August 02, 2023

QCD phase diagram: where models meet the lattice QCD simulations

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Universidade Federal de Santa Maria
SEDES SAPIENTIÆ
1960

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

FAPERGS

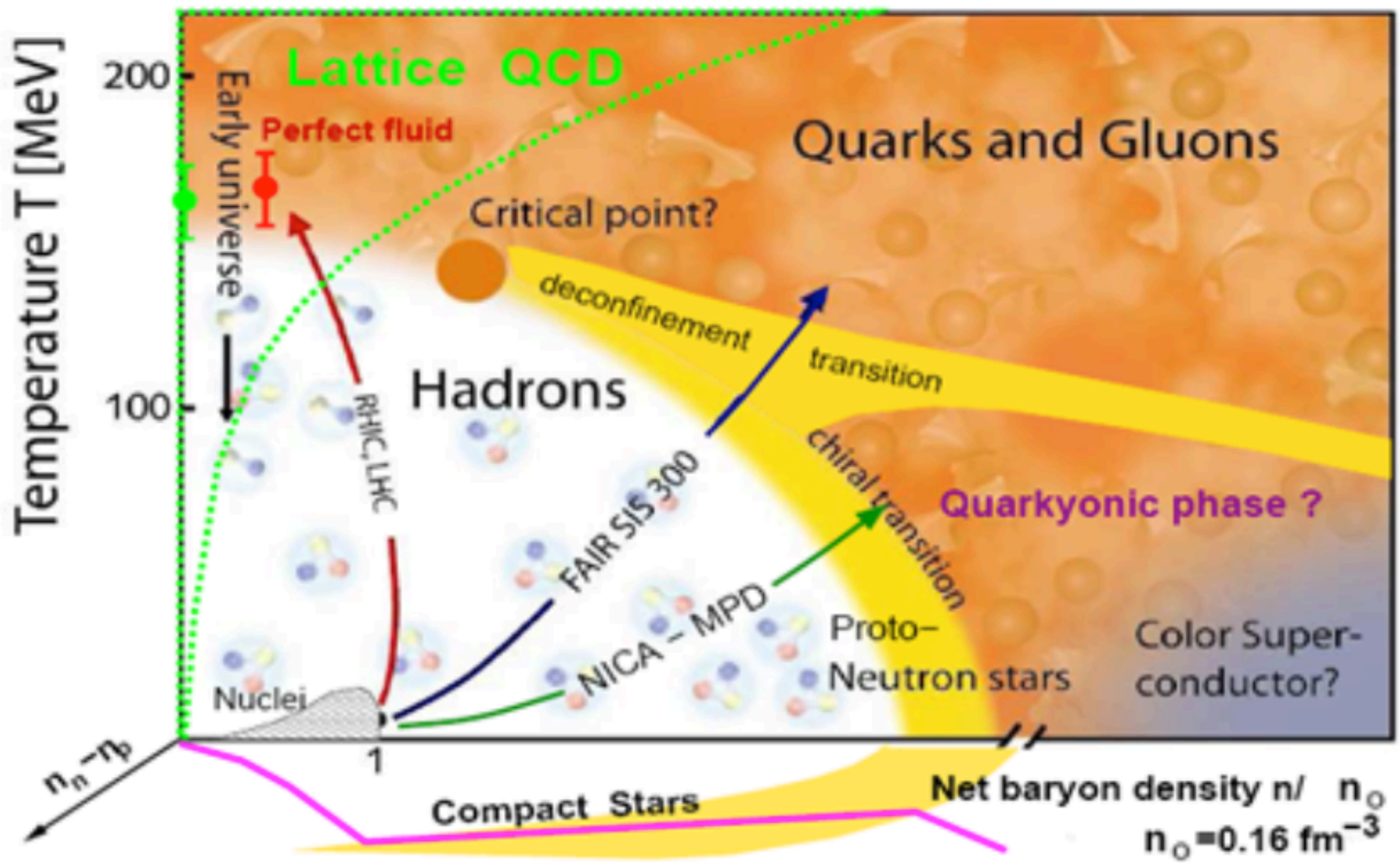
In Collaboration with:

- D.C. Duarte, B. Lopes, R.M. Nunes, A. Pasqualotto, F. A. Xavier - UFSM - Brazil
- S. Avancini, M.B. Pinto, W.R. Tavares - UFSC - Brazil
- Aritra Bandyopadhyay - Heidelberg University - Germany
- G. Krein - IFT - Brazil
- R.O Ramos - UERJ - Brazil
- **A. Ayala, J.L. Hernandez - UNAM - Mexico**
- **L. A. Hernandez - Universidad Autónoma Metropolitana Iztapalapa - Mexico**
- R. Zamora - Fuerza Aerea, Universidad Diego Portales- Chile
- N. Scoccola - Argentina
- V. Dexheimer - KSU - USA
- …

Outline

- Motivation
- i) How a isospin asymmetry can influence the phase diagram of QCD?
- ii) How a chiral imbalance of right-handed and left-handed quarks can influence the phase diagram of QCD?
- iii) How a strong magnetic field can influence the phase diagram of QCD?
- Conclusions and perspectives

QCD Phase Diagram



NICA - Nuclotron-based
Ion Collider fAcility

FAIR - Facility for Antiproton and
Ion Research

Lattice Results: Sign Problem

- fermion determinant is complex

$$[\det M(\mu)]^* = \det M(-\mu^*) \in \mathbb{C}$$

- no positive weight in path integral

$$Z = \int \mathcal{D}U \, e^{-S_{YM}} \det M(\mu)$$

- standard lattice methods base on importance sampling cannot be used!

To make progress

QCD is nonperturbative at relevant scales!

- We use Quantum Field Theory (in medium)
- DSE (beyond RL truncation…)
- Lattice (limitations…)
- Effective models (just a few degrees of freedom): NJL/PNJL, Linear σ model, MIT, Chiral perturbation theory…

QCD with isospin asymmetry : NO Sign Problem!

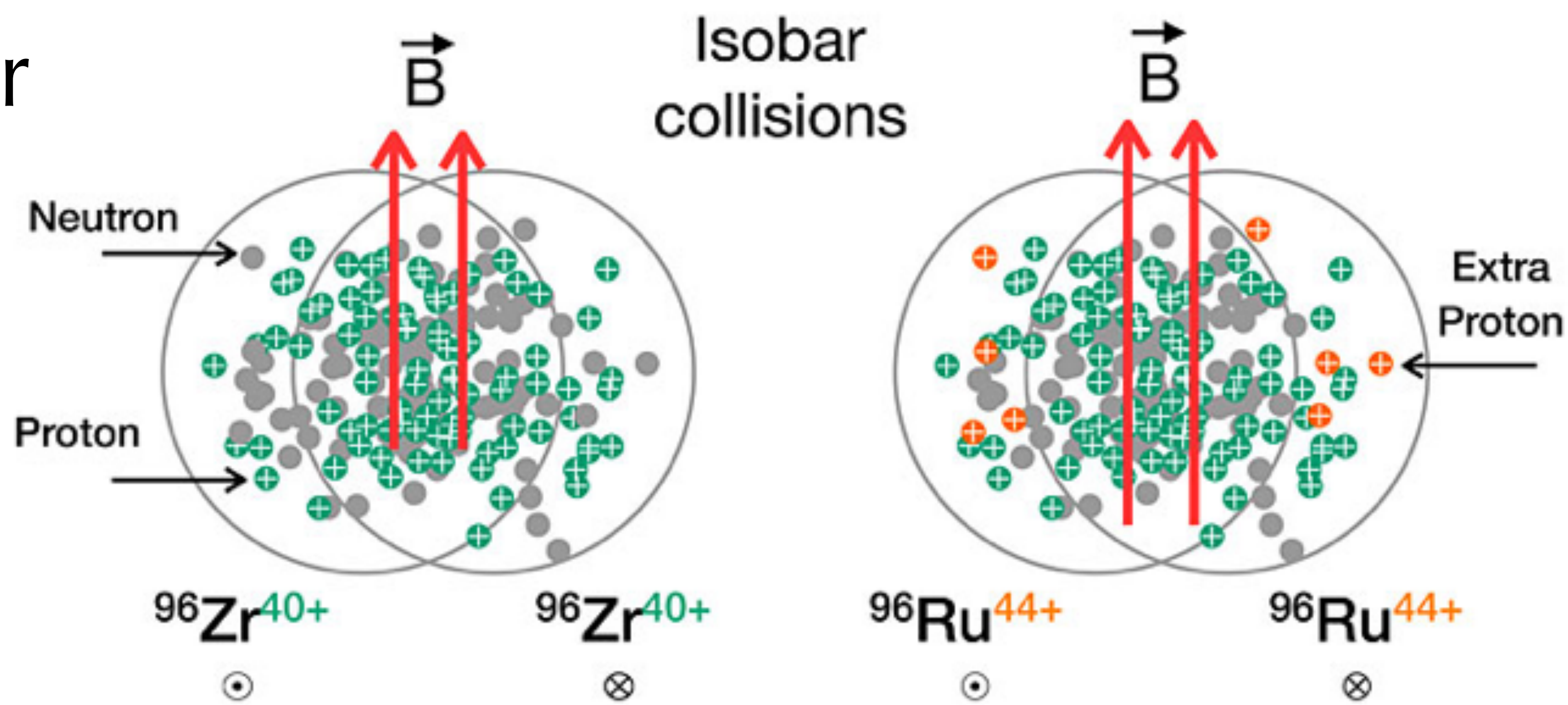
There is hope that lattice simulations of QCD with μ_I can be used as a benchmark platform for comparing different effective models used in the literature.

i) How a isospin asymmetry can influence the phase diagram of **QCD**?

Motivation

Why relevant?

- In RHIC isobar program



- Excess of neutrons over protons
- Excess of π^- over π^+
- Neutron star interiors

$$n_I < 0$$

$$n_I = n_u - n_d$$

Isospin chemical potential

- Grand canonical ensemble
- Chemical potentials

$$\mu_u = \frac{\mu_B}{3} + \mu_I$$
$$\mu_d = \frac{\mu_B}{3} - \mu_I$$

- Zero baryon number:

$$\begin{aligned} \mu_u &= \mu_I \\ \mu_d &= -\mu_I \end{aligned}$$

- Isospin density

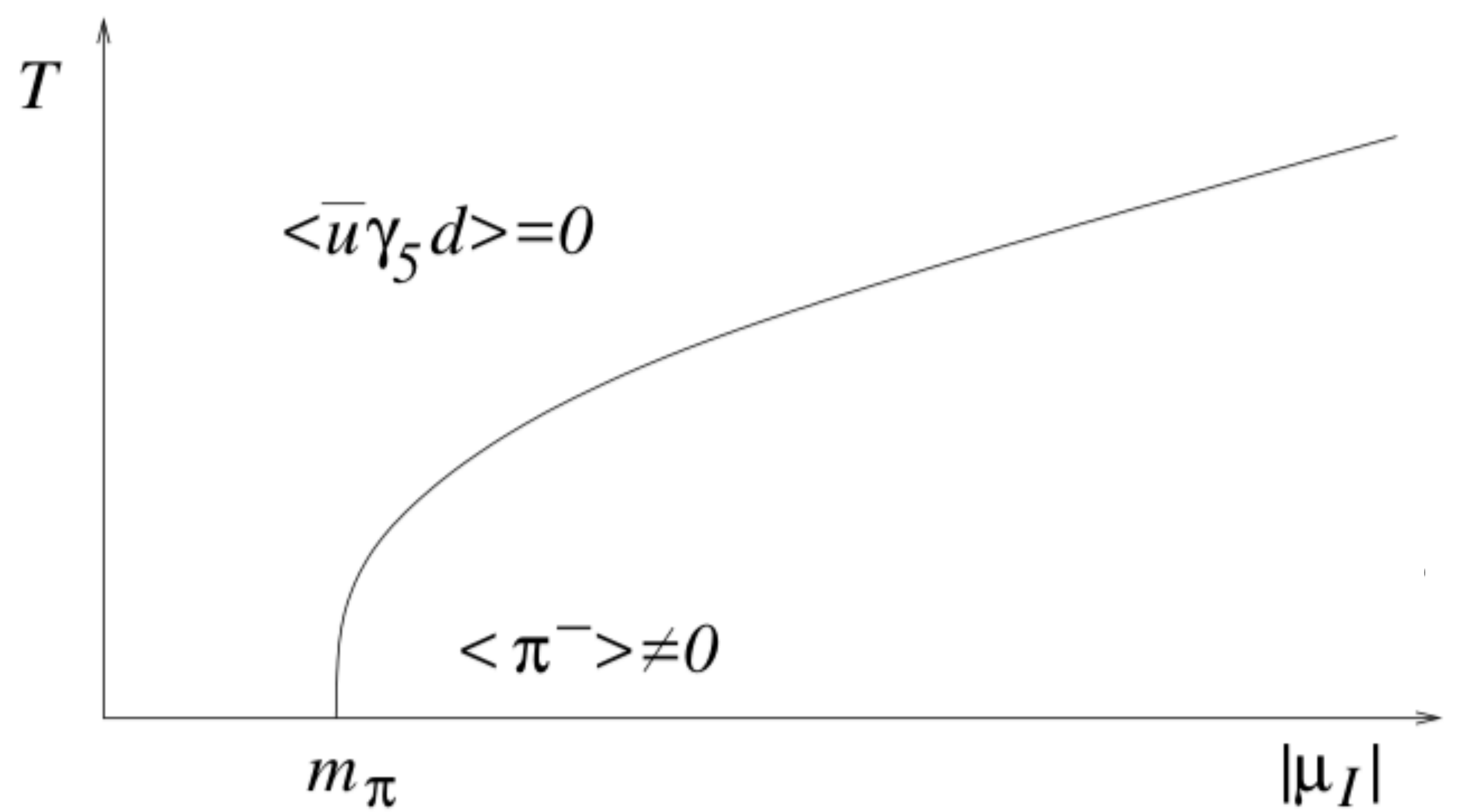
$$n_I = n_u - n_d$$

Pion Condensation

- Low energy QCD: degrees of freedom → pions

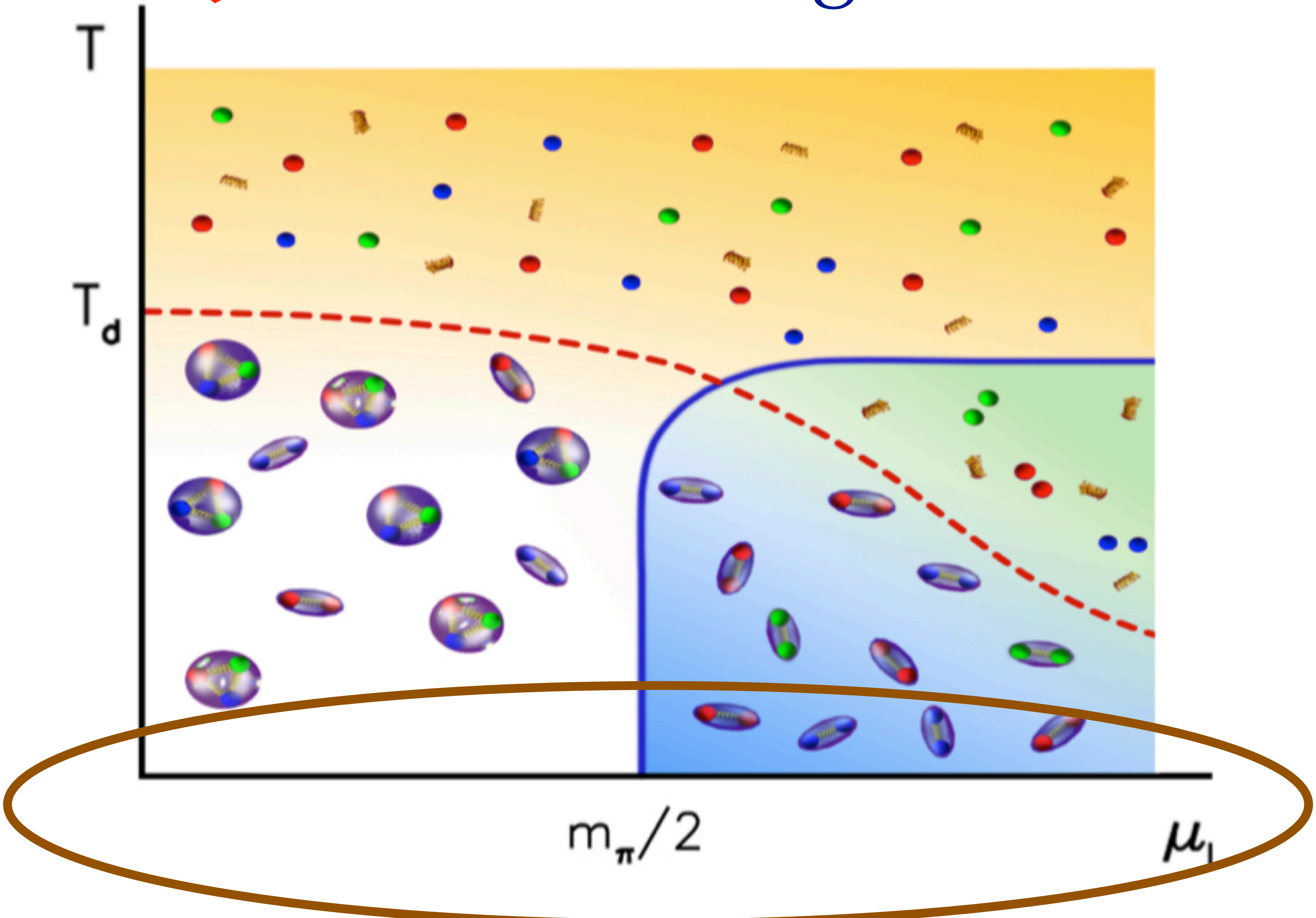
Chpt

- At T=0 $\mu_I < m_\pi$ → **Vacuum**
  $\mu_I \geq m_\pi$ → **BEC**



Son and Stephanov, PRL 86, 592 (2001)

QCD Phase Diagram



[Brandt, Endrődi, Schmalzbauer 1712.08190]

Pions Stars

Eur. Phys. J. A (2017) **53**: 35
DOI 10.1140/epja/i2017-12221-x

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Scrutinizing the pion condensed phase*

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[2] Dipartimento di Fisica e Astronomia, Università di Padova, Via Marzolo 8, I-35131 Padova, Italy
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PHYSICAL REVIEW D **98**, 094510 (2018)

New class of compact stars: Pion stars

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(Received 8 September 2018; published 26 November 2018)

Pions Stars

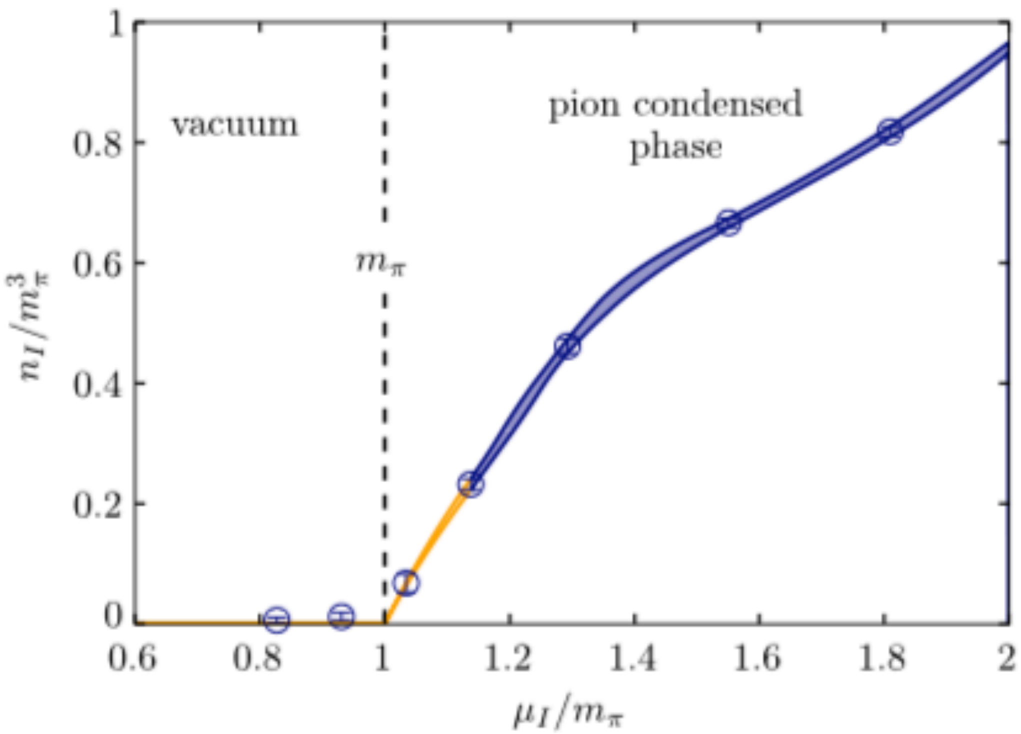
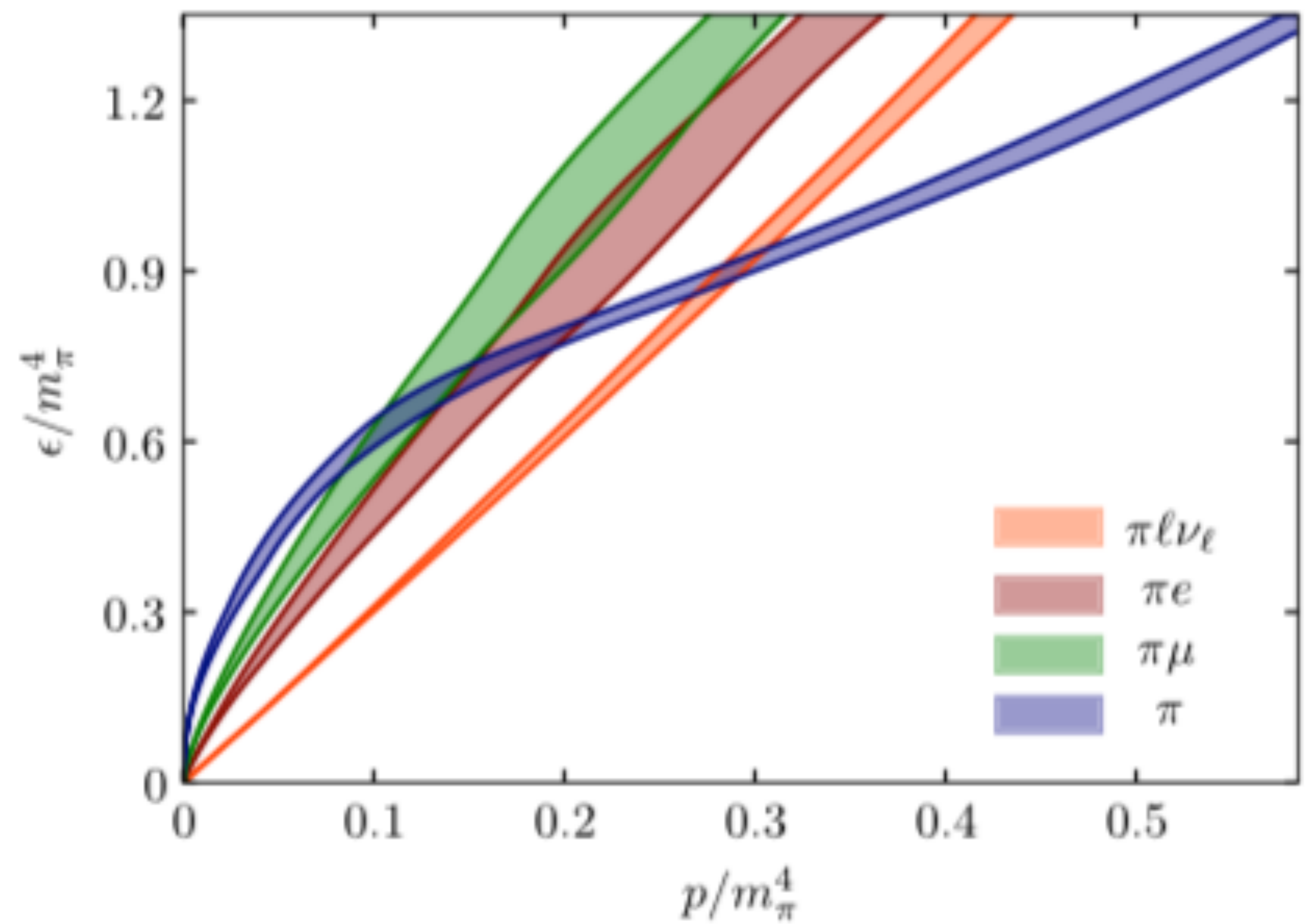
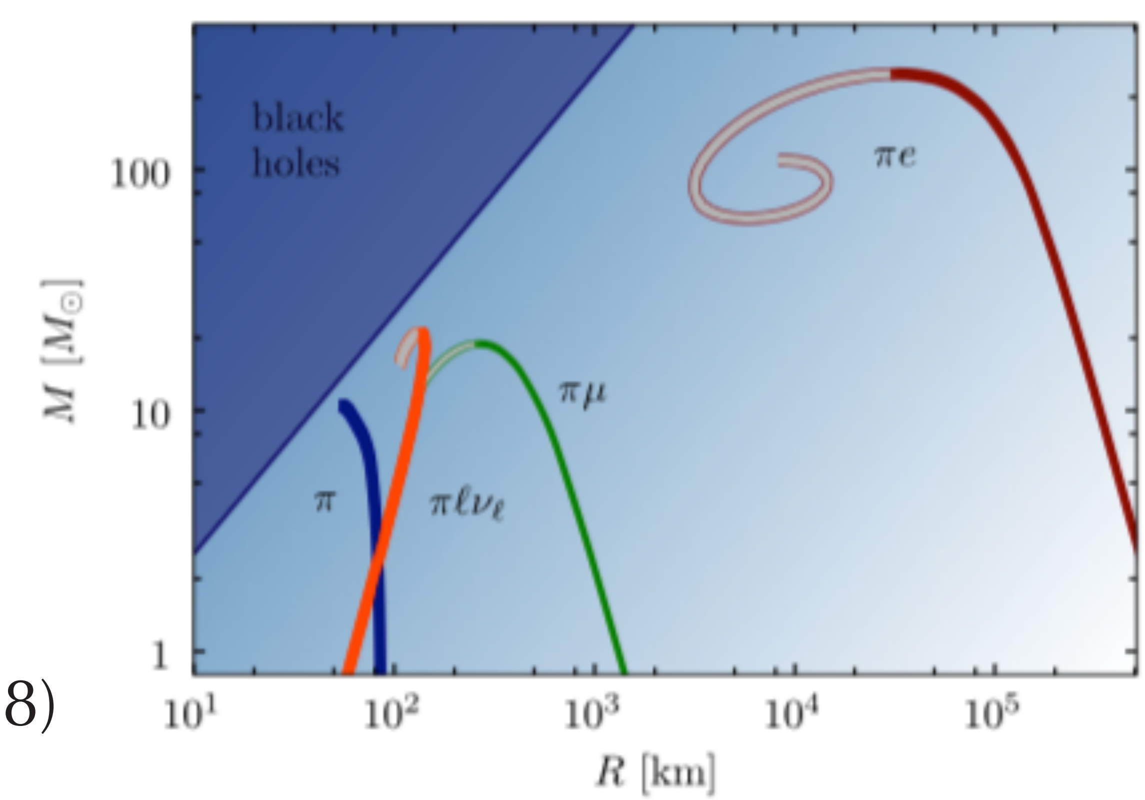


FIG. 1. Phase transition between the vacuum and the pion condensed phase, as exhibited by the isospin density. The lattice data (blue points) are fitted using χPT (yellow curve) and matched to a spline interpolation (blue curve).





Brandt at all , Phys. Rev. D 98, 094510 (2018)

SU(2) NJL model

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\partial_\mu\gamma^\mu - m\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau_3\psi\right)^2 + 2\left(\bar{\psi}i\gamma_5\tau_+\psi\right)\left(\bar{\psi}i\gamma_5\tau_-\psi\right)\right]$$

τ's are the generator matrices for the pseudoscalar interactions, which corresponds to the pionic excitations π_1, π_2, π_3 or equivalently π_+, π_-, π_3, with $\tau_\pm = (\tau_1 \pm \tau_2)/\sqrt{2}$.

Now we can introduce the chiral condensate $\sigma = -2G\langle\bar{\psi}\psi\rangle$ and pion condensates

$$\begin{aligned}\sqrt{2}\pi_+ &= -2\sqrt{2}G\langle\bar{\psi}i\gamma_5\tau_+\psi\rangle = \Delta e^{i\theta},\\ \sqrt{2}\pi_- &= -2\sqrt{2}G\langle\bar{\psi}i\gamma_5\tau_-\psi\rangle = \Delta e^{-i\theta},\end{aligned}$$

where the phase factor θ indicates the direction of the $U_I(1)$ symmetry breaking.

SU(2) NJL model

$$Z_{\rm NJL}(T,\mu_B,\mu_I) = \int [d\bar{\psi}][d\psi]\exp\left[\int_0^\beta dt \int d^3x \left(\mathcal{L}_{\rm NJL} + \bar{\psi}\hat{\mu}\gamma_0\psi\right)\right]$$

$$\hat{\mu} = \begin{pmatrix} \mu_u & 0 \\ 0 & \mu_d \end{pmatrix}$$

$$\mu_u = \frac{\mu_B}{3} + \mu_I$$

$$\mu_d = \frac{\mu_B}{3} - \mu_I$$

$$\mu_B/3 = (\mu_u + \mu_d)/2$$

$$\mu_I = (\mu_u - \mu_d)/2$$

SU(2) Nambu-Jona-Lasinio model (NJL)

$$\Omega^{TRS}_{\mathrm{NJL}}(\sigma,\Delta) = \frac{\sigma^2+\Delta^2}{4G} - 2N_c \int_0^\Lambda \frac{k^2 dk}{2\pi^2}\left[E_k^+ + E_k^-\right]$$

where $E_k^\pm = \sqrt{(E_k \pm \mu_I)^2 + \Delta^2}$

$E_k = \sqrt{k^2+M^2}$

$M = m + \sigma$

UV divergent! Needs regularization!

$$\left.\frac{\partial \Omega_{\mathrm{NJL}}}{\partial \sigma}\right|_{\sigma=\sigma_m} = \left.\frac{\partial \Omega_{\mathrm{NJL}}}{\partial \Delta}\right|_{\Delta=\Delta_m} = 0.$$

From these equations we obtain

$$\sigma = 4GN_c M\, I_\sigma$$
$$\Delta = 4GN_c\Delta\, I_\Delta$$

with the definitions

$$I_\sigma = \sum_{s=\pm1}\int_\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{E_k}\frac{E_k+s\mu_I}{\sqrt{(E_k+s\mu_I)^2+\Delta^2}}$$

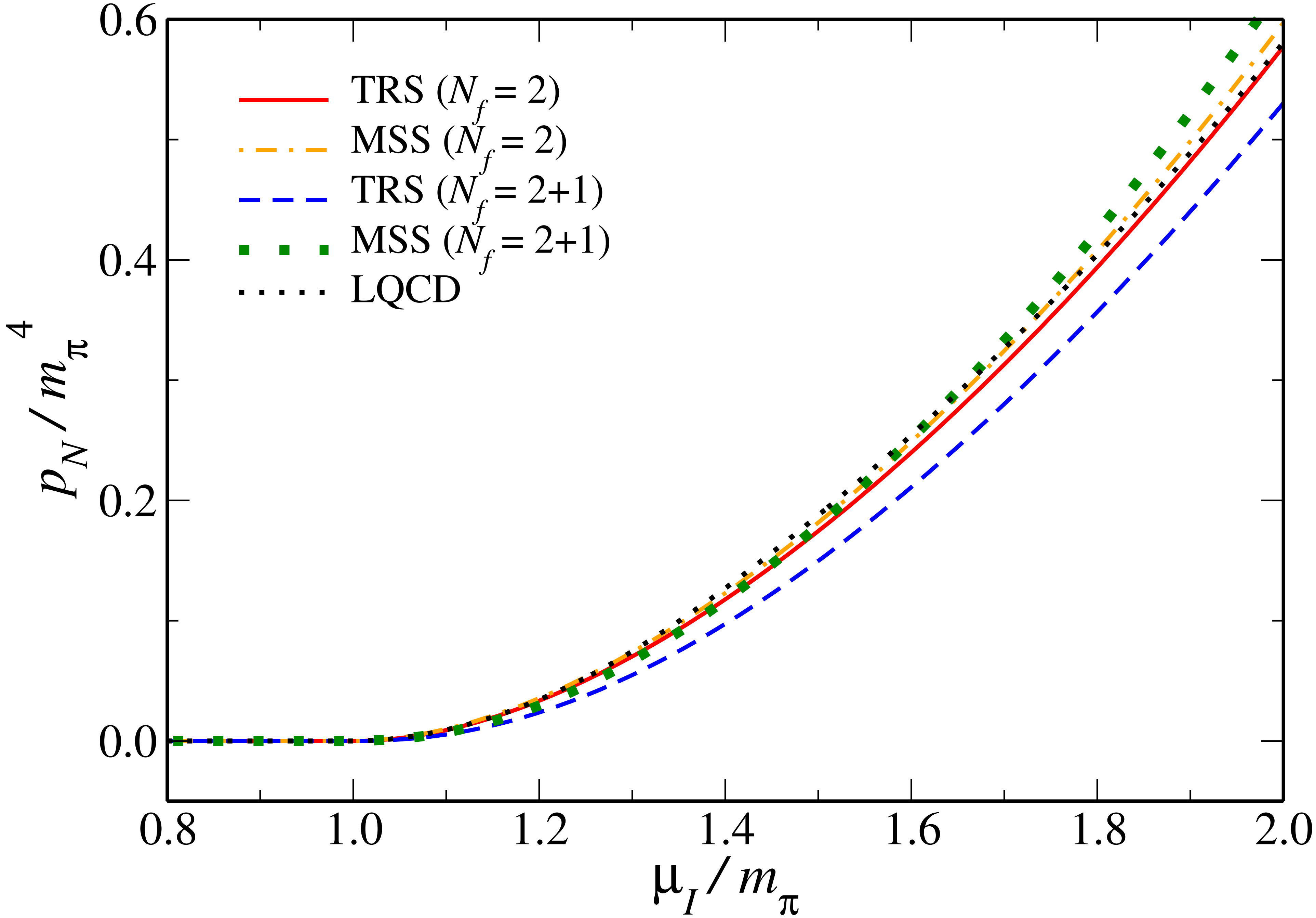
$$I_\Delta = \sum_{s=\pm1}\int_\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{(E_k+s\mu_I)^2+\Delta^2}}$$

Regularization schemes:

TRS - Traditional regularization scheme

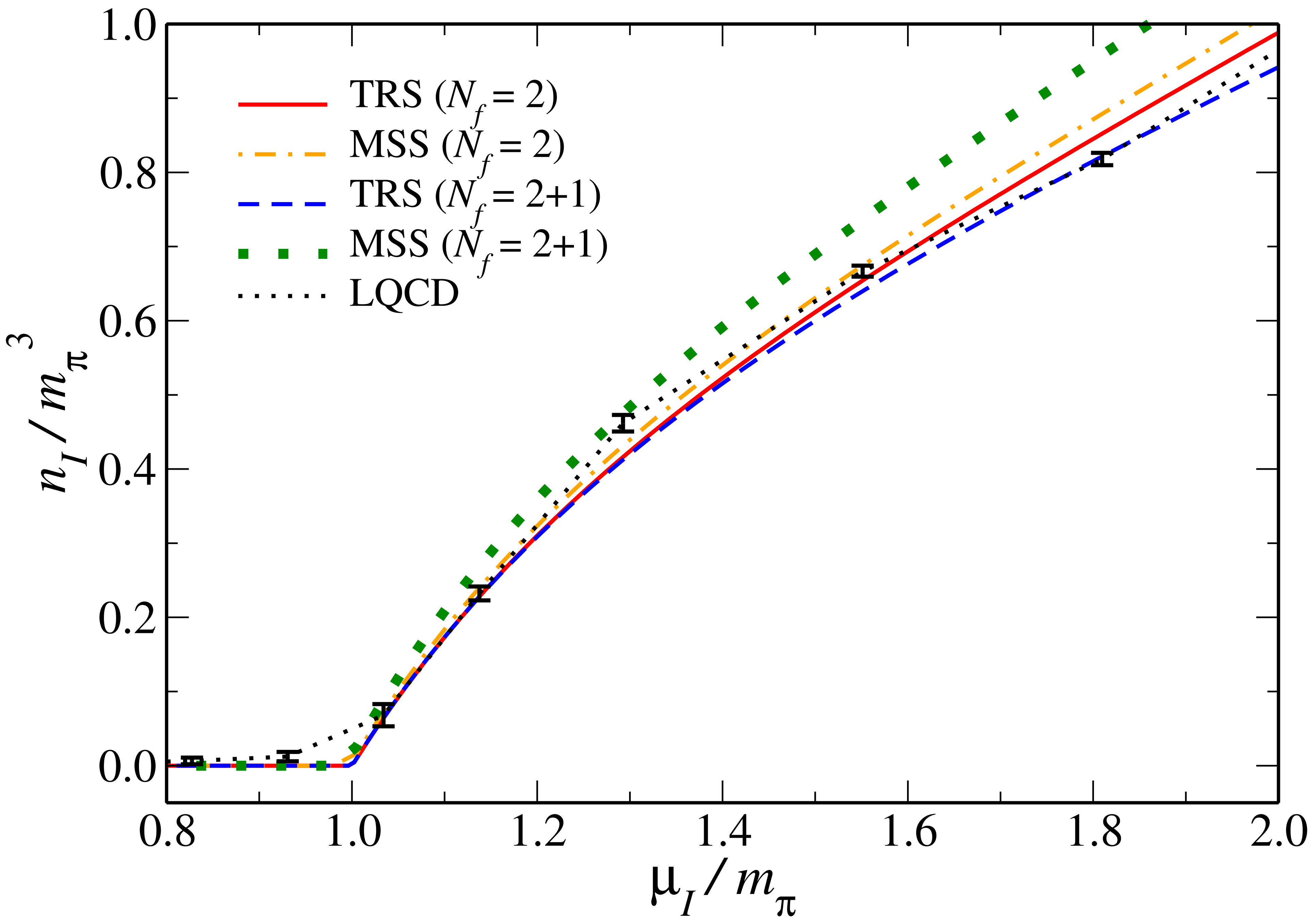
MSS - medium separation scheme: Proper separation of medium effects from divergent integrals

NJL X Lattice T=0



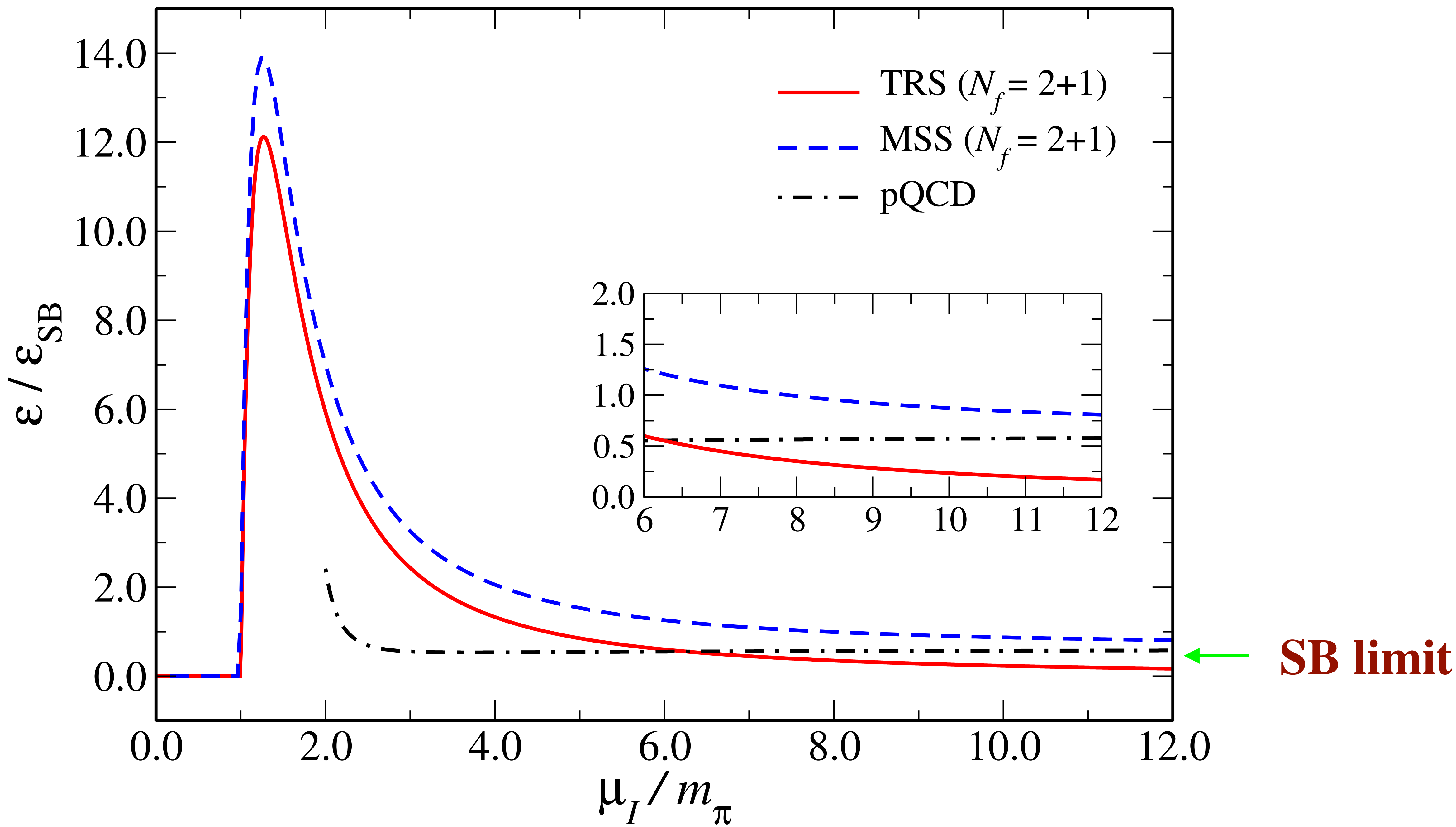
RLSF, S. Avancini, A.Bandyopadhyay, B.Lopes, D.C.Duarte, Phys. Rev D **103**, 076023 (2021)

NJL X Lattice T=0



RLSF, S. Avancini, A.Bandyopadhyay, B.Lopes, D.C.Duarte, Phys. Rev D **103**, 076023 (2021)

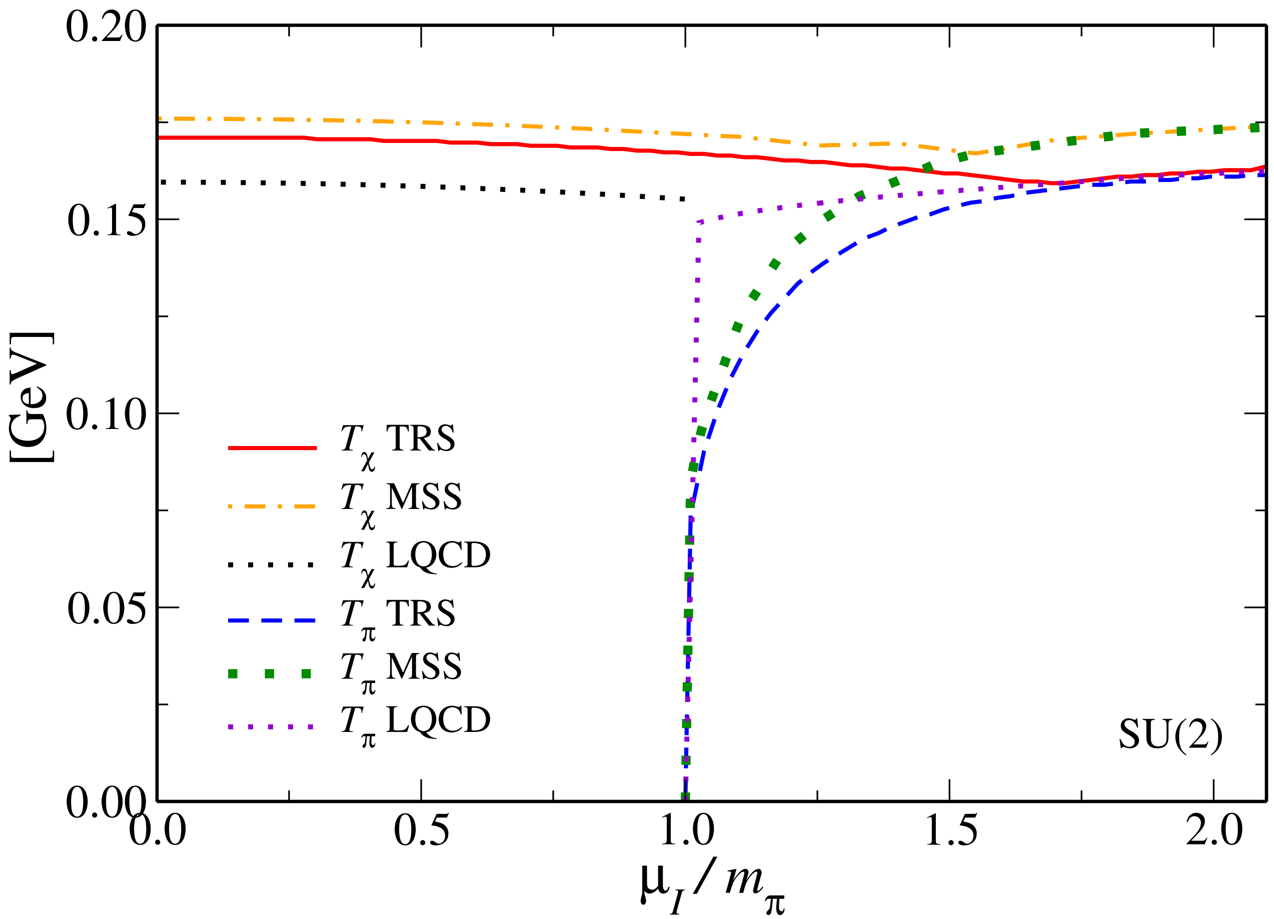
NJL X Lattice T=0

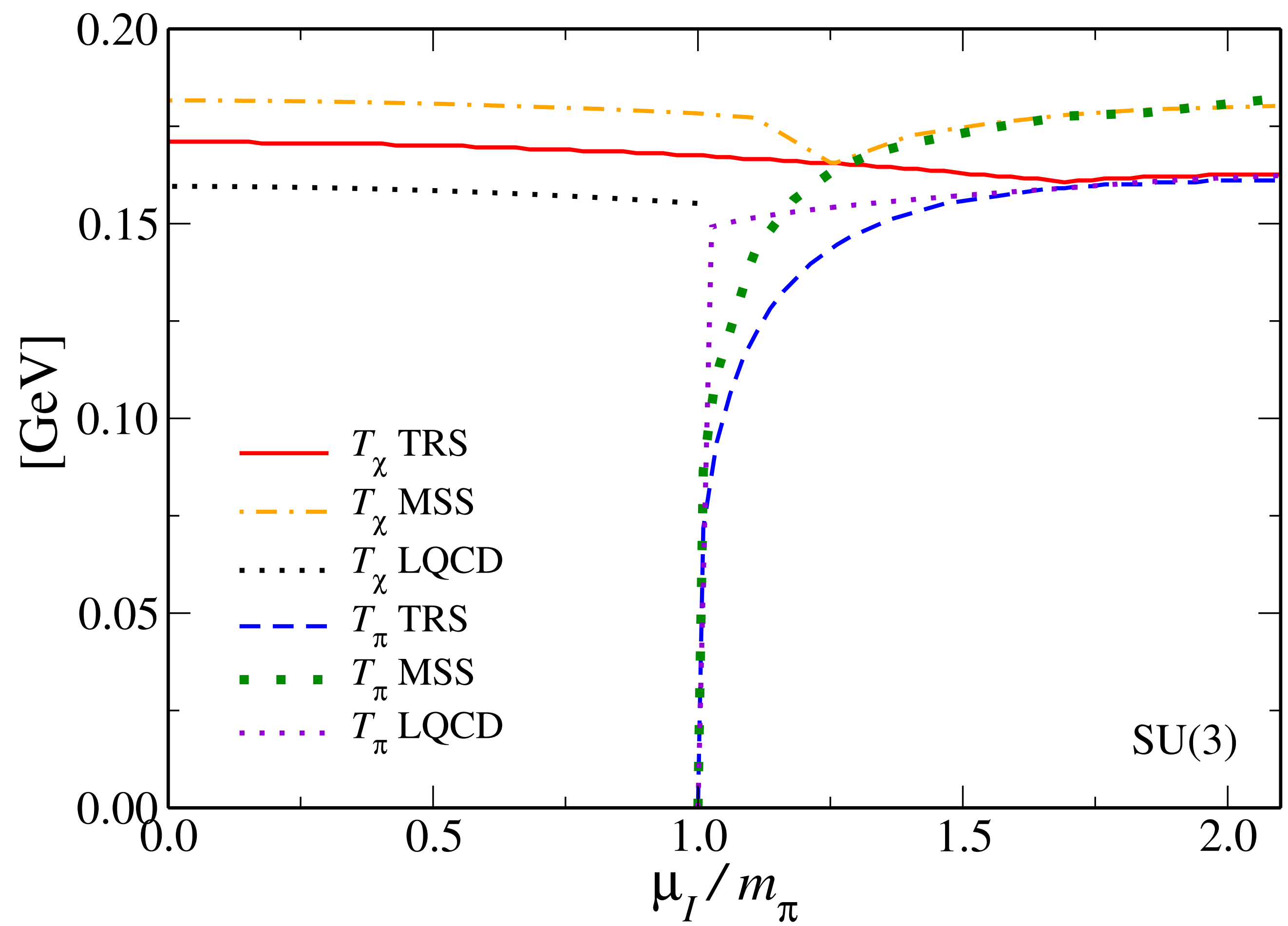


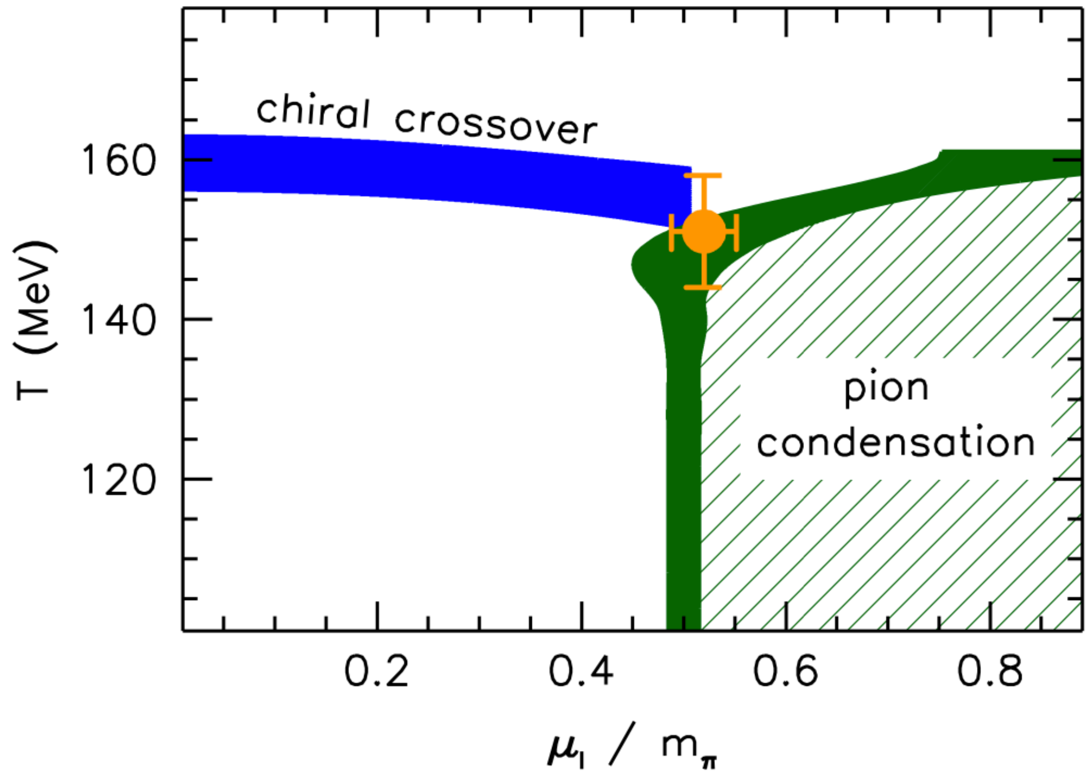
T. Graf, J. Schaffner-Bielich and E. S. Fraga, Phys. Rev. D 93, 085030 (2016)

RLSF, S. Avancini, A.Bandyopadhyay, B.Lopes, D.C.Duarte, Phys. Rev D **103**, 076023 (2021)

Phase Diagram







B.B. Brandt, G. Endrodi, S. Schmalzbauer, *Phys. Rev. D* **97** (2018) 5, 054514

RLSF, S. Avancini, A.Bandyopadhyay, B.Lopes, D.C.Duarte, Phys. Rev D **103**, 076023 (2021)

QCD equation of state at finite isospin density from the linear sigma model with quarks: The cold case

Alejandro Ayala,[1,2,3] Aritra Bandyopadhyay,[3,4,5] Ricardo L. S. Farias,[3]
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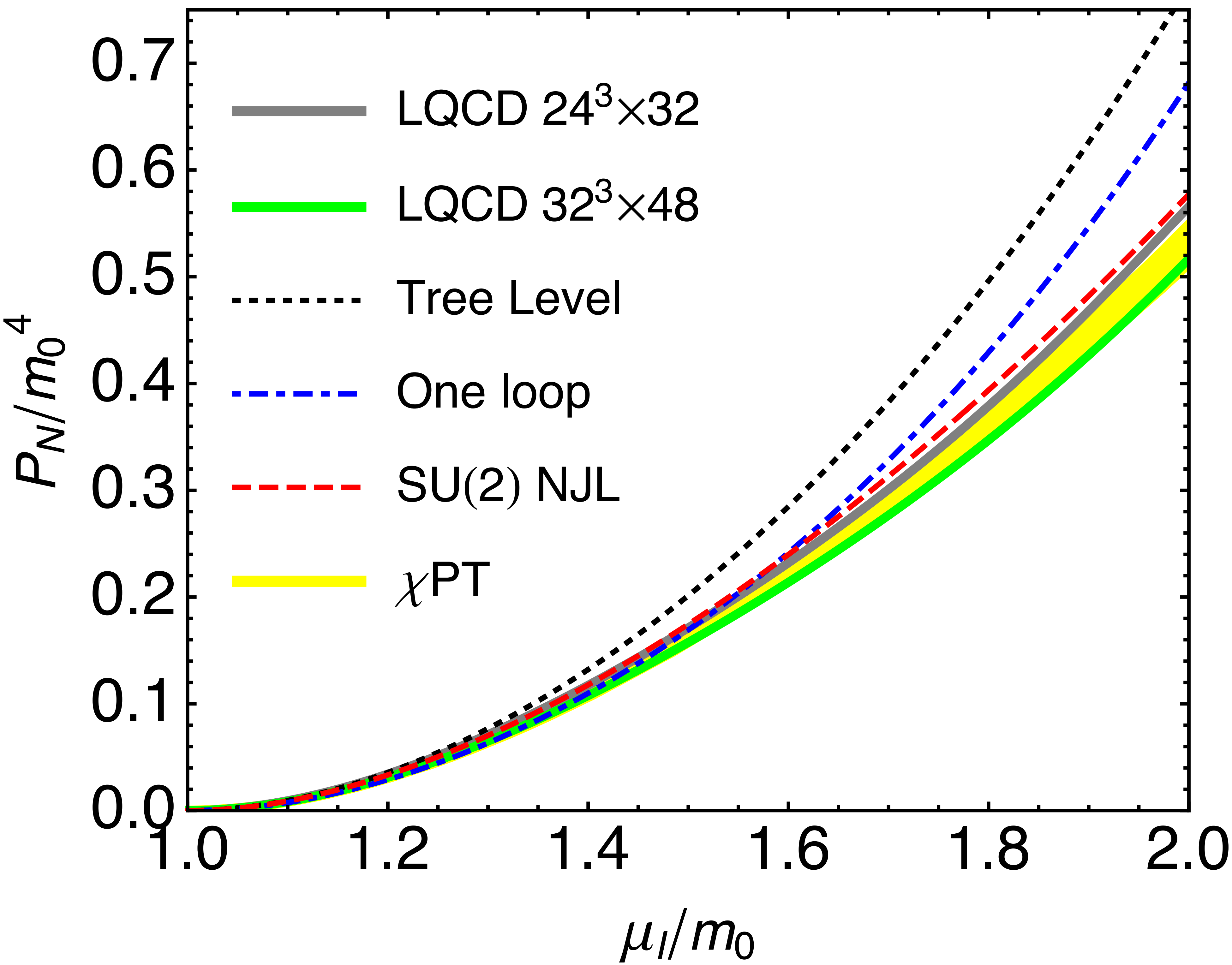
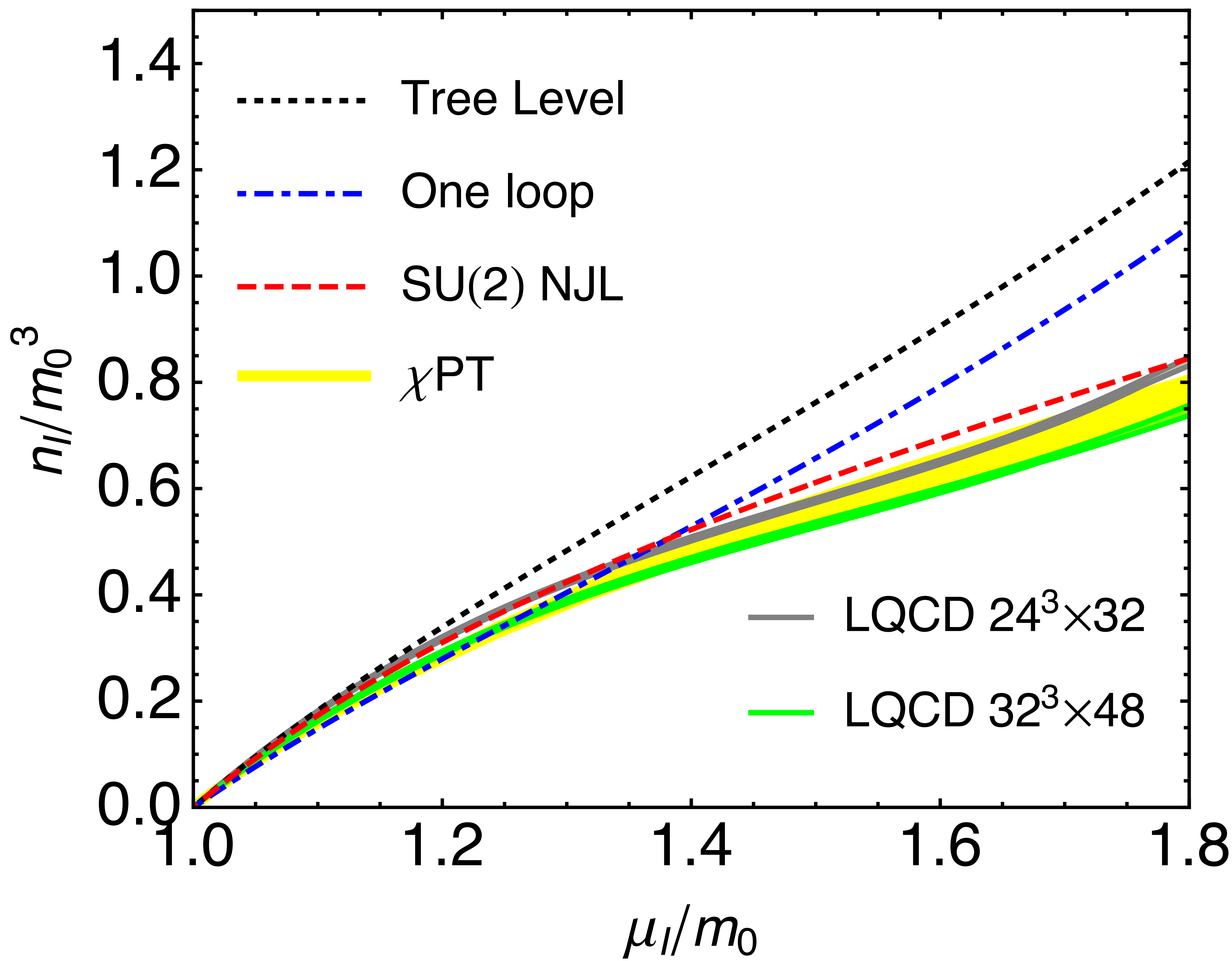
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$
$$+ i\bar{\psi}\gamma^\mu \partial_\mu \psi - ig\bar{\psi}\gamma^5 \vec{\tau} \cdot \vec{\pi}\psi - g\bar{\psi}\psi\sigma,$$

Pressure



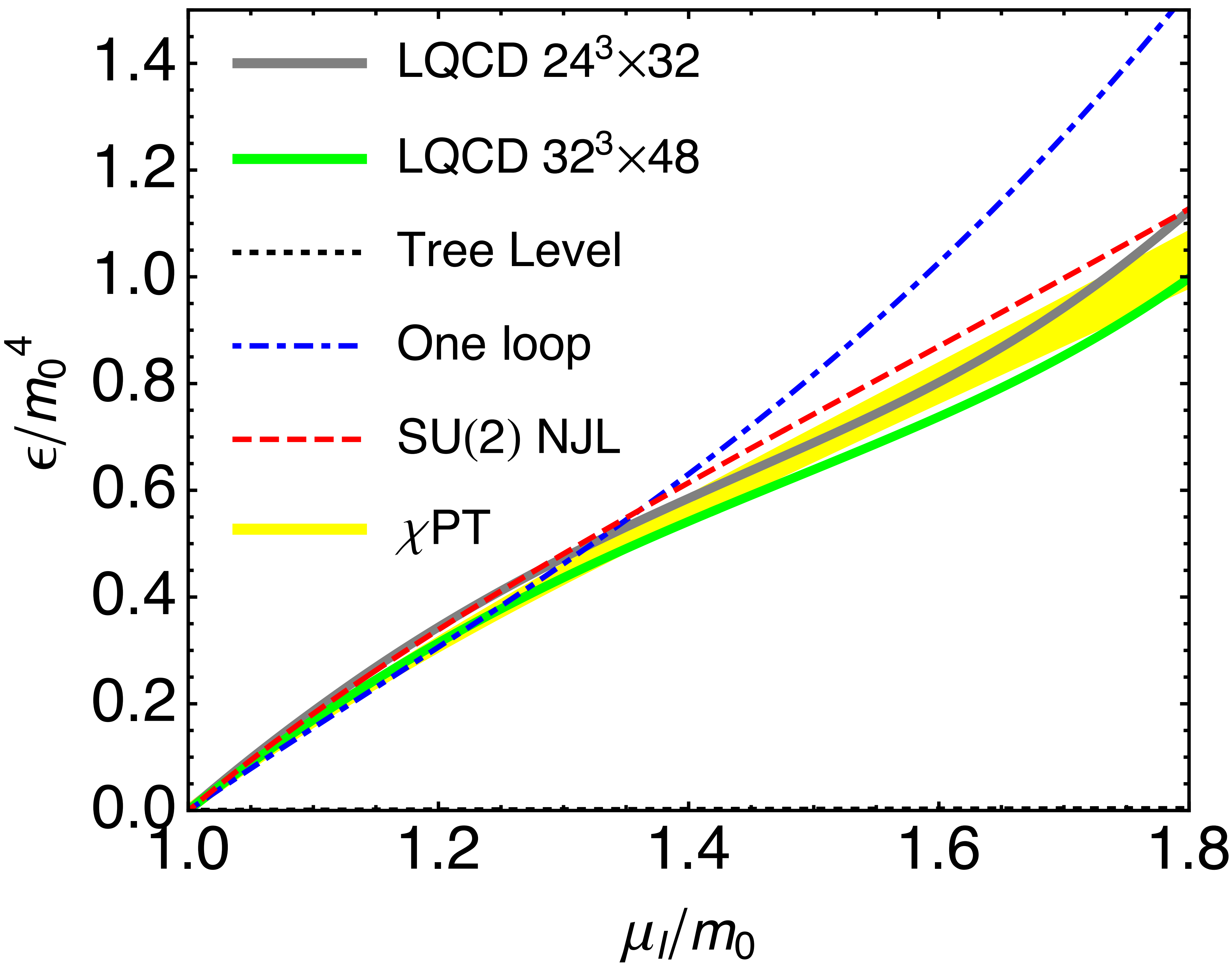
RLSF, A. Ayala, A.Bandyopadhyay, L.A. Hernández and José Luiz Hernández, Phys. Rev D **107**, 074027 (2023)

Isospin density



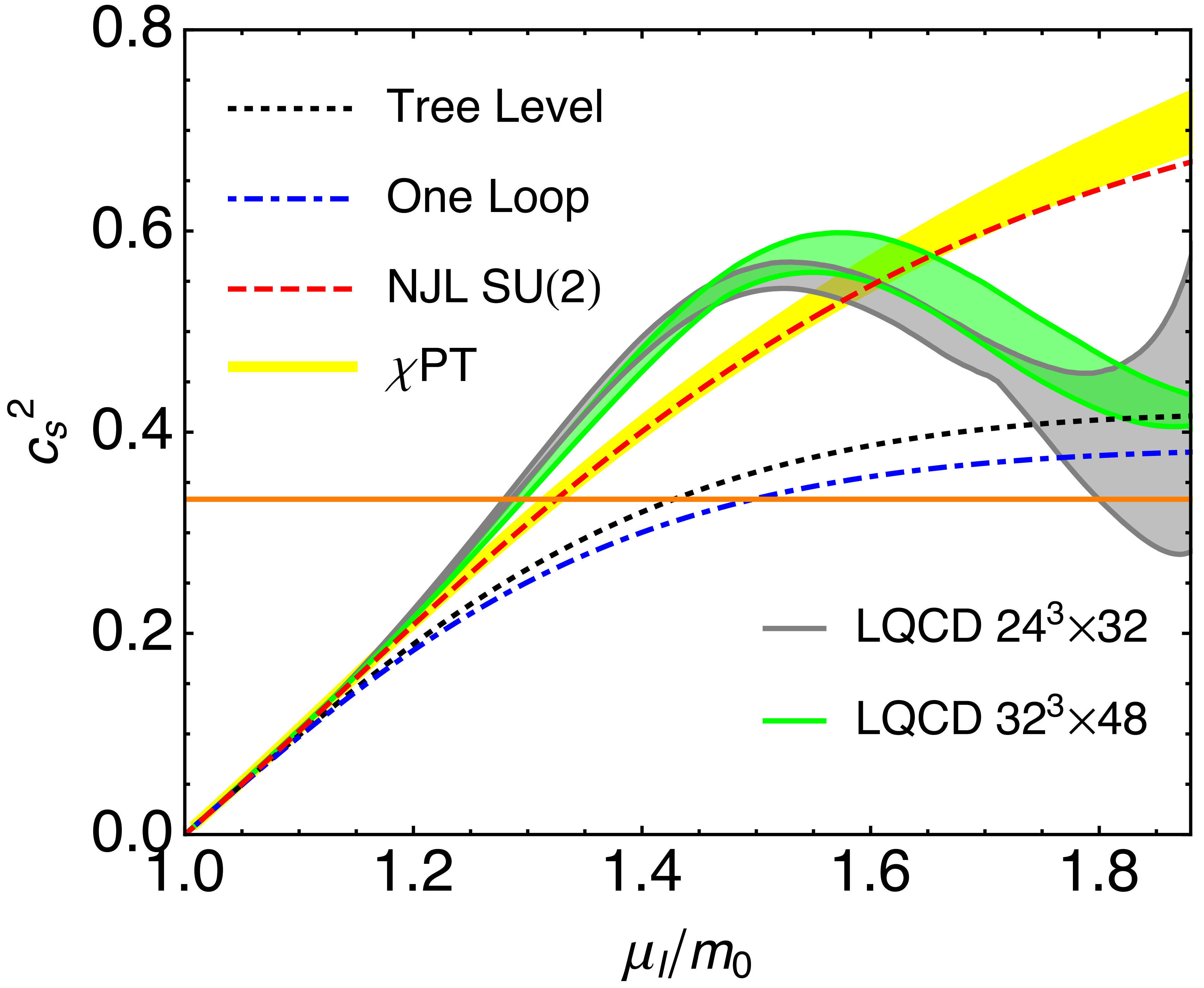
RLSF, A. Ayala, A.Bandyopadhyay, L.A. Hernández and José Luiz Hernández, Phys. Rev D **107**, 074027 (2023)

Energy density



RLSF, A. Ayala, A.Bandyopadhyay, L.A. Hernández and José Luiz Hernández, Phys. Rev D **107**, 074027 (2023)

Speed of Sound square



RLSF, A. Ayala, A.Bandyopadhyay, L.A. Hernández and José Luiz Hernández, Phys. Rev D **107**, 074027 (2023)

How a chiral imbalance of right-handed and left-handed quarks can influence the phase diagram of QCD?

Motivation

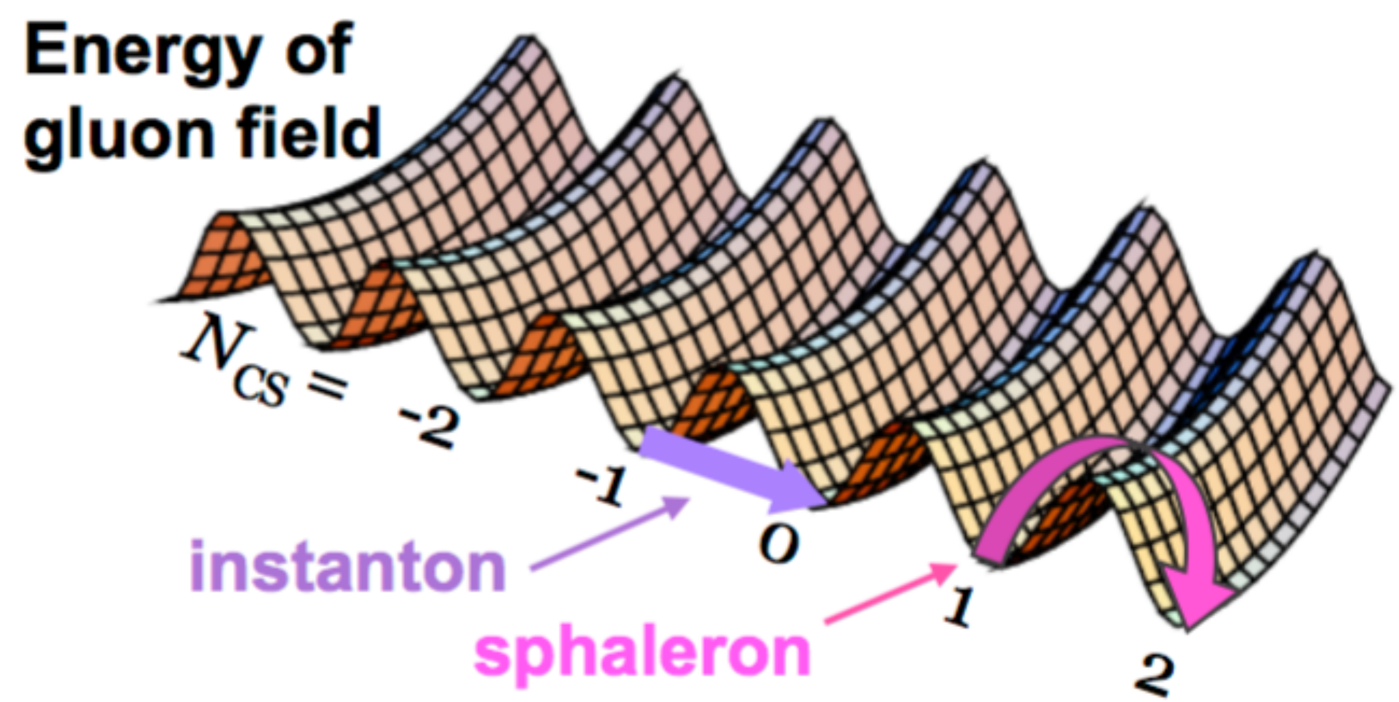
- The nontrivial nature of the vacuum of non-Abelian gauge theories in general, and of QCD in particular, allows for the existence of topological solutions like **instantons** and **sphalerons**.

- 1] G. 't Hooft, Phys. Rev. D 14, 3432 (1976).
- [2] F. R. Klinkhamer and N. S. Manton, Phys. Rev. D 30, 2212 (1984).

Motivation

Topology-induced change of chirality

Right ⟷ Left



- The QCD vacuum has a non-trivial topological structure characterized by an integer-valued Chern-Simons number Ncs
- At zero and low temperatures, the different Chern-Simons sectors are connected by quantum tunneling transitions, i.e. the instantons.
- At finite temperature, the gauge configurations that change the Chern- Simons number can also be activated thermally through sphaleron transitions.

Motivation

- From the Adler-Bell-Jackiw anomaly[&] in the context of QCD, they can generate an asymmetry between the number of left- and right-handed quarks.

Axial anomaly
Roman W. Jackiw (MIT). 2010. 9 pp.
Published in Int.J.Mod.Phys. A25 (2010) 659-667

- Such a chirality imbalance is expected to occur in event-by-event C and CP violating processes in heavy-ion collisions

Selected for a Viewpoint in *Physics*

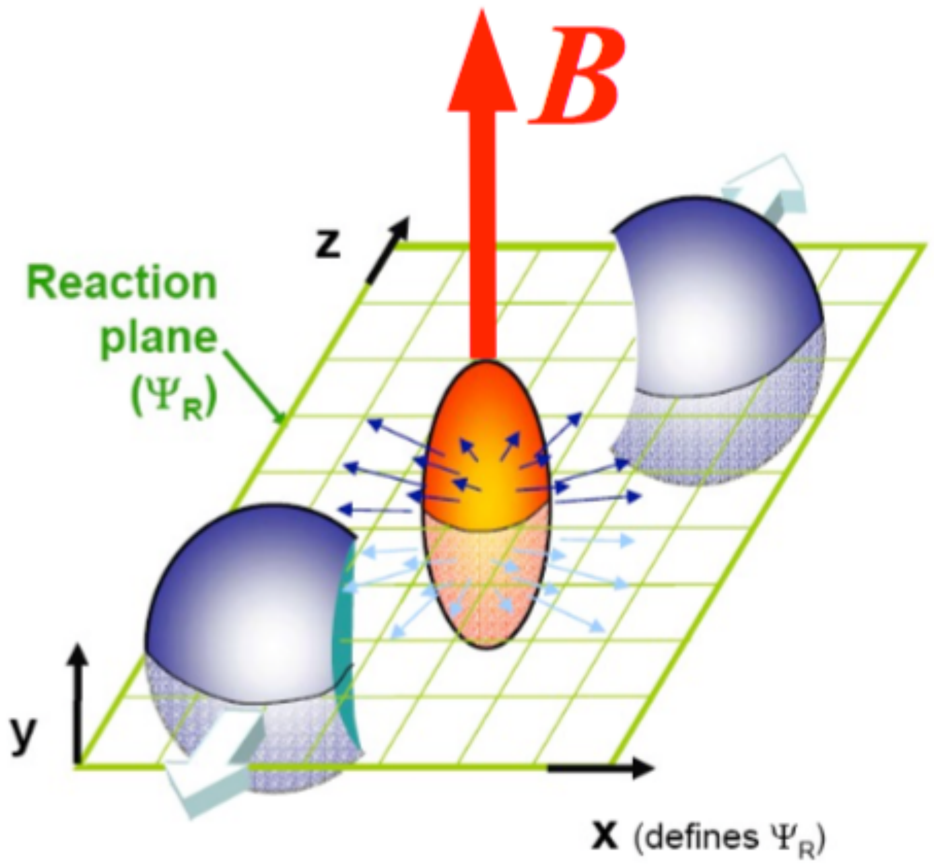
PRL **103,** 251601 (2009) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2009

Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

(STAR Collaboration)

Motivation

- In off-central collisions a magnetic field is created and the presence of a chiral imbalance gives rise to an electric current along the magnetic field, whose effect is to produce charge separation, an effect dubbed **chiral magnetic effect** (CME)



Motivation

CME effect is not restricted to QCD, it extends over a wide range of systems, e.g., in

- hydrodynamics and condensed matter systems#
- has been actually observed in recent condensed matter experiments
- which makes it of much wider interest in physics.

#More Refs. in RLSF, D.C.Duarte, G.Krein and R.O. Ramos, Phys. Rev. D **94**, 074011 (2016)

Motivation

NATURE PHYSICS | LETTER

Chiral magnetic effect in $ZrTe_5$

Qiang Li, Dmitri E. Kharzeev, Cheng Zhang, Yuan Huang, I. Pletikosić, A. V. Fedorov, R. D. Zhong, J. A. Schneeloch, G. D. Gu & T. Valla

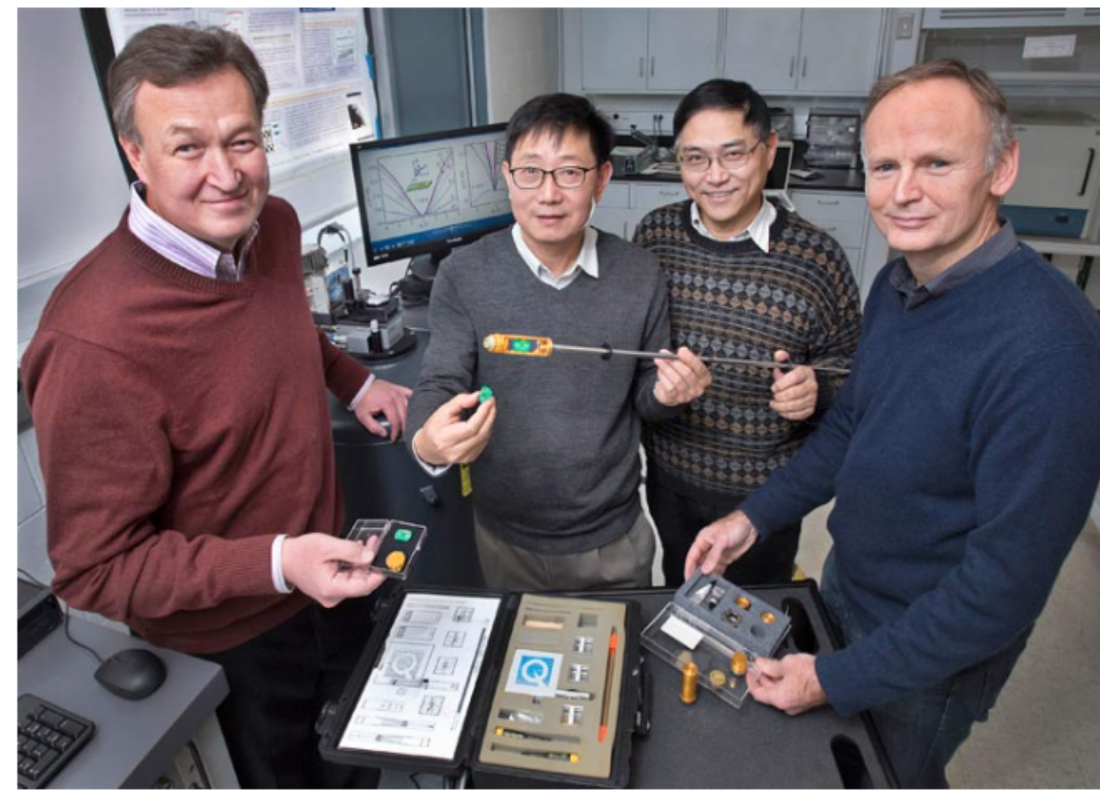
Affiliations | Contributions | Corresponding authors

Nature Physics (2016) | doi:10.1038/nphys3648
Received 19 December 2014 | Accepted 04 January 2016 | Published online 08 February 2016

Chiral Magnetic Effect Generates Quantum Current

Separating left- and right-handed particles in a semi-metallic material produces anomalously high conductivity

February 8, 2016



Nuclear theorist Dmitri Kharzeev of Stony Brook University and Brookhaven Lab with Brookhaven Lab materials scientists Qiang Li, Genda Gu, and Tonica Valla in a lab where the team measured the unusual high conductivity of zirconium pentatelluride. + ENLARGE

- CME recently has been observed in zirconium pentatelluride

From a BNL webpage

Motivation

- The effects of a chiral imbalance in the phase diagram of QCD can be studied in the grand canonical ensemble by the introduction of a chiral chemical potential μ_5

$$\mu_5 \bar{\psi}\gamma_0\gamma_5\psi \longrightarrow$$ in the QCD Lagrangian density

PHYSICAL REVIEW D **78,** 074033 (2008)

Chiral magnetic effect

Kenji Fukushima,[1,*] Dmitri E. Kharzeev,[2,+] and Harmen J. Warringa[2,‡]
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[2]*Department of Physics, Brookhaven National Laboratory, Upton New York 11973, USA*
(Received 2 September 2008; published 31 October 2008)

PHYSICAL REVIEW D **81,** 114031 (2010)

Chiral magnetic effect in the Polyakov–Nambu–Jona-Lasinio model

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Raoul Gatto‡
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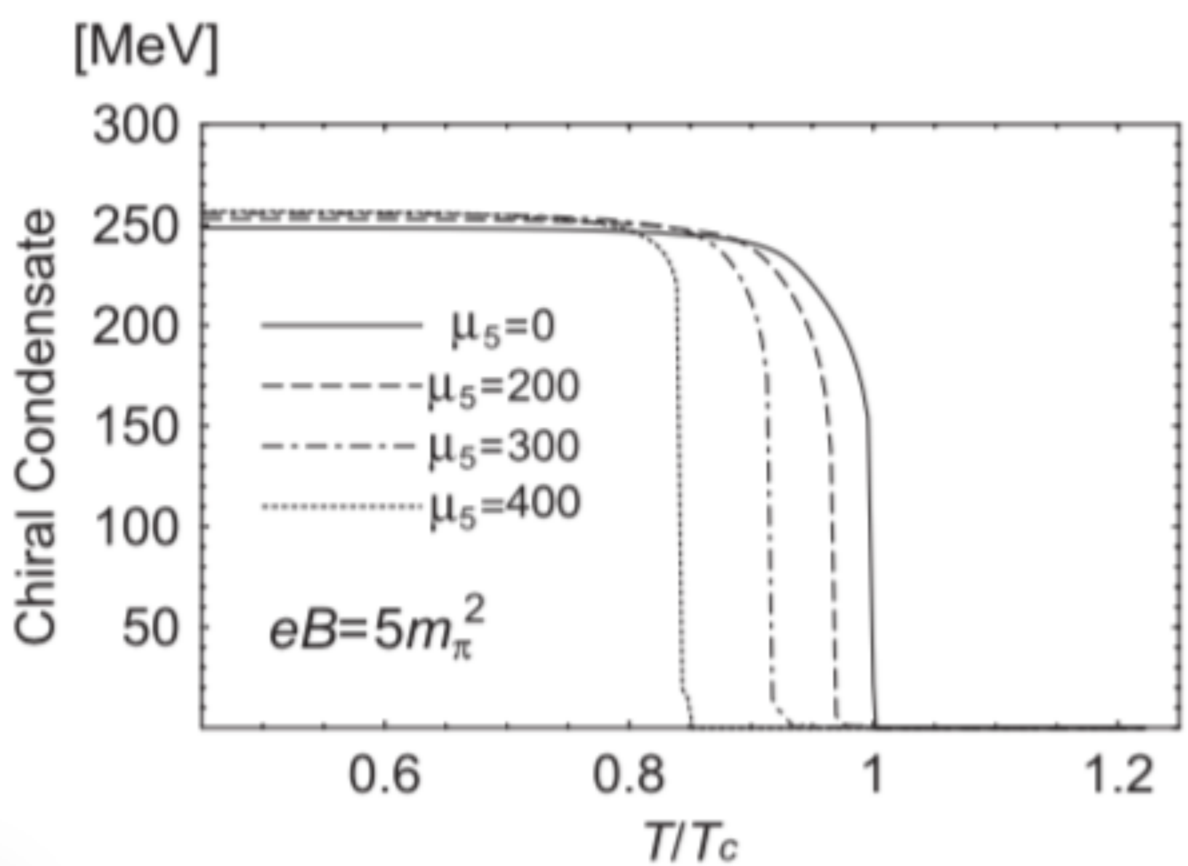
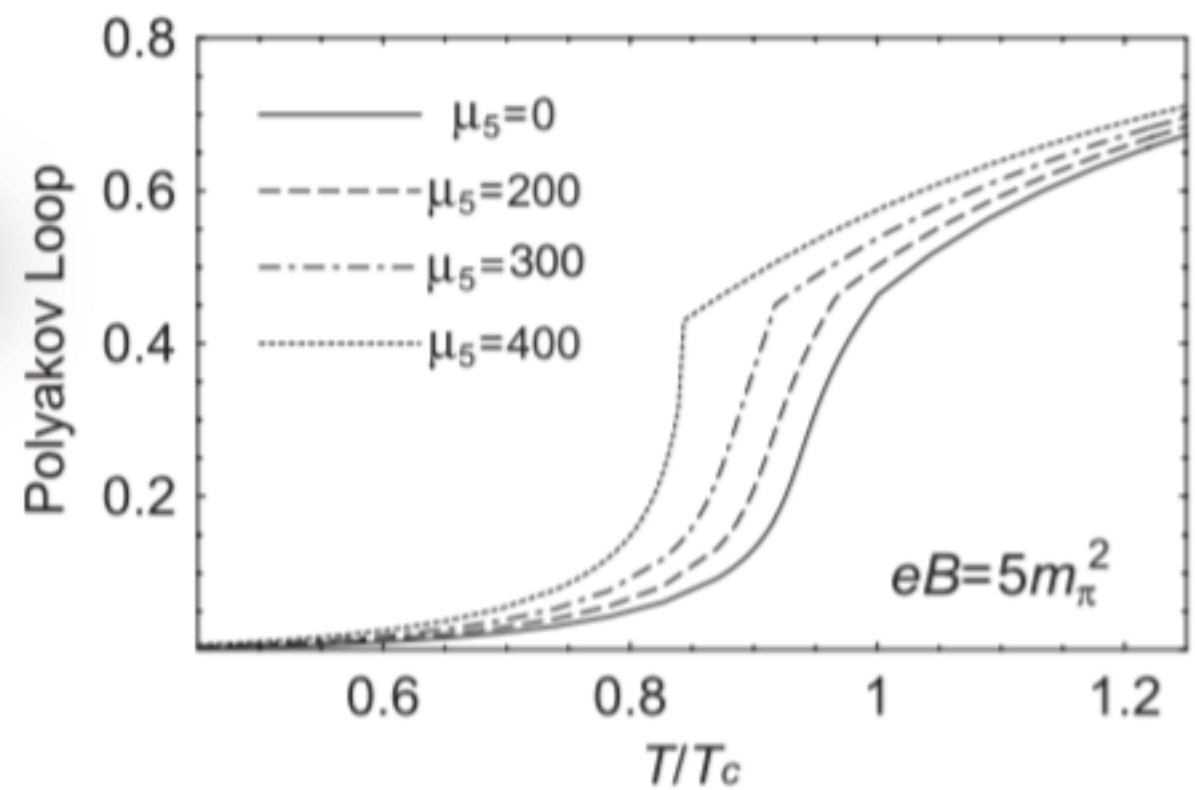
(Received 4 March 2010; published 21 June 2010)

We study the two-flavor Nambu–Jona-Lasinio model with the Polyakov loop in the presence of a strong magnetic field and a chiral chemical potential μ_5, which mimics the effect of imbalanced chirality due to QCD instanton and/or sphaleron transitions. First, we focus on the properties of chiral symmetry breaking and deconfinement crossover under the strong magnetic field. Then we discuss the role of μ_5 on the phase structure. Finally, the chirality charge, electric current, and their susceptibility, which are relevant to the chiral magnetic effect, are computed in the model.

DOI: 10.1103/PhysRevD.81.114031 PACS numbers: 12.38.Aw, 12.38.Mh





PHYSICAL REVIEW D **81**, 114031 (2010)

Chiral magnetic effect in the Polyakov–Nambu–Jona-Lasinio model

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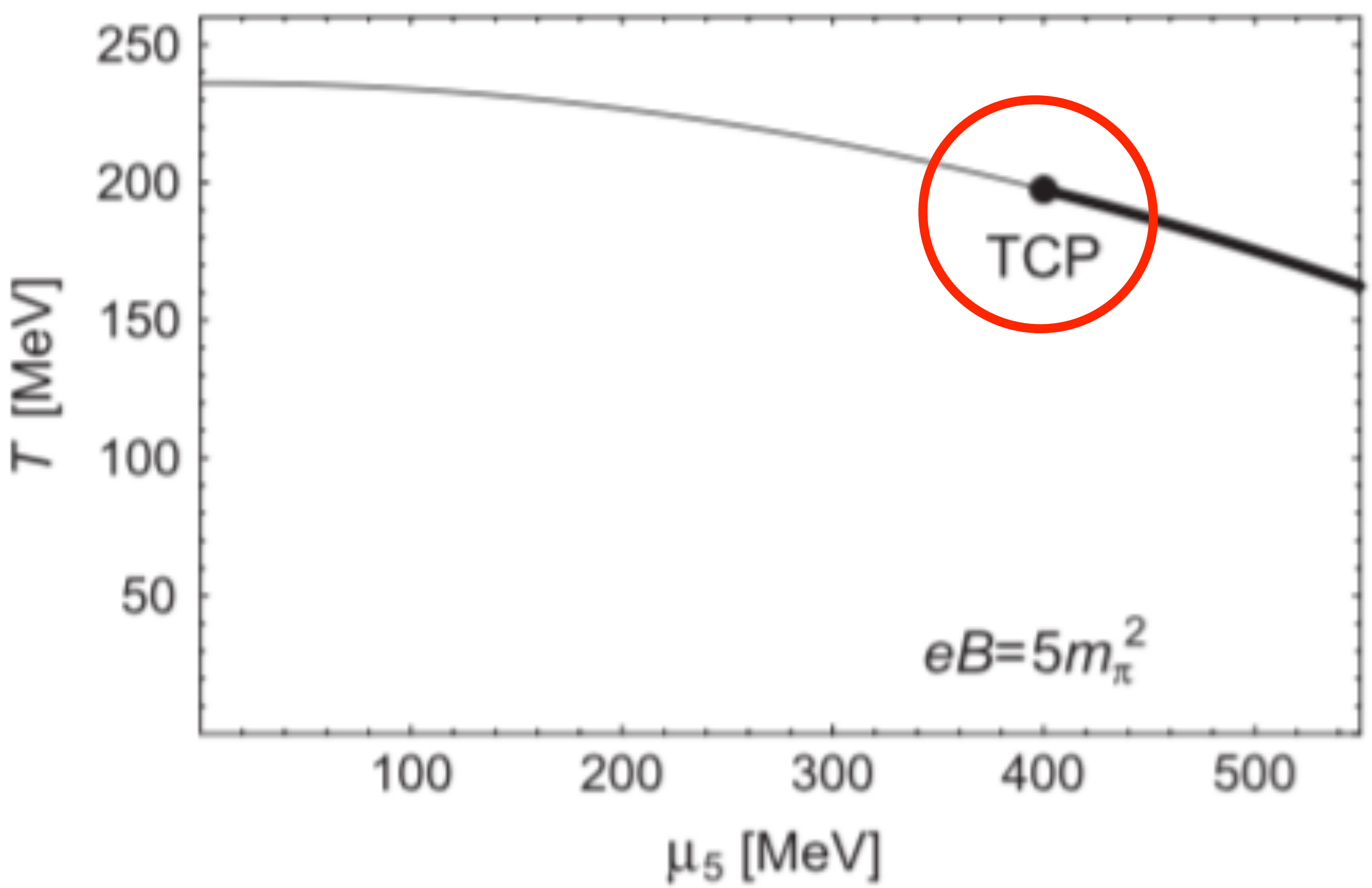

- Critical temperature decrease with μ_5
- Appear a Tricritical point TCP

PHYSICAL REVIEW D **84**, 014011 (2011)

Critical end point of quantum chromodynamics detected by chirally imbalanced quark matter

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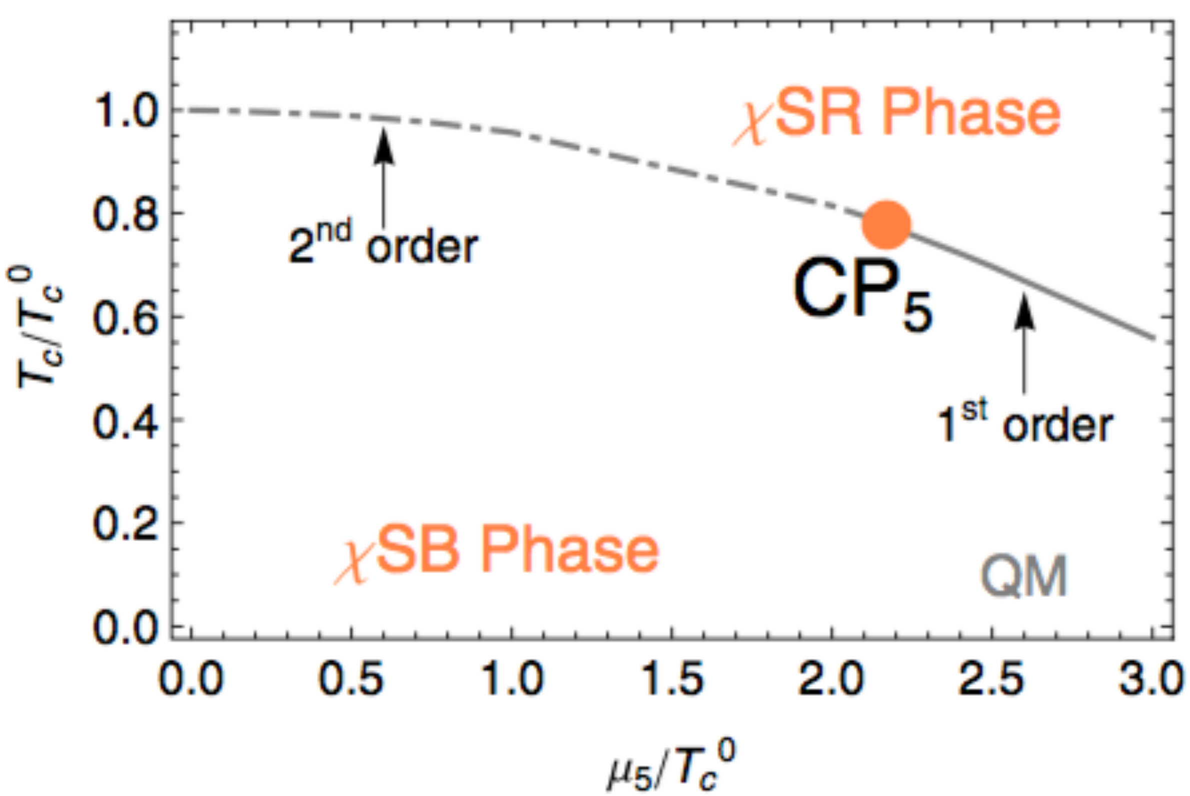
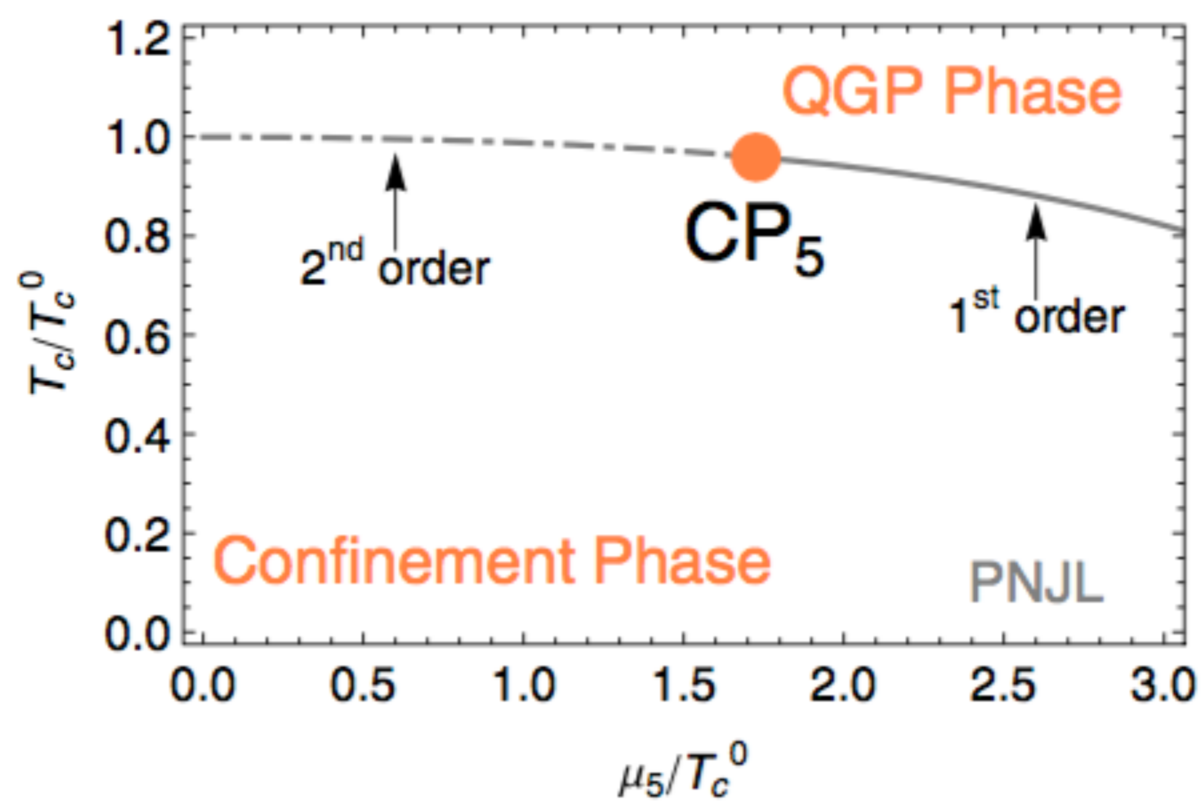
(Received 5 April 2011; published 7 July 2011)

We suggest the idea, supported by concrete calculations within chiral models, that the critical end point of the phase diagram of quantum chromodynamics with three colors can be detected, by means of lattice simulations of grand-canonical ensembles with a chiral chemical potential, μ_5, conjugated to chiral charge density. In fact, we show that a continuation of the critical end point of the phase diagram of quantum chromodynamics at finite chemical potential, μ, to a critical end point in the temperature-chiral chemical potential plane, is possible. This study paves the way for the mapping of the phases of quantum chromodynamics at finite μ, by means of the phases of a fictitious theory in which μ is replaced by μ_5.

DOI: 10.1103/PhysRevD.84.014011 PACS numbers: 12.38.Aw, 12.38.Mh





PHYSICAL REVIEW D **91,** 034017 (2015)

Effect of the chiral chemical potential on the position of the critical endpoint

Bin Wang,[1,2,*] Yong-Long Wang,[3,4] Zhu-Fang Cui,[3,6] and Hong-Shi Zong[3,5,6,†]
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[2]*Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, China*
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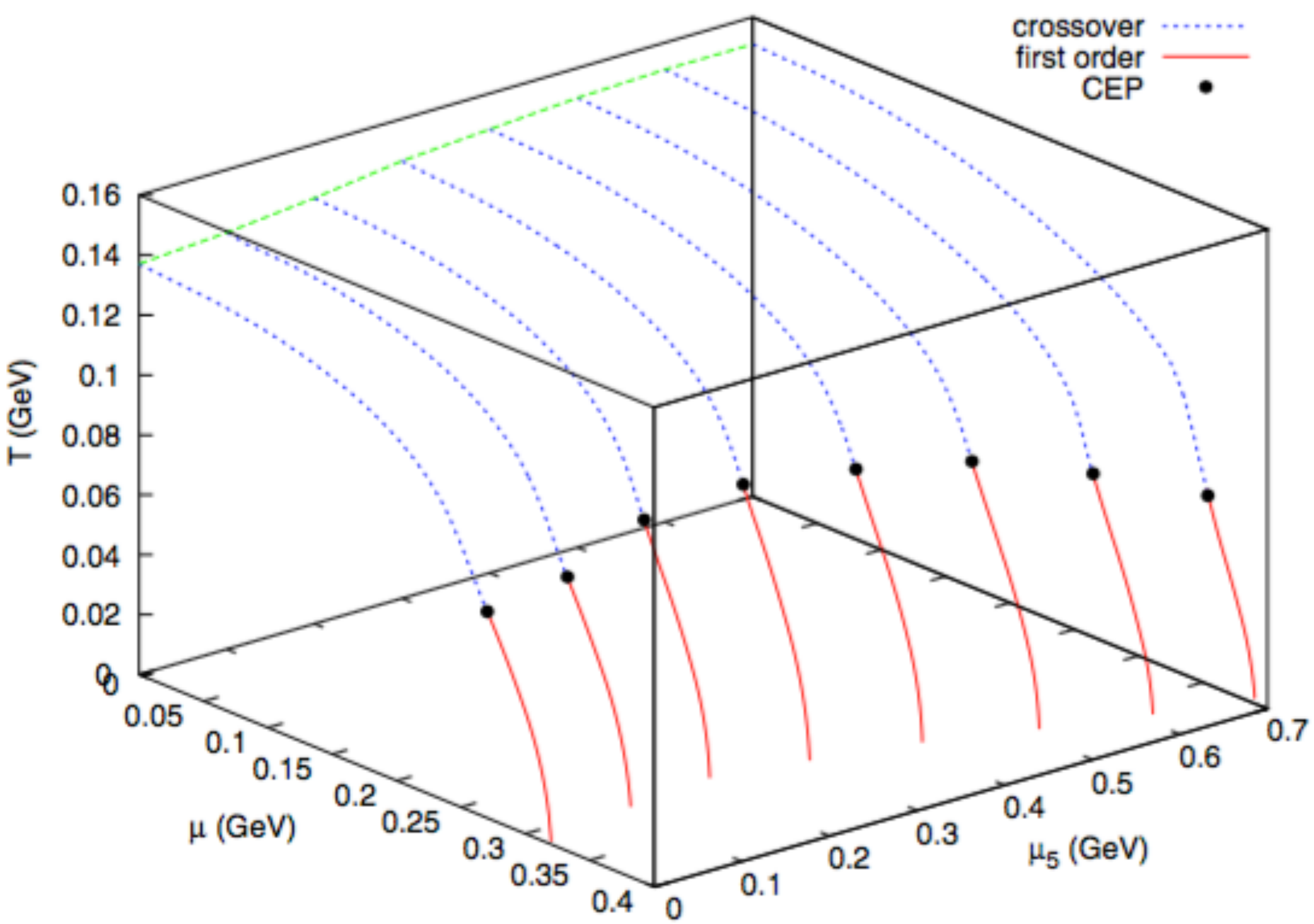
(Received 20 November 2013; published 19 February 2015)

Rank-2 confining separable model Gluon propagator



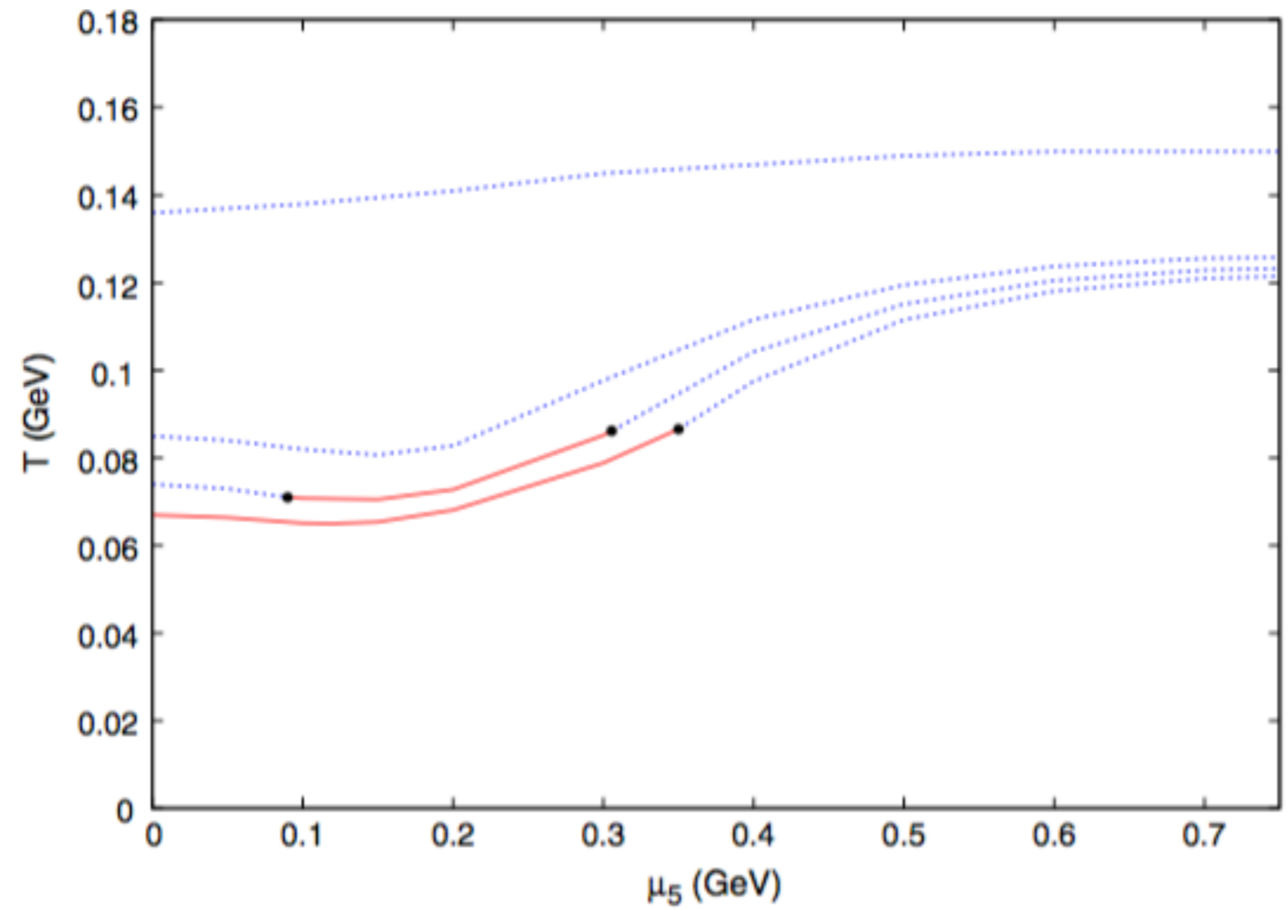


FIG. 6 (color online). The phase diagram in the $T - \mu_5$ plane when μ is at 0, 253.5, 265, and 273 MeV (from top to bottom). The solid line represents the first-order phase transition, while the dashed line represents the crossover.

Lattice Results + μ_5

- QCD in the presence of a chiral chemical potential is **free from the sign problem** and, therefore, amenable to Monte Carlo sampling in lattice simulations

- There is hope that lattice simulations of QCD with μ_5 can be used as a benchmark platform for comparing different effective models used in the literature.

Lattice Results: Nc=3 and Nf=2

PHYSICAL REVIEW D **93**, 034509 (2016)

Study of the QCD phase diagram with a nonzero chiral chemical potential

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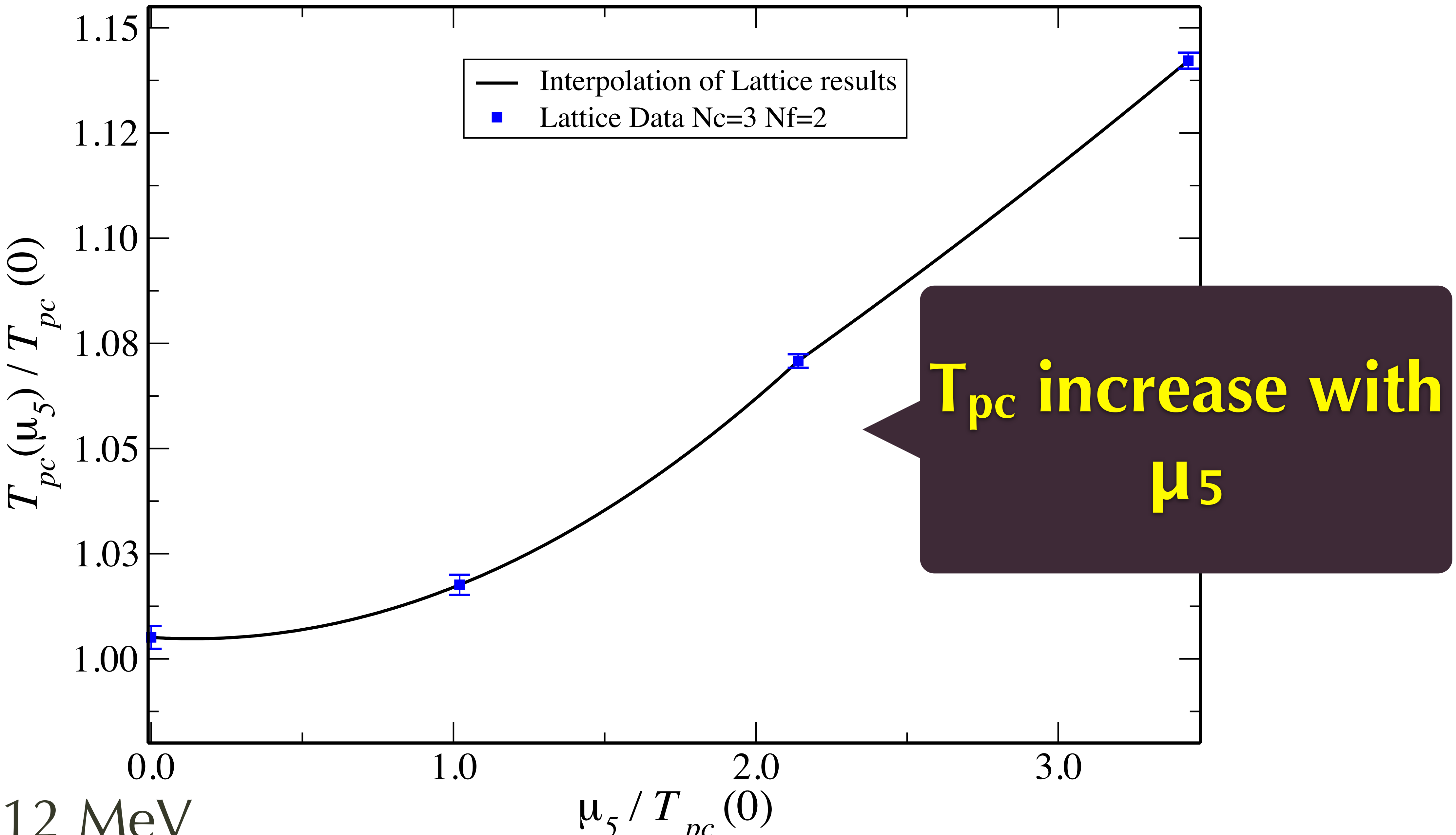
(Received 15 January 2016; published 25 February 2016)

Lattice Results: Nc=3 and Nf=2



PRD 93, 034509 (2016)

- $m_c = 12$ MeV
- No CP
- Always crossover transition!

Thanks Prof. Andrey Kotov for lattice unpublished data and comments!

Motivation

- It is a surprise to the QCD community the four fermions models failure to describe recent lattice results for the critical temperature for the QCD chiral phase transition when there is a chiral imbalance

- Given the prominent role played by NJL type of models in providing insight into the problem of the chiral phase transition, it is important to identify the sources of their failure in reproducing the qualitative features of lattice simulations for the μ_5 dependence of T_c

- Our work identifies the source of the failure of such models and offers a way to cure the problem.

SU(2) Nambu—Jona-Lasinio model (NJL)

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_c + \mu_5 \gamma^0 \gamma^5)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2\right]$$

good **chiral** physics, pions,…
BUT no confinement and no AF

G, Λ and m_c → m_π, f_π and $\langle \bar{\psi}\psi \rangle$

$$\Omega(M, T, \mu_5) = \Omega_0(M, \mu_5) - 2 N_f N_c T \sum_{s=\pm 1} \int \frac{d^3k}{(2\pi)^3} \ln\left[1 + e^{-\omega_s(k)/T}\right]$$

$$M = m_c - 2G \langle \bar{\psi}\psi \rangle$$

Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961)

SU(2) Nambu—Jona-Lasinio model (NJL)

$$\Omega_0(M,\mu_5) = \frac{(M-m_c)^2}{4G} - N_f N_c \sum_{s=\pm 1} \int \frac{d^3k}{(2\pi)^3} \omega_s(k)$$

UV divergent!
Needs regularization!

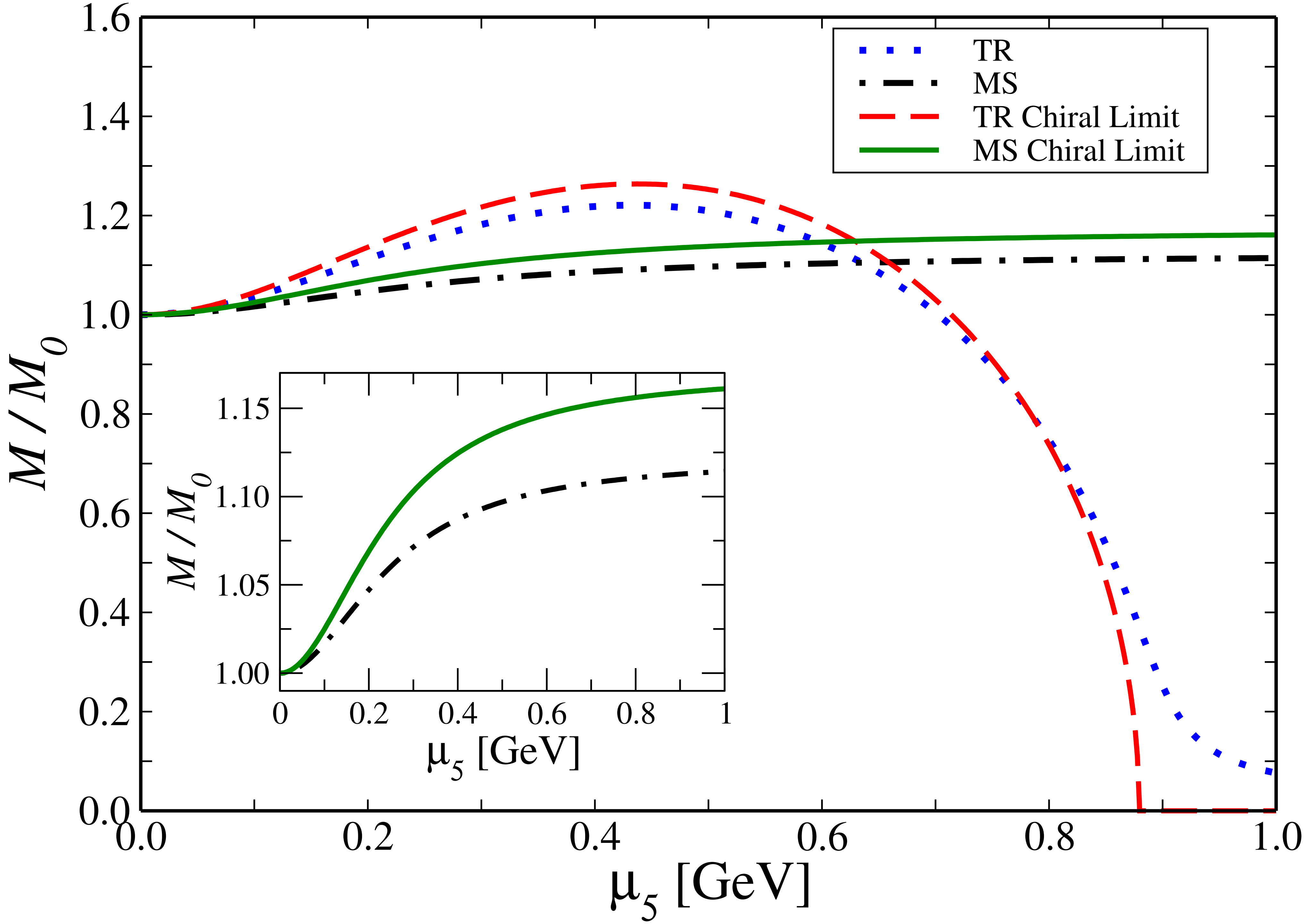
$$\omega_s(k) = \sqrt{(|\mathbf{k}| + s\mu_5)^2 + M^2}$$

are the eigenstates of the Dirac operator with helicity $s = \pm 1$

We fix the parameters of the model by using as input $f_\pi = 92.3$ MeV, $m_\pi = 0.140$ GeV and $\langle \bar{q}q \rangle^{\frac{1}{3}} = -0.250$ GeV, and use a 3d-cutoff to evaluate the vacuum divergent integrals. A good fit is obtained with $m_c = 5.37$ MeV, $G = 4.75\ \mathrm{GeV}^{-2}$ and $\Lambda = 0.660$ GeV. The constituent quark mass is $M_0 = 0.302$ GeV.

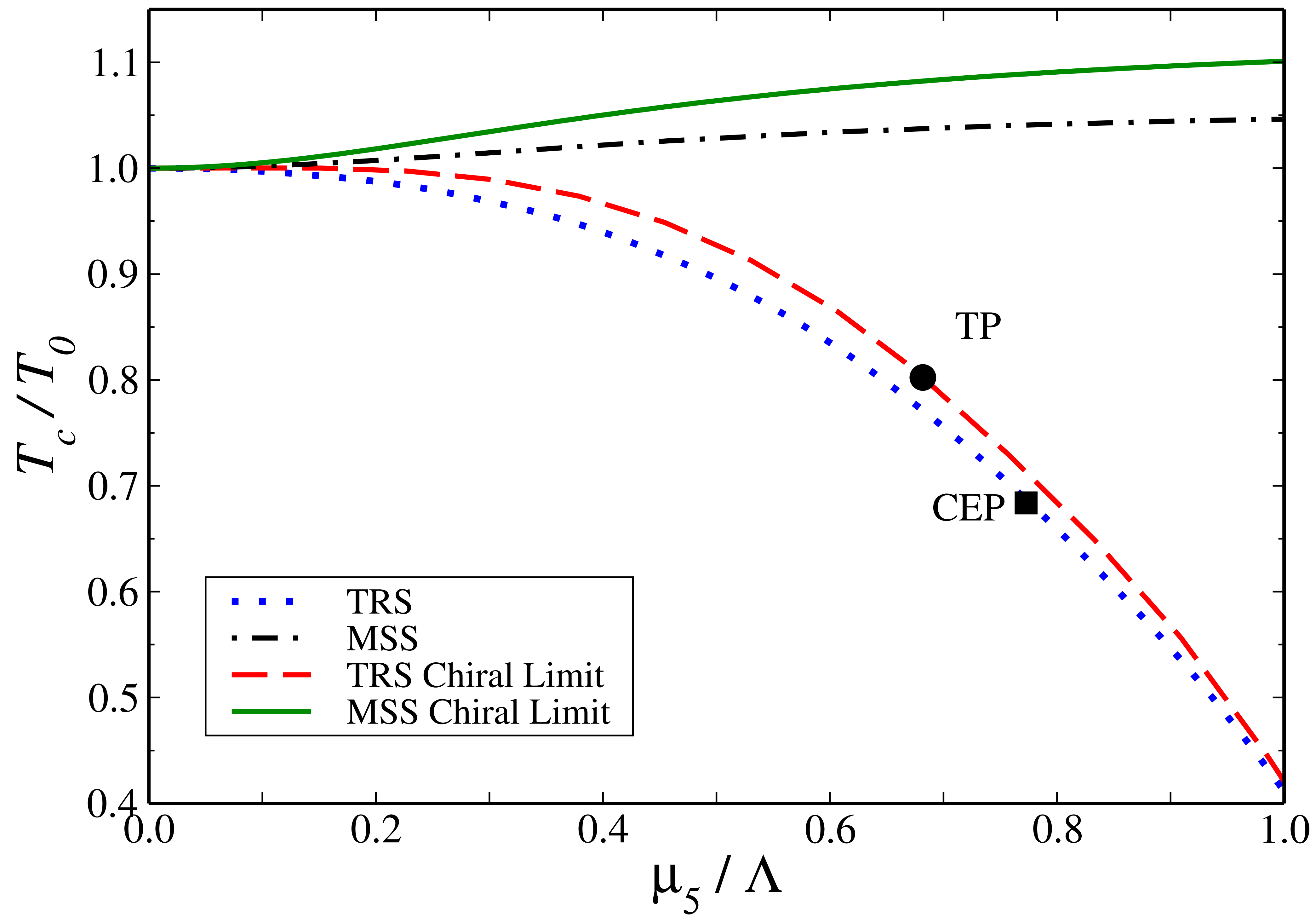
SU(2) NJL: MSS X TR

T=0



R.L.S.Farias, D.C. Duarte, G. Krein and R.O. Ramos, Phys. Rev. D **94**, 074011 (2016)

SU(2) NJL: MSS X TRS



R.L.S.Farias, D.C. Duarte, G. Krein and R.O. Ramos, Phys. Rev. D **94**, 074011 (2016)

QCD + T + B

- QCD in the presence of an external magnetic field and finite temperature is **free from the sign problem** and, therefore, amenable to Monte Carlo sampling in lattice simulations!

How a strong magnetic field can influence the phase diagram of QCD?

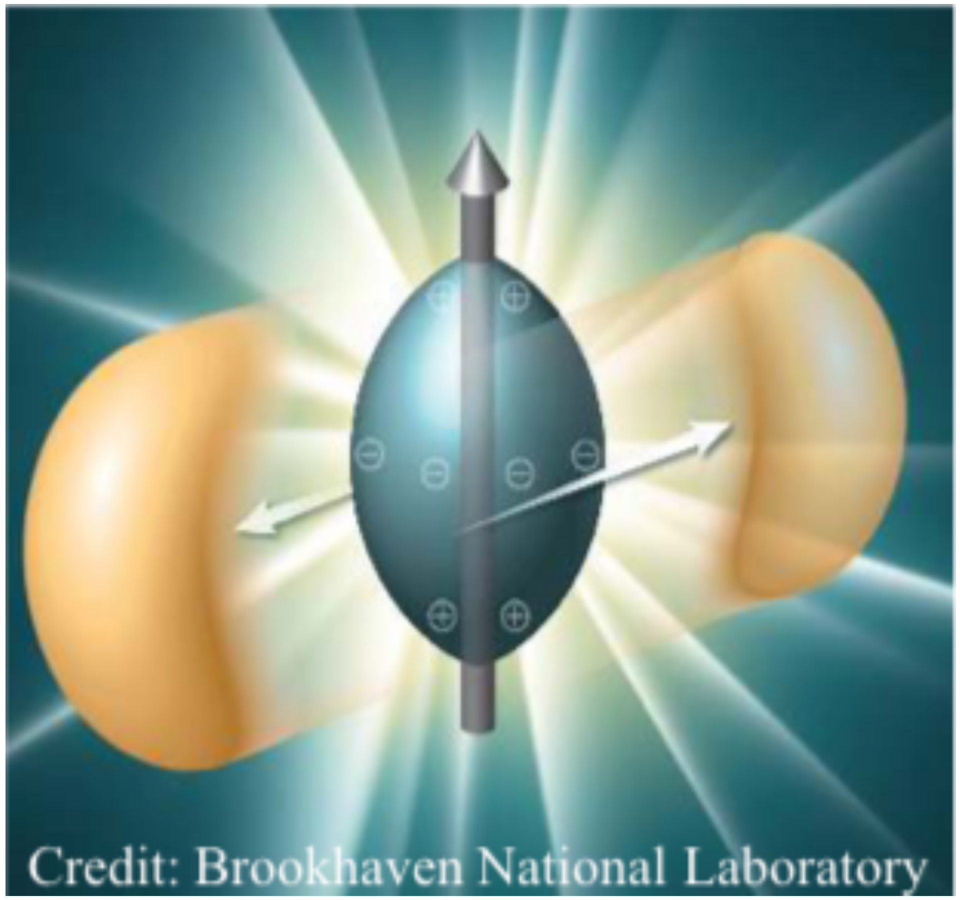
Strong magnetic fields may be produced in off-central heavy ion collisions

- Chiral magnetic effect
- Effects in EoS
- Anisotropies

heavy-ion collisions:

temporarily $B \lesssim 10^{19}\,\mathrm{G}$

Skokov, Illarionov, Toneev,
Int. J. Mod. Phys. A 24, 5925 (2009)



K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008). D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 80, 0304028 (2009). D. E. Kharzeev, Nucl. Phys. A 830, 543c (2009).

Neutron Stars

- **Strong magnetic fields are also present in magnetars:** C. Kouveliotou et al., Nature 393, 235 (1998).

magnetars:

at surface $B \lesssim 10^{15}\,\mathrm{G}$

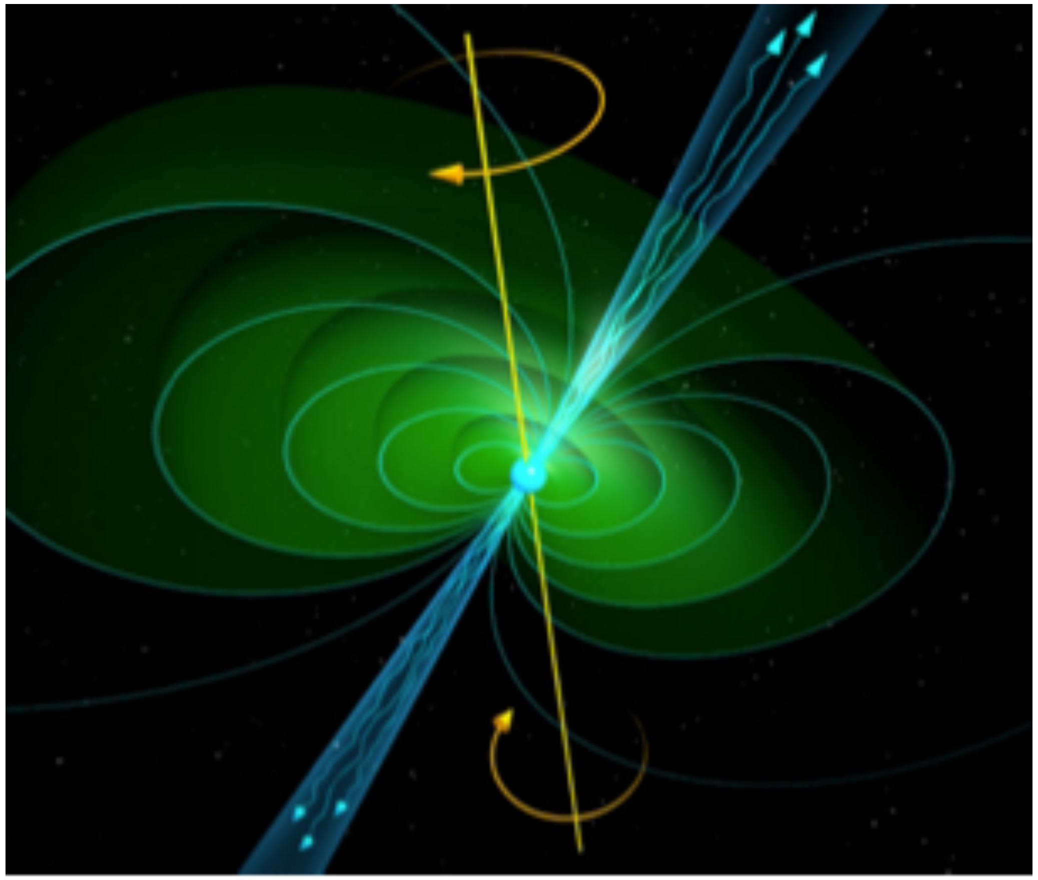
Duncan, Thompson, Astrophys.J. 392, L9 (1992)

larger in the interior,

$B \sim 10^{18-20}\,\mathrm{G}$?

Lai, Shapiro, Astrophys.J. 383, 745 (1991)

E. J. Ferrer *et al.*, PRC 82, 065802 (2010)



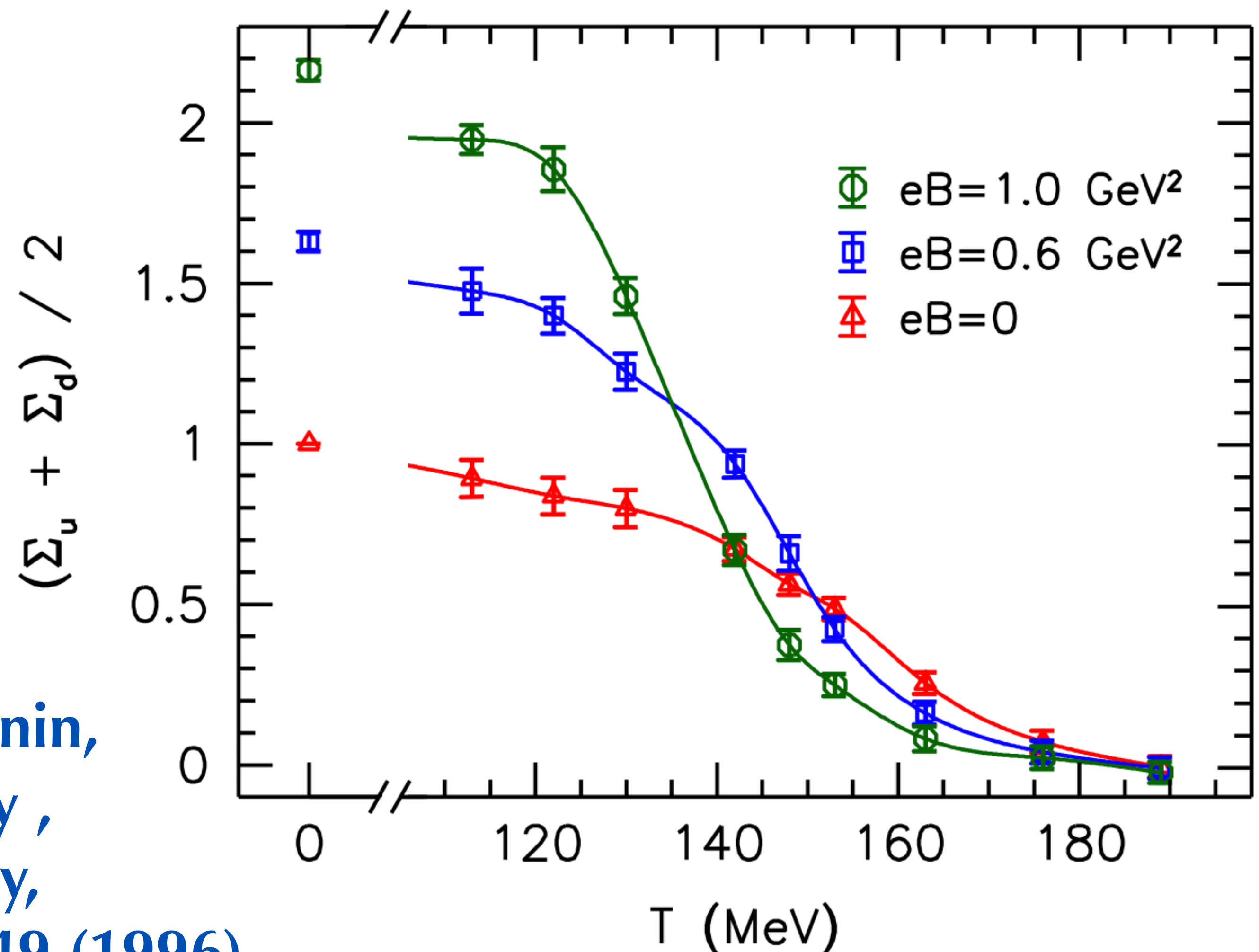
A. K. Harding, D. Lai, Rept. Prog. Phys. 69, 2631 (2006)

- **and might have played an important role in the physics of the early universe.** T. Vaschapati, Phys. Lett. B 265, 258 (1991).

D. Grasso and H.R. Rubinstein, Phys. Rep. 348, 163 (2001).

B Effects on QCD phase transitions?

$$\Lambda_{QCD}^2 \sim (200\,\mathrm{MeV})^2 \sim 2 \times 10^{18}\,\mathrm{G}$$

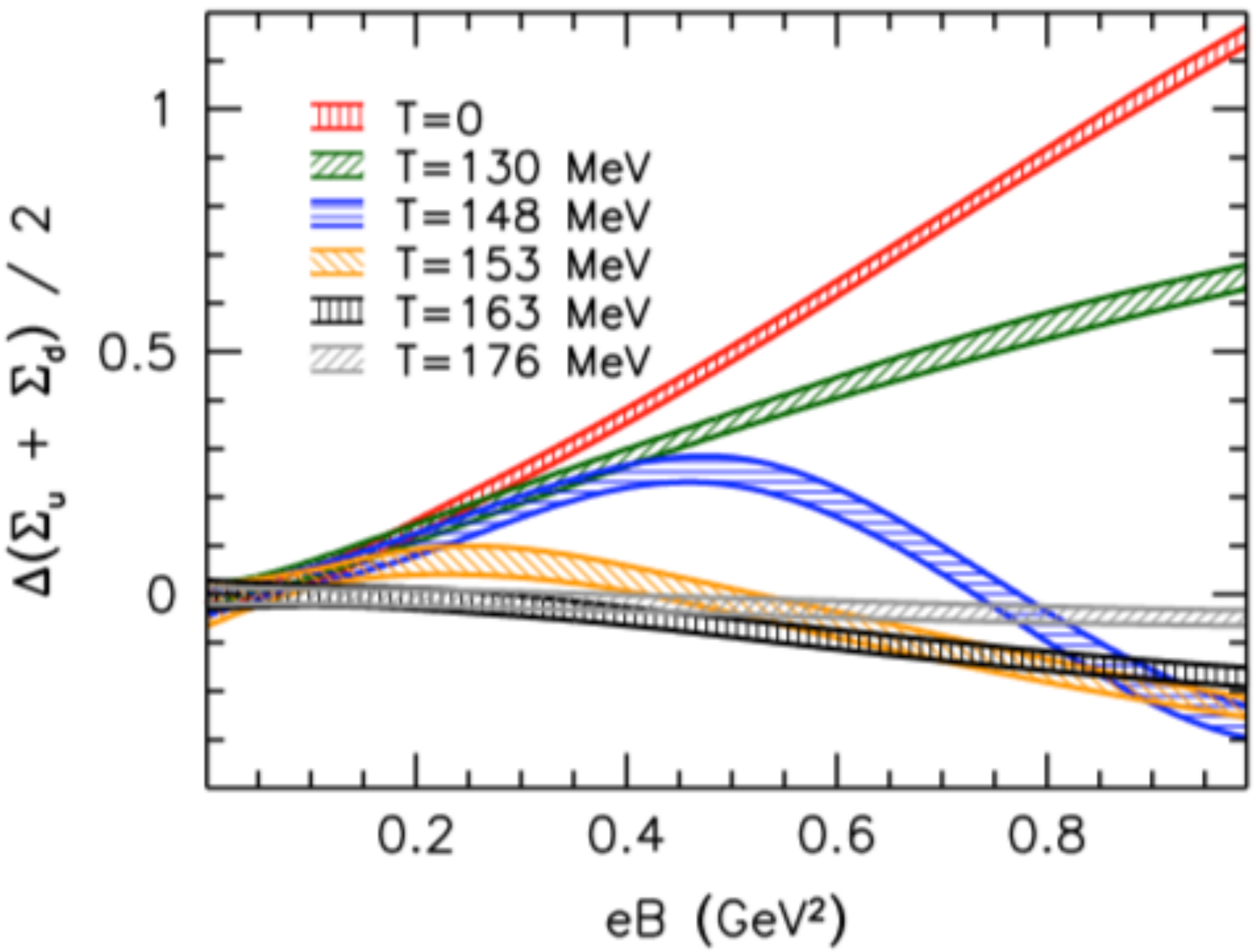


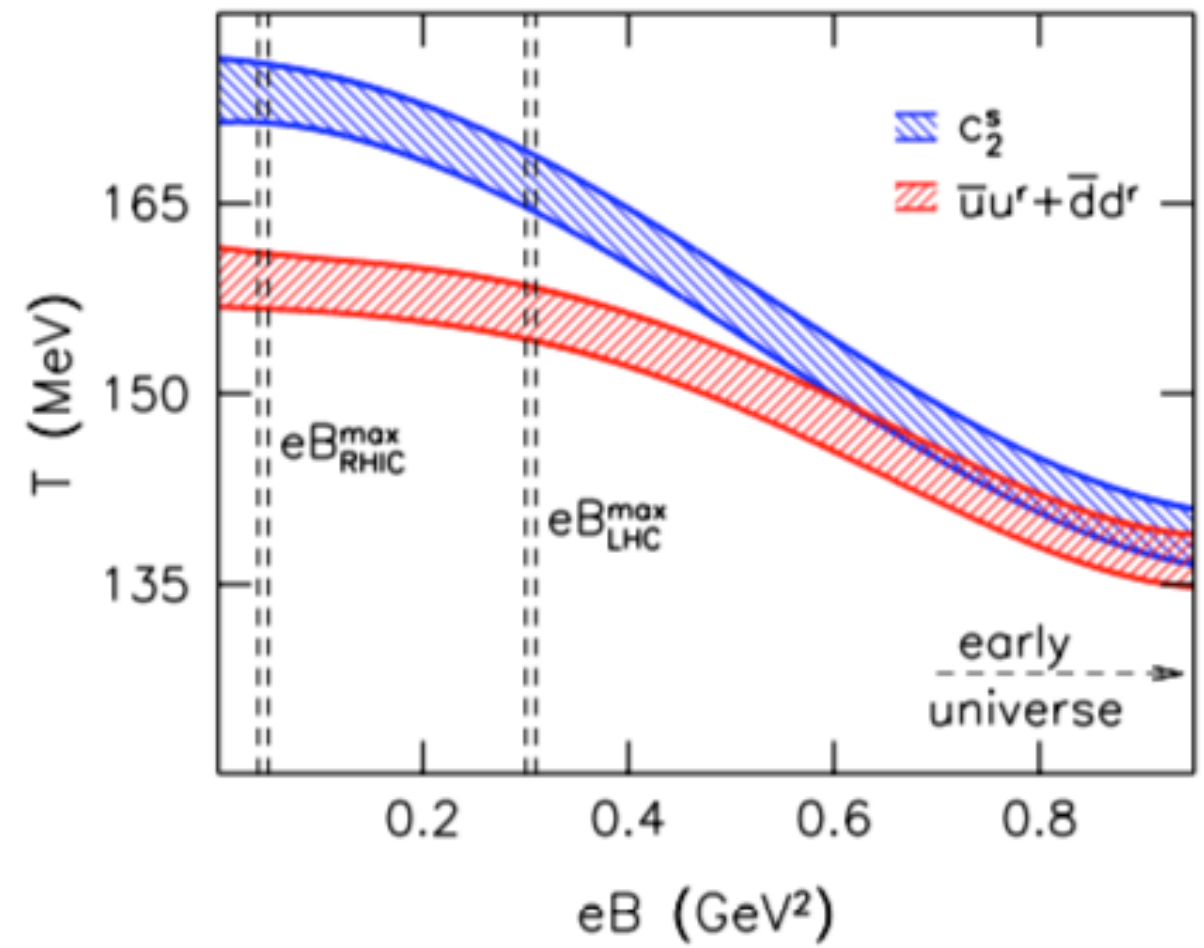
IMC: Bali, Bruckmann, Endrodi, Fodor, Katz et al. *JHEP* 02 (2012) 044 ***Phys.Rev.D*** **86 (2012) 071502**

MC: V.P. Gusynin, V.A. Miransky , I.A. Shovkovy, *Nucl. Phys. B* 462 249 (1996)

B Effects on QCD phase transitions?

MC and IMC

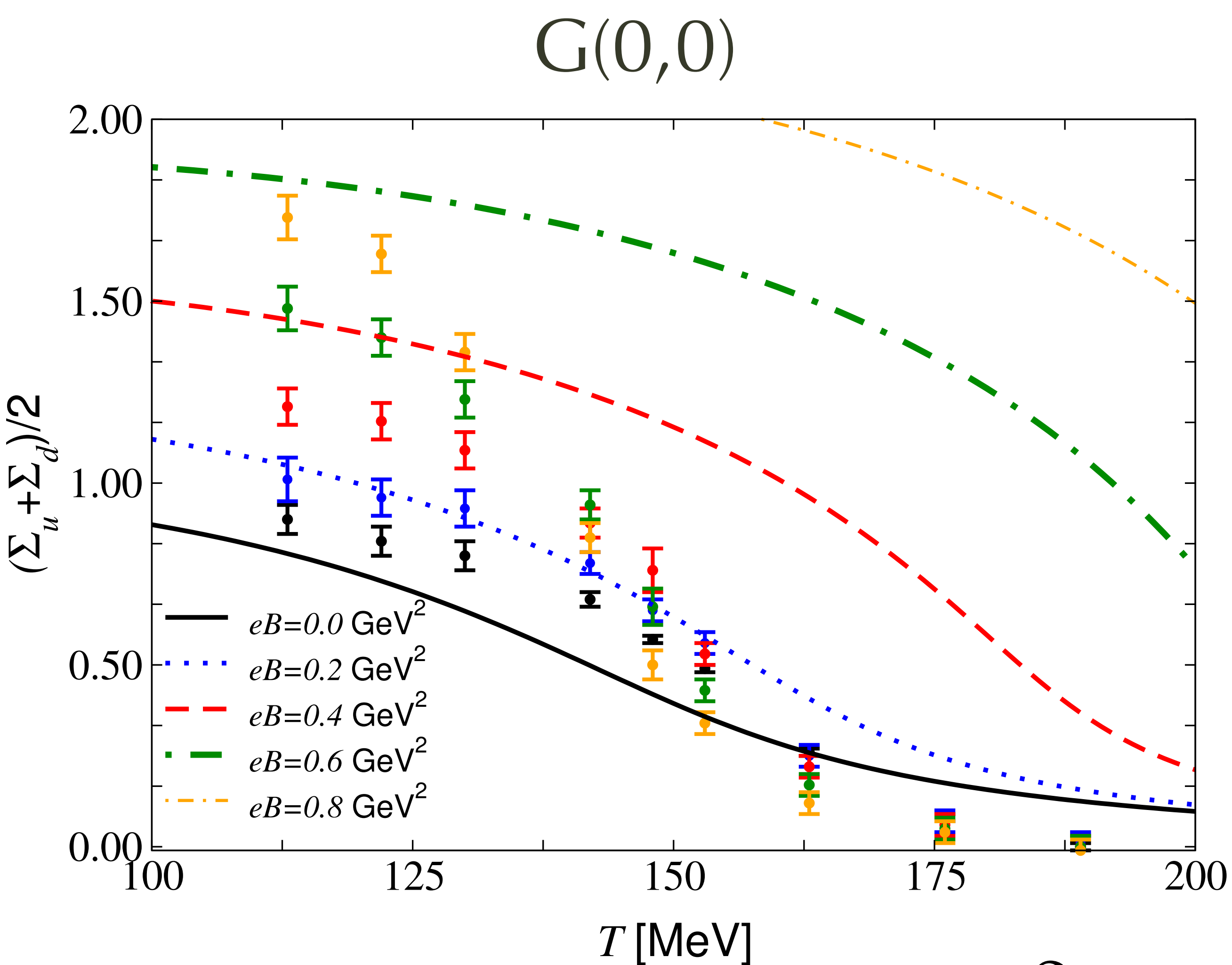


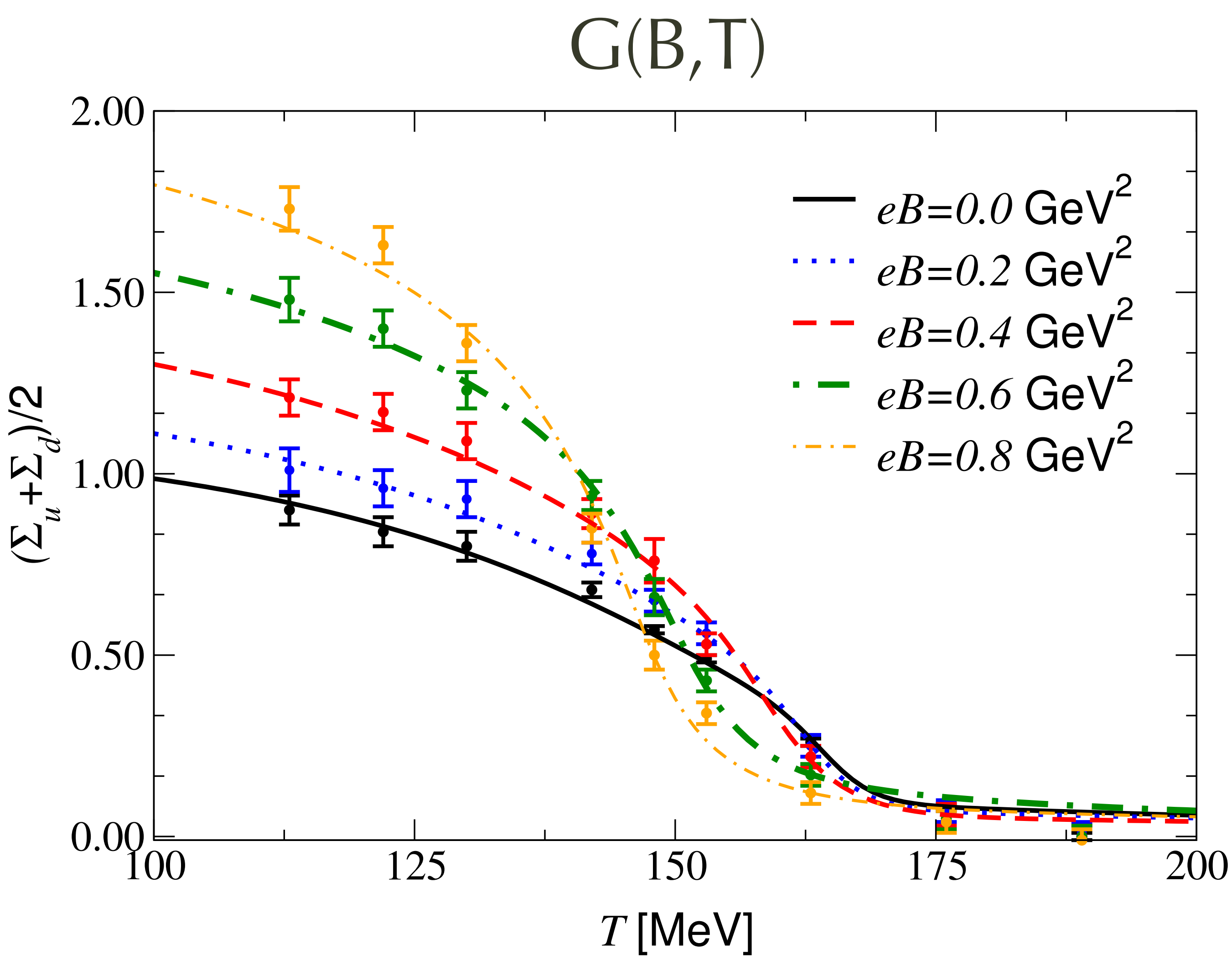


Failure of ALL effective models in providing inverse magnetic catalysis!

G. S. Bali et al., JHEP **1202**, 044 (2012)

SU(2) NJL + Thermo-Magnetic effects G(B,T)





$$\Sigma_f(B,T) = \frac{2m_f}{m_\pi^2 f_\pi^2}\left[\langle\bar{\psi}_f\psi_f\rangle - \langle\bar{\psi}_f\psi_f\rangle_0\right] + 1$$

RLSF, K.P. Gomes, M.B. Pinto, G. Krein, Phys. Rev. C 90, 025203 (2014).

RLSF, V.S. Timoteo, S.S. Avancini, M.B. Pinto and G. Krein Eur. Phys. J. A (2017) 53: 101

RLSF, W. Tavares, S.S. Avancini, V.S. Timoteo, G. Krein and M.B. Pinto, Eur. Phys. J. A (2021) 57: 278

B Effects on QCD phase transitions?

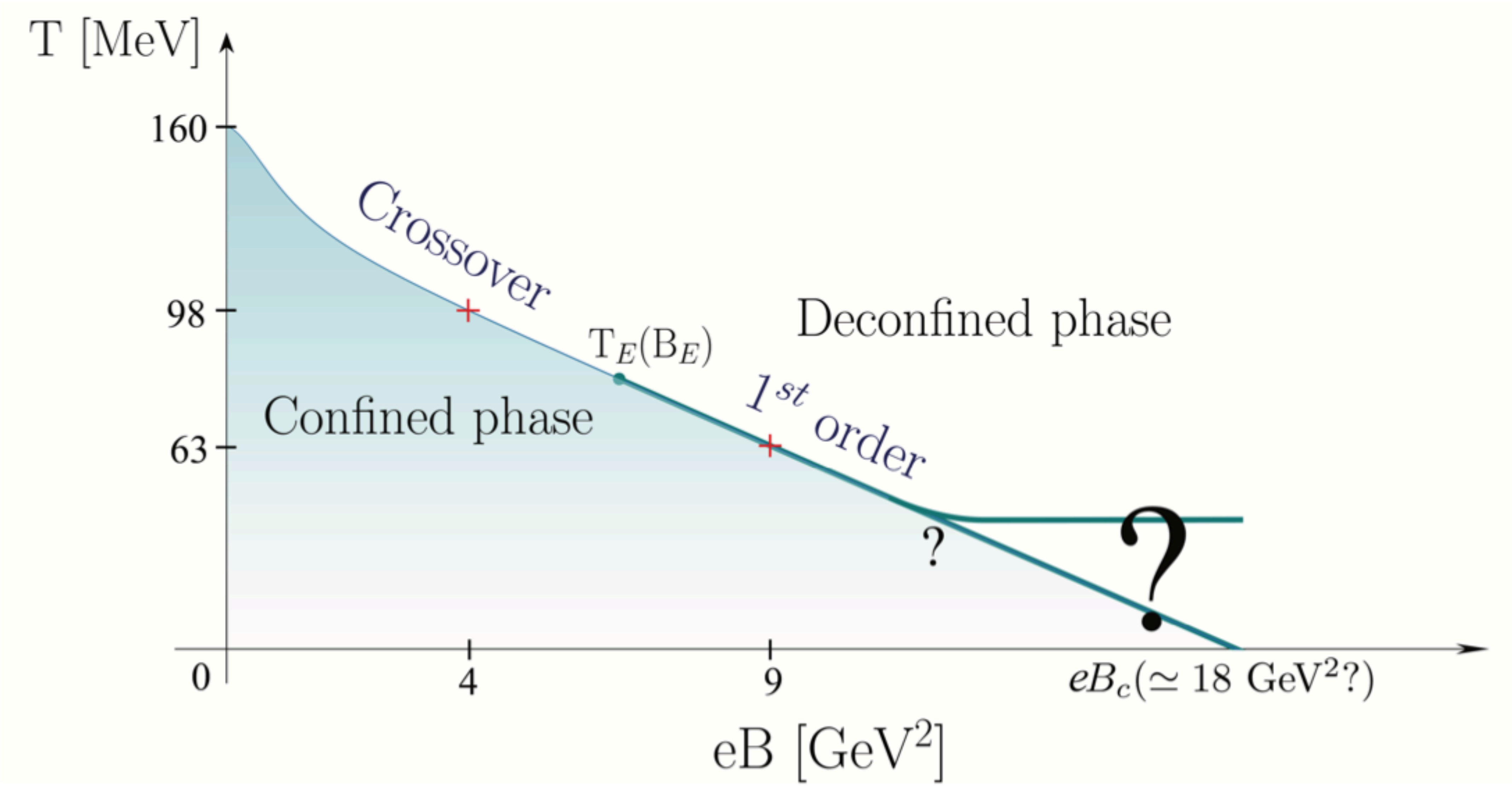
Inverse magnetic catalysis: how much do we know about?
A. Bandyopadhyay, R.L.S. Farias, *Eur. Phys. J. ST* 230 (2021) 3, 719-728,
B. e-Print: 2003.11054 [hep-ph]

Other possible explanations for IMC:

- Competition of B effects on sea and valence quarks, F. Bruckmann, G. Endrodi, T. G. Kovacs, *JHEP* 04 (2013) 112

- Inclusion of plasma screening effects that capture the physics of collective, long-wave modes, and thus describe a prime property of plasmas near transition lines, namely, long distance correlations. A.Ayala, L.A. Hernadez, M.Lowe, C, Villavicencio, EPJA, 57, 234 (2021)

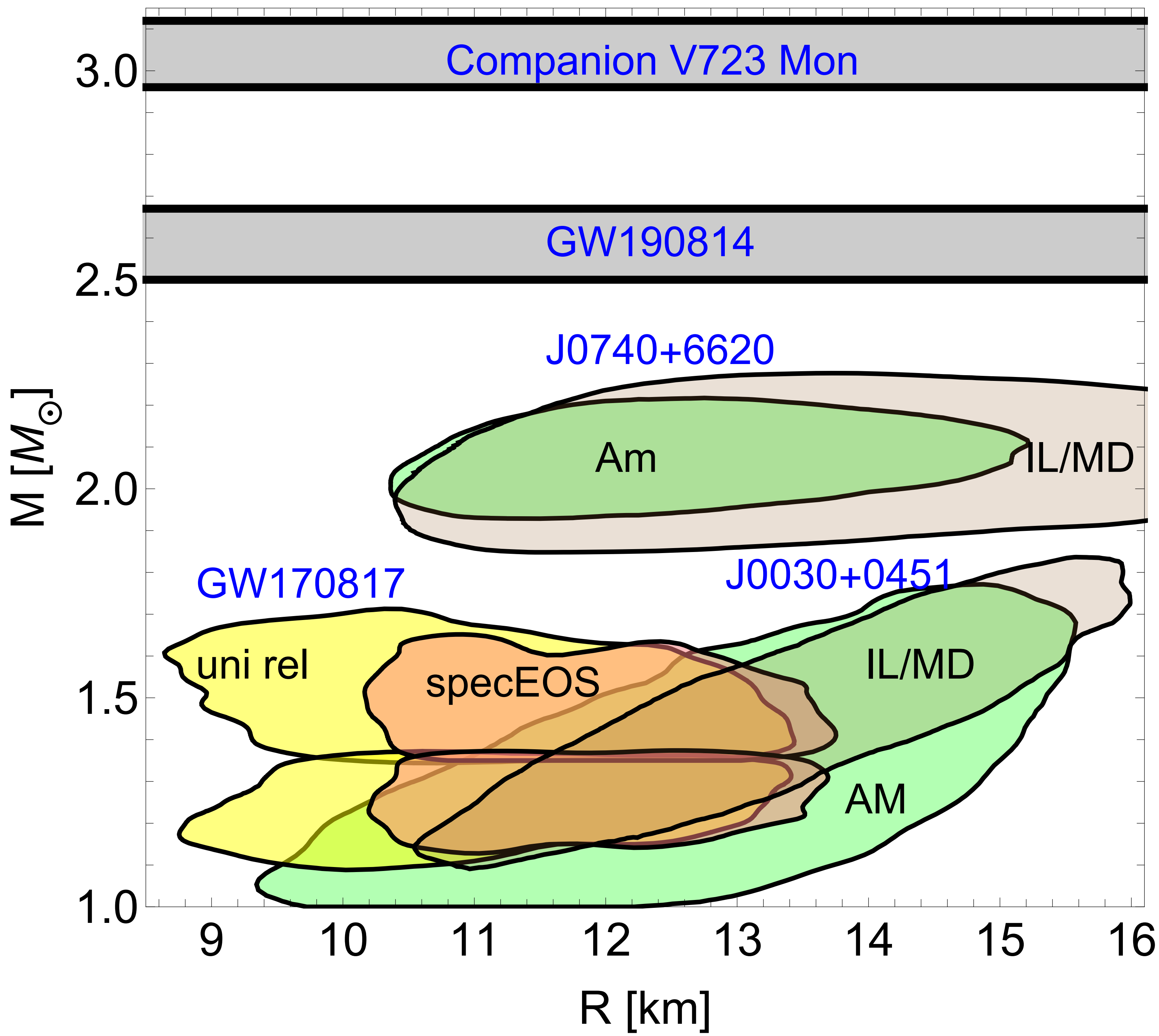
B Effects on QCD phase transitions?



M. D'Elia , L. Maio, F. Sanfilippo, A. Stanzione, *Phys. Rev. D* **105** , 034511 (2022).

Finite baryon density ?

Perspectives



Observational constraints on the neutron star mass-radius plane from LIGO/Virgo and NICER data.

H. Tan, T.Dore, V.Dexheimer, J.Noronha-Hostler, and N. Yunes, Phys. Rev. D **105**, 023018 (2022).

Conclusion

- We can use different lattice results to verify the predictions from effective models of QCD

Perspectives:

- Beyond mean field approximation
- Cross-talk between models
- Confinement effects

Thank you for your attention!