Renormalization of the U(1) gauge theory for tensor dark matter: preliminary results

Armando De La Cruz Rangel Pantoja,
Dr. Mauro Napsuciale Mendivil.

Departamento de Física. Universidad de Guanajuato.

adlc.rangelpantoja@ugto.mx
mauro@fisica.ugto.mx

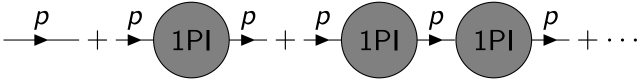
June 12, 2023

Outline

Outline

Introduction: Renormalization

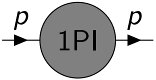
Renormalization is the process by which a theory is restated in terms of the physical fields (ψ, A^μ) and parameters (m, q), which are measured in the experiment. As a consequence of this restatement, the apparent divergences that arise in the amplitudes of some diagrams containing loops are avoided in some theories. For example, for the electron mass in QED:

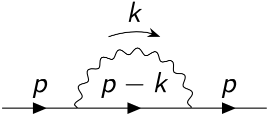


(1)

$$= \frac{i}{\not{p} - (m_0 + \Sigma(\not{p})) + i\varepsilon}, \qquad m_{phys} \equiv m_0 + \Sigma(\not{p}), \qquad (2)$$

where

$-i\Sigma(p) =$ 

, p. ej. : $-i\Sigma^{(2)}(p) =$ 

(3)

Outline

Introduction: Tensor dark matter

Tensor dark matter (TDM) is a proposal to describe DM as a spin-one field in the $(1,0) \oplus (0,1)$ representation of the HLG. In a hidden DM scheme, it is reasonable to hypothesize that interactions with the SM are of the type singletDM-singletSM. Under this hypothesis, the lowest dimensional operators for the effective Lagrangian of interaction are

$$\mathcal{L}_{int} = \bar{\Psi}(g_s \mathbb{1} + i g_p \chi)\Psi \phi^\dagger \phi + g_t \bar{\Psi} M_{\mu\nu} \Psi B^{\mu\nu}. \tag{4}$$

It has been found that this proposal is compatible with the experimental results if $m_{TDM} \approx m_{Higgs}/2$ and $g_s \approx 1.00 \times 10^{-3}$ [1–5]. In the equation (4) the operators are of dimension 4, which opens the possibility that there might be a fundamental theory of TDM behind. The study of its renormalization is important to find out if there might be a fundamental theory with high predictive power or if it is, on the contrary, only an effective theory.

Outline

Free dynamics

In [6] it is developed a formalism from first principles with which it is possible to derive wave equations of the wave functions for free massive particles of spin j in a given representation of the HLG. This formalism arises, essentially, from the identification of parity as a good quantum number for free particles. Its application to the $(1/2,0)\oplus(0,1/2)$ representation produces the Dirac equation; its application to $(1/2,1/2)$ produces the Proca equation; and applying it to the $(1,0)\oplus(0,1)$ representation produces the following equation

$$\left(\Sigma^{\mu\nu}\partial_\mu\partial_\nu + m^2\mathbb{1}\right)\Psi(x) = 0, \qquad \Sigma_{\mu\nu} = \frac{1}{2}\left(\eta_{\mu\nu}\mathbb{1} + S_{\mu\nu}\right). \tag{5}$$

$$\begin{aligned} S^{00} &= \Pi = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad S^{0i} = \begin{pmatrix} 0 & J^i \\ -J^i & 0 \end{pmatrix}, \\ S^{ij} &= \begin{pmatrix} 0 & -\delta^{ij} + \{J^i, J^j\} \\ -\delta^{ij} + \{J^i, J^j\} & 0 \end{pmatrix}, \end{aligned} \tag{6}$$

where J^i are the $SU(2)$ generators for $j = 1$.

Outline

The Lagrangian of the field that obeys this equation is

$$\mathcal{L}=\partial_\mu\bar{\Psi}\Sigma^{\mu\nu}\partial_\nu\Psi-m^2\bar{\Psi}\Psi, \tag{7}$$

where $\bar{\Psi}=\Psi^\dagger\Pi$. The classical aspects and the canonical quantization of this field were studied in [7]. In the massless case this Lagrangian is non-chiral, so it cannot have chiral gauge interactions. This implies that this field cannot have electro-weak interactions, which motivates its study as a candidate for DM.

Outline

Renormalization Massive case

The Lagrangian (7) minimally coupled to the $U(1)$ gauge field B^μ in terms of the bare quantities is

$$\mathcal{L} = D_b^{\dagger\mu}\bar{\Psi}_b\Sigma_{\mu\nu}D_b^\nu\Psi_b - m_b^2\bar{\Psi}_b\Psi_b - \frac{1}{4}B_b^{\mu\nu}B_{b\mu\nu} - \frac{1}{2\xi_b}(\partial_\mu B_b^\mu)^2, \tag{8}$$

written in terms of physical quantities and counterterms we get

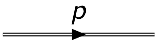
$$\begin{aligned}\mathcal{L} =& \partial^\mu\bar{\Psi}\Sigma_{\mu\nu}\partial^\nu\Psi - m^2\bar{\Psi}\Psi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2\xi}\left(\partial^\mu B_\mu\right)^2 \\ &+ ig\left[\partial^\mu\bar{\Psi}\Sigma_{\mu\nu}\Psi - \bar{\Psi}\Sigma_{\nu\mu}\partial^\mu\Psi\right]B^\nu + g^2\bar{\Psi}\Sigma_{\mu\nu}\Psi B^\mu B^\nu \\ &+\delta_{Z_2}\left(\partial^\mu\bar{\Psi}\Sigma_{\mu\nu}\partial^\nu\Psi - m^2\bar{\Psi}\Psi\right) - \delta_m\bar{\Psi}\Psi - \frac{1}{4}\delta_{Z_3}B^{\mu\nu}B_{\mu\nu} \\ &+ig\delta_g\left[\partial^\mu\bar{\Psi}\Sigma_{\mu\nu}\Psi - \bar{\Psi}\Sigma_{\nu\mu}\partial^\mu\Psi\right]B^\nu + g^2\delta_3\bar{\Psi}\Sigma_{\mu\nu}\Psi B^\mu B^\nu,\end{aligned} \tag{9}$$

where $\Psi = Z_2^{-\frac{1}{2}}\Psi_b$, $B^\mu = Z_3^{-\frac{1}{2}}B_b^\mu$ and the constants are

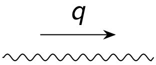
$$\delta_{Z_2} = Z_2 - 1, \qquad \delta_{Z_3} = Z_3 - 1, \qquad \delta_m = Z_2(m_b^2 - m^2), \tag{10}$$

$$\delta_g = \frac{g_b}{g}Z_2 Z_3^{\frac{1}{2}} - 1, \qquad \delta_3 = \frac{g_b^2}{g^2}Z_2 Z_3 - 1, \qquad \xi = \frac{\xi_b}{Z_3}. \tag{11}$$

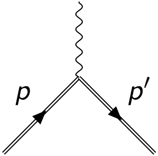
Feynman rules for physical terms are



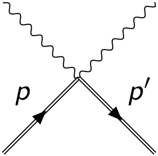
$$=\frac{i}{\Sigma(p)-m^2+i\epsilon},$$



$$=\frac{-i[g^{\mu\nu}-(1-\xi)q^\mu q^\nu/q}{q^2+i\epsilon}$$

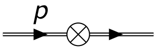


$$=-ig\Sigma_{\mu\nu}(p'+p)^\nu,$$

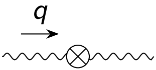


$$=2ig^2\Sigma_{\mu\nu},$$

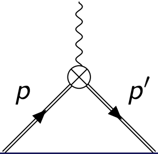
where $\Sigma(p)=\Sigma_{\mu\nu}p^\mu p^\nu$, and the rules for counterterms are



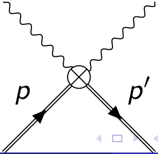
$$=i\delta_{Z_2}(\Sigma(p)-m^2)-i\delta_m,$$



$$=-i\delta_{Z_3}(q^2g^{\mu\nu}-q^\mu q$$



$$=-ig\delta_g\Sigma_{\mu\nu}(p'+p)^\nu,$$

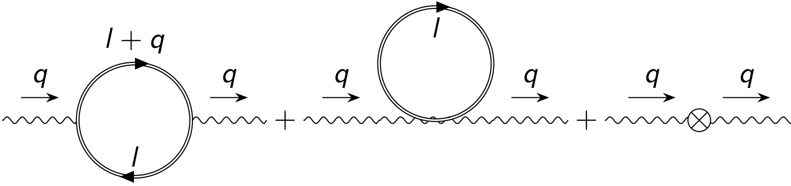


$$=2ig^2\delta_3\Sigma_{\mu\nu}.$$

Renormalization Massive case

With this, the autoenergy of the field B^μ is

$-i\Pi_{\mu\nu}(q) =$



$$= -\, i\Pi^*_{\mu\nu}(q) - i\delta_{Z_3}(q^2 g_{\mu\nu} - q_\mu q_\nu),$$

where

$$-\, i\Pi^*_{\mu\nu}(q) = \int \frac{d^4 l}{(2\pi)^4} \mathrm{Tr}\left\{ [-ig\Sigma_{\mu\alpha}(2l+q)^\alpha] \frac{i}{\Sigma(l+q) - m^2 + i\epsilon} \right.$$
$$\left. \times\, [-ig\Sigma_{\nu\beta}(2l+q)^\beta] \frac{i}{\Sigma(l) - m^2 + i\epsilon} + [2ig^2\Sigma_{\mu\nu}] \frac{i}{\Sigma(l) - m^2 + i\epsilon} \right\}.$$

The result has the structure $-i\Pi_{\mu\nu}(q) = -i\Pi(q^2)(q^2 g_{\mu\nu} - q_\mu q_\nu)$.

The result has the structure $-i\Pi_{\mu\nu}(q) = -i\Pi(q^2)(q^2 g_{\mu\nu} - q_\mu q_\nu)$, whose divergent part is

$$\text{Div}\left[-i\Pi(q^2)\right] = -\, i\left[\frac{g^2\left(78m^4 - 12m^2 q^2 + q^4\right)}{384\pi^2 m^4 \epsilon} + \delta_{Z_3}\right], \qquad (12)$$

which cannot be canceled by the counterterm δ_{Z_3}. The origin of this can be traced back to the propagator of the field Ψ, which can be rewritten as

$$\frac{i}{\Sigma(p) - m^2 + i\epsilon} = \frac{\mathbb{P}_+(p) - \frac{p^2 - m^2}{m^2}\mathbb{P}_-(p)}{p^2 - m^2 + i\epsilon}, \qquad (13)$$

where

$$\mathbb{P}_\pi = \frac{1}{2}\left(\mathbb{1} + \pi\frac{S(p)}{p^2}\right) \qquad (14)$$

are parity projectors, reveals an UV-divergent term off-shell.

In [6] it is shown that the propagator of a massive vector field V^μ has the same structure as (13)

$$iS_\Psi = \frac{\mathbb{P}_+(p) - \frac{p^2-m^2}{m^2}\mathbb{P}_-(p)}{p^2 - m^2 + i\epsilon}, \qquad iS_V = \frac{-\mathbb{P}_-(p) + \frac{p^2-m^2}{m^2}\mathbb{P}_+(p)}{p^2 - m^2 + i\epsilon}, \qquad (15)$$

where the parity projectors $\mathbb{P}_\pm(p)$ of the vector field V^μ are defined in an analogous manner to the ones of Ψ.

In the SM the solution to the renormalizability of the massive vectors bosons $W^\pm$ y Z comes from considering them initially as massless vectors and then giving them mass through the Higgs mechanism. This leads us to look for an analogous solution for the renormalizability of the Ψ interactions, so now we turn to the study of the massless case and eventually look for some mechanism to give it mass.

Outline

Renormalization Massless case

The massless Lagrangian is $\mathcal{L} = \partial_\mu \bar{\Psi} \Sigma^{\mu\nu} \partial_\nu \Psi$. The kintetic operator is not invertible beacause it is esentially an projector $\Sigma(p) = p^2 \mathbb{P}_+(p)$. On the other hand, it has a gauge symmetry

$$\Psi \to \Psi' = \Psi + R_{\mu\nu} \partial^\mu \partial^\nu \Phi, \qquad R_{\mu\nu} = \frac{1}{2} \left(\eta_{\mu\nu} \mathbb{1} - S_{\mu\nu} \right). \tag{16}$$

Adding a gauge-fixing term, the Lagrangian is

$$\begin{aligned} \mathcal{L} =& \partial^\mu \bar{\Psi} \Sigma_{\mu\nu} \partial^\nu \Psi + \frac{1}{\xi^\Psi} \partial^\mu \bar{\Psi} R_{\mu\nu} \partial^\nu \Psi \\ =& \partial^\mu \bar{\Psi} \left(\Sigma_{\mu\nu} + \frac{1}{\xi^\Psi} \right) \partial^\nu \Psi. \end{aligned} \tag{17}$$

Minimally coupling this Lagrangian with the $U(1)$ gauge field B^μ in terms of bare quantities is

$$\mathcal{L} = D_b^{\dagger\mu} \bar{\Psi}_b \left(\Sigma_{\mu\nu} + \frac{1}{x_b} R_{\mu\nu} \right) D_b^\nu \Psi_b - \frac{1}{4} B_b^{\mu\nu} B_{b\mu\nu} - \frac{1}{2 y_b} (\partial_\mu B_b^\mu)^2, \tag{18}$$

where $x_b \equiv \xi_b^\Psi$, $y_b \equiv \xi_b^B$.

Written in terms of the physical quantities

$$
\begin{aligned}
\mathcal{L} =&\partial^{\mu}\bar{\Psi}\left(\Sigma_{\mu\nu}+\frac{1}{x}R_{\mu\nu}\right)\partial^{\nu}\Psi-\frac{1}{4}B^{\mu\nu}B_{\mu\nu}-\frac{1}{2y}\left(\partial^{\mu}B_{\mu}\right)^{2}\\
&+ig\left[\partial^{\mu}\bar{\Psi}\left(\Sigma_{\mu\nu}+\frac{1}{x}R_{\mu\nu}\right)\Psi-\bar{\Psi}\left(\Sigma_{\mu\nu}+\frac{1}{x}R_{\mu\nu}\right)\partial^{\mu}\Psi\right]B^{\nu}\\
&+g^{2}\bar{\Psi}\left(\Sigma_{\mu\nu}+\frac{1}{x}R_{\mu\nu}\right)\Psi B^{\mu}B^{\nu}\\
&+\delta_{Z_2}\partial^{\mu}\bar{\Psi}\left(\Sigma_{\mu\nu}+\frac{1}{x}R_{\mu\nu}\right)\partial^{\nu}\Psi-\frac{1}{4}\delta_{Z_3}B^{\mu\nu}B_{\mu\nu}\\
&+\delta_{g}\left[\partial^{\mu}\bar{\Psi}\left(\Sigma_{\mu\nu}+\frac{1}{x}R_{\mu\nu}\right)\Psi-\bar{\Psi}\left(\Sigma_{\mu\nu}+\frac{1}{x}R_{\mu\nu}\right)\partial^{\mu}\Psi\right]B^{\nu}\\
&+g^{2}\delta_{3}\bar{\Psi}\left(\Sigma_{\mu\nu}+\frac{1}{x}R_{\mu\nu}\right)\Psi B^{\mu}B^{\nu},
\end{aligned}
\tag{19}
$$

where

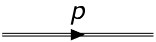
$$\Psi = Z_2^{-\frac{1}{2}} \Psi_b, \qquad B^\mu = Z_3^{-\frac{1}{2}} B_b^\mu, \tag{20}$$

and the constants are

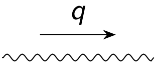
$$\delta_{Z_2} = Z_2 - 1, \qquad \delta_{Z_3} = Z_3 - 1, \qquad \delta_g = \frac{g_b}{g} Z_2 Z_3^{\frac{1}{2}} - 1, \tag{21}$$

$$\delta_3 = \frac{g_b^2}{g^2} Z_2 Z_3 - 1, \qquad y = \frac{y_b}{Z_3}, \qquad x = x_b. \tag{22}$$

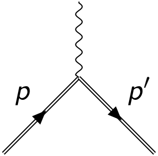
Feynman rules for physical terms are



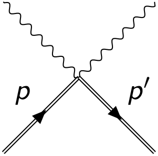
$$=i\frac{\mathbb{1}+(x-1)\mathbb{P}_-(p)}{p^2+i\epsilon},$$



$$=\frac{-i[g^{\mu\nu}-(1-y)q^\mu q^\nu/q^2]}{q^2+i\epsilon},$$



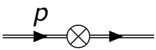
$$=-ig\left(\Sigma_{\mu\nu}+\frac{1}{x}R_{\mu\nu}\right)(p'+p)^\nu,$$



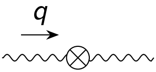
$$=2ig^2\left(\Sigma_{\mu\nu}+\frac{1}{x}R_{\mu\nu}\right).$$

Renormalización Massless case

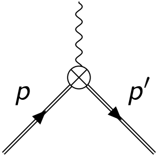
And the rules for counterterms are



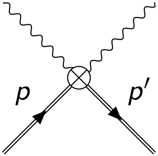
$$= i\delta_{Z_2}\left(\Sigma(p) + \frac{1}{x}R(p)\right) = i\delta_{Z_2}\left(\frac{x+1}{2x}p^2\mathbb{1} + \frac{x-1}{2x}S(p)\right),$$



$$= -\, i\delta_{Z_3}(q^2 g^{\mu\nu} - q^\mu q^\nu),$$



$$= -\, ig\delta_g\left(\Sigma_{\mu\nu} + \frac{1}{x}R_{\mu\nu}\right)(p' + p)^\nu,$$

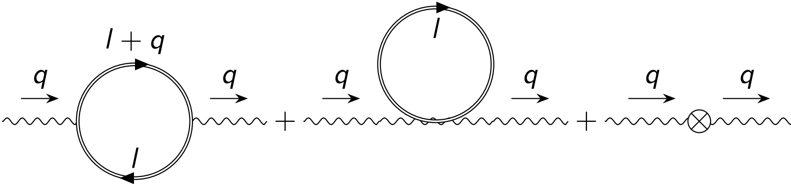


$$= 2ig^2\delta_3\left(\Sigma_{\mu\nu} + \frac{1}{x}R_{\mu\nu}\right).$$

Renormalización Massless case

With this, the autoenergy of the field B^μ is

$$-i\Pi_{\mu\nu}(q) =$$



$$= -\, i\Pi^*_{\mu\nu}(q) - i\delta_{Z_3}(q^2 g_{\mu\nu} - q_\mu q_\nu),$$

where now

$$-\, i\Pi^*_{\mu\nu}(q) = \int \frac{d^4 l}{(2\pi)^4} \mathrm{Tr}\left\{ \left[-ig\left(\Sigma_{\mu\alpha} + \frac{1}{x}R_{\mu\alpha}\right)(2l+q)^\alpha \right] \right.$$
$$\times\, i\frac{\mathbb{1} + (x-1)\mathbb{P}_-(l+q)}{(l+q)^2 + i\epsilon} \left[-ig\left(\Sigma_{\nu\beta} + \frac{1}{x}R_{\nu\beta}\right)(2l+q)^\beta \right]$$
$$\left. \times\, i\frac{\mathbb{1} + (x-1)\mathbb{P}_-(l)}{l^2 + i\epsilon} + \left[2ig^2\left(\Sigma_{\mu\nu} + \frac{1}{x}R_{\mu\nu}\right)\right] i\frac{\mathbb{1} + (x-1)\mathbb{P}_-(l)}{l^2 + i\epsilon} \right\}.$$

Renormalization Massless case

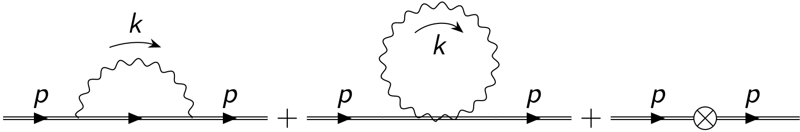
The result has the structure $-i\Pi_{\mu\nu}(q) = -i\Pi(q^2)(q^2 g_{\mu\nu} - q_\mu q_\nu)$, whose divergent part is

$$\text{Div}\left[-i\Pi(q^2)\right] = -\, i\left[\frac{g^2\left(x^4 - 8x^3 + 6x^2 - 8x + 1\right)}{64\pi^2 \epsilon x^2} + \delta_{Z_3}\right], \qquad (23)$$

which can be canceled by the counterterm δ_{Z_3}.

Renormalization Massless case

The autoenergy of the field Ψ is

$-iM^2(p) =$ 

$$= -\, iM^{*2}(p) + i\delta_{Z_2}\left(\Sigma(p) + \frac{1}{x}R(p)\right),$$

$$-\, iM^{*2}(p) = \int \frac{d^4k}{(2\pi)^4}\left\{\left[-ig\left(\Sigma_{\mu\alpha} + \frac{1}{x}R_{\mu\alpha}\right)(2p-k)^{\alpha}\right]\right.$$

$$\times\, i\frac{\mathbb{1} + (x-1)\mathbb{P}_{-}(p-k)}{(p-k)^2 + i\epsilon}\left[-ig\left(\Sigma_{\nu\beta} + \frac{1}{x}R_{\nu\beta}\right)(2p-k)^{\beta}\right]$$

$$\times\, i\frac{-g^{\mu\nu} + (1-y)k^{\mu}k^{\nu}/k^2}{k^2 + i\epsilon} + \left[2ig^2\left(\Sigma_{\mu\nu} + \frac{1}{x}R_{\mu\nu}\right)\right]$$

$$\left.\times\, i\frac{-g^{\mu\nu} + (1-y)k^{\mu}k^{\nu}/k^2}{k^2 + i\epsilon}\right\}.$$

Its divergent part is

$$\begin{aligned}\text{Div}\left[-iM^2(p)\right] =& -i(x+1)\left[\frac{3x^2-2xy+3}{64\pi^2x^2\epsilon}-\frac{\delta_{Z_2}}{2x}\right]p^2\mathbb{1}\\ &-i(x-1)\left[\frac{4x^2+x(7-3y)+4}{96\pi^2x^2\epsilon}-\frac{\delta_{Z_2}}{2x}\right]S(p).\end{aligned} \tag{24}$$

The counterterm δ_{Z_2} can cancel the divergences of both terms only for the following values of the gauge parameter x of the field Ψ.

$$x = \pm 1, \qquad\qquad x = 7 \pm 4\sqrt{3}, \tag{25}$$

for any value of the gauge parameter y of the field B^μ.

Conclusions

For massive TDM:

- The $U(1)$ gauge theory is non-renormalizable due to the divergent behaviour of the Ψ propagator in the UV region.

For massless TDM:

- Massless TDM has a gauge symmetry. Its propagator does not have the bad behaviour in the UV region that the massive propagator has.
- The autoenery of the B^μ field is renormalizable for any x, y values of the gauge parameters.
- The autoenergy of the Ψ field is renormalizable only for the values $x = \pm 1$ or $x = 7 \pm 4\sqrt{3}$, both for any value y.
- We are still working on the study of the other diagrams.
- In the case that TDM would found to be renormalizable we still have to look for some mechanism to give it mass.

References

[1] H. Hernández-Arellano, M. Napsuciale, and S. Rodríguez, "Spin portal to dark matter", Phys. Rev. D **98**, 015001 (2018).

[2] H. Hernández-Arellano, M. Napsuciale, and S. Rodríguez, "Spin-one dark matter and gamma ray signals from the galactic center", (2019).

[3] H. Hernandez-Arellano, M. Napsuciale, and S. Rodriguez, "Cosmic-ray antiproton excess from annihilating tensor dark matter", (2021).

[4] M. Napsuciale, S. Rodríguez, and H. Hernández-Arellano, "Kinetic mixing, custodial symmetry, and a lower bound on the mass of a dark gauge boson", PTEP **2022**, 093E01 (2022).

[5] M. Napsuciale, S. Rodriguez, and H. Hernandez-Arellano, "Kinetic mixing, custodial symmetry, Z, Z' interactions and Z' production in hadron colliders", (2021).

[6] M. Napsuciale, "Space-time origin of gauge symmetry", (2022).

[7] M. Napsuciale, S. Rodríguez, R. Ferro-Hernández, and S. Gómez-Ávila, "Spin one matter fields", Phys. Rev. D **93**, 076003 (2016).