New modular flavor phenomenology from $2D_3$ symmetries

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Contents

The Standard Model

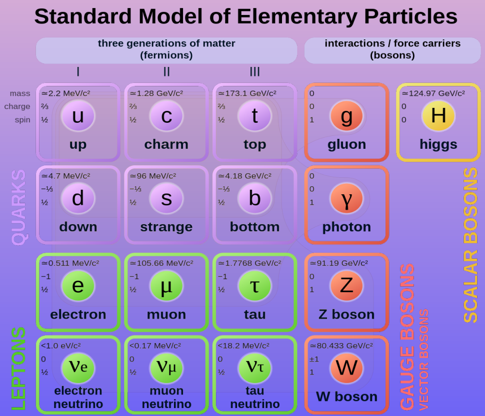

Figure: The Standard Model of Elementary Particles (SM).

The CKM matrix

$$V_{\text{CKM}} =$$

$$\begin{pmatrix} \cos\theta_{12}\cos\theta_{13} & \sin\theta_{12}\cos\theta_{13} & \sin\theta_{13}e^{-i\delta} \\ -\sin\theta_{12}\cos\theta_{23} - \sin\theta_{13}\sin\theta_{23}\cos\theta_{12}e^{i\delta} & \cos\theta_{12}\cos\theta_{23} - \sin\theta_{12}\sin\theta_{13}\sin\theta_{23}e^{i\delta} & \sin\theta_{23}\cos\theta_{13} \\ \sin\theta_{12}\sin\theta_{23} - \sin\theta_{13}\cos\theta_{12}\cos\theta_{23}e^{i\delta} & -\sin\theta_{23}\cos\theta_{12} - \sin\theta_{12}\sin\theta_{13}\cos\theta_{23}e^{i\delta} & \cos\theta_{13}\cos\theta_{23} \end{pmatrix}.$$

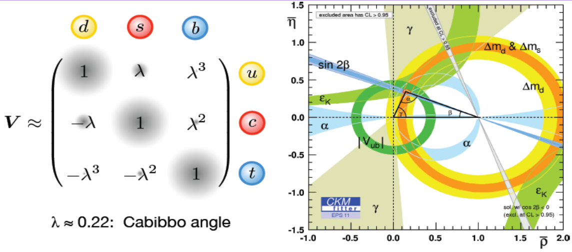

Figure: Parametrization of the CKM matrix.

What is the flavor puzzle?

1. The origin of the 3 fermion generations.
2. Hierchical pattern of the charged lepton masses $m_e << m_\mu << m_\tau$. $m_e/m_\mu \approx 1/200$ and $m_\mu/m_\tau \approx 1/17$.
3. Lightness of neutrinos $m_{\nu i} \lesssim 0.5\text{eV}$, $m_l \gtrsim 0.511\text{MeV}$ and $m_q \gtrsim 2\text{MeV}$.
4. Masses and mixings of fermions from V_{CKM} and U_{PMNS}.

Vector-Valued Modular Forms

The Modular Group

$$\Gamma := \mathrm{SL}(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \,\middle|\, ad - bc = 1\,, \quad a,b,c,d \in \mathbb{Z} \right\}\,. \tag{1}$$

Γ has two generators S and T obeying

$$S^4 = (ST)^3 = 1\,, \quad S^2T = TS^2\,, \tag{2}$$

which can be representated as

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\,, \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\,. \tag{3}$$

VVMF

The group acts on $\mathcal{H} = \{\tau \in \mathbb{C} \mid \mathrm{i}\tau > 0\}$ as the transformations

$$\tau \to \gamma\tau := \frac{a\tau + b}{c\tau + d}, \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma\,. \tag{4}$$

We consider the holomorphic functions $Y(\tau) = (Y_1(\tau), \ldots, Y_d(\tau))^T$ in $\mathcal{H}$ transforming as [2112.14761]

$$Y(\gamma\tau) = (c\tau + d)^{k_Y} \rho_Y(\gamma) Y(\tau)\,, \tag{5}$$

where ρ_Y is a d-dimensional representation of $\gamma \in \Gamma$ with finite image.

VVMF

Under the irrep ρ, all VVMFs constitute a free module $\mathcal{M}(\rho)$ over the ring $\mathcal{M}(1) = \mathbb{C}[E_4, E_6]$. It is possible to generate a basis through the operators

$$D_k^n := D_{k+2(n-1)} \circ D_{k+2(n-2)} \circ \cdots \circ D_k\,, \tag{6}$$

acting on the VVMFs with minimal weight, where

$$D_k := \frac{1}{2\pi\mathrm{i}}\frac{d}{d\tau} - \frac{kE_2(\tau)}{12}\,, \quad k \in \mathbb{N}^+\,, \tag{7}$$

and $E_2(\tau)$ is the quasi-modular Eisenstein series

$$E_2(\tau) = 1 - 24\sum_{n=1}^{\infty}\sigma_1(n)q^n\,. \tag{8}$$

MLDE

A desirable basis of $\mathcal{M}(\rho)$ is $\{Y^{(k_0)}, D_{k_0}Y^{(k_0)}, \ldots, D_{k_0}^{d-1}Y^{(k_0)}\}$.
For a VVMF with weight $k_0 + 2d$, the ring $\mathcal{M}(1) = \mathbb{C}[E_4, E_6]$ is a linear combination of the previous basis such that

$$(D_{k_0}^{d} + M_4 D_{k_0}^{d-2} + \cdots + M_{2(d-1)} D_{k_0} + M_{2d}) Y^{(k_0)} = 0\,, \tag{9}$$

where $M_k \in \mathbb{C}[E_4, E_6]$ is is the scalar modular form of weight k.

In summary, $\mathcal{M}(\rho)$ only depends on the VVMFs under the irrep ρ with minimal weight.

Binary dihedral group

$2D_3$

The binary dihedral group $2D_3$ is the preimage of the dihedral group D_3 under the maping $\mathrm{SU}(2) \cong \mathrm{Spin}(3) \to \mathrm{SO}(3)$.
$2D_3$ has genetrators S y T such that

$$S^4 = (ST)^3 = S^2T^2 = 1, \quad S^2T = TS^2 \,. \tag{10}$$

$2D_3$ has 12 elements and its GAP ID is $[12, 1]$.
Its center is $Z_2^{S^2} := \langle 1, S^2 \rangle$.
$2D_3 \cong Z_3 \rtimes Z_4$.

Character table of $2D_3$

Classes	$1C_1$	$1C_2$	$3C_4$	$2C_3$	$3C_4$	$2C_6$
Representative	1	S^2	T	TS	TS^2	TS^3
$\mathbf{1}$	1	1	1	1	1	1
$\mathbf{1}'$	1	1	-1	1	-1	1
$\hat{\mathbf{1}}$	1	-1	$-i$	1	i	-1
$\hat{\mathbf{1}}'$	1	-1	i	1	$-i$	-1
$\mathbf{2}$	2	2	0	-1	0	-1
$\hat{\mathbf{2}}$	2	-2	0	-1	0	1

Table: The character table of the binary dihedral group $2D_3 \cong Z_3 \rtimes Z_4$.

VVMFs in $2D_3$ with minimal weight

The VVMFs with minimal weight are

$$Y_{1'}^{(6)}(\tau) = \eta^{12}(\tau)\,, \qquad Y_{\hat{1}}^{(9)}(\tau) = \eta^{18}(\tau)\,, \qquad Y_{\hat{1}'}^{(3)}(\tau) = \eta^{6}(\tau)\,,$$
$$Y_{2}^{(2)}(\tau) = \begin{pmatrix} \eta^4(\tau)(\frac{K(\tau)}{1728})^{-\frac{1}{6}}\ {}_2F_1(-\frac{1}{6},\frac{1}{6};\frac{1}{2};K(\tau)) \\ -8\sqrt{3}\eta^4(\tau)(\frac{K(\tau)}{1728})^{\frac{1}{3}}\ {}_2F_1(\frac{1}{3},\frac{2}{3};\frac{3}{2};K(\tau)) \end{pmatrix},$$
$$Y_{\hat{2}}^{(5)}(\tau) = \begin{pmatrix} 8\sqrt{3}\eta^{10}(\tau)(\frac{K(\tau)}{1728})^{\frac{1}{3}}\ {}_2F_1(\frac{1}{3},\frac{2}{3};\frac{3}{2};K(\tau)) \\ \eta^{10}(\tau)(\frac{K(\tau)}{1728})^{-\frac{1}{6}}\ {}_2F_1(-\frac{1}{6},\frac{1}{6};\frac{1}{2};K(\tau)) \end{pmatrix}, \qquad (11)$$

where ${}_2F_1$ is the generalized hypergeometric series and $K(\tau) = 1728/j(\tau)$ is the inverse of Klein-j function $j(\tau)$.

VVMFs in $2D_3$

The VVMFs for $2D_3$ are:

$$
\begin{aligned}
k=2: &\quad Y_{2}^{(2)}\,,\\
k=3: &\quad Y_{\hat{1}'}^{(3)}\,,\\
k=4: &\quad Y_{1}^{(4)}\equiv E_4\,,\quad Y_{2}^{(4)}\equiv -6D_2Y_{2}^{(2)}\,,\\
k=5: &\quad Y_{\hat{2}}^{(5)}\,,\\
k=6: &\quad Y_{1}^{(6)}\equiv E_6\,,\quad Y_{1'}^{(6)}\,,\quad Y_{2}^{(6)}\equiv E_4Y_{2}^{(2)}\,,\\
k=7: &\quad Y_{\hat{2}}^{(7)}\equiv 3D_5Y_{\hat{2}}^{(5)}\,,\quad Y_{\hat{1}'}^{(7)}\equiv E_4Y_{\hat{1}'}^{(3)}\,,\\
&\ldots\,.
\end{aligned}
\tag{12}
$$

SUSY with $2D_3$ invariance

Supersymmetry

We consider the global supersymmetry theory $\mathcal{N} = 1$.
The action is given as

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta} \mathcal{K}(\Psi_I, \bar{\Psi}_I, \tau, \bar{\tau}) + \int d^4x d^2\theta \mathcal{W}(\Psi_I, \tau).$$

The superpotential is

$$\mathcal{W}(\Phi_I, \tau) = \sum_n Y_{I_1 \ldots I_n}(\tau) \Phi_{I_1} \ldots \Phi_{I_n} \,. \tag{13}$$

SUSY $2D_3$ invariance

Every term in the superpotential satisfies

$$-k_{I_1} - k_{I_2} - \cdots - k_{I_n} + k_Y = -1 \,. \tag{14}$$

We assume

$$\Phi_I \to (c\tau + d)^{-k_I} \rho_I(\gamma) \Phi_I \,, \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \,, \tag{15}$$

where $k_I \in \mathbb{Q}$ is the modular weight of Φ_I and $\rho_I(\gamma)$ is a $2D_3$ irrep.
We choose

$$k_I \in \left\{ \frac{-16}{4}, \frac{-15}{4}, \ldots, \frac{15}{4}, \frac{16}{4} \right\} . \tag{16}$$

General fermion mass matrices

All the mass matrices have a structure

$$M_{\psi/\psi^c} = \begin{pmatrix} M_{DD} & \vdots & M_{D3} \\ \cdots & \cdot & \cdots \\ M_{3D} & \vdots & M_{33} \end{pmatrix}, \tag{17}$$

Phenomenology

Quark model with 9 parameters

We obtained a model whose predicted flavor observables are

$$\theta_{12}^q = 0.227392\,,\ \theta_{13}^q = 0.00349376\,,\ \theta_{23}^q = 0.0400987\,,$$
$$\delta_{CP}^q = 70.2738^\circ\,,\ m_u/m_c = 0.00193121\,,\ m_c/m_t = 0.00287386\,,$$
$$m_d/m_s = 0.0230934\,,\ m_s/m_b = 0.018243\,. \tag{18}$$

$$\chi^2_{\text{min}} \sim 19.9.$$

Only m_d/m_s is about the edge of 4σ.

The VEV of the modulus is close to $\omega := e^{\frac{2\pi i}{3}}$.

Quark model with 10 parameters

We obtained a model whose predicted flavor observables are

$$\theta^q_{12} = 0.227368\,,\ \theta^q_{13} = 0.00349295\,,\ \theta^q_{23} = 0.0401456\,,$$
$$\delta^q_{CP} = 69.2212^\circ\,,\ m_u/m_c = 0.00192771\,,\ m_c/m_t = 0.00282204\,,$$
$$m_d/m_s = 0.0505254\,,\ m_s/m_b = 0.0182414\,. \qquad (19)$$

$$\chi^2_{\text{min}} \sim 0.0002.$$

All predictions fall within 1σ.

The VEV of the modulus is close to i.

Dirac neutrino model with 8 parameters

We obtained a model whose predicted flavor observables are

$$\sin^2\theta^l_{12} = 0.302989\,,\ \sin^2\theta^l_{13} = 0.0220300\,,\ \sin^2\theta^l_{23} = 0.571993\,,$$
$$\delta^l_{CP} = 1.49060\pi\,, m_e/m_\mu = 0.00473701\,,\ m_\mu/m_\tau = 0.058568\,,$$
$$m_1 = 17.5723\text{ meV}\,,\ m_2 = 19.5675\text{ meV}\,,\ m_3 = 53.1012\text{ meV}$$
$$m_\beta = 19.6484\text{ meV}\,. \tag{20}$$

$$\chi^2_{\text{min}} \sim 10^{-6}.$$

All predictions fall within 1σ.

Majorana neutrino model for Type-I seesaw mechanism with 9 parameters

We obtained a model whose predicted flavor observables are

$$\begin{aligned}
&\sin^2\theta^l_{12} = 0.302991\,,\ \sin^2\theta^l_{13} = 0.02203\,,\ \sin^2\theta^l_{23} = 0.572022\,,\\
&\delta^l_{CP} = 1.27887\pi\,,\ \alpha_{21} = 1.27782\pi\,,\ \alpha_{31} = 0.418302\pi\,,\\
&m_e/m_\mu = 0.00473698\,,\ m_\mu/m_\tau = 0.0585681\,,\ \frac{\Delta m^2_{21}}{\Delta m^2_{31}} = 0.0295108\,,\\
&m_1 = 39.09\text{ meV}\,,\ m_2 = 40.02\text{ meV}\,,\ m_3 = 63.55\text{ meV}\,. \qquad (21)
\end{aligned}$$

$$\chi^2_{\min} \approx 10^{-6}.$$

All predictions fall within 3σ level.

Majorana neutrino model for Weinberg operator with 7 parameters

We obtained a model whose predicted flavor observables are

$$\begin{aligned}
&\sin^2\theta^l_{12} = 0.302725\,,\ \sin^2\theta^l_{13} = 0.0220538\,,\ \sin^2\theta^l_{23} = 0.656858\,,\\
&\delta^l_{CP} = 1.46771\pi\,,\ \alpha_{21} = 1.93205\pi\,,\ \alpha_{31} = 0.951594\pi\,,\\
&m_e/m_\mu = 0.00473702\,,\ m_\mu/m_\tau = 0.0585681\,,\ \frac{\Delta m^2_{21}}{\Delta m^2_{31}} = 0.0295217\,,\\
&m_1 = 30.15\ \text{meV}\,,\ m_2 = 31.35\ \text{meV}\,,\ m_3 = 58.47\ \text{meV}\,.
\end{aligned} \tag{22}$$

$$\chi^2_{\text{min}} = 17.14.$$

All predictions fall within 3σ level except $\sin^2\theta^l_{23}$.

An integrated model of quarks and leptons

For the model

$$d^c \sim u^c \sim , Q \sim E^c \sim L \sim \hat{2} \oplus \hat{1}' , H_d \sim H_u \sim \mathbf{1}' .$$
$$k_{d^c_D} = k_{u^c_D} = -k_{E^c_3} = 1/2 , \; k_{d^c_3} = k_{u^c_D} = k_{Q_D} = k_{Q_3} = 5/2,$$
$$k_{E^c_3} = k_{L_D} = 7/2, \; k_{L_3} = 3/2 , k_{H_d} = k_{H_u} = 0 .$$

The best-fit input parameters are

$$\begin{aligned}
&\langle\tau\rangle = -0.0613689 + 2.68637\mathrm{i} , \; \beta^d/\alpha^d = 20.3565 , \\
&\delta^d/\alpha^d = 309.699 , \; \gamma^d/\alpha^d = -1040.08 , \; \alpha^u_1/\gamma^u = -6422.49 , \\
&\alpha^u_2/\gamma^u = -6413.76 , \; \beta^u/\gamma^u = 4383.68 , \; \delta^l/\alpha^l = -0.0808854 , \\
&\gamma^l/\alpha^l = 17.0445 , \; \alpha^\nu_1/\beta^\nu = 2.3088 , \; \alpha^\nu_2/\beta^\nu = -11784.2 , \\
&\alpha^d v_d = 0.891348 \text{ MeV} , \; \gamma^u v_u = 6.44174 \text{ MeV} , \\
&\alpha^l v_d = 76.207 \text{ MeV} , \; \frac{\beta^\nu v_u^2}{\Lambda} = 0.010066 \text{ eV} .
\end{aligned} \tag{22}$$

An integrated model of quarks and leptons

The predicted flavor observables are

$$\begin{aligned}
&\sin^2\theta^l_{12} = 0.31818\,,\ \sin^2\theta^l_{13} = 0.021746\,,\ \sin^2\theta^l_{23} = 0.527212\,,\\
&\delta^l_{CP} = 1.59044\pi\,, \alpha_{21} = 0.901365\pi\,,\ \alpha_{31} = 0.526527\pi\,,\\
&m_e/m_\mu = 0.00473731\,,\ m_\mu/m_\tau = 0.058568\,,\ \frac{\Delta m^2_{21}}{\Delta m^2_{31}} = 0.030349\,,\\
&m_1 = 5.23272\text{ meV}\,,\quad m_2 = 10.0738\text{ meV}\,,\ m_3 = 49.6888\text{ meV}\,,\\
&\theta^q_{12} = 0.227464\,,\ \theta^q_{13} = 0.00339533\,,\ \theta^q_{23} = 0.0403661\,,\\
&\delta^q_{CP} = 69.1464^\circ\,,\ m_u/m_c = 0.00192463\,,\ m_c/m_t = 0.00282265\,,\\
&m_d/m_s = 0.0505174\,,\ m_s/m_b = 0.0182406\,. \qquad (24)
\end{aligned}$$

$$\chi^2_{\text{min}} = 8.4.$$

All predictions fall within 3σ.

Conclusions

Conclusion

We have succesful model best-fit models with $2D_3$ symmetry!!

Conclusions

- We investigated the (yet) unexplored phenomenology of $2D_3$.
- $2D_3$ as a flavor group only allows structures $2+1$ with VVMFs.
- After a exhaustive numerical analysis, benchmark models with new phenomenology were provided.
- Several fermion scenarios were taken into account.
- For the first time, Higgs fields in non-trivial representations in modular flavor building.
- We allowed fractional modular weights for the fields.
- Our integrated models can be naturally embedded in a Grand unified theory.
- This phenomenology could arise from some $\mathbb{T}^2/\mathbb{Z}_4$ heterotic orbifold.

Thank you for your attention.