Supersymmetry and Lorentz Violation in 5D.

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Abstract. We present a study for a Supersymmetric field theory with Lorentz-Violation terms in 5D. We perform the analysis in the context of the Berger-Kostelecky model (BK), adding one compactified dimension that explicitly breaks the Lorentz invariance. We introduce terms that encode this breaking, and find non trivial restrictions over boundary conditions of fields that one needs to close the supersymmetric algebra.

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INTRODUCTION

Since their discovery, supersymmetry (SUSY) and theories with extra dimensions (XD) have fascinated many physicists. This has happened despite the absence, so far, of even the slightest experimental indication that they could be relevant for the description of nature, although they both are theoretically motivated. On the other hand, the only fact we know for sure is that supersymmetry has to be badly broken in the energy range explored by the presently available accelerators, whereas in the same energy range extra dimensions could also manifest itself, which makes them very attractive for phenomenological considerations.

SUSY is a radically new type of space-time symmetry that transforms a bosonic state into a fermionic state, or vice versa, with $\Delta S = \pm 1/2$, where S is the spin, such that

$$Q|Boson\rangle = |Fermion\rangle, \qquad Q|Fermion\rangle = |Boson\rangle, \tag{1}$$

where, according to Hagg-Lopuszanski-Sohius theorem [1], the generators Q and $Q^\dagger$ must satisfy an algebra of anticommutation and commutation relations with the schematic form

$$\left\{Q, Q^\dagger\right\} = P^\mu, \qquad \{Q,Q\} = 0, \qquad [P^\mu, Q] = \left[P^\mu, Q^\dagger\right] = 0, \tag{2}$$

where P^μ is the four-momentum generator of spacetime translations. An example of a quantum field theory with supersymmetry is the Wess-Zumino model, which can be extended to extra dimensions. One of the interesting outcomes of SUSY is the possibility of solving the old standing hierarchy problem. XD, on the other hand, found a similar motivation to be consider as a possible new physics at the TeV scale [2], asides from the fact that they are predicted by String Theory.

Historically, interest in Lorentz violation increased dramatically after Kostelecky and Samuel pointed out that string theory has mechanisms that could cause spontaneous breaking of Lorentz symmetry [3]. This idea led to the exploration of some Standard Model extensions. In particular, in the work by Colladay and Kostelecky [4], it was found that many desirable features of standard quantum field theories, as gauge invariance, energy momentum conservation, observer Lorentz invariance, hermiticity, the validity of conventional quantization methods, and power-counting renormalizability, could still be present in a gauge field theory with Lorenz invariance violation, which give them some credibility. Later on, Berger and Kostelecky [5], shown the possibility of building supersymmetric field theories that violate Lorentz and CPT symmetry.

Implicit breaking of Lorentz symmetry is, on the other hand, an important feature of XD theories with compactified dimensions. This, however, essentially occurs only for the higher dimensional Lorentz symmetry, preserving standard Lorentz invariance of the non compact dimensions. An early study of the parameterization of such breaking in non SUSY 5D theories can be found in Ref. [6]. Yet, the question of weather SUSY is consistent with such an scenario remains open, and it is the goal of the present work to explore this question. To simplify matters, for the present discussion we will only consider the case of one compactified spacelike XD.

SUSY AND LORENTZ VIOLATION IN 4D

As a guide, we consider what happens in the case of 4D SUSY field theories, when Lorentz violation is introduced. There, the most general Lorentz violation terms are found [5], in order to close the SUSY algebra, to have nontrivial relationships among scalar and fermionic terms, which is reminiscent of the common masses and couplings of the standard SUSY field theory. As it was found by Berger and Kostelecky, the closure of SUSY requires a general deformation of the momentum operator, such that

$$i\partial_\mu \to i\partial_\mu + ik_{\mu\nu}\partial^\nu .$$

The attempted to eliminate Lorentz violation terms by a choice of coordinates such that $x^\mu \to x^\mu + k^{\mu\nu}x_\nu$, reflects on the metric, which would then not longer have the usual Minkowsky form. Lorenz violation is simple moved into the metric structure. Therefoe, one is compelled to concluded that the resulting Lorentz violation of the deformed SUSY theory is indeed physical. A similar scenario is expected in the case at hand.

A MODEL IN FIVE DIMENSIONS

Let us start by considering the 5D version of the Wess-Zumino model Lagrangian

$$\mathscr{L} = \partial_M \left(\phi^i\right)^\dagger \partial^M \phi^i + i\bar{\psi}\gamma^M\partial_M\psi + \left(F^i\right)^\dagger F^i, \tag{3}$$

where ψ is a Dirac fermion, ϕ_i is a doublet under $SU(2)_R$ of complex scalars and F^i are two auxiliary complex scalars transforming as a doublet under $SU(2)_R$. In 5-dimensional

SUSY, it is convenient to rewrite 4-component Dirac spinors as symplectic-Majorana spinors, which are defined as

$$\xi^i = \begin{pmatrix} \xi^i_L \\ \varepsilon^{ij}\bar{\xi}_{jL} \end{pmatrix}, \qquad \bar{\xi}_{jL} \equiv -i\sigma^2 \left(\xi^j_L\right)^*, \tag{4}$$

where ξ_L are Weyl spinors and ε_{ij} is the total antisymmetric 2-tensor with the convention $\varepsilon^{12} = 1$. For this massless fields case we have the off-shell multiplets and supersymmetric transformations [8],

$$\begin{aligned}
\Phi &= (\phi^i, \psi, F^i), \\
\delta_\xi \phi^i &= -\varepsilon^{ij}\bar{\xi}^j \psi, \\
\delta_\xi \psi &= i\gamma^M \partial_M \phi^i \varepsilon^{ij} \xi^j + F^i \xi^i, \\
\delta_\xi F^i &= -i\bar{\xi}^i \gamma^M \partial_M \psi.
\end{aligned}$$

Next, assuming that the fifth dimension is compact, we are free to add to the Lagrangian the most general terms with explicit Lorentz violation along the extra dimension, but which preserve 4D Lorentz invariance [6], to get

$$\mathscr{L}' = \mathscr{L} + k\partial_5 \left(\phi^i\right)^\dagger \partial^5 \phi^i + ib\bar{\psi}\Gamma^5 \partial_5 \psi. \tag{5}$$

Above Lagrangian is clearly not invariant anymore under the SUSY transformations is Eq. (5). Question is weather a deformation on the SUSY algebra could restore the symmetry. The answer is in the positive, provided there exist a relationship between the coefficients of the Lorentz violation terms, $k = b^2$, which is equivalent to insure the equality of the 4D effective masses for scalar and fermionic field components. Indeed, our analysis shows that t he deformed SUSY algebra,

$$\begin{aligned}
\delta_\xi \phi^i &= -\varepsilon^{ij}\bar{\xi}^j \psi, \\
\delta_\xi \psi &= i\gamma^M \partial_M \phi^i \varepsilon^{ij} \xi^j + ib\gamma^5 \partial_5 \phi^i \varepsilon^{ij} \xi^j + F^i \xi^i \\
&= \left(i\gamma^M \partial_M + ib\gamma^5 \partial_5\right) \left(\phi^i \varepsilon^{ij} \xi^i\right) + F^i \xi^i, \\
\delta_\xi F^i &= -i\bar{\xi}^i \gamma^M \partial_M \psi - ib\bar{\xi}^i \gamma^5 \partial_5 \psi \\
&= -\bar{\xi}^i \left(i\gamma^M \partial_M + ib\gamma^5 \partial_5\right) \psi \, ,
\end{aligned}$$

keeps the Lagrangian $\mathscr{L}'$ invariant, up to a total derivative. Besides, it is not difficult to check that the commutator of two deformed SUSY transformations gives

$$[\delta_1, \delta_2] = 2i\bar{\xi_1}^j \gamma^M \xi_2^j \partial_M + 2ib\bar{\xi_1}^j \gamma^5 \xi_2^j \partial_5 + \bar{\xi_1}^j \xi_2^j \delta_z, \tag{6}$$

which involves the generator of translations. Here δ_z generates the transformations

$$\begin{aligned}
\delta_z \phi^i &= F^i, \\
\delta_z \psi &= \left(i\gamma^M \partial_M + ib\gamma^5 \partial_5\right) \psi, \\
\delta_z F^i &= \left(\Box - b^2 \partial_5^2\right) \phi^i,
\end{aligned}$$

which correspond to $SU(2)_R$ central charge:

$$\left[\delta_\xi, \delta_z\right] = 0.$$

This model is an explicit example of a XD model with exact SUSY and explicit Lorentz Violation on one compactified dimension.

As a final note, let us emphasize that, so far, we have not given any explicit restriction on the extra dimension. If one decides, for instance, to use the $S^1/\mathbb{Z}_2$ orbifold, as it is well known, all involved fields would be expressed in the effective 4D theory in terms of their Kaluza Klein tower decomposition. Each level of the tower would exhibit an N=1 SUSY, as a result of the chosen boundary conditions on fields that break the general 4D N=2 SUSY at the fixed points. Also, the involved fields would have, apart from the usual Kaluza Klein mass originated from the standard kinetic terms, a mass contribution generated directly from the dimensional reduction of the Lorentz violating terms.

CONCLUSIONS

In our model, we found that to close SUSY we do need to deform the SUSY transformations, involving the γ^M matrices, spatial derivatives and a specific relation among Lorentz violation coefficients, $i\gamma^M\partial_M \to i\gamma^M\partial_M + ib\gamma^5\partial_5$. Yet, such a deformation can be see as a redefinition of only the fifth momentum component operator, $i\partial_5 \to i\partial_5 + ib\partial_5$ in a similar fashion as in the Berger-Kostelecky model. It is worth stressing that preserving SUSY requires an algebraic relation among parameters that states that $b^2 = k$. This relationship is easily understood when looking at orbifolding models, where Lorentz violating terms translate into field mass contributions. The survival of the equivalent 4D $N = 2$ SUSY does request that masses of hypermultiplet components be equal, which is only guaranteed for $k = b^2$.

Since supermultiplet fields are doublets under $SU(2)_R$, our model has an internal group, and therefore, central charges. These central charges have the same form as the hypermultiplet in four dimensions, and thus, our model exhibits, as expected, the same structure as $N = 2$ SUSY models in 4D.

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