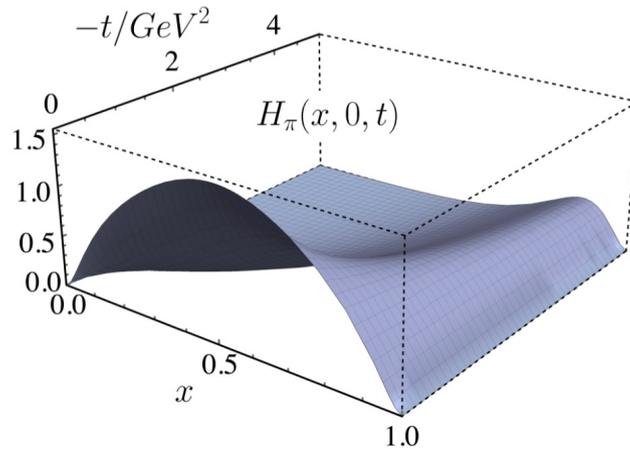
From the PDAs to the GPDs



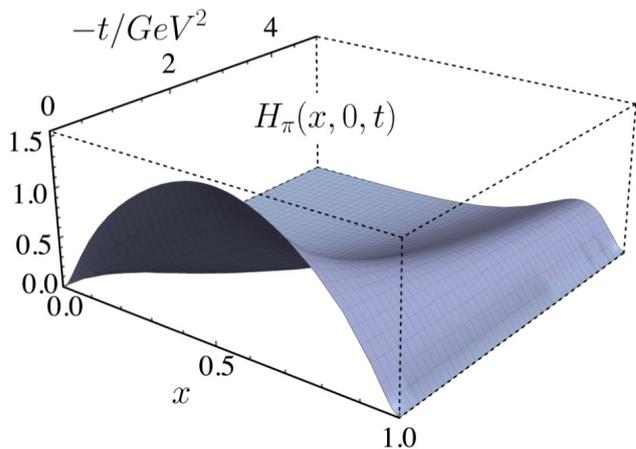
$$\psi_M^q(x, k_\perp^2) = 16\pi^2 f_M$$

$$\frac{\nu \Lambda_{1-2x}^{2\nu}}{(k_\perp^2 + \Lambda_{1-2x}^2)^{\nu+1}} \phi_M^q(x)$$

Overlap Rep. GPDs

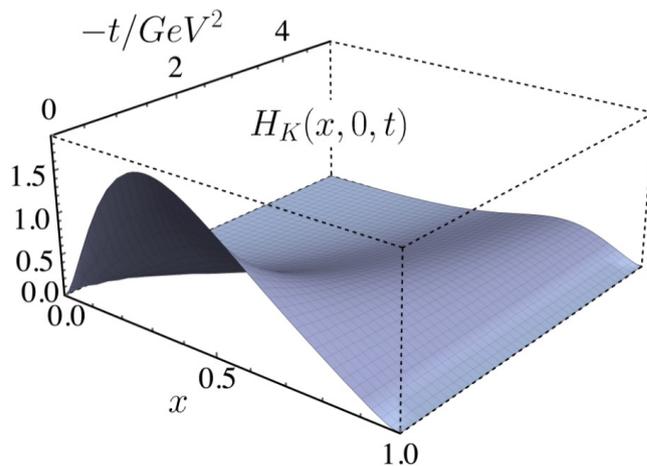
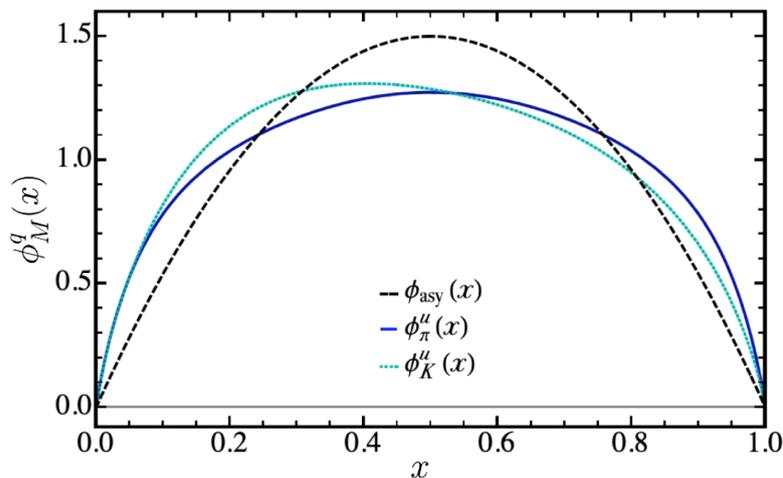


From the GPDs to the PDFs



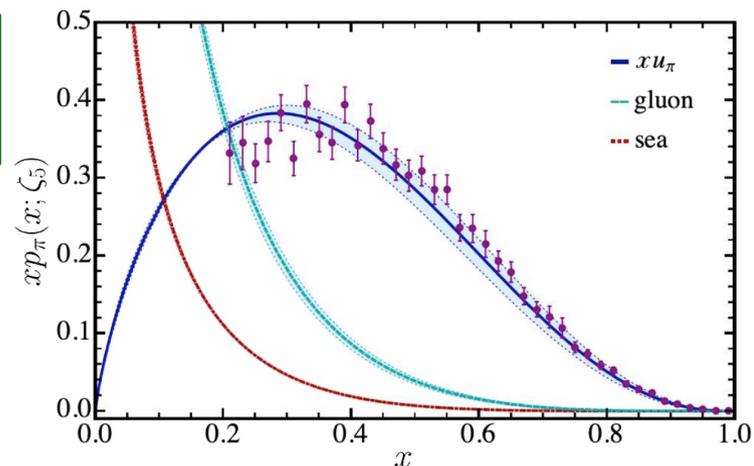
$$q_M(x) \equiv H_M^q(x, 0, 0)$$

**DGLAP Evolution
Equations**



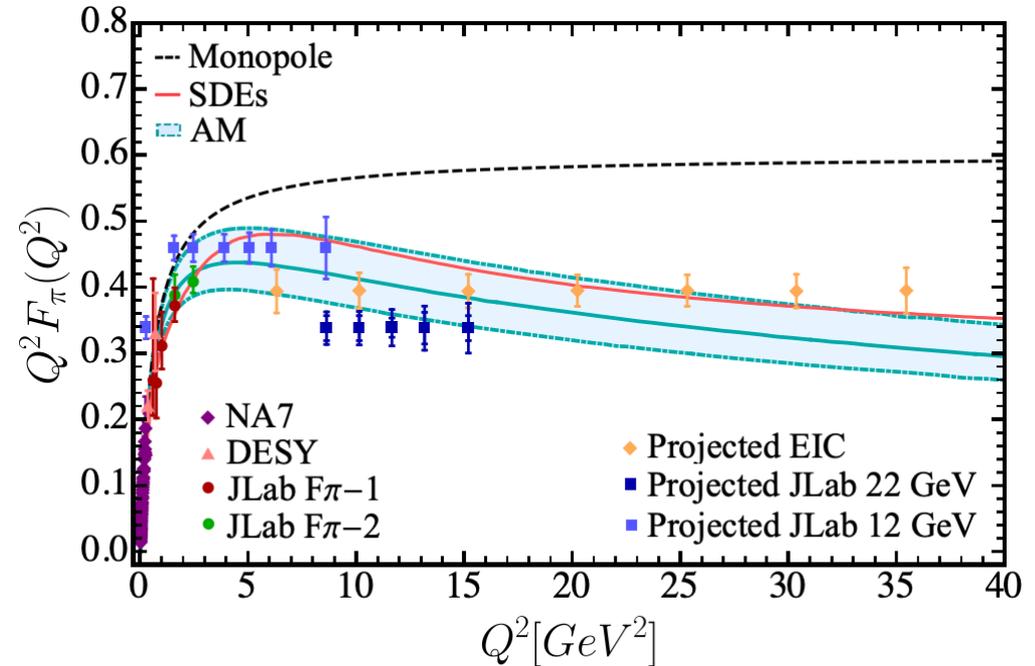
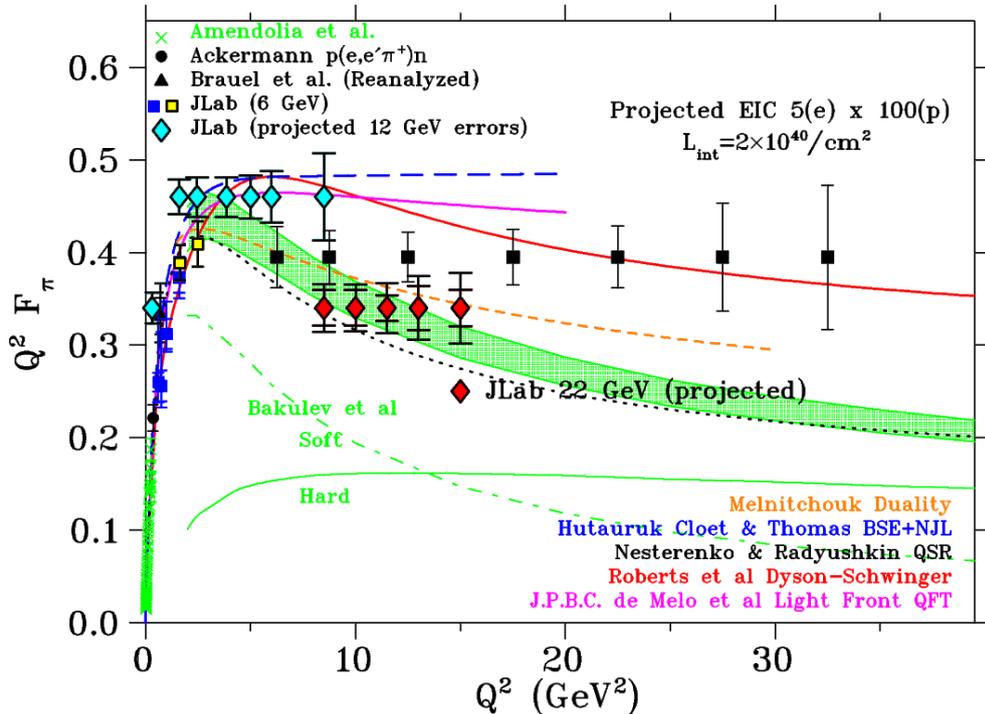
L. Albino, M. Higuera, K. Raya, AB
Phys. Rev. D 106 (2022) 3, 034003

Aicher et. al.
Phys. Rev. Lett. 105, 252003 (2010)



Completing the Cycle – Back to Form Factors

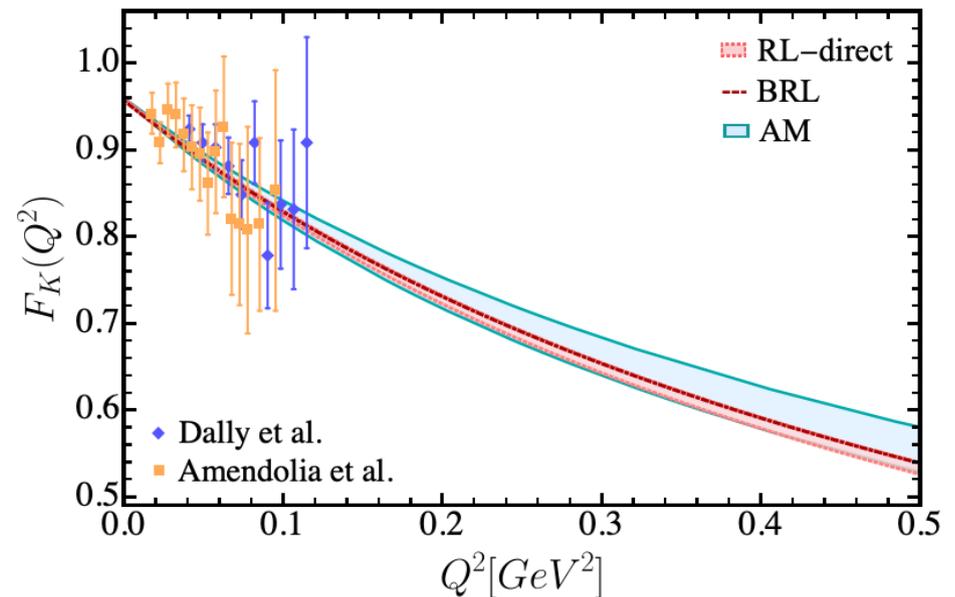
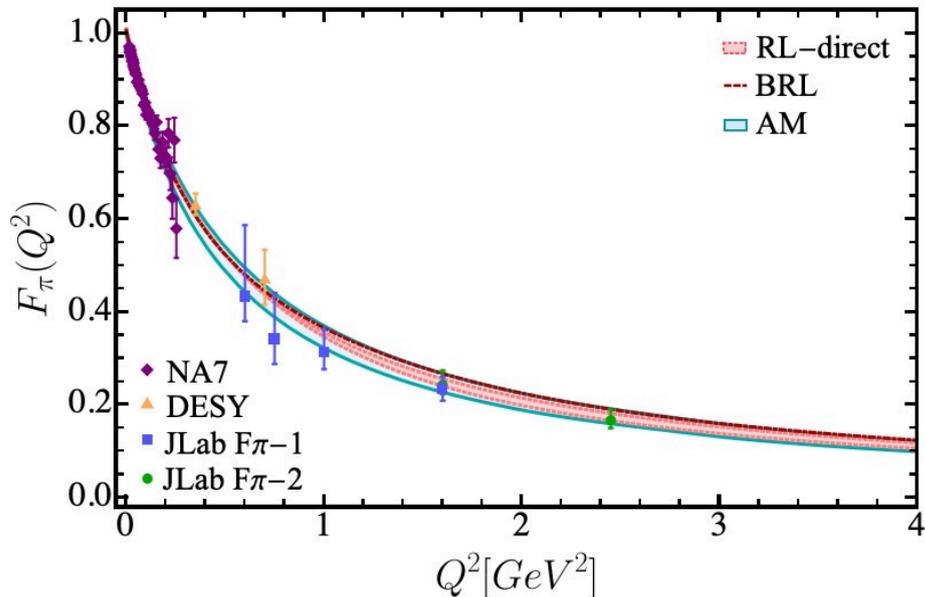
We can extend this analysis of the **Algebraic Model** to compute the **pion electromagnetic form factors** to larger Q^2 range: **0-40 GeV²** which would likely cover the photon virtualities accessible to the **JLab12**, **JLab22** and **EIC** programs:



Completing the Cycle – Back to Form Factors

The **electromagnetic form factors** using our **algebraic model** can be obtained either through the knowledge of the **GPDs** or the direct evaluation of the **triangle diagram**.

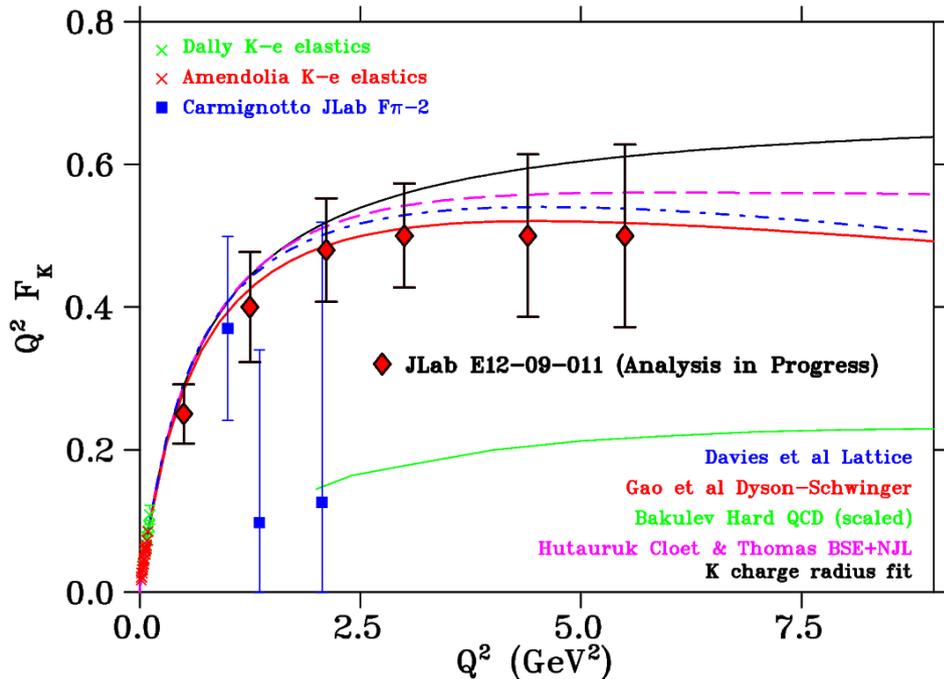
Such an exercise provides stringent constraints on the efficacy of the **algebraic model** we have constructed by direct comparison with the refined calculation of these **form factors**.



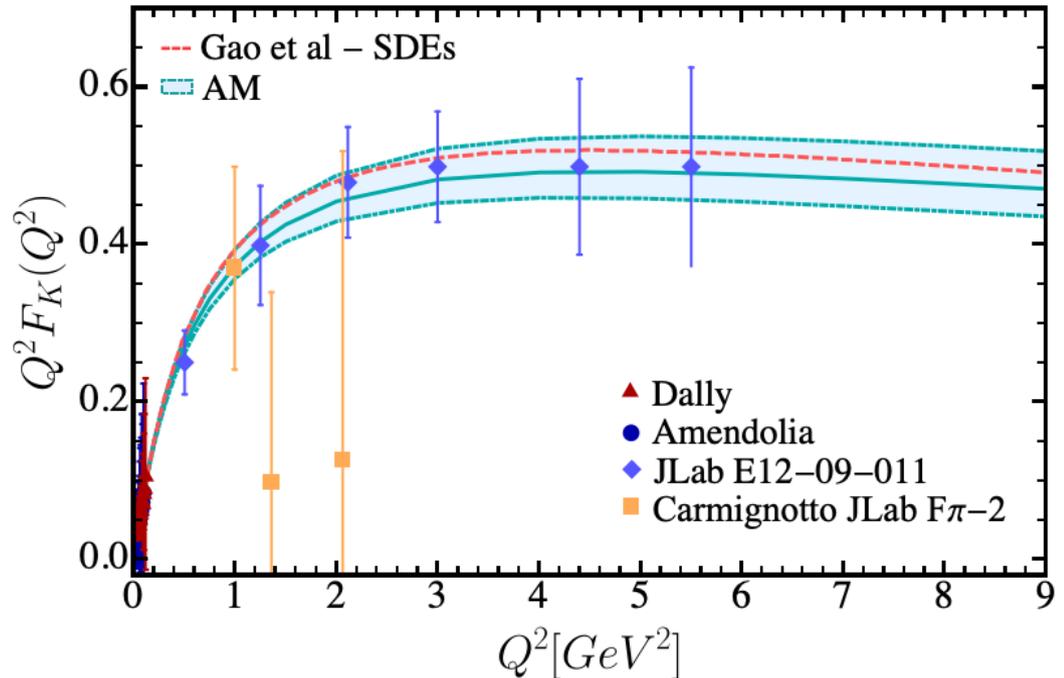
Completing the Cycle – Back to Form Factors

There is an analysis underway of the **kaon electromagnetic form factor** till **5.5 GeV²** of the data obtained in **JLab E12-09-011** experiment.

Courtesy Garth Huber



Algebraic Model results



Summary and Outlook

- The interplay of **QCD akin** truncations of **Schwinger-Dyson equations** and **algebraic model** based upon these studies shed important light on the **internal structure** of **pion** and **kaon**.
- **QCD akin** refined computation of **pion** and **kaon electromagnetic form factors** at low and intermediate virtualities of the probing photon in electroproduction processes:

A. Miramontes AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013

L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802

- Results for the **pion electromagnetic form factor** at large photon virtualities accessible to the potential **22GeV upgrade** of the **JLab** and **EIC** are also available:

L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802

J. Arrington, et al. (Feb 23, 2021, J.Phys. G 48 (2021) 7, 075106

- More recently, **pion** and **kaon form factors** have been computed in the the **time-like region**

A.S. Miramontes, H. Sanchis Alepuz, R. Alkofer, Phys. Rev. D 103 (2021) 11, 116006

A.S. Miramontes, AB, Phys. Rev. D 107 (2023) 1, 014016

Summary and Outlook

- Carefully constructed **Algebraic Models** can enable computation of the **GPDs**, **PDFs** and **EFF** with relative ease while mimicking the reliability of **QCD akin** refined truncations of **Schwinger-Dyson equations**.

L. Albino, M. Higuera, K. Raya, AB Phys. Rev. D 106 (2022) 3, 034003

B. Almeida, J. Cobos, AB, K. Raya, J. Rodríguez, J. Segovia, Phys. Rev. D 107 (2023) 7, 074037

- Despite these encouraging results and synergy with experimental endeavors at **JLab** and **EIC**, further improvements and extensions in the **continuum QCD approach** are desirable.
- More work into the theoretical foundations of the truncations involved at the level of the **Green functions** of the fundamental degrees of freedom, i.e., **quarks**, **gluons**, as well as **quark-gluon** and **gluon-gluon** interactions continues vigorously.
- **Schwinger-Dyson equations** have also been of substantial success in the studies of **baryons** such as the **transition form factors** of **nucleon** to its **excited states** which is a hallmark of **CLAS**, **CLAS12** and **CLAS22** programs at **JLab** and hold the promise to offer a reliable tool for the current and future **JLab** and **EIC era** research into the heart of **hadronic matter**.

Thank you for your attention

Upcoming Events

XVI Escuela de Física Fundamental

16-20 October 2023

Mexico/General timezone

Overview

Scientific Programme

Timetable

Registration

[Registration Form](#)

[Participant List](#)

La ***XVI Escuela de Física Fundamental***, se llevará a cabo en la Ciudad ***Culiacán de Rosales, Sinaloa del 16-20 de octubre de 2023***. En esta ocasión, la organización local del evento está a cargo de la ***Facultad de Ciencias Físico-Matemáticas de la Universidad Autónoma de Sinaloa*** (FCFM-UAS). La Escuela de Física Fundamental inició en 2005 en la ciudad de Morelia para celebrar el Año Internacional de la Física.

Esta Escuela de Física se realizó en la ciudad de Morelia en 2005, 2007, 2009 y 2015. Durante los años 2006, 2008, 2010, 2011, 2013, 2014, 2016 y 2017, 2018, 2019 y 2020, las Universidades de Sonora, Guanajuato, Autónoma de Puebla, UNAM, Sonora, Sinaloa, Veracruzana y Autónoma de Estado de Hidalgo, MCTP en Chiapas, CINVESTAV-IPN y la Universidad de Querétaro, respectivamente, fueron las sedes de la Escuela.

Upcoming Events

XX Mexican School on Particles and Fields 2023

from 30 October 2023 to 3 November 2023

Mexico/General timezone

[XX MSPF webpage](#)

[Overview](#)

[Scientific Programme](#)

[Timetable](#)

[Call for Abstracts](#)

[View my Abstracts](#)

[Submit Abstract](#)

[Registration](#)

The **XX Mexican School on Particles and Fields** takes place from **30 Oct – 3 Nov, 2023**, in the colonial city of Mérida, Yucatán, México. The format of the school is such that the morning sessions are devoted to theoretical and experimental reviews, whereas parallel thematic sessions shall be held in the afternoons. All the reviews and seminars are delivered by experts of international prestige on subjects which are of current interest to the global scientific community and are also actively pursued within México.

In order to equip graduate students and attending postdocs with the necessary tools to perform fully in the school, courses of interest are taught, with English being the language of instruction. An informal and friendly atmosphere is encouraged during the courses so that the students can overcome their inhibitions and actively participate in the discussions.

Upcoming Events

From: Ross Corliss <ross.corliss@stonybrook.edu>

Subject: CFNS Workshop Approved

Date: 17 July 2023 17:53:34 GMT-3

To: bennich@unifesp.br

Cc: Abhay Deshpande <abhay.deshpande@stonybrook.edu>, cfns_contact@stonybrook.edu

Reply-To: cfns_contact@stonybrook.edu

Dear Bruno,

Thank you for submitting a workshop proposal for the 2023-2024 cycle. We have completed our review and, based on the scientific merit, have approved "From quarks and gluons to the internal dynamics of hadrons" as an official CFNS workshop. Congratulations! Please forward this email to the rest of your organizing committee.

You have requested an in-person workshop in May, 2024, either 15-17 or 21-24. We are able to accommodate May 15-17. Once you confirm these dates, we will add the workshop to the official calendar. As you develop final speaker and attendee lists, please aim for a total attendance of no more than 40 people, to avoid overcrowding the seminar room.

We will create an indico page for your workshop and give you editing permission (to come in a future email). We can also create a ZOOM connection if desired; please let us know if we should do so or if you prefer to do that on your own. If you have any further questions, please don't hesitate to reach out.

Once again, congratulations. We look forward to the workshop.

Best Regards,
Ross Corliss
Abhay Deshpande

Upcoming Events

16th Conference on Quark Confinement and the Hadron Spectrum (Confinement XVI)

19-24 August 2024. Cairns Convention Centre, Cairns, Queensland, Australia (C24-08-05)

Theory-HEP

Phenomenology-HEP

Theory-Nucl

Lattice

Experiment-HEP

Experiment-Nucl

16th conference in the [Confinement](#) series

Contact: [Kizilersu, Ayse \(ayse.kizilersu@adelaide.edu.au\)](mailto:ayse.kizilersu@adelaide.edu.au)

XV Simposio Internacional en Física de Altas Energías (SILAF AE), Ciudad de México, México, 4-8 Nov., 2024.