

# Functional approaches to infrared Yang-Mills theory in the Coulomb gauge

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**Abstract.** We present the current status of ongoing efforts to use functional methods, Dyson-Schwinger equations and functional renormalization group equations, for the description of the infrared regime of nonabelian (pure) gauge theories. In particular, we present a new determination of the color-Coulomb potential with the help of the functional renormalization group that results in an almost linearly rising potential between static color charges at large spatial distances.

## 1. Introduction

Important progress has been achieved over the last decade in the description of the deep infrared region of nonabelian gauge theories with the help of functional methods, employing Coulomb gauge fixing. By functional methods we refer to semi-analytical tools that do *not* make use of the discretization of space-time as does lattice gauge theory. Specifically, equations of Dyson-Schwinger-type arising from a variational principle have been used, and more recently functional renormalization group equations. In this contribution, we will report on the current status of these investigations. We will focus exclusively on pure gauge theories, more specifically SU(N) Yang-Mills theory, but include static color charges so as to obtain a description of the heavy quark potential, as in quenched approximations.

We will start by briefly describing the general theoretical setup: the Hamiltonian framework is used, where the Weyl *and* Coulomb gauge conditions,  $A_0^a(\mathbf{x}) = 0$  and  $\nabla \cdot \mathbf{A}^a(\mathbf{x}) = 0$ , are imposed on the SU(N) gauge fields. Physical states are described by wave functionals of  $\mathbf{A}^a(\mathbf{x})$  with scalar product

$$\langle \phi | \psi \rangle = \int D[\mathbf{A}] J[\mathbf{A}] \phi^*[\mathbf{A}] \psi[\mathbf{A}]. \quad (1)$$

Here,  $J[\mathbf{A}]$  stands for the Faddeev-Popov (FP) determinant  $J[\mathbf{A}] = \text{Det}(-\nabla \cdot \mathbf{D})$  with the spatial covariant derivative  $\mathbf{D}^{ab} = \delta^{ab} \nabla + g f^{abc} \mathbf{A}^c(\mathbf{x})$ . The functional integral in (1) is understood to be

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restricted to spatially transverse gauge fields, i.e. those that fulfill the gauge fixing conditions.

The dynamics is defined by the Christ-Lee Hamiltonian  $H$  [1] that we do not write out. In the presence of a static color charge density  $\rho_q^a(\mathbf{x})$ ,  $H$  contains the interaction term

$$H_q = \frac{1}{2} \int d^3x d^3y \rho_q^a(\mathbf{x}) F^{ab}(\mathbf{x}, \mathbf{y}) \rho_q^b(\mathbf{y}) \quad (2)$$

with the integral kernel

$$F^{ab}(\mathbf{x}, \mathbf{y}) = \langle \mathbf{x}, a | (-\nabla \cdot \mathbf{D})^{-1} (-\nabla^2) (-\nabla \cdot \mathbf{D})^{-1} | \mathbf{y}, b \rangle. \quad (3)$$

The vacuum expectation value  $\langle F^{ab}(\mathbf{x}, \mathbf{y}) \rangle$  is called the color-Coulomb potential. It is supposed to give the dominant contribution to the confining interaction between color charges. More precisely, for large spatial distances the color-Coulomb potential provides an upper bound for the Wilson potential [2].

For the following, it will be convenient to write the FP determinant in a local form by introducing ghost fields,

$$J[\mathbf{A}] = \text{Det}(-\nabla \cdot \mathbf{D}) = \int D[\bar{c}, c] \exp\left(-\int d^3x \bar{c}^a(\mathbf{x}) (-\nabla \cdot \mathbf{D}^{ab}) c^b(\mathbf{x})\right). \quad (4)$$

In our analysis, we will focus on the equal-time correlation functions, i.e. the vacuum expectation values of products of the field operators  $\mathbf{A}^a(\mathbf{x})$  (transverse),  $c^a(\mathbf{x})$  and  $\bar{c}^a(\mathbf{x})$ . We can easily write down an expression for the generating functional of these correlation functions,

$$\begin{aligned} Z[\mathbf{J}, \eta, \bar{\eta}] &= \int D[\bar{c}, c, \mathbf{A}] e^{-\int d^3x \bar{c}(-\nabla \cdot \mathbf{D})c} |\psi[\mathbf{A}]|^2 \\ &\times \exp\left(\int d^3x [\mathbf{J}^a(\mathbf{x}) \cdot \mathbf{A}^a(\mathbf{x}) + \bar{c}^a(\mathbf{x}) \eta^a(\mathbf{x}) + \bar{\eta}^a(\mathbf{x}) c^a(\mathbf{x})]\right), \end{aligned} \quad (5)$$

where  $\psi[\mathbf{A}]$  is the vacuum wave functional. If we formally define an ‘‘action’’  $S[\mathbf{A}]$  through  $|\psi[\mathbf{A}]|^2 = e^{-S[\mathbf{A}]}$ , (5) looks like the usual generating functional of Euclidean Green’s functions in the covariant Lagrangian formulation of the theory, only in three instead of four dimensions. Of course,  $S[\mathbf{A}]$  is a complicated and a priori unknown functional of  $\mathbf{A}^a(\mathbf{x})$ . We will parametrize the ‘‘propagators’’, the equal-time two-point correlation functions of the theory, in the most general way (restricted by symmetries) as follows:

$$\langle A_i^a(\mathbf{p}) A_j^b(-\mathbf{q}) \rangle = \frac{1}{2\omega(p)} \delta^{ab} \left( \delta_{ij} - \frac{p_i p_j}{p^2} \right) (2\pi)^3 \delta(\mathbf{p} - \mathbf{q}), \quad (6)$$

$$\langle c^a(\mathbf{p}) \bar{c}^b(-\mathbf{q}) \rangle = \langle \langle \mathbf{p}, a | (-\nabla \cdot \mathbf{D})^{-1} | \mathbf{q}, b \rangle \rangle = \frac{d(p)}{p^2} \delta^{ab} (2\pi)^3 \delta(\mathbf{p} - \mathbf{q}). \quad (7)$$

Here and in the following, we use the notation  $p = |\mathbf{p}|$ . The functions  $\omega(p)$  and  $d(p)$  will be of central interest in the rest of this contribution. Notice that the ghost propagator (7) is just the vacuum expectation value of the inverse FP operator (or rather, its integral kernel).

## 2. Variational principle: Dyson-Schwinger equations

A set of equations of Dyson-Schwinger-type for the equal-time correlation functions of the theory was obtained in Ref. [3] from the variational principle, using a Gaussian ansatz for the vacuum

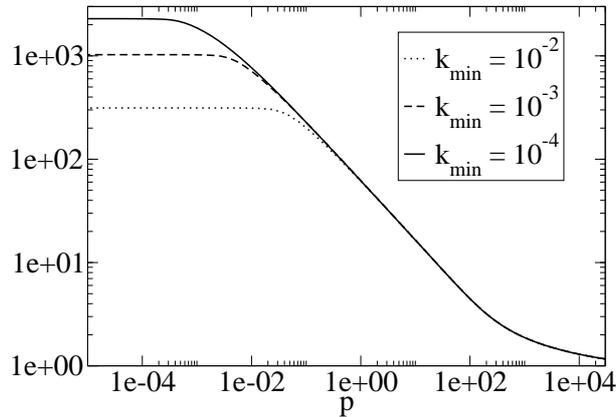












**Figure 3.** The Coulomb form factor  $f(p)$  from (27).

close to  $V_c(p) \propto p^{-4}$  which would correspond to a linearly rising potential in position space.

In summary, we find that functional methods are a powerful tool for the description of the nonperturbative infrared regime of nonabelian gauge theories. The formulation of these theories in the Coulomb gauge is particularly convenient, mainly because it gives direct access to the color-Coulomb potential. The Gribov-Zwanziger confinement scenario provides a conceptual framework to understand the confinement mechanism. It can be conveniently implemented via the horizon condition. In particular, we have seen that an almost linearly rising color-Coulomb potential is obtained from the functional renormalization group equations (and the factorization hypothesis is violated). It has also become clear that the approximations employed still have to be improved in order to achieve a quantitatively reliable description of the infrared region.

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