3D O(2) non-linear sigma model on the lattice and cosmic strings

Edgar López Contreras, Jaime Fabián Nieto Castellanos, Elías Natanael Polanco Euán and Wolfgang Bietenholz

Instituto de Ciencias Nucleares
Universidad Nacional Autónoma de México

RADPyC 23

Non-linear sigma model 3D O(2)

- 3D Euclidean space
- Field variables in S^1.
- 3D O(2) model, 3D XY model or 3D plane rotator.
- Relevant to the critical behavior of superfluids and planar magnets.

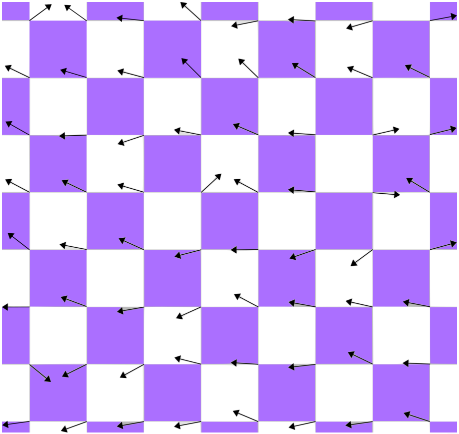
Partition Function

$$Z = \int \mathcal{D}\vec{\sigma} \; e^{-\beta \mathcal{H}[\vec{\sigma}]}.$$

- Where $\beta = 1/T$, with $k_B = 1$.

Non-linear sigma model 3D O(2)

- In the lattice regularization, at each site there is a spin $\vec{\sigma}_x = (\cos\theta_x, \sin\theta_x)$, where $\theta_x \in (-\pi, \pi]$ and $x = (x_1, x_2, x_3)$.
- We employ a cubic lattice Λ with periodic boundary conditions.
- Example of a 2D O(2) configuration:



Non-linear sigma model 3D O(2)

- We consider two distinct Hamiltonians.

Standard Ferromagnetic Hamiltonian

$$\mathcal{H}[\vec{\sigma}] = -\sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- $\sum_{\langle ij \rangle}$ denotes sum over nearest-neighbors.

Topological Hamiltonian

$$\mathcal{H}[\vec{\sigma}] = \begin{cases} 0 & \text{if} \ \ |\Delta\varphi_{x,x+\hat{\mu}}| < \delta, \quad \forall x, \hat{\mu} \\ +\infty & \text{otherwise} \end{cases}$$

- $\Delta\varphi_{x,x+\hat{\mu}} = \varphi_{x+\hat{\mu}} - \varphi_x \mod 2\pi \in (-\pi, \pi)$. $|\hat{\mu}| = 1$.
- φ_x is the angle of $\vec{\sigma}_x$ with respect to a fixed point in S^1.
- Most configurations are invariant under infinitesimal transformations.

String defects within the Standard Model

- In the Early Universe, the unbroken electroweak symmetry was,

$$\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}.$$

- Electroweak symmetry "breaks spontaneously", at $\approx$ 159 GeV[1], when the Higgs field acquires a vacuum expectation value (VEV),

$$\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}} \longrightarrow \mathrm{U}(1)_{\mathrm{EM}}.$$

[1] d'Onofrio, Michela; Rummukainen, Kari (2016). "Standard model cross-over on the lattice". Physical Review D. 93 (2): 025003.

String defects within the Standard Model

- When the temperature of the Universe dropped below 159 GeV, the Higgs field in most regions will acquire a VEV, but for causality reasons the phases in widely separated regions will be uncorrelated (Kibble mechanism[2]).
- String defects are trapped in interfaces where the phase change around a loop is 2π.

[2]Kibble, Tom W K (1976). "Topology of cosmic domains and strings". Journal of Physics A: Mathematical and General. 9 (8): 1387–1398.

Time evolution of string defects

- A random network of strings may arise.
- The characteristic scale of this network is the correlation length ξ.
- At a short time after string formation the Universe was very dense, which damps the string fluctuations.
- The damping force on strings increases ξ, but for causality reasons, ξ can never grow faster than ct. This requires a mechanism for energy dissipation, which involves gravitational radiation and particle emission.

3D O(2) string defects

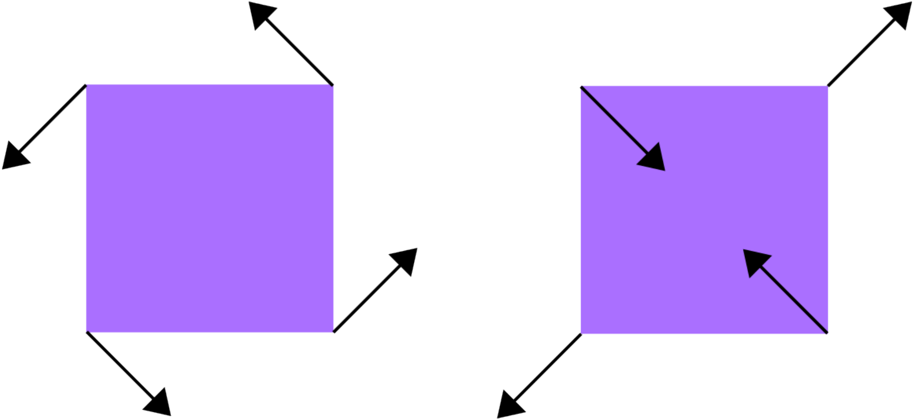
- The 3D O(2) model undergoes a second-order phase transition, where the O(2) symmetry breaks.
- The vacuum manifold is isomorphic to S^1,

$$\pi_1(S^1) = \mathbb{Z},$$

  which characterizes one-dimensional topological defects or "string defects".
- Vortex lines.

Vortices

- Field configurations with vortices and anti-vortices.



Vorticity

- Each plaquette has a vorticity,

$$v_{\mu\nu}(x) = \frac{1}{2\pi}\left(\Delta\varphi_{x,x+\hat{\mu}} + \Delta\varphi_{x+\hat{\mu},x+\hat{\mu}+\hat{\nu}} + \Delta\varphi_{x+\hat{\mu}+\hat{\nu},x+\hat{\nu}} + \Delta\varphi_{x+\hat{\nu},x}\right),$$

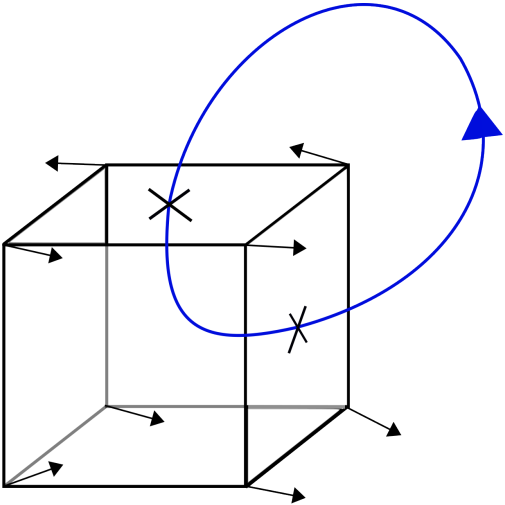
  where $|\hat{\mu}|, |\hat{\nu}| = 1$.

- There are only 3 values for the plaquettes vorticity,

$$v_{\mu\nu}(x) \in \{-1 \text{ (antivortex)}, 0, 1 \text{ (vortex)}\}.$$

Vortex lines

- Vortices tend to form vortex lines.



Observables

- Magnetization density,

$$m = \frac{1}{L^3}\langle|\vec{M}[\vec{\sigma}]|\rangle, \; \vec{M}[\vec{\sigma}] = \sum_{x\in\Lambda}\vec{\sigma}_x.$$

- Magnetic susceptibility,

$$\chi_M = \frac{1}{L^3}\left(\langle|\vec{M}[\vec{\sigma}]|^2\rangle - \langle|\vec{M}[\vec{\sigma}]|\rangle^2\right).$$

- Connected correlation function,

$$C(x,y) = \langle\vec{\sigma}_x\cdot\vec{\sigma}_y\rangle - \langle\vec{\sigma}_x\rangle\cdot\langle\vec{\sigma}_y\rangle.$$

- At large distance $|x| \gg 1$,

$$C(0,x) = C(x) \propto \cosh\left(\frac{x - L/2}{\xi}\right).$$

- ξ correlation length.

Critical exponents

In a cubic lattice of volume L^3, at T_c, observables grow as follows,

$$\xi \propto |T - T_c|^{-\nu},$$

$$m \propto L^{-\beta/\nu},$$

$$\chi_M \propto L^{\gamma/\nu},$$

Models in the same universality class share same values of critical exponents.

Critical exponents

- For the standard action $\beta_c = 0.454168(5)$.
- For the topological action $\delta_c = 2.5156(2)$.
- Critical exponents,

Standard action

β/ν	γ/ν	ν
0.512(3)	1.97(1)	0.68(2)

Topological action

β/ν	γ/ν	ν
0.506(4)	1.99(1)	0.63(4)

- Compatible critical exponents.
- Both actions fall into the same universality class.

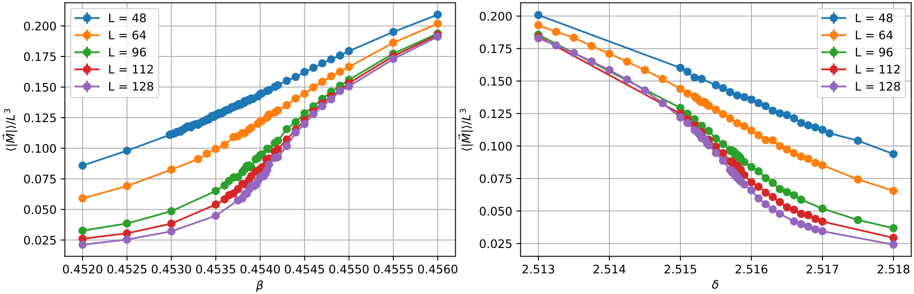
Step scaling function

- At some fixed value u_0 of $\xi(L)/L$ we define,

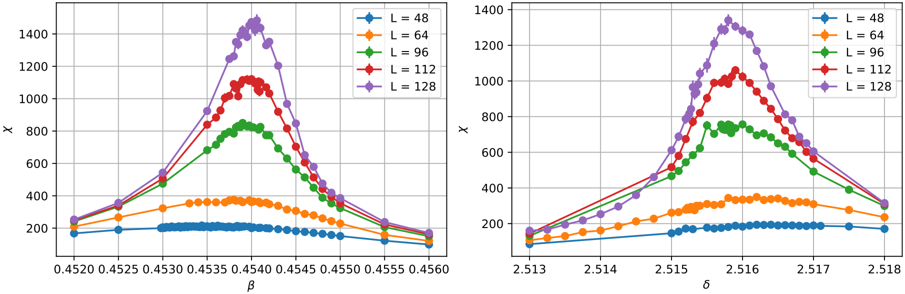
$$\sigma(2, u_0) = \frac{2L}{\xi(2L)}.$$

- σ is a universal quantity, based on finite-size effects.
- It allows us to study lattice artifacts.
- $\sigma(2, 1.3) = 0.95(1)$, with the standard action.
- $\sigma(2, 1.3) = 0.94(4)$, with the topological action.

Magnetization



Magnetic susceptibility



Gradient Flow Equation

- We explore the persistence of topological defects after evolution of field configurations through gradient flow equation.
- The general gradient flow equation reads[3],

$$\frac{\delta\sigma_t^a(x)}{\delta t} = -g^{ab}\frac{\delta S(\sigma_t)}{\delta\sigma_t^b(x)},$$

  where S is the action and g^{ab} is non-trivial for non-linear sigma models.
- The evolution of the field through this equation implies the reduction of the action,

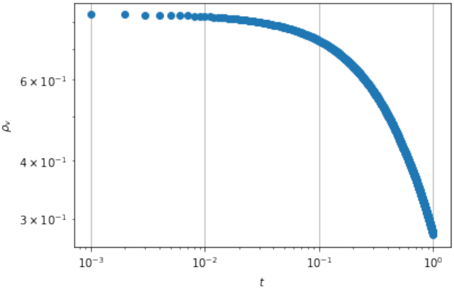
$$\partial_t S(\sigma_t) \leq 0.$$

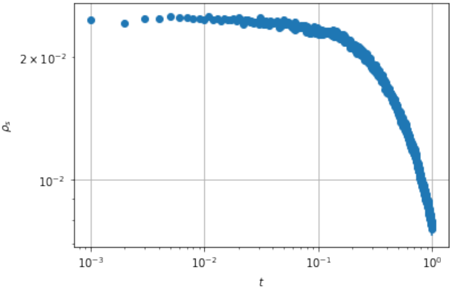
  In the 3D O(2) model, this flow causes vorticity and string density suppression.

[3] Kikuchi, K., Onogi, T. (2014). "Generalized gradient flow equation and its application to super Yang-Mills theory". J. High Energ. Phys. 2014, 94.

Vorticity and String density suppression

- Suppression of densities of topological defects in a volume of 16^3 initially at $T = 2T_c$.





Conclusions

- There is evidence that the proposed topological action is in the same universality class as the standard action.
- The persistence of topological defects can be explored with the evolution through gradient flow equation.
- Vortex lines in the 3D O(2) non-linear sigma model bear strong analogies with global cosmic strings.